① $y(t) = t^2 x(t-1)$

Linearity :

$x_1(t) \quad y_1(t) = t^2 x_1(t-1)$

$y_2(t) = t^2 x_2(t-1)$

$y(t) = y_1(t) + y_2(t) = t^2(x_1(t-1) + x_2(t-1)) \rightarrow ①$

$x_1(t) + x_2(t) \longrightarrow y(t) = t^2(x_1(t-1) + x_2(t-1)) \longrightarrow ②$

} Superposition ✓

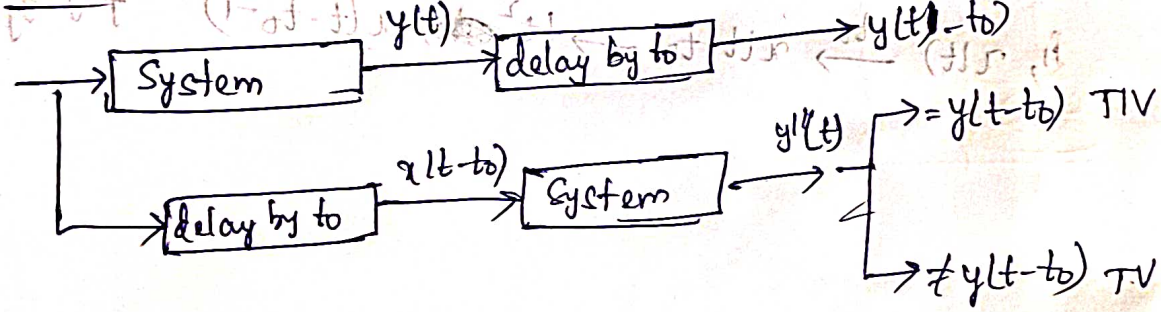
$y(t) \longleftarrow x(t)$

$ay(t) \longleftarrow ax(t)$

$a(x(t)) \rightarrow y'(t) = at^2 x(t-1) = ay(t)$ ✓

∴ System is linear

Time Invariant :



i) $y(t) \xrightarrow{to} y(t-to) \longrightarrow (t-to)^2 x(t-to-1)$

ii) $x(t) \xrightarrow{to} x(t-to) \xrightarrow{Sys} t^2 x(t-to-1)$

} ≠ TV system

② $y[n] = x[n+1] - x[n-1]$

$y_1[n] = x_1[n+1] - x_1[n-1]$

$y_2[n] = x_2[n+1] - x_2[n-1]$

$y_1[n]+y_2[n]$

$y[n] = (x_1[n+1]+x_2[n+1]) - (x_1[n-1]+x_2[n-1])$ → (1)

$x_1[n]+x_2[n] \longrightarrow y[n] = (x_1[n+1]+x_2[n+1]) - (x_1[n-1]+x_2[n-1])$ → (2)

① = ② Superposition ✓

Scaling: $ax(n) \longrightarrow a(x[n+1]-x[n-1]) \longrightarrow ay[n]$ ✓ scaling

∴ Linear ✓

Time Invariant.

~~i) $y[n] \xrightarrow{t_0} y(t-t_0)$~~

i) $y[n] \xrightarrow{n_0} y[n-n_0] \longrightarrow x[n-n_0+1] - x[n-n_0-1]$ → ①

ii) $x[n] \xrightarrow{n_0} x[n-n_0] \xrightarrow{sys} y[n] = x[n-n_0+1] - x[n-n_0-1]$ → ②

① = ②

TIV ✓

③ $y(t) = x(2t)$

$x_1(t) \longrightarrow y_1(t) \Rightarrow y_1(t) = x_1(2t)$

$\Rightarrow y_2(t) = x_2(2t)$

$y_1(t)+y_2(t) = x_1(2t)+x_2(2t)$ → ①

$x_1(t)+x_2(t) \longrightarrow x_1(2t)+x_2(2t)$ → ②

① = ②

$ax(t) \longrightarrow ax(2t) \longrightarrow ay(t)$ scaling ✓

Linear ✓

TIV:

i) $y(t) \xrightarrow{t_0} y(t-t_0) \longrightarrow x(2(t-t_0))$

ii) $x(t) \xrightarrow{t_0} x(t-t_0) \xrightarrow{sys} x[2t-t_0)$ } ≠

∴ TV system.