House Prices and Negative Nominal Interest Rates*

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Abstract

When borrowers have access to two types of debt, and the cost of one type of debt declines relative to the other, borrowers should substitute towards the type of debt that has become relatively cheaper. When household borrowers can choose between uncollateralized debt and mortgage debt any shock that drives up the spread of the interest rate on uncollateralized debt over the interest rate on mortgage debt will incentivize households to substitute towards mortgage debt. In turn households will demand more housing to collateralize this mortgage debt, pushing house prices up. I build a model that captures this novel *debt substitution channel* linking debt substitution to house prices. I study this channel in the context of negative nominal interest rate policy (NIRP) in Denmark. I show empirically that NIRP is associated with an increase in the spread between the interest rate on uncollateralized debt and the interest rate on mortgage debt. Using the model I show that the debt substitution channel amplifies the impact of monetary policy rate cuts on house prices and reduces the ability of monetary policy to stimulate the aggregate consumption of borrowers. Lastly I show that monetary policy rate hikes from negative levels have a smaller impact on inflation due to the debt substitution channel.

Keywords: Negative Interest Rates, Reversal Rate, Mortgage Credit, House Prices, Non-Banks.
JEL Codes: E43, E44, E51, G21, G23.

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1 Introduction

Recent experience with negative nominal interest rate policy (NIRP) suggests that once the nominal policy rate goes below zero, an erosion in bank profits means that there is an attenuation or even a reversal of the pass-through of monetary policy rate cuts to the interest rates on bank lending. This impacts the relative cost of bank loans vs other types of debt which are not intermediated by banks (e.g. mortgages in certain financial systems). In this paper I examine how households respond to this relative price change and how this impacts the transmission of monetary policy below zero. I find that below zero the transmission of monetary policy rate cuts to house prices and aggregate consumption changes. The impact on house prices is amplified but monetary policy is less effective at simulating consumption. Looking forward, I find that monetary policy rate hikes might be less effective at reducing inflation, when the monetary policy rate lifts off from a negative nominal level.

In July 2012 Denmark was the first country to introduce a negative nominal policy rate. Denmark is a particularly useful setting to study the implications of the pass-through of the negative interest rate policy to different consumer loan products. This is because, like the US, mortgages are provided by financial entities ("mortgage credit institutions") that do not issue deposits and therefore are not subject to the adverse impacts of the NIRP the way commercial banks are.

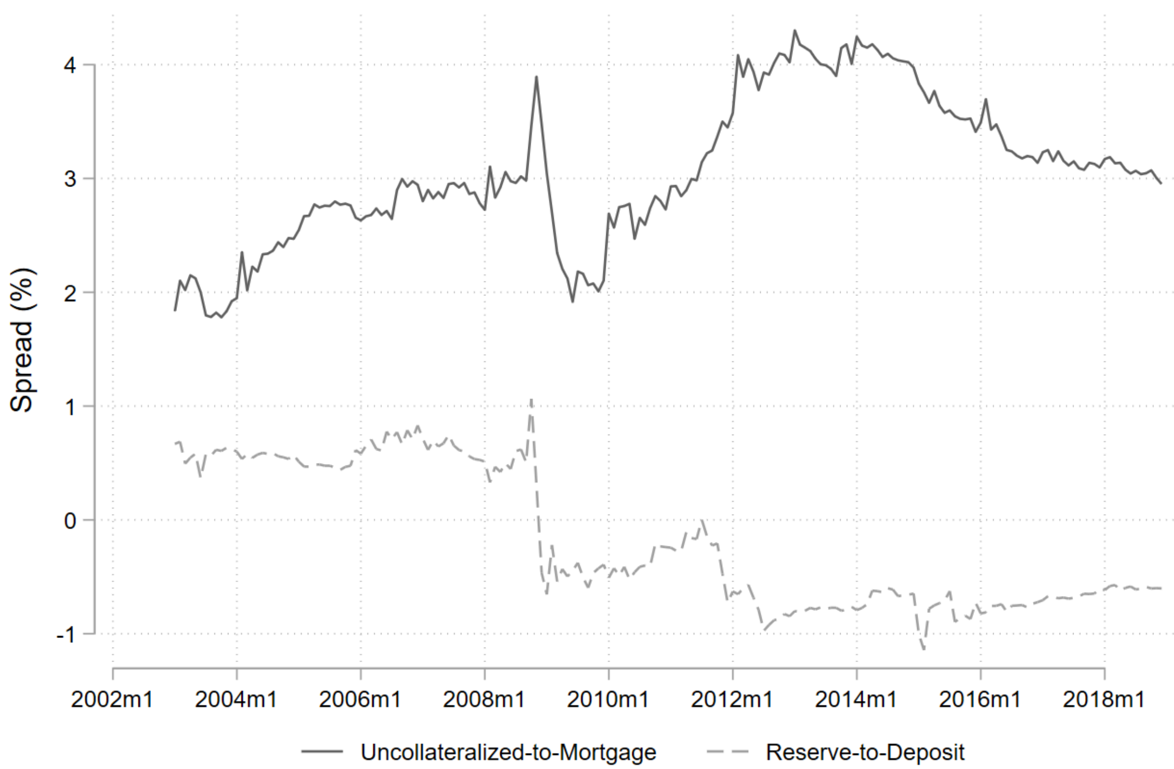
Figure 1 suggests that under very low or negative nominal policy rates the relative cost of uncollateralized bank loans vs mortgage loans is negatively correlated with banks' net interest margin. The solid line in this figure measures the spread between the average lending rate on uncollateralized bank loans (to households) minus the average interest rate on mortgages provided by mortgage credit institutions (MCIs). As this spread increases mortgages become relatively more attractive compared to alternative forms of lending. As I show in this paper this drives a substitution into mortgage debt and house prices go up because housing is more demanded as collateral.

The dashed line in figure 1 is the spread between the weighted average interest rate banks earn on central bank reserves (the interest rate on reserves is the policy rate[1]) minus

[1]Technically *rates*, as there is more than one type of reserve in Denmark. See section 2 for an explanation of this and of the tiered reserve system in Denmark

a weighted average of the interest rate banks pay on household and firm deposits. Normally the reserve-deposit spread is positive, so it contributes positively to a bank's net interest margin[2] and so is a source of bank profits. The lower the reserve-deposit spread the less profitable deposits are for banks. Because the nominal interest rate on deposits did not adjust as quickly as the nominal interest rate on reserves the reserve-deposit spread fell below zero in 2008 as the nominal policy rate dropped to very low and then negative levels (in July 2012). This had the marginal effect of eroding the profitability of Danish banks.

Figure 1: Evolution of Spreads



Note: "Uncollateralized-to-Mortgage" is the spread between the weighted average interest rate charged by banks on uncollateralized lending to households minus the weighted average interest rate charged by mortgage credit institutions on mortgage lending (both including fees). "Reserve-to-Deposit" is an approximation of the weighted average of the interest rate banks earn on reserves minus the weighted average interest rate banks pay on firm and household deposits.
Source: Danmarks Nationalbank's MFI Statistics.

When banks face constraints (e.g. capital regulation) the erosion of bank profits from

[2]The difference between what a bank earns on assets (including central bank reserves) vs what a bank pays on its liabilities (including deposits).

the negative reserve-deposit spread will push banks against these constraints. As this effect intensifies the response of banks to further monetary policy rate cuts will attenuate or even reverse: i.e. interest rates on bank loans may not drop by much or may even increase. Ulate (2021) refers to this effect as the "net-worth channel of monetary policy". This channel generates a breakdown in the traditional bank lending channel of monetary policy. I will refer to this channel as the "bank net-worth channel" to emphasize that it applies to Danish banks but not Danish mortgage credit institutions.

Financial intermediaries that do not fund themselves via deposits do not experience the same erosion of profitability. Mortgage credit institutions in Denmark are funded by mortgage bonds. The interest rate on these bonds is not sticky downward in the same way as bank deposit rates. This means that Danish mortgage credit institutions do not face the net-worth channel of monetary policy under the negative interest rate policy. Monetary policy passes through to mortgage rates in the same way above zero as below zero, driving the increase in the spread between uncollateralized bank loans and mortgage loans in figure 1.

Using Danish bank-level data on lending to households before and during the negative nominal policy rate period (July 2012 and after) in Denmark I find the following empirical results consistent with monetary policy driving the widening of the uncollateralized bank loan to mortgage loan spread during the negative interest rate period. One, the pass-through of monetary policy rate cuts to lending interest rates is positive across both uncollateralized bank loans and mortgage loans before July 2012. Two, the pass-through to mortgage loan interest rates is not statistically significantly different after July 2012. Three, the pass-through of monetary policy rate cuts to uncollateralized household loans is statistically significantly lower after July 2012. These results are consistent with a widening of the spread between uncollateralized lending and mortgage lending in response to monetary policy rate cuts when the nominal policy rate is already negative.

Using Danish household-level microdata I find a similar attenuation in the pass-through of monetary policy rate changes to bank lending interest rates but not mortgage rates after July 2012. Using the microdata I am able to control for a range of household observables, suggesting the dynamics in the data are driven by credit supply changes not credit demand changes.

I build a model in which borrowers can access uncollateralized loans and mortgage loans. When uncollateralized loans become relatively more expensive borrowers substitute towards mortgage debt. Using this model I find that in response to a shock that increases the uncollateralized-to-mortgage spread borrowers demand more mortgage debt (as its relative price has declined). As a result borrowers demand more housing to collateralize the increase in mortgage debt. This drives house prices up. Because the increase in the spread is a marginal tightening of credit conditions to households, this shock mutes consumption. This is the *debt substitution channel*.

I also find using the model that the quantitative size of these results is increasing in the duration of the shock. This suggests important policy implications: minimizing the duration of the negative interest rate policy will minimize the unintended consequences on house prices and consumption of the debt substitution channel.

Lasting, embedding the simple model into a standard DSGE model with nominal frictions I find that the addition of the *debt substitution channel* reduces the effect of monetary policy on inflation. This effect is symmetric: rate cuts are less inflationary, and rate hikes reduce inflation by less. Contractionary monetary policy, when lifting-off from a very low or negative nominal policy rate environment, has a quantitatively smaller impact on fighting inflation.

The paper proceeds as follows. Section 2 reviews the institutional context of negative nominal interest rate policy and the mortgage sector in Denmark. Section 3 presents empirical evidence on the pass-through of monetary policy rate cuts during the negative interest rate period in Denmark. Section 4 presents a simple model that illustrates the *debt substitution channel*. Section 5 presents the model calibration. Section 6 presents and discusses the simulation results using the simple model. Section 7 summarizes the results with the model extended to include nominal frictions. Section 8 presents initial results in a further version of the model that endogenizes the uncollateralized-to-mortgage spread by explicitly modeling financial frictions faced by commercial banks. Section 9 concludes.

Related Literature This paper relates to the recent theoretical literature on the impacts of negative nominal interest rate policy.

The bank lending channel captures that if bank loan interest rates decline with policy rates this decline stimulates the economy via more consumption and investment due to lower borrowing costs. The mechanism in this paper relies on the break-down of the traditional bank lending channel of monetary policy driving an asymmetric impact of monetary policy on uncollateralized bank loan interest rates vs mortgage interest rates. When negative nominal interest rate policy erodes bank profits, this leads to a breakdown in the bank lending channel. The interest rate on bank loans either fall less or even increase in response to monetary policy rate cuts in negative territory (the "bank net-worth channel"). This effect dominates, breaking down the bank lending channel. In models where banks are the sole provider of credit this is purely contractionary. In the model in this paper the breakdown of the bank lending channel drives up the uncollateralized-to-mortgage spread, because mortgage credit is not provided by traditional banks. This shifts borrowers towards mortgage credit, changing the transmission of monetary policy, but not in a purely contractionary way. Adding and explaining this additional *deb substitution channel* is the key contribution of this paper.

The recent theoretical literature on negative interest rate policy establishes how negative policy rates break down the bank lending channel. Ulate (2021) shows, using a DSGE model, that the erosion in bank capital due to the bank net-worth channel of monetary policy will eventually push bank interest rates up in response to monetary policy rate cuts when policy rates are sufficiently low. He also finds empirical evidence supporting the bank net-worth channel using bank-level data from multiple advanced economies (including Denmark). Eggertsson, Juelsrud, Summers and Wold (2019) show using Swedish data a reversal in the pass-through of monetary policy rate cuts to certain bank lending interest rates, and similarly they find theoretically a break-down in the bank lending channel. Abadi et al. (2022) introduce a trade-off: a monetary policy rate cut increases the value of long-term fixed rate assets that banks hold but reduce banks' expected net interest income in the future. When the latter effect dominates the bank lending channel breaks down.

Groot and Haas (2022) find that the breakdown in the bank lending channel from the erosion of capital due to the change in net interest margin is counteracted by the signaling effect of negative interest rates: to the extent to which there is inertia in the policy rate, going negative makes maintaining low interest rates for longer more credible. In turn this stimulates the economy today. Darracq Paris, Kok and Rottner (2021) find that the breakdown in the bank lending channel can be mitigated by macroprudential policy: they show that building up capital buffers reduces the risk of hitting the reversal rate.

To my knowledge is there no theoretical paper that looks at the debt substitution channel examined in this paper and its implications for the transmission of negative nominal interest rate policy.

There is also an active empirical literature on negative interest rates. Ampudia and van den Heuvel (2019) find evidence that under low rates further monetary policy rate cut have adverse impacts on bank profitability. This is key to the breakdown in the bank lending channel essential to generate the dynamics in the spread between uncollateralized bank loans and mortgage loans studied here. Heider et al. (2019) find that high deposit banks experience a reduction in net worth relative to low deposit banks, low deposit banks also tend to lend less but to more risky borrowers. Bittner et al. (2021) examine risk taking induced by negative nominal policy rates by comparing the experience of Portuguese and German Banks. Adolfsen and Spange (2020) find an attenuation of the pass-through of monetary policy rate cuts to lending rates in Denmark. Abildgren and Kuchler (2020) examine how Danish firms react to the introduction of negative deposit rates on firm deposits.

2 Institutional Context

2.1 Danish Mortgage Credit System

In Denmark residential mortgages are provided by mortgage credit institutions (MCIs) which operate differently to banks (see Gundersen et al. (2011) for a in depth explanation of the Danish system). Mortgage credit institutions raise funds in the bond market, instead of from depositors. In this way Danish mortgage credit institutions are similar to government sponsored enterprises (GSEs) in the United States. See Kjeldsen (2003) for a more detailed description of the similarities and differences between the Danish and American mortgage systems. There is a close relationship between the interest rate on mortgage bonds and the interest rate on mortgages provided to borrowers.

In addition to mortgage credit, which households can access up to 80% of the value of their house, households can take out bank loans that are collateralized by housing. These top-up type loans are either accessible at the time of house purchase or later on for refinancing. They are usually no more than 15% of the value of the house. In the model I present later I abstract from the existence of these loans.

2.2 Danish Negative Interest Rate Policy:

Because of the fixed exchange rate regime, the policy rate in Denmark is not a traditional instrument of monetary policy. Rather it targets the exchange rate. Therefore the negative interest rate policy (NIRP) is not designed to stimulate the economy, rather it is intended to import the ECB's monetary policy via the fixed exchange rate and open capital markets. Nonetheless, the prolonged Danish experience with negative policy rates provides a useful environment to examine the transmission of NIRP.

In Denmark there are two types of monetary policy reserves: certificates of deposit, which have a original maturity of 7 days, and current-account deposits, which are redeemable on demand. Normally the longer maturity certificates of deposit have a higher interest rate, however, July 2012 the certificate of deposit rate was reduced from 0.05 to -0.2. This introduced a negative nominal policy rate into Denmark for the first time. The certificate of deposit rate briefly return above zero between late April and early September 2014, but otherwise it has been negative.

Up until March 2021 the interest rate on the current-account deposits remained at zero. This was effectively a tiering system: banks could hold up to a certain individual amount in current-account deposits, beyond which they were required to convert current-account deposits to certificates of deposit (incurring the negative interest rate). This individual limit on current-account deposits only applied if the banking system in aggregate hit a specific limit on total current-account deposits. Jorgensen and Risbjerg (2012) for an in depth explanation of the Danish reserve system under negative interest rates. In March 2021 the interest rate on current-account deposits was reduced from zero to -0.5 (to match the interest rate on certificates of deposit), this effectively ended the tiering of reserves.

2.3 Pass-through to Deposit Rates

In Denmark deposit rates have been slower to fall than the interest rate on reserves, particularly for household deposits, but not stuck at zero. There was relatively quick introduction of negative nominal rates on deposits for firms. Abildgren and Kuchler (2020) study the response of firms to the introduction of negative interest rates on their deposits. Household deposits remained at zero for longer, and even as of early 2022 most banks only apply negative nominal interest rates on marginal household deposits above 100,000 DKK (about 13,400 EUR). Danish banks have not passed on the cost of negative nominal rates on reserves to their customers 1-for-1. This means that their net interest margin (the rate earned on reserves minus the rate paid on deposits) is negative and errodes bank profits.

3 Empirical

In this section I use both bank-level data and household-level microdata to examine the pass-through of monetary policy rate changes to interest rates on bank loans vs mortgage loans. I find evidence consistent with a substantial reduction in the pass-through of policy rate changes to the interest rate on uncollateralized bank loans during the negative rate period. There is no reduction in the pass-through of policy rate changes to mortgage rates during the negative period. This evidence is consistent with a widening of the spread between uncollateralized debt and mortgage debt during the negative period (as illustrated by figure 1).

3.1 Interest Rate Pass-Through: Bank-Level Data

The data used here is drawn from the MFI statistics collected at the bank level and monthly frequency by Danmarks Nationalbank.

The regression in equation 3.1 tests if the pass-through of policy rate changes to lending rates has changed during the negative nominal interest rate period.

$$\Delta i^b_{i,t} = \alpha + \eta I^{negative}_t + \beta \Delta i^r_t + \gamma \Delta i^r_t \times I^{negative}_t + \delta_i + \epsilon_{i,t}, \tag{3.1}$$

where $\Delta i^b_{i,t}$ is the monthly change in lending rate for loan type b (either bank loans not associated with housing, bank loans associated with housing, or mortgage loans) from bank i at time t. $I^{negative}_t$ is an indicator variable equal to one if the month is July 2012 or later (the month during which the policy rate went negative in Denmark for the first time). Δi^r_t is the monthly change in the monetary policy rate (certificate of deposit rate) and δ_i captures bank fixed effects. Standard errors are clustered at the bank level. If γ is negative and statistically significant this is associated with a decline in the pass-through of policy rate changes to the relevant lending rate during the negative interest rate period.

Table 1: Lending Rates, Comparison of Pass-Through

	Bank Loans	Housing Related Bank Loans	Mortgage Loans
Δi^r_t	0.365***	0.408***	0.086***
	(0.02)	(0.02)	(0.02)
$I^{negative}_t$=1 × Δi^r_t	-0.317***	-0.310***	0.068*
	(0.06)	(0.07)	(0.03)
$I^{negative}_t$=1	-0.025***	-0.013**	0.011***
	(0.00)	(0.01)	(0.00)
Constant	0.002	-0.007**	-0.023***
	(0.00)	(0.00)	(0.00)
Bank FE	Yes	Yes	Yes
F statistic	346.26	263.60	16.17
Observations	4,074	3,837	1,874

Note: ***p < .01, **p < .05, *p < .1 . "Bank loans" are loans from banks to households, unrelated to housing. "Housing related bank loans" are loans from banks to households that are related to housing. "Mortgage Loans" are loans collateralized by housing from mortgage credit institutions to households. The interest rate on each lending type is the average interest rate in the stock of loans of that type. The bank level rates are weighted by volume of lending.

The results in table 1 are consistent with attenuation of the pass-through from changes in the policy rate to changes in the lending rate, but only for loans provided by commercial banks, not for mortgages issued by mortgage banks in Denmark. For the interest rate on bank loans the γ coefficient is negative and large enough to suggest a potential reversal of the pass-through of monetary policy. This is consistent with the aggregate evidence in figure 1 which shows that the spread between the interest rate on uncollateralized bank loans relative to the interest rate on mortgage loans increased during the negative interest rate period. These results suggest that the increase in the uncollateralized - mortgage spread is being driven by a differential attenuation (or even reversal) in the pass-through of monetary policy rate cuts during the negative interest rate period. These results stand up when including more lags of the change in the policy rate (see appendix E)

However, these results do not rule out other potential drivers of the increase in this spread. For example it is possible that other factors either drove or exacerbated the attenuation of monetary policy pass-through to bank lending interest rates. Adolfsen and Spange (2020) examine a wider range of loan types from Danish banks, they also find attenuation of the pass-through of monetary policy rate changes to this broader aggregate of interest rates on loans (including firm loans). Their analysis includes firm loans and finds slightly less attenuation in aggregate, suggesting the attenuation of pass-through is particularly strong for household uncollateralized loans. They suggest that post-financial crisis adjustment of Danish banks to risk as an alternative factor driving the change in pass-through.

The results in this section are suggestive that there may be a relationship between negative interest rates and the attenuation of pass-through of policy rate changes to uncollateralized lending rates. But the empirical evidence I present here is not causal.

3.2 Interest Rate Pass-Through: Household-Level Microdata

Figure 2 shows that the spread between the interest rate on bank debt and the interest rate on mortgage debt, measured at the household level, increased during the negative interest rate period. In this section I examine the pass-through of monetary policy to bank loan interest rates vs mortgage interest rates, using the household-level micro data to control for changing household characteristics. The regression in equation 3.2 tests if the pass-through of policy rate changes to lending rates has changed during the negative nominal interest rate period:

$$\Delta i^b_{j,t} = \alpha + \eta I^{negative}_t + \beta \Delta i^r_t + \gamma \Delta i^r_t \times I^{negative}_t + \delta_i + z'_{j,t}\theta + \epsilon_{j,t}, \tag{3.2}$$

where $\Delta i^b_{j,t}$ is the yearly change in lending rate for loan type b (either bank loans or mortgage loans) taken out by household j at time t. $I^{negative}_t$ is an indicator variable equal to one if the month is July 2012 or later (the month during which the policy rate went negative in Denmark for the first time). Δi^r_t is the yearly change in the monetary policy rate (certificate of deposit rate) and δ_j captures household fixed effects. $z'_{j,t}$ is a vector of household-specific characteristics[3]. Standard errors are clustered at the household level. If γ is negative and statistically significant this is associated with a decline in the pass-through of policy rate changes to the relevant lending rate.

Table 2: Comparison of Pass-Through: Household Microdata

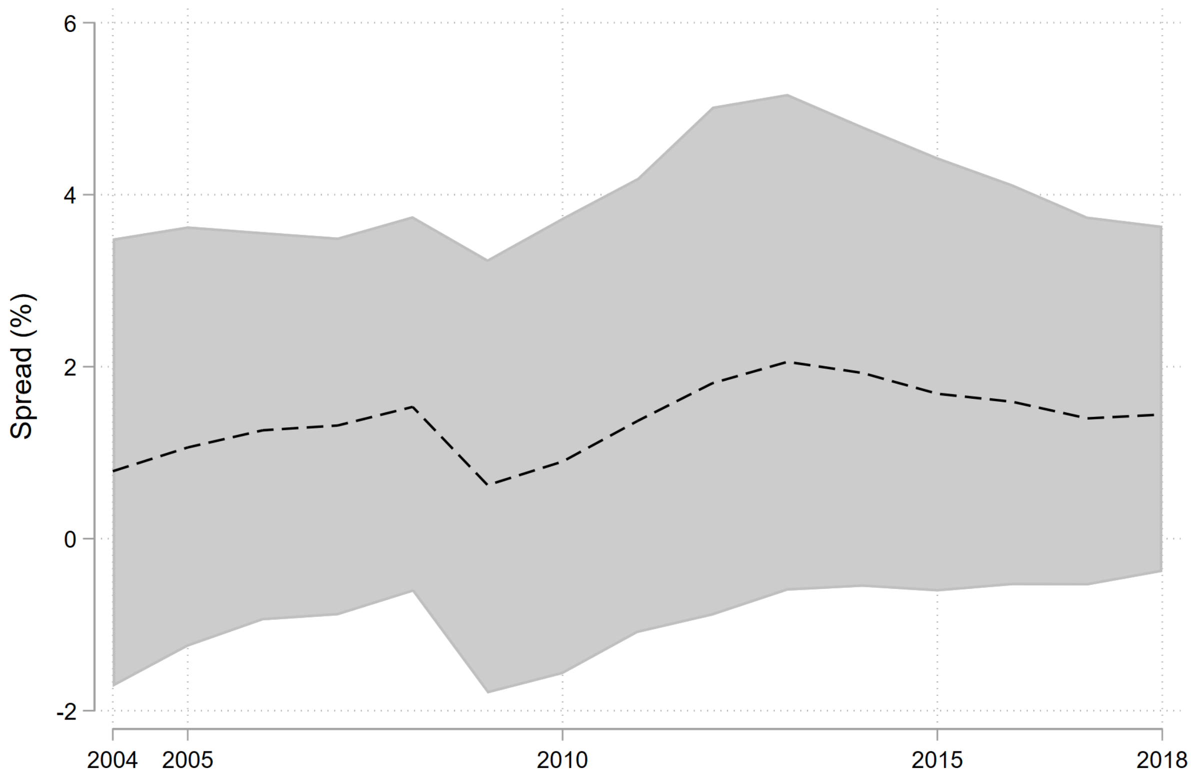
	(1)	(2)
	Bank Loans	Mortgage Loans
Δi_t^r	0.271***	0.040***
	(0.00)	(0.00)
$I_t^{negative}$=1 × Δi_t^r	-0.299***	0.068***
	(0.00)	(0.00)
$I_t^{negative}$=1	-0.047***	0.112***
	(0.00)	(0.00)
Constant	0.032	0.084***
	(0.02)	(0.01)
Household FE	Yes	Yes
Household Controls	Yes	Yes
F statistic	7,314	2,726
Observations	12507418	10517143

Note: ***p $<$ 0.01, **p $<$ 0.05, *p $<$ 0.1. "Bank loans" are loans from banks to households. "Mortgage Loans" are loans collateralized by housing from mortgage credit institutions to households.

As in the bank level regressions the γ coefficient is negative and statistically significant only for bank loans. This suggests that, even when controlling for changes in borrower characteristics, there was an attenuation in the pass-through of monetary policy to bank loan interest rates but not mortgage interest rates. In fact the pass-through of monetary policy to mortgage loan interests rates seems to even *strengthen* (a positive and statistically significant γ coefficient) during the NIRP period.

[3]Household controls include: An indicator of the household containing a single member, the ages of the first two household members, the number of children, municipality, total wage income, an indicator of a household member currently unemployed, an indicator of a household member unemployed in the last two years, total debt, total housing wealth, total mortgage debt, and total bank debt.

Figure 2: Evolution of Spreads



Note: This is the annual spread between the interest rate on bank debt and the interest rate on mortgage debt. These data are at the household level, for all households in the data that have both positive mortgage and bank debt in a given year. The spread is based on imputed interest rates[a]. The black dashed line is the median (by year) spread between the imputed interest rate on bank debt and the imputed interest rate on mortgage debt, at the household level. The shaded area covers the 25th-75th percentiles of the within household spread, also by year.
Source: Statistics Denmark, IND register.

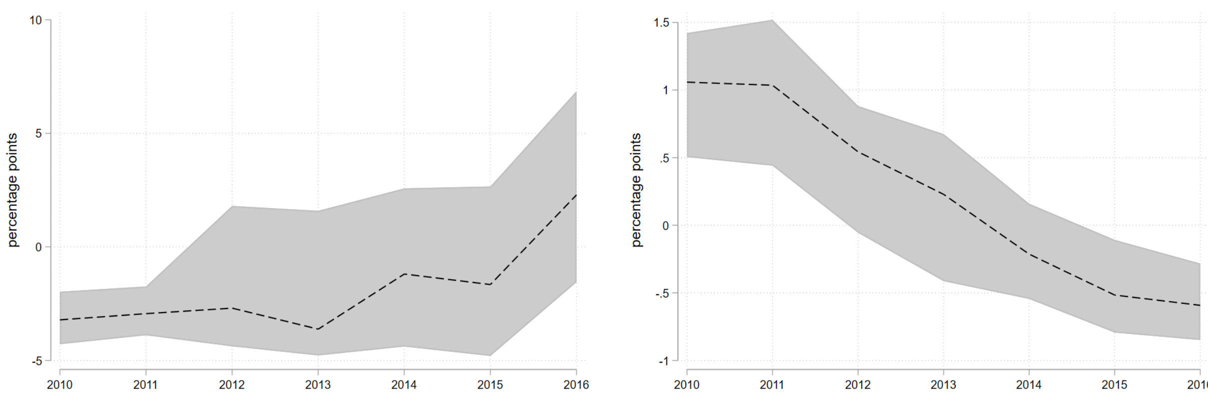
[a]In the Indkomst (IND, i.e. income) register total interest expenses and end of year debt balances are reported. Following Jensen and Johannesen (2017) I use the following formula to impute the interest rate from these data:

$$interestRate_t = \frac{annualinterestexpenses}{0.5 * balance_t + 0.5 * balance_{t-1}}.$$

3.3 Consistent Interest Rate Series

The proceeding section and figure 2 shows that the spread between the interest on bank debt and the interest on mortgage debt at the *household* level also rose during the negative interest rate period in Denmark. However, it is still possible that this was driven by changes in loan characteristics. In this section I use loan-level data from the Danish register data to control for changes in both borrower and loan characteristics[4]. This loan level data on mortgage loans only goes back to 2009.

Figure 3: Consistent Interest Rates



Bank Loans | Mortgage Loans

Note: The black dashed line is the median (by year) for each consistent interest rate series. The shaded area covers the 25th-75th percentiles of each consistent interest rate series, also by year. The consistent interest rates are based on residual (mean zero) series, so their level is not comparable.

I develop an interest rate series that is consistent across borrowers and time in the spirit of Justiniano et al. (2022). The idea is to produce a residual series that essentially captures the variation in interest rates, on mortgages and bank loans, that is *not* driven by changes in loan or borrower characteristics.

The empirical model is as follows:

$$r_{i,j,t} = c + x'_{i,t}\gamma + z'_{j,t}\alpha + \varepsilon_t, \tag{3.3}$$

where $r_{i,j,t}$ is the imputed interest rate on account i owned by household j in year t. $x'_{i,t}$ is a vector of loan-specific characteristics. $z_{j,t}$ is a vector of household-specific characteristics.

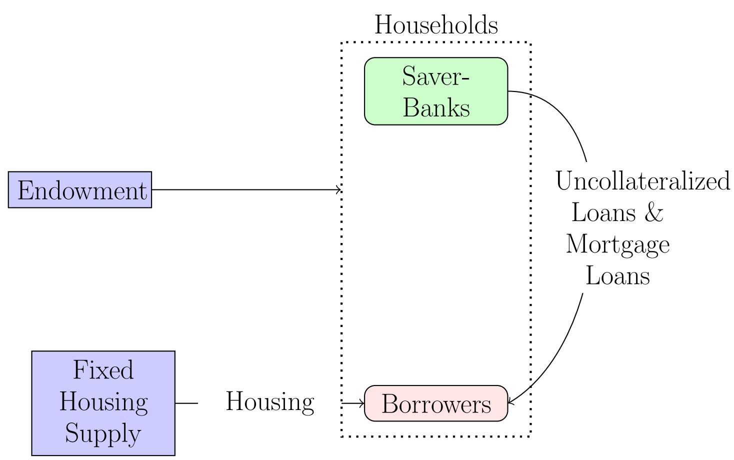
[4]This is the REAL register for mortgage loans, and the URTEPERS register for bank loans.

Appendix F reports the regression table for these results.

These results are only illustrative of the trend in the two interest rate series, controlling for non-credit supply factors. Figure 3 indicates that the bank loan series did not decline and in fact seems to increase. While the mortgage rate series seems to decline over the NIRP. This is consistent with a widening of the spread between the uncollateralized bank loan interest rates and mortgage loan interest rates widening due to the interaction of monetary policy and the bank net-worth channel.

4 Stylized Model

This section sets out a stylized model with an exogenous spread between the rate on uncollateralized lending and the rate on mortgage lending (i.e. lending collateralized by housing). This model illustrates the *debt substitution channel* that links borrower substitution between the two types of debt to fluctuations in house prices.



4.1 Households

There are two representative households: the saver and the borrower. Each household is of mass 1. Borrowers are relatively impatient individuals who value housing and face a collateral constraint when obtaining mortgage debt.

4.1.1 Savers' Problem

The savers have an un-modeled rigid demand for housing. As a result borrowers are the marginal buyers of housing. This reflects Geanakoplos (2010)'s idea that the asset is priced by the most levered individuals (the borrowers). Similar assumptions about segmentation are made by Justiniano et al. (2019), Ferrante (2019), and Greenwald (2018).

The savers are relatively patient (their discount factor $\tilde{\beta}_t$ is larger than the borrowers' discount factor), they consume and make loans directly to households. Their problem is:

$$\max_{\{\tilde{c}_t, l_t, b_t\}} \; E_0 \sum_{t=0}^{\infty} (\tilde{\beta}_t)^t \Big[\tilde{u}(\tilde{c}_t) - \tilde{v}(l_t) \Big] \tag{4.1}$$

subject to their budget constraint:

$$\tilde{c}_t + l_t + b_t \leq \tilde{y} + R^l_{t-1} l_{t-1} + R^b_{t-1} b_{t-1}. \tag{4.2}$$

Saver specific notation is denoted with tildes: $\tilde{c}_t$ denotes consumption of non-durable goods, $\tilde{y}$ is the saver's per period endowment. b_t are mortgage loans to borrowers (which pay the interest rate R^b_t). l_t are uncollateralized loans to borrowers (which pay the interest rate R^l_t). $\tilde{v}(l_t)$ is a utility cost for holding uncollateralized debt. The functional form is:

$$\tilde{v}(l_t) = \tilde{\beta}_t \tau_{l,t} l_t. \tag{4.3}$$

$\tau_{l,t}$ determines the spread spread between uncollateralized lending and mortgage lending ($R^l_t - R^b_t$). This spread captures in reduced form that mortgage lending is generally cheaper or less risky than uncollateralized forms of household lending. Shocks to the savers' discount factor ($\tilde{\beta}_t$) impact the level of the mortgage rate, but because $\tilde{\beta}_t$ enters equation (4.3) the savers' discount factor ($\tilde{\beta}_t$) shock does not impact the spread[5].

Mapping to a Richer Model with Financial Intermediaries: Shocks to $\tau_{l,t}$ capture exogenous fluctuations in the savers' required excess return on uncollateralized lending over

[5] Savers have linear utility, so the savers' first order conditions w.r.t mortgage loans (A.1) and uncollateralized loans (A.2) together mean that the uncollateralized-to-mortgage spread only depends on $\tau_{l,t}$: $R^l_t - R^b_t = \tau_{l,t}$.

their return on mortgage lending. This is to compensate for the change in dis-utility of holding uncollateralized loans. This is a reduced form way to capture the dynamics of the uncollateralized-to-mortgage spread in a richer model with banks.

This hypothetical richer model would contain two financial institutions: a commercial bank and a mortgage credit institution. The commercial bank in this setup could be modeled after the financial sector in Eggertsson et al. (2019), Ulate (2021), or Abadi et al. (2022). Meaning that the commercial bank would have to face a lower bound or friction on the interest rate they pay on deposits and a capital or other regulatory constraint. As shown in these papers, this generates an attenuation or reversal in the pass-through of monetary policy to the interest rate on bank loans (the bank net-worth channel). The model would also have to include a mortgage credit institution that does not face a friction on the interest rate they pay on liabilities. The dynamics generated in such a model by a monetary policy rate cut in negative territory would generate upward pressure on the uncollateralized-to-mortgage spread.

4.1.2 Borrowers' Problem

Borrowers are relatively impatient (discount factor: $\hat{\beta}_t < \tilde{\beta}_t$), they receive loans directly from savers, consume, and purchase housing. Their problem is:

$$\max_{\{\hat{c}_t, l_t, b_t, \hat{h}_t\}} \quad E_0 \sum_{t=0}^{\infty} (\hat{\beta}_t)^t \Big[\hat{u}(\hat{c}_t) + \hat{v}(\hat{h}_t)\Big], \tag{4.4}$$

subject to their budget constraint:

$$\hat{c}_t + R^l_{t-1} l_{t-1} + R^b_{t-1} b_{t-1} + p_{h,t}\hat{h}_t = l_t + b_t + p_{h,t}\hat{h}_{t-1} + \hat{y}, \tag{4.5}$$

a collateral constraint on mortgage loans:

$$b_t \leq m_b \frac{E_t p_{h,t+1} \hat{h}_t}{R^b_t}, \qquad (\hat{\mu}_{b,t} \geq 0) \tag{4.6}$$

and an overall borrowing limit:

$$l_t + b_t \leq m_y \hat{y}, \qquad (\hat{\mu}_{y,t} \geq 0) \tag{4.7}$$

Borrower specific notation is denoted with hats: $\hat{c}_t$ denotes consumption of non-durable goods, $\hat{h}_t$ denotes borrower housing, and $\hat{y}$ the borrower's per period endowment. m_b is the exogenous collateral value of housing and $p_{h,t}$ is the price of housing. m_y captures the extent that borrowers can borrow against their endowment income. l_t are uncollateralized loans (with interest rate R_t^l) and b_t are mortgage loans (with interest rate R_t^b).

4.2 Exogenous Shocks & Market Clearing

Housing is in fixed supply and held entirely by borrowers:

$$\hat{h}_t = H \tag{4.8}$$

The resource constraint is:

$$\hat{c}_t + \tilde{c}_t = \hat{y} + \tilde{y} \tag{4.9}$$

The shock process for the dis-utility cost to savers of holding uncollateralized loans evolves according to:

$$\tau_{l,t} = \tau_l + \epsilon_{\tau_l,t}, \tag{4.10}$$

where τ_l is calibrated to match the target spread between the interest rate on uncollateralized lending and mortgage lending ($R^l - R^b$) in steady state. The shock process for the savers' time preference evolves according to:

$$\tilde{\beta}_t = \tilde{\beta} + \epsilon_{\tilde{\beta},t}. \tag{4.11}$$

4.3 Functional Forms

Savers have linear utility:

$$\tilde{u}(\tilde{c}_t) = \tilde{c}_t \tag{4.12}$$

Borrowers have log utility in consumption and housing:

$$\hat{u}(\hat{c}_t) = \log(\hat{c}_t) \tag{4.13}$$

$$\hat{v}(\hat{h}_t) = \log(\hat{h}_t) \tag{4.14}$$

4.4 House Pricing Equation

Appendix B derives the borrowers' pricing equation[6] for housing from the borrowers' optimality conditions for housing, uncollateralized loans and mortgage loans (eq A.8,A.7,A.6):

$$p_{h,t} = j\hat{c}_t + j\sum_{i=0}^{\infty}\hat{c}_{t+i+1}\left\{\Pi_{k=0}^{i}\left[\frac{\hat{c}_{t+k}}{\hat{c}_{t+k+1}}\hat{\beta}\left(1 + m_b\frac{R^l_{t+k} - R^b_{t+k}}{R^b_{t+k}}\right)\right]\right\} \tag{4.15}$$

Other things equal, this expression makes it clear that house prices increase when the spread between the interest rates on uncollateralized debt and mortgage debt $(R^l_{t+k} - R^b_{t+k})$ increases. An increase in this spread decreases the relative price of mortgage debt which incentivizes borrowers to substitute from uncollateralized debt to mortgage debt. In order to access mortgage debt households need collateral (i.e. housing) so an increase in the demand for mortgage debt also drives house prices up because its role as collateral is now more valuable. This is the *debt substitution channel*.

This debt substitution behavior is supported by the empirical literature, for example Pennington-Cross and Chomsisengphet (2010) find that subprime households in the United States are more likely to refinance their mortgages or take up other types of debt collateralized by housing when uncollateralized forms of debt become relatively more expensive.

The house pricing equation also indicates that (keeping the spread constant) house prices are inversely related to the level of the mortgage rate. Intuitively: borrowers demand more (less) housing when the level of the interest rate on mortgage credit is lower (higher).

Shifts in consumption also impact house prices. Households desire a balanced basket between consumption of housing and consumption of non-durable goods ($\hat{c}$). Therefore as consumption of non-durable goods go up housing demand also goes up, pushing up house prices. It is clear to see in equation (4.15) that the level of consumption ($\hat{c}_{t+i+1}$) enters

[6] Assuming perfect foresight and that housing consumption is normalized to unity $\forall t$.

positively. Additionally house prices are inversely related to the growth rate of consumption (the inverse growth rate $(\hat{c}_{t+k}/\hat{c}_{t+k+1})$ enters positively into equation 4.15).

5 Parameter Values

Table 3: Calibrated Parameters

Parameter	Value	Description
$\tilde{\beta}$	0.9943	Saver's discount factor
$\hat{\beta}$	0.97	Borrower's discount factor
H	1	Total inelastic supply of housing
$\hat{y}$	0.5	Borrower Endowment
$\tilde{y}$	0.5	Saver Endowment

The parameter values here represent an initial calibration. The level and distance between the saver and borrower discount factors is similar to other saver-borrower models (e.g. Iacoviello and Neri (2010)). Housing supply is normalized to 1. The borrowers and savers are assumed to have an equal share of the per-period endowment, which is also normalized to 1.

The calibration period is from 2003-2006[7]. The uncollateralized-to-mortgage spread target $(R^l_t - R^b_t)$ is set to target the annualized spread in the interest rate between bank loans unrelated housing and mortgage loans[8] (see figure 1). The loan-to-value target matches the maximum loan-to-value ratio on mortgages made by Danish mortgage credit institutions.

Using the Danish microdata borrower's are defined as those individuals with liquid assets[9] less than or equal to 1 month of their annual salary.The ratio of mortgage debt to annual GDP is calibrated to match the total stock of outstanding mortgage debt of borrowers'. The ratio of bank debt to mortgage debt is calibrated to match the outstanding stock of non-housing related bank loans to mortgage loans, again only for those individuals defined as borrowers based on liquid assets.

[7]The start date is based the availability of bank level and microdata, and the end in 2006 is intended to target a period before the start of the Global Financial Crisis.

[8]Mortgage loans are calculated inclusive of administration fees ("Bidrag" in Danish).

[9]Defined as deposits in credit institutions.

Table 4: Steady State Targets

Target	Value	Description
$R^l - R^b$	233	Spread between uncollateralized and mortgage debt (annual basis points)
$B/(p_h H)$	80%	loan-to-value ratio
$B/(4(\hat{y}+\tilde{y}))$	75.32%	ratio of mortgage debt to annual GDP
L/B	17.71%	ratio of bank debt to mortgage debt

Note: The parameters τ_l, m_b, j, and m_y are calibrated to match these targets..

6 Simulation Results & Discussion

To review, the credit conditions faced by borrowers can be characterized by three different model objects: i) the interest rate on uncollateralized lending (R_t^l), ii) the interest rate on mortgage lending (R_t^b), and iii) the uncollateralized-to-mortgage spread ($R_t^l - R_t^b$). The simulations presented here explore how changes in this spread impact house prices and mortgage credit.

I find that an increase in the uncollateralized-to-mortgage spread (keeping the mortgage rate constant) drives households to substitute towards mortgage credit. This substitution pushes house prices up but borrower consumption down. This is the *debt substitution channel*. The size of this effect on house prices is increasing in the expected persistence of the shock that drives the spread up.

6.1 Stylized Negative Interest Rate Shock

The first set of simulations (figure 4) compares three different shocks to credit conditions faced by borrower households. They are i) a 50 basis point drop in the mortgage rate + the mortgage-to-uncollateralized spread ($R_t^l - R_t^b$) increases by 50 basis points (black solid line), ii) a 50 basis point drop in both lending rates + spread unchanged (dashed blue line), and iii) a 50 basis point increase in the spread (mortgage rate unchanged, uncollateralized lending rate increases by 50 basis points, red dotted line).

The first simulation (a 50 basis point drop in the mortgage rate + spread increases by 50 basis points, black solid line) captures in reduced form the dynamics of the hypothetical

Figure 4: Easing Shocks, 10 quarters

Note: The shocks last 10 periods, on the realization of the first shock the agents also realize news about the future shock process."mortgage rate drops, spread increases" (sold black line) is a 50 basis point drop in the mortgage rate, keeping the interest rate on uncollateralized bank lending constant. "mortgage rate drops, spread unchanged" (dashed blue line) is a 50 basis point drop in both the mortgage rate and interest rate on uncollateralized bank lending. "mortgage rate unchanged, spread increases" (dotted red line) keeps the mortgage rate unchanged, with a 50 basis point increase in the interest rate on uncollateralized bank lending.

richer model (discussed in section 4.1.1) in response to a negative monetary policy shock when the policy rate is already negative. Under such a shock mortgage rates should follow the policy rate down (because mortgage credit institutions are not subject to the adverse impacts of negative rates). However, the pass-through of monetary policy to bank lending rates is inhibited due to the net margin channel.

The second and third simulations decompose this shock. The second simulation (Mortgage Rate Drop + Spread Unchanged, dashed blue line), captures purely the effect of lower lending rates with the spread unchanged. This captures in reduced form a negative monetary policy shock when the policy rate is above zero, so the policy rate cut is transmitted equally to both lending rates. The third simulation (Mortgage Rate Unchanged + Spread

Increase, red dotted line), purely captures the effect of an increase in the spread between uncollateralized debt and mortgage debt: the debt substitution channel.

All three shocks drive house prices up, however only the first two shocks loosen credit conditions for borrowers. The third shock actually tightens credit conditions. Due to the relative increase in the price of uncollateralized debt, borrowers substitute toward mortgage debt. This increases the house price, because housing is now more valuable as a form of collateral for the now relatively cheaper type of debt. This is the *debt substitution channel*: any shock that increases the uncollateralized-to-mortgage spread increases borrower demand for housing and pushes house prices up. When monetary policy rate cuts below zero increase the uncollateralized-to-mortgage spread, the transmission of monetary policy to house prices is amplified.

6.1.1 Mortgage Rate Drop, Spread Unchanged

The path of consumption drives the dynamics of the house price under the mortgage rate shock (keeping the spread constant - blue dashed line). This shock eases the cost of both types of debt, giving borrowers the incentive to move consumption forward in time. This applies to both non-durable consumption ($\hat{c}_t$) and consumption of housing ($\hat{h}_t$). Non-durable consumption jumps in the period following the shock, stays at a higher level, and then returns close to steady state when the shock series subsides. As is clear from the house pricing equation (section 4.4, eq 4.15) the level of non-durable consumption, and the inverse of the growth in non-durable consumption ($\hat{c}_{t+k}/\hat{c}_{t+k+1}$) both effect the house price.

On impact of the shock (and realization of future shocks) there are two counteracting effects: i) R_t^b has dropped (spread unchanged) which pushes the house price up, ii) non-durable consumption is expected to increase in the next period, meaning there is an on impact decline in the inverse growth rate of non-durable consumption which has a negative impact on the house price. In the following periods (while the shock persists) the growth rate of non-durable consumption is approximately zero so the effect of the drop in the mortgage rate wins out and house prices experience a level shift up. When the mortgage rate reverts back to its old (higher) level there is an expected drop in non-durable consumption, so these effects again balance each other out and the drop in house prices (driven by the inverse of consumption growth) lags the end of the shock series by one period.

6.1.2 Mortgage Rate Unchanged, Spread Increase

The debt substitution effect in isolation (i.e. an increase in the uncollateralized lending rate keeping the mortgage rate fixed - the red dashed line) has the following dynamics. House prices and mortgage debt increase on impact. The effect of the shock on house prices and mortgage debt decays over time, and in the last period of the shock the house price actually drops below the steady state level.

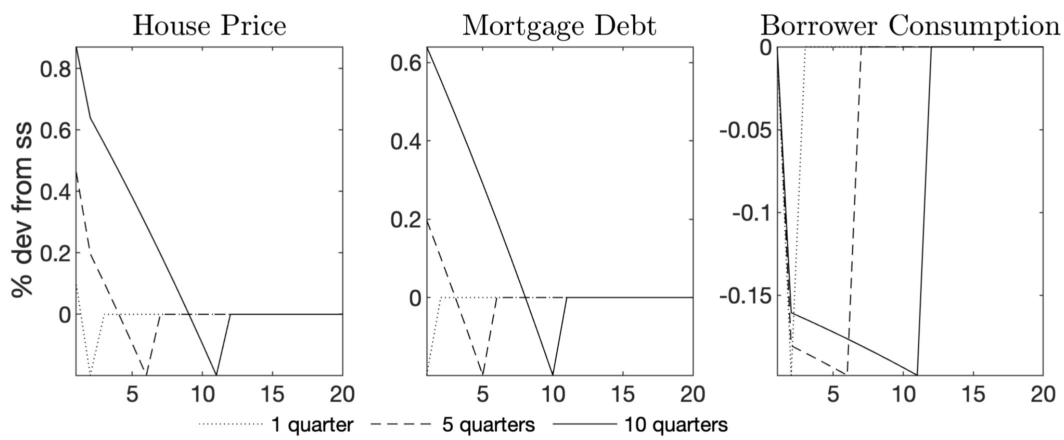
The driver of the on-impact jump in house prices is clear from the house pricing equation (eq 4.15 in section 4.4). This expression shows that house prices depend on the sum of future spreads (between the uncollateralized debt and mortgage debt). Intuitively a shock that drives the spread up for a number of periods will cause the house price to jump on impact (when the initial shock and announcement of future shocks are realized). The effect on the house price will slowly decline as the sum of future shocks declines.

This shock tightens credit conditions for borrowers: now the cost of bringing consumption forward in time has increased, so it has a negative impact on non-durable consumption. The drop in consumption somewhat mutes the impact on house prices because borrowers want to consume a balanced basket of non-durable consumption and housing (see section 4.4 for further discussion). The final drop in house prices is driven by this negative consumption effect dominating. The next section will show that this negative consumption effect becomes more dominant if the persistence of the shock to the uncollateralized-to-mortgage spread is less.

6.2 Shock Persistence Amplifies the Debt Substitution Channel's Impact on House Prices & Mortgage Debt

Figure 5 plots the "mortgage rate unchanged, spread increases" simulation (in the red dotted line in figure 4 above) but with varying shock persistence. Comparing the simulations in figure 5 it is clear to see that the shock persistence matters highly for the quantitative importance of the debt substitution channel. The shorter the shock persistence, the more the effect of the drop in consumption due to credit tightening dominates the debt substitution effect. In the 1-period shock the negative consumption effect is so dominate that mortgage debt actually declines and house prices only slightly increase on impact and then drop. In the 5 and 10 period simulations this effect does not dominate (house prices and mortgage debt increase) but following the end of the shock series, house prices and mortgage credit drop before converging towards steady state (due to the negative consumption effect).

Figure 5: Easing Shocks – Expected Persistence Matters



Note: This figure shows the "mortgage rate unchanged, spread increases" shocks: the mortgage rate is unchanged with a 50 basis point increase in the interest rate on uncollateralized bank lending. There are three simulations, with the shocks lasting 1, 5, and 10 periods respectively. The 1-period shock is purely unexpected. For the 5 and 10 period shock simulations the agents also realize news about the future shock process on realization of the first shock .

The impact of persistence has relevant policy implications. The existing literature already tells us that the bank net-worth channel of monetary policy creeps up over time: Eggertsson et al. (2019), Abadi et al. (2022), and Ulate (2021). Specifically these papers tell us that negative interest rates erode bank profits slowly, and as these losses accumulate over time there is an attenuation (or even reversal) of the pass-through of monetary policy rate cuts to bank lending interest rates (i.e. the uncollateralized interest rate in this model). The spread shocks here capture this effect in reduced form.

In addition, the persistence of the shock that increases the uncollateralized-to-mortgage spread amplifies the increase in house prices and and mortgage debt. The debt substitution channel here introduces a new unintended consequence of negative nominal interest rate policy: a housing boom with increased household leverage. Households' expectations matter a lot. Forward guidance when policy rates are negative will further amplify the housing market boom via the debt substitution channel.

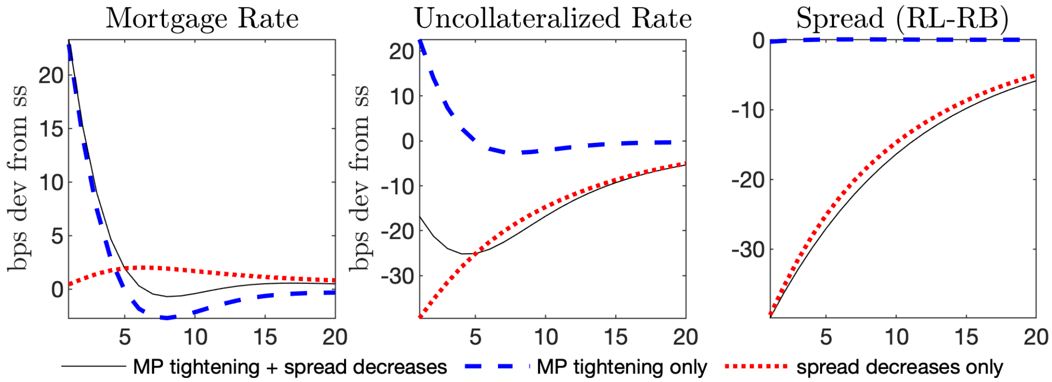
7 Model with New Keynesian Frictions

In order to examine the impact of the debt substitution channel on inflation I embed the stylized model into a simple New Keynesian DSGE framework. Appendix C outlines this model. The following section compares a monetary policy shock with and without an exogenous change in the uncollateralized-to-mortgage spread. I focus on the case of monetary policy tightening. When there is a monetary policy tightening I move the spread down, to mimic monetary policy liftoff from a very low or negative nominal policy rate.

Monetary policy hikes from very low or negative levels allow banks to increase their reserve-to-deposit spread. Drechsler et al. (2017) find that when the Federal Funds Rate rises banks allow the spread between the Federal Funds Rate and their deposit rates to rise. In the context of negative nominal interest rate policy, when the reserve-to-deposit spread has been negative, allowing this spread to rise would move it closer to or above zero. This would have a normalizing effect, meaning that deposits would start to contribute positively to bank profits again, and as bank profits rise banks would move further away from their capital constraints. This would alleviate the bank net-worth channel, allowing bank lending spreads to fall.

Following this normalization, monetary policy pass-through via the traditional bank lending channel should resume (so the interest rate on bank loans would rise with monetary policy rate cuts). However, this would be somewhat counteracted by the fall in bank lending spreads (putting downward pressure on the interest rate on bank loans). In contrast the pass-through of interest rates on mortgage loans would not change. So there is potential for the uncollateralized-to-mortgage spread to *fall* as monetary policy rates rise.

Figure 6: Monetary Policy Tightening 1



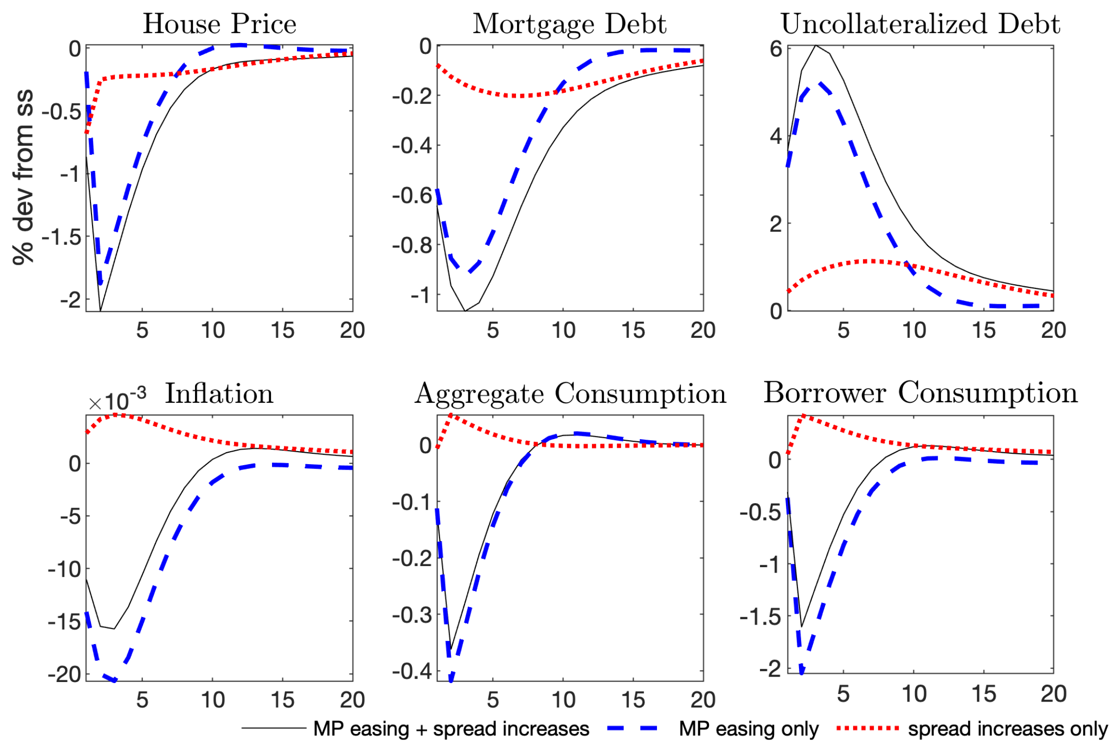
Note: Here the monetary policy tightening shocks are one-off. The shocks to the uncollateralized-to-mortgage spread ($\tau_{l,t}$) follows an AR(1) process. "monetary policy tightening + spread decreases" (sold black line) is a one-off positive monetary policy shock plus a negative shock to the uncollateralized-to-mortgage spread. "monetary policy tightening only" (dashed blue line) is just a one-off monetary policy shock. "spread decreases only" (dotted red line) is just a negative shock to the uncollateralized-to-mortgage spread.

7.1 The Debt Substitution Channel Moderates the Effect of Monetary Policy on Inflation

In this simulation (figures 6 and 7) there is a monetary policy hike. The blue dashed line reflects the simulation where the monetary policy hike is equally passed through to both interest rates. In the solid black line, the monetary policy hike is passed through to the mortgage rate, but is not equally passed through to the interest rate on uncollateralized bank loans. In the scenario where the policy rate is lifting off from very low or negative levels this would reflect the fact that the increase in the policy rate allows banks to increase their net interest margin by maintaining deposit rates at the same level. In this case while mortgage rates rise with the policy rate, uncollateralized bank rates would rise by less.

Figure 6 shows the path of the two interest rates and the spread between the uncollateralized rate and the mortgage rate. Figure 7 illustrates the response of house prices, borrowing, inflation, and consumption. In figure 7 under the monetary policy tightening plus spread decline scenario monetary policy tightening is less effective at fighting inflation compared to the monetary policy tightening simulation where monetary policy is equally passed through to both interest rates. This is because of the *debt substitution effect*: the decline in the spread drives borrowers to substitute *towards* uncollateralized borrowing. They demand less

Figure 7: Monetary Policy Tightening 2



Note: Here the monetary policy tightening shocks are one-off. The shocks to the uncollateralized-to-mortgage spread ($\tau_{l,t}$) follows an AR(1) process. "monetary policy tightening + spread decreases" (sold black line) is a one-off positive monetary policy shock plus a negative shock to the uncollateralized-to-mortgage spread. "monetary policy tightening only" (dashed blue line) is just a one-off monetary policy shock. "spread decreases only" (dotted red line) is just a negative shock to the uncollateralized-to-mortgage spread.

housing, and the marginal effect of the debt substitution channel is to push up consumption. This increase in borrower consumption leads to inflation falling by less than it otherwise would.

8 Model with Banks

This section endogenizes the uncollateralized-to-mortgage spread by explicitly modeling commercial banks which face financial frictions. Figure 8 provides an overview of the baseline model with banks (appendix D outlines this model). In the simulations below the effect of a monetary policy cut below zero in this model is compared to an alternative model where the debt substitution channel is switched off (only mortgage lending exists and all lending is intermediated via commercial banks).

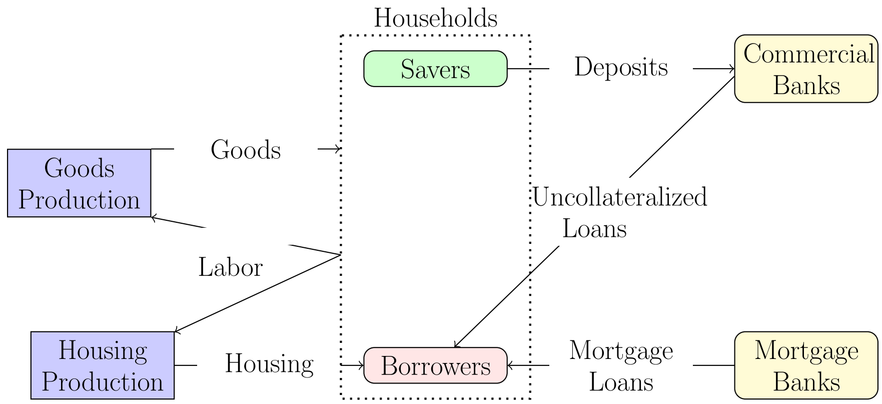


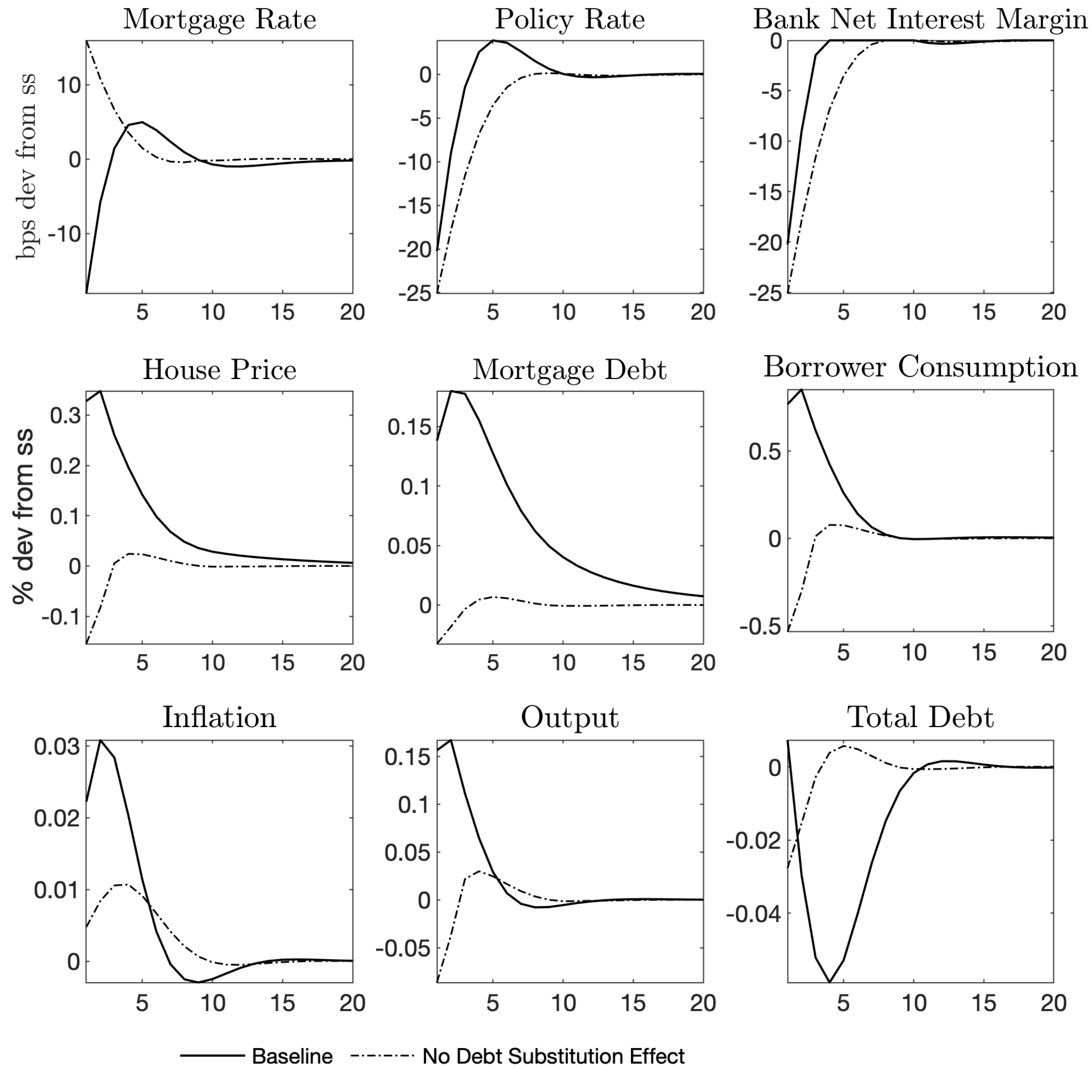
Figure 8: Baseline - Banks Model

Figure 9 shows the response of a one-off easing monetary policy shock, in the baseline banks model ("Baseline") vs al alternative model where all loans are related to housing and intermediated via banks ("No Debt Substitution Effect"). In both models the commercial banking sector faces an erosion of their net interest margin, pushing them against leverage limits and driving an increase in the spread on commercial bank loans. This generates reversal rate conditions in the commercial banking sector, as discussed in Eggertsson et al. (2019) and Abadi et al. (2022).

In the baseline model however mortgage lending is outside of the commercial banking sector so monetary policy easing loosens credit conditions in the mortgage sector while the adverse impact on commercial banks tightens the alternative sources of household credit (uncollateralized credit). Conventional monetary policy still stimulates output and consumption, even though commercial banks face reversal rate conditions.

In the "No Debt Substitution Effect" model all credit is intermediated via commercial banks, and it is very clear that the impact of conventional monetary policy is substantially reduced.

Figure 9: Monetary Policy Cut Below Zero



9 Conclusion

In this paper I show that substitution from uncollateralized debt to mortgage debt driven by negative interest rate policy (NIRP) amplifies the impact of expansionary monetary policy on house prices but mutes the ability of monetary policy to stimulate consumption. This is the *debt substitution channel*. Furthermore the quantitative impact of this channel is increasing in the expected duration of NIRP policy. I have also shown that this channel is symmetric: monetary policy hikes from very low or negative levels have a smaller impact on inflation in the presence of the debt substitution channel vs without it (when monetary policy is equally passed-through to all lending rates).

I have focused on negative nominal interest rate policy as a driver of the debt substitution effect. However this channel could be triggered by any shock that drives the spread between uncollateralized debt and mortgage debt up. In financial systems where uncollateralized debt is primarily provided by traditional commercial banks and mortgage debt is provided by other credit institutions, then any shock that asymmetrically impacts the profitability of commercial banks will also trigger the debt substitution channel. For example a change in market power in the commercial banking sector, a change in regulation of commercial banks, or a release of the capital buffer could also trigger the debt substitution channel. This would be an interesting direction for future study.

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A Model Equations

Savers' FoC wrt housing collateralized loans:

$$\tilde{u}_{c,t} = \tilde{\beta}_t E_t \big[\tilde{u}_{c,t+1} R_t^b \big] \tag{A.1}$$

Savers' FoC wrt uncollateralized loans:

$$\tilde{u}_{c,t} + \tilde{\beta}_t \tau_{l,t} = \tilde{\beta}_t E_t \big[\tilde{u}_{c,t+1} R_t^l \big] \tag{A.2}$$

Borrowers' budget constraint:

$$\hat{c}_t + R_{t-1}^l l_{t-1} + R_{t-1}^b b_{t-1} + p_{h,t} \hat{h}_t = l_t + b_t + p_{h,t} \hat{h}_{t-1} + \hat{y}. \tag{A.3}$$

Borrowers' collateral constraint on housing collateralized loans:

$$b_t \leq m_b \frac{E_t p_{h,t+1} \hat{h}_t}{R_t^b}, \qquad (\hat{\mu}_{b,t} \geq 0). \tag{A.4}$$

Borrowers' total borrowing limit:

$$l_t + b_t \leq m_y \hat{y}, \qquad (\hat{\mu}_{y,t} \geq 0). \tag{A.5}$$

Borrowers' FoC wrt housing collateralized loans credit:

$$\hat{u}_{c,t} - \hat{\beta} E_t \big[\hat{u}_{c,t+1} R_t^b \big] = \hat{\mu}_{b,t} + \hat{\mu}_{y,t}. \tag{A.6}$$

Borrowers' FoC wrt uncollateralized loans:

$$\hat{u}_{c,t} - \hat{\beta} E_t \big[\hat{u}_{c,t+1} R_t^l \big] = \hat{\mu}_{y,t}. \tag{A.7}$$

Borrowers' FoC wrt housing:

$$\frac{j}{\hat{h}_t} - \hat{u}_{c,t} p_{h,t} + \hat{\beta} E_t \big[\hat{u}_{c,t+1} p_{h,t+1} \big] + \frac{m_b E_t p_{h,t+1}}{R_t^b} \hat{\mu}_{b,t} = 0 \tag{A.8}$$

Resource Constraint:

$$\hat{c}_t + \tilde{c}_t = \hat{y} + \tilde{y}. \tag{A.9}$$

Housing market clearing:

$$\hat{h}_t = H \tag{A.10}$$

Shock to savers' dis-utility for uncollateralized lending:

$$\log\left(\frac{\tau_{l,t}}{\tau_l}\right) = \rho_{\tau_l} \log\left(\frac{\tau_{l,t}}{\tau_l}\right) + \epsilon_{\tau_l,t}, \tag{A.11}$$

Shock to savers' time preference:

$$\log\left(\frac{\tilde{\beta}_t}{\tilde{\beta}}\right) = \rho_{\tilde{\beta}} \log\left(\frac{\tilde{\beta}_{t-1}}{\tilde{\beta}}\right) + \epsilon_{\tilde{\beta},t}, \tag{A.12}$$

B Borrowers' House Pricing Equation

Assuming perfect foresight, log utility at that $\hat{h}_t = 1 \forall t$ the borrower's first order condition w.r.t. housing can be written as:

$$\begin{aligned} p_{h,t} &= \hat{c}_t\left[j + p_{h,t+1}\left(\frac{\hat{\beta}}{\hat{c}_{t+1}} + \frac{m_b}{R_t^b}\hat{\mu}_{b,t}\right)\right] && \text{(B.1)} \\ &= j\hat{c}_t + p_{h,t+1}\underbrace{\left(\hat{\beta}\frac{\hat{c}_t}{\hat{c}_{t+1}} + \frac{m_b}{R_t^b}\hat{\mu}_{b,t}\hat{c}_t\right)}_{\equiv\Psi_t} && \text{(B.2)} \\ &= j\hat{c}_t + p_{h,t+1}\Psi_t && \text{(B.3)} \end{aligned}$$

Iterate this expression forward:

$$\begin{aligned} p_{h,t} &= j\hat{c}_t + \Psi_t\left[j\hat{c}_{t+1} + \Psi_{t+1}p_{h,t+2}\right] && \text{(B.4)} \\ &= j\hat{c}_t + j\hat{c}_{t+1}\Psi_t + \Psi_t\Psi_{t+1}p_{h,t+2} && \text{(B.5)} \\ &= j\hat{c}_t + j\hat{c}_{t+1}\Psi_t + j\hat{c}_{t+2}\Psi_t\Psi_{t+1} + \Psi_t\Psi_{t+1}\Psi_{t+2}p_{h,t+3} && \text{(B.6)} \\ &= j\hat{c}_t + j\sum_{i=0}^{\infty}\hat{c}_{t+i+1}\left\{\Pi_{k=0}^{i}\Psi_{t+k}\right\} && \text{(B.7)} \end{aligned}$$

Substituting the expression for Ψ_t back in:

$$p_{h,t} = j\hat{c}_t + j\sum_{i=0}^{\infty}\hat{c}_{t+i+1}\Big\{\Pi_{k=0}^{i}\Big[\hat{c}_{t+k}\Big(\frac{\hat{\beta}}{\hat{c}_{t+k+1}} + \frac{m_b}{R^b_{t+k}}\hat{\mu}_{b,t+k}\Big)\Big]\Big\} \tag{B.8}$$

Subtracting the borrowers optimality condition for uncollateralized debt from the optimality condition for mortgage debt (A.6-A.7):

$$\hat{\mu}_{b,t+k} = \frac{\hat{\beta}(R^l_{t+k} - R^b_{t+k})}{\hat{c}_{t+k+1}} \tag{B.9}$$

Plugging this into the house pricing expression results in:

$$p_{h,t} = j\hat{c}_t + j\sum_{i=0}^{\infty}\hat{c}_{t+i+1}\Big\{\Pi_{k=0}^{i}\Big[\frac{\hat{c}_{t+k}}{\hat{c}_{t+k+1}}\hat{\beta}\Big(1 + m_b\frac{R^l_{t+k} - R^b_{t+k}}{R^b_{t+k}}\Big)\Big]\Big\} \tag{B.10}$$

C Stylized Model with New Keynesian Frictions

This appendix outlines the model described in section 7. This is essentially the stylized model described in section 4 embedded in a DSGE framework (i.e. Christiano, Eichenbaum and Evans (2005)).

C.1 Model

C.1.1 Households

Savers' Problem

$$\max_{\{\tilde{c}_t, l_t, b_t, \tilde{n}_{ict}, \tilde{n}_{iht}\}} E_0 \sum_{t=0}^{\infty} (\tilde{\beta})^t \left\{ \Gamma_c \log(\tilde{c}_t - \epsilon \tilde{c}_{t-1}) - \tilde{v}(l_t) - \tilde{\psi}_L \int_0^1 \frac{\tilde{n}_{ict}^{1+\sigma_L}}{1+\sigma_L} - \tilde{\psi}_L \int_0^1 \frac{\tilde{n}_{iht}^{1+\sigma_L}}{1+\sigma_L} \right\} \tag{C.1}$$

subject to their budget constraint:

$$\tilde{c}_t + l_t + b_t + p_{l,t} land_t = \frac{R^l_{t-1} l_{t-1}}{\pi_t} + \frac{R^b_{t-1} b_{t-1}}{\pi_t} + \tilde{w}_t \tilde{n}_t + (p_{l,t} + R^{land}_t) land_{t-1} + Div_t. \tag{C.2}$$

Saver specific notation is denoted with tildes: $\tilde{c}_t$ denotes consumption of non-durable goods. Saver's have habits in consumption, where $\Gamma_c \equiv \frac{1-\epsilon}{1-\tilde{\beta}\epsilon}$. $\tilde{\psi}_L$ captures the savers dis-utility of work. σ_L is the Frisch elasticity of labor supply. $\tilde{n}_{ict}$ and $\tilde{n}_{iht}$ are the labor supply of saver labor type i for goods and housing production respectively. b_t are mortgage loans to borrowers (which pay the interest rate R^b_t). Savers' own and rent land to housing producers. $land_t$ is the quantity of land, $p_{l,t}$ is the price of land, and R^{land}_t is the rental rate on land. Div_t is dividends from commercial banks (which are owned by savers).

l_t are uncollateralized loans to borrowers (which pay the interest rate R^l_t). $\tilde{v}(l_t)$ is a utility cost for holding uncollateralized debt. As in the stylized model the functional form is:

$$\tilde{v}(l_t) = \tilde{\beta} \tau_{l,t} l_t. \tag{C.3}$$

Borrowers' Problem

$$\max_{\{\hat{c}_t, l_t, b_t, \hat{h}_t, \hat{n}_t\}} E_0 \sum_{t=0}^{\infty} (\hat{\beta})^t \left\{ \log\left(\hat{c}_t + j \log(\hat{h}_t)\right) - \hat{\psi}_L \int_0^1 \frac{\hat{n}_{ict}^{1+\sigma_L}}{1+\sigma_L} - \hat{\psi}_L \int_0^1 \frac{\hat{n}_{iht}^{1+\sigma_L}}{1+\sigma_L} \right\}, \tag{C.4}$$

subject to their budget constraint:

$$\hat{c}_t + \frac{R^l_{t-1} l_{t-1}}{\pi_t} + \frac{R^b_{t-1} b_{t-1}}{\pi_t} + p_{h,t}\hat{h}_t = l_t + b_t + p_{h,t}(1-\delta_h)\hat{h}_{t-1} + \hat{w}_t \hat{n}_t, \tag{C.5}$$

a collateral constraint on mortgage loans:

$$b_t \leq m_b \frac{E_t p_{h,t+1} \pi_{t+1} \hat{h}_t}{R^b_t}, \qquad (\hat{\mu}_{b,t} \geq 0) \tag{C.6}$$

and an overall borrowing limit:

$$l_t + b_t \leq m_y \hat{w} \hat{n}, \qquad (\hat{\mu}_{y,t} \geq 0) \tag{C.7}$$

Borrower specific notation is denoted with hats: $\hat{c}_t$ denotes consumption of non-durable goods, $\hat{h}_t$ denotes borrower housing. Borrower's have GHH style preferences in housing and consumption. $\hat{\psi}_L$ captures the savers dis-utility of work. σ_L is the Frisch elasticity of labor supply. $\hat{n}_{ict}$ and $\hat{n}_{iht}$ are the labor supply of borrower labor type i for goods and housing production respectively. $\hat{w}_t$ is the wage of the borrower household. m_b is the exogenous collateral value of housing and $p_{h,t}$ is the price of housing. m_y captures the extent that borrowers can borrow against their steady state level of income.

C.1.2 Production

The firms face standard pricing frictions a la Christiano et al. (2005).

Intermediate Goods Intermediate goods $Y_{i,t}$, $j \in \left[0,1\right]$ are produced by monopolistic firms according to the following technology:

$$Y_{jt} = \tilde{n}_{jct}^{\alpha} \hat{n}_{jct}^{1-\alpha} \tag{C.8}$$

where $\alpha \in (0,1)$ is the share of output going to savers. $\tilde{n}_{jct}$ and $\hat{n}_{jct}$ are labor of savers and borrower households used by firm j.

Intermediate goods firms face a Calvo friction in the pricing of their goods. Each period a random fraction $1-\xi_p$ can reoptimize their price P_{jt}. The remaining firms (fraction ξ_p) index their price to their last period's price:

$$P_{j,t} = \tilde{\pi}_t P_{j,t-1}, \tag{C.9}$$

where the inflation indexation term is:

$$\tilde{\pi}_t \equiv (\pi_t^{target})^{\iota} (\pi_{t-1})^{1-\iota}, \tag{C.10}$$

and $\pi_{t-1} \equiv P_{t-1}/P_{t-2}$ is gross inflation. π_t^{target} is the central bank's target inflation rate. ι is the price indexing weight on the inflation target.

Final Goods Firms The homogeneous final good Y_t, is produced by a competitive representative firm (with Dixit-Stiglitz technology):

$$Y_t = \left[\int_0^1 Y_{jt}^{\frac{1}{\lambda_f}} dj \right]^{\lambda_f}, \; 1 \leq \lambda_f < \infty, \; j \in \big[0,1\big]. \tag{C.11}$$

One unit of Y_t can be converted into one unit of the consumption good and thus (given perfect competition in the use of this technology) consumption has the price P_t.

Labor Market There are different labor unions for housing production and goods production[10]. For simplicity the subscripts c, h are suppressed here. Each differentiated saver labor type type $i \in [0,1]$ provides labor services $\tilde{n}_{it}$ and is represented by a monopoly union that sets its wage rate $\tilde{W}_{it}$ while facing a Calvo friction. Each period a fraction $1-\xi_w$ of the monopoly unions can update the wage. The remaining fraction ξ_w set their wage as follows:

$$\tilde{W}_{it} = \pi_{wt}^{index} \tilde{W}_{i,t-1}, \tag{C.12}$$

[10]This differentiation of labor follows the simplifying assumption in Iacoviello and Neri (2010).

where:

$$\pi_{wt}^{index} \equiv \left(\pi_t^{target}\right)^{\iota_w} \left(\pi_{t-1}\right)^{1-\iota_w}, \qquad 0 < \iota_w < 1. \tag{C.13}$$

Labor is aggregated via a Dixit-Stiglitz style aggregator by a competitive and representative labor contractor:

$$\tilde{l}_t = \left[\int_0^1 \left(\tilde{h}_{it}\right)^{\frac{1}{\lambda_w}} di \right]^{\lambda_w}, \qquad 1 \leq \lambda_w. \tag{C.14}$$

The homogeneous saver labor aggregate $\tilde{l}_t$ is sold to intermediate goods producers at the nominal wage $\tilde{W}_t$.

Borrowers are similarly organized into monopoly unions, which also face the same Calvo fraction. Each period a fraction $1 - \xi_w$ of the monopoly unions can update the wage. The remaining fraction ξ_w set their wage as follows:

$$\hat{W}_{it} = \pi_{wt}^{index} \hat{W}_{i,t-1}, \tag{C.15}$$

where the indexation term is the same. The borrower labor aggregate is:

$$\hat{l}_t = \left[\int_0^1 \left(\hat{h}_{it}\right)^{\frac{1}{\lambda_w}} di \right]^{\lambda_w}, \qquad 1 \leq \lambda_w. \tag{C.16}$$

The homogeneous saver labor aggregate $\hat{l}_t$ is sold to intermediate goods producers at the nominal wage $\hat{W}_t$.

Housing Production Housing producers face the following production function:

$$IH_t = (\tilde{n}_{h,t}^{\alpha} \hat{n}_{h,t}^{1-\alpha})^{1-\mu_l} (land_{t-1})^{\mu_l} \tag{C.17}$$

They maximize:

$$p_{h,t} IH_t - \left(\tilde{W}_t \tilde{n}_{h,t} + \hat{W}_t \hat{n}_{h,t} + R_t^{land} l_{t-1} \right), \tag{C.18}$$

where $land_t$ is land, which is in fixed supply.

C.1.3 Monetary Policy

The monetary policy authority operates according to a standard taylor rule:

$$\log\left(\frac{R_t}{R}\right) = \rho_m \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_m)\left[\phi_\pi \log\left(\frac{\pi_t}{\pi^{\text{target}}}\right) + \frac{\phi_y}{4}\log\left(\frac{Y_t}{Y}\right)\right] + u_{m,t} \quad \text{(C.19)}$$

alternative monetary policy rules: Either $GDP_t = Y_t$, or (as in Iacoviello and Neri (2010)) GDP includes housing production: $GDP_t \equiv Y_t + p_h IH_t$.

C.1.4 Exogenous Shocks & Market Clearing

Housing evolves according to:

$$H_t = (1-\delta_h)H_{t-1} + IH_t \quad \text{(C.20)}$$

Housing is held by borrowers (saver's have a fixed un-modeled demand for housing):

$$\hat{h}_t = H_t \quad \text{(C.21)}$$

The resource constraint is:

$$\hat{c}_t + \tilde{c}_t = Y_t \quad \text{(C.22)}$$

The shock process for the dis-utility cost to savers of holding uncollateralized loans evolves according to:

$$\tau_{l,t} = \tau_l + \epsilon_{\tau_l,t}, \quad \text{(C.23)}$$

where τ_l is calibrated to match the target spread between the interest rate on uncollateralized lending and mortgage lending $(R^l - R^b)$ in steady state. The shock process for the monetary policy shock evolves according to:

$$u_{m,t} = u_m + \epsilon_{m,t}. \quad \text{(C.24)}$$

where u_m is the steady state value of the shock (which is zero).

C.2 Parameter Values

The calibration of parameters that also enter the stylized model are as in section 5. The additional parameters from adding New Keynesian frictions are from Christiano et al. (2005) and Christiano et al. (2014). The new steady-state target (the ratio of total housing wealth to annual GDP) is from Iacoviello and Neri (2010).

Table 5: Calibrated Parameters

Parameter	**Value**	**Description**
$\tilde{\beta}$	0.9943	Saver's discount factor
$\hat{\beta}$	0.97	Borrower's discount factor
H	1	Total inelastic supply of housing
ι	0.5	pricing indexing weight on inflation target
π^{target}	1.005	steady-state inflation target
η	1	curvature on disutility of labor
λ_f	1.2	the steady-state mark-up of intermediate goods firms
λ_w	1.05	steady-state markup of labor unions
ξ_p	0.74	Calvo price stickiness
ξ_p	0.74	Calvo wage stickiness
ϕ_π	1.5	Taylor Rule weight on inflation
ϕ_y	0.36	Taylor Rule weight on output
ρ_m	0.81	Taylor Rule inertia
μ_l	0.3	Land share in housing production

Table 6: Steady State Targets

Target	**Value**	**Description**
$R^l - R^b$	233	Spread between uncollateralized and mortgage debt (annual basis points)
$B/(p_h H)$	80%	loan-to-value ratio
$B/(4(\hat{y}+\tilde{y}))$	75.32%	ratio of mortgage debt to annual GDP
L/B	17.71%	ratio of bank debt to mortgage debt
$\frac{\hat{y}}{\hat{y}+\tilde{y}}$	0.25	Fraction of households with liquid assets less than monthly income.

Note: The parameters τ_l, m_b, j, α and m_y are calibrated to match these targets.

D Banks Model with NK Frictions

This appendix outlines the model described in section 8. The model is similar to the model in appendix C, except that now banks intermediate all lending to borrowers. The changes from the model in C are described below.

D.1 Model

D.1.1 Households

Savers' Problem

$$\max_{\{\tilde{c}_t,\tilde{n}_{ict},\tilde{n}_{iht}\}} E_0 \sum_{t=0}^{\infty} (\tilde{\beta})^t \left\{ \Gamma_c \log(\tilde{c}_t - \epsilon \tilde{c}_{t-1}) - \tilde{\psi}_L \int_0^1 \frac{\tilde{n}_{ict}^{1+\sigma_L}}{1+\sigma_L} - \tilde{\psi}_L \int_0^1 \frac{\tilde{n}_{iht}^{1+\sigma_L}}{1+\sigma_L} \right\} \tag{D.1}$$

subject to their budget constraint:

$$\tilde{c}_t + p_{l,t} land_t = \tilde{w}_t \tilde{n}_t + (p_{l,t} + R_t^{land}) land_{t-1} + Div_t. \tag{D.2}$$

Borrowers' Problem The borrowers' problem is identical to the model with an exogenous spread (in appendix C).

D.1.2 Production

The production section is identical to the model with an exogenous spread (appendix C.1.2).

D.1.3 Commercial Banking Sector

Commercial banks face a Gerali et al. (2010) style capital constraint.

The commercial bank's problem is to make loans (l_t), choose the amount of central bank reserves (a_t) to hold, and issue deposits (d_t) to maximize their discounted sum of future profits:

$$\max_{\{l_t,d_t,a_t\}} E_0 \sum_{t=0}^{\infty} \tilde{\Lambda}_{t,t+1} \left[n_{t+1} - \frac{\kappa}{2} \left(\frac{n_t}{l_t} - \nu \right)^2 n_t \right], \tag{D.3}$$

subject to their balance sheet identity:

$$l_t + a_t = n_t + d_t, \tag{D.4}$$

the banker's demand for reserves:

$$a_t = \Xi_t d_t, \tag{D.5}$$

and an individual commercial bank's evolution of net worth:

$$n_{t+1} = \frac{R^l_t l_t}{\pi_{t+1}} + \frac{R_t a_t}{\pi_{t+1}} - \frac{R^d_t d_t}{\pi_{t+1}}. \tag{D.6}$$

Where the second term in D.3 is the cost banks pay for deviating from their target leverage $(1/\nu)$. This captures regulatory constraints faced by commercial banks.

Aggregate commercial banking sector net worth evolves according to:

$$N_t = R^l_{t-1} L_{t-1} + R_{t-1} A_{t-1} - R^d_{t-1} D_{t-1} - \xi_c N_{t-1}, \tag{D.7}$$

where the second term is dividends paid to saver households:

$$Div_t = \xi_c N_{t-1}. \tag{D.8}$$

The lower bound on deposit rates is introduces as in Groot and Haas (2022). The fully non-linear version of the model is used in simulations so that in the simulation of the monetary policy rate cut above zero the following holds for the deposit rate:

$$R^d_t = R_t \tag{D.9}$$

And in the simulations of the policy rate cut below zero the deposit rate is constrained to not fall below its steady state level:

$$R^d_t = \max(R_t, R^d) \tag{D.10}$$

D.1.4 Monetary Policy

The production section is identical to the model with an exogenous spread (appendix C.1.3).

D.1.5 Exogenous Shocks & Market Clearing

This is identical to the model with an exogenous spread (appendix C.1.4).

D.2 Parameter Values - New to Banks Block

κ is set equal to 11, to match its value in Gerali et al. (2010). ν is calibrate to match jointly the commercial bank's spread $R^l - R$ and target commercial bank leverage L/N in steady state. These targets are respectively 233 annualized basis points and 5.8[11].

E Additional Regression Results

This appendix reports the results of a regression of bank level lending rates on changes in the monetary policy rate, including lags.

$$\Delta i^b_{i,t} = \alpha + \eta I^{negative}_t + \sum_{k=0}^{3} \beta_k \Delta i^r_{t-1} + \sum_{k=0}^{3} \gamma_k \Delta i^r_{t-1} \times I^{negative}_t + \delta_i + \epsilon_{i,t}, \tag{E.1}$$

[11]This matches commercial bank leverage, adjusted for the fraction of assets that are household loans on commercial banks balance sheets, over the calibration period 2003-06.

Table 7: Lending Rates, Comparison of Pass-Through, With Lags

	Bank Loans	Housing Related Bank Loans	Mortgage Loans
Δi^r_t	0.229***	0.287***	0.027
	(0.02)	(0.02)	(0.02)
Δi^r_{t-1}	0.314***	0.335***	0.083***
	(0.02)	(0.02)	(0.02)
Δi^r_{t-2}	-0.006	0.054*	0.007
	(0.02)	(0.03)	(0.01)
Δi^r_{t-3}	0.125***		0.098***
	(0.02)		(0.01)
$I^{negative}_t$=1 $\times$ Δi^r_t	-0.217***	-0.238***	0.117***
	(0.07)	(0.06)	(0.03)
$I^{negative}_t$=1 $\times$ Δi^r_{t-1}	-0.134***	-0.105***	-0.048*
	(0.04)	(0.04)	(0.02)
$I^{negative}_t$=1 $\times$ Δi^r_{t-2}	-0.182***	-0.170***	0.040***
	(0.05)	(0.05)	(0.01)
$I^{negative}_t$=1 $\times$ Δi^r_{t-3}		-0.001	-0.012
		(0.04)	(0.04)
$I^{negative}_t$=1	-0.031***	-0.019***	0.008**
	(0.00)	(0.01)	(0.00)
Constant	0.008***	-0.002	-0.019***
	(0.00)	(0.00)	(0.00)
Bank FE	Yes	Yes	Yes
F statistic	784.84	994.74	.
Observations	4,014	3,780	1,850

Note: ***p < .01, **p < .05, *p < .1 . "Bank loans" are loans from banks to households, unrelated to housing. "Housing related bank loans" are loans from banks to households that are related to housing. "Mortgage Loans" are loans collateralized by housing from mortgage credit institutions to households. The interest rate on each lending type is the average interest rate in the stock of loans of that type. The bank level rates are weighted by volume of lending.

F Regression Tables - Consistent Interest Rate Series

This appendix reports the regression results of the empirical model in equation (3.3). For the regression on bank loan interest rates the loan characteristic controls are: loan maturity, indicator for borrower being resident in Denmark, an indicator of the loan being in arrears in the current year, an indicator of wider debt stress, and indicator of the loan being housing related[12]. For the regression on mortgage interest rates the loan characteristic controls are:

[12]This would be a loan that is potentially collateralized by housing but provided by a traditional bank, not a mortgage credit institution. This primarily characterizes 2nd lien or "top-up" loans that make up no more that 15% of the value of the house.

an indicator of being within the interest-only period (in an IO loan), an indicator for fixed rate mortgages, an indicator for adjustable rate mortgages, an indicator for adjustable rate with interest rate ceiling, the loan to value, the log of the value of the loan at origination, the log of the amount disbursed at origination, the debt-to-income ratio at origination, an indicator of the loan carrying a public guarantee, an indicator of whether or not the loan has an interest rate ceiling, an indicator of the loan being in arrears in the current year.

Borrower characteristics (used both in the regressions on bank loan interest rates and mortgage loan interest rates) are: an indicator for 1-person households, the age of the first two individuals in the household, the number of children in the household, the kommune (municipality) of the household, the total wage income of the household, indicators for unemployment of more than 3 months in the past year and more than 6 months in the past two years. The total debt of the household, the total value of all properties owned by the household, the total mortgage debt of the household, and the total bank debt of the household.

Dependent variables: the interest rates used are the contractual interest rates reported in the register data, not imputed rates. The regression results reported here and in the main text are based on new lending (loans with a new loan number). This will also capture existing mortgages that are refinanced without equity extraction to update the interest rate.

Table 8: Consistent Mortgage Rate Regression

	(1)
singlehh	0.000
	(.)
Age Person 1	-0.004***
	(0.00)
Age Person 2	-0.002***
	(0.00)
Number of Children	0.006***
	(0.00)
Municipality	-0.000***
	(0.00)
Total Wage Income	-0.000***
	(0.00)
Total debt	0.000***
	(0.00)
Total housing wealth	-0.000***
	(0.00)
Total mortgage debt	0.000***
	(0.00)
Total bank debt	-0.000*
	(0.00)
In interest only period	0.360***
	(0.00)
FRM	1.395***
	(0.01)
ARM with ceiling	-1.045***
	(0.01)
ARM	-0.259***
	(0.01)
LTV	0.004***
	(0.00)
log mortgage value at origination	-0.190***
	(0.00)
log disbursement at origination	0.086***
	(0.01)
DTI at origination	0.000*
	(0.00)
Public Guarantee	0.000
	(.)
Rate Ceiling	1.102***
	(0.01)
In arrears	0.134***
	(0.03)
Constant	4.104***
	(0.07)
F statistic	69,154
Observations	1251192.00

Note: ***p $<$ 0.01, **p $<$ 0.05, *p $<$ 0.1.

Table 9: Consistent Bank Loan Interest Rate Regression

	(1)
Single Household	0.000
	(.)
Age Person 1	-0.012***
	(0.00)
Age Person 2	-0.005***
	(0.00)
Number of Children	-0.192***
	(0.01)
Municipality	-0.000***
	(0.00)
Total Wage Income	-0.000***
	(0.00)
Total debt	0.000**
	(0.00)
Total housing wealth	-0.000
	(0.00)
Total mortgage debt	-0.000***
	(0.00)
Total bank debt	-0.000**
	(0.00)
Maturity	-0.105***
	(0.00)
DK resident	0.486***
	(0.10)
In arrears	6.842***
	(0.28)
Debt Stress	0.938***
	(0.27)
Housing Related bank debt	0.852***
	(0.04)
Constant	5.647***
	(0.11)
F statistic	2,683
Observations	614051.00

Note: ***p < 0.01, **p < 0.05, *p < 0.1.