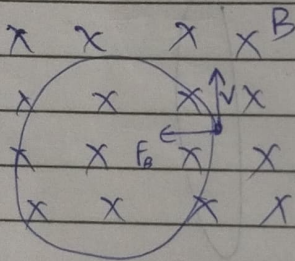


Stationary charges $\rightarrow E$
 Moving charges $\rightarrow B \& E$

$$F_{\text{mag}} = q(\vec{v} \times \vec{B})$$

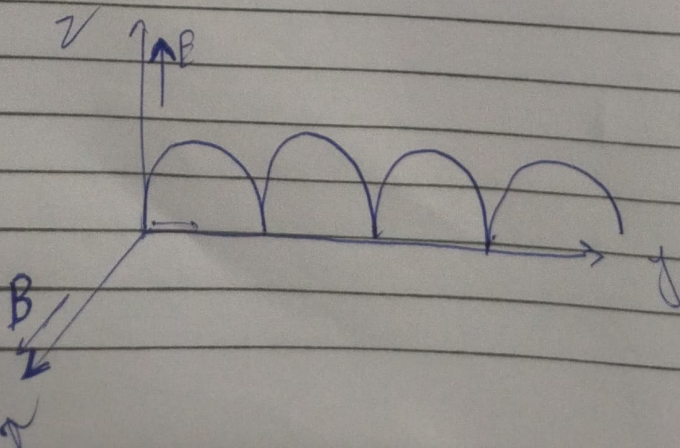
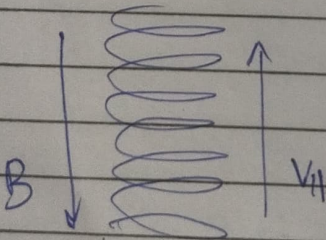
\hookrightarrow Magnetic force on charge q moving with velocity \vec{v} in field \vec{B}

$$F_{\text{net}} = q(\vec{E} + \vec{v} \times \vec{B})$$



$$q v B = m v^2 / R$$

$$R = \frac{m v}{q B}$$



$$dW_{\text{mag}} = \vec{F}_{\text{mag}} \cdot d\vec{l}$$

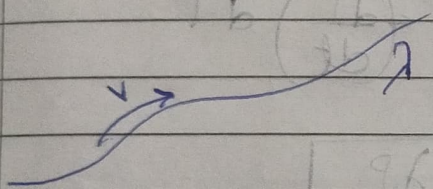
$$= q(\vec{v} \times \vec{B}) \cdot \vec{v} dt$$

$$= 0$$

$$\therefore \boxed{W_{\text{mag}} = 0}$$

Current

Charge per unit time passing at given point.



$$I = qv$$

$$\vec{F}_{\text{mag}} = \int (\vec{v} \times \vec{B}) dq = \int (\vec{I} \times \vec{B}) \frac{dq}{I}$$

$$\boxed{\vec{F}_{\text{mag}} = \int \vec{I} (\vec{I} \times \vec{B})} \quad \vec{I}, \vec{dl} \text{ point in the same direction}$$

When charge flows over a surface, we describe it by the surface current density K .

$$K = \frac{dq}{dl_{\perp}}$$

$K \rightarrow$ current per unit width

$$\boxed{K = \sigma V}$$

$$\vec{F}_{\text{mag}} = \int (\vec{V} \times \vec{B}) \sigma da$$

$$\therefore \boxed{\vec{F}_{\text{mag}} = \int (\vec{K} \times \vec{B}) da}$$

Volume current density

$$J = \frac{di}{dA}$$

curr / Area

$$J = \rho V$$

$$J = \int (\rho \times B) d\tau$$

$$I = \int_S J \cdot da = \int (\nabla \cdot J) d\tau$$

$$\begin{aligned} \int (\nabla \cdot J) d\tau &= - \frac{d}{dt} \int \rho d\tau \\ &= - \int \left(\frac{d\rho}{dt} \right) d\tau \end{aligned}$$

$$\therefore \nabla \cdot J = - \frac{\partial \rho}{\partial t}$$

$\rho \rightarrow$ Charge density

Dictionary

$$\sum_{i=1}^n \int_{line} () Idl \approx \int_{surface} () K da \approx \int_{vol} () J d\tau$$

Steady currents \rightarrow constant magnetic fields

(magnetostatics)

$$\therefore \frac{\partial \rho}{\partial t} = 0 \quad \frac{\partial \mathbf{J}}{\partial t} = 0$$

$$\boxed{\nabla \cdot \mathbf{J} = 0} \text{ for steady currents}$$

$$\boxed{B(\mathbf{r}) = \frac{\mu_0}{4\pi} i \int \frac{d\mathbf{l} \times \hat{\mathbf{r}}}{r^2}}$$

$d\mathbf{l} \rightarrow$ along wire in direction of current

unit Tesla (T)

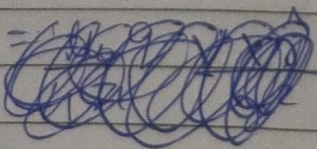
$\hat{\mathbf{r}} \rightarrow$ point at which we want B

$$\boxed{B_{\infty} = \frac{\mu_0 i}{2\pi s}} \rightarrow \text{Due to } \infty \text{ length wire}$$

$$f = \frac{\mu_0 i_1 i_2}{2\pi d}$$

Force per unit length b/w two ∞ current carrying wires

$$B(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{K} \times \hat{\mathbf{r}}}{r^2} d\mathbf{a} = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J} \times \hat{\mathbf{r}}}{r^2} d\mathbf{r}$$



Divergence & curls of B

$$\boxed{\int \vec{B} \cdot d\vec{l} = \mu_0 \text{enc}l}$$

$$\boxed{\text{enc}l = \int J \cdot da}$$

$$\int \vec{B} \cdot d\vec{l} = \int (\nabla \times \vec{B}) \cdot d\vec{a} = \mu_0 \int J \cdot d\vec{a}$$

$$\therefore \boxed{\nabla \times \vec{B} = \mu_0 \vec{J}}$$

$$\boxed{\nabla \cdot \vec{B} = 0}$$

Magnetic vector potential

$$\boxed{\vec{B} = \nabla \times \vec{A}}$$

→ vector potential

$$\boxed{\nabla \cdot \vec{A} = 0}$$

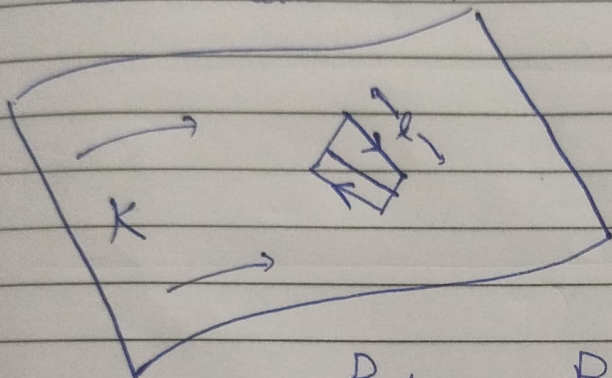
$$\boxed{\nabla^2 \vec{A} = -\mu_0 \vec{J}}$$

$$A(r) = \frac{\mu_0}{4\pi} \int \frac{J(r')}{r} d\tau' = \frac{\mu_0}{4\pi} \int \frac{K da'}{r}$$

$$= \frac{\mu_0 i}{4\pi} \int \frac{dl'}{r}$$

$$\oint B \cdot dl = (B_{\text{above}} - B_{\text{below}}) l = \mu_0 K l$$

$$\therefore B_{\text{above}} - B_{\text{below}} = \mu_0 K$$



Since

$$\oint B \cdot da = 0$$

$$B_{\text{above}}^{\perp} = B_{\text{below}}^{\perp}$$

$$B_{\text{above}} - B_{\text{below}} = \mu_0 (K \times \hat{n})$$

$$A_{\text{above}} = A_{\text{below}}$$

$$A_{\text{dip}}(r) = \frac{\mu_0}{4\pi} \frac{m \times \hat{r}}{r^2}$$

$$m = I \int da = I a$$

↓
Magnetic dipole
moment