IIIT-Bangalore
Statistics
Problem Set 1

(Sampling Distributions)

1. The distribution of a population random variable X is given by $P(X = 0) = P(X = 1) = \frac{1}{2}$. A random sample of size 4 is drawn from the hypothetical population of X. Show that the sampling distribution of the statistic $t = x_1 + x_2 + x_3 + x_4$ is a binomial $(4, \frac{1}{2})$.

2. Show that sample mean is asymptotically normal $(m, \frac{\sigma}{\sqrt{n}})$, where m is the population mean and σ is the population standard deviation.

3. Find the sampling distribution of the sample mean for a normal population.

4. Find the sampling distribution of the statistic $\chi^2 = \frac{nS^2}{\sigma^2}$ for a normal population whose variance is σ^2. Show that the sample mean $\bar{X}$ and sample variance S^2 are independent variates.

5. Show that the sampling distribution of the statistic $t = \frac{(\bar{x}-m)\sqrt{n}}{s}$ where $s^2 = \frac{n}{n-1}S^2$ is t-distributed with $(n-1)$-degrees of freedom.

6. Find the sampling distribution of the sample variance for a normal (m, σ) population.

7. Find the sampling distribution of the sample mean for the (a) Binomial, (b) Poisson and (c) Gamma distribution.

8. Find the expression of the Standard Error of the sample mean.

9. The variable X is normally distributed with mean 68 cms and s.d. 2.5 cms. What should be the size of the sample whose mean shall not differ from the population mean by more than 1 cm. with probability 0.95?
   (Given that the area under standard normal curve to the right of the ordinate at 1.96 is 0.025)

10. The mean of a certain normal distribution is equal to the standard error of the mean of samples of size 100 from that distribution. Find the

probability that the mean of a sample of size 25 from the population will be negative. (Given $\frac{1}{\sqrt{2\pi}}\int_{1/2}^{\infty} e^{-t^2}dt = 0.3085$)