The authors of the following three articles make a strong case for math being fun, relevant, and an ongoing part of children's lives—not a dreaded four-letter word.

• In "Children Are Born Mathematicians: Promoting the Construction of Early Mathematical Concepts in Children Under Five," **Eugene Geist** argues that math literacy begins at birth. All that children need to construct math concepts for themselves are a stimulating environment and receptive adults.

• **Debra Thatcher,** in her article "Reading in the Math Class," suggests that teachers make use of children's literature to stimulate the learning of mathematical concepts. She provides examples of appropriate books and questions teachers might pose to spark children's thinking about math.

• **Anne Murray's** "Ideas on Manipulative Math" presents a case study of her kindergarten class, where the math center is the most popular area in her room. Through studies and investigations with manipulatives, children construct math concepts on their own. "Not only are the children learning, but they know it and love every moment," says Murray.

You will find below a list of suggested readings and Websites for exploring these approaches further.

Suggested readings

Anderson, T.L. 1996. "They're trying to tell me something": A teacher's reflection on primary children's construction of mathematical knowledge. *Young Children* 51 (4): 34–42.

Baratta-Lorton, M. 1995. *Mathematics their way.* 20th anniv. ed. Saratoga, CA: Center for Innovation in Education.

Baroody, A.J. 2000. Research in Review. Does mathematics instruction for three- to five-year-olds really make sense? *Young Children* 55 (4): 61–67.

Brown, S. 1997. First graders write to discover mathematics' relevancy. *Young Children* 52 (4): 51–53.

Copley, J.V. 2000. *The young child and mathematics.* Washington, DC: NAEYC.

Forman, G.E., & D.S. Kuschner. 1983. *The child's construction of knowledge: Piaget for teaching children.* Washington, DC: NAEYC.

Fosnot, C. 1989. *Enquiring teachers, enquiring learners: A constructivist approach for teaching.* New York: Teachers College Press.

Greenberg, P. 1993. Ideas that work with young children. How and why to teach all aspects of preschool and kindergarten math naturally, democratically, and effectively (for teachers who don't believe in academic programs, who do believe in educational excellence, and who find math boring to the max)—Part 1. *Young Children* 48 (4): 75–84.

Greenberg, P. 1994. Ideas that work with young children—Part 2. *Young Children* 49 (2): 12–18, 88.

Hinnant, H.A. 1999. Growing gardens and mathematicians: More books and math for young children. *Young Children* 54 (2): 23–26.

Kamii, C. 1993. Primary arithmetic: Children inventing their own procedures. *Arithmetic Teacher* 41 (4): 200–03.

Kroll, L., & M. Halaby. 1997. Writing to learn mathematics in the primary school. *Young Children* 52 (4): 54–60.

Meriwether, L. 1997. Math at the snack table. *Young Children* 52 (5): 69–73.

Mills, H., D.J. Whitin, & T. O'Keffe. 1993. Teaching math concepts in a K–1 class doesn't have to be like pulling teeth—But maybe it should be! *Young Children* 48 (2): 17–20.

Reggio Children. 1997. *Shoe and meter. Children and measurement. First approaches to the discovery, function, and use of measurement.* Reggio Emilia, Italy: Author.

O'Brien, T.C. 1999. Parrot math. *Phi Delta Kappan* 80 6: 434–38.

Richardson, K., & L. Salked. 1995. Transforming mathematics curriculum. In *Reaching potentials: Transforming early childhood curriculum and assessment—Volume 2,* eds. S. Bredekamp & T. Rosegrant, 23–42. Washington, DC: NAEYC.

Salisbury, G.R. 1971. Attribute blocks with infants. *Visual Education* (March): 27–28.

Unglaub, K.W. 1997. What counts in learning to count? *Young Children* 52 (4): 48–50.

Whitin, D.J. 1994. Literature and mathematics in preschool and primary: The right connection. *Young Children* 49 (2): 4–11.

Websites

Conference on Standards for Preschool and Kindergarten Mathematics Education—www.gse.buffalo.edu/org/conference/

National Council of Supervisors of Mathematics (NCSM)—http://ncsmonline.org/OtherResources/resources.html

National Council of Teachers of Mathematics (NCTM)—http://nctm.org and www.standards.nctm.org

U.S. Department of Education, Office of Educational Research and Improvement (OERI) National Institute on Early Childhood Development and Education—www.ed.gov/pubs/EarlyMath/

Children Are Born Mathematicians:

Promoting the Construction of Early Mathematical Concepts in Children under Five

Eugene Geist

Children are mathematicians from the day they are born. They are constructing knowledge constantly as they interact mentally, physically, and socially with their environment and with other people. Young children may not be able to add or subtract, but their relationships with people and their interactions with a stimulating environment set the stage for the development of mathematical concepts (Sinclair et al. 1989). There is even some evidence that the ability to comprehend some mathematical concepts may be innate (Starkey & Cooper 1980; Wynn 1995; Koechlin, Dehaene, & Mehler 1997).

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Innate ability to acquire mathematics

Perhaps, just as Chomsky (1999) has shown strong evidence for an innate *language acquisition device* that provides humans with a framework for learning language, there is a *mathematics acquisition device* that provides a framework for mathematical concepts. If such a mathematics acquisition device were present, we would expect children to naturally acquire mathematical concepts without direct teaching, to follow a standard sequence of gradual development, and most important, to show evidence of construction of mathematical concepts from a very early age. With careful examination of infants, toddlers, and preschoolers, one can see evidence for all of these criteria.

Emergent mathematical understanding

Perhaps it is time to begin looking at the construction of mathematical concepts the same way we look at literacy development—as emergent. The idea that literacy learning begins the day that children are born is widely accepted in the early childhood field. Children learn language by listening and by eventually speaking and writing. This language learning is aided by an innate language acquisition device that acts as a foundation for grammatical development and language learning (Chomsky 1999). Reading to infants, toddlers, and preschoolers is known to be an early positive step toward literacy success because it promotes and supports learning to read and write by immersing children in language and giving them an opportunity to interact with it (Ferreiro & Teberosky 1982).

Mathematical learning can be viewed in a similar way. Children begin to construct the foundations for future mathematical concepts during the first few months of life. Before children can add or even count, they must construct ideas about mathematics that cannot be directly taught. Ideas that will support formal mathematics later in life include order and sequence, seriation, and classification.

The seemingly simple idea that a number represents a specific quantity actually involves a complex relationship that children must construct. Quantification is the basis for formal mathematics, and it is a synthesis of order (the basic understanding that objects are counted in a specific sequence and each object is counted only once). Seriation is the ability to place an object or group of objects in a logical series based on a property of the object or objects. Classification is the ability to group like objects in sets by a specific characteristic. This synthesis takes place by children interacting with objects and putting them in many different types of relationships.

Courtesy of the author

> It is time to begin looking at the construction of mathematical concepts the same way we look at literacy development—as emergent.

A child constructing math concepts

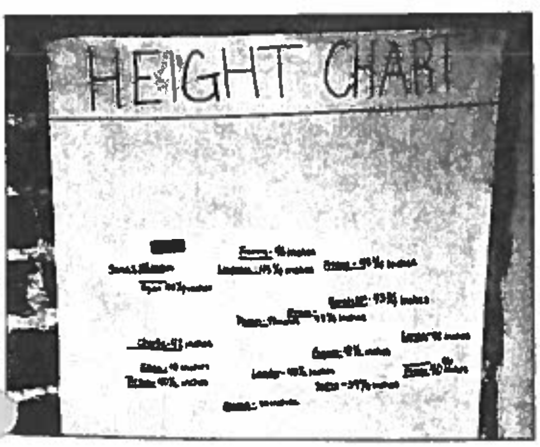
An example of a child not yet capable of this synthesis, but beginning the process of constructing math understanding, is the three-year-old with whom I shared the following interactions. The girl's parents had asked her to say her numbers for me and she correctly counted to 20 with no errors. I then pulled out 20 pennies that I had in my pocket. I asked her if she could figure out a way to make sure we both got the same number of pennies. She looked at the pile of pennies, split the pile down the middle, slid a handful over to me, and took the rest. My pile contained 12 pennies and hers contained 8. I asked her how she knew we had the same amount, and she attempted to count the pennies in front of me by pointing at the pile and saying, "One, two, three, four, five, six, seven." However, she did not have an understanding of the importance of order, and therefore counted some pennies twice and missed some completely. I then asked her to count her pennies, and she counted 10. When I again asked her if we both had the same amount, she made another quick visual inventory and replied "yes."

I then lined up eight pennies in a row and asked her to make a row with as many pennies as I had laid out. She took the rest of the pennies (12) and made a row below mine. I again asked her if there were the same number of pennies in each row. She counted her row and replied, "Yes. See—one, two, three, four, five, six, seven, eight, nine, ten." I asked her to count mine, and she came up with eight. I asked her again if they had the same number, and she again replied "yes."

This child was not yet able to coordinate order, classification, and seriation and therefore could not compare the penny quantities. Children as young as two may be able to count to 10 or even 20, but if they do not link their counting to quantification it is no different from memorizing their ABCs or a list of names like Bob, Joe, and Sara. Because she did not understand that the numbers she recited each represented a specific quantity, this child could not perceive a numerical relationship between the two sets of objects.

This three-year-old used visual cues to estimate the sameness and difference of the sets instead of using number. Her logic and problem-solving ability were still perceptually bound. However, as she continually interacted with the objects and with other children and adults, she would come to realize the limits of her method and begin to construct new ways of reaching a solution. This type of confusion, or what Piaget called *disequilibrium* (Piaget & Inhelder [1964] 1969), is what leads a child to make further constructions and to strengthen her understanding of mathematical concepts.

Courtesy of the author



The development of more complex understanding

Eight months later I had an opportunity to interact with this girl again. We repeated the game with the pennies. This time when I asked her to divide the pennies, she used a one-to-one correspondence method. She gave me a penny and then one to herself until all the pennies were distributed. When I asked her how she knew we had the same number, she counted each penny only once and in a specific order to get the correct answer.

I collected all the pennies in one pile, then showed the girl one more penny and added it to the pile. I asked her if she saw what I had done, and she said, "Yes, you added one more penny!" I then asked her to figure out a way to divide the pennies and make sure we both got the same number. She used the same method of one-to-one correspondence she had used previously. I asked her if we had the same number of pennies and she replied "yes." When I asked her to count them, and it turned out that I had one more penny, she was quite perplexed. She could not figure out how that had happened.

The child had made significant progress in her understanding of basic mathematical concepts. Her method of dividing the pennies was no longer visual. She used number concepts to solve her problem. However, her understanding of quantification was still weak and broke down when strongly challenged. Again, we see the disequilibrium that will lead the child to search for better ways of solving a problem. Through the process the child will construct new mathematical understanding.

Even very young children construct math knowledge

Children even younger than three years of age can use their developing understanding of order, seriation, and classification and their natural problem-solving ability. I observed an 18-month-old child playing in a large pit filled with different-colored balls. The child dropped one ball, then a second ball, and then a third ball over the side of the pit. The child then went to the opposite side of the pit and dropped two balls. He returned to the first side, reexamined the grouping of balls, then moved again to the second side and dropped

Before children can add or even count, they must construct ideas about mathematics that cannot be directly taught.

another ball over the side to make the second grouping a set of three.

This may seem unimpressive by adult standards, but for an 18-month-old child, the coordination and comparison of threes on opposite sides of a structure is evidence of making a mathematical relationship. It is not yet a numerical relationship because the child is using visual perception to make the judgment of same or different. However, the coordination of dropping three balls each time is evidence of an understanding of more and less and basic equality. The child may not be developmentally ready for counting and quantification, but this simple task shows that children as young as 18 months can understand some rudimentary mathematical relationships.

Teachers of infants and toddlers need to be aware of these actions and abilities and help provide activities to encourage construction of mathematical concepts. Activities that provide children with concrete experiences manipulating objects and interacting with other children and adults, such as distributing snacks or sorting items by color or size, promote this type of construction.

Promoting emergent math

Although these basic mathematical concepts cannot and should not be directly taught, educators of young children need to emphasize and encourage children's interaction with their environment as a means of promoting and encouraging emergent math concepts. Children's logical and mathematical thinking develops by being exercised and stimulated. Teachers who encourage children to put objects into all kinds of relationships are promoting children's emergent understanding of mathematics.

Making sure that children from birth through age four have a stimulating environment and opportunities to explore many different kinds of relationships can support their emerging understanding of mathematics. Teachers in infant, toddler, and preschool programs can do a number of things, like offering objects to compare, using rhythm activities and music, modeling mathematical behavior, and incorporating math into everyday activities to facilitate the emergent mathematical learning within every child. The basic framework for math cannot be directly taught but can be easily promoted in the classroom.

Birth to two

Infants and toddlers explore their environments using their senses. Piaget and Inhelder ([1964] 1969) called this time the sensorimotor stage because children explore and learn about their environments through motor activity and by touching, seeing, tasting, and hearing. It may not seem that any mathematical construction is going on during this time. However, children begin to perceive relationships between and among objects as they begin to construct ways to classify, seriate, compare, and order objects. Classification begins with a child's ability to match objects and evolves into a system of organizing objects into groups with similar characteristics. Classification is an important foundation for future mathematical concepts such as comparing sets of numbers and quantification.

Rhythm and music. Rhythm and music activities and materials are excellent for promoting emergent mathematics. Using bongo drums with infants and toddlers can help children experience mathematics. Teacher and child take turns repeating each other's beats; the teacher beats the drum twice, and the child beats the drum twice. If the child takes the lead, the teacher echoes the child's playing. This helps support the child's understanding of a one-to-one correspondence. It also demonstrates a matching relationship, which helps refine the child's ability to classify.

Using synthesizers with automatic beat generators is another good way to promote math through music. Let children play notes on the keyboard along with the generated beat. These synthesizers come with headphones so children can play whatever they feel like and not bother other children in the classroom.

I observed one teacher encouraging her children to organize a marching band using the musical instruments and items in the room. The children decided how to march. One child even insisted that he say "one, two, one, two" as they marched. The children for the most part coordinated their beat as they marched through the hall of the center, outside, and back to their room with one child saying "one, two" the whole time to keep them all together.

Activities to Promote Mathematical Construction in Infants, Toddlers, and Preschoolers

Music

- Call and response using beats on bongo drums
- Marching to music
- Playing a xylophone (relation of size to tone)
- Clapping
- Playing music on a synthesizer along with a beat generator
- Creating their own music

Shapes

- Sorting attribute blocks
- Playing with different-colored blocks, balls, and other items
- Building with blocks
- Completing puzzles
- Using manipulatives

Everyday activities

- *Snack time*
  - Distributing snack
  - Counting the number of people at the table
  - Estimating
- *Circle time*
  - Voting
  - Making time relationships (e.g., "Did we get into our circle slower or faster than yesterday?")
  - Calendar activities
- *Playground*
  - Counting (e.g., the number of times a child rides the tricycle around the course)
  - Measuring (e.g., "How high do you think you can build your sand structure?")
- *Feeding classroom pets*
  - Measuring amounts of food
  - Exploring "more" and "less" relationships

Project approach

- Using math to solve problems
- Using counting to solve disputes
- Using math to explain and demonstrate an idea
- Using math to plan

Using numbers, counting, and quantification in everyday activities. Even children under the age of two can be exposed to math during everyday tasks and activities such as snack time or circle time. Any opportunity to count should be taken advantage of to help the children understand one-to-one correspondence and quantity. Teachers should count and use math whenever possible and even ask children questions about simple mathematical relationships. This type of interaction helps children to recognize the importance of numbers and promotes the construction of emergent mathematics. Even children of this age can understand the concept of more. Asking children to compare groups of objects or quantities encourages the development of *more, less,* and *the same* concept.

Although children in this group have not yet constructed an understanding of number, this is no reason not to use math around them. Just as reading to infants and toddlers helps them develop literacy skills, using math around children helps them construct number concepts.

Blocks and shapes. Children who are surrounded with interesting objects are naturally led to make relationships between those objects. Determining whether objects are the same or different, matching, and classifying all require a child to focus on a certain quality of the object in order to make the comparison. The more frequently children make comparisons, the more complex their comparisons become. The simple act of adding an increasing variety of colored balls or blocks to the child's choices can facilitate more and more complex mathematical relationships. These activities support the concepts of seriation and classification.

Shapes can be used to show matching relationships. In infant and toddler rooms there should be an abundance of different-shaped blocks and tiles for children to match and compare. Because their mathematical development is still in its early stages, infants and toddlers naturally look for exact matches. This is the level of classification that they can handle. Infants and toddlers cannot see something as the same and different at the same time.

I observed a teacher working with a 12-month-old. They were examining a group of blue and yellow triangle blocks. The child gave a yellow triangle to the teacher and then picked up another yellow triangle and gave it to the teacher. The teacher then picked up a blue triangle and showed it to the child. The child grasped it and threw it back in the pile, found another yellow triangle and gave it to the teacher. To the child, the yellow and blue triangles are not matches because they are different colors.

Construction using cardboard boxes can also help children make relationships. In my experience, infants and toddlers love to play with cardboard boxes. Boxes of all sizes can be made available for the children to stack and arrange to make structures. Larger boxes can have doors or holes in them for the children to crawl in and out. These boxes can be put together in a variety of ways, and each combination or sequence is another relationship that the child has made. In the process of arranging the boxes, children have discussions and social interactions that also promote the making of new social relationships.

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As children develop their matching and classifying skills, they will be able to see more complex relationships. But to construct these concepts, children need time and interaction with objects and other people. Even if children are "prewired" for math, they still have to construct the concepts piece by piece. Children slowly construct formal mathematics understanding step by step, over the infant, toddler, and preschool years. This is why it is so vitally important to offer children as young as a few months old opportunities to match, classify, and compare.

Three- and four-year-olds

As children begin to move out of their sensorimotor stage of development and into what Piaget and Inhelder ([1964] 1969) called the preoperational stage, the big change is that children are able to think representationally and begin to acquire a certain degree of abstract thinking. Due to these abilities, children can think about objects that are not right in front of them, and they can begin to make connections between current and previous experiences. Children of this age can make much more complex relationships between objects. This is important for emerging mathematical concepts because it is during this time that the mental structures allowing a child to understand quantity are constructed.

The concepts of seriation, classification, and order take on a new dimension as children begin to understand more abstract relationships. Three- and four-year-olds can make comparisons to objects that are not present or events that took place in the past. This allows children to synthesize order, seriation, and classification to construct abstract mental structures that will support quantification and formal mathematics.

Children begin to make mental mathematical relationships that build on and refine the idea of *more* into "one more" or "two more." This refinement eventually leads to the child's being able to understand that three is one more than two and two more than one. This is the core idea behind quantification.

Manipulatives. An easy way to promote math to three- and four-year-olds is simply to ask children to use mathematical concepts in their activities. If a child is using blocks, a teacher can ask, "How many blocks do you have?" or "How many more do you need?" Children are willing and even excited to count objects and make mathematical relationships if the teacher encourages them.

A four-year-old child was making a chain out of different-colored

© Rick Fatica

Although he made mistakes and showed an incomplete understanding of number concepts, he was getting closer to using mathematics in a conventional manner. His actions were like those of children who move from drawing squiggles to writing words in the process of learning to write conventionally.

plastic links. I asked him how long he was trying to make the chain. He did not respond, so I tried a more direct question, "How many links do you have so far?" He put on the next link and then proceeded to count each link. There were eight. After he put on another link, I asked again, "How many do you have now?" He went back to the beginning and counted each link again and got an answer of nine. When he again added a link, I asked him a more leading question: "You had nine and you put one more on. How many do you have now?" Again the child counted all the links until he got the answer of 10. After that I did not have to ask him again. Each time he put on a new link, he would count all the links. He eventually made a chain with 27 links.

After 15 links this child's counting became erratic. Sometimes he counted carefully and got the correct answer, and other times he missed some links in his counting. For example, after correctly counting 26 and adding one more, the boy counted again and missed a few. After completing the counting he triumphantly announced, "Fifteen!" The fact that he now had fewer than before did not seem to trouble him.

Everyday activities. Just as with the infants and toddlers, everyday activities such as snack and circle times can be used with three- and four-year-olds to promote the use of math. Dividing up snack, counting plates, and other activities can be assigned to children. They then have to use their own mathematical problem-solving ability to figure out the best way to perform the tasks. A child who is assigned to put out the plates for his table of five may do it by going to the stack of plates, getting one plate and placing it in front of one child, then going back to the plates to get another plate for the next child, and so on until everyone has a plate. Eventually the child will realize that he can count the children, then go to the plates, count out five plates, and distribute them accordingly. Allowing the child to use her own method of solving a problem such as this allows emergent understanding of math to develop in a child-centered developmental pattern.

Everyday activities such as snack and circle times can be used to promote the use of math.

Assigning two children to figure out how to solve an everyday problem as described above promotes problem solving even more. The children can discuss, plan, and even debate the best way to solve the problem. This give-and-take will push both children to construct new ways of seeing the problem (Kamii & Lewis 1990; Kamii 1991). In an argument, the children must clearly communicate their ideas to another person and at the same time evaluate the other person's ideas. In the process, both children examine and perhaps modify their own ideas.

Whenever a decision needs to be made in which children can have input, voting allows the teacher to use math in an integrated way. Not only does it offer an opportunity to count, but to compare numbers as well. Children can be asked to vote on which book to read first. The teacher asks the children to vote for one book. As the teacher counts the hands, she encourages the children to count with her. If the vote is six to five, the teacher can ask the children which book has won.

The project approach

The project approach to early childhood education allows children to explore their world and construct knowledge through genuine interaction with their environment. Lilian Katz (1989) states that young children should have activities that engage their minds fully in the quest for knowledge, understanding, and skill. When engaging in the project approach, children are not just gathering knowledge from a worksheet, structured activity, or teacher but are actively making decisions about not only what to learn, but also how and where to learn it. Through this method, children construct problem-solving techniques, research methods, and questioning strategies.

When children work on projects, a number of opportunities arise for them to use math. In a recent project on construction and transportation, children had an opportunity to use measurement to help them

Questioning strategies, activities, and simple games offer a great opportunity for teachers to help children construct basic mathematical concepts.

build a truck. They measured how long, tall, and wide they wanted it and then transferred their numbers to the cardboard they were using to make their truck. Their measurements were not accurate, and they did not really understand the concept of using a measuring tape. Still, these activities were at the beginning on the continuum of math learning, just as writing scribbles is a beginning step in literacy learning.

The children also learned about blueprints, and when they made their own the teacher asked them how many windows they wanted in their house, how many bathrooms, and how many rooms in total. They discussed the layout of the house, which rooms would have windows, and how the rooms were located in the house. The children had to plan, count, use number, and measure to complete the activity.

Conclusion

There are many easy things that teachers can do to encourage the emerging mathematician in every child. Questioning strategies, activities, and simple games offer a great opportunity for teachers to help children construct basic mathematical concepts. An active, stimulating environment and a teacher who is willing to see the child's ability to construct mathematical concepts are invaluable to a child's construction of mathematics.

If we are to view the development of mathematics as emergent, we must understand that construction of mathematical concepts begins the day a child is born. Children construct the basic concepts of mathematics such as quantification, seriation, order, and classification without much interference or direct teaching from adults. The understanding of these concepts is not something that *can* be taught to children; they must construct it for themselves. The role of the teacher is to facilitate learning by offering infants, toddlers, and preschoolers opportunities and materials to promote their construction of mathematical thinking.

References

Chomsky, N. 1999. On the nature, use, and acquisition of language. In *Handbook of child language acquisition*, eds. W.C. Ritchie & T.K. Bhatia, 33–54. San Diego, CA: Academic Press. (Note: This reprinted chapter originally appeared in 1987, Special Issue [11] *Sophia Linguistica*.)

Ferreiro, E., & A. Teberosky. 1982. *Literacy before schooling*. Exeter, NH: Heinemann.

Katz, L.G. 1989. *Engaging children's minds: The project approach*. Norwood, NJ: Ablex.

Kamii, C., & B.A. Lewis. 1990. Research into practice. Constructivist learning and teaching. *Arithmetic Teacher* 38 (1): 34–35.

Kamii, C., & others. 1991. Reform in primary mathematics education: A constructivist view. *Educational Horizons* 70 (1): 19–26.

Koechlin, E., S. Dehaene, & J. Mehler. 1997. Numerical transformations in five-month-old human infants. *Mathematical Cognition* 3 (2): 89–104.

Piaget, J., & B. Inhelder. [1964] 1969. *The early growth of logic in the child: Classification and seriation*. New York: Norton.

Sinclair, H., M. Stambak, I. Lezine, & S. Rayna. 1989. *Infants and objects: The creativity of cognitive development*. San Diego, CA: Academic Press.

Starkey, P., & R.G. Cooper, Jr. 1980. Perception of numbers by human infants. *Science* 210 (4473): 1033–35.

Wynn, K. 1995. Origins of numerical knowledge. *Mathematical Cognition* 1 (1): 35–60.