The Portfolio Balance Channel of Quantitative Easing in a DSGE Model with Financial Frictions*

Genevieve Nelson†
Danmarks Nationalbank

December 18, 2020

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Abstract

Investors who arbitrage between long term government debt and corporate debt expand the Portfolio Balance Channel in that the effects of QE spill over to the overall cost of corporate borrowing. I find that overall the Federal Reserve's second round of Large-Scale Asset Purchases (LSAPII) boosts output between 0.51% - 1.69%, the equivalent of a 83 - 279 basis point cut in the Federal Funds rate. The long term maturity preference of investors increases output growth by between 0.4 and 1.34% points, and inflation between 20 and 68 annualized basis points more than the model without this expanded channel.

Keywords: Quantitative Easing, Financial Frictions

JEL Codes: E13, E52

* This paper is a revised version of Chapter 1 of my PhD dissertation at the University of Oxford. I am very grateful to my advisor Andrea Ferrero. For insightful comments I thank: Matteo Iacoviello, Francesco Zanetti, Michael McMahon, Philipp Lieberknecht, Romanos Priftis, and seminar participants at Danmarks Nationalbank and the Bank of Canada.

† Postdoctoral Economist, Research Unit, Danmarks Nationalbank, Langelinie Allé 47, 2100 Copenhagen, Denmark; Email: gne@nationalbanken.dk; Web: https://www.genevievenelson.com. This paper represents solely the view of the author and does not in any way reflect the opinions of Danmarks Nationalbank.

1 Introduction

The Portfolio Balance Channel of Quantitative Easing (QE) captures the idea that purchases of longer maturity US Treasuries compress the term premium. In addition the portfolio balance channel includes the idea that purchases of longer maturity US Treasuries increase the price of other, substitute assets. Krishnamurthy and Vissing-Jorgensen (2011) emphasize that this second part of the portfolio balance effect can be driven in multiple ways, depending on which assets are substitutes for long term US Treasuries.

The main contribution of this paper is to add one specific mechanism to expand the portfolio balance channel: investors who arbitrage between the return on investing in corporate debt and the return on long term government bonds. I find that the second round of the Federal Reserve's Large Scale Asset Purchase Program (LSAPII) boosted output between 0.51% and 1.69%. The expanded portfolio balance channel drives 78% of the boost to output and 71% of the boost to inflation.

Longstaff, Mithal and Neis (2005) use information from credit default swaps to quantify the portion of the yield spread not explained by default risk. The non-default related yield spread increases particularly for bonds rated below the investment grade threshold. Krishnamurthy and Vissing-Jorgensen (2011) emphasize that this is evidence of a "safety channel": i.e. there are investors who arbitrage between long term US Treasuries and corporate debt rated Baa and above.

Vector autoregression based estimates of the macroeconomic impacts of Quantitative Easing generally find larger quantitative impacts of QE than the early DSGE literature on QE did. For example Baumeister and Benati (2013) find that the median impact of the Federal Reserve's second round of Large Scale Asset Purchases (LSAPII) was to boost GDP by 3% and increase inflation by 1%. For the UK Kapetanios, Mumtaz, Stevens and Theodoridis (2012) find that the peak effects of the Bank of England's first round of QE were a 1.5% increase in GDP and a 1.25% increase in inflation. In contrast the quantitative results in the early DSGE literature on QE are muted. For example Chen, Curdia and Ferrero

Table 1: 1% of GDP QE Shock

Paper	Peak Output Effect	Peak Inflation Effect (bps)
Chen, Curdia and Ferrero (2012)	0 - 0.12	0.7 - 2.4
Gertler and Karadi (2013)	0.25	4
Falagiarda (2014)	0.16 - 0.31	5.6 - 14
Graeve and Theodoridis (2016)	0.22	9
Carlstrom, Fuerst and Paustian (2017)	0.40	3 - 9
this paper	0.12 - 0.4	6.8 - 16.6

Here I standardized the size of the QE shock to 1% of GDP, i.e. about $146 billion in purchases. Appendix C.1 explains the standardization approach in detail. The peak output effect is stated in percentage deviation from steady state, and the peak inflation effect is stated in annualized basis points deviation from its steady state level.

(2012) find that the Federal Reserve's second round of asset purchases had a slightly smaller effect than a surprise 25 basis point cut in the Federal Funds Rate. Their median results are that GDP increased by 0.13% and inflation increased by 3 basis points (both annualized). Harrison (2012) similarly finds small impacts of the Bank of England's QE program. Table 1 indicates that more recent DSGE studies suggest QE has a larger quantitative role, and in particular that the expanded portfolio balance channel is an important driver of the inflationary aspects of QE.

This paper can also shed light on the debate over the effectiveness of monetary policy at the zero lower bound (ZLB). If unconventional monetary policy is equally effective (relative to conventional policy) in moving output and inflation then the ZLB does not impact a central bank's ability to obtain its objectives: this is the "irrelevance hypothesis", characterized by Debortoli, Galí and Gambetti (2020). On the side of the debate that supports the irrelevance hypothesis: Swanson and Williams (2014) find that longer term yields continued to react as sensitively to macroeconomic news during the ZLB period (2008-2012) as the benchmark period (1990-2000). Debortoli et al. (2020) find that the response of output, inflation and long term interest rates are "hardly" impacted by the ZLB binding. On the other side of the debate: Ikeda et al. (2020) find evidence against the irrelevance hypothesis. I find that a one-off Federal Funds Rate cut calibrated to match the peak in output driven by QE suggests that the second round of Federal Reserve Large Scale Asset Purchases was the equivalent of a substantial Federal Funds Rate cut: between 83 and 279 basis points.

The model in this paper builds quantitative easing into a model based on Christiano, Motto and Rostagno (2014). The Christiano et al. (2014) model builds a costly state verification financial friction into a standard DSGE model a la Smets and Wouters (2007) and Christiano, Eichenbaum and Evans (2005). The costly state verification friction comes from Bernanke, Gertler and Gilchrist (1999): lenders can only view the balance sheet of a defaulted non-financial firm by paying a cost. This generates an external finance premium. Firms receive idiosyncratic productivity shocks. A shock to the standard deviation of the productivity distribution is called a "risk shock". Christiano et al. (2014) find that the variation of the risk shock over time is the most important driver of US business cycles. Del Negro and Schorfheide (2013) find the risk shock to have been an important factor during the Great Recession.

Using the model I can quantify the role of the financial friction in amplifying the portfolio balance channel of quantitative easing. I find that the costly state verification financial friction is a quantitatively important amplifier of QE: it drives about 25% of the boost to output and 21% of the boost to inflation from LSAPII.

Quantitative Easing works in the model because of the addition of households who have a preference for longer term assets - the "preferred habitat" characterized by Vayanos and Vila (2009): so that changes in the relative supply of long-to-short term government bonds will impact the term premium. The model captures this with household who have a preferred ratio of long-to-short term government bonds - following Andrés, López-salido and Nelson (2004). This preference is crucial to breaking Wallace's Irrelevance result (Wallace, 1981). This reduced-form approach to capturing the elasticity of the term premium to QE purchases is standard in the QE literature, see for example: Harrison (2012), Falagiarda (2014), Hohberger, Priftis and Vogel (2020). And related to the approach in Chen et al. (2012)

In the model I add the expanded portfolio balance channel by including investors who have a preferred habitat for long term assets: i.e. who arbitrage between holding a diversified set of loans to entrepreneurs (i.e. corporate bonds) and holding long term government bonds. Adding this channel is the key contribution of this paper. This is a departure from the Christiano et al. (2014) model, where the cost of entrepreneurial credit depends on the short-term rate plus a spread. I find that LSAPII boosts output between 0.4% - 1.34% points more and inflation between 20% and 68% under the expanded portfolio balance channel relative to the model where investors have short maturity preferences.

This paper relates to the work of Gertler and Karadi (2013) and Carlstrom, Fuerst and Paustian (2017) in that both examine the relationship between the financing conditions of investment activity and the yields on long term government bonds. Gertler and Karadi (2013) build a model where banks face a diversion constraint but government bonds are less divertible than claims on non-financial firms; notably they find that the zero lower bound is responsible for more than 80% of the effect of QE. In contrast I find that, thought QE is boosted by the zero lower bound constraint, away from the ZLB it still maintains about 50% of its effect.

Carlstrom et al. (2017), though their primary focus is not on QE, build a model where financial intermediaries are the sole holders of long term debt and consider investment projects and long term government debt perfect substitutes. The term premium is positive because intermediaries face a simple"hold-up" problem financial constraint[1].

In this paper investors who arbitrage between long term government bonds and diversified corporate debt link QE directly to the cost of financing investment activity. Embedding this channel in the costly state verification setup[2] generates new lessons about QE: QE reduces both default rates and credit spreads, and boosts the stock market.

These financial market moments are in line with the empirical literature. Foley-Fisher, Ramcharan and Yu (2016) examine firms that are more dependent on long term debt. They

[1] Carlstrom et al. (2017) mention that in their "hold-up" problem financial friction spreads will respond more to leverage relative to a costly state verification (CSV) framework. The model developed here shows that the effect of QE holds up quantitatively in a CSV framework.

[2] The CSV framework provides additional financial moments over the alternative frameworks.

find that around Maturity Extension Program (MEP)[3] announcement dates those firms experienced an increase in their stock prices and issued more long term debt. They find that the dynamics of the risk premium on A- rated bonds was consistent with the pricing of long term preferred habitat investors. Additionally they find that firms increased employment and investment in response to the MEP and the increase was greater the more dependent the firm was on long term debt. Rogers, Scotti and Wright (2014) find that QE has a substantial impact on US stock prices: for an unconventional monetary policy surprise corresponding to a 25 basis point decline in the 10-year Treasury yield they find that the S&P increased 0.94%.

The paper proceeds as follows. Section 2 presents the model. Section 3 presents the data and calibration method. Section 4 presents the simulation of the Federal Reserve's second round of Large Scale Asset Purchases. Section 5 concludes.

2 The Model

2.1 Households

All households are identical and large in number. Each household holds a large number of entrepreneurs and every type of differentiated labor. Households consume, invest to produce raw capital (which is then sold to entrepreneurs), buy and hold long and short term government bonds, supply labor, and receive labor income.

The representative household has the following utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log(C_t - bC_{t-1}) - \psi_L \int_0^1 \frac{h_{it}^{1+\sigma_L}}{1+\sigma_L} di - \frac{\tilde{\nu}}{2} \left(\frac{B_t^L}{B_t} - \delta^b \right)^2 \right\}, \tag{1}$$

where β is the household's discount rate, C_t is per capita consumption, b is the habit parameter, ψ_L is the dis-utility weight on labor, σ_L is the Frisch elasticity of labor supply, and h_{it}

[3] The Maturity Extension Program involved the Federal Reserve selling short term US Treasuries to buy long term US Treasuries. The effect of the MEP is comparable to QE in that it changed the ratio of long to short term government bonds held by the public.

is labor supplied by labor type i. δ^b is the preferred ratio of long-to-short term government debt, $\tilde{\nu}$ governs the dis-utility from deviating from the preferred portfolio, and finally B_t and B_t^L are the market value of privately held short and long term government debt.

The final term in the household's utility function is the household's portfolio preference. Households have quadratic disutility over deviations from the preferred ratio of long-to-short term debt (δ^b). I follow Harrison (2012), which builds on the work of Andrés et al. (2004). I calibrate δ^b to match the steady state ratio of long-to-short term government debt in the hands of the public. $\tilde{\nu} > 0$ implies that the term premium responds to changes in the household's relative holdings of short and long term government debt (this breaks Wallace's Irrelevance Result).

The household's budget constraint is:

$$\begin{aligned} & P_t C_t + B_t + B_t^L + \frac{P_t}{\Upsilon^t \mu_{\Upsilon t}} I_t + Q_{\bar{K},t}(1-\delta)\bar{K}_t \\ & \leq \int_0^1 W_{it} h_{i,t} di + R_{t-1} B_{t-1} + R_t^L B_{t-1}^L + Q_{\bar{K},t}\bar{K}_{t+1} + \Pi_t, \end{aligned} \tag{2}$$

where P_t is the price of the consumption good, I_t is the quantity of investment goods purchased by the household for a price $P_t/(\Upsilon^t \mu_{\Upsilon t})$ ($\mu_{\Upsilon t}$ is a shock to investment technology, and Υ^t is trend growth in investment technology), $\bar{Q}_{\bar{K},t}$ is the price of raw capital, $\bar{K}_{t+1}$ is end of period t raw capital, δ is the depreciation rate of capital, W_{it} is the wage rate for labor type i, R_{t-1} is the rate paid on short term bonds issued in $t-1$ maturing in t, and R_t^L is the rate paid on long term bonds issued in $t-1$ maturing in t and Π_t includes transfers from entrepreneurs and intermediate goods producers, and transfers to & from investors.

I base the model's treatment of the bond market on Harrison (2012). There are two types of government bonds: short, and long. Short bonds sell for a unit price at time t, and return R_t units of currency at time t+1. Long bonds are perpetuities that exist for an infinite number of periods (unless the government removes them from the market). They provide a coupon payment of 1 unit of currency each period, and have a value V_t at time t. In each period t, after making the coupon payment, the government rolls over its debt by purchasing the entire stock of long term debt (B^c_{t-1}) at the market price V_t and issuing new consol bonds B^c_t which are purchased by households for the market price V_t. $R^L_t \equiv (1+V^L_t)/V^L_{t-1}$ is the gross return at time t on a long bond sold at time $t-1$. $B^L_t \equiv V_t B^c_t$ is the total nominal value of long bonds at time t. Note the timing: V^L_t is unknown at time $t-1$; therefore R^L_t is not known for certain until time t.

When investing in raw capital the household faces the following law of motion:

$$\bar{K}_{t+1} = (1-\delta)\bar{K}_t + \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t. \tag{3}$$

The investment adjustment cost function S has the functional form:

$$S(x_t) \equiv \frac{1}{2}\left\{ \exp\left[\sqrt{S''}(x_t - x)\right] + \exp\left[-\sqrt{S''}(x_t - x)\right] - 2\right\}, \tag{4}$$

where $x_t \equiv \frac{I_t}{I_{t-1}}$ and $S'' \equiv S''(x)$ is a parameter calibrated to match the dynamics of investment.

2.2 Production Markets

2.2.1 Goods Market

Each intermediate good, $Y_{jt},\ j \in [0,1]$ is produced by a different monopolist according to the following production function:

$$Y_{jt} = \begin{cases} \epsilon_t K^{\alpha}_{jt}\big(z_t l_{jt}\big)^{(1-\alpha)} - \Phi z^*_t, & \text{if } \epsilon_t K^{\alpha}_{jt}\big(z_t l_{jt}\big)^{(1-\alpha)} > \Phi z^*_t \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

where the capital share $\alpha \in (0,1)$ and ϵ_t is a technology shock (that is covariance stationary). K_{jt} is the quantity of effective capital used by monopolist producer j, and l_{jt} the quantity of homogeneous labor employed by monopolist producer j. z_t is an effective labor shock which has a stationary growth rate. The proportional fixed cost Φz_t^* is such that the intermediate monopolistic producer earns zero profits in steady state. Below I describe the detrending term z_t^* in more detail.

There is a Calvo friction in the pricing of intermediate goods. Each period a random fraction of intermediate firms, $1 - \xi_p$, can reoptimize their price P_{jt}. The remaining fraction ξ_p set their price as follows:

$$P_{jt} = \tilde{\pi}_t P_{j,t-1}, \tag{6}$$

where inflation indexation is as follows:

$$\tilde{\pi}_t = \left(\pi_t^{target}\right)^{\iota} \left(\pi_{t-1}\right)^{1-\iota}. \tag{7}$$

$\pi_{t-1} \equiv P_{t-1}/P_{t-2}$ is gross inflation. And π_t^{target} is the central bank's target inflation rate. ι is the price indexing weight on the inflation target.

The homogeneous final good Y_t, is produced by a competitive representative firm (with Dixit-Stiglitz technology):

$$Y_t = \left[\int_0^1 Y_{jt}^{\frac{1}{\lambda_f}} dj \right]^{\lambda_f}, \; 1 \leq \lambda_f < \infty, \; j \in [0, 1]. \tag{8}$$

The homogeneous final good has two uses: consumption and investment. One unit of Y_t can be converted into one unit of the consumption good C_t, and thus (given perfect competition in the use of this technology) consumption has the price P_t. One unit of Y_t can also be converted into $\Upsilon^t \mu_{\Upsilon,t}$ units of the investment good, and thus (again given perfect competition in the use of the technology) has the price $P_t/(\Upsilon^t \mu_{\Upsilon,t})$, where $\Upsilon > 1$.

There are two sources of growth in the model. First, the trend rise in the aforementioned technology for producing investment goods, Υ^t, and second, the effective labor shock z_t which has a stationary growth rate. The de-trending term z_t^* is a combination of both sources of growth:

$$z_t^* = z_t \Upsilon^{\left(\frac{\alpha}{1-\alpha}\right)t}. \tag{9}$$

z_t^* is used to normalize variables to find a non stochastic steady state. z_t^* is such that Y_t/z_t^* will converge to a constant in the nonstochastic steady state of the model. $\mu_{z,t}^* \equiv z_t^*/z_{t-1}^*$ is the growth rate of z_t^*, which has the stationary growth rate μ_z^*.

2.2.2 Labor Market

Each differentiated labor type type $i \in [0,1]$ provides labor services h_{it} and is represented by a monopoly union that sets its wage rate W_{it} while facing a Calvo friction. Each period a fraction $1-\xi_w$ of the monopoly unions can update the wage. The remaining fraction ξ_w set their wage as follows:

$$W_{it} = \left(\mu_{z,t}^*\right)^{\iota_\mu} \left(\mu_z^*\right)^{1-\iota_\mu} \tilde{\pi}_{wt} \, W_{i,t-1}, \tag{10}$$

where:

$$\tilde{\pi}_{wt} \equiv \left(\pi_t^{target}\right)^{\iota_w} \left(\pi_{t-1}\right)^{1-\iota_w}, \qquad 0 < \iota_w < 1. \tag{11}$$

Labor is aggregated via a Dixit-Stiglitz style aggregator by a competitive and representative labor contractor:

$$l_t = \left[\int_0^1 \left(h_{it}\right)^{\frac{1}{\lambda_w}} di\right]^{\lambda_w}, \qquad 1 \leq \lambda_w. \tag{12}$$

The homogeneous labor aggregate l_t is sold to intermediate goods producers at the nominal wage W_t.

2.3 Entrepreneurs

Entrepreneurs have the role of turning raw capital (purchased from households) into effective capital (to be then sold to intermediate goods producers). Entrepreneurs experience idiosyncratic productivity shocks in their ability to turn raw capital into productive capital. Entrepreneurs finance the purchase of raw capital via debt, and entrepreneurs who experience low idiosyncratic productivity shocks default on their debt. Entrepreneurs' creditors can only observe the state of a defaulted entrepreneur's balance sheet by paying a proportional recovery cost. The expectation of paying this cost introduces a spread on entrepreneurial debt. This is the Costly State Verification (CSV) financial friction characterized in Bernanke, Gertler and Gilchrist (1999) and Christiano, Motto and Rostagno (2014).

Entrepreneurs' aggregate net worth is considered a proxy for the value of the stock market. Entrepreneurs can either be interpreted as being firms in the non-financial sector, or financial institutions with non-diversified holdings.

Entrepreneurs are classified by their net worth. An entrepreneur with net worth $N \geq 0$ is called an 'N-type' entrepreneur. The timing of one cycle in the life of an entrepreneur is as follows. Following production in period t, each entrepreneur gets a loan from a mutual fund. Each mutual fund is specialized. They make loans only to entrepreneurs of a specific level of net worth, but perfectly diversify by holding a large number of those loans. The entrepreneur combines the loan $B_t^{N,credit}$ (issued in period t and due in period $t+1$) with their own net worth to purchase raw capital ($\bar{K}_{t+1}^N$) at price $Q_{\bar{K},t}$:

$$B_t^{N,credit} + N = Q_{\bar{K},t}\bar{K}_{t+1}^N. \tag{13}$$

After raw capital is purchased each entrepreneur receives an idiosyncratic shock ω that determines the amount of effective capital they have, $\omega\bar{K}_{t+1}^N$. As in Bernanke et al. (1999) and Christiano et al. (2014), ω is distributed (independently across entrepreneurs and time) log-normally with a unit mean and a standard deviation $\sigma_t \equiv \sqrt{var(\log\omega)}$. The *risk shock* σ_t is simply the extent of cross-sectional dispersion of idiosyncratic productivity shocks experienced by entrepreneurs.

After realizing the risk shock entrepreneurs choose the utilization rate of effective capital u_{t+1}^N to maximize their return on capital: ωR_{t+1}^k at the end of period $t+1$. Entrepreneurs supply $u_{t+1}^N\omega\bar{K}_{t+1}^N$ units of effective capital to intermediate goods producers at the market rental rate r_{t+1}^k. The following defines the return on capital:

$$R_{t+1}^k \equiv \frac{[u_{t+1}r_{t+1}^k - a(u_{t+1})]\Upsilon^{-(t+1)}P_{t+1} + (1-\delta)Q_{\bar{K},t+1}}{Q_{\bar{K},t}}. \tag{14}$$

The choice of utilization is independent of net worth, so the N superscript is dropped. The utilization cost of capital, $a(u_t)$, is increasing and convex:

$$a(u_t) \equiv \frac{r_t^k}{\sigma_a}\Big[\exp\Big(\sigma_a(u_t-1)\Big)-1\Big]. \tag{15}$$

In addition to the utilization choice entrepreneurs must also choose the type of debt contract to accept. It is each entrepreneur's objective to maximize their expected net worth in the next period (t+1), which is as follows:

$$\begin{aligned} & E_t\Bigg\{\int_{\bar{\omega}_{t+1}}^{\infty}\Big[R_{t+1}^k\omega Q_{\bar{K},t}\bar{K}_{t+1}^N - B_t^{N,credit}Z_{t+1}\Big]dF(\omega,\sigma_t)\Bigg\} \\ & = E_t\Big[1-\Gamma(\bar{\omega}_{t+1})\Big]R_{t+1}^k L_t N, \end{aligned} \tag{16}$$

where $\Gamma_t(\bar{\omega}_{t+1}) \equiv \Big[1-F_t(\bar{\omega}_{t+1})\Big]\bar{\omega}_{t+1} + G_t(\bar{\omega}_{t+1})$ is the fraction of expected earnings paid to the investor, $1-F_t(\bar{\omega}_{t+1})$ is the probability the entrepreneur experiences a idiosyncratic shock over the default threshold $\bar{\omega}_{t+1}$, and $F(.)$ is the cumulative distribution function of ω. Z_{t+1} is the gross nominal interest rate on debt. $G_t(\bar{\omega}_{t+1})$ is the expected value of the idiosyncratic shock in the population of defaulting entrepreneurs:

$$G_t(\bar{\omega}_{t+1}) \equiv \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega), \tag{17}$$

where $L_t \equiv (Q_{\bar{K},t}\bar{K}_{t+1}^N)/N$ is the entrepreneur's leverage. Entrepreneurs maximize (16) by choosing: (1) the level of the idiosyncratic shock ω below which they will default ($\bar{\omega}_{t+1}$), or equivalently the gross nominal interest rate on debt to be paid next period (Z_{t+1}), and (2) the amount of leverage L_t they will take on, subject to the set of debt contracts that mutual funds are willing to provide (equation 20). The following defines the default threshold:

$$\bar{\omega}_{t+1} \equiv \frac{B_t^{N,credit}Z_{t+1}}{R_{t+1}^k Q_{\bar{K},t}\bar{K}_{t+1}^N}. \tag{18}$$

2.3.1 Mutual Funds

As in Christiano et al. (2014) each mutual funds specializes in loans to N-type entrepreneurs, and is perfectly diversified by holding a large number of N-type loans. They sell packaged entrepreneurial debt to investors, which has the return $R_{t+1}^{N,credit}$.

$$\Big[1 - F_t(\bar{\omega}_{t+1})\Big] Z_{t+1} B_t^{N,credit} + (1-\mu) \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega) R_{t+1}^k Q_{\bar{K},t} \bar{K}_{t+1}^N = R_{t+1}^{N,credit} B_t^{N,credit}. \quad (19)$$

If an entrepreneur experiences an idiosyncratic shock ω below the threshold $\bar{\omega}_{t+1}$, then they will not be able to repay their debt to the investor and will declare bankruptcy. In this instance the mutual fund only knows that the entrepreneur is bankrupt, but does not observe the value of ω. Without further action by the mutual fund the entrepreneur could decide to transfer only a fraction of their remaining assets, $\omega R_{t+1}^k Q_{\bar{K},t} \bar{K}_{t+1}^N$, back to the mutual fund. In order to become fully informed about the assets a bankrupt entrepreneur has, the mutual fund must pay a cost that is a proportion μ of the final assets recovered. Thus the mutual fund only receives a fraction $(1-\mu)$ of the total assets of bankrupt entrepreneurs.

The following condition (20) characterizes the available menu of contracts entrepreneurs can choose from. This comes from using the definition of the default threshold (18) to substitute out $Z_{t+1} B_t^{N,credit}$ in the mutual funds' zero profit condition (19).

$$\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1}) = \frac{L_t - 1}{L_t} \frac{R_{t+1}^{N,credit}}{R_{t+1}^k}. \quad (20)$$

Note that net worth N does not enter in (20): all entrepreneurs select the same contract, irrespective of net worth, and the return on their packaged debt is the same for all N:

$$R_{t+1}^{N,credit} = R_{t+1}^{credit}. \quad (21)$$

2.3.2 Investors

Investors are agents in the economy who arbitrage between long term government bonds and perfectly diversified entrepreneurial debt (B_t^{credit}):

$$B_t^{credit} \equiv \int_0^\infty B_t^{N,credit} f_t(N) dN. \tag{22}$$

Each investor i receives a transfer τ_t^i from their household each period and transfers their profits back to the household in the following period. Investor i's problem is to maximize their expected profit (23) choosing long term government bonds ($B_t^{L,i}$) and diversified entrepreneurial debt ($B_t^{credit,i}$):

$$\max_{B_t^{L,i}, B_t^{credit,i}} \quad E_t \left\{ R_{t+1}^L B_t^{L,i} + R_{t+1}^{credit} B_t^{credit,i} \right\} \tag{23}$$

subject to their balance sheet identity:

$$\tau_t^i = B_t^{L,i} + B_t^{credit,i}. \tag{24}$$

The investor's optimality condition implies that the expected spread between the return on diversified entrepreneurial debt and the return on the long term government bond is zero[4]:

$$E_t \Lambda_{t,t+1} \Big[R_{t+1}^{credit} - R_{t+1}^L \Big] = 0, \tag{25}$$

where $\Lambda_{t,t+1}$ is the household's stochastic discount factor.

The existence of these investors in the model captures the expanded portfolio balance channel: when the rate on long term government debt falls, investors are willing to hold packaged entrepreneurial debt (i.e. corporate debt) at a lower rate. This in turn relaxes

[4]Clearly this is a strong assumption for two reasons. One, in the data there is a spread between corporate debt yields and US Treasury yields - meaning that there are likely frictions between government debt and corporate debt, and two, the empirical evidence that supports a preferred habitat preference over bond maturity does not extend to all corporate debt. The evidence in Longstaff, Mithal and Neis (2005) supports the preferred habitat only extending to corporate debt rated above Baa. Appendix E.2 tests the robustness of this assumption.

the credit conditions that mutual funds are able to provide to entrepreneurs. Though the entrepreneurial debt contracts themselves are not long term this captures in reduced form the idea that there are investors in the economy who have a preferred habitat preference for long duration assets.

There is empirical evidence that certain investors have a preferred habitat for long term assets, particularly for investment grade corporate bonds. Insurance companies are long term preferred habitat investors because they must match the duration of their assets and liabilities. The capital equity requirements that they face increase between bonds rated A- and above, and bonds below that threshold. Foley-Fisher et al. (2016) find that in response to the Maturity Extension Program (MEP) the risk premium on A- rated bonds fell, whereas the risk premium on BBB+ rated bonds did not, suggesting a direct preferred habitat spillover effect from the MEP program[5]. Furthermore they find support for the "gap-filling" hypothesis: that is the idea that firms that are able to issue longer term debt respond to the combination of QE purchases and demand for long term debt from investors by increasing their issuance of long term debt. Longstaff et al. (2005) find evidence for preferred habitat preferences in the bond market extending to corporate bonds rated BAA and above.

This setup is a departure from Christiano et al. (2014), where the assumption is that mutual funds package entrepreneurial debt and sell it directly to households. In their model the relative outside rate is the short-term policy rate, this is equivalent to a setup where investors arbitrage between diversified entrepreneurial debt and short term government bonds ("Short Maturity Investor"). Section 4.6 shows the simulation of LSAPII in the baseline model versus the "Short Maturity Investor" model.

[5]The Maturity Extension Program involved the Federal Reserve selling short term US Treasuries to buy long term US Treasuries. The effect of the MEP is comparable to QE in that it changed the ratio of long to short term government bonds held by the public.

2.3.3 Accelerator Effect

The characterization of the financial friction is from Christiano et al. (2014) and is based on the costly state verification (CSV) financial accelerator mechanism in Bernanke et al. (1999). Bernanke et al. (1999) emphasize the following intuition behind the accelerating affects of the financial friction in the CSV class of models. The basic idea is that a fall in entrepreneurial net worth means that the entrepreneurs will have less inside funds to invest in the project. Therefore the mutual funds that make loans to entrepreneurs face a greater agency cost when they finance the entrepreneurs. Essentially the entrepreneur has less "skin in the game". The higher agency cost means mutual funds charge a higher interest rate, so that the premium on external finance faced by entrepreneurs increases. Faced with a larger interest rate on loans, other things being equal, entrepreneurs will choose to purchase less capital. Because entrepreneurs play a key role in turning raw capital into effective capital used by producers the increase in the external finance premium will decrease output. The net worth of entrepreneurs is pro-cyclical. Therefore the external finance premium is counter-cyclical. Thus the interactions between mutual funds and entrepreneurs via the hike in the external finance premium will serve to amplify the business cycle.

This also means that to the extent to which QE boosts entrepreneurial net worth it can have the reverse effect - lowering the external finance premium, boosting capital, and increasing output.

2.3.4 Aggregates

Aggregate raw capital:

$$\bar{K}_{t+1} = \int_0^\infty \bar{K}_{t+1}^N f_t(N) dN. \tag{26}$$

Aggregate effective capital:

$$K_t = \int_0^\infty \int_0^\infty u_t^N \omega \bar{K}_t^N f_{t-1}(N) dF(\omega) dN = u_t \bar{K}_t. \tag{27}$$

Aggregate net worth:

$$N_{t+1} = \int_0^\infty N f_t(N) dN. \tag{28}$$

Aggregate credit:

$$B_t^{credit} = \int_0^\infty B_t^{N,credit} f_t(N) dN = \int_0^\infty \left[Q_{\bar{K},t} \bar{K}_{t+1}^N - N \right] f_t(N) dN = Q_{\bar{K},t} \bar{K}_{t+1} - N_{t+1}. \tag{29}$$

Finally the evolution of aggregate net worth is:

$$N_{t+1} = \gamma \left[1 - \Gamma_{t-1}(\bar{\omega}_t) \right] R_t^k Q_{\bar{K},t-1} \bar{K}_t + W^e, \tag{30}$$

where γ is the fraction of entrepreneurs that continue each period (a fraction $1-\gamma$ exit and pay dividends to the household), and W^e is the transfer from the household to new entering entrepreneurs.

2.4 Government Policies

As in Chen et al. (2012) the government has an auto-regressive supply rule for the market value of de-trended long term bonds ($b_t^L \equiv B_t^L/(P_t z_t^*)$):

$$\log \left(\frac{b_t^L}{b^L} \right) = \rho_{bL} \left(\frac{b_{t-1}^L}{b^L} \right) + u_t^{bL}. \tag{31}$$

u_t^{bL} is the sum of unanticipated and anticipated news shocks to the supply of long term government bonds:

$$u_t^{bL} \equiv \epsilon_t^{bL} + \xi_{1,t-1}^{bL} + ... + \xi_{8,t-8}^{bL}. \tag{32}$$

The government budget constraint is:

$$B_t + B_t^L = R_{t-1} B_t + R_t^L B_{t-1}^L + G_t - T_t, \tag{33}$$

where G_t is nominal government spending, and T_t is nominal government taxation. Government spending is:

$$\frac{G_t}{P_t z_t^*} = g, \tag{34}$$

where the level of real de-trended government spending, g, is fixed to be 20% of steady state output.

I adapted the fiscal rule from Davig and Leeper (2006) and Eusepi and Preston (2011):

$$\frac{T_t}{P_t z_t^*} - \frac{G_t}{P_t z_t^*} = \kappa \left(\frac{b_{t-1} + b_{t-1}^L}{b^L + b} \right)^{\phi_T} \epsilon_t^T, \tag{35}$$

where κ is the steady state primary fiscal surplus, and ϕ_T is set high enough so that the primary surplus adjusts to satisfy the government inter-temporal budget constraint and where b_{t-1} and b_{t-1}^L are the real de-trended market value of short and long term bonds respectively ($b_t \equiv B_t/(P_t z_t^*)$).

2.5 Monetary Policy & Resource Constraint

2.5.1 Monetary Policy Rule

The central bank sets the policy rate according to a backward-looking Taylor Rule:

$$\log\left(\frac{R_t}{R}\right) = \rho_m \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_m)\left[\phi_\pi \log\left(\frac{\pi_t}{\pi_t^{target}}\right) + \frac{\phi_y}{4}\left(\log\frac{Y_t}{Y_{t-1}} - \log \mu_z^*\right)\right] + \frac{1}{400} u_t^m, \tag{36}$$

where μ_z^* is the steady state growth of output. u_t^m is the sum of unanticipated and anticipated (news) monetary policy shocks:

$$u_t^m \equiv \epsilon_t^m + \xi_{1,t-1}^m + \ldots + \xi_{8,t-8}^m, \tag{37}$$

where ϵ_t^m is the unanticipated monetary policy shock. And $\sum_{p=1}^{p=8} \xi_{p,t-p}^m$ is the sum of anticipated monetary policy shocks.

2.5.2 The Resource Constraint

$$Y_t = G_t + C_t + \frac{I_t}{\Upsilon^t \mu_{\Upsilon,t}} + a(u_t)\Upsilon^{-t}\bar{K}_t + \Theta\frac{1-\gamma}{\gamma}(N_{t+1} - W^e) + D_t, \tag{38}$$

where $a(u_t)\Upsilon^{-t}\bar{K}_t$ is the aggregate capital utilization cost of entrepreneurs. $\Theta(1-\gamma)(N_{t+1} - W^e)/\gamma$ are the resources consumed by exiting entrepreneurs. The resources expended on monitoring entrepreneurs are:

$$D_t \equiv \frac{\mu G_{t-1}(\bar{\omega}_t)R_t^k + Q_{\bar{K},t-1}\bar{K}_t}{P_t}. \tag{39}$$

3 Data and Calibration

The target period for calibration is 1985Q1 to 2007Q3. Appendix C describes the data.

Table 2: Steady State Targets

Target	**Value**
Nominal Federal Funds Rate, annualized (R)	5.11
Ratio of long term government bonds to annual output, $B^L/(4 \times Y)$	19.3%
Ratio of long to short bond holdings, B^L/B	1.86
Ratio of government spending to quarterly output, G/Y	20%
Hours worked (h)	1
Inflation, APR (π)	2%

Table 2 lists the steady state targets. The calibrations corresponding with these targets are in appendix C table 7. I calibrate the household's discount rate β to match the period average of the Effective Federal Funds Rate. I fix the ratio of government bonds held by the public to output in steady state to match the data. I set δ^b to match the period average of the ratio of long-to-short term government bonds held by the public. I define long term debt as any government bond with over 1 year until maturity, and short-term debt to include reserves. I set government spending in steady state to 20% of quarterly output, in line with the target in Christiano et al. (2014). I set ψ_L to target steady state hours worked (h) equal

Table 3: Calibrated Parameters

Parameter	Description	Calibration
α	capital's share of output	0.4
b	habit parameter	0.74
Θ	fraction of assets consumed by exiting entrepreneurs	0.005
δ	depreciation rate of capital	0.025
ϵ	steady state value of the technology shock	1
$F(\bar{\omega})$	steady state probability of default	0.0056
γ	fraction of entrepreneurs who survive	0.985
ι	price indexing weight on inflation target	0.9
ι_μ	wage indexing weight on persistent technology growth	0.94
ι_w	wage indexing weight on inflation target	0.49
λ_f	markup in the product market	1.2
λ_w	markup in the labor market	1.05
μ	monitoring cost	0.21
μ_Υ	steady state value of $\mu_{\Upsilon,t}$	1
μ_z^*	mean growth rate of the unit root technology shock	1.0041
ϕ_π	parameter on inflation in the Taylor Rule	2.40
ϕ_y	parameter on output in the Taylor Rule	0.36
ϕ_T	fiscal rule feedback parameter (Chen et al., 2012 posterior mean)	1.3147
ρ_m	weighting of lagged short-rate in Taylor Rule	0.85
S''	parameter in the investment adjustment cost function	10.78
σ_a	curvature of utilization cost	2.54
σ_L	Frisch elasticity of labor supply	1
Υ	quarterly rate of investment-specific technological change	1.0042
w^e	lump sum transfer from household to the entrepreneur	0.005
ξ_p	Calvo price stickiness	0.74
ξ_w	Calvo wage stickiness	0.81

to 1. Steady state inflation is 2%.

$\nu \equiv \tilde{\nu}\delta^b(1+\delta^b)/(\lambda_z b)$ is the elasticity of the term premium to changes in the relative supply of long term government bonds (see appendix D). The target range is a 3 to 10 basis point drop in the term premium in response to a $100 billion reduction in the supply of long term government bonds available to the public (with an equal increase in reserves). Section 4.2 discusses this range further.

I fix the parameters in table 3 according to the calibration or posterior modes in Christiano et al. (2014). I specify all shock processes as log AR(1)'s. Table 4 specifies their persistence parameters.

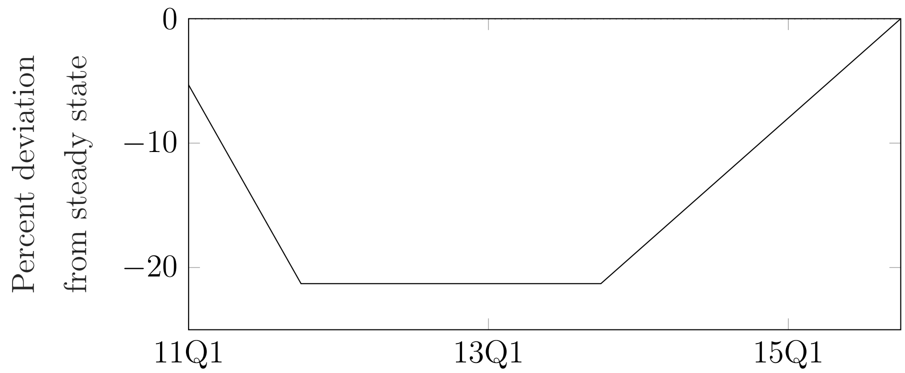
Table 4: Shock Autocorrelations

Parameter	Description	Calibration
ρ_{bL}	long term bond supply	0
ρ_{ϵ}	transitory technology	0.81
$\rho_{\mu_z^*}$	persistent technology growth	0.15
ρ_{σ}	risk shock	0.97

4 Simulations

4.1 Simulating LSAPII

Figure 1: Simulated Path of Long-Term Government Debt Held by the Public

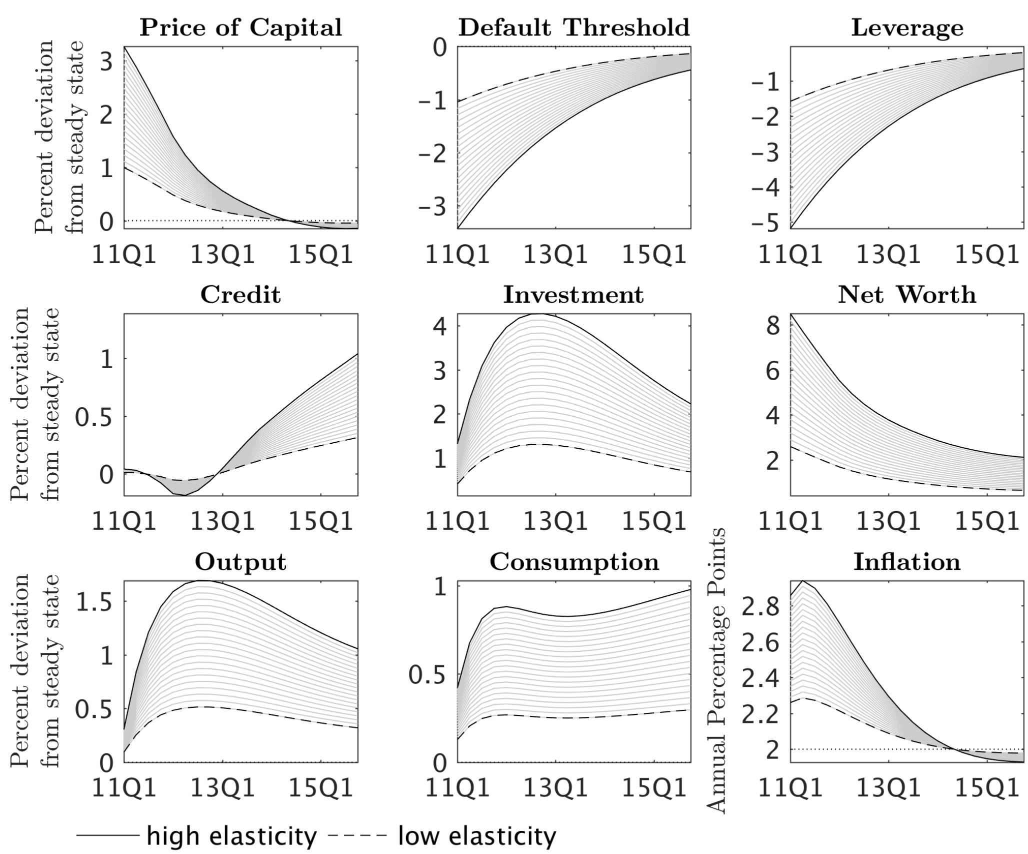


Following Chen et al. (2012) I simulate the second round of the Federal Reserve's large scale asset purchase program (LSAPII) with purchases taking place over two quarters, held for four quarters, and unwound over 8 quarters. The full path of purchases (figure 1) is known upon announcement of the program. I constrain the response of the federal funds rate for four quarters. I implement both the announced path of purchases and forward guidance on the policy rate using news shocks. Appendix C describes the mapping between the size of the \$600 Billion in purchases and long bond supply shocks.

4.2 The Impact of LSAP II

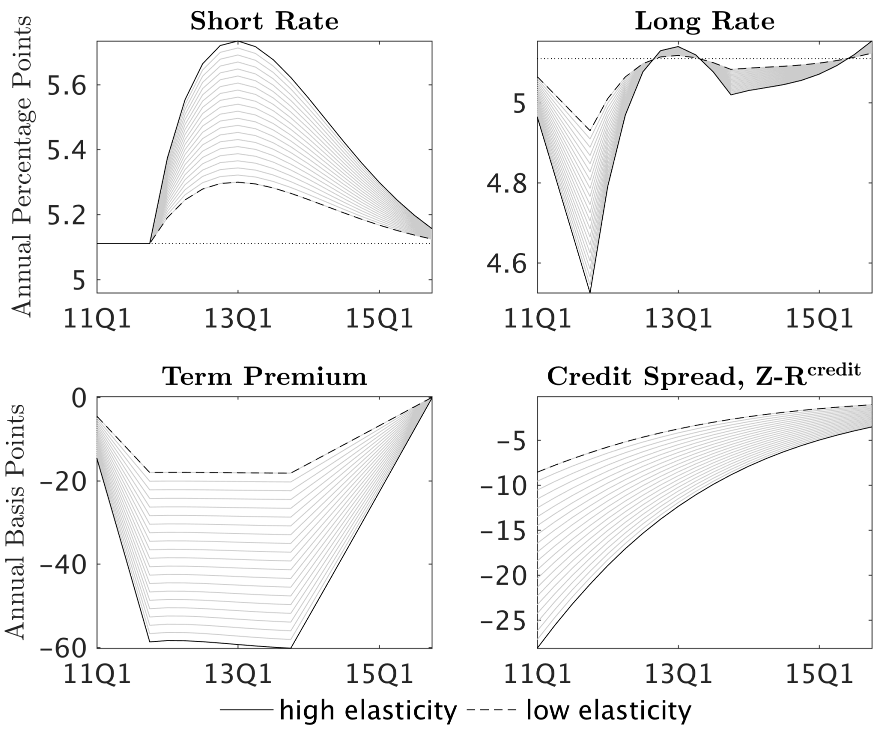
Empirical estimates of QE's "stock effects" - i.e. the impact on bond prices from the semi-permanent reduction of bond supply within a given maturity - suggest that the Federal Reserve's various rounds of LSAP purchases reduced term premiums between 3 and 10 basis points per $100 billion of long term bond purchases. On the upper range of the estimates D'Amico and King (2013) find that LSAPI on average decreased yields within a given maturity by 1 basis point per $10 billion in long term bond purchases. On the lower range Hamilton and Wu (2012) find that a $400 billion purchase of long term maturity government bonds could reduce the 10-year rate by 13 basis points when the policy rate is at the zero lower bound.

Figure 2: The Impact of LSAPII (a)



I calibrate ν to match these estimates of the elasticity of the term premium to purchases. This gives a range of 0.00074- 0.0025. Figures 2 and 3 show the range of results. The output growth peak is between 0.51% - 1.69%. Output 6 years after the start of the LSAPII program is between 0.26% - 0.86% above its steady state level. Inflation increases between 28 and 94 basis points. Comparing these results to the narrow portfolio balance channel results (see section 4.6) indicates that expanding the portfolio balance channel is important to match the empirical evidence on inflation. The response of inflation to LSAPII here is comparable to Baumeister and Benati, 2013 who find LSAPII increased inflation by 1%. Entrepreneurs' aggregate net worth (a proxy for the stock market) increases between 2.6 - 8.5%, exceeding the empirical evidence in Rogers et al. (2014) which suggest[6] an increase between 0.7 - 2.3% (based on the term premium dynamics in the LSAPII simulation). Lastly LSAPII drives the credit spread down by between 8 and 28 basis points.

Figure 3: The Impact of LSAPII (b)



4.2.1 QE Mechanisms

In this paper the household's preference for an ideal ratio of long to short term government bonds in the household's utility function captures the idea that certain agents prefer to hold longer maturity assets ("preferred habitat") as characterized by Vayanos and Vila (2009). This means per period returns on short vs long term assets are not fully arbitraged away, leading to a term premium for long term government bonds. This is a subset of the "Safety

[6]They estimate that the S&P increased 0.94% for 25 basis point reduction in the 10 year Treasury yield. Here the long term government bond rate falls between 18 - 60 basis points.

Channel" described by Krishnamurthy and Vissing-Jorgensen (2011): the preferred habitat is restricted to the near zero default risk assets, in this case only US Treasury bonds. This channel breaks Wallace's Irrelevance result, meaning that the term premium changes in response to changes in the relative supply of long vs short term government bonds. The term premium elasticity ν captures the strength of this channel and, as clearly can be seen in the lower left panel of figure 3, impacts how much the term premium drops in response to the LSAPII purchases. In a model with only this effect, for example Chen et al. (2012), QE acts solely via changing household's consumption and investment decisions via the long term bond Euler. As Chen et al. (2012) find this effect alone is quantitatively limited.

Adding the Entrepreneurial sector, and investors who arbitrage between long term government debt and packaged entrepreneurial debt, expands the Portfolio Balance Channel and the role of preferred habitat. A decline in the long term government bond yield spills over into a general decline in the cost of corporate borrowing. Longstaff, Mithal and Neis (2005) show that this effect probably only exists for investment grade corporate debt (i.e. above Baa), that is: Krishnamurthy and Vissing-Jorgensen (2011)'s Safety channel including investment grade corporate debt. Krishnamurthy and Vissing-Jorgensen (2011) classify a general spillover effect as the "Duration Risk Channel". Investor preferences in the model here capture both the Safety Channel and Duration Risk Channel: because investors hold a perfectly diversified aggregate of corporate (i.e. entrepreneurs') debt. Given the relatively weak empirical support for the Duration Risk Channel, I explore including a friction in the arbitrage between corporate debt and US Treasuries in appendix E.2.

The expanded Portfolio Balance channel interacts with a "Default Risk Channel" characterized by Krishnamurthy and Vissing-Jorgensen (2011). Lower borrowing costs mean that fewer entrepreneurs default. This, via the costly-state verification friction, lowers the spread charged on individual entrepreneurial debt contracts further, boosting net worth and accelerating the effects of QE via a reduction in the external finance premium (see section 4.5).Additionally QE has an inflationary impact, which means that *real* rates fall further.

A partial and a general equilibrium effect drive the dynamics of credit in different directions. QE reduces the term premium, which reduces the long term interest rate on government bonds especially when the policy rate is constrained. Because investors arbitrage between corporate and long term government debt this lowers the cost of borrowing for entrepreneurs.

The partial equilibrium effect is to stimulate credit demand, because credit is cheaper. Furthermore, because credit is cheaper more entrepreneurs can repay their debt, so in aggregate entrepreneurial net worth increases. Now entrepreneurs have more "skin in the game", so due to the costly state verification friction the credit spread compresses, amplifying further the effect of QE.

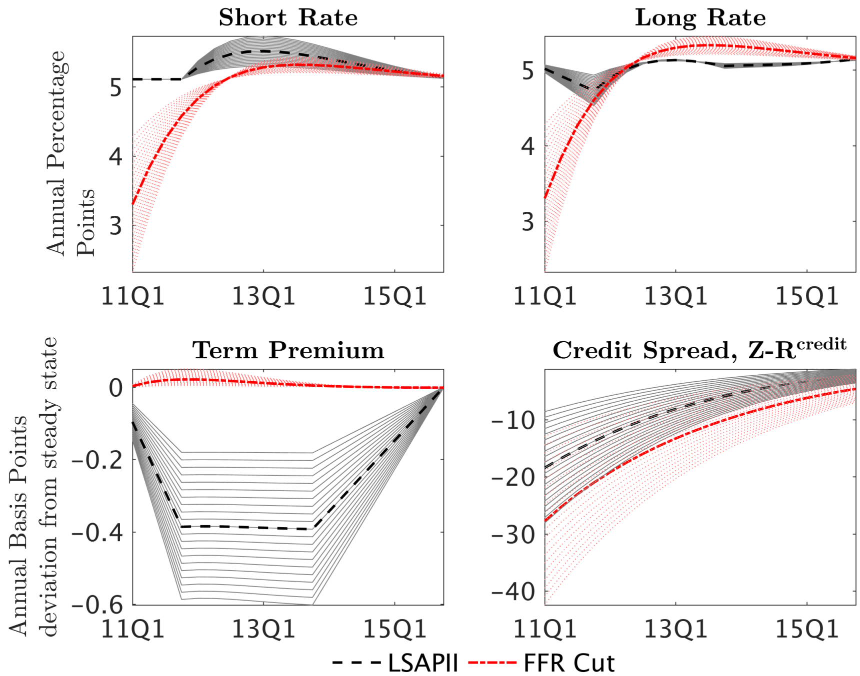
In general equilibrium the lower cost of borrowing in addition to higher net worth drives entrepreneurs to invest more. This leads to a jump in the price of capital upon impact of the QE shock, with the expectation that the price of capital will eventually return to steady state. Christiano et al. (2014) emphasize that this expected decline in the price of capital mutes the entrepreneurs incentives to take on credit. This effect is particularly pronounced initially. That is why credit responds by less than net worth, and so leverage falls in response to shocks that improve conditions for entrepreneurs (including the QE shock).

4.3 LSAPII Impact vs FFR Cut Equivalence

The simulated LSAPII program produces the same boost to output as a 83 to 279 basis point cut to the federal funds rate, corresponding to the lower and upper ranges for the elasticity of the term premium to QE purchases (i.e. ν calibration). Figures 4 and 5 show the response of the economy to the LSAPII program versus a federal funds rate cut.

By target, output growth is the same across the LSAPII and the federal funds rate cut simulations. Unsurprisingly inflation growth is roughly comparable across the two simulations. Investment responds slightly more to the federal funds rate cut (between 0.12 and 0.39 percentage points).

Figure 4: Quantitative Easing vs a FFR Cut (a)



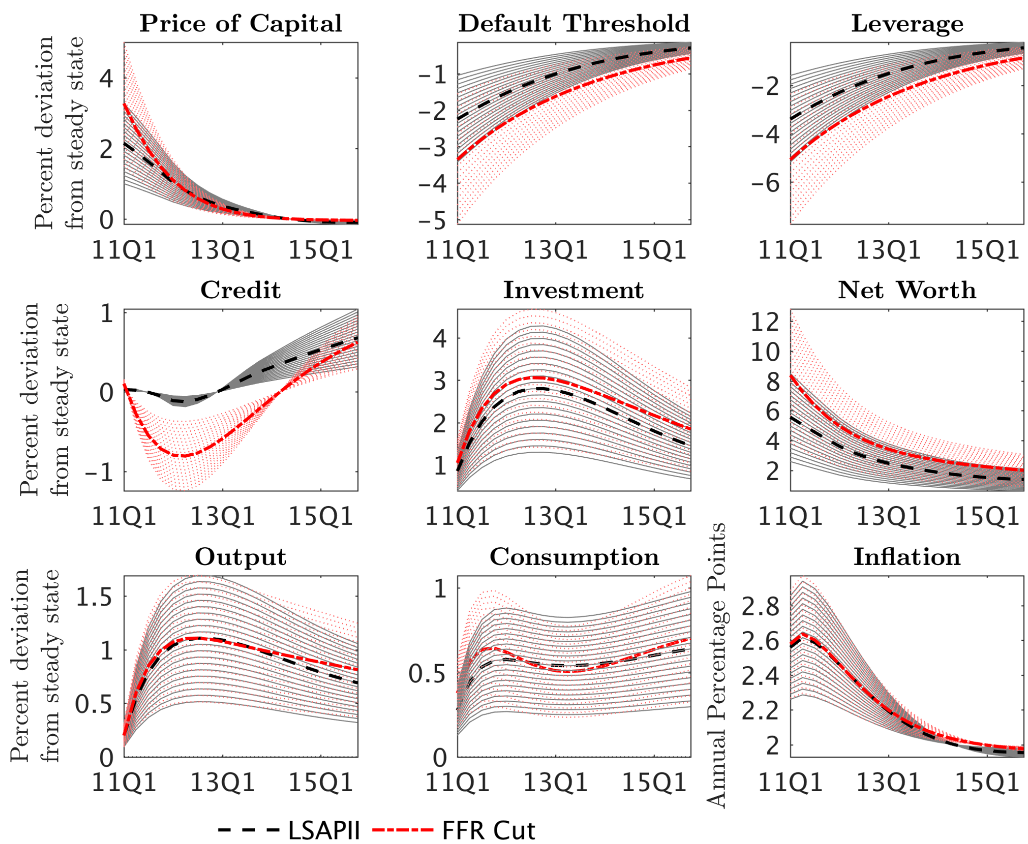
Note: The thick dashed lines correspond to the simulation at the median value for the elasticity of the term premium (ν).

The impact on entrepreneurial credit conditions is substantially different between QE and the equivalent federal funds rate cut, highlighting the different channel through which these two policies act. Under quantitative easing credit drops slightly (between 0.06 - 0.19% below its steady state level) once the forward guidance constraint on the federal funds rate is lifted. In contrast under the federal funds rate cut simulation credit drops between 0.37 - 1.24% below its steady state value (as the policy rate is normalized). The drop in the long rate is front-loaded under the FFR cut. This mean that the general equilibrium effect driven by the expected decline in the price of capital is stronger, muting credit more.

4.4 Importance of the ZLB Constraint

In the baseline LSAPII simulation ("Forward Guidance") news shocks to the Taylor Rule keep the federal funds rate at its steady state level for four quarters . In the "FFR Unconstrained" simulation the federal funds rate is unconstrained. In figures 6 and 7 it is clear that

Figure 5: Quantitative Easing vs a FFR Cut (b)

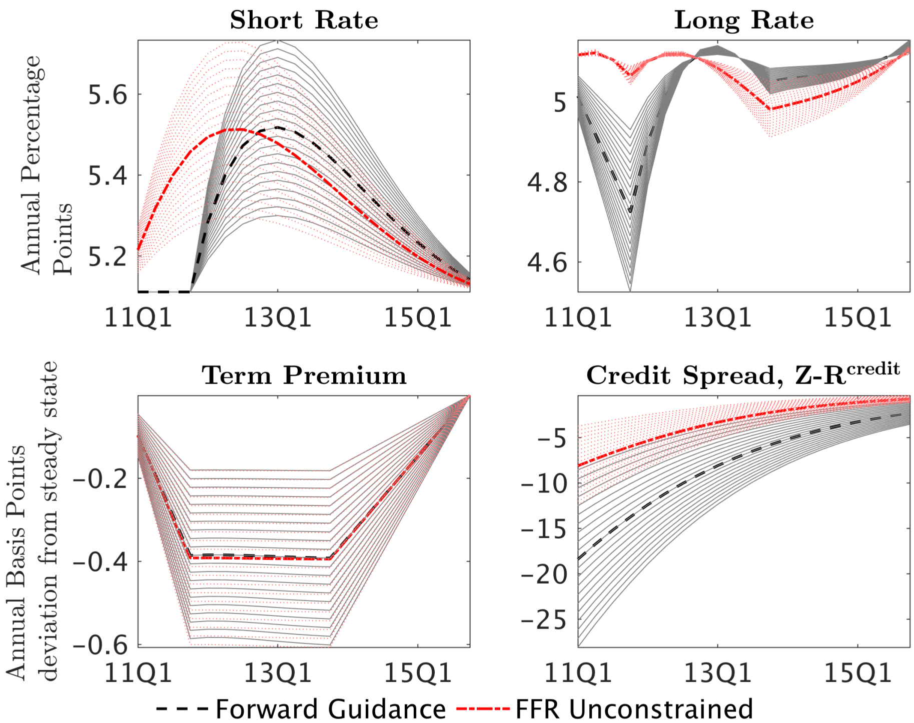


Note: The thick dashed lines correspond to the simulation at the median value for the elasticity of the term premium (ν).

the forward guidance amplifies the stimulus effect of QE. Output growth is between 0.24 and 0.79 percentage points higher under forward guidance than no forward guidance. Inflation is between 14 and 47 annualized basis points higher under QE with forward guidance than QE without forward guidance. Forward guidance amplifies QE's effect on the credit spread: it falls by about 10 basis points more at the median value for the elasticity of the term premium with forward guidance than without.

The intuition is simply that QE is inflationary and expansionary. If the short rate is not constrained, the Taylor Rule drives the central bank to raise rates in response to the effects of the QE program: moderating QE's stimulus effect but does not qualitatively change the impact of QE on most series.

Figure 6: QE With and Without Forward Guidance (a)



Note: The thick dashed lines correspond to the simulation at the median value for the elasticity of the term premium (ν).

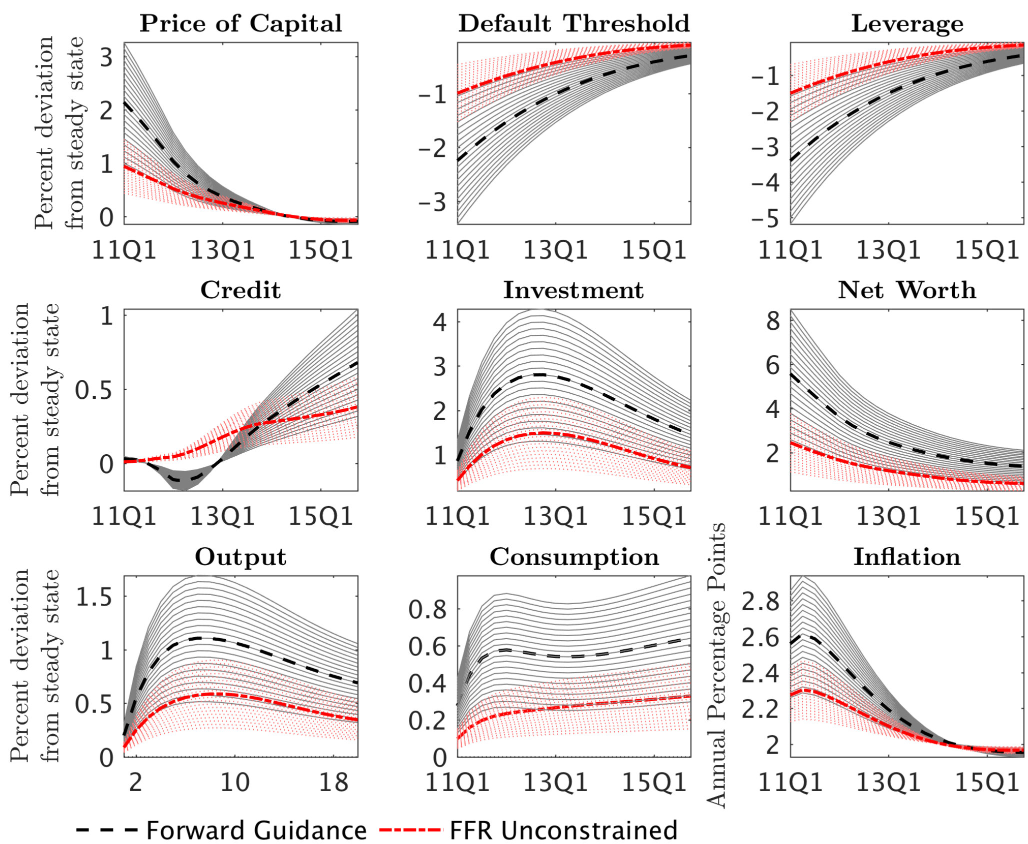
The exception here is the response of credit. Without forward guidance ("FFR Unconstrained") the increase in the federal funds rate is front loaded. And because the federal funds rate is never constrained the long term government bond rate does not experience as dramatic a decline. This substantially mutes the on impact response of entrepreneurial net worth (a 2.5% jump over steady state as compared to 5.6%, at the median ν value). A lower net worth means that the entrepreneurs rely initially more on credit.

4.5 The Costly-State Verification Friction Amplifies QE

The CSV friction boosts output growth between 0.13 and 0.44 percentage points, relative to the LSAPII simulation without the friction ($\mu = 0$, "No CSV"). The CSV friction boosts inflation between 6 and 20 annualized basis points and boosts investment between 0.59 and 1.94 percentage points.

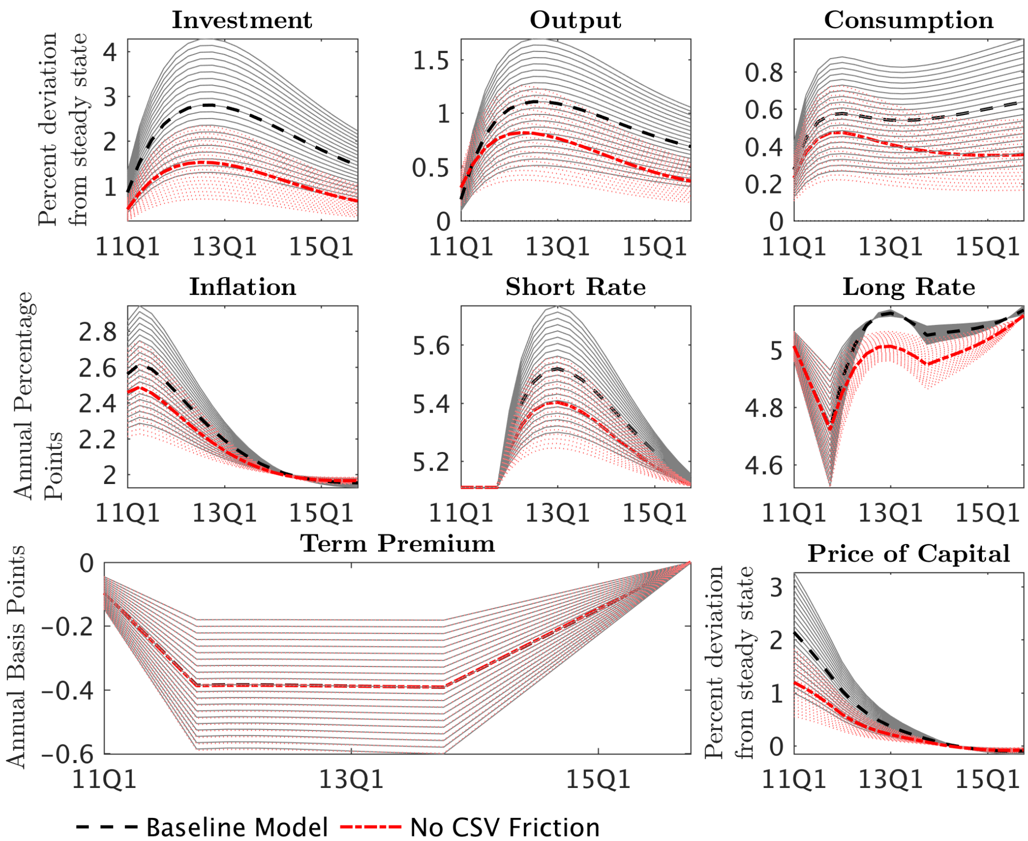
Note that in the "No CSV" model the spread between the expected return on capital and the long term government bond rate is zero[7]. So even in the "No CSV" model results presented in figures 8 QE has a strong quantitative impact because the long term interest rate is directly related to the return on capital, and QE directly depresses the long term interest rate.

Figure 7: QE With and Without Forward Guidance (b)



Note: The thick dashed lines correspond to the simulation at the median value for the elasticity of the term premium (ν).

Figure 8: The Costly State Verification Friction Amplifies the Portfolio Balance Channel



Note: The thick dashed lines correspond to the simulation at the median value for the elasticity of the term premium (ν).

4.6 Impact of Investors' Preferred Habitat Preferences

The following repeats the baseline QE simulation (LSAPII + ZLB constraint) in a version of the model where investors arbitrage between diversified entrepreneurial debt and short term government bonds ("Short Maturity Investor"). These results indicate that expanding the portfolio balance channel to include corporate debt[8] boosts the quantitative impact of QE substantially. In the baseline model output growth is between 0.4 - 1.34 % points more

[7]The entrepreneur's optimal choice of the default rate is reduced to $E\left[R^k_{t+1} - R^L_t\right] = 0$. In the Christiano et al. (2014) model this condition will be $E\left[R^k_{t+1} - R_t\right] = 0$. Without the portfolio preference, this would be equivalent to a standard DSGE model (eg Christiano et al. (2005)) where households choose capital investment.

[8]In the language of Krishnamurthy and Vissing-Jorgensen (2011): the Safety Channel and Duration Risk Channel.

than in the Short Maturity Investor model. In the baseline model inflation is between 20 and 68 annualized basis points more than in the Short Maturity Investor model. Dropping investors' preferred habitat for long term assets completely eliminates the lagged positive response of credit.

The only series that is not substantially dampened is consumption. QE does have a role without the expanded portfolio balance channel – in impacting the inter-temporal consumption decisions of households via the Euler equation for long bonds. But as shown by Chen et al. (2012) this effect is quantitatively limited. These results indicate that the addition of the costly state verification financial friction alone does not substantially boost this effect.

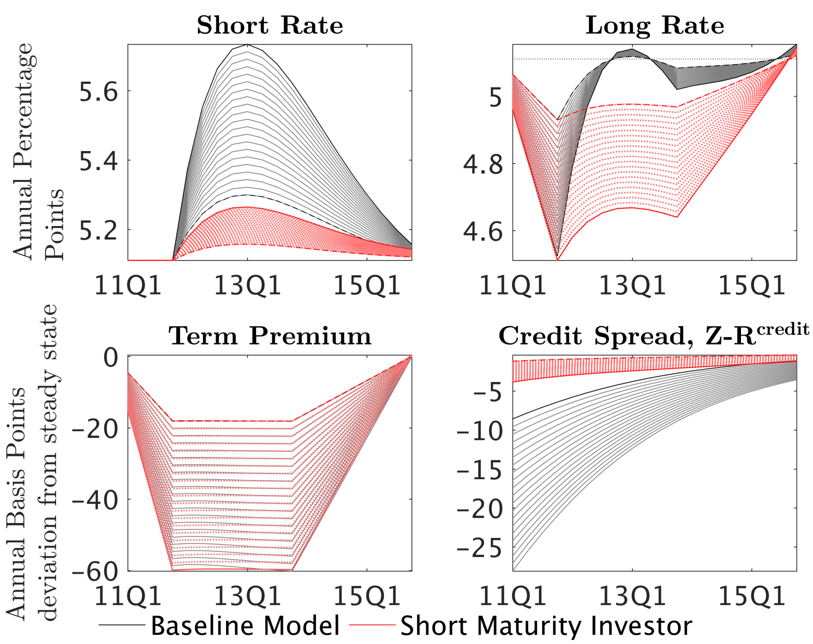
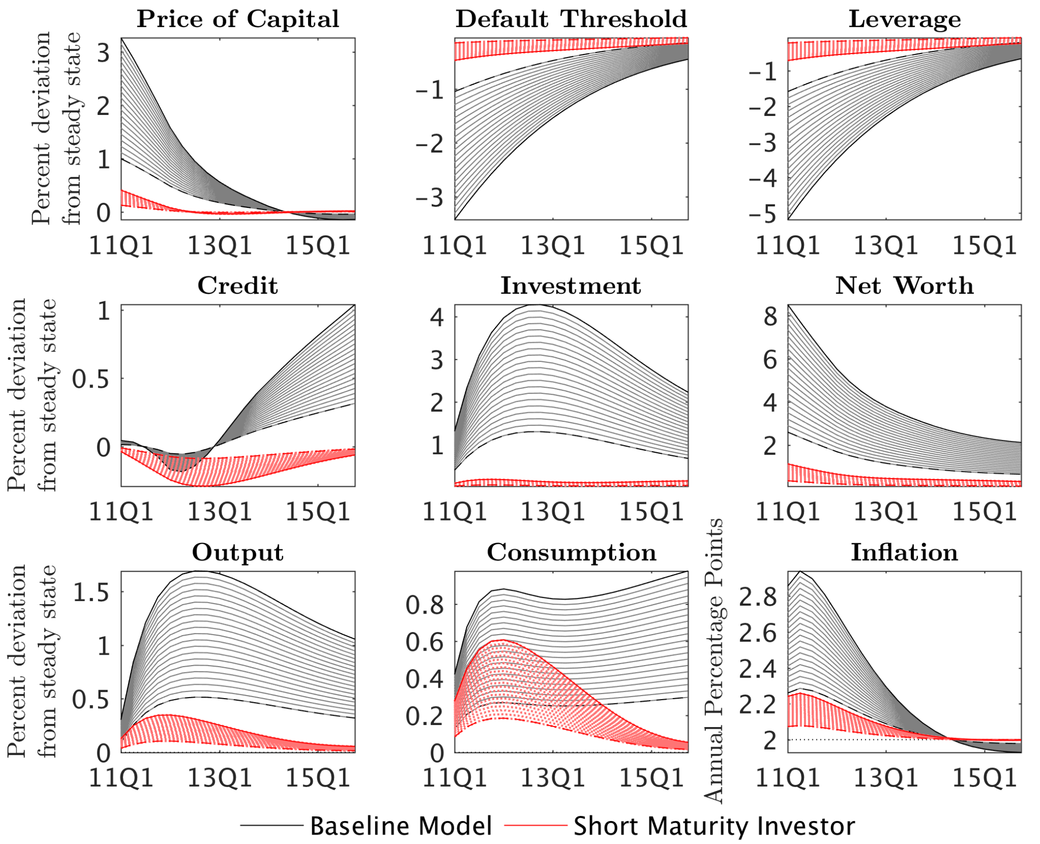


Figure 9: Impact of Investor Preferences (a)

Note: the credit spread in the short model is $Z - R$ and the credit spread in the long model is $Z - R^L$. The solid lines correspond to the models simulated at the high end of the range for the elasticity of the term premium (ν). The dashed lines correspond to the models simulated at the low end of the range for the elasticity of the term premium.

Figure 10: Impact of Investor Preferences (b)



Note: The solid lines correspond to the models simulated at the high end of the range for the elasticity of the term premium (ν). The dashed lines correspond to the models simulated at the low end of the range for the elasticity of the term premium.

5 Conclusion

I find that the second round of the Federal Reserve's Large Scale Asset Purchase Program (LSAPII) boosted output between 0.51% and 1.69% and inflation by 28 to 94 annualized basis points. Expanding the portfolio balance channel to include investors who have preferred habitat preferences for long term assets (both government and corporate debt) drives a substantial portion of the effect of QE (75% of the boost to output and 71% of the boost to inflation). Financial frictions (in this case the costly state verification friction developed by Bernanke et al., 1999) play an important quantitative role in amplifying the effect of QE (about 25% of the boost to output and 21% of the boost to inflation). QE is more effective when the short rate is constrained (i.e. at the zero lower bound) but still has a quantitatively important impact on output and inflation away from the zero lower bound: 46% of the boost to output and 50% of the boost to inflation remains. Lastly, LSAPII is the equivalent of a substantial cut to the Federal Funds Rate (between 83 - 279 annualized basis points). When the zero lower bound binds central banks can continue to achieve their objectives via the use of unconventional monetary policy.

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A Characterization of the Equilibrium

Lowercase variables are real de-trended variables. So if X_t is a nominal variable then $x_t \equiv X_t/(P_t z_t^*)$.

Table 5: Notation Key

$q_t \equiv \Upsilon^t \frac{Q_{\bar{K},t}}{P_t}$	$y_{z,t} \equiv \frac{Y_t}{z_t^*}$	$i_t \equiv \frac{I_t}{z_t^* \Upsilon^t}$
$\tilde{w}_t \equiv \frac{W_t}{z_t^* P_t}$	$\bar{k}_t \equiv \frac{\bar{K}_t}{z_{t-1}^* \Upsilon^{t-1}}$	$\mu_{z,t}^* \equiv \frac{z_t^*}{z_{t-1}^*}$
$c_t \equiv \frac{C_t}{z_t^*}$	$b_t \equiv \frac{B_t}{z_t^* P_t}$	$b_t^L \equiv \frac{B_t^L}{z_t^* P_t}$
$g_t \equiv \frac{G_t}{z_t^*}$	$t_t \equiv \frac{T_t}{z_t^* P_t}$	$n_{t+1} \equiv \frac{N_{t+1}}{z_t^* P_t}$
$b_t^{credit} \equiv \frac{B_t^{credit}}{z_t^* P_t}$	$\lambda_{z,t} \equiv \lambda_t P_t z_t^*$	$v_t^L \equiv \frac{V_t^L}{z_t^* P_t}$
$\Lambda_{zt,t+1} \equiv \frac{\beta}{\pi_{t+1}\mu_{z,t+1}^*} \frac{\lambda_{z,t+1}}{\lambda_{z,t}}$		

A.1 Auxiliary Expressions:

Aux 1: Index term in price updating for firms who cannot re-optimize

$$\tilde{\pi}_t \equiv \left(\pi_t^{\text{target}}\right)^{\iota} \pi_{t-1}^{1-\iota}.$$

Aux 2: $\tilde{\pi}_t$ ahead 1 period

$$\tilde{\pi}_{t+1} \equiv \left(\pi_{t+1}^{\text{target}}\right)^{\iota} \pi_t^{1-\iota}.$$

Aux 3: Definition of $K_{p,t}$:

$$K_{p,t} \equiv F_{p,t} \left[\frac{1 - \xi_p \left(\frac{\tilde{\pi}_t}{\pi_t}\right)^{\frac{1}{1-\lambda_{f,t}}}}{1 - \xi_p} \right]^{1-\lambda_{f,t}}.$$

Aux 4: $K_{p,t}$ ahead 1 period

$$K_{p,t+1} \equiv F_{p,t+1} \left[\frac{1 - \xi_p \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_{f,t+1}}}}{1 - \xi_p} \right]^{1-\lambda_{f,t+1}}.$$

Aux 5: Index term in wage updating for non-reoptimizing unions

$$\tilde{\pi}_{w,t} \equiv \left(\pi_t^{\text{target}}\right)^{\iota_w} \left(\pi_{t-1}\right)^{1-\iota_w}.$$

Aux 6: $\tilde{\pi}_{w,t}$ ahead 1 period

$$\tilde{\pi}_{w,t+1} \equiv \left(\pi_{t+1}^{\text{target}}\right)^{\iota_w} \left(\pi_t\right)^{1-\iota_w}.$$

Aux 7: Wage Inflation

$$\pi_{w,t} \equiv \pi_t \mu^*_{z,t} \frac{\tilde{w}_t}{\tilde{w}_{t-1}}.$$

Aux 8: $\pi_{w,t}$ ahead 1 period

$$\pi_{w,t+1} \equiv \pi_{t+1} \mu^*_{z,t+1} \frac{\tilde{w}_{t+1}}{\tilde{w}_t}.$$

Aux 9: Definition of $K_{w,t}$

$$K_{w,t} \equiv \frac{\tilde{w}_t F_{w,t}}{\psi_L} \left[\frac{1 - \xi_w \left(\frac{\tilde{\pi}_{w,t} (\mu^*_z)^{\iota_\mu} (\mu^*_{z,t})^{1-\iota_\mu}}{\pi_{w,t}} \right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w} \right]^{1-\lambda_w(\sigma_L+1)}.$$

Aux 10: $K_{w,t}$ ahead 1 period

$$K_{w,t+1} \equiv \frac{\tilde{w}_{t+1} F_{w,t+1}}{\psi_L} \left[\frac{1 - \xi_w \left(\frac{\tilde{\pi}_{w,t+1} (\mu^*_z)^{\iota_\mu} (\mu^*_{z,t+1})^{1-\iota_\mu}}{\pi_{w,t+1}} \right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w} \right]^{1-\lambda_w(\sigma_L+1)}.$$

A.2 Distributions

$$\begin{aligned}
F_t(\bar{\omega}_{t+1}) &\equiv CDF\left(\frac{\log(\bar{\omega}_{t+1}) + \frac{1}{2}\sigma_t^2}{\sigma_t} \right), \\
G_t(\bar{\omega}_{t+1}) &\equiv CDF\left(\frac{\log(\bar{\omega}_{t+1}) + \frac{1}{2}\sigma_t^2}{\sigma_t} - \sigma_t \right), \\
G'_t(\bar{\omega}_{t+1}) &\equiv PDF\left(\frac{\log(\bar{\omega}_{t+1}) + \frac{1}{2}\sigma_t^2}{\sigma_t} - \sigma_t \right) \frac{1}{\sigma_t}, \\
\Gamma_t(\bar{\omega}_{t+1}) &\equiv \bar{\omega}_{t+1} \Big[1 - F_t(\bar{\omega}_{t+1}) \Big] + G_t(\bar{\omega}_{t+1}), \\
\Gamma'_t(\bar{\omega}_{t+1}) &= 1 - F_t(\bar{\omega}_{t+1}).
\end{aligned}$$

A.3 Model Equations

Equation 1 (First order condition with respect to consumption):

$$E_t \left\{ \frac{\mu^*_{z,t}}{c_t \mu^*_{z,t} - b c_{t-1}} - \frac{b\beta}{c_{t+1}\mu^*_{z,t+1} - b c_t} - \lambda_{z,t} \right\} = 0. \tag{A.1}$$

Equation 2 (First order condition with respect to the short bond):

$$E_t \left\{ \nu \left(\frac{b_t^L}{b_t} - \delta^b \right) \frac{b_t^L}{b_t^2} - \lambda_{z,t} + \beta \frac{\lambda_{z,t+1}}{\pi_{t+1}\mu^*_{z,t+1}} R_t \right\} = 0. \tag{A.2}$$

Equation 3 (First order condition with respect to the long bond):

$$E_t\Big\{-\nu\Big(\frac{b_t^L}{b_t}-\delta^b\Big)\frac{1}{b_t}-\lambda_{z,t}+\beta\frac{\lambda_{z,t+1}}{\pi_{t+1}\mu^*_{z,t+1}}R^L_{t+1}\Big\}=0. \tag{A.3}$$

Equation 4 (First order condition with respect to investment):

$$\begin{aligned}&E_t\Big\{\lambda_{z,t}\Big(q_t-\frac{1}{\mu_{\Upsilon,t}}\Big)-\lambda_{z,t}q_t\Big[S\Big(\frac{\mu^*_{z,t}\Upsilon i_t}{i_{t-1}}\Big)+S'\Big(\frac{\mu^*_{z,t}\Upsilon i_t}{i_{t-1}}\Big)\frac{\mu^*_{z,t}\Upsilon i_t}{i_{t-1}}\Big] \\ &+\beta\frac{\lambda_{z,t+1}q_{t+1}}{\mu^*_{z,t}\Upsilon}S'\Big(\frac{\mu^*_{z,t+1}\Upsilon i_{t+1}}{i_t}\Big)\Big(\frac{\mu^*_{z,t+1}\Upsilon i_{t+1}}{i_t}\Big)^2\Big\}=0.\end{aligned} \tag{A.4}$$

Equation 5 (Firm's Production Function):

$$y_{z,t}=(p_t^*)^{\frac{\lambda_{f,t}}{\lambda_{f,t}-1}}\Bigg[\epsilon_t\Bigg(\frac{u_t\bar{k}_t}{\mu^*_{z,t}\Upsilon}\Bigg)^{\alpha}\Bigg(h_t(w_t^*)^{\frac{\lambda_w}{\lambda_w-1}}\Bigg)^{1-\alpha}-\phi\Bigg]. \tag{A.5}$$

Equation 6 (Resource Constraint):

$$y_{z,t}=g_t+c_t+\frac{i_t}{\mu_{\Upsilon,t}}+\Theta\frac{1-\gamma}{\gamma}\big(n_{t+1}-w^e\big)+d_t+\frac{a(u_t)\bar{k}_t}{\Upsilon\mu^*_{z,t}}, \tag{A.6}$$

where $d_t\equiv\dfrac{\mu G_{t-1}(\bar{\omega}_t)R_t^k q_{t-1}\bar{k}_t}{\pi_t\mu^*_{z,t}}$.

Equation 7 (Rental Rate of Capital):

$$r_t^k=\alpha\epsilon_t\Bigg(\frac{\mu^*_{z,t}\Upsilon h_t(w_t^*)^{\frac{\lambda_w}{\lambda_w-1}}}{u_t\bar{k}_t}\Bigg)^{1-\alpha}s_t. \tag{A.7}$$

Equation 8 (Marginal Cost):

$$s_t=\frac{1}{\epsilon_t}\Bigg(\frac{r_t^k}{\alpha}\Bigg)^{\alpha}\Bigg(\frac{\tilde{w}_t}{1-\alpha}\Bigg)^{1-\alpha}. \tag{A.8}$$

Equation 9 (Optimal utilization of capital):

$$r_t^k=a'(u_t)=r^k\exp(\sigma_a(u_t-1)), \tag{A.9}$$

where $a(u_t)\equiv\frac{r^k}{\sigma_a}\big[\exp(\sigma_a(u_t-1))-1\big]$.

Equation 10 (Law of motion for capital):

$$\bar{k}_{t+1} = \frac{1-\delta}{\mu^*_{z,t}\Upsilon}\bar{k}_t + \left[1 - S\Big(\frac{i_t \mu^*_{z,t}\Upsilon}{i_{t-1}}\Big)\right] i_t. \tag{A.10}$$

Equation 11 (Rate of return on capital):

$$R^k_t = \frac{u_t r^k_t - a(u_t) + (1-\delta) q_t}{\Upsilon q_{t-1}} \pi_t. \tag{A.11}$$

Equation 12 (Entrepreneurs' FoC wrt $\bar{\omega}_{t+1}$):

$$E_t \Bigg\{ \big[1 - \Gamma_t(\bar{\omega}_{t+1})\big] \frac{R^k_{t+1}}{R^{credit}_{t+1}} + \frac{\Gamma'_t(\bar{\omega}_{t+1})}{\Gamma'_t(\bar{\omega}_{t+1}) - \mu G'_t(\bar{\omega}_{t+1})} \left[\frac{R^k_{t+1}}{R^{credit}_{t+1}} \Big(\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1}) \Big) - 1 \right] \Bigg\} = 0. \tag{A.12}$$

Equation 13 (Evolution of Entrepreneurs' Net Worth):

$$n_{t+1} = \frac{\gamma}{\pi_t \mu^*_{z,t}} \Bigg\{ R^k_t \Big(1 - \mu G_{t-1}(\bar{\omega}_t)\Big) - R^{credit}_t \Bigg\} \bar{k}_t q_{t-1} + w^e + \gamma \frac{R^{credit}_t}{\pi_t \mu^*_{z,t}} n_t. \tag{A.13}$$

Equation 14 (Mutual Funds Zero-Profit Condition):

$$\frac{q_t \bar{k}_{t+1}}{n_{t+1}} \frac{R^k_{t+1}}{R^{credit}_{t+1}} \Big[\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})\Big] - \frac{q_t \bar{k}_{t+1}}{n_{t+1}} + 1 = 0. \tag{A.14}$$

Equation 15 (AR(1) for the supply of long term government bonds):

$$\log\left(\frac{b^L_t}{b^L}\right) = \rho_{bL} \log\left(\frac{b^L_{t-1}}{b^L}\right) + u^{bL}_t. \tag{A.15}$$

Equation 16 (Taylor Rule):

$$\log\left(\frac{R_t}{R}\right) = \rho_m \log\left(\frac{R_{t-1}}{R}\right) + \big(1 - \rho_m\big) \left[\phi_\pi \log\left(\frac{\pi_t}{\pi^{target}_t}\right) + \frac{\phi_y}{4} \left(\log\frac{y_{z,t}}{y_z} - \log\frac{\mu^*_z}{\mu^*_{z,t}} \right) \right] + \frac{1}{400}\epsilon^m_t. \tag{A.16}$$

Equation 17 (Shock equation for σ_t):

$$\log\left(\frac{\sigma_t}{\sigma}\right) = \rho_\sigma \left(\frac{\sigma_{t-1}}{\sigma}\right) + \epsilon_{\sigma,t}. \tag{A.17}$$

Equation 18 (Φ_t = its value in steady state):

$$\phi = steady\_state(\Phi). \tag{A.18}$$

<u>Equations Related to Price Setting:</u>

Equation 19 (Law of motion for p_t^*):

$$p_t^* = \left[(1-\xi_p)\left(\frac{K_{p,t}}{F_{p,t}}\right)^{\frac{\lambda_{f,t}}{1-\lambda_{f,t}}} + \xi_p\left(\frac{\tilde{\pi}_t}{\pi_t}p_{t-1}^*\right)^{\frac{\lambda_{f,t}}{1-\lambda_{f,t}}}\right]^{\frac{1-\lambda_{f,t}}{\lambda_{f,t}}}. \tag{A.19}$$

Equation 20 (Law of motion for $F_{p,t}$, relates to Calvo Frictions):

$$F_{p,t} = E_t\left\{\lambda_{z,t}y_{z,t} + \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_{f,t+1}}}\beta\xi_p F_{p,t+1}\right\}. \tag{A.20}$$

Equation 21 (Law of motion for $K_{p,t}$):

$$K_{p,t} = E_t\left\{\lambda_{z,t}\lambda_{f,t}y_{z,t}s_t + \beta\xi_p\left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}}\right)^{\frac{\lambda_{f,t}}{1-\lambda_{f,t}}}K_{p,t+1}\right\}. \tag{A.21}$$

Equation 22 (Law of motion for $F_{w,t}$, characterizes optimal wage setting):

$$F_{w,t} = E_t\left\{\lambda_{z,t}(w_t^*)^{\frac{\lambda_w}{\lambda_w-1}}\frac{h_t}{\lambda_w} + \beta\xi_w(\mu_z^*)^{\frac{1-\iota_\mu}{1-\lambda_w}}(\mu_{z,t+1}^*)^{\frac{\iota_\mu}{1-\lambda_w}-1}\left(\frac{1}{\pi_{w,t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}}\frac{\tilde{\pi}_{w,t+1}^{\frac{1}{1-\lambda_w}}}{\pi_{t+1}}F_{w,t+1}\right\}. \tag{A.22}$$

Equation 23 (Law of motion for $K_{w,t}$):

$$K_{w,t} = E_t\left\{\left[(w_t^*)^{\frac{\lambda_w}{\lambda_w-1}}h_t\right]^{1+\sigma_L} + \beta\xi_w K_{w,t+1}\left(\frac{\tilde{\pi}_{w,t+1}(\mu_{z,t+1}^*)^{\iota_\mu}(\mu_z^*)^{1-\iota_\mu}}{\pi_{w,t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)}\right\}. \tag{A.23}$$

Equation 24 (Law of motion for w_t^*):

$$w_t^* = \left[(1-\xi_w)\left(\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}(\mu_z^*)^{1-\iota_\mu}(\mu_{z,t}^*)^{\iota_\mu}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right)^{\lambda_w} + \xi_w\left(\frac{\tilde{\pi}_{w,t}(\mu_{z,t}^*)^{\iota_\mu}(\mu_z^*)^{1-\iota_\mu}}{\pi_{w,t}}w_{t-1}^*\right)^{\frac{\lambda_w}{1-\lambda_w}}\right]^{\frac{1-\lambda_w}{\lambda_w}}. \tag{A.24}$$

Equation 25 (Entrepreneurs' balance sheet):

$$q_t\bar{k}_{t+1} = B_t^{credit} + n_{t+1}. \tag{A.25}$$

Equation 26 (Definition of leverage, L_t):

$$L_t = \frac{n_{t+1} + B_t^{credit}}{n_{t+1}}. \tag{A.26}$$

Equation 27 (Real government bonds):

$$b_t^L = v_t^L B_t^c. \tag{A.27}$$

Equation 28 (Long Rate):

$$R_{t+1}^L = \frac{1 + V_{t+1}^L}{V_t^L}. \tag{A.28}$$

Equation 29 (Entrepreneurial debt rate):

$$Z_{t+1} = R_{t+1}^k \bar{\omega}_{t+1} \frac{L_t}{L_t - 1}. \tag{A.29}$$

<u>Fiscal Policy Block:</u>

Equation 30 (Government Spending Rule):

$$g_t = g_{yz} steady\_state(y_z). \tag{A.30}$$

Equation 31 (Government Budget Constraint):

$$b_t + b_t^L = \frac{R_{t-1} b_{t-1}}{\pi_t \mu_{z,t}^*} + \frac{R_t^L b_{t-1}^L}{\pi_t \mu_{z,t}^*} + g_t - t_t. \tag{A.31}$$

Equation 32 (Fiscal Rule):

$$t_t - g_t = \kappa \left(\frac{b_{t-1} + b_{t-1}^L}{b + b^L} \right)^{\phi_T} \epsilon_t^T. \tag{A.32}$$

Equation 33 (Investor Arbitrage Condition):

$$\Lambda_{zt,t+1} \Big[R_{t+1}^{credit} - R_{t+1}^L \Big] = 0. \tag{A.33}$$

Equation 34 (Shock equation for ϵ_t):

$$\log \left(\frac{\epsilon_t}{\epsilon} \right) = \rho_\epsilon \left(\frac{\epsilon_{t-1}}{\epsilon} \right) + \epsilon_{\epsilon,t}. \tag{A.34}$$

Equation 35 (Shock equation for $\mu^*_{z,t}$):

$$\log\left(\frac{\mu^*_{z,t}}{\mu^*_z}\right) = \rho_{\mu^*_z}\left(\frac{\mu^*_{z,t-1}}{\mu^*_z}\right) + \epsilon_{\mu^*_z,t}. \tag{A.35}$$

B News Shocks

The calibration method of the anticipated news shocks is based on Del Negro, Giannoni and Patterson (2013) and Laséen and Svensson (2011). This appendix section specifies where news shocks appear in the model, the news shock structure, and the process of calibrating news shocks to match a given path for the short rate.

B.1 Where is the News?

News in this model appears as shocks to the Taylor rule:

$$\log\left(\frac{R_t}{R}\right) = \rho_m \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_m)\left[\phi_\pi \log\left(\frac{\pi_t}{\pi_t^{target}}\right) + \frac{\phi_y}{4}\left(\log\frac{y_{z,t}}{y_s} - \log\frac{\mu^*_z}{\mu^*_{z,t}}\right)\right] + \frac{1}{400}u^m_t, \tag{B.1}$$

and the long term government bond supply rule:

$$\log\left(\frac{b^L_t}{b^L}\right) = \rho_{BL}\log\left(\frac{b^L_{t-1}}{b^L}\right) + u^{BL}_t. \tag{B.2}$$

In the model simulations $\rho_{BL} = 0$, so that the entire path of bond purchases, holding, and unwinding in the quantitative easing program is set via surprise and news shocks.

B.2 News Shock Structure

A generic news shock has the following representation:

$$u_t = \epsilon_t + \xi_{1,t-1} + \xi_{2,t-2} + ... + \xi_{p,t-p}, \tag{B.3}$$

where ϵ_t is the unanticipated shock and $\xi_{p,t-p}$ for $p \geq 1$ are the anticipated news shocks. The shock $\xi_{p,t-p}$ is observed by agents in period $t-p$, but does not affect the relevant sum of shocks until period t.

B.3 Forward Guidance Implementation

Dynare has the following state-space representation of the model:

- s_t = m x 1 vector of states. (m = M_.npred + M_.nboth).
- x_t = n x 1 vector of controls. (n = M_.nstatic + M_.nfwrd).
- ϵ_t = w x 1 vector of shocks. (w = M_.exo_nbr).
- Φ = (n x m), the policy function.

State-space representation:

$$s_t = \underset{mxm}{A} s_{t-1} + \underset{mxw}{B} \epsilon_t, \tag{B.4}$$

$$x_t = \Phi s_t. \tag{B.5}$$

Substitute (B.4) into (B.5):

$$s_t = As_{t-1} + B\epsilon_t,$$
$$x_t = \Phi As_{t-1} + \Phi B\epsilon_t.$$

Define $\underset{nxm}{C} \equiv \Phi A$ and $\underset{nxw}{D} \equiv \Phi B$. And rewrite the state-space system:

$$s_t = As_{t-1} + B\epsilon_t,$$
$$x_t = Cs_{t-1} + D\epsilon_t.$$

Stack the system and collapse: $Y_t = \begin{bmatrix} s_t \\ x_t \end{bmatrix}, \qquad \Psi = \begin{bmatrix} A \\ C \end{bmatrix}, \qquad \Omega = \begin{bmatrix} B \\ D \end{bmatrix}.$

$$Y_t = \Psi s_{t-1} + \Omega\epsilon_t. \tag{B.6}$$

- $\Psi = oo_.dr.ghx$ = (m+n) $\times$ m, matrix of coefficients that appears in the Dynare generated transition rule. (# of endogenous variables = m+n, by # of state variables = m).

- $\Omega = oo_.dr.ghu$ = (m+n) $\times$ w, matrix of coefficients that appears in the Dynare generated transition rule. It has dimension (# of endogenous variables by # of shocks).

Let Z be a matrix ($m \times (m+n)$) that selects the state variables from the Y_t matrix, so that $s_t = ZY_t$. And define $M \equiv \Psi Z$. So we can rewrite B.6 as:

$$Y_t = MY_{t-1} + \Omega\epsilon_t.$$

Split the shock vector:

$$\epsilon_t = \epsilon_t^1 + \epsilon_t^2, \tag{B.7}$$

where ϵ_t^1 is a $w \times 1$ vector and where all shocks, except the monetary policy forward guidance shocks are replaced with zeros. And ϵ_t^2 is a $w \times 1$ vector where all but the monetary policy forward guidance shocks are replaced with zeros. So can further rewrite B.6 as:

$$Y_t = MY_{t-1} + \Omega\Big[\epsilon_t^1 + \epsilon_t^2\Big]. \tag{B.8}$$

Both the forward guidance shocks and the QE shock, that hit at $t = 1$, so $\epsilon_t = 0\ \forall t > 1$. Note that ϵ_1^1 is known (this vector contain the shocks to the long bond supply rule that introduce QE). Further note that ϵ_1^2 is the vector of shocks to be calibrated to produce the target path for the policy rate.

Using equation B.8 note that:

$$Y_1 = MY_0 + \Omega\Big[\epsilon_1^1 + \epsilon_1^2\Big],$$

where Y_0 is the steady state (also known). Iterating forward:

$$Y_2 = MY_1 = M^2Y_0 + M\Omega\Big[\epsilon_1^1 + \epsilon_1^2\Big].$$

Generally:

$$Y_t = M^tY_0 + M^{t-1}\Omega\Big[\epsilon_1^1 + \epsilon_1^2\Big].$$

Let $\mathbf{R}^{FG}$ be the $t_{FG} \times 1$ target vector for the path of the policy rate, where t_{FG} is the number of periods that the policy rate is constrained (in the LSAPII simulation $t_{FG} = 4$). Let $\tilde{Z}$ be a $1 \times (n + m)$ row vector that selects the row of Y_t corresponding to the policy rate (in Dynare this is the DR ordering of R_t).

$$
\begin{aligned}
R_1 &= \tilde{Z} M Y_0 + \tilde{Z}\Omega\epsilon_1^1 + \tilde{Z}\Omega\epsilon_1^2, \\
R_2 &= \tilde{Z} M^2 Y_0 + \tilde{Z} M\Omega\epsilon_1^1 + \tilde{Z} M\Omega\epsilon_1^2, \\
R_3 &= \tilde{Z} M^3 Y_0 + \tilde{Z} M^2\Omega\epsilon_1^1 + \tilde{Z} M^2\epsilon_1^2, \\
R_4 &= \tilde{Z} M^4 Y_0 + \tilde{Z} M^3\Omega\epsilon_1^1 + \tilde{Z} M^3\Omega\epsilon_1^2.
\end{aligned}
$$

Stack:

$$
\begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix} = \underbrace{\begin{bmatrix} \tilde{Z} M Y_0 + \tilde{Z}\Omega\epsilon_1^1 \\ \tilde{Z} M^2 Y_0 + \tilde{Z} M\Omega\epsilon_1^1 \\ \tilde{Z} M^3 Y_0 + \tilde{Z} M^2\Omega\epsilon_1^1 \\ \tilde{Z} M^4 Y_0 + \tilde{Z} M^3\Omega\epsilon_1^1 \end{bmatrix}}_{\mathbf{K}} + \underbrace{\begin{bmatrix} \tilde{Z}\Omega \\ \tilde{Z} M \\ \tilde{Z} M^2 \\ \tilde{Z} M^3\Omega \end{bmatrix}}_{\mathbf{J}} \epsilon_1^2. \tag{B.9}
$$

Set the vector of policy rates equal to the target path $\mathbf{R}_{FG}$:

$$
\begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix} = \begin{bmatrix} R_1^{FG} \\ R_2^{FG} \\ R_3^{FG} \\ R_4^{FG} \end{bmatrix} \equiv \mathbf{R}^{FG}. \tag{B.10}
$$

$$
\mathbf{R}^{FG} = \mathbf{K} + \mathbf{J}\epsilon_1^2. \tag{B.11}
$$

and solve for ϵ_1^2:

$$
\epsilon_1^2 = \mathbf{J}^{-1}\Big(\mathbf{R}^{FG} - \mathbf{K}\Big). \tag{B.12}
$$

C Data Appendix

I use US quarterly data from 1985Q1 to 2007Q3 (the period before the NBER recession start date). The following series come from the Federal Reserve Bank of St. Louis' Federal Reserve Economic Data (FRED): Gross Domestic Product (GDP), and Effective Federal Funds Rate (FEDFUNDS). Data on the nominal value of privately-held marketable interest-bearing US public debt come from the Haver/DLX USECON database. The data are broken down by time until maturity. The long term government bonds are defined as bonds with over 1 year until maturity, and the short term government bonds are defined as bonds with less than 1 year left until maturity.

Table 6: Haver/DLX USECON Data Codes

Haver/DLX Code	**Data Series**
PDIMP	Total
PDIMPL	Less than 1 year left until maturity
PDIMP1	1 to 5 years left until maturity
PDIMP5	5 to 10 years left until maturity
PDIMP10	10 to 20 years left until maturity
PDIMP20	over 20 years left until maturity

The size of a \$100 billion USD purchase of long term bonds is x% of the steady state quantity of long term bonds, where x is calibrated as follows:

$$x = \frac{100}{bL\_yz \times \frac{\text{2007q3 GDP}}{4}}, \tag{C.1}$$

where bL_yz is the steady state ratio of long term bonds to (quarterly) output, and 2007Q3 GDP is in annual terms. \$100 billion USD purchase of long term bonds is equivalent to 3.55% of steady state de-trended long bonds (b^L). So the \$600 billion USD purchase of long term bonds in LSAPII is equivalent to 21.3% of steady state de-trended long bonds.

Table 7: Parameters Corresponding to Targets

Parameter	Description	Calibration
β	discount rate	0.9964
b^L_yz	steady state ratio of long term government bonds to output	0.77
δ^b	steady state ratio of long to short bond holdings	1.86
g_yz	steady state ratio of government spending to output	0.2
κ	steady state primary fiscal surplus	0.0143
π^{target}	steady state target inflation	1.005
ψ_L	disutility weight on labor	1.2126
ν	elasticity of the term premium to the bond ratio	0.00074-0.0025

C.1 Calibrating the 1% of GDP QE Shock

To standardize the QE shocks in table 1 I do the following. First, the long-run ratio of long term government bonds to quarterly GDP implies that long term government bonds are 19.25% of annual GDP. This means that 1% of GDP is equivalent to 5.19% of steady state detrended long term government bonds. Given the preceding calculations (\$100 bn $\approx$ 3.55% of steady state de-trended long term government bonds) this means 5.19% $\approx$ \$146 bn. I standardize the results reported in the papers in table 1 using this number. The long term US Treasuries purchases in LSAPI are approximately a 2% of GDP QE shock, and LSAPII is approximately a 4% of GDP QE shock.

D Derivation of ν

The $\nu \equiv \tilde{\nu}\delta^b(1+\delta^b)/(\lambda_z b)$ is the elasticity of the term premium with respect to the relative supply of long to short term government debt. The ν parameter governs the responsiveness of the term premium to changes in the relative supply of long versus short term bonds. The following shows the log-linearization of the key equations (Model equations 2 & 3) around the steady state used to calibrate the partial equilibrium response.

First order condition with respect to the short bond:

$$f^a \equiv E_t\left\{\tilde{\nu}\left(\frac{b_t^L}{b_t} - \delta^b\right)\frac{b_t^L}{b_t^2} - \lambda_{z,t} + \beta\frac{\lambda_{z,t+1}}{\pi_{t+1}\mu^*_{z,t+1}}R_t\right\} = 0.$$

First order condition with respect to the long bond:

$$f^b \equiv E_t\Big\{ - \tilde{\nu}\Big(\frac{b_t^L}{b_t} - \delta^b\Big)\frac{1}{b_t} - \lambda_{z,t} + \beta\frac{\lambda_{z,t+1}}{\pi_{t+1}\mu^*_{z,t+1}}R^L_{t+1}\Big\} = 0.$$

Log-linearizing around the steady state:

$$f^a \approx \lambda_z\Big[\hat{\lambda}_{z,t+1} - \hat{\lambda}_{z,t} + \hat{R}_t - \hat{\pi}_{t+1} - \hat{\mu}^*_{z,t+1}\Big] + \frac{\tilde{\nu}(\delta^b)^2}{b}(\hat{b}_t^L - \hat{b}_t), \tag{D.1}$$

$$f^b \approx \lambda_z\Big[\hat{\lambda}_{z,t+1} - \hat{\lambda}_{z,t} + \hat{R}^L_{t+1} - \hat{\pi}_{t+1} - \hat{\mu}^*_{z,t+1}\Big] - \frac{\tilde{\nu}\delta^b}{b}(\hat{b}_t^L - \hat{b}_t). \tag{D.2}$$

This implies:

$$\hat{R}^L_{t+1} - \hat{R}_t = \frac{1}{\lambda_z}\frac{\tilde{\nu}\delta^b}{b}(1+\delta^b)\big[\hat{b}_t^L - \hat{b}_t\big] = \nu\big[\hat{b}_t^L - \hat{b}_t\big]. \tag{D.3}$$

E Additional Results

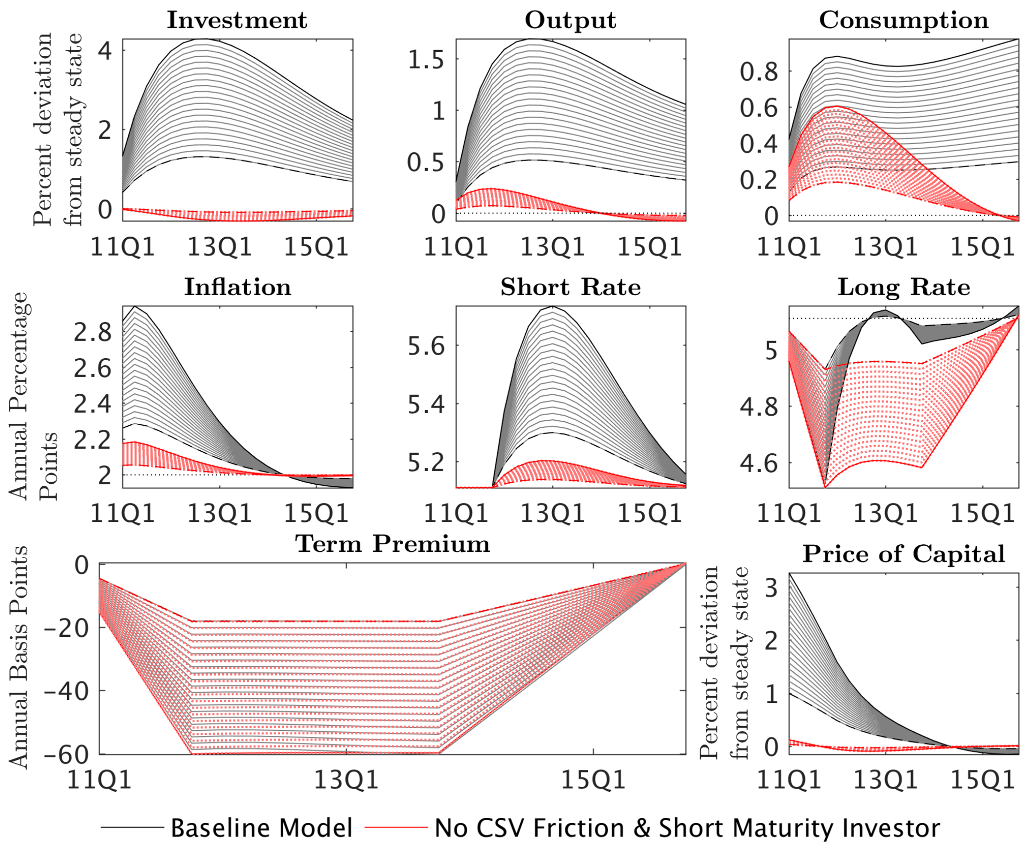
E.1 Narrow Portfolio Balance Channel Without CSV Friction

The "No CSV & Short Maturity Investor" model is a nested version of the baseline model. The state verification cost is set to zero ($\mu = 0$), meaning that there is no external finance premium for entrepreneurs. This removes the accelerator effect from variations in entrepreneurial net worth. Additionally the assumption that investors have a preferred habitat preference for long term assets is dropped. This means investors arbitrage between diversified entrepreneurial debt and short term government bonds[9]. This shuts off the Default Risk Channel, the Duration Risk Channel, and restricts the Safety Channel.

Output is boosted in the baseline model of this paper relative to the No CSV & Short Maturity Investor model by 0.44% to 1.45%. Inflation is boosted between 23 to 77 annualized basis points. Investment growth is boosted between 1.3- 4.3 percentage points. In the No CSV & Short Maturity Investor model the peak growth of output is between 0.07% - 0.23%,

[9]The two changes imply that the entrepreneurs' first order condition with respect to the default threshold becomes $E\Big[R^k_{t+1} - R_t\Big] = 0$, making this model comparable to a model in which households directly invest in capital - eg Chen et al. (2012)

Figure 11: Baseline vs Narrow PBC Without CSV Friction



Note: The solid lines correspond to the models simulated at the high end of the range for the elasticity of the term premium (ν). The dashed lines correspond to the models simulated at the low end of the range for the elasticity of the term premium.

and inflation grows between 5 and 18 annualized basis points. These results are in the range of the small quantitative impact of LSAPII found in Chen et al. (2012).

E.2 Convenience Yield

This section tests the robustness of the quantitative role of QE to an attenuated pass-through from long term government bonds to corporate bond yields. Krishnamurthy and Vissing-Jorgensen (2012) find that US Treasuries have attributes such that the spread between US Treasuries and AAA rated corporate debt depends on the supply of US Treasuries. In particular they find that for a 1% increase in long term debt-to-GDP the AAA to 10 Year US Treasury spread increases by 75 basis points. I capture that by incorporating a "convenience yield" preference (ζ_t^p) in the investor's linear utility function (E.4). Where cy is calibrated to match the empirical evidence in Krishnamurthy and Vissing-Jorgensen (2012).

$$\max_{B_t^{L,i}, B_t^{credit,i}} \quad E_t \left\{ (R_{t+1}^L + \zeta_t^p) B^{L,t} + R_{t+1}^{credit} B_t^{credit} \right\}, \tag{E.4}$$

subject to their balance sheet identity:

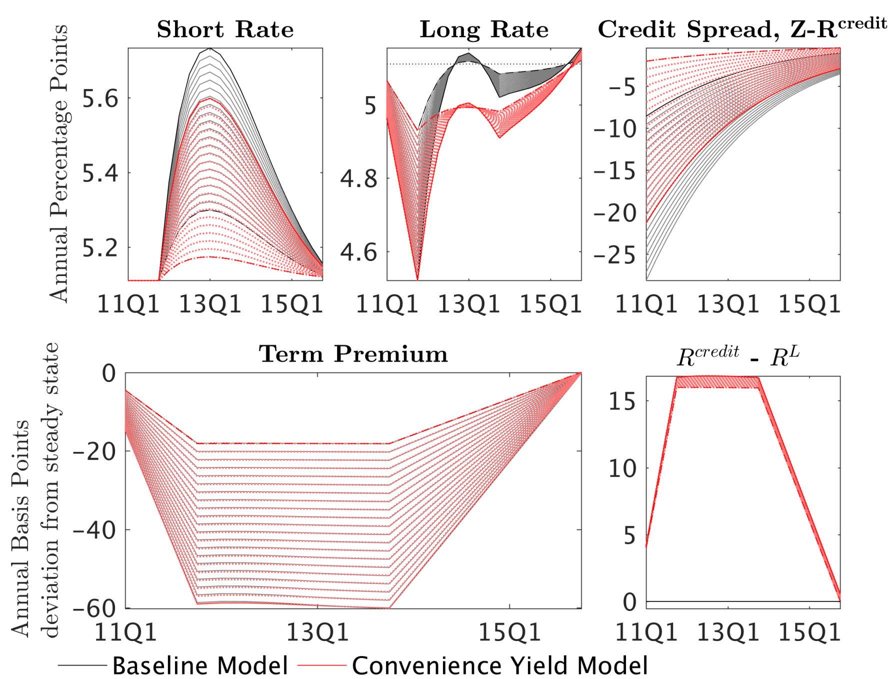
$$\tau_t = B_t^{L,i} + B_t^{credit,i}, \tag{E.5}$$

where:

$$\zeta_t^p \equiv - \underbrace{cy}_{>0} \left[\log \frac{B_t^L}{Y_t} - \log \frac{B^L}{Y} \right]. \tag{E.6}$$

The investor's optimality condition (E.7) shows that ζ_t^p is the spread between diversified

Figure 12: Convenience Yield (a)



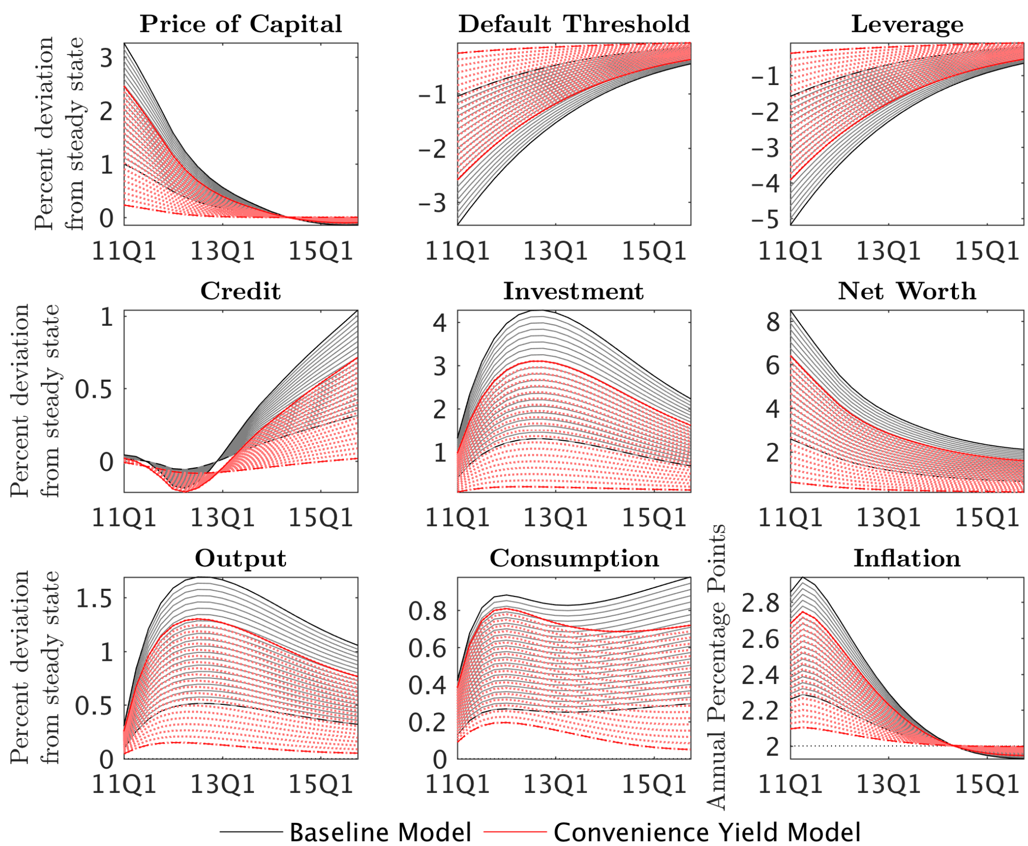
Note: The solid lines correspond to the models simulated at the high end of the range for the elasticity of the term premium (ν). The dashed lines correspond to the models simulated at the low end of the range for the elasticity of the term premium.

corporate debt (which can be thought of as the AAA rate) and the long term government bond rate.

$$\Lambda_{t,t+1}\Big[R^{credit}_{t+1} - R^{L}_{t+1} - \zeta^{p}_{t}\Big] = 0. \tag{E.7}$$

The headline result here is that LSAPII boosts output between 0.15-1.3% (cf 0.51-1.69% in the baseline model) and boosts inflation between 10 - 74 basis points (cf 28-94 basis points in the baseline model).

Figure 13: Convenience Yield (b)



Note: The solid lines correspond to the models simulated at the high end of the range for the elasticity of the term premium (ν). The dashed lines correspond to the models simulated at the low end of the range for the elasticity of the term premium.