DAA Problem Set 4

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1 Greedy Algorithms

1. You are given a bag of capacity W and n items such that for $1 \leq i \leq n$, c_i is the maximum available quantity of item i and p_i is the total value of item i. Choose an amount $x_i \leq c_i$ of each item i to be packed in the bag subject to
$$\sum_i x_i \leq W$$
and maximizing the total value of the packed items.

2. Let T be a tree. You need to find a subset of edges E in T such that no two edges in E share an endpoint. Give a greedy algorithm to find such a set of maximum cardinality.

3. A matching in a tree is a collection of edges such that no pair of edges share a common node. The matching with the most edges is known as a maximum matching. Given a tree, give an algorithm to find the maximum matching in the given tree.

4. Your friend is working as a camp counselor, and he is in charge of organizing activities for a set of junior-high-school-age campers. One of his plans is the following mini-triathalon exercise: each contestant must swim 20 laps of a pool, then bike 10 miles, then run 3 miles. The plan is to send the contestants out in a staggered fashion, via the following rule: the contestants must use the pool one at a time. In other words, first one contestant swims the 20 laps, gets out, and starts biking. As soon as this first person is out of the pool, a second contestant begins swimming the 20 laps; as soon as he or she is out and starts biking, a third contestant begins swimming and so on.)

   Each contestant has a projected swimming time (the expected time it will take him or her to complete the 20 laps), a projected biking time (the expected time it will take him or her to complete the 10 miles of bicycling), and a projected running time (the time it will take him or her to complete the 3 miles of running). Your friend wants to decide on a schedule for the triathalon: an order in which to sequence the starts of the contestants.

Let's say that the completion time of a schedule is the earliest time at which all contestants will be finished with all three legs of the triathalon, assuming they each spend exactly their projected swimming, biking, and running times on the three parts. (Again, note that participants can bike and run simultaneously, but at most one person can be in the pool at any time.) What's the best order for sending people out, if one wants the whole competition to be over as early as possible? More precisely, give an efficient algorithm that produces a schedule whose completion time is as small as possible.

5. (a) Given a value of V Rs and an infinite supply of each of the denominations 1, 2, 5, 10, 20, 50, 100, 500, 1000 valued coins/notes, give a greedy algorithm to find the minimum number of coins and/or notes needed to make the change.

   (b) Does this algorithm work for all denominations and values? If yes, prove it. If not, provide a counter example for it.