The Portfolio Balance Channel of Quantitative Easing in a DSGE Model with Financial Frictions

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Abstract

Financial frictions amplify the portfolio balance effect of QE. A costly state verification friction increases output growth by between 0.13 - 0.41 percentage points and increases inflation between 6 - 18 basis points more than the model without the friction. I find that overall that the Federal Reserve's second round of Large-Scale Asset Purchases (LSAPII) boosts output between 0.51% - 1.62%, which is the equivalent of a 83 - 278 basis point cut in the Federal Funds rate. Investors who arbitrage between long term government debt and corporate debt create a Portfolio Balance Channel in that the effects of QE spill over to the overall cost of corporate borrowing. This long term maturity preference of investors increases output growth by between 0.4 and 1.27% points, and inflation between 20 and 64 annualized basis points more than the model without this channel.

Keywords: Quantitative Easing, Financial Frictions

JEL Codes: E13, E52

1 Introduction

There is a tension between the generally small effects of the Federal Reserve's programs of Quantitative Easing[1] (QE) found in DSGE models and the relatively larger effects of QE shown in VAR/empirical work. This paper asks if the presence of financial frictions can amplify the portfolio balance effect of QE. I find that the second round of the Federal Reserve's Large Scale Asset Purchase Program (LSAPII) boosted output between 0.51% and 1.62%, the equivalent of a 83 - 278 basis point cut in the Federal Funds Rate.

At its most general the Portfolio Balance Channel of QE captures the idea that purchases of longer maturity US Treasuries increase the price of other, substitute assets. Krishnamurthy and Vissing-Jorgensen (2011) emphasize that the portfolio balance effect can be driven in multiple ways. This paper focuses on one specific mechanism: investors who arbitrage between the return on investing in corporate debt and the return on long term government bonds.

Quantitative Easing works in the model because households and investors have a preference for longer term assets - the "preferred habitat" characterized by Vayanos and Vila (2009). So the relative supply of long-to-short term bonds will impact the relative price of bonds. Krishnamurthy and Vissing-Jorgensen (2011) emphasize that the evidence for preferred habitat is strongest for near-zero default risk assets. Longstaff, Mithal and Neis (2005) find that this includes corporate bonds of investment grade quality (rated above Baa). A narrow application of preferred habitat implies that a reduction in the relative supply of long vs short term government debt will reduce the term premium on government bonds. The model captures this with household who have a preferred ratio of long-to-short term government bonds. This preference is crucial to breaking Wallace's Irrelevance result (Wallace, 1981) meaning that QE has a role in this model. A wider interpretation of preferred habitat (extending it to corporate borrowing) implies that QE should also put downward pressure on corporate borrowing costs. In the model this is captured by investors who substitute between longer term government debt and longer term corporate debt. This means that QE will drive yields on corporate bonds. This is then amplified via a default risk channel - lower corporate borrowing costs and better economic conditions decrease default rates.

[1] I.e the Large Scale Asset Purchase Programs (LSAPs)

Vector autoregression (VAR) based estimates of the macroeconomic impacts of Quantitative Easing generally find larger quantitative impacts of QE than DSGE literature does. For example Baumeister and Benati (2013) find that the median impact of the Federal Reserve's second round of Large Scale Asset Purchases (LSAPII) was to boost GDP by 3% and increase inflation by 1%. For the UK Kapetanios, Mumtaz, Stevens and Theodoridis (2012) find that the peak effects of the Bank of England's first round of QE were a 1.5% increase in GDP and a 1.25% increase in inflation. In contrast the quantitative results in the DSGE literature on QE are muted. For example Chen, Curdia and Ferrero (2012) find that the Federal Reserve's second round of asset purchases had a slightly smaller effect than a surprise 25 basis point cut in the Federal Funds Rate. Their median results are that GDP increased by 0.13% and inflation increased by 3 basis points (both annualized). Harrison (2012) similarly finds small impacts of the Bank of England's QE program. One exception is Graeve and Theodoridis (2016): in a DSGE model with a complex fiscal block the authors find the Federal Reserve's 2011 Maturity Extension Program increased GDP by 0.6%. Key to their results is the response of the maturity structure of government debt to the QE policy. In contrast here I consider only the marginal change in the government's debt maturity structure (the reduction in long term bonds and the corresponding increase in reserves coming directly from purchases).

The model in this paper builds quantitative easing into a model based on Christiano, Motto and Rostagno (2014). The Christiano et al. (2014) model builds a costly state verification financial friction into a standard DSGE model a la Smets and Wouters (2007) and Christiano et al. (2005). The costly state verification friction comes from Bernanke, Gertler and Gilchrist (1999): lenders can only view the balance sheet of a defaulted non-financial firm by paying a cost. This generates an external finance premium that lenders must pay. Firms receive idiosyncratic productivity shocks. A shock to the standard deviation of the productivity distribution is called a "risk shock". Christiano et al. (2014) find that the variation of the risk shock over time is the most important driver of US business cycles. Del Negro and Schorfheide (2013) find the risk shock to have been an important factor during the Great Recession.

The model allows for QE to be non-neutral by incorporating preferred habitat preferences in two ways. One, households have a preferred ratio of long to short term government bonds - following Harrison (2012). Two, there are investors who only invest in long term government bonds or corporate debt. Broadening the role of preferred habitat in driving QE is motivated by the evidence in Krishnamurthy and Vissing-Jorgensen (2011) and Longstaff et al. (2005) which suggests that preferred habitat preferences for long term maturity bonds apply to high grade corporate debt in addition to long term Treasuries. Widening the preferred habitat preferences adds a Portfolio Balance Channel of QE and is the key contribution of this paper.

The paper proceeds as follows. Section 2 presents the model. Section 3 presents the data and calibration method. Section 4 presents the simulation of the Federal Reserve's second round of Large Scale Asset Purchases. Section 5 concludes.

2 The Model

2.1 Households

All households are identical and large in number. Each household holds a large number of entrepreneurs and every type of differentiated labor. Households consume, invest to produce raw capital (which is then sold to entrepreneurs), buy and hold long and short term government bonds, supply labor, and receive labor income.

The representative household has the following utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log(C_t - bC_{t-1}) - \psi_L \int_0^1 \frac{h_{it}^{1+\sigma_L}}{1+\sigma_L} di - \frac{\tilde{\nu}}{2} \left(\frac{B_t^L}{B_t} - \delta^b \right)^2 \right\}, \tag{1}$$

where β is the household's discount rate, C_t is per capita consumption, b is the habit parameter, ψ_L is the dis-utility weight on labor, σ_L is the Frisch elasticity of labor supply, and h_{it} is labor supply by labor type i. δ^b is the preferred ratio of long-to-short term government debt, $\tilde{\nu}$ governs the dis-utility from deviating from the preferred portfolio, and finally B_t and B_t^L are the market value of privately held short and long term government debt.

The final term in the household's utility function is the household's portfolio preference. Households have quadratic disutility over deviations from the preferred ratio of long-to-short term government debt (δ^b). I follow Harrison (2012), which builds on the work of Andrés et al. (2004). I calibrate δ^b to match the steady state ratio of long-to-short term government debt in the hands of the public. $\tilde{\nu} > 0$ implies that the term premium responds to changes in the household's relative holdings of short and long term government debt (this breaks Wallace's Irrelevance Result). I calibrate $\tilde{\nu} > 0$ to match the elasticity of the term premium to changes in the maturity structure of government debt.

The household's budget constraint is:

$$\begin{aligned} & P_t C_t + B_t + B_t^L + \frac{P_t}{\Upsilon^t \mu_{\Upsilon t}} I_t + Q_{\bar{K},t}(1-\delta)\bar{K}_t \\ & \leq \int_0^1 W_{it} h_{i,t} di + R_{t-1} B_{t-1} + R_t^L B_{t-1}^L + Q_{\bar{K},t}\bar{K}_{t+1}, \end{aligned} \tag{2}$$

where P_t is the price of the consumption good, I_t is the quantity of investment goods purchased by the household for a price $\frac{P_t}{\Upsilon^t \mu_{\Upsilon t}}$ ($\mu_{\Upsilon t}$ is a shock to investment technology, and Υ^t is trend growth in investment technology), $\bar{Q}_{\bar{K},t}$ is the price of raw capital, $\bar{K}_{t+1}$ is end of period t raw capital, δ is the depreciation rate of capital, W_{it} is the wage rate for labor type i, R_{t-1} is the rate paid on short term bonds issued in $t-1$ maturing in t, and R_t^L is the rate paid on long term bonds issued in $t-1$ maturing in t.

The model's treatment of the bond market is based on Harrison (2012). There are two types of government bonds: short, and long. Short bonds sell for a unit price at time t, and return R_t units of currency at time t+1. Long bonds are modeled as perpetuities that exist for an infinite number of periods (unless the government removes them from the market). They provide a coupon payment of 1 unit of currency each period, and have a value V_t at time t. In each period t, after making the coupon payment, the government rolls over its debt by purchasing the entire stock of long-term debt (B_{t-1}^c) at the market price V_t and issuing new consol bonds B_t^c which are purchased by households for the market price V_t. $R_t^L \equiv \frac{1+V_t^L}{V_{t-1}^L}$ is the gross return on a long bond sold at time $t-1$. $B_t^L \equiv V_t B_t^c$ is the total nominal value of long bonds at time t. Note the timing: V_t^L is unknown at time $t-1$; therefore R_t^L (the gross return on a long bond purchased at time $t-1$) is not known for certain until time t.

When investing in raw capital the household faces the following law of motion:

$$\bar{K}_{t+1} = (1-\delta)\bar{K}_t + \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t. \tag{3}$$

The investment adjustment cost function S has the functional form:

$$S(x_t) \equiv \frac{1}{2}\left\{ \exp\left[\sqrt{S''}(x_t - x)\right] + \exp\left[-\sqrt{S''}(x_t - x)\right] - 2 \right\}, \tag{4}$$

where $x_t \equiv \frac{I_t}{I_{t-1}}$ and $S'' \equiv S''(x)$ is a parameter calibrated to match the dynamics of investment.

2.2 Production Markets

2.2.1 Goods Market

Each intermediate good, $Y_{jt},\ j \in [0,1]$ is produced by a different monopolist according to the following production function:

$$Y_{jt} = \begin{cases} \epsilon_t K_{jt}^{\alpha}\left(z_t l_{jt}\right)^{(1-\alpha)} - \Phi z_t^*, & \text{if } \epsilon_t K_{jt}^{\alpha}\left(z_t l_{jt}\right)^{(1-\alpha)} > \Phi z_t^* \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

where the capital share $\alpha \in (0,1)$ and ϵ_t is a technology shock (that is covariance stationary). K_{jt} is the quantity of effective capital used by monopolist producer j, and l_{jt} the quantity of homogeneous labor employed by monopolist producer j. z_t is an effective labor shock which has a stationary growth rate. The proportional fixed cost Φz_t^* is such that the intermediate monopolistic producer earns zero profits in steady state. The detrending term z_t^* is described in more detail below.

There is a Calvo friction in the pricing of intermediate goods. Each period a random fraction of intermediate firms, $1 - \xi_p$, can reoptimize their price P_{jt}. The remaining fraction ξ_p set their price as follows:

$$P_{jt} = \tilde{\pi}_t P_{j,t-1}, \tag{6}$$

where inflation indexation is as follows:

$$\tilde{\pi}_t = \left(\pi_t^{target}\right)^{\iota}\left(\pi_{t-1}\right)^{1-\iota}. \tag{7}$$

$\pi_{t-1} \equiv \frac{P_{t-1}}{P_{t-2}}$ is gross inflation. And π_t^{target} is the central bank's target inflation rate. ι is the price indexing weight on the inflation target.

The homogeneous final good Y_t, is produced by a competitive representative firm (with Dixit-Stiglitz technology):

$$Y_t = \left[\int_0^1 Y_{jt}^{\frac{1}{\lambda_f}} dj \right]^{\lambda_f}, \; 1 \leq \lambda_f < \infty, \; j \in [0,1]. \tag{8}$$

The homogeneous final good has two uses: consumption and investment. One unit of Y_t can be converted into one unit of the consumption good C_t, and thus (given perfect competition in the use of this technology), consumption has the price P_t. One unit of Y_t can also be converted into $\Upsilon^t \mu_{\Upsilon,t}$ units of the investment good, and thus (again given perfect competition in the use of the technology) has the price $\frac{P_t}{\Upsilon^t \mu_{\Upsilon,t}}$, where $\Upsilon > 1$.

There are two sources of growth in the model. First, the trend rise in the aforementioned technology for producing investment goods, Υ^t, and second, the effective labor shock z_t which has a stationary growth rate. The de-trending term z_t^* is a combination of both sources of growth:

$$z_t^* = z_t \Upsilon^{\left(\frac{\alpha}{1-\alpha}\right)t}. \tag{9}$$

z_t^* is used to normalize variables to find a non stochastic steady state. z_t^* is such that $\frac{Y_t}{z_t^*}$ will converge to a constant in the nonstochastic steady state of the model. $\mu_{z,t}^* \equiv \frac{z_t^*}{z_{t-1}^*}$ is the growth rate of z_t^*, which has the stationary growth rate μ_z^*.

2.2.2 Labor Market

Each differentiated labor type type $i \in [0,1]$ provides labor services h_{it} and is represented by a monopoly union that sets its wage rate W_{it} while facing a Calvo friction. Each period a fraction $1 - \xi_w$ of the monopoly unions can update the wage. The remaining fraction ξ_w set their wage as follows:

$$W_{it} = \left(\mu_{z,t}^*\right)^{\iota_\mu} \left(\mu_z^*\right)^{1-\iota_\mu} \tilde{\pi}_{wt} W_{i,t-1}, \tag{10}$$

where:

$$\tilde{\pi}_{wt} \equiv \left(\pi_t^{target}\right)^{\iota_w} \left(\pi_{t-1}\right)^{1-\iota_w}, \qquad 0 < \iota_w < 1. \tag{11}$$

Labor is aggregated via a Dixit-Stiglitz style aggregator by a competitive and representative labor contractor:

$$l_t = \left[\int_0^1 \left(h_{it} \right)^{\frac{1}{\lambda_w}} di \right]^{\lambda_w}, \qquad 1 \leq \lambda_w. \tag{12}$$

The homogeneous labor aggregate l_t is sold to intermediate goods producers at the nominal wage W_t.

2.3 Entrepreneurs

Entrepreneurs have the role of turning raw capital (purchased from households) into effective capital (to be then sold to intermediate goods producers). They experience idiosyncratic productivity shocks in their ability to turn raw capital into productive capital. They finance the purchase of raw capital via debt, and entrepreneurs who experience low idiosyncratic productivity shocks default on their debt. Entrepreneurs' creditors can only observe the state of a defaulted entrepreneur's balance sheet by paying a proportional recovery cost. The expectation of paying this cost introduces a spread on entrepreneurial debt. This is the Costly State Verification (CSV) financial friction characterized in Bernanke, Gertler and Gilchrist, 1999 (henceforth BGG) and Christiano et al. (2014) (henceforth CMR).

Entrepreneurs' aggregate net worth is considered a proxy for the value of the stock market. Entrepreneurs can either be interpreted as being firms in the non-financial sector, or financial institutions with non-diversified holdings.

Entrepreneurs are classified by their net worth. An entrepreneur with net worth $N \geq 0$ is called an 'N-type' entrepreneur. The timing of one cycle in the life of an entrepreneur is as follows. Following production in period t, each entrepreneur gets a loan from a mutual fund. Each mutual fund is specialized. They make loans only to entrepreneurs of a specific level of net worth, but perfectly diversify by holding a large number of those loans. The entrepreneur combines the loan $B_{t+1}^{N,credit}$ (issued in period t and due in period $t+1$) with their own net worth to purchase raw capital ($\bar{K}_{t+1}^N$) at price $Q_{\bar{K},t}$:

$$B_{t+1}^{N,credit} + N = Q_{\bar{K},t} \bar{K}_{t+1}^N. \tag{13}$$

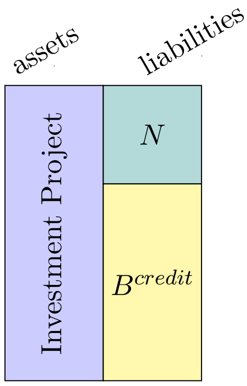


Figure 1: Entrepreneur's Balance Sheet

After raw capital is purchased each entrepreneur receives an idiosyncratic shock ω that determines the amount of effective capital they have, $\omega \bar{K}^N_{t+1}$. As in BGG and CMR, ω is distributed (independently across entrepreneurs and time) log-normally with a unit mean and a standard deviation $\sigma_t \equiv \sqrt{var(\log \omega)}$. The *risk shock* σ_t is simply the extent of cross-sectional dispersion of idiosyncratic productivity shocks experienced by entrepreneurs.

After realizing the risk shock entrepreneurs choose the utilization rate of effective capital u^N_{t+1} to maximize their return on capital: ωR^k_{t+1} at the end of period $t+1$. Entrepreneurs supply $u^N_{t+1} \omega \bar{K}^N_{t+1}$ units of effective capital to intermediate goods producers at the market rental rate r^k_{t+1}. The return on capital is defined as follows:

$$R^k_{t+1} \equiv \frac{[u_{t+1} r^k_{t+1} - a(u_{t+1})]\Upsilon^{-(t+1)} P_{t+1} + (1-\delta) Q_{\bar{K},t+1}}{Q_{\bar{K},t}}. \tag{14}$$

The choice of utilization is independent of net worth so the N superscript is dropped. The utilization cost of capital, $a(u_t)$, is increasing and convex:

$$a(u_t) \equiv \frac{r^k_t}{\sigma_a} \Big[\exp\Big(\sigma_a (u_t - 1)\Big) - 1 \Big]. \tag{15}$$

In addition to the utilization choice entrepreneurs must also choose the type of debt contract to accept. It is each entrepreneur's objective to maximize their expected net worth in the next period (t+1), which is as follows:

$$E_t\Bigg\{\int_{\bar{\omega}_{t+1}}^{\infty}\Big[R^k_{t+1}\omega Q_{\bar{K},t}\bar{K}^N_{t+1} - B^{N,credit}_{t+1}Z_{t+1}\Big]dF(\omega,\sigma_t)\Bigg\}$$
$$= E_t\Big[1-\Gamma(\bar{\omega}_{t+1})\Big]R^k_{t+1}L_tN. \tag{16}$$

where $\Gamma_t(\bar{\omega}_{t+1}) \equiv \Big[1 - F_t(\bar{\omega}_{t+1})\Big]\bar{\omega}_{t+1} + G_t(\bar{\omega}_{t+1})$ is the fraction of expected earnings paid to the investor, $1-F_t(\bar{\omega}_{t+1})$ is the probability the entrepreneur experiences a idiosyncratic shock over the default threshold $\bar{\omega}_{t+1}$, and $F(.)$ is the cumulative distribution function of ω. Z_{t+1} is the gross nominal interest rate on debt. $G_t(\bar{\omega}_{t+1})$ is the expected value of the idiosyncratic shock in the population of defaulting entrepreneurs:

$$G_t(\bar{\omega}_{t+1}) \equiv \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega), \tag{17}$$

where $L_t \equiv \frac{Q_{\bar{K},t}\bar{K}^N_{t+1}}{N}$ is the entrepreneur's leverage. Entrepreneurs maximize (16) by choosing the conditions that characterize the debt contract from the available set of debt contracts that mutual funds are willing to provide. That is equation (19) described below. The conditions are: one, the level of the idiosyncratic shock ω below which they will default (this is $\bar{\omega}_{t+1}$), or equivalently the gross nominal interest rate on debt to be paid next period: Z_{t+1}, and two, the amount of leverage L_t they will take on.

The default threshold is defined as follows:

$$\bar{\omega}_{t+1} \equiv \frac{B^{N,credit}_{t+1}Z_{t+1}}{R^k_{t+1}Q_{\bar{K},t}\bar{K}^N_{t+1}}. \tag{18}$$

2.3.1 Mutual Funds & Investors

Specialized mutual funds make loans to N-type entrepreneurs, diversifying over type. They sell packaged entrepreneurial debt to investors. Investors are only willing to hold long term debt - they have preferred habitat preferences. Because of their preferences investors will only hold packaged entrepreneurial debt if its return is equal to the return on long term government debt[2]. Investors' preferences mean the long term government

[2]Clearly this is a strong assumption for two reasons. One, in the data there is a spread between corporate debt yields and US Treasury yields - meaning that there are likely frictions between government debt and corporate debt, and two, the empirical evidence that supports a preferred habitat preference over bond maturity does not extend to all corporate debt. The evidence in Longstaff, Mithal and Neis

bond rate enters the mutual fund's zero-profit condition:

$$\left[1-F_t(\bar{\omega}_{t+1})\right]Z_{t+1}B_{t+1}^{N,credit}+(1-\mu)\int_0^{\bar{\omega}_{t+1}}\omega dF_t(\omega)R_{t+1}^kQ_{\bar{K},t}\bar{K}_{t+1}^N \equiv E_tR_{t+1}^LB_{t+1}^{N,credit}. \quad (19)$$

Putting the long term government bond rate into the zero profit condition (19) captures the portfolio balance channel in this model. When the rate on long term government debt falls, investors are willing to hold packaged entrepreneurial debt at a lower rate. This in turn relaxes the credit conditions that mutual funds are able to provide to entrepreneurs.

If an entrepreneur experiences an idiosyncratic shock ω below the threshold $\bar{\omega}_t$, then they will not be able to repay their debt to the investor and will declare bankruptcy. In this instance the mutual fund only knows that the entrepreneur is bankrupt, but does not observe the value of ω. Without further action by the mutual fund the entrepreneur could decide to transfer only a fraction of their remaining assets, $\omega R_{t+1}^kQ_{\bar{K},t}\bar{K}_{t+1}^N$, back to the mutual fund. In order to become fully informed about the assets a bankrupt entrepreneur has, the mutual fund must pay a cost that is a proportion μ of the final assets recovered. Thus the mutual fund only receives a fraction $(1-\mu)$ of the total assets of bankrupt entrepreneurs.

The following condition (20) characterizes the available menu of contracts entrepreneurs can choose from. This comes from using the definition of the default threshold (18) to substitute out $Z_{t+1}B_{t+1}^{N,credit}$ in the mutual funds' .

$$\Gamma_t(\bar{\omega}_{t+1})-\mu G_t(\bar{\omega}_{t+1})=\frac{L_t-1}{L_t}\frac{R_{t+1}^L}{R_{t+1}^k}. \quad (20)$$

2.3.2 Accelerator Effect

The characterization of the financial friction is from Christiano et al. (2014) (CMR) and is based on the costly state verification (CSV) financial accelerator mechanism in Bernanke, Gertler and Gilchrist, 1999 (BGG). BGG emphasize the following intuition behind the accelerating affects of the financial friction in the CSV class of models. The basic idea is that a fall in entrepreneurial net worth means that the entrepreneurs will

(2005) supports the preferred habitat only extending to corporate debt rated above Baa. This suggests that in future work adding a friction to investor arbitrage is important.

have less inside funds to invest in the project. Therefore the investors that make loans to entrepreneurs face a greater agency cost when they finance the entrepreneurs. Essentially the entrepreneur has less “skin in the game”. The higher agency cost means investors charge a higher interest rate, so that the premium on external finance faced by entrepreneurs increases. Faced with a larger interest rate on loans, other things being equal, entrepreneurs will choose to purchase less capital. Because entrepreneurs play a key role in turning raw capital into effective capital used by producers the increase in the external finance premium will decrease output. The net worth of entrepreneurs is pro-cyclical. Therefore the external finance premium is counter-cyclical. Thus the interactions between investors and entrepreneurs via the hike in the external finance premium will serve to amplify the business cycle.

This also means that to the extent to which QE boosts entrepreneurial net worth it can have the reverse effect - lowering the external finance premium, boosting capital, and increasing output.

2.3.3 Aggregates

The aggregates are as follows:

Aggregate raw capital:

$$\bar{K}_{t+1} = \int_0^\infty \bar{K}_{t+1}^N f_t(N) dN. \tag{21}$$

Aggregate effective capital:

$$K_t = \int_0^\infty \int_0^\infty u_t^N \omega \bar{K}_t^N f_{t-1}(N) dF(\omega) dN = u_t \bar{K}_t. \tag{22}$$

Aggregate net worth:

$$N_{t+1} = \int_0^\infty N f_t(N) dN. \tag{23}$$

Aggregate credit:

$$B_{t+1}^{credit} = \int_0^\infty B_{t+1}^N f_t(N) dN = \int_0^\infty \left[Q_{\bar{K},t} \bar{K}_{t+1}^N - N \right] f_t(N) dN = Q_{\bar{K},t} \bar{K}_{t+1} - N_{t+1}. \tag{24}$$

Finally the evolution of aggregate net worth is:

$$N_{t+1} = \gamma\Big[1 - \Gamma_{t-1}(\bar{\omega}_t)\Big] R_t^k Q_{\bar{K},t-1} \bar{K}_t + W^e, \tag{25}$$

where γ is the fraction of entrepreneurs that continue each period (a fraction $1-\gamma$ exit and pay dividends to the household), and W^e is the transfer from the household to new entering entrepreneurs.

2.4 Government Policies

As in Chen et al. (2012) the government has an auto-regressive supply rule for the market value of de-trended long-term bonds ($b_t^L \equiv \frac{B_t^L}{P_t z_t^*}$):

$$\log\left(\frac{b_t^L}{b^L}\right) = \rho_{bL}\left(\frac{b_{t-1}^L}{b^L}\right) + u_t^{bL}. \tag{26}$$

u_t^{bL} is the sum of unanticipated and anticipated news shocks to the supply of long term government bonds:

$$u_t^{bL} \equiv \epsilon_t^{bL} + \xi_{1,t-1}^{bL} + ... + \xi_{8,t-8}^{bL}. \tag{27}$$

ϵ_t^b is the unanticipated shock to the supply of short term bonds, and $\sum\limits_{p=1}^{p=8} \xi_{p,t-p}^b$ is the sum of anticipated shocks to the supply of short term bonds.

The government budget constraint is:

$$B_t + B_t^L = R_{t-1}B_t + R_t^L B_{t-1}^L + G_t - T_t, \tag{28}$$

where G_t is nominal government spending, and T_t is nominal government taxation.

Government spending is:

$$\frac{G_t}{P_t z_t^*} = g_t, \tag{29}$$

where the steady state level of real de-trended government spending, g, is calibrated to be 20% of steady state output.

The fiscal rule is adapted from Davig and Leeper (2006) and Eusepi and Preston

(2011):

$$\frac{T_t}{P_t z_t^*} - \frac{G_t}{P_t z_t^*} = \kappa \left(\frac{b_{t-1} + b_{t-1}^L}{b^L + b} \right)^{\phi_T} \epsilon_t^T, \tag{30}$$

where κ is the steady state primary fiscal surplus, and ϕ_T is set high enough so that the primary surplus adjusts to satisfy the government inter-temporal budget constraint and where b_{t-1} and b_{t-1}^L are the real de-trended market value of short and long term bonds respectively ($b_t \equiv \frac{B_t}{P_t z_t^*}$).

2.5 Monetary Policy & Resource Constraint

2.5.1 Monetary Policy Rule

The central bank sets the short rate according to a backward-looking Taylor Rule:

$$\log\left(\frac{R_t}{R}\right) = \rho_m \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_m)\left[\phi_\pi \log\left(\frac{\pi_t}{\pi_t^{target}}\right) + \frac{\phi_y}{4}\left(\log\frac{Y_t}{Y_{t-1}} - \log\mu_z^*\right)\right] + \frac{1}{400}u_t^m, \tag{31}$$

where μ_z^* is the steady state growth of output. u_t^m is the sum of unanticipated and anticipated (news) monetary policy shocks:

$$u_t^m \equiv \epsilon_t^m + \xi_{1,t-1}^m + ... + \xi_{8,t-8}^m, \tag{32}$$

where ϵ_t^m is the unanticipated monetary policy shock. And $\sum_{p=1}^{p=8} \xi_{p,t-p}^m$ is the sum of anticipated monetary policy shocks.

2.5.2 The Resource Constraint

$$Y_t = G_t + C_t + \frac{I_t}{\Upsilon^t \mu_{\Upsilon,t}} + a(u_t)\Upsilon^{-t}\bar{K}_t + \Theta\frac{1-\gamma}{\gamma}(N_{t+1} - W^e) + D_t, \tag{33}$$

where $a(u_t)\Upsilon^{-t}\bar{K}_t$ is the aggregate capital utilization cost of entrepreneurs. $\Theta\frac{1-\gamma}{\gamma}(N_{t+1} - W^e)$ are the resources consumed by exiting entrepreneurs. $D_t \equiv \frac{\mu G_{t-1}(\bar{\omega}_t)R_t^k + Q_{\bar{K},t-1}\bar{K}_t}{P_t}$ are the resources expended on monitoring entrepreneurs.

Table 1: Steady State Targets

Target	**Value**
Nominal Federal Funds Rate, annualized (R)	5.11
Ratio of long-term government bonds to annual output $\left(\frac{B^L}{4\times Y}\right)$	19.3%
Ratio of long to short bond holdings $\left(\frac{B^L}{B}\right)$	1.86
Ratio of government spending to quarterly output $\left(\frac{G}{Y}\right)$	20%
Hours worked (h)	1
Inflation, APR (π)	2%

3 Data and Calibration

The target period for calibration is 1985Q1 to 2007Q3. The data series used and mapping between the data and steady state targets are described in appendix D.

The household's discount rate β is calibrated to match the period average of the Effective Federal Funds Rate. δ^b is set to match the period average of the ratio of long-to-short term government bonds held by the public, where long-term debt is defined as any government bond with over 1 year until maturity, and short-term debt includes reserves. Government spending in steady state is set to 20% of quarterly output, in line with the target in Christiano et al. (2014). ψ_L is set to target steady state hours worked (h) equal to 1. Steady state inflation is 2%. The calibrations corresponding with these targets are appendix D table 6. The κ parameter in the fiscal rule (equation 30) is set to equal the steady state primary fiscal surplus.

$\nu \equiv \frac{1}{\lambda_z}\frac{\tilde{\nu}\delta^b}{b}(1+\delta^b)$ is the elasticity of the long rate to changes in the relative supply of long bonds (see appendix E for further detail). The target range is a 3 to 15 basis point drop in the term premium in response to a \$100 billion reduction in the supply of long-term bonds available to the public (holding short bonds constant). This range is discussed further in section 4.2.

The parameters in table 2 are fixed according to the calibration in CMR, or CMR posterior modes. All shock processes are specified as log AR(1)'s. Their persistence parameters are specified in table 3:

Table 2: Calibrated Parameters

Parameter	Description	Calibration
α	capital's share of output	0.4
b	habit parameter	0.74
Θ	fraction of assets consumed by exiting entrepreneurs	0.005
δ	depreciation rate of capital	0.025
ϵ	steady state value of the technology shock	1
$F(\bar{\omega})$	steady state probability of default	0.0056
γ	fraction of entrepreneurs who survive	0.985
ι	price indexing weight on inflation target	0.9
ι_μ	wage indexing weight on persistent technology growth	0.94
ι_w	wage indexing weight on inflation target	0.49
λ_f	markup in the product market	1.2
λ_w	markup in the labor market	1.05
μ	monitoring cost	0.21
μ_Υ	steady state value of $\mu_{\Upsilon,t}$	1
μ_z^*	mean growth rate of the unit root technology shock	1.0041
ϕ_π	parameter on inflation in the Taylor Rule	2.40
ϕ_y	parameter on output in the Taylor Rule	0.36
ϕ_T	feedback parameter in the fiscal rule, set according to Chen et al. (2012) posterior mean	1.3147
ρ_m	weighting of lagged short-rate in Taylor Rule	0.85
S''	parameter in the investment adjustment cost function	10.78
σ_a	curvature of utilization cost	2.54
σ_L	Frisch elasticity of labor supply	1
Υ	quarterly rate of investment-specific technological change	1.0042
w^e	lump sum transfer from household to the entrepreneur	0.005
ξ_p	Calvo price stickiness	0.74
ξ_w	Calvo wage stickiness	0.81

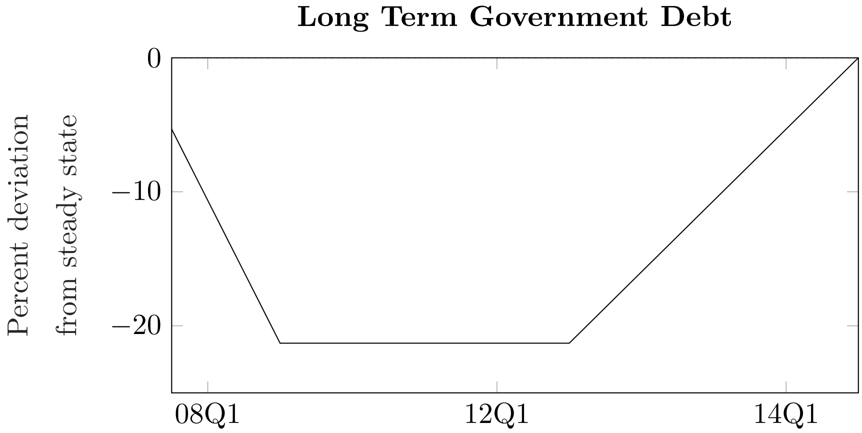
Table 3: Shock Autocorrelations

Parameter	Description	Calibration
ρ_{bL}	long-term bond supply	0
ρ_ϵ	transitory technology	0.81
$\rho_{\mu_z^*}$	persistent technology growth	0.15
ρ_σ	risk shock	0.97

4 Simulations

4.1 Simulating LSAPII

Figure 2: Simulated Path of Long-Term Government Debt During LSAPII



Following Chen et al. (2012) I simulate the second round of the Federal Reserve's quantitative easing program (LSAPII) with purchases taking place over two quarters, held for four quarters, and QE2 unwound over 8 quarters. The full path of purchases (figure 2) is known upon announcement of the program. I constrain the response of the federal funds rate for four quarters. I implement both the QE announced path and the forward guidance using news shocks. The mapping between the size of the $600 Billion in purchases and long bond supply shocks is described in appendix D.

4.2 The Impact of LSAP II

Empirical estimates of QE's "stock effects" - i.e. the impact on bond prices from the semi-permanent reduction of bond supply within a given maturity - suggest that the Federal Reserve's various rounds of LSAP purchases reduced term premiums between 3 and 10 basis points per $100 billion of long term bond purchases. On the upper range of the estimates D'Amico and King (2013) find that LSAPI on average decreased yields within a given maturity by 1 basis point per $10 billion in long term bond purchases. On the lower range Hamilton and Wu (2012) find that a $400 billion purchase of long-term maturity government bonds could reduce the 10-year rate by 13 basis points when the policy rate is at the zero lower bound.

I calibrate ν to match these estimates of the elasticity of the term premium to purchases. This gives a range of 0.00074- 0.0025. Figures 3 and 4 show the range of results. The output growth peak is between 0.51% - 1.62%. Output 6 years after the start of the LSAPII program is between 0.26% - 0.80% above its steady state level. Inflation increases between 28 and 88 basis points. In terms of federal funds rate cut equivalence (the rate cut needed to achieve the same peak growth in output) the model suggests the LSAPII program was equal to between a 83 - 278 annualized basis point cut in the federal funds rate.

Figure 3: The Impact of LSAPII (a)

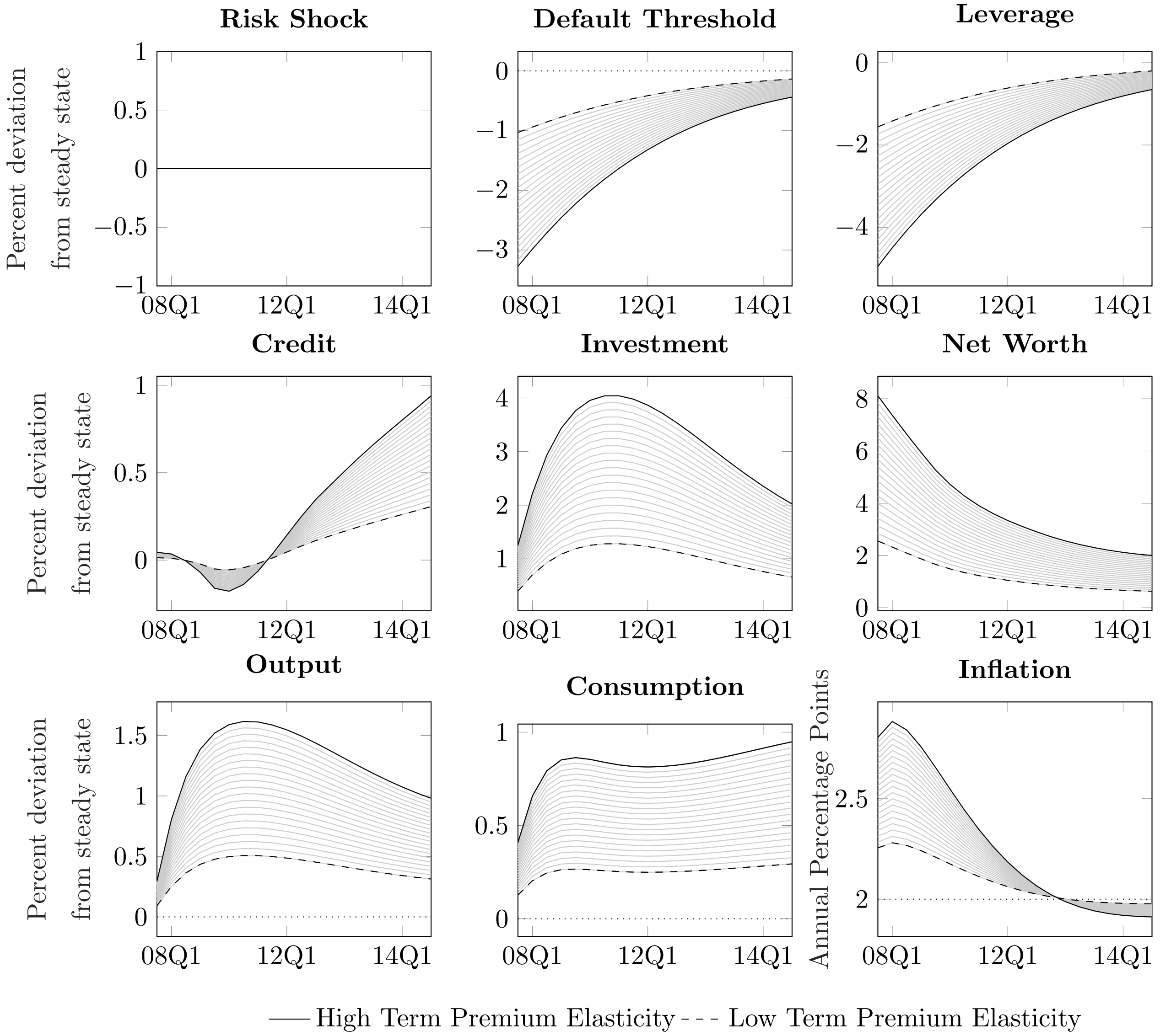
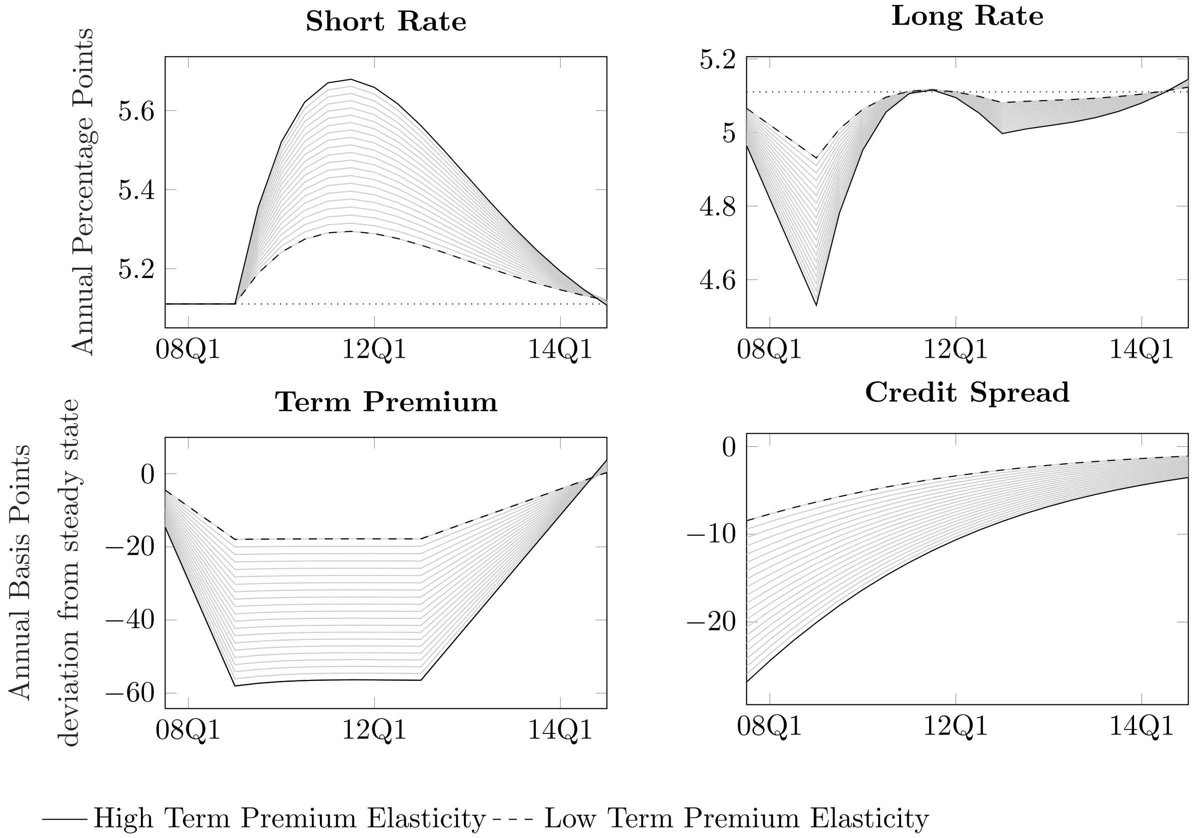


Figure 4: The Impact of LSAPII (b)



4.2.1 QE Mechanisms

In this paper the household's preference for an ideal ratio of long to short term government bonds in the household's utility function captures the idea that certain agents prefer to hold longer maturity assets ("preferred habitat") as emphasized by Vayanos and Vila (2009). This means per period returns on short vs long term assets are not fully arbitraged away, leading to a term premium for long term government bonds. The households have a narrow preferred habitat, restricted to only US Treasury bonds. This is the "Safety Channel" described by Krishnamurthy and Vissing-Jorgensen (2011): the preferred habitat is restricted to the near zero default risk assets. This channel breaks Wallace's Irrelevance result, meaning that the term premium falls in response to changes in the relative supply of long vs short term government debt. The term premium elasticity ν captures the strength of this channel and, as clearly can be seen in the lower left panel of figure 4, impacts how much the term premium drops in response to the LSAPII purchases. In a model with only this effect, for example Chen et al. (2012), QE acts solely via changing household's consumption and investment decisions via the long term bond Euler. As Chen et al. (2012) find this effect alone is quantitatively limited.

Adding the Entrepreneurial sector, and investors who arbitrage between long term government debt and packaged entrepreneurial debt, expands the role of preferred habitat. A decline in the long term government bond yield spills over into a general decline in the cost of corporate borrowing. Longstaff, Mithal and Neis (2005) show that this effect probably only exists for highly rated corporate debt (i.e. above Baa). In contrast to the Safety Channel (the spillover from Treasury yields to other near zero default risk assets, including high grade corporate debt) Krishnamurthy and Vissing-Jorgensen (2011) classify a general spillover effect as the "Duration Risk Channel". Corporate debt in this model does not distinguish between high and low rated debt because investors hold a perfectly diversified aggregate of debt. So by adding investors with strong long term maturity preferences I capture both the Safety Channel and Duration Risk Channel. Given the relatively weak empirical support for the Duration Risk Channel, including a friction in the arbitrage between corporate debt and US Treasuries could be an important dimension for future work.

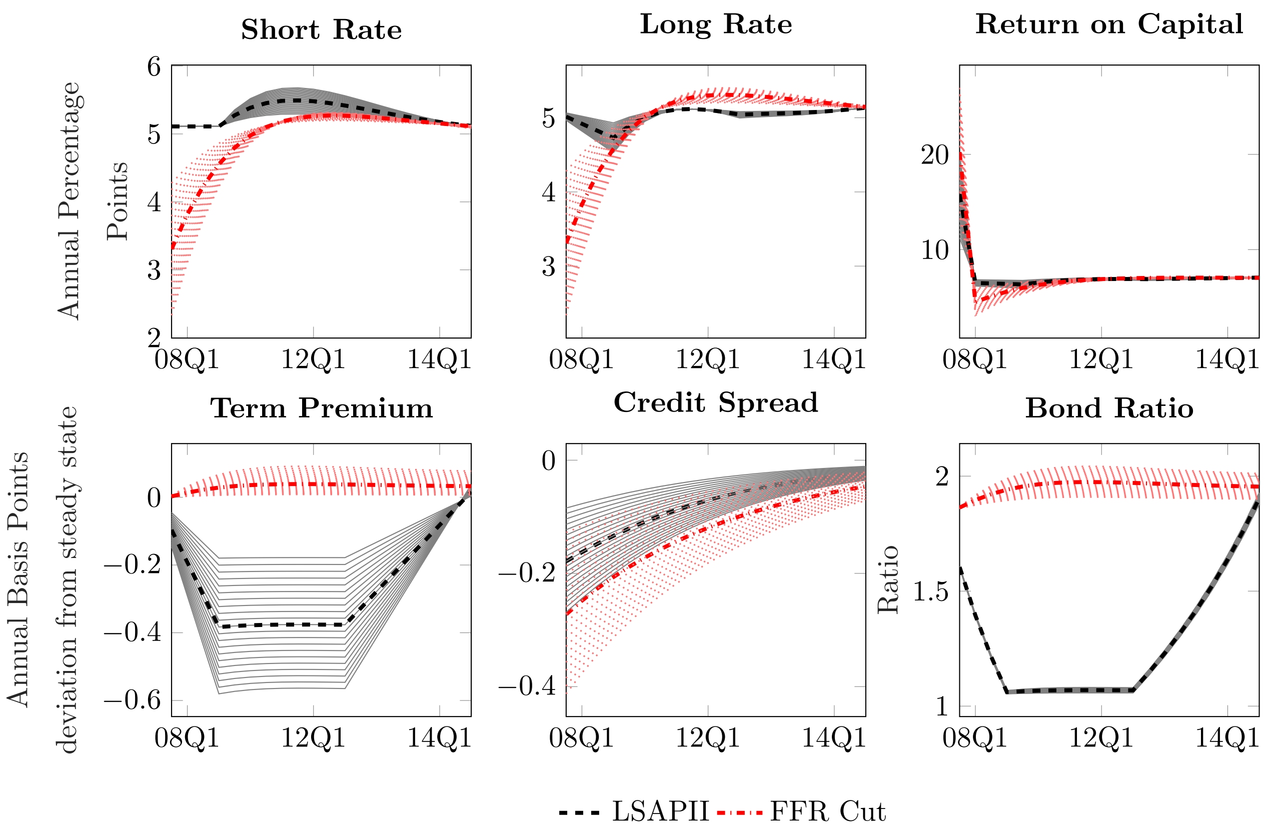
The Portfolio Balance channel also interacts with a "Default Risk Channel". Lower borrowing costs mean that fewer entrepreneurs default. This in turn, via the costly-state verification friction, lowers the spread charged on individual entrepreneurial debt contracts further, boosting net worth and accelerating the effects of QE via a reduction in the external finance premium (see section 4.5).

Borrowing cost declines stimulate entrepreneurial net worth and further amplify the impact of QE via accelerator effects. And QE has an inflationary impact, which means that *real* rates fall further.

I have a few additional comments about the dynamics of the entrepreneurs' credit conditions during QE. Both the leverage level and the default threshold fall. This on the surface suggests two opposite effects on the credit conditions, both easing (lower default threshold) and tightening (lower leverage). However, as pointed out in Christiano et al. (2014) the expectation that the price of capital will return to steady state has a muting effect on the response of credit under any shock. That is why credit responds by less than net worth, and so leverage falls in response to shocks that improve conditions for entrepreneurs.

4.3 LSAPII Impact vs FFR Cut Equivalence

Figure 5: Quantitative Easing vs a FFR Cut (a)



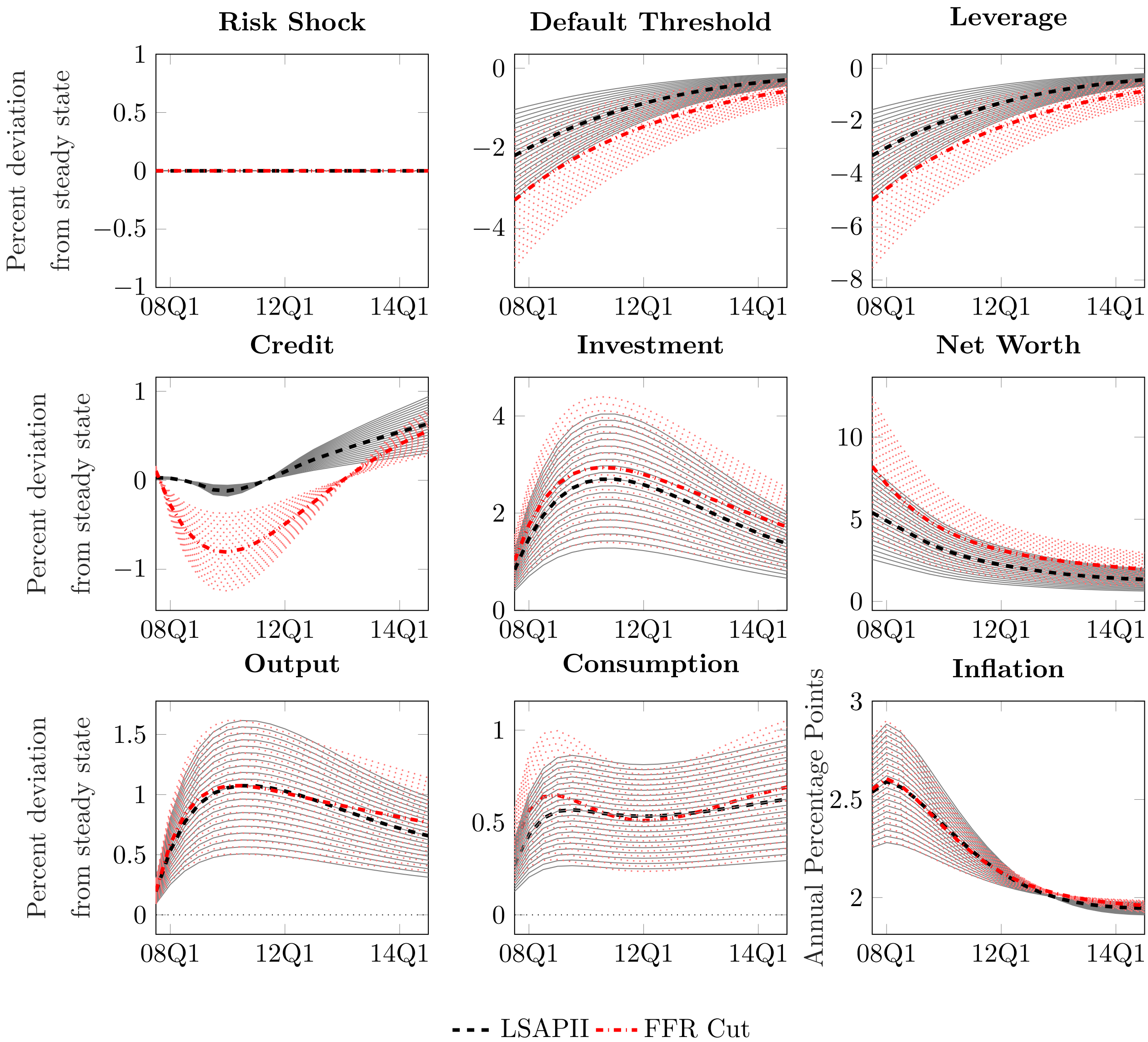
Note: The thick dashed lines correspond to the simulation at the median value for the portfolio preference term ν.

The simulated LSAPII program produces the same boost to output as a 83 to 278 basis point cut to the federal funds rate, corresponding to the lower and upper ranges for the strength of the portfolio preference (i.e. ν parameter calibration). Figures 5 and 6 show the response of the economy to the LSAPII program versus a federal funds rate cut (over the range of the portfolio preference parameter).

By target, output growth is the same across the LSAPII and the federal funds rate cut simulations. Note the thick dashed lines are the simulations at the median value of the term premium elasticity ν. Unsurprisingly inflation growth is roughly compatible across the two simulations. Investment responds slightly more to the federal funds rate cut (between 0.12 and 0.36 percentage points).

The impact on entrepreneurial credit conditions is substantially different between QE

Figure 6: Quantitative Easing vs a FFR Cut (b)

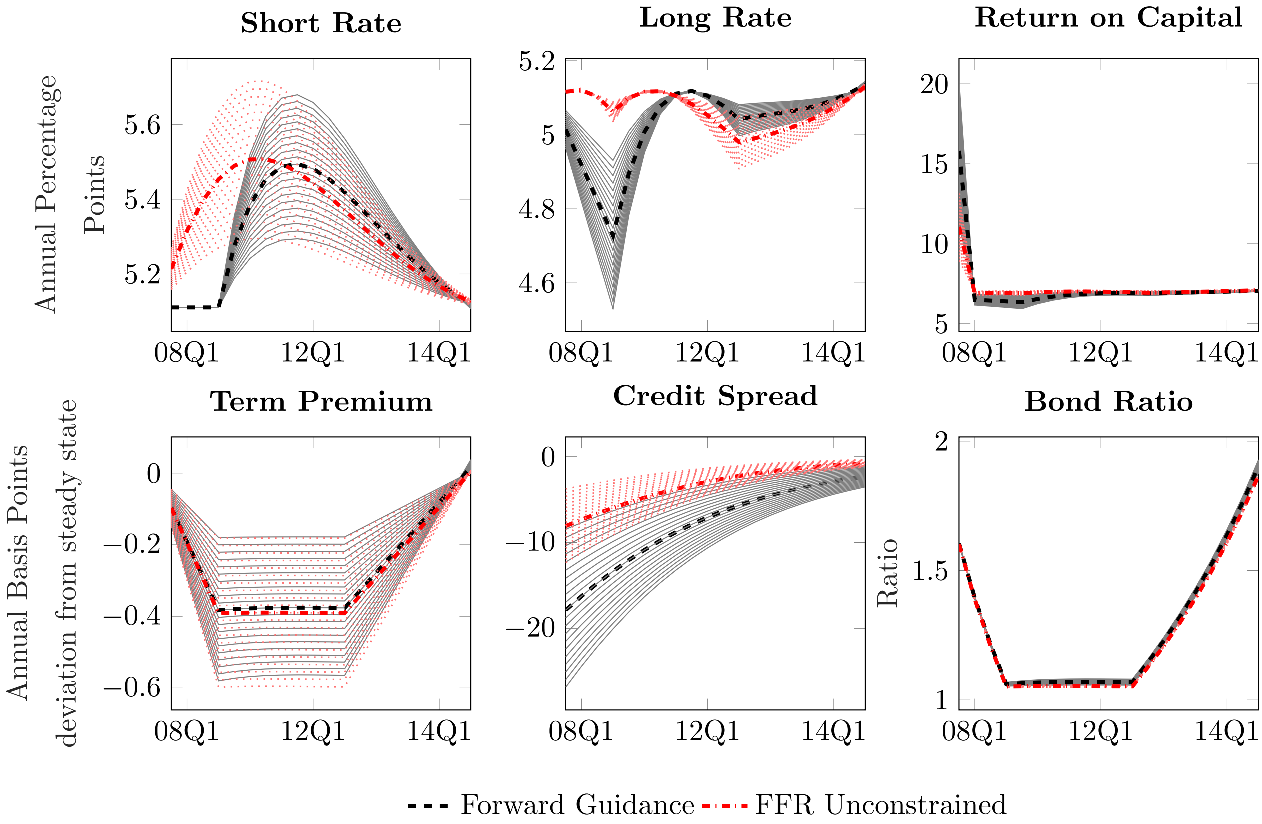


Note: The thick dashed lines correspond to the simulation at the median value for the portfolio preference term ν.

and the equivalent federal funds rate cut, highlighting the different channel through which these two policies act. Under quantitative easing credit drops slightly (between 0.06 - 0.18 % below its steady state level) once the forward guidance constraint on the federal funds rate is lifted. In contrast under the federal funds rate cut simulation credit drops between 0.37 - 1.24% below its steady state value (as the policy rate is normalized). The difference is that quantitative easing keeps long rates depressed for longer. This stimulates entrepreneurs' access to credit via the Portfolio Balance Channel, boosting credit quantity more than the federal funds rate cut scenario.

4.4 Importance of the ZLB Constraint

Figure 7: QE With and Without Forward Guidance (a)

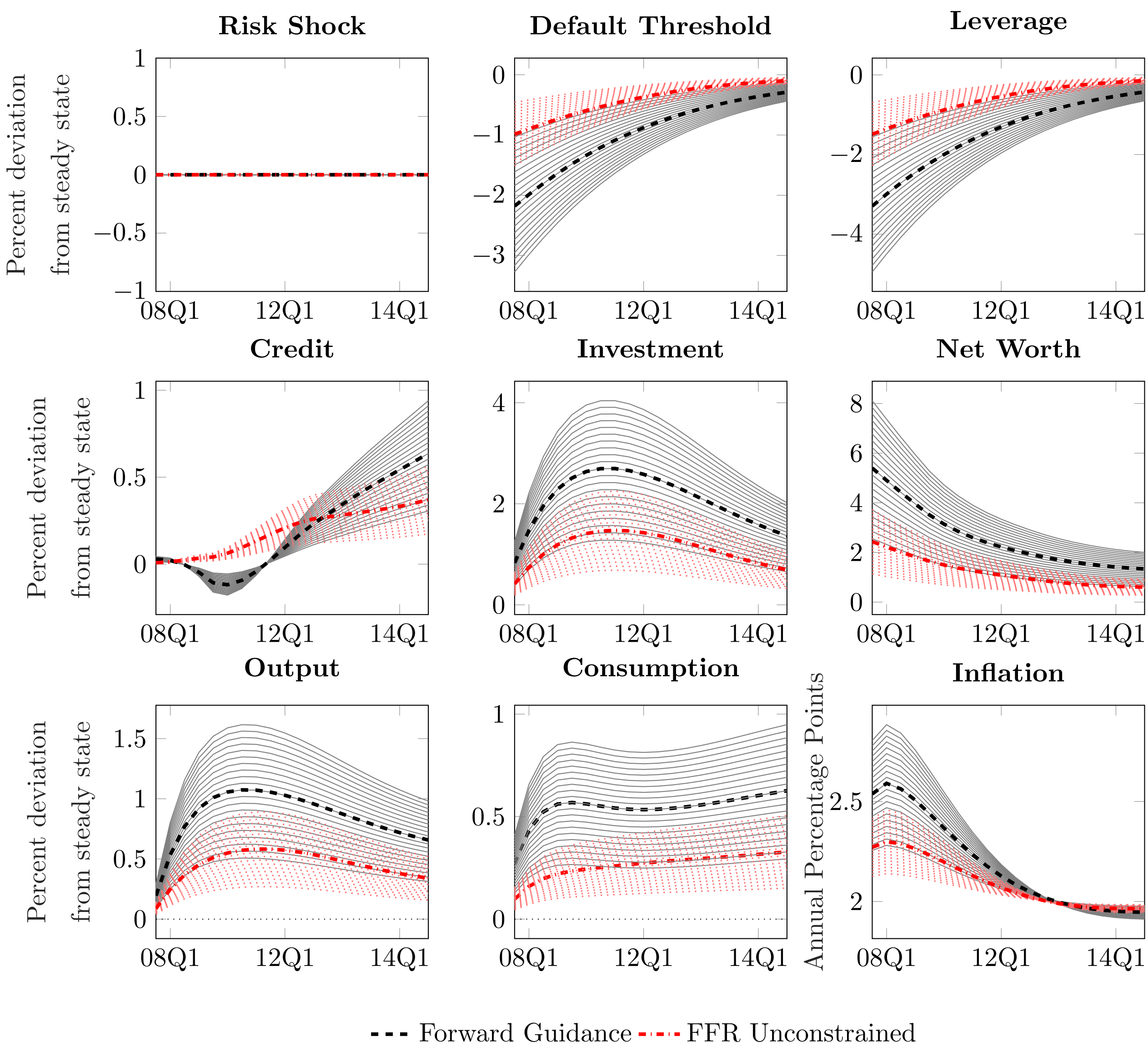


Note: The thick dashed lines correspond to the simulation at the median value for the portfolio preference term ν.

In the baseline LSAPII simulation ("Forward Guidance") news shocks are used to keep the federal funds rate at its steady state level. In the "FFR Unconstrained" simulation the federal funds rate is unconstrained. In figures 7 and 8 it is clear that the forward guidance magnifies the stimulus effect of QE. Output growth is between 0.24 and 0.73 percentage points higher under forward guidance than no forward guidance. Inflation is between 14 and 43 annualized basis points higher under QE with forward guidance than QE without forward guidance.

The intuition is simply that QE is inflationary and expansionary. This means that if the short rate is not constrained the Taylor Rule drives the central bank to raise rates in response to the effects of QE program. So removing the Forward Guidance policy which constrains monetary policy moderates QE's stimulus effect but does not qualitatively change the impact of QE on most series.

Figure 8: QE With and Without Forward Guidance (b)

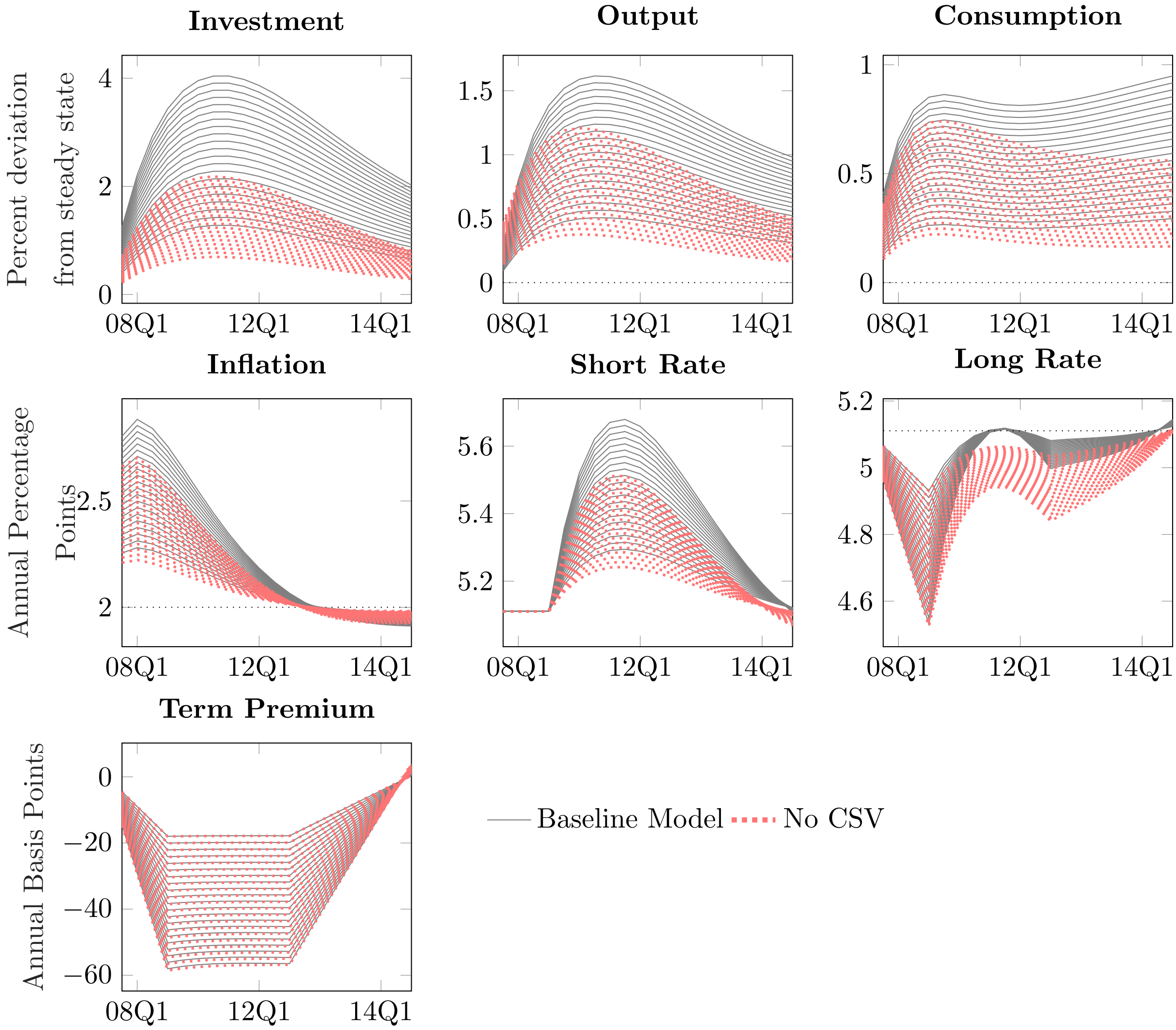


Note: The thick dashed lines correspond to the simulation at the median value for the portfolio preference term ν.

The exception here is the response of credit. Credit has a different response under the QE with forward guidance ("Forward Guidance") than it does just under QE ("FFR Unconstrained"). Under the FFR unconstrained simulation the increase in the federal funds rate is front-loaded. And because the federal funds rate is never constrained the long term government bond rate does not experience as dramatic a decline. This substantially mutes the on impact response of entrepreneurial net worth (a 2.4% jump over steady state as compared to 5.4%, at the median ν value). A lower net worth means that, in the FFR unconstrained simulation, the entrepreneurs rely initially more on credit (and the drop in leverage is less pronounced).

4.5 Amplifying of Quantitative Easing: Costly-State Verification

Figure 9: The Costly State Verification Friction Amplifies the Portfolio Balance Channel



Output growth is between 0.13 and 0.41 percentage points greater with the Costly-State Verification (CSV) friction than without ($\mu = 0$, "No CSV"). Inflation is boosted between 6 and 18 annualized basis points by the CSV friction. Investment is between 0.58 and 1.86 percentage points higher with the CSV friction than without.

Note that in the No CSV model the spread between the expected return on capital and the long term government bond rate is zero[3]. So even in the No CSV results presented in figures 9 QE has a strong quantitative impact because the long term interest rate is directly related to the return on capital, and QE directly depresses the long term interest rate. Exploring the extent to which preferred habitat preferences link the long term interest rate to the return on capital, in future work, could be an important part of understanding QE transmission.

[3]The entrepreneur's optimal choice of the default rate is reduced to $E\left[R^k_{t+1} - R^L_t\right] = 0$. In the Christiano et al. (2014) model this condition will be $E\left[R^k_{t+1} - R_t\right] = 0$. Without the portfolio preference, this would be equivalent to a standard DSGE model (eg Christiano et al. (2005)) where households choose capital investment.

4.6 Impact of Investors' Preferred Habitat Preferences

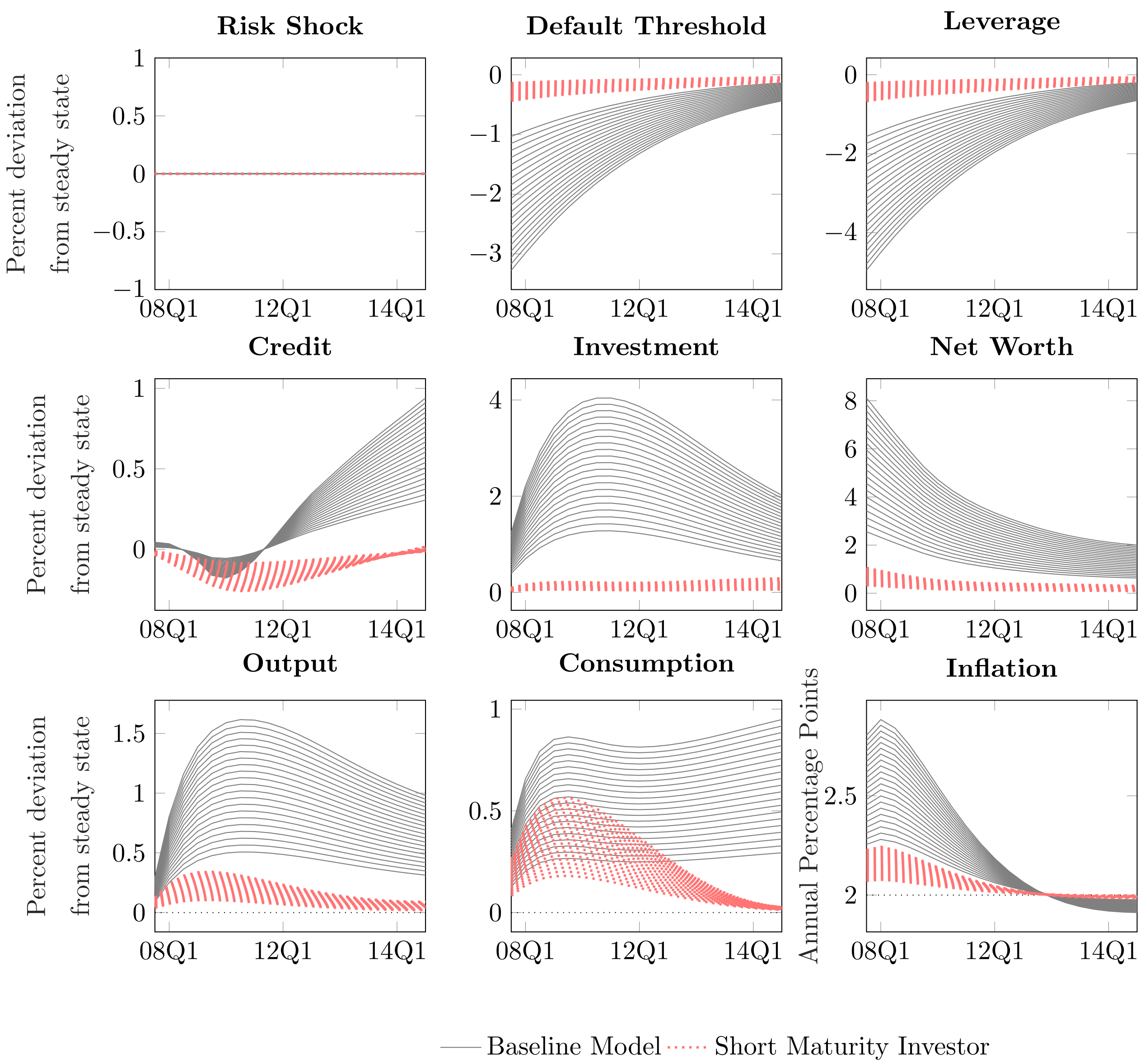


Figure 10: Impact of Investor Preferences (a)

The following repeat the baseline QE simulation (LSAPII + ZLB constraint) in the long vs short model. These results further emphasize the importance of the Duration Risk Channel to the quantitative impact of QE. Output growth is between 0.4 - 1.27 % points more when investors have a preferred habitat preference for long term assets ('the baseline model), versus when investors arbitrage between packaged corporate debt and the short term rate ("Short Maturity Investor"). Inflation is boosted between 20 and 64 annualized basis points more under the assumption of duration risk. Dropping investors' preferred habitat completely eliminates the lagged positive response of credit.

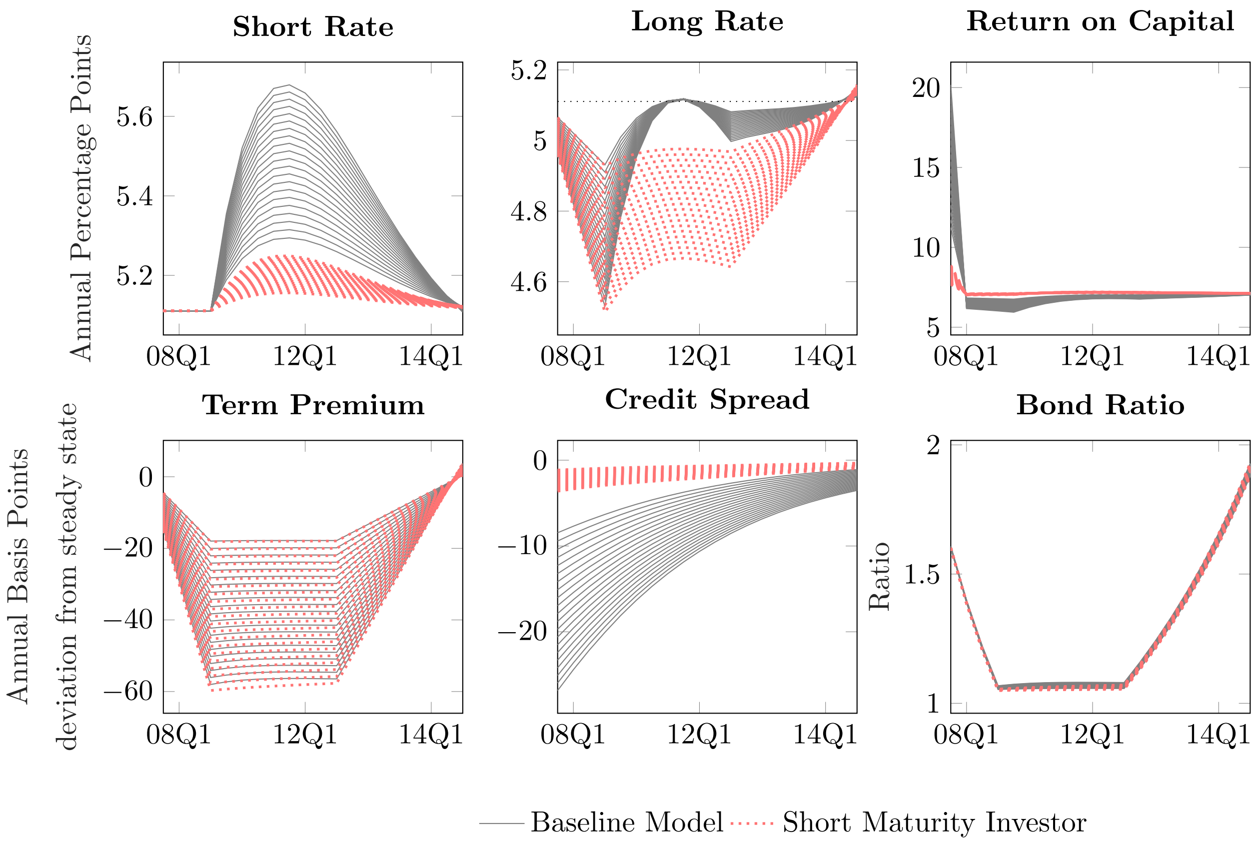


Figure 11: Impact of Investor Preferences (b)

Note: the credit spread in the short model is $Z - R$ and the credit spread in the long model is $Z - R^L$.

The only series that is not substantially dampened is consumption. QE does have a role without duration risk - in impacting the inter-temporal consumption decisions of households via the Euler equation for long bonds. But as shown by Chen et al. (2012) this effect is quantitatively limited. And these results indicate that the addition of the CSV financial friction alone does not substantially boost this effect.

5 Conclusion

In this paper I find that the second round of Federal Reserve Large Scale Asset Purchases (LSAPII) boosted output between 0.51% and 1.62% and inflation by 28 to 88 annualized basis points. Quantitatively this is closer to the larger macroeconomic effects found in vector autoregression assessments of QE than in the DSGE literature on QE. The key mechanism that contributes to the larger quantitative impact is that investors arbitrage between long term government debt and corporate bonds. This arbitrage exists because investors have preferred habitat preferences for long term bonds. This means that when Treasury yields fall as a result of QE the cost of corporate borrowing also falls, generating a portfolio balance effect.

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A Characterization of the Equilibrium

The equilibrium in this model requires equilibrium in the bond market, goods market (intermediate and final), and labour market. The resource constraint, household's budget constraint, and the investor arbitrage condition must hold. Lastly, interest rates must be such that households are willing to hold the available portfolio of bonds.

Lowercase variables are real detrended variables. So if X_t is a nominal variable then $x_t \equiv \frac{X_t}{P_t z_t^*}$.

Table 4: Notation Key

$q_t \equiv \Upsilon^t \frac{Q_{\bar{K},t}}{P_t}$	$y_{z,t} \equiv \frac{Y_t}{z_t^*}$	$i_t \equiv \frac{I_t}{z_t^* \Upsilon^t}$
$\tilde{w}_t \equiv \frac{W_t}{z_t^* P_t}$	$\bar{k}_t \equiv \frac{\bar{K}_t}{z_{t-1}^* \Upsilon^{t-1}}$	$\mu_{z,t}^* \equiv \frac{z_t^*}{z_{t-1}^*}$
$c_t \equiv \frac{C_t}{z_t^*}$	$b_t \equiv \frac{B_t}{z_t^* P_t}$	$b_t^L \equiv \frac{B_t^L}{z_t^* P_t}$
$g_t \equiv \frac{G_t}{z_t^*}$	$t_t \equiv \frac{T_t}{z_t^* P_t}$	$n_{t+1} \equiv \frac{N_{t+1}}{z_t^* P_t}$
$b_{t+1}^{credit} \equiv \frac{B_{t+1}^{credit}}{z_t^* P_t}$	$\lambda_{z,t} \equiv \lambda_t P_t z_t^*$	$v_t^L \equiv \frac{V_t^L}{z_t^* P_t}$

A.1 Auxiliary Expressions:

Aux 1: Index term in price updating for firms who cannot re-optimize

$$\tilde{\pi}_t \equiv \left(\pi_t^{\text{target}}\right)^{\iota} \pi_{t-1}^{\iota-1}.$$

Aux 2: $\tilde{\pi}_t$ ahead 1 period

$$\tilde{\pi}_{t+1} \equiv \left(\pi_{t+1}^{\text{target}}\right)^{\iota} \pi_{t}^{\iota-1}.$$

Aux 3: Definition of $K_{p,t}$:

$$K_{p,t} \equiv F_{p,t} \left[\frac{1 - \xi_p \left(\frac{\tilde{\pi}_t}{\pi_t}\right)^{\frac{1}{1-\lambda_{f,t}}}}{1 - \xi_p} \right]^{1-\lambda_{f,t}}.$$

Aux 4: $K_{p,t}$ ahead 1 period

$$K_{p,t+1} \equiv F_{p,t+1}\left[\frac{1-\xi_p\left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_{f,t+1}}}}{1-\xi_p}\right]^{1-\lambda_{f,t+1}}.$$

Aux 5: Index term in wage updating for non-reoptimizing unions

$$\tilde{\pi}_{w,t} \equiv \left(\pi_t^{\text{target}}\right)^{\iota_w}\left(\pi_{t-1}\right)^{\iota_w-1}.$$

Aux 6: $\tilde{\pi}_{w,t}$ ahead 1 period

$$\tilde{\pi}_{w,t+1} \equiv \left(\pi_{t+1}^{\text{target}}\right)^{\iota_w}\left(\pi_t\right)^{\iota_w-1}.$$

Aux 7: Wage Inflation

$$\pi_{w,t} \equiv \pi_t \mu_{z,t}^* \frac{\tilde{w}_t}{\tilde{w}_{t-1}}.$$

Aux 8: $\pi_{w,t}$ ahead 1 period

$$\pi_{w,t+1} \equiv \pi_{t+1}\mu_{z,t+1}^* \frac{\tilde{w}_{t+1}}{\tilde{w}_t}.$$

Aux 9: **Definition of $K_{w,t}$:**

$$K_{w,t} \equiv \frac{\tilde{w}_t F_{w,t}}{\psi_L}\left[\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t}(\mu_z^*)^{\iota_\mu}(\mu_{z,t}^*)^{1-\iota_\mu}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right]^{1-\lambda_w(\sigma_L+1)}.$$

Aux 10: $K_{w,t}$ ahead 1 period

$$K_{w,t+1} \equiv \frac{\tilde{w}_{t+1} F_{w,t+1}}{\psi_L}\left[\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t+1}(\mu_z^*)^{\iota_\mu}(\mu_{z,t+1}^*)^{1-\iota_\mu}}{\pi_{w,t+1}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right]^{1-\lambda_w(\sigma_L+1)}.$$

A.2 Distributions

$$F_t(\bar{\omega}_{t+1}) \equiv CDF\left(\frac{\log(\bar{\omega}_{t+1})+\frac{1}{2}\sigma_t^2}{\sigma_t}\right),$$

$$G_t(\bar{\omega}_{t+1}) \equiv CDF\left(\frac{\log(\bar{\omega}_{t+1}) + \frac{1}{2}\sigma_t^2}{\sigma_t} - \sigma_t\right),$$

$$G'_t(\bar{\omega}_{t+1}) \equiv PDF\left(\frac{\log(\bar{\omega}_{t+1}) + \frac{1}{2}\sigma_t^2}{\sigma_t} - \sigma_t\right)\frac{1}{\sigma_t},$$

$$\Gamma_t(\bar{\omega}_{t+1}) \equiv \bar{\omega}_{t+1}\Big[1 - F_t(\bar{\omega}_{t+1})\Big] + G_t(\bar{\omega}_{t+1}),$$

$$\Gamma'_t(\bar{\omega}_{t+1}) = 1 - F_t(\bar{\omega}_{t+1}).$$

A.3 Model Equations

Equation 1 (First order condition with respect to consumption):

$$E_t\left\{\frac{\mu^*_{z,t}}{c_t\mu^*_{z,t} - bc_{t-1}} - \frac{b\beta}{c_{t+1}\mu^*_{z,t+1} - bc_t} - \lambda_{z,t}\right\} = 0. \tag{A.34}$$

Equation 2 (First order condition with respect to the short bond):

$$E_t\left\{\nu\left(\frac{b_t^L}{b_t} - \delta^b\right)\frac{b_t^L}{b_t^2} - \lambda_{z,t} + \beta\frac{\lambda_{z,t+1}}{\pi_{t+1}\mu^*_{z,t+1}}R_t\right\} = 0. \tag{A.35}$$

Equation 3 (First order condition with respect to the long bond):

$$E_t\left\{-\nu\left(\frac{b_t^L}{b_t} - \delta^b\right)\frac{1}{b_t} - \lambda_{z,t} + \beta\frac{\lambda_{z,t+1}}{\pi_{t+1}\mu^*_{z,t+1}}R^L_{t+1}\right\} = 0. \tag{A.36}$$

Equation 4 (First order condition with respect to investment):

$$E_t\Big\{\lambda_{z,t}\Big(q_t-\frac{1}{\mu_{\Upsilon,t}}\Big)-\lambda_{z,t}q_t\Big[S\Big(\frac{\mu^*_{z,t}\Upsilon i_t}{i_{t-1}}\Big)+S'\Big(\frac{\mu^*_{z,t}\Upsilon i_t}{i_{t-1}}\Big)\frac{\mu^*_{z,t}\Upsilon i_t}{i_{t-1}}\Big] \\ +\beta\frac{\lambda_{z,t+1}q_{t+1}}{\mu^*_{z,t}\Upsilon}S'\Big(\frac{\mu^*_{z,t+1}\Upsilon i_{t+1}}{i_t}\Big)\Big(\frac{\mu^*_{z,t+1}\Upsilon i_{t+1}}{i_t}\Big)^2\Big\}=0. \tag{A.37}$$

Equation 5 (Firm's Production Function):

$$y_{z,t}=(p^*_t)^{\frac{\lambda_{f,t}}{\lambda_{f,t}-1}}\left[\epsilon_t\left(\frac{u_t\bar{k}_t}{\mu^*_{z,t}\Upsilon}\right)^{\alpha}\left(h_t(w^*_t)^{\frac{\lambda_w}{\lambda_w-1}}\right)^{1-\alpha}-\phi\right]. \tag{A.38}$$

Equation 6 (Resource Constraint):

$$y_{z,t}=g_t+c_t+\frac{i_t}{\mu_{\Upsilon,t}}+\Theta\frac{1-\gamma}{\gamma}\big(n_{t+1}-w^e\big)+d_t+\frac{a(u_t)\bar{k}_t}{\Upsilon\mu^*_{z,t}}, \tag{A.39}$$

where $d_t\equiv\frac{\mu G_{t-1}(\bar{\omega}_t)R^k_t q_{t-1}\bar{k}_t}{\pi_t\mu^*_{z,t}}$.

Equation 7 (Rental Rate of Capital):

$$r^k_t=\alpha\epsilon_t\left(\frac{\mu^*_{z,t}\Upsilon h_t(w^*_t)^{\frac{\lambda_w}{\lambda_w-1}}}{u_t\bar{k}_t}\right)^{1-\alpha}s_t. \tag{A.40}$$

Equation 8 (Marginal Cost):

$$s_t=\frac{1}{\epsilon_t}\left(\frac{r^k_t}{\alpha}\right)^{\alpha}\left(\frac{\tilde{w}_t}{1-\alpha}\right)^{1-\alpha}. \tag{A.41}$$

Equation 9 (Optimal utilization of capital):

$$r^k_t=a'(u_t)=r^k\exp(\sigma_a(u_t-1)), \tag{A.42}$$

where $a(u_t)\equiv\frac{r^k}{\sigma_a}\big[\exp(\sigma_a(u_t-1))-1\big]$.

Equation 10 (Law of motion for capital):

$$\bar{k}_{t+1} = \frac{1-\delta}{\mu^*_{z,t}\Upsilon}\bar{k}_t + \Big[1 - S\Big(\frac{i_t \mu^*_{z,t}\Upsilon}{i_{t-1}}\Big)\Big] i_t, \tag{A.43}$$

Equation 11 (Rate of return on capital):

$$R^k_t = \frac{u_t r^k_t - a(u_t) + (1-\delta) q_t}{\Upsilon q_{t-1}} \pi_t, \tag{A.44}$$

Equation 12 (Entrepreneurs' FoC wrt $\bar{\omega}_{t+1}$):

$$\begin{aligned} E_t \Bigg\{ & \big[1 - \Gamma_t(\bar{\omega}_{t+1})\big] \frac{R^k_{t+1}}{R^L_{t+1}} \\ & + \frac{\Gamma'_t(\bar{\omega}_{t+1})}{\Gamma'_t(\bar{\omega}_{t+1}) - \mu G'_t(\bar{\omega}_{t+1})} \Bigg[\frac{R^k_{t+1}}{R^L_{t+1}} \Big(\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1}) \Big) - 1 \Bigg] \Bigg\} = 0. \end{aligned} \tag{A.45}$$

Equation 13 (Evolution of Entrepreneurs' Net Worth):

$$n_{t+1} = \frac{\gamma}{\pi_t \mu^*_{z,t}} \Bigg\{ R^k_t \Big(1 - \mu G_{t-1}(\bar{\omega}_t) \Big) - R^L_t \Bigg\} \bar{k}_t q_{t-1} + w^e + \gamma \frac{R^L_t}{\pi_t \mu^*_{z,t}} n_t. \tag{A.46}$$

Equation 14 (Mutual Funds Zero-Profit Condition):

$$\frac{q_t \bar{k}_{t+1}}{n_{t+1}} \frac{R^k_{t+1}}{R^L_{t+1}} \Big[\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1}) \Big] - \frac{q_t \bar{k}_{t+1}}{n_{t+1}} + 1 = 0. \tag{A.47}$$

Equation 15 (AR(1) for the supply of long term government bonds):

$$\log \left(\frac{b^L_t}{b^L} \right) = \rho_{bL} \log \left(\frac{b^L_{t-1}}{b^L} \right) + u^{bL}_t. \tag{A.48}$$

Equation 16 (Taylor Rule):

$$\log\left(\frac{R_t}{R}\right) = \rho_m \log\left(\frac{R_{t-1}}{R}\right) +$$
$$\big(1-\rho_m\big)\left[\phi_\pi \log\left(\frac{\pi_t}{\pi_t^{target}}\right) + \frac{\phi_y}{4}\left(\log\frac{y_{z,t}}{y_z} - \log\frac{\mu_z^*}{\mu_{z,t}^*}\right)\right] + \frac{1}{400}\epsilon_t^m. \tag{A.49}$$

Equation 17 (Shock equation for σ_t):

$$\log\left(\frac{\sigma_t}{\sigma}\right) = \rho_\sigma\left(\frac{\sigma_{t-1}}{\sigma}\right) + \epsilon_{\sigma,t}. \tag{A.50}$$

Equation 18 (Φ_t = its value in steady state):

$$\phi = steady\_state(\Phi). \tag{A.51}$$

Equations Related to Price Setting:

Equation 19 (Law of motion for p_t^*):

$$p_t^* = \left[(1-\xi_p)\left(\frac{K_{p,t}}{F_{p,t}}\right)^{\frac{\lambda_{f,t}}{1-\lambda_{f,t}}} + \xi_p\left(\frac{\tilde{\pi}_t}{\pi_t}p_{t-1}^*\right)^{\frac{\lambda_{f,t}}{1-\lambda_{f,t}}}\right]^{\frac{1-\lambda_{f,t}}{\lambda_{f,t}}}. \tag{A.52}$$

Equation 20 (Law of motion for $F_{p,t}$, relates to Calvo Frictions):

$$F_{p,t} = E_t\left\{\lambda_{z,t}y_{z,t} + \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_{f,t+1}}}\beta\xi_p F_{p,t+1}\right\}. \tag{A.53}$$

Equation 21 (Law of motion for $K_{p,t}$):

$$K_{p,t} = E_t\left\{\lambda_{z,t}\lambda_{f,t}y_{z,t}s_t + \beta\xi_p\left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}}\right)^{\frac{\lambda_{f,t}}{1-\lambda_{f,t}}}K_{p,t+1}\right\}. \tag{A.54}$$

Equation 22 (Law of motion for $F_{w,t}$, characterizes optimal wage setting):

$$F_{w,t} = E_t \left\{ \lambda_{z,t} (w_t^*)^{\frac{\lambda_w}{\lambda_w - 1}} \frac{h_t}{\lambda_w} + \beta \xi_w (\mu_z^*)^{\frac{1-\iota_\mu}{1-\lambda_w}} (\mu_{z,t+1}^*)^{\frac{\iota_\mu}{1-\lambda_w} - 1} \left(\frac{1}{\pi_{w,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \frac{\tilde{\pi}_{w,t+1}^{\frac{1}{1-\lambda_w}}}{\pi_{t+1}} F_{w,t+1} \right\}. \tag{A.55}$$

Equation 23 (Law of motion for $K_{w,t}$):

$$K_{w,t} = E_t \left\{ \left[(w_t^*)^{\frac{\lambda_w}{\lambda_w - 1}} h_t \right]^{1+\sigma_L} + \beta \xi_w K_{w,t+1} \left(\frac{\tilde{\pi}_{w,t+1} (\mu_{z,t+1}^*)^{\iota_\mu} (\mu_z^*)^{1-\iota_\mu}}{\pi_{w,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} \right\}. \tag{A.56}$$

Equation 24 (Law of motion for w_t^*):

$$w_t^* = \left[(1-\xi_w) \left(\frac{1 - \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} (\mu_z^*)^{1-\iota_\mu} (\mu_{z,t}^*)^{\iota_\mu} \right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w} \right)^{\lambda_w} + \xi_w \left(\frac{\tilde{\pi}_{w,t} (\mu_{z,t}^*)^{\iota_\mu} (\mu_z^*)^{1-\iota_\mu}}{\pi_{w,t}} w_{t-1}^* \right)^{\frac{\lambda_w}{1-\lambda_w}} \right]^{\frac{1-\lambda_w}{\lambda_w}}. \tag{A.57}$$

Equation 25 (Entrepreneurs' balance sheet):

$$q_t \bar{k}_{t+1} = B_{t+1}^{credit} + n_{t+1}. \tag{A.58}$$

Equation 26 (Definition of leverage, L_t):

$$L_t = \frac{n_{t+1} + B_{t+1}^{credit}}{n_{t+1}}. \tag{A.59}$$

Equation 27 (Real government bonds):

$$b_t^L = v_t^L B_t^c. \tag{A.60}$$

Equation 28 (Long Rate):

$$R^L_{t+1} = \frac{1 + V^L_{t+1}}{V^L_t}. \tag{A.61}$$

Equation 29 (Entrepreneurial debt rate):

$$Z_{t+1} = R^k_{t+1}\bar{\omega}_{t+1}\frac{L_t}{L_t - 1}. \tag{A.62}$$

<u>Fiscal Policy Block:</u>

Equation 30 (Government Spending Rule):

$$g_t = g_{yz} steady\_state(y_z). \tag{A.63}$$

Equation 31 (Government Budget Constraint):

$$b_t + b^L_t = \frac{R_{t-1}b_{t-1}}{\pi_t \mu^*_{z,t}} + \frac{R^L_t b^L_{t-1}}{\pi_t \mu^*_{z,t}} + g_t - t_t. \tag{A.64}$$

Equation 32 (Fiscal Rule):

$$t_t - g_t = \kappa \left(\frac{b_{t-1} + b^L_{t-1}}{b + b^L} \right)^{\phi_T} \epsilon^T_t. \tag{A.65}$$

B News Shocks

The calibration method of the anticipated news shocks is based on Del Negro, Giannoni and Patterson (2013) and Lasen and Svensson (2011). This appendix section specifies where news shocks appear in the model, the news shock structure, and the process of calibrating news shocks to match a given path for the short rate.

B.1 Where is the News?

News in this model appears as shocks to the Taylor rule and shocks to the AR(1) rule for exogenous long-term bond supply. The backward-looking Taylor rule:

$$\log\left(\frac{R_t}{R}\right) = \rho_m \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_m)\left[\phi_\pi \log\left(\frac{\pi_t}{\pi_t^{target}}\right) + \frac{\phi_y}{4}\left(\log\frac{y_{z,t}}{y_s} - \log\frac{\mu_z^*}{\mu_{z,t}^*}\right)\right] + \frac{1}{400}u_t^m. \tag{B.66}$$

The long term government bond supply rule:

$$\log\left(\frac{b_t^L}{b^L}\right) = \rho_{BL}\log\left(\frac{b_{t-1}^L}{b^L}\right) + u_t^{BL}. \tag{B.67}$$

In the model simulations $\rho_{BL} = 0$, so that the entire path of bond purchases, holding, and unwinding in the quantitative easing program is set via surprise and news shocks.

B.2 News Shock Structure

A generic news shock has the following representation:

$$u_t = \epsilon_t + \xi_{1,t-1} + \xi_{2,t-2} + ... + \xi_{p,t-p}, \tag{B.68}$$

where ϵ_t is the unanticipated shock (eg the surprise monetary policy shock or long bond supply shock). And $\xi_{p,t-p}$ for $p \geq 1$ are the anticipated news shocks. The shock $\xi_{p,t-p}$ is observed by agents in period $t-p$, but does not affect the relevant sum of shocks until period t. In the AR(1) for long bonds these news shocks represent pre-announced QE purchases. In the Taylor rule they are a pre-announced path for the short rate targeted by the central bank (i.e. Forward Guidance).

B.3 Forward Guidance Implementation

Dynare has the following state-space representation of the model:

- s_t = m x 1 vector of states. (m = M_.npred + M_.nboth).
- x_t = n x 1 vector of controls. (n = M_.nstatic + M_.nfwrd).
- ϵ_t = w x 1 vector of shocks. (w = M_.exo_nbr).

- Φ = (n x m), the policy function.

State-space representation:

$$s_t = \underset{mxm}{A} s_{t-1} + \underset{mxw}{B} \epsilon_t, \tag{B.69}$$

$$x_t = \Phi s_t. \tag{B.70}$$

Substitute (B.69) into (B.70):

$$s_t = As_{t-1} + B\epsilon_t,$$

$$x_t = \Phi As_{t-1} + \Phi B\epsilon_t.$$

Define $\underset{nxm}{C} \equiv \Phi A$ and $\underset{nxw}{D} \equiv \Phi B$. And rewrite the state-space system:

$$s_t = As_{t-1} + B\epsilon_t,$$

$$x_t = Cs_{t-1} + D\epsilon_t.$$

Stack the system and collapse: $Y_t = \begin{bmatrix} s_t \\ x_t \end{bmatrix}, \qquad \Psi = \begin{bmatrix} A \\ C \end{bmatrix}, \qquad \Omega = \begin{bmatrix} B \\ D \end{bmatrix}.$

$$Y_t = \Psi s_{t-1} + \Omega\epsilon_t. \tag{B.71}$$

- $\Psi = oo_.dr.ghx$ = (m+n) $\times$ m, matrix of coefficients that appears in the Dynare generated transition rule. (# of endogenous variables = m+n, by # of state variables = m).

- $\Omega = oo_.dr.ghu$ = (m+n) $\times$ w, matrix of coefficients that appears in the Dynare generated transition rule. It has dimension (# of endogenous variables by # of shocks).

Let Z be a matrix ($m \times (m+n)$) that selects the state variables from the Y_t matrix, so that $s_t = ZY_t$. And define $M \equiv \Psi Z$. So we can rewrite B.71 as:

$$Y_t = MY_{t-1} + \Omega\epsilon_t.$$

Split the shock vector:

$$\epsilon_t = \epsilon_t^1 + \epsilon_t^2. \tag{B.72}$$

Where ϵ_t^1 is a $w \times 1$ vector where all shocks, except the monetary policy forward guidance shocks are replaced with zeros. And ϵ_t^2 is a $w \times 1$ vector where all but the monetary policy forward guidance shocks are replaced with zeros. So can further rewrite B.71 as:

$$Y_t = MY_{t-1} + \Omega\Big[\epsilon_t^1 + \epsilon_t^2\Big]. \tag{B.73}$$

Both the forward guidance shocks and the QE shock, that hit at $t = 1$, so $\epsilon_t = 0 \ \forall t > 1$. Note that ϵ_1^1 is known (this vector contain the shocks to the long bond supply rule that introduce QE). Further note that ϵ_1^2 is the vector of shocks to be calibrated to produce the target path for the policy rate.
Using equation B.73 note that:

$$Y_1 = MY_0 + \Omega\Big[\epsilon_1^1 + \epsilon_1^2\Big],$$

where Y_0 is the steady state (also known). Iterating forward:

$$\begin{aligned} Y_2 &= MY_1 \\ &= M^2 Y_0 + M\Omega\Big[\epsilon_1^1 + \epsilon_1^2\Big]. \end{aligned}$$

Generally:

$$Y_t = M^t Y_0 + M^{t-1}\Omega\Big[\epsilon_1^1 + \epsilon_1^2\Big]. \tag{B.74}$$

Let $\mathbf{R}^{FG}$ be the $t_{FG} \times 1$ target vector for the path of the policy rate, where t_{FG} is the number of periods that the policy rate is constrained (in the LSAPII simulation $t_{FG} = 4$). Let $\tilde{Z}$ be a $1 \times (n + m)$ row vector that selects the row of Y_t corresponding to the policy rate (in Dynare this is the DR ordering of R_t).

$$
\begin{aligned}
R_1 &= \tilde{Z}MY_0 + \tilde{Z}\Omega\epsilon_1^1 + \tilde{Z}\Omega\epsilon_1^2,\\
R_2 &= \tilde{Z}M^2Y_0 + \tilde{Z}M\Omega\epsilon_1^1 + \tilde{Z}M\Omega\epsilon_1^2,\\
R_3 &= \tilde{Z}M^3Y_0 + \tilde{Z}M^2\Omega\epsilon_1^1 + \tilde{Z}M^2\epsilon_1^2,\\
R_4 &= \tilde{Z}M^4Y_0 + \tilde{Z}M^3\Omega\epsilon_1^1 + \tilde{Z}M^3\Omega\epsilon_1^2.
\end{aligned}
$$

Stack:

$$
\begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix} = \underbrace{\begin{bmatrix} \tilde{Z}MY_0 + \tilde{Z}\Omega\epsilon_1^1 \\ \tilde{Z}M^2Y_0 + \tilde{Z}M\Omega\epsilon_1^1 \\ \tilde{Z}M^3Y_0 + \tilde{Z}M^2\Omega\epsilon_1^1 \\ \tilde{Z}M^4Y_0 + \tilde{Z}M^3\Omega\epsilon_1^1 \end{bmatrix}}_{\mathbf{K}} + \underbrace{\begin{bmatrix} \tilde{Z}\Omega \\ \tilde{Z}M \\ \tilde{Z}M^2 \\ \tilde{Z}M^3\Omega \end{bmatrix}}_{\mathbf{J}} \epsilon_1^2. \tag{B.75}
$$

Set the vector of policy rates equal to the target path $\mathbf{R}_{FG}$:

$$
\begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix} = \begin{bmatrix} R_1^{FG} \\ R_2^{FG} \\ R_3^{FG} \\ R_4^{FG} \end{bmatrix} \equiv \mathbf{R}^{FG}. \tag{B.76}
$$

$$
\mathbf{R}^{FG} = \mathbf{K} + \mathbf{J}\epsilon_1^2. \tag{B.77}
$$

and solve for ϵ_1^2:

$$
\epsilon_1^2 = \mathbf{J}^{-1}\Big(\mathbf{R}^{FG} - \mathbf{K}\Big). \tag{B.78}
$$

C The Role of Government Spending Rules

In this appendix I examine alternative government spending rules and show that changing the government spending rule has minimal quantitative impact on the results and qualitatively does not change the results. I present two alternative rules: one the Graeve and Theodoridis (2016) government spending rule, or two, a simple auto-regressive supply rule for the market value of de-trended short-term bonds

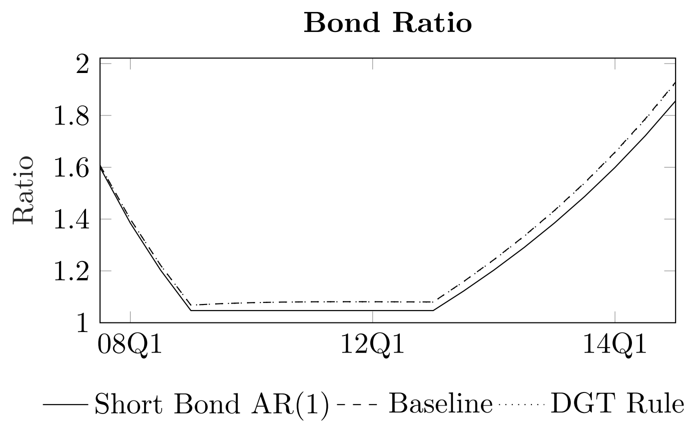

Figure 12: Effect of Government Spending Rules (a)

($b_t \equiv \frac{B_t}{P_t z_t^*}$):

$$\log\left(\frac{b_t}{b}\right) = \rho_b\left(\frac{b_{t-1}}{b}\right) + u_t^b, \tag{C.79}$$

where b is the steady state level of de-trended short-term government debt. Under the AR(1) for short term bonds a quantitative easing shock is captured as a combination of negative shocks that reduce the quantity of long-term government debt in the hands of the public, and positive shocks (u_t^b) to the supply of short term government debt in the hands of the public. The increase in short-term debt is set to be equal to the magnitude of the decrease in the market value of long-term government debt, reflecting the increased creation of reserves used to purchase long-term bonds.

The Graeve and Theodoridis (2016) rule ($\phi_T = 0.04.$):

$$\frac{T_t}{P_t z_t^*} = \kappa\left(\frac{\frac{B_{t-1}}{P_{t-1}z_{t-1}^*} + \frac{B_{t-1}^L}{P_{t-1}z_{t-1}^*}}{b + b^L}\right)^{\phi_T} \epsilon_t^T. \tag{C.80}$$

Figure 12 shows that the fiscal policy rule has a minimal impact on the response of short bonds to QE purchases, and therefore a minimal impact on the impact of QE, as figures 13 and 14 indicate.

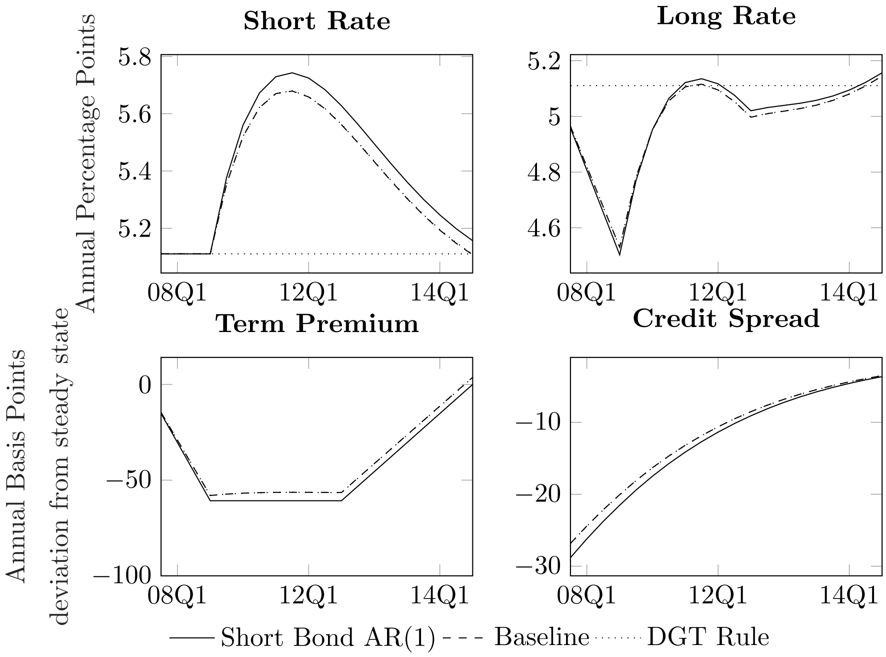

Figure 13: Effect of Government Spending Rules (b)

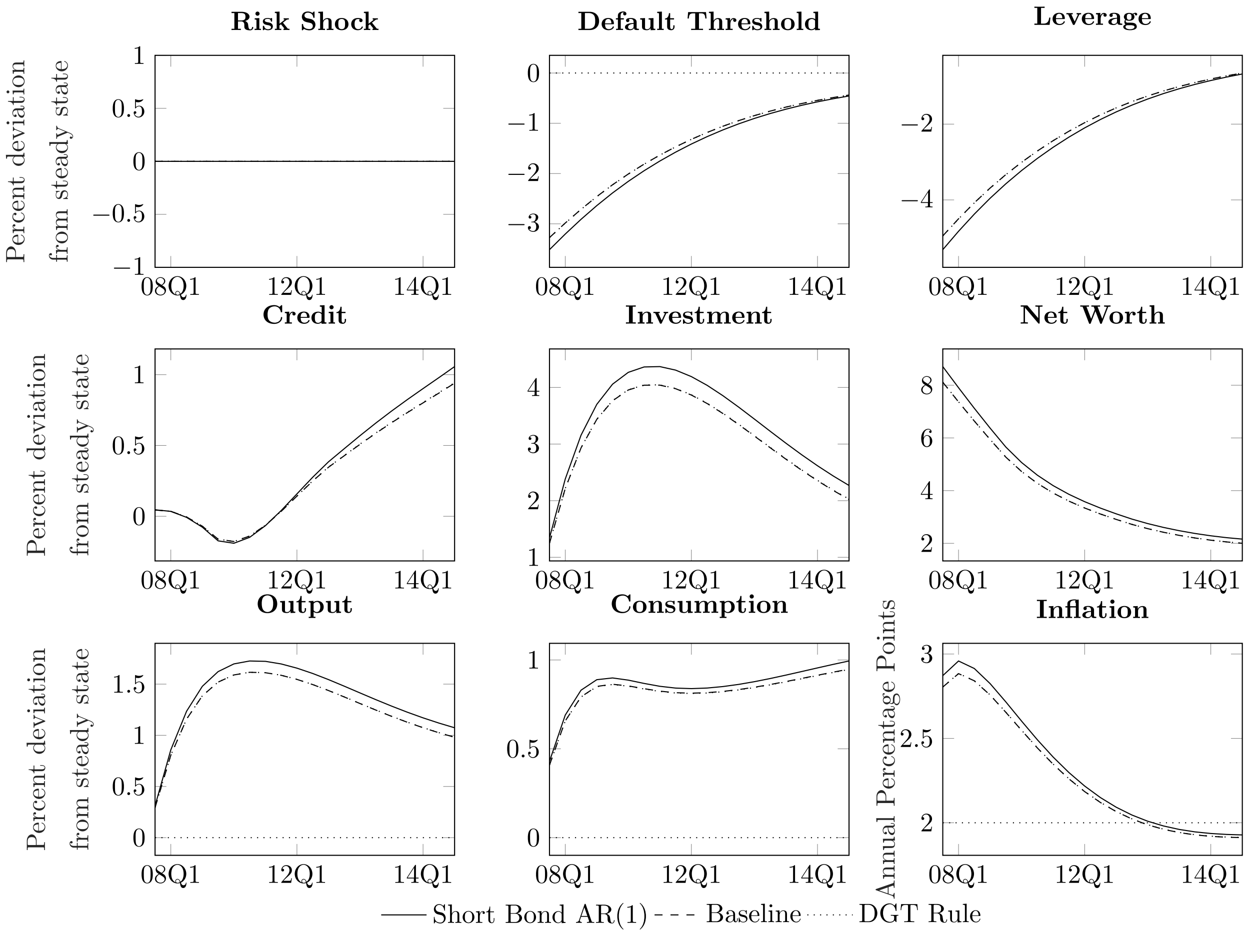

Figure 14: Effect of Government Spending Rules (c)

D Data Appendix

I use US quarterly data from 1985q1 to 2007q3 (the period before the NBER recession start date). The following series come from the Federal Reserve Bank of St. Louis' Federal Reserve Economic Data (FRED): Gross Domestic Product (GDP), and Effective Federal Funds Rate (FEDFUNDS). Data on the nominal value of privately-held marketable interest-bearing US public debt come from the Haver/DLX USECON database. The data are broken down by time until maturity. The long term government bonds are defined as bonds with over 1 year until maturity, and the short term government bonds are defined as bonds with less than 1 year left until maturity.

Table 5: Haver/DLX USECON Data Codes

Haver/DLX Code	**Data Series**
PDIMP	Total
PDIMPL	Less than 1 year left until maturity
PDIMP1	1 to 5 years left until maturity
PDIMP5	5 to 10 years left until maturity
PDIMP10	10 to 20 years left until maturity
PDIMP20	over 20 years left until maturity

The size of a \$100 billion USD purchase of long term bonds is x% of the steady state quantity of long term bonds, where x is calibrated as follows:

$$x = \frac{100}{bL\_yz \times \frac{\text{2007q3 GDP}}{4}}, \tag{D.81}$$

where bL_yz is the steady state ratio of long term bonds to (quarterly) output, and 2007q3 GDP is in annual terms. \$100 billion USD purchase of long term bonds is equivalent to 3.55% of steady state de-trended long bonds (b^L). So the \$600 billion USD purchase of long term bonds in LSAPII is equivalent to 21.3% of steady state de-trended long bonds.

Table 6: Parameters Corresponding to Targets

Parameter	**Description**	**Calibration**
β	discount rate	0.9964
b^L_{yz}	steady state ratio of long-term government bonds to output	0.77
δ^b	steady state ratio of long to short bond holdings	1.86
g_{yz}	steady state ratio of government spending to output	0.2
κ	steady state primary fiscal surplus	0.0143
π^{target}	steady state target inflation	1.005
ψ_L	disutility weight on labor	1.2126
ν	elasticity of the term premium to the bond ratio	0.00074-0.0025

E Derivation of ν

The $\nu \equiv \frac{1}{\lambda_z}\frac{\tilde{\nu}\delta^b}{b}(1+\delta^b)$ is the elasticity of the term premium with respect to the relative supply of long to short term government debt. The ν parameter governs the responsiveness of the term premium to changes in the relative supply of long versus short term bonds. The following shows the log-linearization of the key equations (Model equations 2 & 3) around the steady-state used to calibrate the partial equilibrium response.

First order condition with respect to the short bond:

$$f^a \equiv E_t\left\{\tilde{\nu}\left(\frac{b^L_t}{b_t}-\delta^b\right)\frac{b^L_t}{b^2_t}-\lambda_{z,t}+\beta\frac{\lambda_{z,t+1}}{\pi_{t+1}\mu^*_{z,t+1}}R_t\right\}=0.$$

First order condition with respect to the long bond:

$$f^b \equiv E_t\left\{-\tilde{\nu}\left(\frac{b^L_t}{b_t}-\delta^b\right)\frac{1}{b_t}-\lambda_{z,t}+\beta\frac{\lambda_{z,t+1}}{\pi_{t+1}\mu^*_{z,t+1}}R^L_{t+1}\right\}=0.$$

Log-linearizing around the steady state:

$$f^a \approx \lambda_z\Big[\hat{\lambda}_{z,t+1}-\hat{\lambda}_{z,t}+\hat{R}_t-\hat{\pi}_{t+1}-\hat{\mu}^*_{z,t+1}\Big]+\frac{\tilde{\nu}(\delta^b)^2}{b}(\hat{b}^L_t-\hat{b}_t), \tag{E.82}$$

$$f^b \approx \lambda_z\Big[\hat{\lambda}_{z,t+1}-\hat{\lambda}_{z,t}+\hat{R}^L_{t+1}-\hat{\pi}_{t+1}-\hat{\mu}^*_{z,t+1}\Big]-\frac{\tilde{\nu}\delta^b}{b}(\hat{b}^L_t-\hat{b}_t). \tag{E.83}$$

This implies:

$$\hat{R}^L_{t+1} - \hat{R}_t = \frac{1}{\lambda_z}\frac{\tilde{\nu}\delta^b}{b}(1+\delta^b)\big[\hat{b}^L_t - \hat{b}_t\big]$$
$$= \nu\big[\hat{b}^L_t - \hat{b}_t\big]. \tag{E.84}$$

F Additional Results

F.1 Baseline Model vs Nested Model without the Default Risk or Wider Duration Preference

The "No CSV & Short Maturity Investor" model is a nested version of the baseline model. The state verification cost is set to zero ($\mu = 0$), meaning that there is no external finance premium for entrepreneurs. This removes the accelerator effect from variations in entrepreneurial net worth. Because credit conditions are neutral to the condition of the entrepreneurs' balance sheet, this shuts off the Default Risk Channel. Also in this nested model version the assumption that investors have a preferred habitat preference for long term assets is dropped. This means the long rate is replaced with the short rate in the mutual fund's zero profit condition[4].

Output is boosted in the baseline model of this paper relative to the No CSV & Short Maturity Investor model by 0.44% to 1.38%. Inflation is boosted between 23 to 71 annualized basis points. Investment growth is boosted between 1.29 - 3.97 percentage points. In the No CSV & Short Maturity Investor model the peak growth of output is between 0.07% - 0.23%, and inflation grows between 5 and 17 annualized basis points. These results are in the range of the small quantitative impact of LSAPII found in Chen et al. (2012), and further underline the importance not only of the financial friction in amplifying the impact of QE but also the Duration Risk Channel component of the Portfolio Balance Channel.

[4]The two changes imply that the entrepreneurs' first order condition with respect to the default threshold becomes $E\Big[R^k_{t+1} - R_t\Big] = 0$, making this model comparable to a model in which households directly invest in capital - eg Chen et al. (2012)

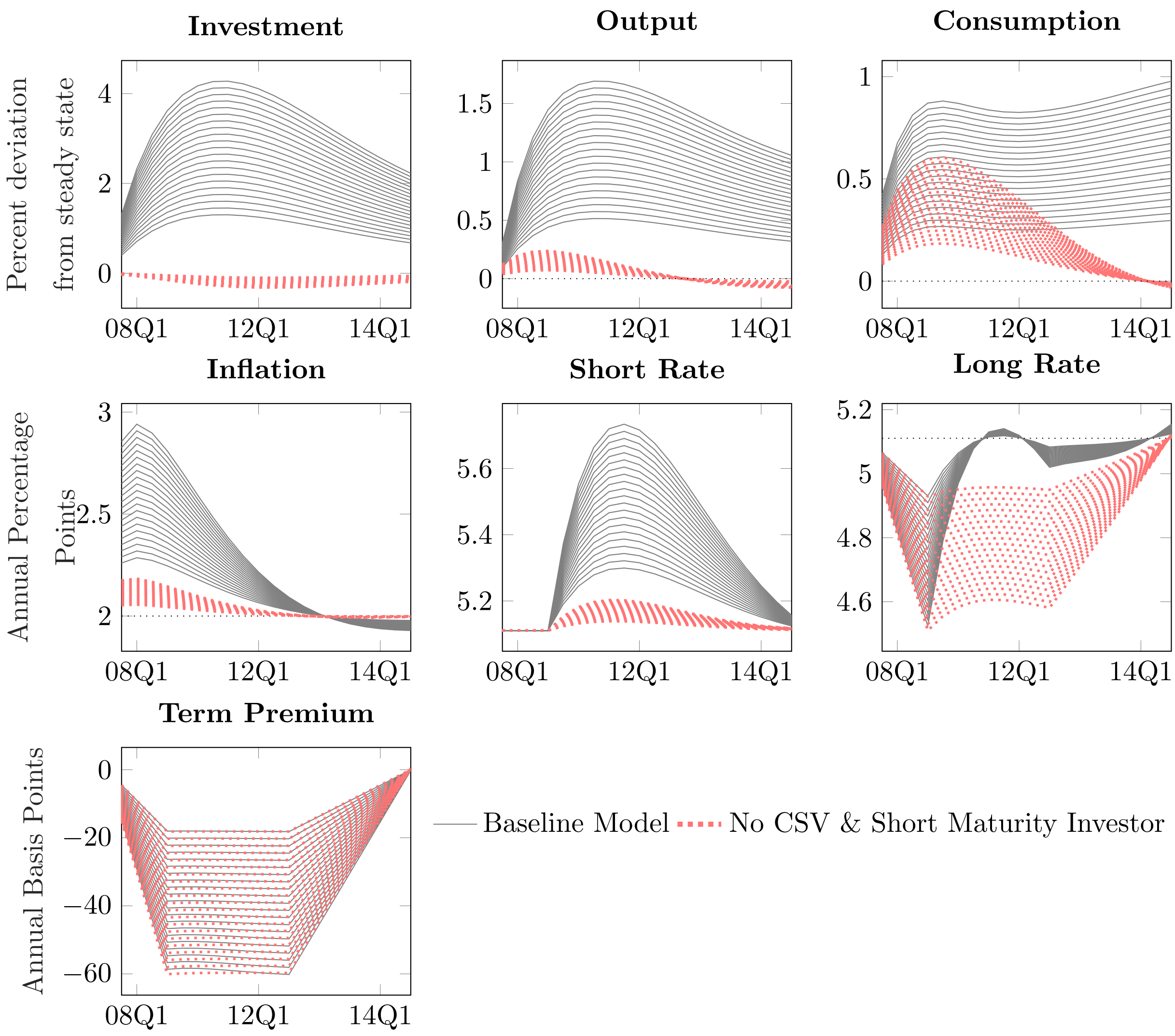

Figure 15: Baseline vs No Default Risk or Duration Preference