IIIT-Bangalore
BS 109 - Probability and Statistics
Problem Set 5

(Probability Distribution: Transformation of Random Variable)

1. Find the distribution of the square $Y = X^2$ where X is a Poisson-μ variate. (Ans. $P(Y = i^2) = \frac{e^{-\mu}\mu^i}{i!}, i = 0, 1, 2, \ldots$)

2. The probability density function of the random variable X is given by,

$$f_X(x) = \begin{cases} 2xe^{-x^2}, & x > 0 \\ 0, & \text{elsewhere.} \end{cases}$$

   Find the distribution of $Y = X^2$. (Ans. $f_Y(y) = e^{-y}$, $y > 0$)

3. If X is normal $(0, 1)$, then find the distribution of $Y = e^X$.
   (Ans. $f_Y(y) = \frac{1}{\sqrt{2\pi}} \frac{e^{-\frac{1}{2}(\log y)^2}}{y}$, $y > 0$)

4. If X is $\gamma(l)$ variate, find the p.d.f. of $Y = \sqrt{X}$.
   (Ans. $f_Y(y) = \frac{2e^{-y^2} y^{2l-1}}{\Gamma(l)}$, $y > 0$)

5. If X has $\beta_1(l, m)$ distribution, then $Y = \frac{X}{1-X}$ has $\beta_2(l, m)$ distribution.

6. A point is chosen at random on a semi-circle having centre at the origin and radius unity. The point is projected on the diameter. Prove that the distance of the point of projection from the centre has the probability density $\frac{1}{\pi\sqrt{1-x^2}}$ for $-1 < x < 1$ and zero elsewhere.
   (Ans. $f_X(x) = \frac{1}{\pi} \frac{1}{\sqrt{1-x^2}}$, $-1 < x < 1$)

7. A straight line i drawn through a fixed point (λ, μ) $(\lambda > 0)$ making an angle X, which is chosen at random in the interval $(0, \pi)$, with the y-axis. Prove that the intercept on the y-axis, Y has a Cauchy distribution with parameters λ, μ.

8. In the equation $t^2 + 2t - X = 0$, X is a random variable uniformly distributed over the interval $(0, 2)$. Find the p.d.f. of the larger root.
   (Ans. $f_Y(y) = \frac{1}{4(1+y)}$, $y \in (0, -1 + \sqrt{3})$

9. If X is a normal (m, σ) variate, prove that

$$P(a < X < b) = \Phi\left(\frac{b-m}{\sigma}\right) - \Phi\left(\frac{a-m}{\sigma}\right)$$

and

$$P(|X - m| > a\sigma) = 2[1 - \Phi(a)]$$

where $\Phi(x)$ denotes the standard normal distribution function.

*10 Let X be a standard normal variate. Show that $Y = \frac{X^2}{2}$ is $\gamma(\frac{1}{2})$ variate.

*11 If X is uniformly distributed in the interval $(-1, 1)$ find the distribution of $|X|$.
(Ans. $f_Y(y) = \begin{cases} 1, & 0 < y < 1 \\ 0, & \text{elsewhere.} \end{cases}$)