Problem Set 2

Solutions

September 14, 2021

Contents

Question 5

A class has only 3 students A, B, C who attend the class independently. The probabilities of their attendance in any day being $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$ respectively. Find the probability that the total number of attendances in 2 consecutive days is exactly 3.

Solution 3.5

Let E_1, E_2, E_3, denote the events defined as :

- E_1 : Two trials of observing if A attends the class.
- E_2 : Two trials of observing if B attends the class.
- E_3 : Two trials of observing if C attends the class.

These can be considered as bernoulli (or binomial trials) because only two outcomes are possible, either the student attends the class, which we can deem as a "success", or not, which can be deemed as a "failure".

Solution 3.5

The probability of success of events, E_1, E_2, E_3 are $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$ respectively.
As E_1, E_2, E_3 are independent events associated with our experiment,
$P(E_1 \cdot E_2 \cdot E_3) = P(E_1) \cdot P(E_2) \cdot P(E_3)$

Solution 3.5

$$P_{required} = \left(\frac{1}{2}\right)^2 \binom{2}{1}\frac{2}{3}\cdot\frac{1}{3}\cdot\left(\frac{1}{4}\right)^2 \cdot + \left(\frac{1}{2}\right)^2\cdot\left(\frac{1}{3}\right)^2\cdot\binom{2}{1}\frac{3}{4}\cdot\frac{1}{4} + \binom{2}{1}\frac{1}{2}\cdot\frac{1}{2}\cdot\left(\frac{2}{3}\right)^2\cdot\left(\frac{1}{4}\right)^2$$
$$+ \left(\frac{1}{2}\right)^2\cdot\left(\frac{2}{3}\right)^2\binom{2}{1}\frac{3}{4}\cdot\frac{1}{4} + \binom{2}{1}\frac{1}{2}\cdot\frac{1}{2}\cdot\left(\frac{1}{3}\right)^2\cdot\left(\frac{3}{4}\right)^2 + \left(\frac{1}{2}\right)^2\cdot\binom{2}{1}\frac{2}{3}\cdot\frac{1}{3}\cdot\left(\frac{3}{4}\right)^2$$
$$+ \binom{2}{1}\frac{1}{2}\cdot\frac{1}{2}\cdot\binom{2}{1}\frac{2}{3}\cdot\frac{1}{3}\cdot\binom{2}{1}\frac{3}{4}\cdot\frac{1}{4}$$
$$= \frac{1}{4}$$

Question 6

If a die is thrown n times, find the probability that (i) the greatest and (ii) the least number obtained will have a given number r.

Solution 3.6.1

Let X_r be the event defined as: Obtaining number less than or equal to r in all n throws of a die.
As the throw of a die is independent, and probability of success can be given as $\frac{r}{6}$, therefore $P(X_r) = \left(\frac{r}{6}\right)^n$.
Now, $P_{required} = P(X_r - X_{r-1}) = \left(\frac{r}{6}\right)^n - \left(\frac{r-1}{6}\right)^n$

Solution 3.6.2

Let X_r be the event defined as: Obtaining number greater than or equal to r in all n throws of a die.
As the throw of a die is independent, and probability of success can be given as $\frac{7-r}{6}$, therefore $P(X_r) = \left(\frac{7-r}{6}\right)^n$.
Now, $P_{required} = P(X_r - X_{r-1}) = \left(\frac{7-r}{6}\right)^n - \left(\frac{7-r-1}{6}\right)^n$
(We'll be replacing r with $r+1$ in the second term, i.e, X_{r+1}, thus making the term $P(X_r - X_{r+1})$. X_{r-1} is written only for symmetry purposes.)