# IIIT-Bangalore
## Course: Probability and Statistics
## Problem Set 4

## (Probability Distribution: Univariate)

1. The spectrum of the random variable $X$ consists of the points $1, 2, \ldots, n$ and $P(X = i) \propto \frac{1}{i(i+1)}$. Determine the distribution function of $X$ and compute $P(3 < X \le n)$ and $P(X > 5)$.     (Ans. $P(X = i) = \frac{n+1}{n} \cdot \frac{1}{i(i+1)}$)

2. A number is chosen at random from each of the two sets $\{0, 1, 2, 3\}$ and $\{0, 1, 2, 3\}$. Find the probability distribution of the random variable denoting the sum of the numbers chosen. (Ans. $f_0 = \frac{1}{16} = f_6$, $f_1 = \frac{2}{16} = f_5$, $f_2 = \frac{3}{16} = f_4$, $f_3 = \frac{4}{16}$)

3. Find the probability distribution of the number of failures preceding the first success in an infinite sequence of Bernoulli trials with probability of success $p$.     (Ans. $P(X = i) = (1 - p)^i p$, $i = 0, 1, \ldots$)

4. For what value of $k$ the function given by $f(x) = ke^{-|x-a|}$, $-\infty < x < \infty$ is a possible probability density function (p.d.f.). Calculate the corresponding distribution function (d.f.).     (Ans. $k = \frac{1}{2}$)

5. Show that the function $f(x) = \begin{cases} |x|, & x \in (-1, 1) \\ 0, & \text{elsewhere} \end{cases}$ is a possible p.d.f. and find the corresponding d.f.     (Ans. check!)

6. Show that the function $f(x)$ given by

$$f(x) = \begin{cases} x, & x \in (0, 1) \\ k - x, & x \in (1, 2) \\ 0, & \text{elsewhere} \end{cases}$$

   is a possible p.d.f. for a suitable $k$ and calculate $P(\frac{1}{2} < X < \frac{3}{2})$. (Ans. check!)

7. If $X$ is Poisson distribution with parameter $\mu$, prove that

$$P(X \le n) = \frac{1}{n!} \int_{\mu}^{\infty} e^{-x} x^n dx.$$

8. Three concentric circles of radii $\frac{1}{\sqrt{3}}$, $1$ and $\sqrt{3}$ feet are drawn on a target board. If a shot falls within the innermost circle 3 points are

scored; if it falls within the next two rings, then the scores are respectively 2 and 1. The score is 0 if the shot falls outside the outermost circle. If the probability density of the distance distance of the hit from the centre of the target is $\frac{2}{\pi} \cdot \frac{1}{1+r^2}$, then find the probability distribution of the score.

(Ans. $X$ : Score, $P(X=0)=\frac{1}{3}$, $P(X=1)=\frac{1}{6}$, $P(X=2)=\frac{1}{6}$, $P(X=3)=\frac{1}{3}$.)

9. A point $X$ is chosen at random on a line segment $AB$ whose middle point is $O$. Find the probability that $AX$, $BX$ and $AO$ form the sides of a triangle. (Ans. $\frac{1}{2}$)

10. A point $P$ is chosen at random on a circle of radius 'a' and $A$ is a fixed point on the circle. Find the probability that the chord $AP$ will exceed the length of an equilateral triangle inscribed in the circle. (Ans. $\frac{1}{3}$)

11. If there is a war every 15 years on the average, then find the probability that there will be no war in 25 years. (Ans. $e^{-\frac{5}{3}}$)

12. A radio active source emits on the average 2.5 particles per second. Calculate the probability that 3 or more particles will be emitted in an interval of 4 seconds. (Ans. $1 - 61e^{-10}$)