DAA Problem Set 1

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1 Asymptotic Notations

1. If $f(n) = n!$ and $g(n) = n^n$, prove that $f(n) = \Theta(g(n))$.
2. Order the following functions from asymptotically smallest to asymptotically largest (Indicate ties if any)
   (a) $f_1(n) = \log n$
   (b) $f_2(n) = \log n^3$
   (c) $f_3(n) = \log n^{\log n}$
   (d) $f_4(n) = n^{2.5}$
   (e) $f_5(n) = n^2 \log n$
   (f) $f_6(n) = n^{\log n}$
   (g) $f_7(n) = \log (n \log n)$
   (h) $f_8(n) = \sqrt{n}$
   (i) $f_9(n) = \log \log 2^{\sqrt{n}}$
   (j) $f_{10}(n) = \binom{n}{2}$
   (k) $f_{11}(n) = \lceil (log(logn)) \rceil !$
   (l) $f_{12}(n) = \lceil (log(n)) \rceil !$
3. Order the following functions from asymptotically smallest to asymptotically largest (Indicate ties if any)
   (a) 10^n
   (b) $n^{\frac{1}{3}}$
   (c) n^n
   (d) $\log n$
   (e) $2^{\sqrt{\log n}}$
4. If f(n) = n^2+5n and g(n) = n^2, prove that f(n) = $\Theta(g(n))$
5. Which of the following is/are correct? Provide justification.

(a) If f(x) = $x^2 + 3x + 2$ and g(x) = $5x^2$, then f(x) = $\Omega(g(x))$

(b) If f(x) = $\Theta(g(x))$, then f(x) = O(g(x)) and g(x) = $\Omega(f(x))$

(c) f(x) = $\Theta(g(x))$ iff g(x) = $\Theta(f(x))$

2 Proofs and Time Complexities

1. Prove the correctness of the following algorithms, and provide their time complexities, with formal arguments -

   (a) Bubble Sort

   (**Hint:** Use induction on the index of the outer for loop, and prove a property which holds across the iterations of the loop).

   (b) Algorithm 1 to compute the ceiling of the log of x.

Algorithm 1 Program to compute ceil of log x

Require: $x \geq 0$
Ensure: $ceiling(logx)$
  $i \leftarrow 0$
  $y \leftarrow 1$
  while $y < x$ **do**
    $i \leftarrow i + 1$
    $y = 2 * y$
  end while
  return i

   (c) Algorithm 2 to compute the floor of the square root of x.

Algorithm 2 Find Floor of Square Root

Require: $n \geq 0$
Ensure: $\text{floor}(\sqrt{n})$
  if $n = 0$ or $n = 1$ **then**
    return n
  end if
  low $\leftarrow 1$
  high $\leftarrow n$
  result $\leftarrow 0$
  while low $\leq$ high **do**
    mid $\leftarrow \lfloor (\text{low} + \text{high})/2 \rfloor$
    if $\text{mid}^2 = n$ **then**
      return mid
    else if $\text{mid}^2 < n$ **then**
      result $\leftarrow$ mid
      low $\leftarrow$ mid $+ 1$
    else
      high $\leftarrow$ mid $- 1$
    end if
  end while
  return result