Electrostats

principle of superposition

• $Q(\vec{r})$

q_1 q_2

$q(\vec{r_i})$

$$\vec{r} = \vec{r} - \vec{r_i}$$

$$F_{Total} = \vec{F_1} + \vec{F_2} + \dots \vec{F_i} + \dots$$

coulomb's law

pc $\longrightarrow$ q $\qquad$ $\vec{r}$ $\qquad$ +c Q

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{qQ}{|r|^2} (\hat{r})$$

QF

$$\vec{F} = \vec{F_1} + \vec{F_2} + \dots$$

$$\frac{1}{4\pi\varepsilon_0} \left(\frac{q_1 Q}{r_1^2} + \frac{q_2 Q}{r_2^2} \dots \right)$$

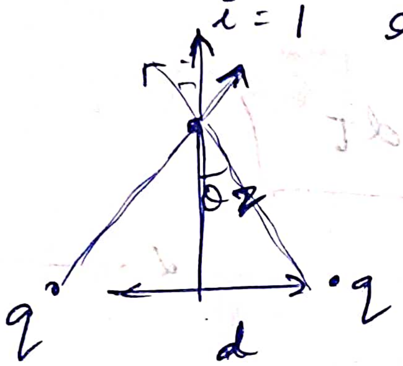
$$\vec{F} = \frac{Q}{4\pi\epsilon_0}\left(\frac{q_1}{r_1^2} + \frac{q_2}{r_2^2} - \right)$$

$\vec{F} = Q\vec{E}$ → electric field.

$$E(\vec{r}) = \sum_{i=1}^{M} \frac{kq_i}{r_i^2}\hat{r}_i \quad \vec{r_i} = \vec{r} - \vec{r_i}$$

$Q(\vec{r})$ $\quad$ $q_i(\vec{r_i})$

eg)



$$E_z = 2\left(\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\cos\theta\right)$$

$$\left(\sqrt{z^2 + d^2/4}\right)^2$$

$\cos\theta = \frac{z}{r}$

$$\frac{1}{4\pi\epsilon_0}\frac{2qz}{\left[z^2 + \left(\frac{d}{2}\right)^2\right]^{3/2}}\hat{z}$$

$z >> d$

$$\frac{1}{4\pi\epsilon_0}\left(\frac{2qz}{z^3}\right)\hat{z}$$

continuous charge distribution

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int\frac{1}{r^2}\hat{r}\,dq$$

line charge density $\rightarrow \lambda$

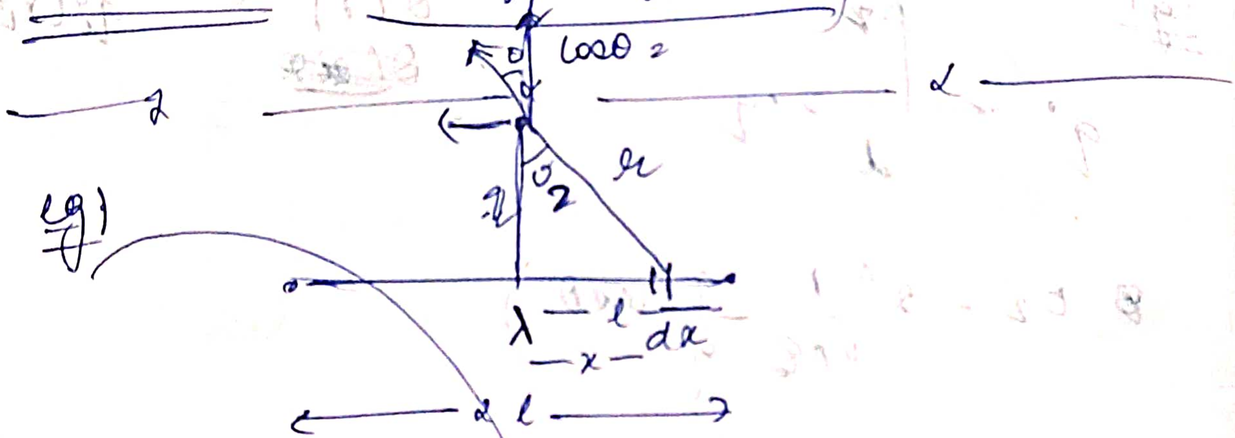
$$\boxed{dq = \lambda\, dl}$$

surface charge distr. $\rightarrow \sigma$

$$\boxed{dq = \sigma\, da}$$

vcd $\Rightarrow$ $\boxed{dq = \rho\, d\tau}$

eg)



$$r = (x^2 + z^2)^{1/2}$$

$$E = \frac{1}{4\pi\varepsilon_0}\int_{-L}^{L} \frac{z}{x^2+z^2}(\lambda\, dx)$$

$x^2 + z^2 = t$

$2x\, dx = dt$

$$\left(\frac{1}{2}\right)\frac{\lambda \hat{r}}{4\pi\varepsilon_0}\int_{-L}^{L} \frac{dt}{t}$$

$\ln(t)$

$$\frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{l^2+z^2}{z^2}\right)$$

$$\frac{2}{4\pi\varepsilon_0}\int_0^L \left(\frac{z}{\mathscr{r}}\right) \times \frac{\lambda}{\mathscr{r}^2}\, dx$$

$$\Rightarrow \frac{2z\lambda}{4\pi\varepsilon_0}\int_0^L \left[\frac{dx}{(x^2+z^2)^{3/2}}\right]$$

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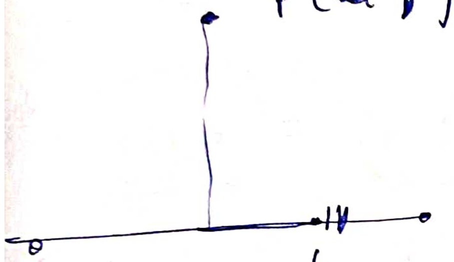
$P(\text{at } \vec{r})$ $\quad \vec{r} = z\hat{z}$



$L, \ \text{at } \vec{r}' = x\hat{x}$

$dl' = dx$

$$\vec{\mathscr{r}} = z\hat{z} - x\hat{x}$$

$$|\vec{\mathscr{r}}| = \sqrt{z^2 + x^2} \Rightarrow \hat{\mathscr{r}} = \frac{z\hat{z} - x\hat{x}}{\sqrt{z^2+x^2}}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_{-L}^{L} \frac{\lambda}{(z^2+x^2)}\, \frac{z\hat{z} - x\hat{x}}{\sqrt{x^2+z^2}}\, dx$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left[z\hat{z}\int_{-L}^{L} \frac{dx}{(x^2+z^2)^{3/2}} - \hat{x}\int_{-L}^{L} \frac{x\,dx}{(x^2+z^2)^{3/2}} \right]$$

$$\int_{-L}^{L} \frac{dx}{(x^2+z^2)^{3/2}}$$

$x^2+z^2=t^2$

$2x\,dx = 2t\,dt$

$\frac{1}{x^2+z^2}$ $\quad z^3 \quad \frac{dx}{\left(\frac{x^2}{z}+z^2\right)}$

$$\left.\frac{x}{z^2\sqrt{x^2+z^2}}\right]_{-L}^{L}$$

$$\frac{\lambda z}{4\pi\varepsilon_0}\left(\frac{2L}{z^2\sqrt{L^2+z^2}}\right)\hat{z} + \left(\left.\frac{1}{\sqrt{z^2+x^2}}\right]_{-L}^{L}\right)$$

$$\frac{1}{4\pi\varepsilon_0}\frac{2\lambda L}{z(L^2+z^2)^{1/2}}$$

— α — α — α —

Electric field lines

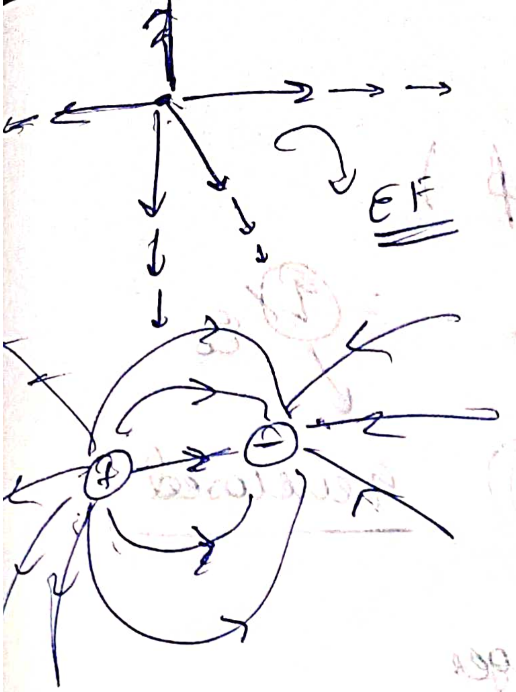
$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

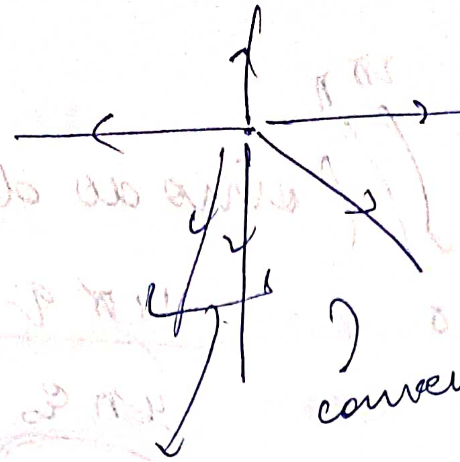
~~proton~~ origin

$\frac{1}{r^2}$

↳ inverse sq law

coulombs law





Flux = (Field) × (area)

Field: when constant.

area: through which field is going

$$\phi_E = \oint_s \vec{E} \cdot \vec{da}$$

$\propto$ charge enclosed in a volume

~~proportional to~~ measure of field lines passing through s

eg: spherical surface = $r^2 \sin\theta \, d\theta \, d\phi \, \hat{r}$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$$

centre of sphere: q

$$\oint_s \vec{E} \cdot \vec{da} = \oint \frac{q}{4\pi\varepsilon_0} \left(\frac{1}{\cancel{r^2}} \hat{r} \right)$$

$\cancel{r^2} \sin\theta \, d\theta \, d\phi$

$$\frac{q}{4\pi\epsilon_0} \int_0^{2\pi}\int_0^{\pi} (\sin\theta \, d\theta \, d\phi)$$

$$\frac{4\pi q}{4\pi \epsilon_0} = q/\epsilon_0$$

$r \neq 0$ q enclosed

Assembly of charges

n charges

$$\vec{E} = \sum_{i=1}^{n} \vec{E_i}$$

Flux of all charges

$$\oint \vec{E} \cdot \vec{da} = \sum_{i=1}^{n} \left(\oint \vec{E_i} \cdot \vec{da} \right) = \sum_{i=1}^{n} \frac{q_i}{\epsilon_0}.$$

Gauss law (integral form)

apply divergence theorem

$$\boxed{\int_V (\nabla \cdot \vec{E}) \cdot d\tau = q/\epsilon_0.}$$

$$Q_{enc} = \underbrace{\int \rho \, d\tau}_{\text{volume integral of volume charge density}}$$

$$Q_{enc} = \int_V \rho \, d\tau$$

$$\iiint_V (\vec{\nabla} \cdot \vec{E} \, d\tau) = \int_V \frac{\rho}{\varepsilon_0} d\tau$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$ KCP → Gauss law differential form

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left(\int \frac{\hat{r}}{r^2}\,\rho(\vec{r}')\,d\tau'\right)$$

all space — source charge ($\vec{r}'$)

$$\vec{\nabla}\cdot\vec{E} = \frac{1}{4\pi\epsilon_0}\int_{-\infty}^{\infty} \vec{\nabla}\cdot\frac{\hat{r}}{r^2}\,\rho(\vec{r})\,d\tau'$$

$$= \frac{1}{4\pi\epsilon_0}\int 4\pi\,\delta(\vec{r})\,\rho(\vec{r}')\,d\tau$$

$$\boxed{\vec{\nabla}\cdot\vec{E} = \frac{\rho(\vec{r})}{\epsilon_0}}$$

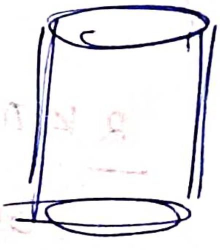
Application ~~to~~ of Gauss law

~~af~~ after applying on surface with symmetrical field.

1. Spherical sym

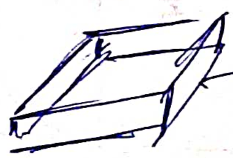
Gaussian surface

2. cylindrical sym

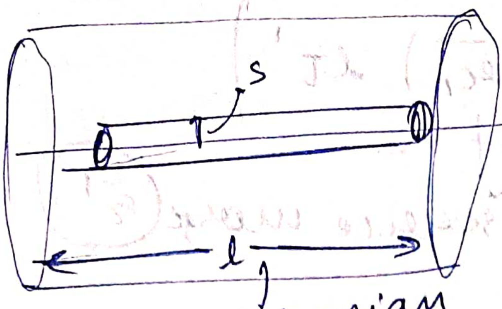


Gaussian surface

3. Plane symmetry



Gaussian



$\rho = ks$ → volume charge density
const

$\cancel{E}$ E F → cylindrically symm

$$\oint_s \vec{E} \cdot da = \frac{Q_{enc}}{\epsilon_0} \quad \int \rho \, d\tau$$

$$= \cancel{k s \, sd \, s \, ds \, d\phi}$$

$\cancel{Q_{en}}$

$k \quad k \, \cancel{2} \pi \frac{s^2}{\cancel{2}} \times l$

$(\pi s^2 l)$

$$\frac{k \pi s^2 l}{\epsilon_0} = \oint \vec{E} \cdot da$$

$$Q_{enc} = \int (ks')(s' \, ds \, d\phi \, dz)$$

$$= \frac{2 k \pi s^3 l}{3 \epsilon_0}$$

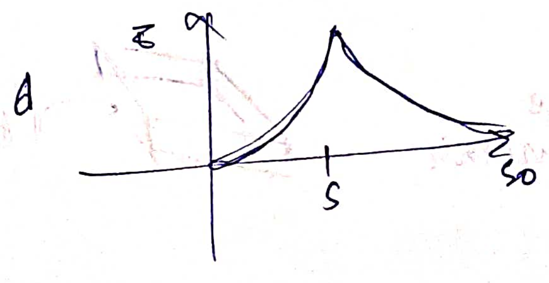
$$\frac{Q_{en}}{\epsilon_0} = \frac{2 k \pi s^3 l}{3 \epsilon_0}$$

out $|E| \, \cancel{2\pi} s_0 l = \cancel{2} k \pi / 3$

$\frac{k s^3}{3 \epsilon_0 s_0}$

$|E| \, \cancel{k \pi s l} = \frac{k l \pi \, s^2 \cancel{l}}{3 \epsilon_0}$

$|E| = \frac{k s_0^2}{3 \epsilon_0}$



$$\oint = \int \oint$$

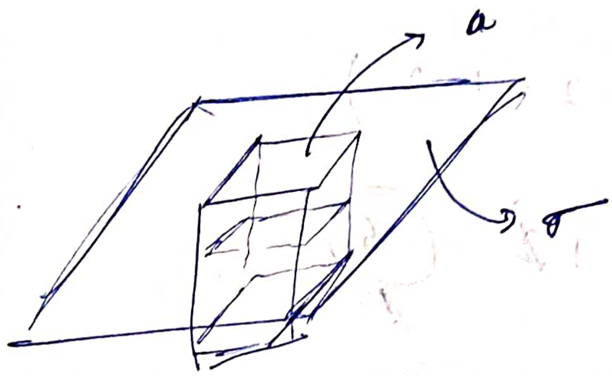
$$\oint \vec{E}\cdot\vec{da} = \underbrace{\int \vec{E}\cdot da}_{\text{curved}} + \underbrace{\cancel{\int \vec{E}\cdot da}}_{\text{flat}\ 0}$$

$$= \int \vec{E}\cdot(s\,dz\,d\phi\,\hat{s})$$

$s\,dz\,d\phi$ $\quad$ $\cancel{2\pi s l}$ $\quad \vec{E}\int da = 2\pi s l$

$$|\vec{E}|\,2\pi s l = \frac{2\pi k s^3 l}{3\epsilon_0}\hat{s}$$

$$\boxed{|\vec{E}| = \frac{ks^2}{3\epsilon_0}\hat{s}}$$



$(2a)$

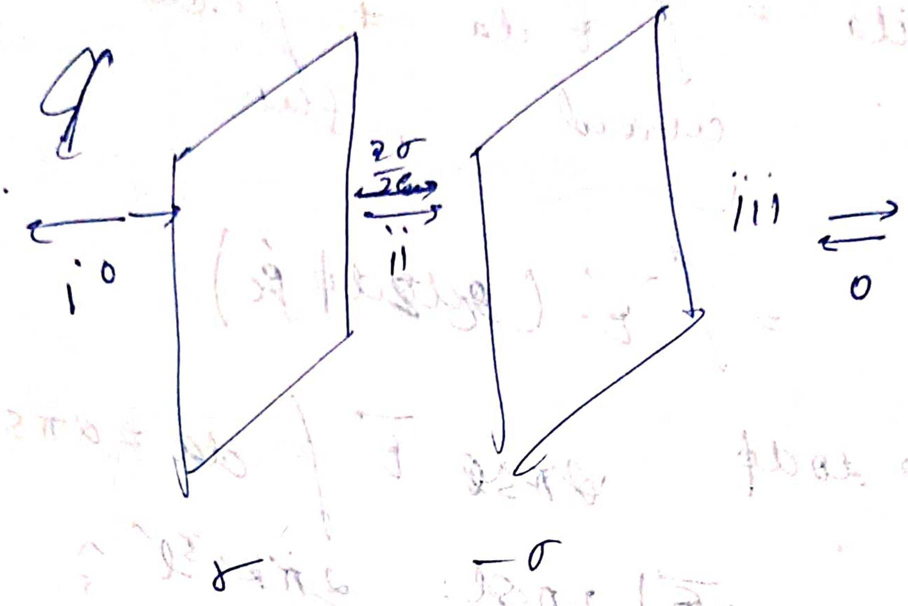
$$|\vec{E}|(2a) = \frac{\sigma a}{\epsilon_0}$$

$$\text{flux} = \frac{q_{enc}}{\epsilon_0} \qquad |\vec{E}| = \frac{\sigma}{2\epsilon_0}$$

$$\sigma = \frac{q}{a} \qquad \frac{q}{a} \qquad \frac{(\sigma a)}{\epsilon_0} = \oint \vec{E}\cdot da = 2aE$$

$$\boxed{\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{n}}$$

$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$



Q1) $I = \int_V \vec{r}\cdot(\vec{d}-\vec{r})\,\delta(\vec{r}-\vec{e})\,dr$

$= \vec{e}\cdot(\vec{d}-\vec{e})$

$= (3,2,1)(-2\ -1\ 2)$

$-6 \cancel{-2} + \cancel{2}$ -6

8

1.5 $\sqrt{9+4+1}$ 0

Q2, 3 −2

1 + $\cancel{12}$

$\sqrt{3}$ 1·

$$\oint \vec{E} \cdot \vec{da} = \frac{Q_{enc}}{\varepsilon_0}$$

$$|\vec{E}| \oint \vec{da} = \frac{\rho}{\varepsilon_0} \iiint (r^2 \sin\theta \, dr \, d\theta \, d\phi)$$

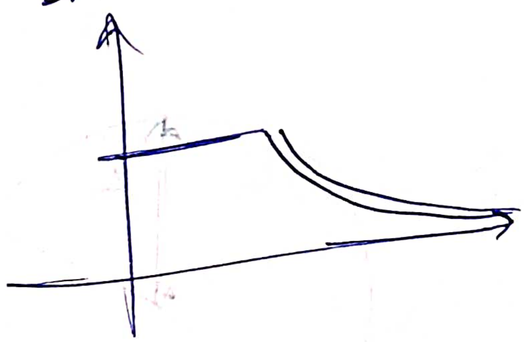
$\cancel{r^2 \sin\theta}$

$(r^2 \sin\theta \, d\theta \, d\phi)$

$$\frac{\left(\frac{4}{3}\pi r^3\right)\rho}{}$$

$$\frac{4\pi r^3 \rho}{3\varepsilon_0}$$

$$\frac{q}{\cancel{\ldots}R^3} \quad \frac{q}{\varepsilon_0}\left(\frac{r}{R}\right)^3$$



$$\frac{q}{\rho 3} = \rho \, 4\pi R$$

$|E| \cancel{\frac{4\pi}{3}}$

$$|\vec{E}| \, \cancel{\oint da} \, \frac{4}{3}\pi r^3 = \frac{\left(\frac{4}{3}\pi r^3\right)\rho}{\varepsilon_0}$$

$$|\vec{E}| = \frac{\rho}{\varepsilon_0}$$

inside

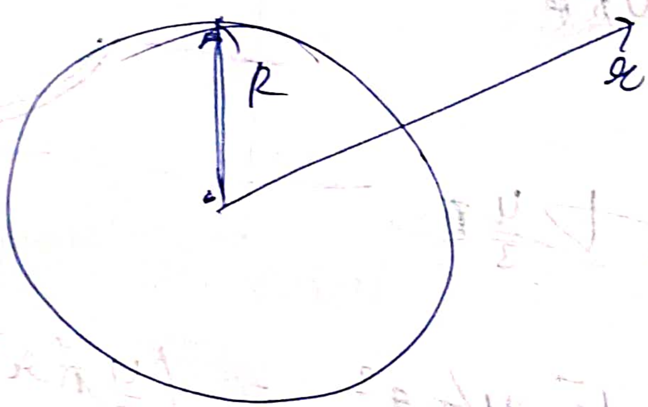
outside $|E| = \frac{\rho \, R^3}{\varepsilon_0 \, r^2}$

$$dq = \rho\, d\tau = \rho(4\pi r^2)\, dr$$

$$d\vec{E} = \frac{dq}{4\pi\varepsilon_0 r^2}\,\hat{r} = \frac{4\pi r^2 \rho\, dr}{4\pi\varepsilon_0 r^2}\,\hat{r}$$

$$\nabla\cdot E = \frac{\rho}{\varepsilon_0}$$

$$(\nabla\cdot E)\cdot d\vec{l} = dE$$

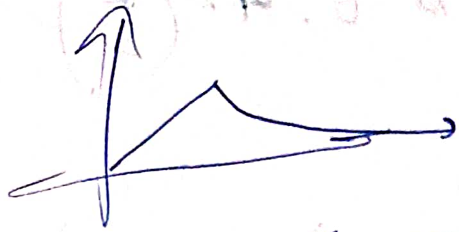


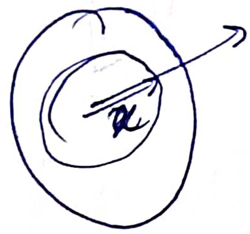
$$\oint_S \vec{E}\cdot d\vec{a} = \int_0^{2\pi}\int_0^{\pi} E_r\, \underbrace{r^2}_{\text{outside}} \sin\theta\, d\theta\, d\phi$$

$$\vec{E}\cdot d\vec{a} = E_r (r^2)\, 4\pi = \frac{Q_t}{\varepsilon_0}$$

$$E_r = \frac{Q_t}{4\pi\varepsilon_0 r^2}$$

$E_x = \dfrac{Q_i}{4\pi\varepsilon_0 x^2} \dfrac{x^3}{R^3} \qquad \dfrac{Q_x}{Q_i} = \dfrac{x^3}{R^3}$

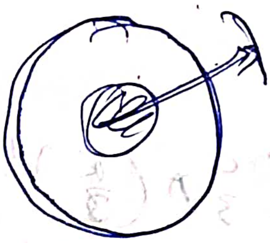




$E(2\pi x l) = \dfrac{\rho(\pi x^2)\, l}{\varepsilon_0}$ — (i)

$E(2\pi x l) = \dfrac{\rho(\pi a^2)\, l}{\varepsilon_0}$

$E(2\pi x l)$

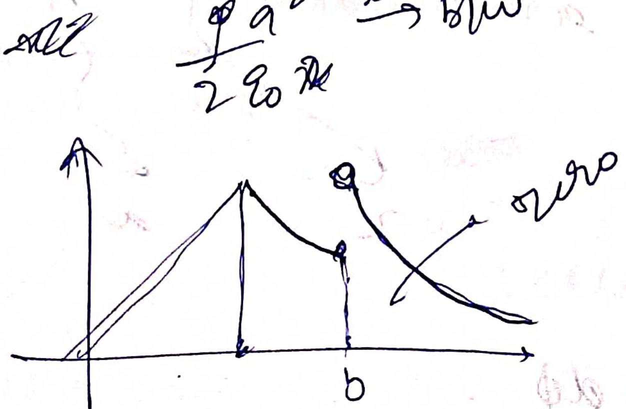


$E(2\pi x l) = \dfrac{\sigma(2\pi b l)}{\varepsilon_0}$

$\frac{1}{x}\left(\dfrac{\rho a^2}{2\varepsilon_0} + \dfrac{\sigma b}{\varepsilon_0}\right) \quad E_{out} = \dfrac{\sigma b}{x\varepsilon_0} + \dfrac{\rho a^2}{2x\varepsilon_0}$

$E_{in\,cy} = \dfrac{\rho x}{2\varepsilon_0}\hat{x}$

$\dfrac{\rho a^2}{2\varepsilon_0 x}\hat{x}$ b/w



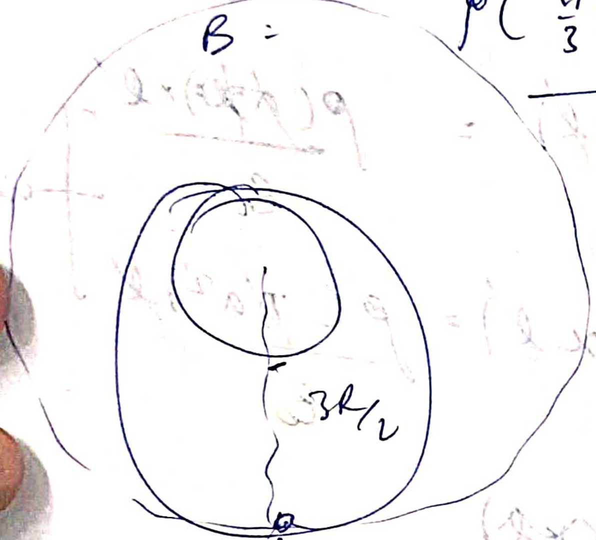
zero

$\dfrac{\rho a^2}{2\varepsilon_0 b} = \dfrac{\sigma}{\varepsilon_0}$

$\sigma = -\dfrac{\rho a^2}{2b}$

$$\frac{-\rho\left(\frac{4}{3}\pi\left(\frac{R}{2}\right)^3\right)}{\varepsilon_0} = E \cdot 4\pi\left(\frac{R}{2}\right)^2$$

B : $\cancel{\dfrac{\rho\left(\frac{4}{3}\pi R^3\right) - \rho\left(\frac{4}{3}\pi\left(\frac{R}{2}\right)^3\right)}{\pi\varepsilon_0} = E(4\pi}$

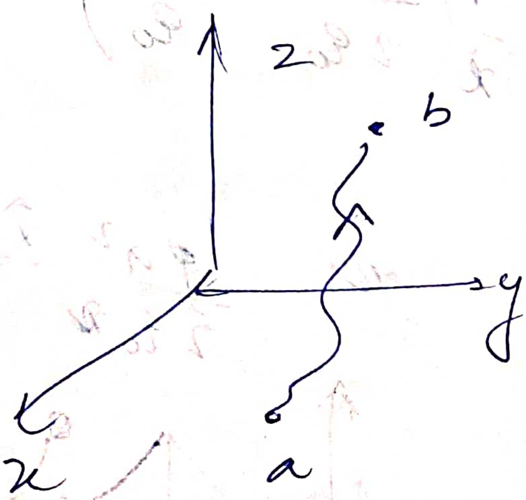


diff gaussian surface

$$\frac{\rho\left(\frac{4}{3}\pi R^3\right)}{4\pi r^2\varepsilon_0} - \frac{\rho\left(\frac{4}{3}\pi\left(\frac{R}{2}\right)^3\right)}{4\pi\left(\frac{3R}{2}\right)^2\varepsilon_0}$$

curl of $\vec{E}$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$



$$d\vec{l} = \hat{r}\,dr + \hat{\theta}\,r\,d\theta + \hat{\phi}\,r\sin\theta\,d\phi \, .$$

$\int \vec{E}\cdot d\vec{l}$ $\quad$ dr $\quad\quad$ $\int_a^b \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}(dr)$

$\frac{q}{4\pi\epsilon_0}\int_{r_a}^{r_b}\frac{q\,dr}{r^2}$

same → path independent

$$\oint \vec{E}\cdot d\vec{l} = 0 = \int_S (\nabla \times E)\cdot d\vec{a}$$

stokes law.

$\vec{\nabla}\times\vec{E} = 0 \Rightarrow$ curl of gradient goes to zero

$\vec{\nabla}\times\vec{E} = 0$

$\vec{\nabla}\times(-\vec{\nabla}V) = 0$

convention

irrotational field

$\vec{E} = -\vec{\nabla}V$ → ~~sco~~ gradient of a scalar field

↳ electric potential

$$(-\nabla \vec{V}) \cdot \vec{dl} = \vec{E} \cdot \vec{dl}$$

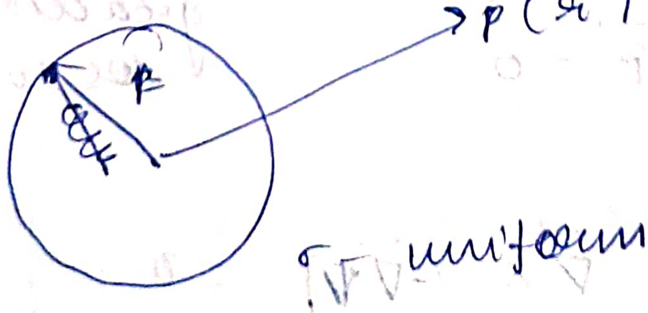
$$V(\vec{r}) = -\int_{ref}^{r} \vec{E} \cdot \vec{dl}$$

$$V(\vec{b}) - V(\vec{a}) = -\int_{ref}^{b} \vec{E} \cdot \vec{dl} + \int_{ref}^{a} \vec{E} \cdot \vec{dl}$$

$$\underbrace{V(\vec{b}) - V(\vec{a})} = -\int_{a}^{b} \vec{E} \cdot \vec{dl}$$

W.D taking q charge from point a to b

eg)



uniform

Field outside

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$$

q → total charge

$q = \sigma A$

$$\vec{E} = \frac{\sigma \times 4\cancel{\pi} R^2}{\cancel{4\pi} r^2 \varepsilon_0}\hat{r}$$

$$V(\vec{r}) = -\int_{\text{ref}}^{r} \vec{E}\cdot \vec{dl}$$

ref - ∞ $\quad V(\vec{r}) = -\int_{\infty}^{r} \frac{\sigma R^2}{\varepsilon_0 r^2} dr$

$r > R$ $\quad V(\vec{r}) = \frac{\sigma R^2}{\varepsilon_0}\left(\frac{1}{r}\right)$ outside

$r < R$ $\quad \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$ → const potential

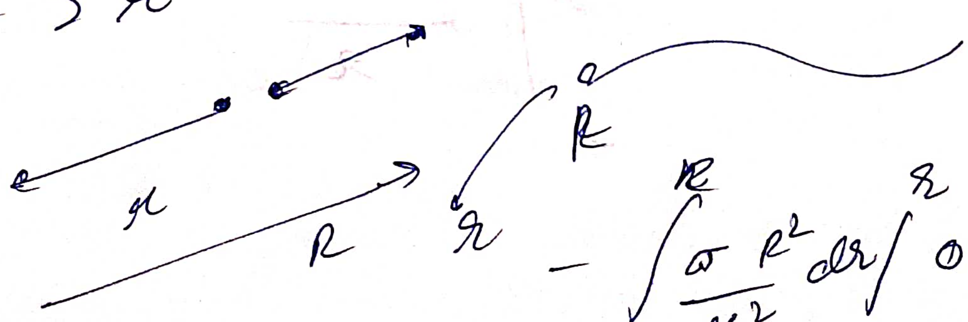
$$V(\vec{r}) = -\frac{1}{4\pi\varepsilon_0}\left(\int^{R} \frac{q}{r^2} dr'\right)$$

$$-\int_{\infty}^{R} \frac{\sigma R^2}{\varepsilon_0 r^2} dr \quad - \int_{\infty}^{r} = 0$$

∞

r

$R > r$



r $\quad R$ $\quad r$ $\quad R$

$$-\int_{\infty}^{R} \frac{\sigma R^2}{\varepsilon_0 r^2} dr \quad \int_{R}^{r} 0$$

POQ

$$\vec{\nabla}\cdot(-\vec{\nabla}V) = -\nabla^2 V = \rho/\varepsilon_0$$

$$\vec{\nabla}\cdot(\vec{E})$$

$$\boxed{\nabla^2 V = -\frac{\rho}{\varepsilon_0}}$$ → poisson eqn

$$V = -\int_{\infty}^{r} \frac{kq}{r^2}\,dr = \frac{kq}{r}$$

n assembly charge

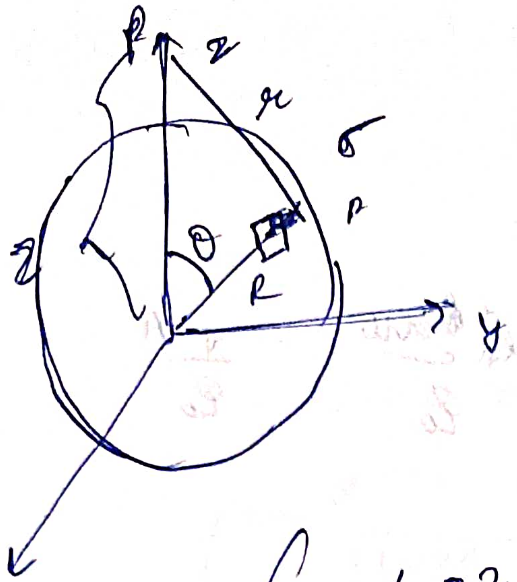
$$V = k\left(\sum_{i=1}^{n} \frac{q_i}{r_i}\right)$$

continuous

$$k\int_{\infty}^{r} \frac{\rho(r_i)\,d\tau}{r}$$

$$k\int_{\infty}^{r} \frac{\sigma(r_i)\,da}{r}$$

$$k\int_{\infty}^{r} \frac{\lambda(r_i)\,dl}{r}$$



$$\frac{\sigma(r^2 \sin\theta)\, d\theta\, d\phi\, \hat{r}}{r}$$

$$K\int \frac{\sigma(R^2 \sin\theta)\, d\theta\, d\phi\, \hat{r}}{R^2+z^2-2Rz\cos\theta}$$

$$r^2 = R^2 + z^2$$

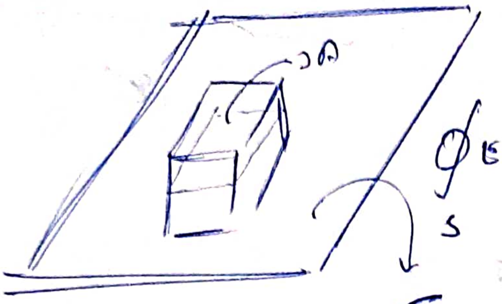
$$V(z) = 2\pi K\sigma R^2 \int \frac{\sin\theta\, d\theta}{\sqrt{R^2+z^2-2Rz\cos\theta}}$$

$$V(z) = \frac{R^2\sigma}{\varepsilon_0 z}$$

$$\frac{R^2\sigma}{\varepsilon_0}$$

$$\frac{R\sigma}{\varepsilon_0} \qquad \frac{q}{4\pi R\varepsilon_0} \qquad \frac{Kq}{R}$$

Boundary conditions

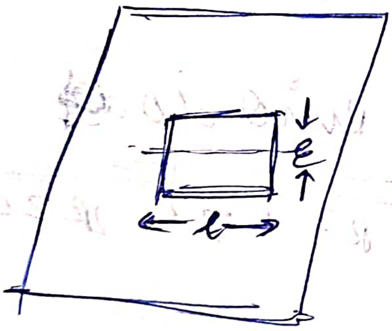


$$\oint_S \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

σ

$$\boxed{E^{\perp}_{above} - E^{\perp}_{below} = \frac{\sigma}{\epsilon_0}}$$

E not continuous



σ

$$\oint \vec{E}\cdot d\vec{l} = 0$$

curl zero

$$\boxed{E^{\parallel}_{above} = E^{\parallel}_{below}}$$

$E^{\parallel}_{above}$

$$\vec{E}_{\perp,a} - \vec{E}_{\perp,b} = \frac{\sigma}{\epsilon_0}\hat{n}$$

$$V_{above} - V_{below} = -\int_a^b \vec{E}\cdot d\vec{l}$$

$l \to 0$. $\lim_{(a \to b)\to 0} \Big] = 0$

$$\boxed{V_{above} = V_{below}}$$

continuous

first order diff of potential gives EF

if V is discont EF wouldn't exist

$$\vec{\nabla} V_{above} - \vec{\nabla} V_{below} = -\frac{\sigma}{\epsilon_0}\hat{n}$$

$$\frac{\partial}{\partial n} V_{above} - \frac{\partial}{\partial n} V_{below} = -\frac{\sigma}{\epsilon_0}$$

$$\boxed{\frac{\partial V}{\partial n} = (\vec{\nabla} V)\cdot\hat{n}}$$

work and energy

$$W = \int_{\vec{a}}^{\vec{b}} \vec{F}\cdot\vec{dl} = -Q\int_{\vec{a}}^{\vec{b}} \vec{E}\cdot\vec{dl}$$

$$= Q\left[V(\vec{b}) - V(\vec{a})\right]$$

$$V(\vec{b}) - V(\vec{a}) = \frac{W}{Q}$$

$$W = Q\left[V(\vec{r}) - V(\infty)\right]$$

Energy of a p c distribution

q_1 ∞ —— $\vec{r_1}$ no work

q_2 —— $\vec{r_2}$ some work

$$W_2 = q_2 V_1(\vec{r_2})$$

$$= q_2 \left(\frac{k q_1}{r_{12}} \right)$$

potential at point r_i due to other charges.

$\frac{1}{2} \sum q_i V(r_i)$

$$W_3 = k q_3 \left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}} \right)$$

total WD =

$$k \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right) = \underline{\underline{WD}}$$

for n point charges

$$TWD = \frac{k}{2} \left(\sum_{j} \sum_{\substack{i \\ i \neq j}} \frac{q_i q_j}{r_{ij}} \right)$$

$$\frac{1}{2} \sum_{i=1}^{n} q_i \left(\sum_{j=1}^{n} \frac{k q_j}{r_{ij}} \right)$$

Energy for a continuous charge distribution

$$W = \frac{1}{2}\int \rho V d\tau$$

$$\frac{1}{2}\int \lambda V dl$$ → diff forms.

$$\frac{1}{2}\oint \sigma V da$$

__volume__

$$\rho = \epsilon_0 \vec{\nabla}\cdot\vec{E}$$ → diff Gauss Law

$$\rho = \frac{1}{2}\int \epsilon_0 (\vec{\nabla}\cdot\vec{E}) V d\tau$$

$$\boxed{W = \frac{\epsilon_0}{2}\int (\vec{\nabla}\cdot\vec{E}) V d\tau}$$

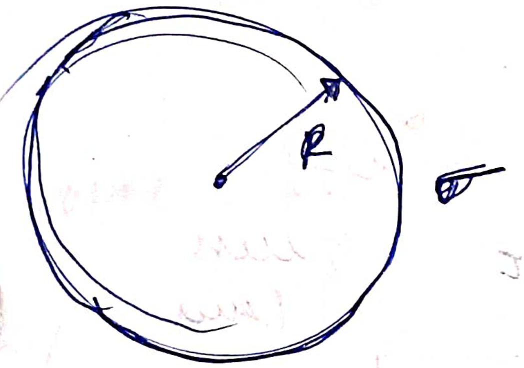
$$W = \frac{\epsilon_0}{2}\left[\underbrace{-\int \vec{E}\cdot(\vec{\nabla}V) d\tau + \oint_S V \vec{E}\cdot da}_{\text{by parts}}\right]$$

$$\nabla \vec{V} = -\vec{E}$$

$$W_D = \frac{\epsilon_0}{2}\left[\int E^2 d\tau + \underbrace{\oint_S V\vec{E}\cdot d\vec{a}}_{\text{zero for } \infty}\right]$$

$$\boxed{W = \frac{\epsilon_0}{2}\int_{\text{all space}} E^2 \, d\tau}$$

eg) uniform charged spherical shell



$W = \frac{1}{2}\int V\sigma \, da$

Total charge q

assemble

potential at surface

$$V(R) = \frac{1}{4\pi\epsilon_0}\frac{q}{R}$$

$$W = \frac{1}{8\pi\epsilon_0}\frac{q}{R}\int \sigma \, da = \frac{q^2}{8\pi\epsilon_0 R}$$

solution 2

$$W = \frac{\epsilon_0}{2} \int_{\text{all space}} E^2 \, d\tau$$

$\vec{E}$ inside $= 0$

$\vec{E}$ outside $= \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$

$$E^2_{out} = \frac{q^2}{16\pi^2 r^4 \epsilon_0^2}$$

$$\frac{\epsilon_0}{2(4\pi\epsilon_0)^2} \int_R^\infty \frac{q^2}{r^{\cancel{4}}} \cancel{r^2} \sin\theta \, dr \, d\theta \, d\phi$$

$$\frac{q^2 \cancel{\epsilon_0}}{2(4\pi\epsilon_0)^{\cancel{2}}} \left[\frac{1}{R} \times 4\pi\right]$$

$$\frac{q^2}{8\pi\epsilon_0 R}$$

$W \rightarrow \frac{\epsilon_0}{2} \int E^2 d\tau \rightarrow$ always +ve

$W \rightarrow \frac{1}{2} \int V dq$ — can be +ve, can be -ve

which is correct ?

Energy of a point charge

$$W = \frac{\epsilon_0 \times 4\pi}{2(4\pi\epsilon_0)^2} \int_0^\infty \frac{q^2}{r^4} r^2 \, dr$$

$$W = \frac{q^2}{8\pi\epsilon_0} \int_0^\infty \frac{1}{r^2} = \infty$$

one expression says ∞

other says 0

depends on the process of bringing charges

"content"

"where is the energy stored"

it is stored in electric field

$\frac{\epsilon_0}{2} E^2 \rightarrow$ energy per unit volume

$\vec{E}_1$ & $\vec{E}_2$

$$w_{total} = \frac{\epsilon_0}{2} \int (\vec{E}_1 + \vec{E}_2)^2 d\tau$$

$$= w_1 + w_2 + \epsilon_0 \int (\vec{E}_1 \cdot \vec{E}_2)\, d\tau$$

more energy than sum of individual.

bunging

point charge in some EF that's why there will be some w.d

In energy superposition principle is not valid.

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conductors

(i) $\vec{E} = 0$ inside ?

free charges move such that $\vec{E}$ inside become zero

(ii) $\rho = 0$ inside a conductor

$$\vec{\nabla} \cdot \vec{E} = 0 = \frac{\rho}{\epsilon_0} \text{ only way is } \rho = 0$$

$E = 0$

(iii) Any charge will reside on the surface of a conductor

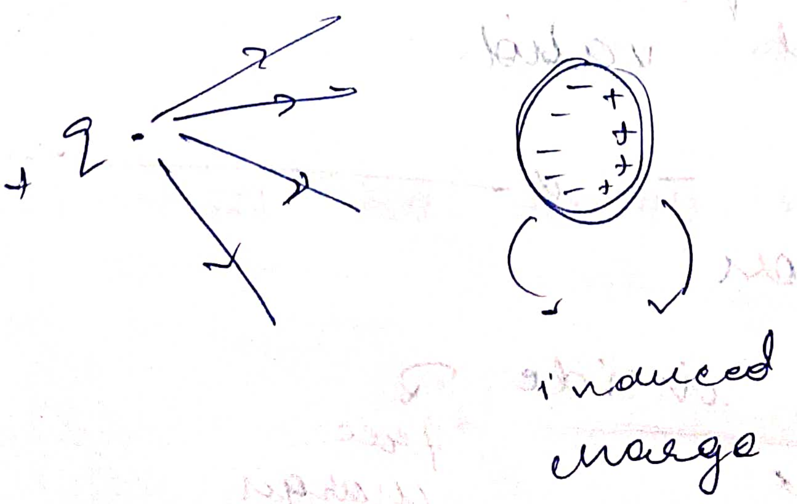
$\vec{E} = 0$

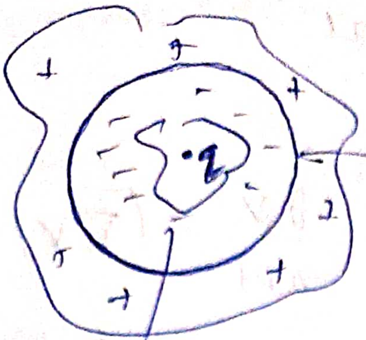
(iv) Equipotential

$$V(\vec{b}) - V(\vec{a}) = -\int_{\vec{a}}^{\vec{b}} \vec{E} \cdot d\vec{l} = 0$$

$$V(\vec{a}) = V(\vec{b})$$

(v) $\vec{E}$ outside the surface is perpendicular to conductor





Gaussian surface

cavity

$$\oint \vec{E}\cdot d\vec{a} = 0$$

$$\frac{Q_{en}}{\epsilon_0} = 0$$

$$\frac{+q + q_{in}}{\epsilon_0} = 0$$

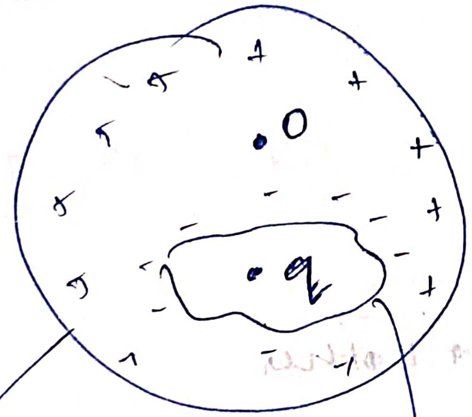
$q_{in} = -q$

↓

inner surface.

q_{in}, outer $= +q$

eg)



cavity

$\vec{E}_{outside} = ?$

$$\vec{E} = \frac{kq}{r^2}\hat{r}$$

Surface charge & force on a conductor

E inside $= 0$

$$\frac{\sigma A}{\epsilon_0} = \oint \vec{E}\cdot d\vec{a}$$

E immediate outside

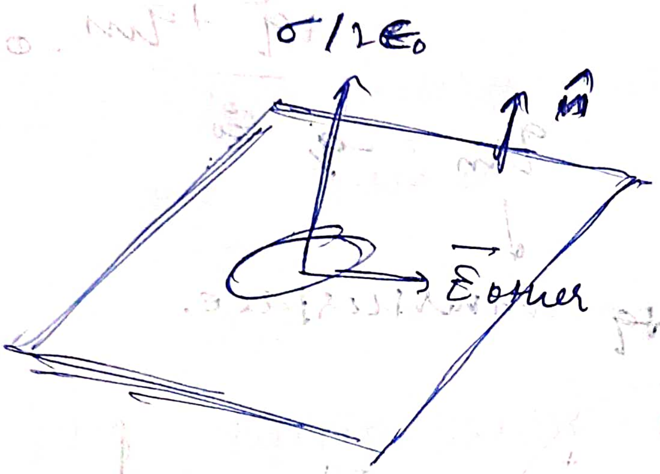
||

$$\frac{\sigma}{\epsilon_0}\hat{n}$$

In terms of potential

$$\sigma = -\epsilon_0 \frac{\partial V}{\partial n} \qquad \frac{\partial V}{\partial n} = (\vec{\nabla} V)\cdot\hat{n}$$

$E_{\perp}$



$$\vec{f} = \sigma \vec{E}_{avg}$$

$$= \frac{1}{2}\sigma(\vec{E}_{above} + \vec{E}_{below})$$

$\cancel{\vec{f} = \sigma \vec{E}_{a}}$

$$\vec{E} = \vec{E}_{patch} + \vec{E}_{other}$$

it will experience EF acting on it parallely if $E_{patch} = 0$

$$\vec{E}_{above} = \vec{E}_{other} + \frac{\sigma}{2\epsilon_0}\hat{n}$$

$$\vec{E}_{below} = \vec{E}_{other} - \frac{\sigma}{2\epsilon_0}\hat{n}$$

$$f = \frac{\sigma}{2\epsilon_0}\hat{n}$$

$$E_{other} = \frac{1}{2}(\vec{E}_a + \vec{E}_b)$$

$$= \vec{E}_{avg}$$

Electrostatic pressure

$$P = \frac{\epsilon_0}{2} E^2$$

capacitor

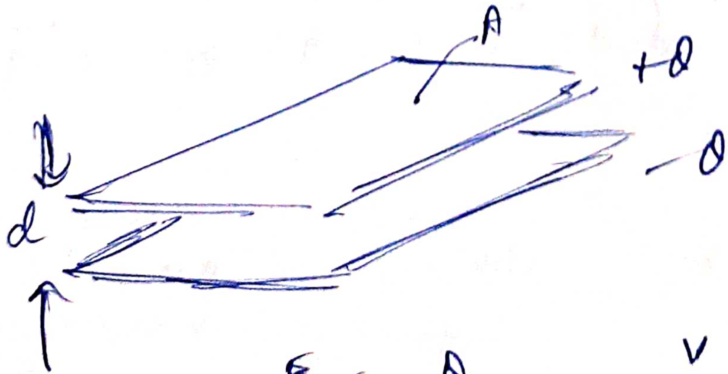
+Q on one conductor

-Q on other conductor

$$V = V_+ - V_- = -\int^{+} \vec{E} \cdot d\vec{l}$$

$$C = \frac{Q}{V} \qquad V = CQ$$

capacitance — Farad

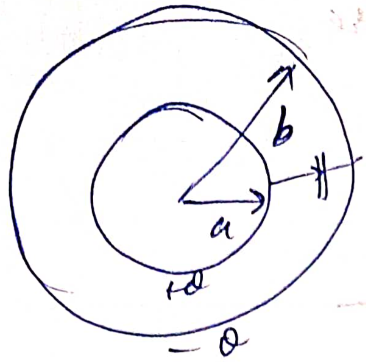


Find capacitance

$$E = \frac{Q}{A\epsilon_0} \qquad V = \frac{Q}{A\epsilon_0} d$$

$$\boxed{\text{Capacitance} \Rightarrow C = \frac{Q}{V} = \frac{A\epsilon_0}{d}}$$

eg)



$b > r$

$$E\left(4\pi r^2\right) = \frac{Q}{\epsilon_0}$$

$$E = \int_a^b \frac{kQ}{r^2} \cdot dr$$

$$V = kQ\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$\frac{kQ(b-a)}{ab(4\pi\epsilon_0)} = V$$

$$C = \frac{4\pi\epsilon_0 ab}{b-a}$$

$a \longrightarrow \infty$