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ABOUT THE COVER: The Integrated Science and Technology/Computer Science building at James Madison University's College of Integrated Science and Technology opened in 1997 on a new portion of JMU being developed east of Interstate 81 from the original campus. Wilson Hall which sits at the head of JMU's historic Quadrangle is seen in the background. A second building in CISAT, the Health and Human Services Building, opened in the fall of 2000 and is connected to the ISAT/CS Building.

From the President

Debbie Haver

Dear Friends,

I wish to offer congratulations to all the mathematics educators who supported the 2001 Math Odyssey Spring Conference this past March at James Madison University. A special thanks and hoorah goes to Maureen Hijar and Joe Hill for working together as a team offering us a technological conference with online information and registration. Maureen, Joe, and all the other volunteers, as a dear friend once said, "You are the Music Maker – You are the Dreamer of Dreams." Let's keep it going!

I want to share with you an enlightening, little, story book, Who Moved My Cheese? by Spencer Johnson, M.D. This story is about four mice, ***Sniff, Scurry, Hem, and Haw,*** who characterize our feelings in the midst of change. ***Sniff*** is able to sniff out the situation and see the change early. ***Scurry*** always goes into action immediately. ***Hem*** doesn't see the need to change and doesn't think he has to change. He ignores what is happening around him. ***Haw*** has the ability to see what he is doing wrong, laughs at himself, changes, and does better.

Which mouse do you relate to? How are you adjusting to having your "Cheese Moved" by the Standards of Accreditation and the Standards of Learning?"

We have three years of SOL data (numbers and notes) to analyze. So, what is the difference between schools with successful SOL scores, schools who show continued improvement, and schools who climb the ropes only to slide back down?

Recently, I attended a meeting where three schools explained how they were able to attain accreditation. One common thread emerged, **teachers and administrators sharing and working together as a team** eliciting help from the community, businesses, churches, colleges, and universities and staying connected with ***change*** through a commitment to their professional organization. I am reminded of a transparency that I have seen showing three mice standing individually in front of a maze. The mice form a mouse-pyramid to tackle the task of deciding the best path concluding Together Everyone Achieves More!

Everyday, as individuals, we put forth our best efforts to provide successful opportunities for SOL achievement. But, working alone is not enough. We need to get together with other professionals to help us maintain our confidence and more importantly, our sense of humor and balance.

Collaborating and networking opportunities are at your fingertips! Be connected to the Internet – generate a list of friends in your email address book who will share ideas and information. Log-onto **www.vctm.org** Sign-up for a course on-line. You are able to work whenever you have the time and in whatever attire is comfortable for you. Most importantly, be connected and stay connected to a friend.

Minutes from the January 24, 2001 Meeting of the Virginia Council of Teachers of Mathematics Board

David and Gail Englert

Debbie Haver called the meeting to order and introductions were made. Constituent Representatives were welcomed, and thanked for attending the meeting. Vandi Hodges was thanked for arranging to have the meeting in the Hanover County School Board offices.

The following points were discussed during this meeting:

- the VCTM Web page
- the upcoming newsletter
- the First Timer's Grant
- a report from VCMS
- an update on the annual scholarship heading the committee.
- an update on the progress of the Teacher of the Year Committee
- Nominations for next year's board
- Constitutional changes affecting board positions
- A report on the annual meeting, to be held March 9-10 on the JMU campus in Harrisonburg, and meeting in subsequent years
- Journal deadlines, possible sponsorship, and publishing information
- Affiliate Group Reports
- FANS updates
- the possible addition of an Activities Committee
- Payment of Dues / possible dues increase
- Travel Costs for Executive Board approved at $50 per year
- International Mathematics Olympiad volunteers
- Appointment of a Webmaster
- Appointment of representative from Best Practice Centers and VCMS
- Indexing availability of Training Opportunities

Lessons from the TIMSS

Eugene A. Geist

The Third International Mathematics and Science Study (TIMSS) reported more than just numerical achievement data. The study also contained information on teachers' lives, students' lives, and curricula, as well as a videotape of eighth-grade mathematics classes (OERI 1996a). These data, especially the videotapes study, go beyond comparisons of achievement scores and allow for cross-cultural comparisons of mathematics instruction in the United States and other countries. In particular, we can learn many lessons from examining instructional methods in the United States and comparing them with those of Japan. This comparison has significant implications for implementing NCTM's Standards in U.S. classrooms and teaching our students as if they were young mathematicians.

TIMSS Achievement Results in Fourth and Eighth Grades

According to the achievement results for mathematics (see Tables 1 and 2), the average score for fourth-grade students in the United States was 545, which is above the international average of 529. Japanese students, however, scored significantly higher than United States students, with an average score of 597.

The results for eighth-grade students in the United States decreased from the positive showing for fourth grade. The United States average was 500, which is 13 points below the international average of 513. Japan again scored significantly higher than the United States. The Japanese score of 605 was 92 points above the international average (OERI 1996b).

TIMSS Classroom Videotape

One part of the TIMSS was an intensive videotaped survey of 231 eighth-grade mathematics lessons in the United States, Japan, and Germany. This part of the study was the first attempt to collect a nationally representative sample of videotaped observations of classroom instruction. According to the TIMSS, "The purpose of gathering this information was to understand better the process of classroom instruction in different cultures to improve student learning in our schools" (OERI 1996a). A collection of six vignettes of representative lessons was assembled to allow teachers and others to examine and compare classroom instruction in the three countries. This article discusses examples from the videotaped vignettes of geometry instruction.

Using the videotape study and other information that the TIMSS gathered, we can begin to look at the aspects of teaching that seem to be productive in Japan and, perhaps, find some answers to the persistent question of why Japanese students perform so much better than United States students do in mathematics. One answer that presents itself from the videotape study is that the Japanese do a much better job of treating their students like mathematicians and implementing the ideas behind the NCTM's Standards. The tradition in United States classrooms is to treat the learning of mathematics as memorization and practice (Miller 1973), and this tendency still exists today. To encourage children to think and act like mathematicians, we must encourage them to interact with the subject as if they *were* mathematicians (Kamii and Ewing 1996). To accomplish this goal, we must know what mathematicians do when they are presented with problems.

TABLE 1

TIMSS Fourth-Grade Achievement Results

Nations with average scores **significantly higher** than that of the U.S.	
Singapore	625
Korea	611
Japan	597
Hong Kong	587
The Netherlands	577
Czech Republic	567
Austria	559
Nations with average scores **not significantly different** from that of the U.S.	
Slovenia	552
Ireland	550
Hungary	548
Australia	546
United States	545
Canada	532
Israel	531
Nations with average scores **significantly lower** than that of the U.S.	
Latvia	525
Scotland	520
England	513
Cyprus	502
Norway	502
New Zealand	499
Greece	492
Thailand	490
Portugal	475
Iceland	474
Iran, Islamic Republic	429
Kuwait	400

NOTE: Not all countries participated in both grade-level assessments. Twenty-six nations participated in the fourth-grade assessment, and forty-one, in the eighth-grade assessment.

Characteristics in Mathematicians

From discussions with mathematicians and graduate students in mathematics, seven characteristics emerged that describe how mathematicians go about solving problems. These seven characteristics can be seen in Japanese classrooms and instructional practices, but the TIMSS videotape suggests that most students in the United States learn mathematics much differently.

(1) Mathematicians often work for a long time on a single problem

Mathematicians may spend months and years thinking about, and working on, a proof to one problem. Students should also be given ample time to work on one problem. Giving students fewer problems and more time to complete them can enhance their problem-solving abilities.

In the videotape of the eighth-grade geometry class in the United States, more than one hundred geometry questions were asked and answered in rapid succession. When students were given time to do problems in class, they were given more than forty problems to complete in just twenty minutes.

By contrast, the Japanese class worked on only two problems for the entire fifty-minute class period. The students were given ample time to think about, and experiment with, different methods of achieving a solution. For homework, the students were also given only one problem, which was derived from the discussion during class.

(2) Mathematicians collaborate with their colleagues and study the work of others.

Social interaction is one of the most important aspects of being a mathematician. A mathematics classroom, especially one that views students as young mathematicians, should include many opportunities for social interaction (Kamii 1985). The NCTM's Communication Standard underscores the importance of collaboration. Traditional mathematics lessons and homework assignments in the United States are designed to be solitary endeavors. Children are not encouraged to defend a solution or collaborate on solving a problem. Instead, they are given individual practice worksheets and asked to complete them quietly (Fosnot 1989). In the videotape of the United States geometry class, children worked alone and were discouraged from talking to other students.

The Japanese classroom was ripe with the exchange of ideas. After posing a problem, the teacher asked the students to think about it for three minutes. Then he asked them to discuss the problem with other students or a teacher for three minutes. After that time, the entire class engaged in a discussion of the problem while two students presented their solutions.

If children are to be viewed as young mathematicians, they must be allowed to defend, question, and explain their mathematical ideas through collaborating, arguing, and consulting with other students in the classroom (Kamii, Lewis, and Jones 1991, 1993). Children construct mathematical understanding through this type of social interaction. Without this interaction, children merely memorize the steps required to arrive at a certain solution without developing understanding (Perry, VanderStoep, and Yu 1993).

TABLE 2

TIMSS Eighth-Grade Achievement Results

Nations with average scores **significantly higher** than that of the U.S.	
Singapore	643
Korea	607
Japan	605
Hong Kong	588
Belgium (Flemish)	565
Czech Republic	564
Slovak Republic	547
Switzerland	545
The Netherlands	541
Slovenia	541
Bulgaria	540
Australia	539
France	538
Hungary	537
Russian Federation	535
Australia	530
Ireland	527
Canada	527
Belgium (French)	526
Sweden	519
Nations with average scores **not significantly different** from that of the U.S.	
Thailand	522
Israel	522
Germany	509
New Zealand	508
England	506
Norway	503
Denmark	502
United States	500
Scotland	498
Latvia	493
Spain	487
Iceland	487
Greece	484
Romania	482
Nations with average scores **significantly lower** than that of the U.S.	
Lithuania	477
Cyprus	474
Portugal	454
Iran, Islamic Republic	428
Kuwait	392
Colombia	385
South Africa	354

NOTE: Not all countries participated in both grade-level assessments. Twenty-six nations participated in the fourth-grade assessment, and forty-one, in the eighth-grade assessment.

(3) Mathematicians must prove for themselves that their solutions are correct.

After the Japanese students had been given time to think [illegible] and discuss their answers, students were asked to present and defend their solutions to the class. The students presented their solutions on the chalkboard. The teacher did not tell the students whether they were correct but asked whether others understood how the solution was derived (Blake, Hurley, and Arenz 1995). The students were required to reason about and prove their answers.

The United States classroom used a call-and-response mode. The teacher called out the problem, and a student responded with an answer. If the answer was incorrect, the teacher would ask rhetorically, "Are you sure about your answer?" The teacher then called on another student to give the correct answer.

Mathematicians must question assumptions and understand the mathematics behind an answer. Mathematicians must prove to themselves and others that their solutions are correct. If students are taught merely to memorize answers and constantly rely on a teacher to tell them whether they are correct, then the important process of proving a solution is removed from the students.

(4) Mathematicians work on complex problems.

Children, like mathematicians, should be immersed in complex problems that require mathematical problem solving and complex numerical thinking. Good problems ask students to find innovative solutions without setting time limits ([illegible]efield 1997). Problems can and should spark discussion and even disagreement among the children.

The problems in the Japanese classroom asked students to use a concept covered in a preceding class period to construct a triangle with the same area as a given quadrilateral. The lesson focused on the concept of area; little or no arithmetic was involved. Several different solutions were possible and were used by the students. The students also found many different ways to proceed.

In the United States classroom, students were given basic arithmetic problems. Students learned the definitions of complementary and supplementary angles. Then they were asked to give the complementary and supplementary angles of a given angle. In other words, students repeatedly subtracted numbers from 90 or 180 to get their answers.

(5) Mathematicians get satisfaction from the process.

Children understand mathematical concepts and procedures more thoroughly when they are allowed to use their own thinking processes to explore mathematics (Kamii, Lewis, and Jones 1993). In the process of discussing and comparing different methods of reach solution, children strengthen their understanding of both concepts and procedures. This approach allows students to make connections with what they already know and with real-life experiences.

(6) Mathematicians gain a sense of pride in attaining solutions.

Children can show enthusiasm about mathematics problems and find pleasure and excitement in problem solving (University of Chicago 1998). If children are allowed to think for themselves and discuss and defend their ideas, mathematics becomes just as much fun as playing a video game to the finish or working diligently to put together a puzzle.

At the end of the lesson in the Japanese classroom, the students discussed the day's problem with the teacher. They asked whether the same transformation would work with five- or six-sided figures. The teacher asked the students to try the transformation at home that evening and promised that they would discuss the question in class the next day. The students were enthusiastic about the process and the content of the day's work and demonstrated a sense of accomplishment.

(7) Mathematicians use unsuccessful attempts as stepping stones to solutions.

For children to think like mathematicians, they must realize that they may have to try many different approaches before they reach a solution. Teachers should emphasize the valuable mathematical thinking going on in the children's minds. Teachers should also model the processes of going through unsuccessful attempts and making errors as stepping stones to solutions.

The Japanese lesson was designed to allow students to experiment with different solutions, then discuss the solutions with other students or the teacher. The students were given time to use different methods, and to prove their answers to themselves and others. In contrast, students in the United States classroom did not have the time to go through this process. They were given one chance to get the solution right, and the thinking behind incorrect solutions was never discussed or examined.

Closing Comments

The TIMSS videotape found that mathematics instruction in the United States eighth-grade classroom focused on teaching students how to complete a certain type of problem. Teachers emphasized rote memorization of formulas and procedures for figuring out the correct answer. In the Japanese classroom, the focus was on teaching concepts and developing students' mathematical thinking abilities.

The NCTM's *Standards* documents (1989, 1998) describe the mathematical processes through which students should acquire and use mathematical knowledge. When the United States classroom and the Japanese classroom were compared using the five criteria in Table 3 as measuring tools, the Japanese classroom came much closer to implementing the recommendations of current research and best practice. The teaching methods in Japan show how the NCTM's Standards can be applied in the classroom and how students can be treated like mathematicians.

On the basis of responses from teachers in the United States, the TIMSS identified three categories of current ideas about mathematics. The ideas involve *(a)* hands-on, real-world mathematics, *(b)* cooperative learning, and *(c)* a focus on thinking (OERI 1996b). Again, the Japanese classroom incorporated these ideas more effectively than the United States classroom did, as shown in Table 4.

The fact that these three categories emerged from re-

TABLE 3

Comparison of application of the ideas behind NCTM's Standards in Japanese and United States classrooms

NCTM Standard	Japanese Classroom	United States Classroom
Problem solving	Students spend fifty minutes working individually and in groups on two problems.	Students solve simple arithmetic problems and follow memorized formulas.
Reasoning and proof	Students share their ideas in small groups and defend their answers in front of the whole class.	Students are not asked about their reasoning.
Communication	Students are encouraged to move around the room and discuss solutions with other students and the teacher.	Students talk only when called on by the teacher. Answers are usually terse.
Connections	The teacher links the current day's activity with the previous day's work, expanding on the concept and explaining practical applications.	No connections are made to real-life applications.
Representation	Students draw their solutions on the chalkboard for the whole class to see.	Students use basic arithmetic to find supplementary and complementary angles.

sponses of teachers in the United States shows that these teachers realize the importance of treating children like mathematicians. The difficulties arise when teachers are asked to cover too much material in a short amount of time. The TIMSS found that teachers in the United States are asked to teach more topics in less depth than teachers in Japan are and that this requirement leads to an unfocused curriculum in the United States (OERI 1996b).

Children have natural curiosity and a zeal for exploration and understanding that apply to learning mathematics. If students are encouraged to act like young mathematicians and use their natural thinking abilities to attack and solve problems, as we see in the classrooms of Japan, mathematics becomes not a chore but a challenge to the student (Wakefield 1997).

Generating excitement about the subject should be the goal of every teacher of mathematics. From early childhood, children must be treated as if they were young mathematicians. This philosophical change in our approach to teaching will not be brought about by emphasizing skill-and-drill methods or adding more mandatory tests (Kelly 1999). We must undertake a deliberate process of change in the way that children are viewed and treated in classrooms (Bay, Reys, and Reys 1999). We can learn some valuable lessons from the TIMSS videotape. If children are to be seen as young mathematicians, many long-held beliefs and practices will have to change. The TIMSS and NCTM have pointed mathematics instruction in the United States in the right direction; now all we have to do is move forward.

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TABLE 4

Comparison of the use of current mathematics education ideas in Japanese and United States classrooms

Current Idea	Japanese Classroom	United States Classroom
Hands-on, real-world math	The teacher relates the topic and concept to a real-world application.	Lesson is abstract, and no links to real life or applications are made.
Cooperative learning	Students work together to find answers to their problems.	No interaction takes place among students.
Focus on thinking	The students focus on problem solving and use their natural thinking abilities to solve problems.	Students sit passively as the teacher explains the steps to solve a specific problem.

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WEB BYTES

NCES Students' Classroom offers games, activities, and information related to finding, using, and appreciating statistics. nces.ed.gov/nceskids.

ParentSmart is a search engine for parents, teachers, and others that offers access to articles on education issues. www.parentsmart.com.

ENC Online recently launchedan all-never version of its Web site that features additional mathematics and science content, faster downloads, and new e-mail services. enc.org.

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