Signals

EGC 113

Source : https://www.princeton.edu/~cuff/ele301/files/lecture1_2.pdf

What are Signals?

- A signal is a pattern of variation of values of a quantity w.r.t. an independent variable such as time, space.
- Signals are variables that carry information
  - Voltages and currents in a circuit
  - Acoustic pressure (sound) over time
  - Velocity of a car over time
  - Intensity level of a pixel (camera, video) over time

What are Signals?

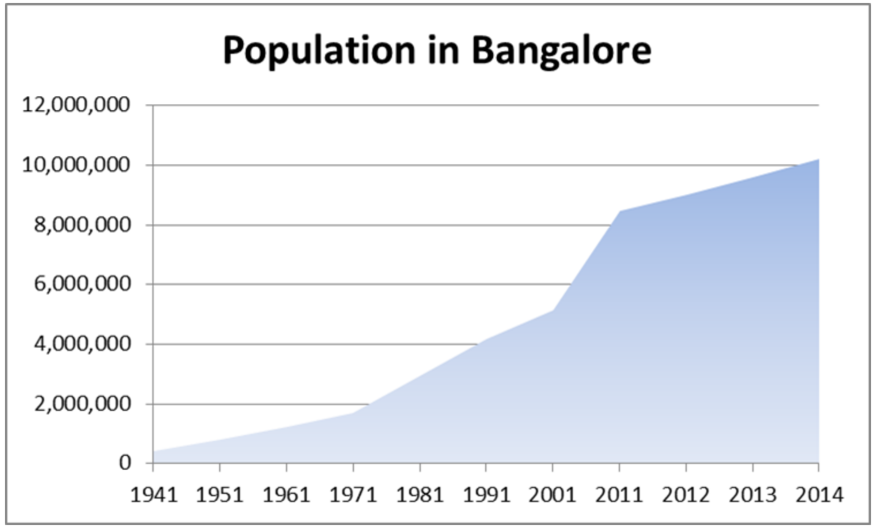
- Typical thinking of signals in terms of communication and information
  - radio signal
  - broadcast or cable TV
  - Audio
  - Electric voltage or current in a circuit

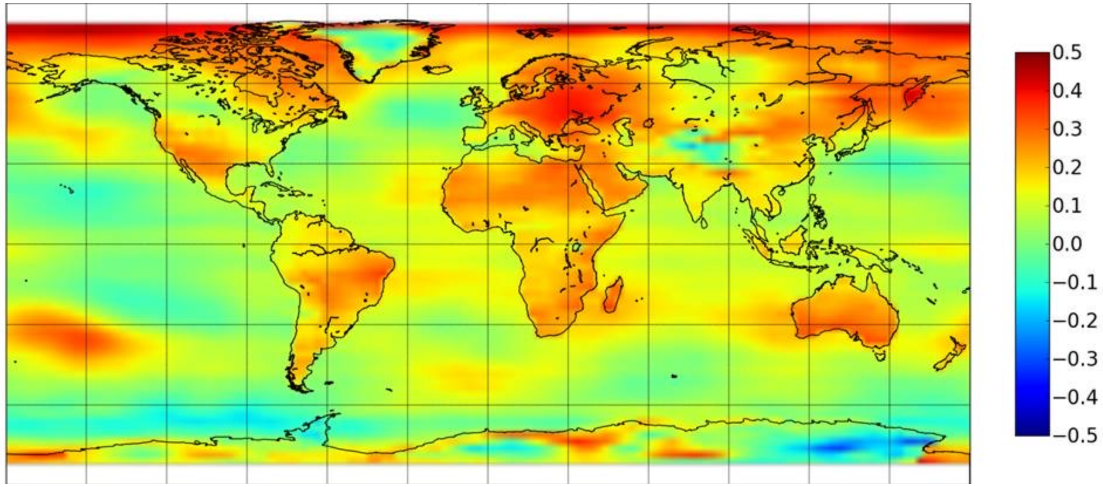
- More generally, any physical or abstract quantity that can be measured, or influences one that can be measured, can be thought of as a signal.
  - Tension on bike brake cable
  - Roll rate of a spacecraft
  - Concentration of an enzyme in a cell
  - The price of dollars in euros
  - The federal deficit
- Very general concept.

Signal ?

- Is it a ....
- Function, Sequence of numbers

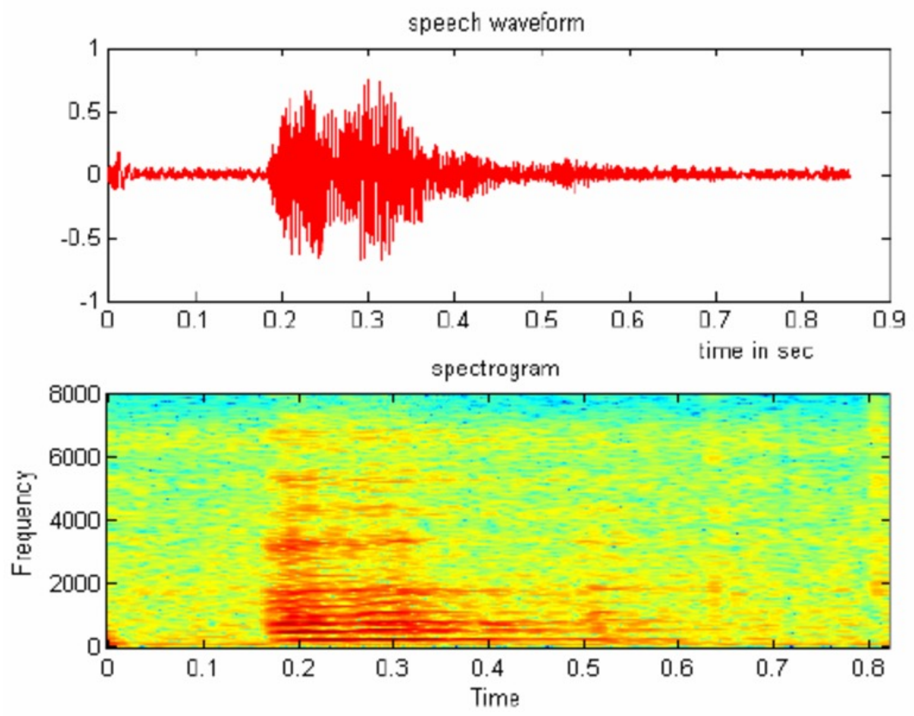
- E.g.
  - Avg. Bangalore population plotted each year
  - Temperature at every spatial location in the room or temperature across the globe



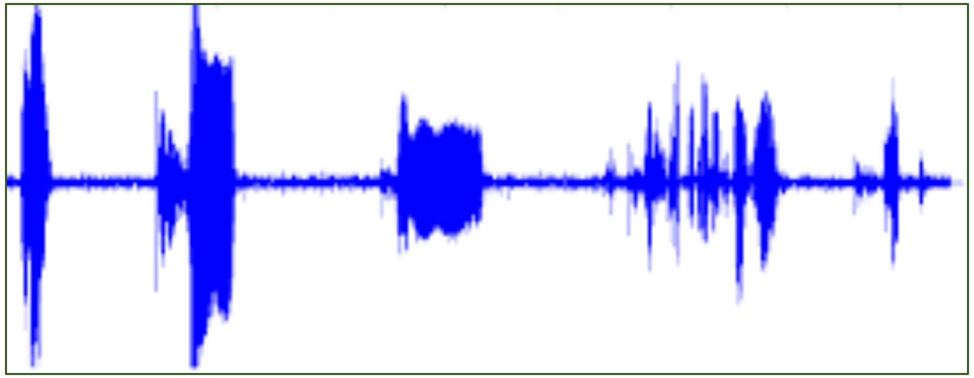


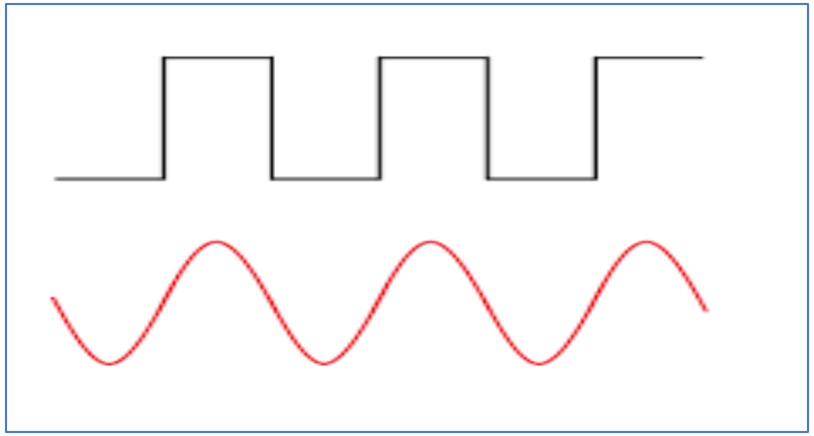
Signal

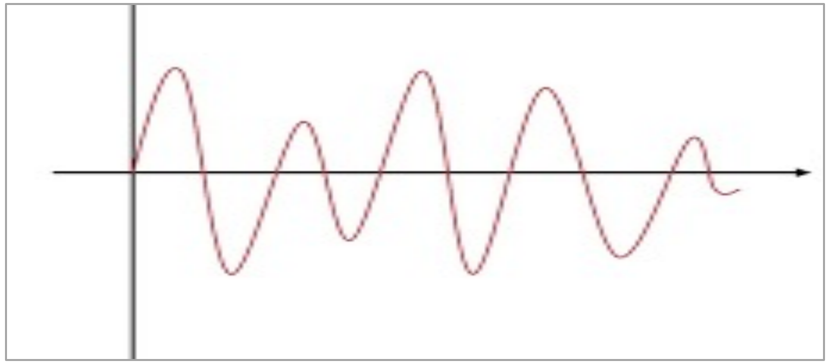
- Signal Emerges from a Physical Phenomenon..
- Representation : Function, Number series…
- Could be 1-D, 2-D, 3-D, 4-D…
- Example : speech = s(t); Here s denotes the amplitude (intensity); it is the dependent variable and ‘t’ is the independent variable.



Examples







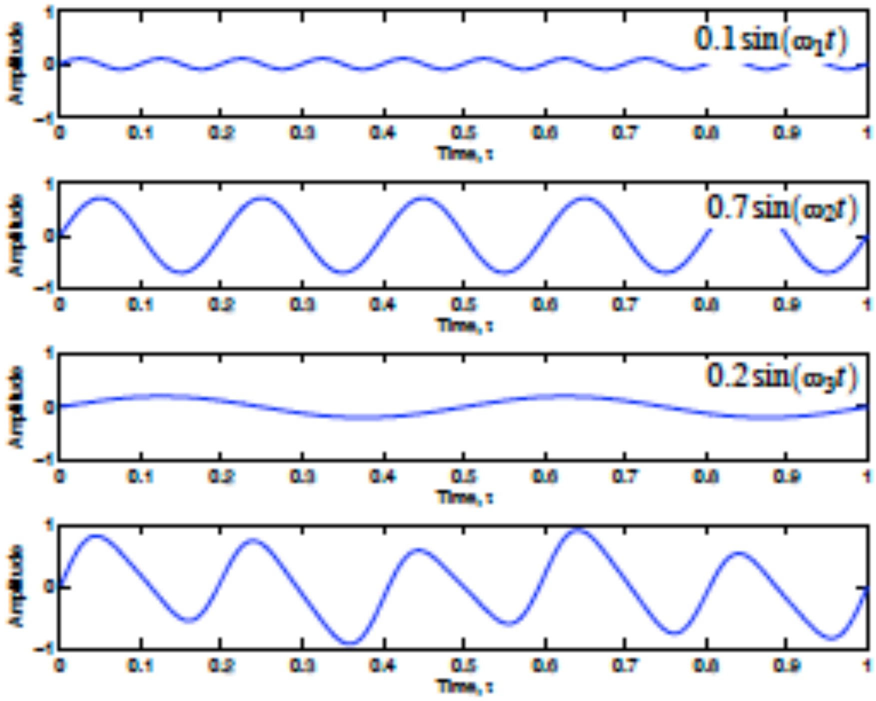
Time Series

- Time-Series Representation of Signals
  Typically think of a signal as a “time series”, or a sequence of values in time

Useful for saying what is happening at a particular time
Not so useful for capturing the overall characteristics of the signal.

Frequency Representation

- Represent signal as a combination of sinusoids



$$f(t) = 0.1\sin(\omega_1 t) + 0.7\sin(\omega_2 t) + 0.2\sin(\omega_3 t)$$

Frequency Representation

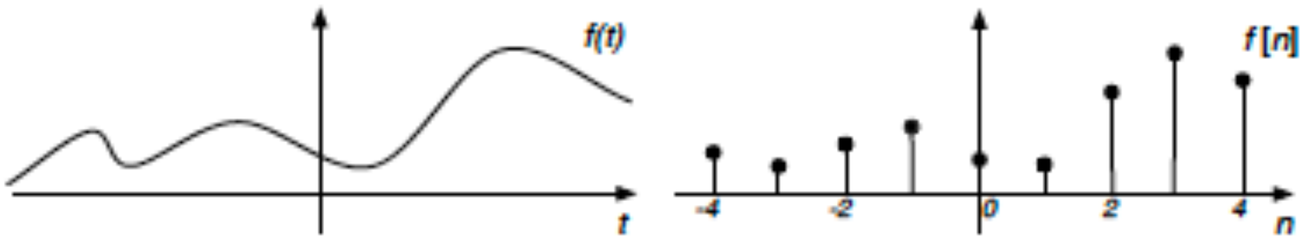
- This example is mostly a sinusoid at frequency ω_2, with small contributions from sinusoids at frequencies ω_1 and ω_3.
  - Very simple representation (for this case).
  - Not immediately obvious what the value is at any particular time.

- Why use frequency domain representation?
  - Simpler for many types of signals (AM radio signal, for example)
  - Many systems are easier to analyze from this perspective (Linear Systems).
  - Reveals the fundamental characteristics of a system.

- *Rapidly becomes an alternate way of thinking about the world.*

Continuous Signals

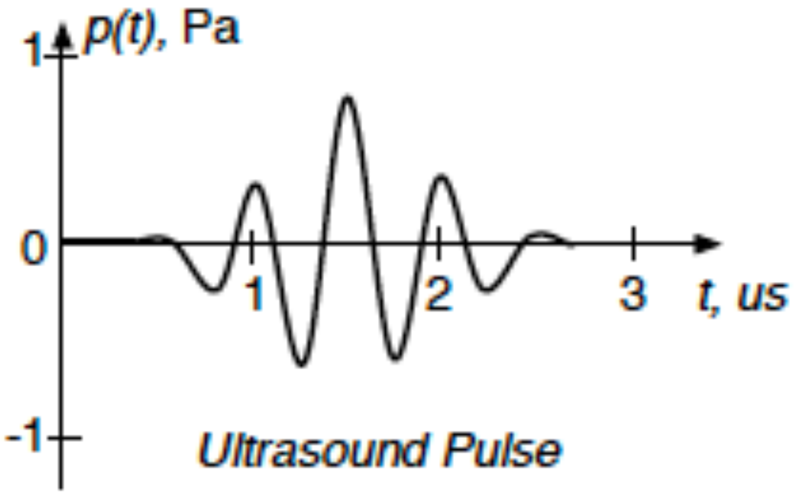
- Most of the signals we will talk about are functions of time.
- There are many ways to classify signals. This class is organized according to whether the signals are continuous in time, or discrete.
- A *continuous-time* signal has values for all points in time in some (possibly infinite) interval.
- A *discrete time* signal has values for only discrete points in time.



- Signals can also be a function of space (images) or of space and time (video), and may be continuous or discrete in each dimension.

Continuous Time Signals

- Function of a time variable, something like t, τ, t_1.
- The entire signal is denoted as v, $v(.)$, or $v(t)$, where t is a dummy variable.
- The value of the signal at a particular time is $v(1.2)$, or $v(t), t = 2$.



Discrete Time Signals

- Fundamentally, a discrete-time signal is sequence of samples, written

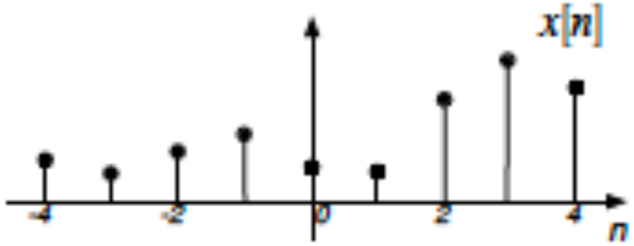
$$x[n]$$

where n is an integer over some (possibly infinite) interval.

- Often, at least conceptually, samples of a continuous time signal

$$x[n] = x(nT)$$

where n is an integer, and T is the *sampling period*.



- Discrete time signals may not represent uniform time samples (NYSE closes, for example)

Continuous Signals

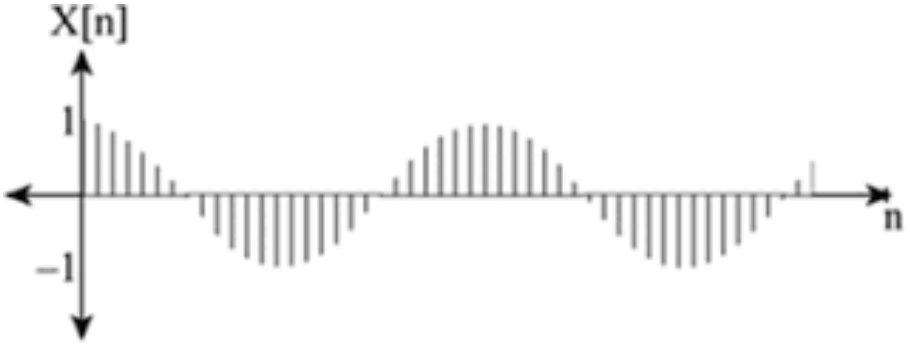
- Can a machine plot a Continuous time signals (CTS) ?
- E.g. s(t) = sin(t)
- What are the values that ‘t’ can take ? It can take infinite values, the range as well as the resolution is infinite.
- What are the values that ‘s’ can take ? Again, Infinite

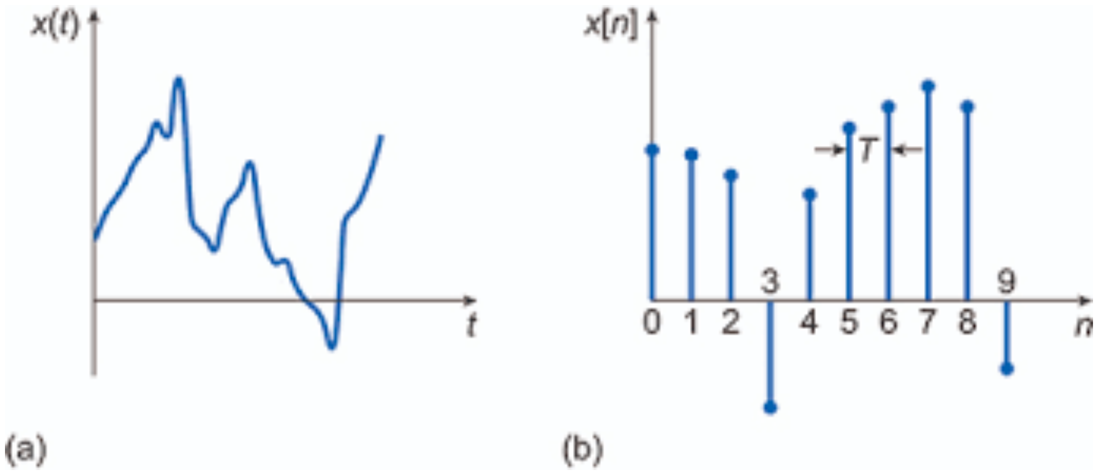
Continuous Signals

In practice, it’s impossible to work with Continuous signals !!

Then, why are Continuous signals important ??

Continuous-time vs Discrete-time signals





Examples of Discrete time signals

- Daily Average Bangalore temperature
- Stock market Hourly index

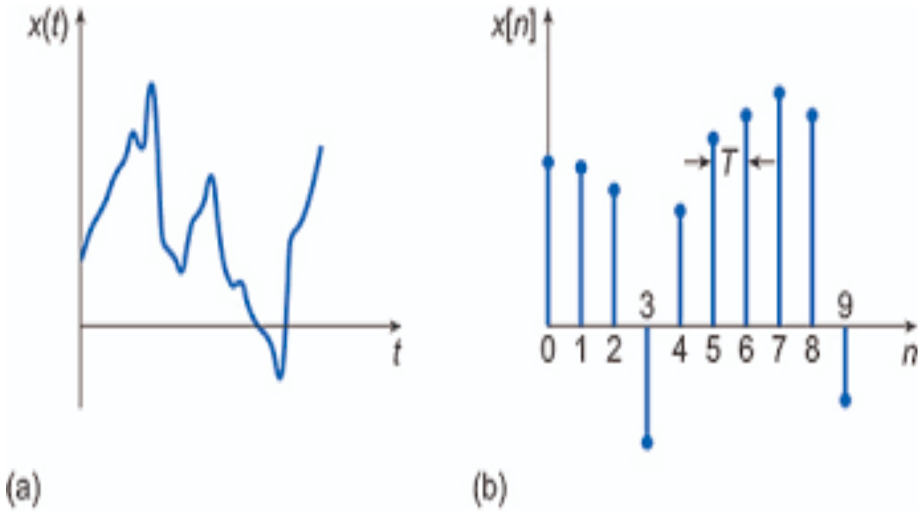
- Can a machine plot a Discrete-time signal ?

(1) x(n) = sin(2*pi*n); Plot over 2 complete cycles

(2) x(n) = sin(2*pi*2*n); Plot over 2 complete cycles

Continuous-time Vs Discrete-time

- Convention :



- x(t) is used for Cts-time signals
- t Ɛ R
- Plotted with solid curves

- x[n] is used for Discrete-time signals
- n Ɛ N
- Plotted with spikes at values takes by n

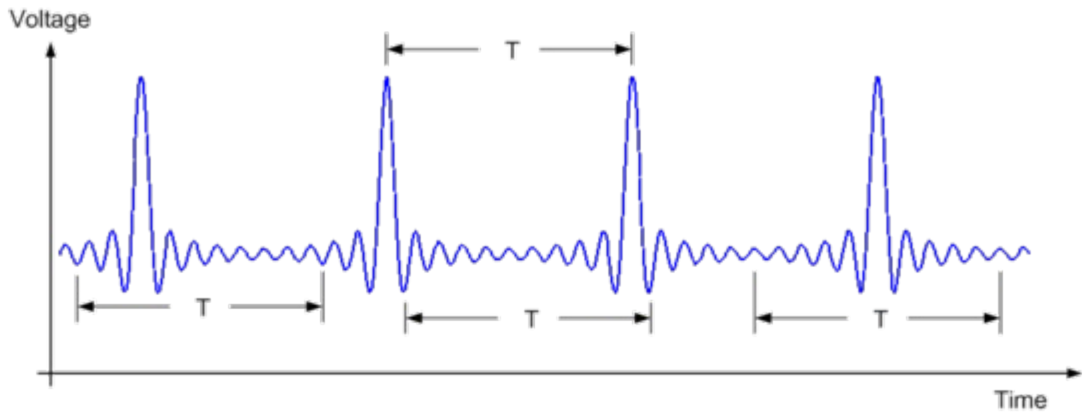
Continuous vs. Discreet Signals

Signal Attributes

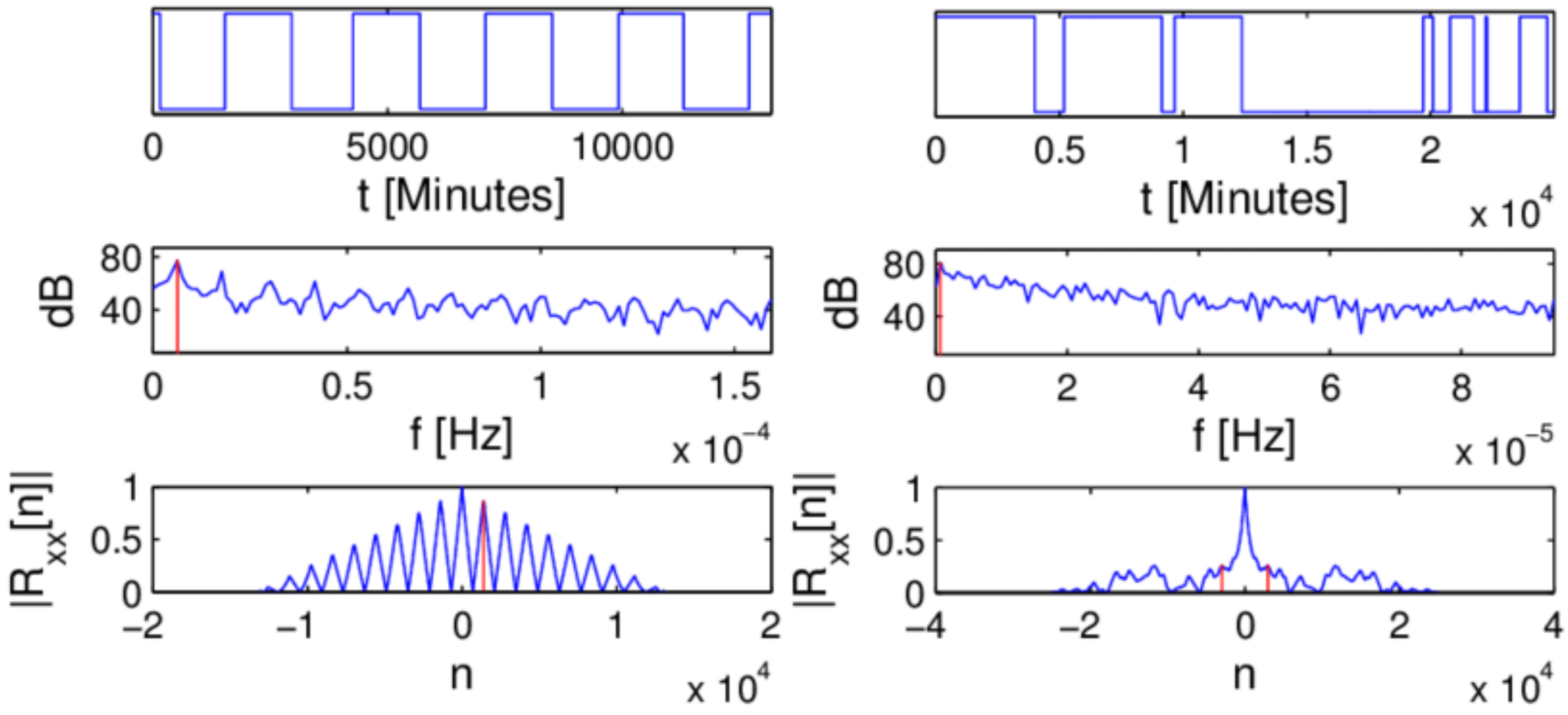
- Periodicity :
- For a cts-time signal to be periodic with period ‘T’ ($T > 0$), it has to satisfy, $x(t) = x(t + T)$, for all values of t

- i.e. $x(t) = x(t + mT)$, $m \in Z$

- Fundamental period ?



Periodic and aperiodic signals



Periodicity

- What is the span of a Periodic signal ?

- Find the period of :

  (a) $\sin(t/2)$

  (b) $\sin(2t)$

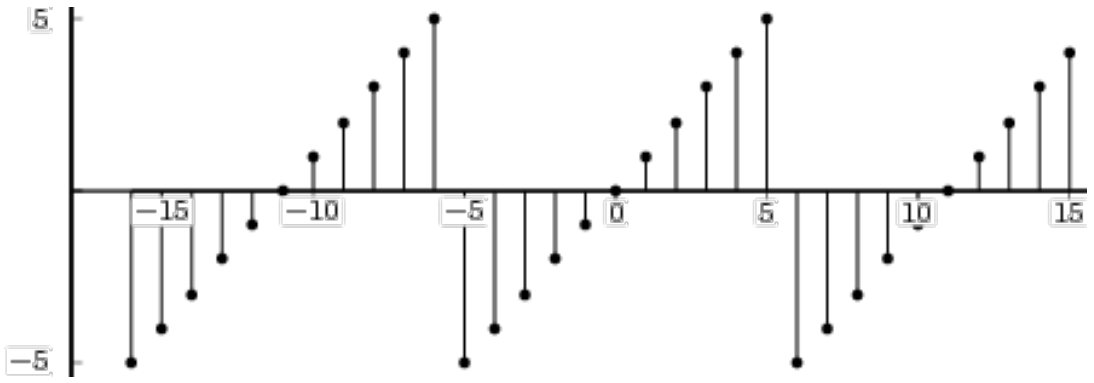
  (c) $\sin(3t) + \sin(t/6)$

Signal Attributes

- Periodicity :
- For a Discrete-time signal to be periodic with period ‘N’ it has to satisfy, x[n] = x[n+N], for all integer values of n

- i.e x [n] = x[n + mN], m Ɛ Z

- Fundamental period ?



Periodicity

- Find whether the given signal is periodic and find fundamental period :

1. $x(t) = sin^2(4\pi t)$
2. $x(t) = \sin(6\pi t) + \cos(5\pi t)$
3. $x[n] = \ e^{j2n}$
4. $x[n] = \cos(\frac{3\pi}{4}n)$
5. $x[n] = \sin(\frac{3\pi}{4}n) + \cos(\frac{5\pi}{7}n)$

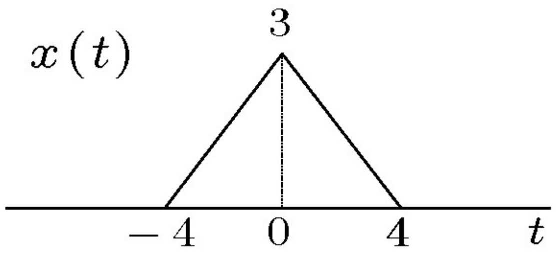
Energy

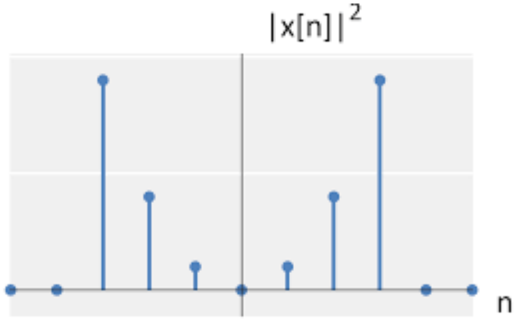
- Signal with finite energy is Energy signal

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$E_\infty = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

Typically, signal with finite energy should be non-zero over a compact interval

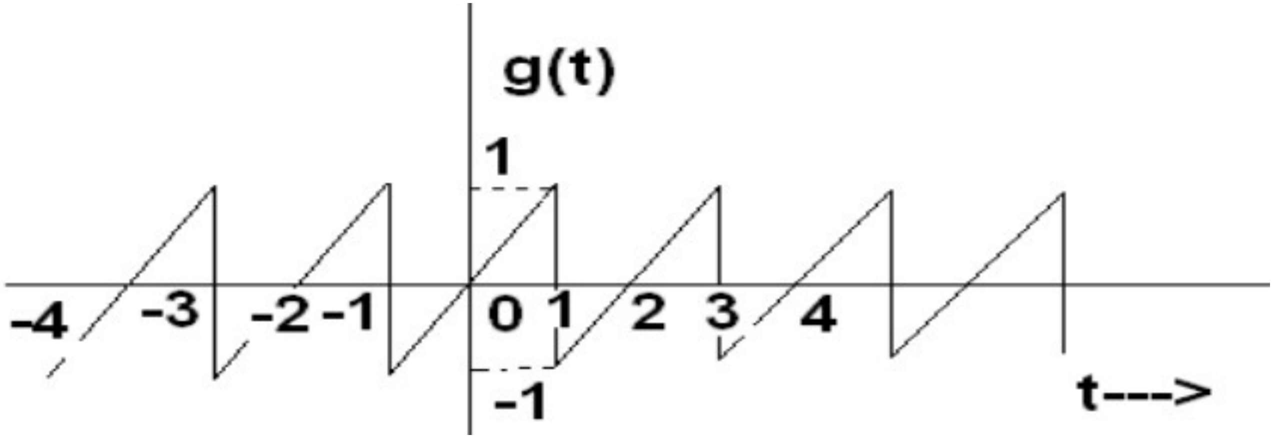




Power

- A signal with finite Power is called Power signal

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2\,\mathrm{d}t \qquad\qquad P_\infty = \lim_{N\to\infty} \frac{1}{2N+1}\sum_{n=-N}^{N} |x[n]|^2$$



Power-Energy signals

- A signal is called an **energy signal** if $E_\infty < \infty$
- A signal is called a **power signal** if $0 < P_\infty < \infty$

Power-Energy signals

- A signal can not be both an energy signal and a power signal

What is the power of an Energy signal ?

Sample problems

Find the energy of the following signal.

$$g(t) = \begin{cases} e^{t/2} & 0 \leq t \\ 0 & t < 0 \end{cases}$$

Find the power of the following signal.

$$g(t) = \begin{cases} t & \frac{-T}{2} \leq t < \frac{T}{2} \\ 0 & o.w. \end{cases}$$

What is the period of the following signals?

a. $\cos\left(\frac{\pi}{2}t\right)$

b. $\sin(t) + \cos\left(\frac{t}{2}\right)$

Determine values of P_∞ and E_∞

1. $x(t) = e^{j(2t+\pi/4)}$
2. $x(t) = \cos(t)$
3. $x[n] = e^{j(\frac{\pi n}{2}+\frac{\pi}{8})}$
4. $x[n] = \cos(\frac{\pi}{4}n)$

Problems

1) $x[n] = 1/n$, for $n \ddagger 0$; Else 0

Find Energy

$E = 2 \times \Sigma(1/n^2) = \pi/3$

Hence Energy signal

What do you observe in the signal ?

Problems

2) $x[n] = (-1)^n$

What is the Energy of this signal ?

Problems

2) $x[n] = (-1)^n$

What is the Energy of this signal ?

Infinity..why ??

What is the Power of this signal ?

P = 1 → This is a Power signal

$$P = \lim_{n \to \infty} \frac{1}{(2n+1)} * \sum_{m=-n}^{n} 1$$

Different versions of a signal

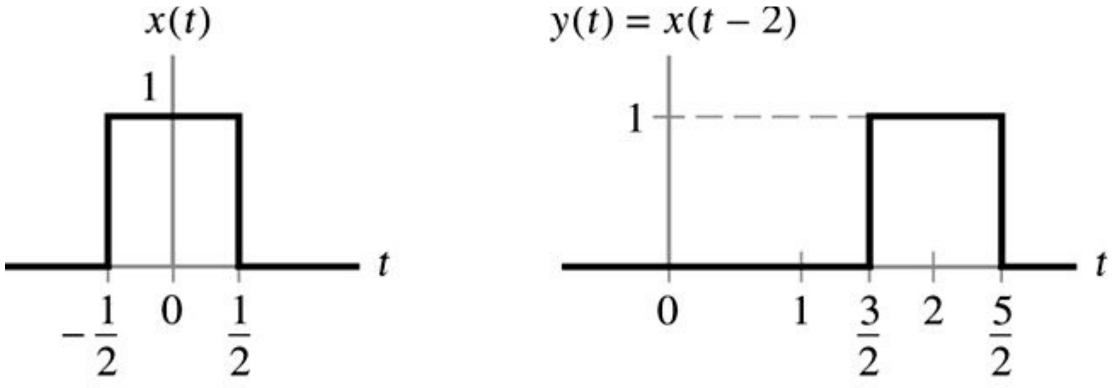
- What are possible Transformations of the Independent Variable ??

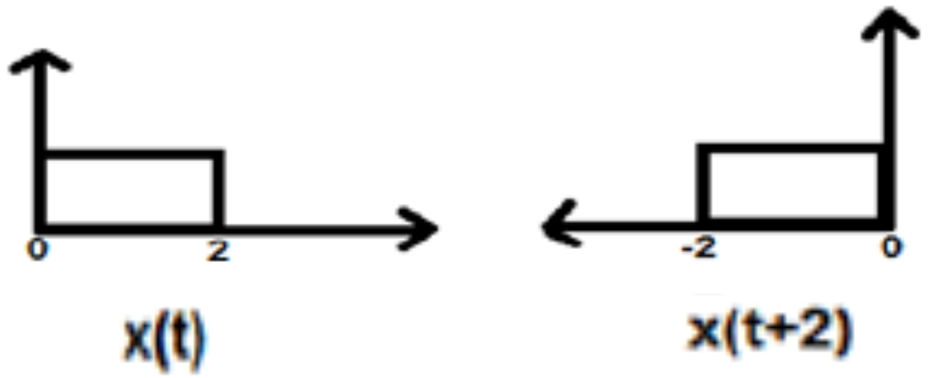
Transformations of the Independent Variable

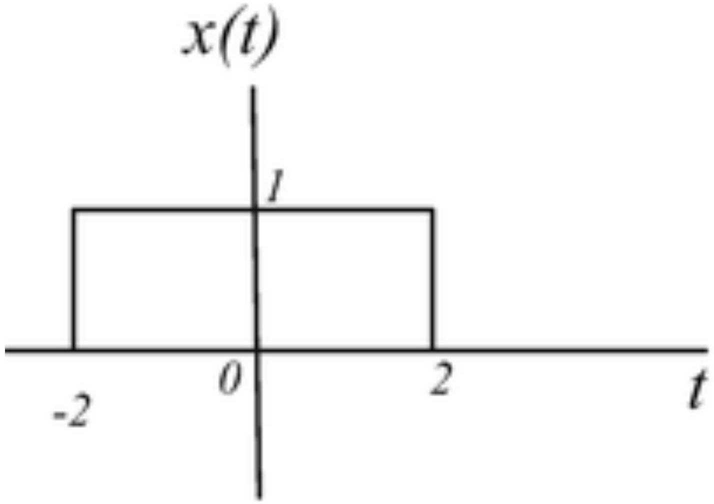
- Time shift

a) For Cts-time signal, x(t), time-shift makes it $x(t-t_0)$

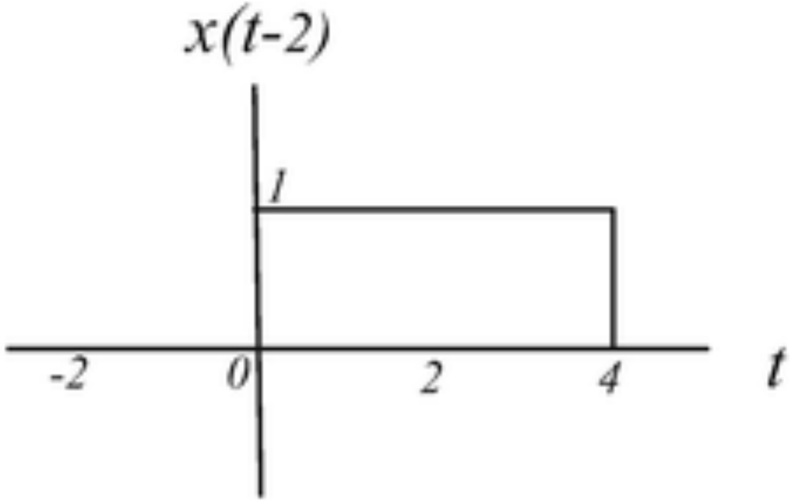
Time-shift preserves the shape of the signal



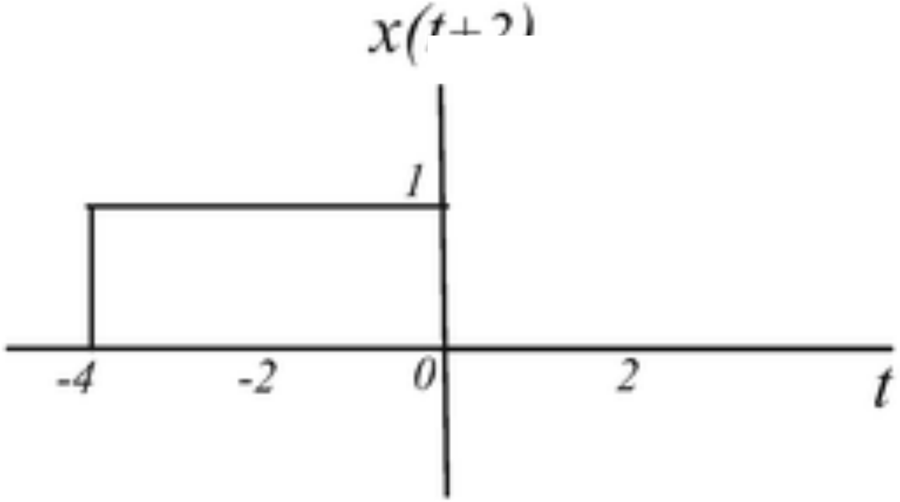

0
2
x(t)
-2
0
x(t+2)



(a)



(b)

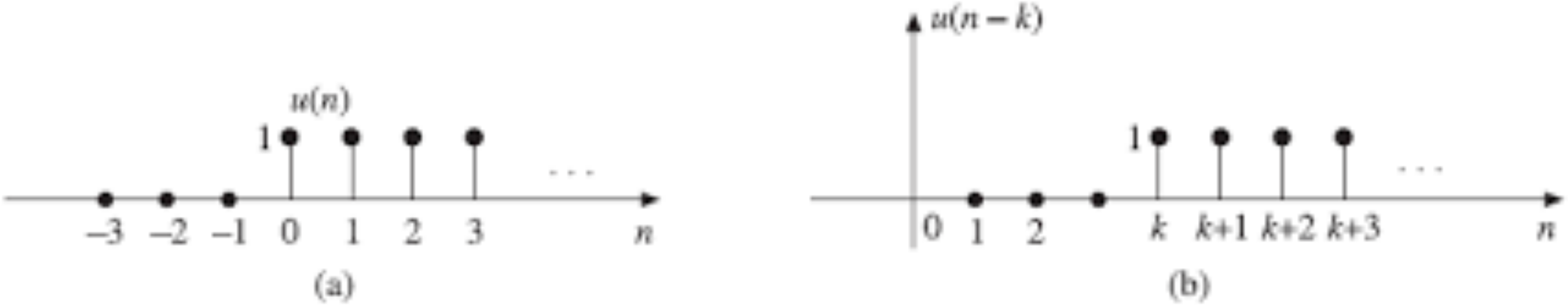


(c)

Time-shift for Discrete-time signals

For Discrete-time signal, x[n], time-shift makes it x[n-n_0]

Time-shift preserves the shape of the signal

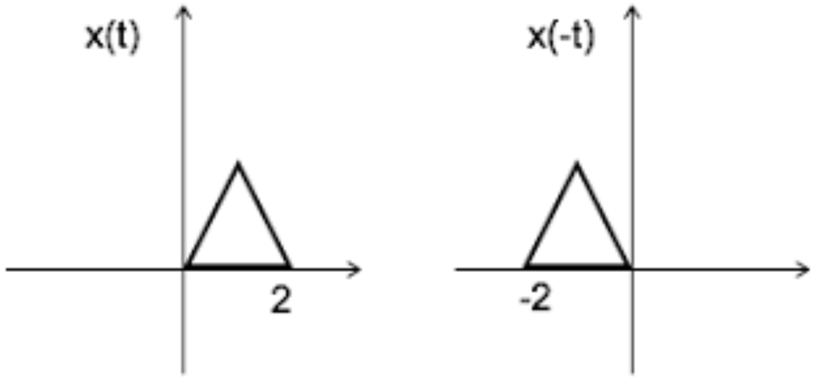


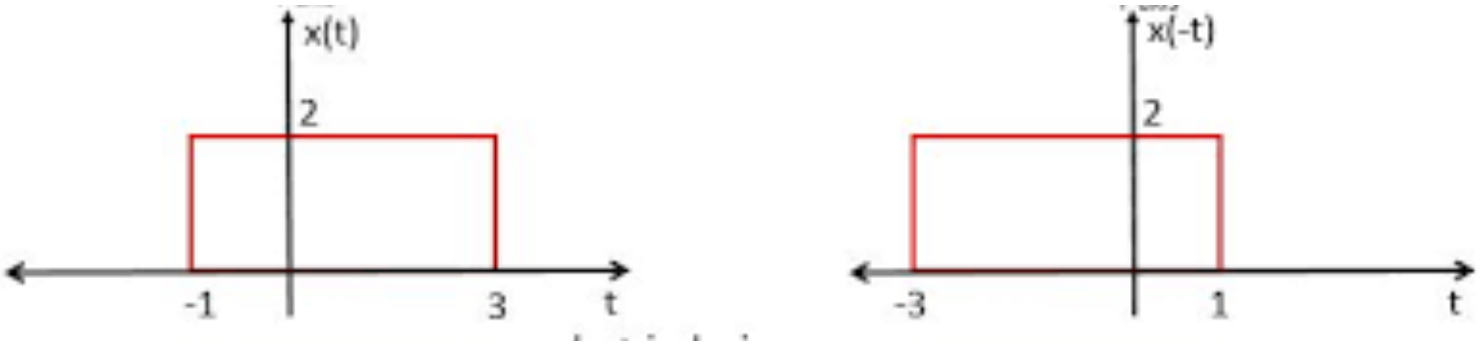
Reflection

- x(t) When time-reversed gives x(-t)
- X[n] when time-reversed gives x[-n]

- Real-life examples :
  - Mirror Reflection
  - Playing audio/video tape in reverse

Examples

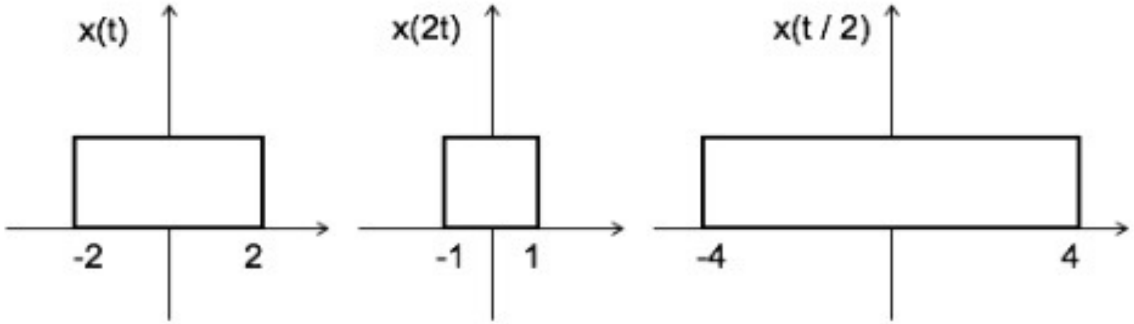




Scaling

- x(t) When time-axis is scaled by “A” gives x(At)

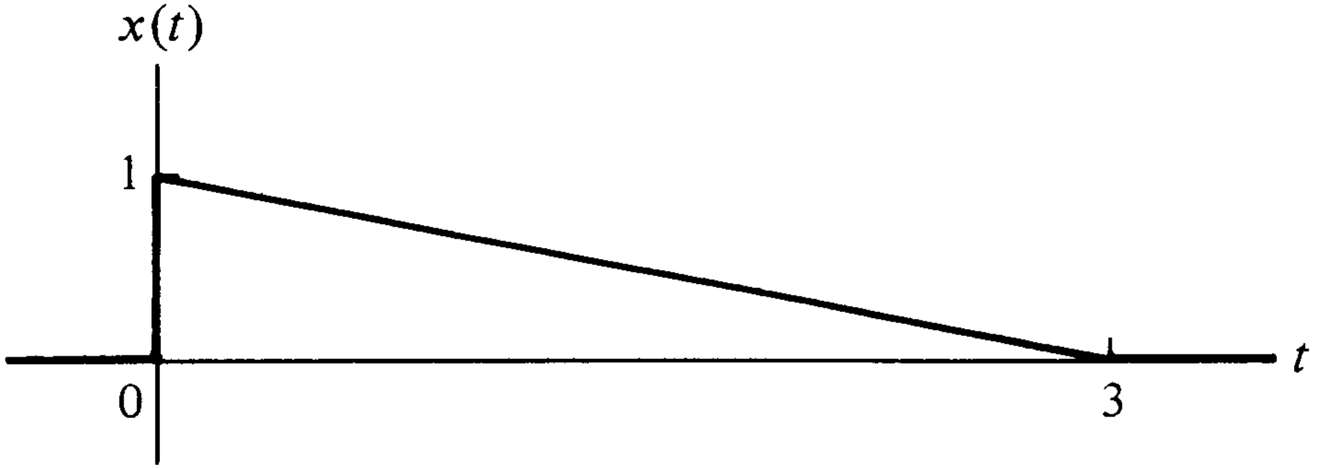
$|A| > 1 \rightarrow$ Compression of the signal
$|A| < 1 \rightarrow$ Expansion of the signal



Problem

For x(t) indicated in Figure, sketch the following:

1. x(-t)
2. x(t+2)
3. x(2t+2)
4. x(1-t)



Plot the following signals

1. Sketch the following signals:

a)

$$x(t) = \begin{cases} 0 & \text{if} \quad t < -4 \\ t+2 & \text{if} \quad -4 \le t < 3 \\ t-2 & \text{if} \quad 3 \le t \end{cases}$$

b) y(t) = x(t-1) where x(t) is defined in part a)

c)

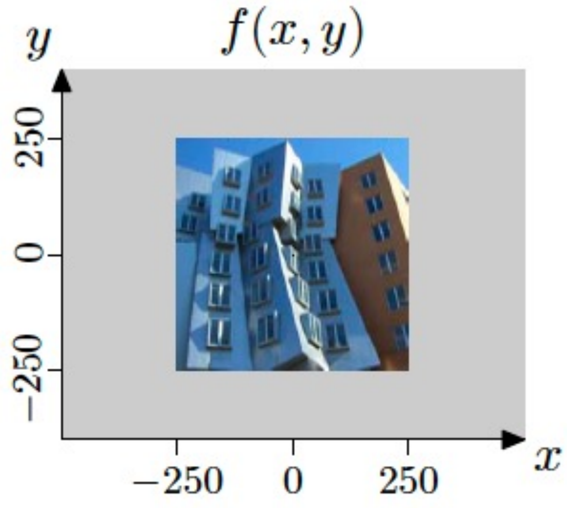
$$x[n] = \begin{cases} 0 & \text{if} \quad n < 2 \\ 2n-4 & \text{if} \quad 2 \le n < 4 \\ 4-n & \text{if} \quad 4 \le n \end{cases}$$

d) y[n] = x[n+1] where x[n] is defined in part c)

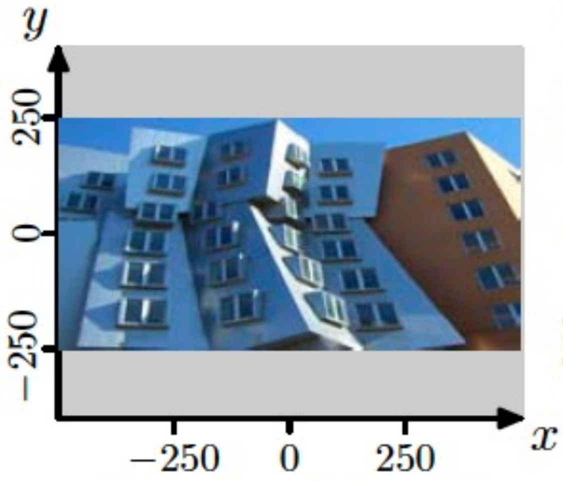
Assignment

- Download any small speech/music file. Make sure it is in .wav format and not longer than 5 seconds in duration. Let this signal be f(t).

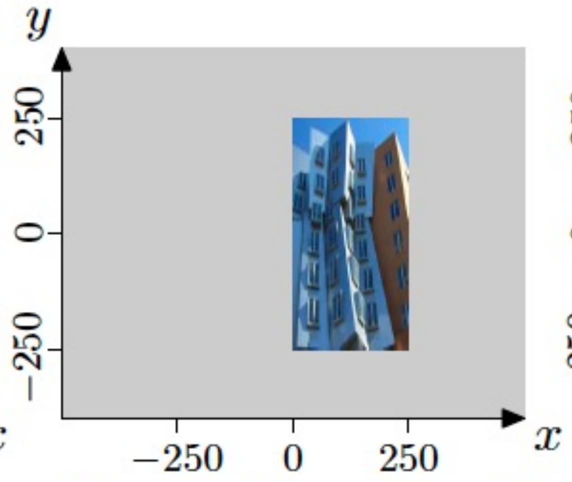
- Listen to
- f1(t) = 2*f(t)
- f2(t) = f(2t)
- f3(t) = f(t)/3
- f4(t) = -f(t)



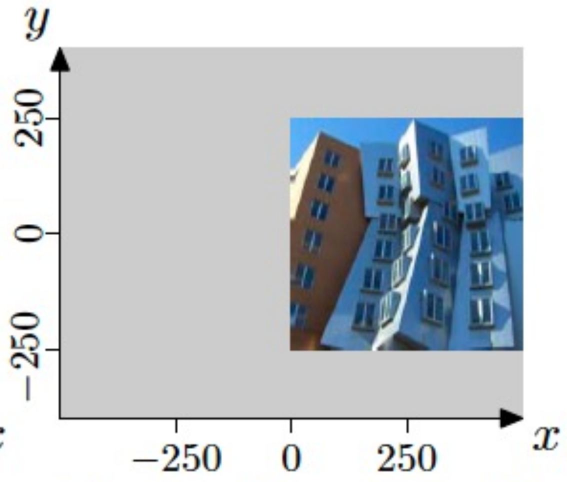
How many images match the expressions beneath them?



$f_1(x,y) = f(2x,y)$?



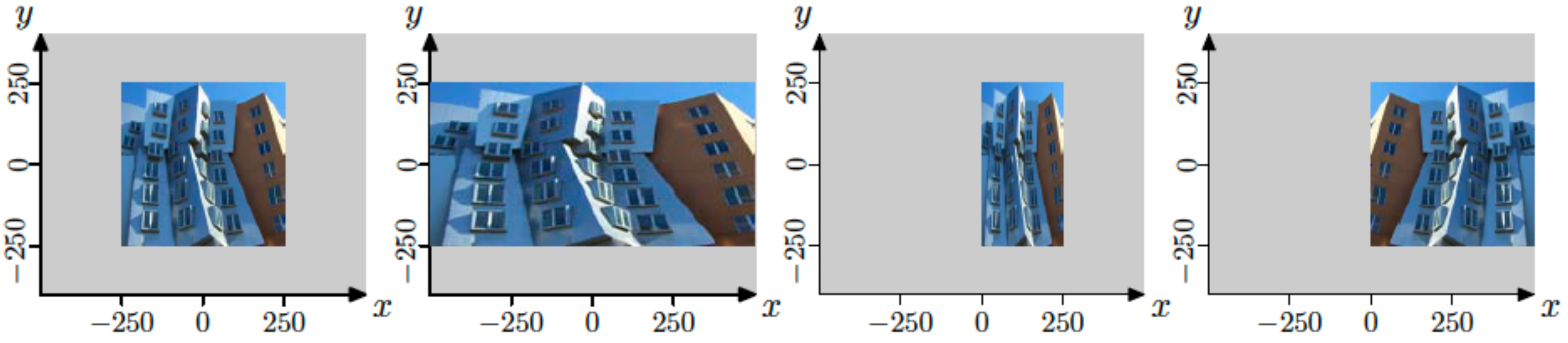
$f_2(x,y) = f(2x-250,y)$?



$f_3(x,y) = f(-x-250,y)$?

Source: MIT, OCW, Lecture notes by Dr. Freeman

Check Yourself



$f(x,y)$ $\quad$ $f_1(x,y) = f(2x,y)$? $\quad$ $f_2(x,y) = f(2x-250,y)$? $\quad$ $f_3(x,y) = f(-x-250,y)$?

$x = 0$	$\rightarrow f_1(0,y) = f(0,y)$	✓
$x = 250$	$\rightarrow f_1(250,y) = f(500,y)$	✗
$x = 0$	$\rightarrow f_2(0,y) = f(-250,y)$	✓
$x = 250$	$\rightarrow f_2(250,y) = f(250,y)$	✓
$x = 0$	$\rightarrow f_3(0,y) = f(-250,y)$	✗
$x = 250$	$\rightarrow f_3(250,y) = f(-500,y)$	✗