$$\delta_{ij} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$$

$$\epsilon_{ijk} = \begin{cases} 1 & \text{symm. / cyc.} \\ -1 & \text{anti " / "} \\ 0 & \text{repetit. of indices} \end{cases}$$

Component A_{ijl} of $\underset{\sim}{A}$

$$\sum_k A_{ijk} B_{klm}$$

$$A_{ij} \cdot \delta_{ij} = A_{ii}$$

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$$\left(\vec{\nabla}(\vec{A}\cdot\vec{B})\right)_i = \partial_i (A_j B_j) \qquad \partial_i \equiv \frac{\partial}{\partial x_i}$$

$$\vec{\nabla}\cdot(\vec{A}\times\vec{B}) = \partial_i(\epsilon_{ijk} A_j B_k) = \epsilon_{ijk}\,\partial_i (A_j B_k)$$

$$= \epsilon_{ijk}\left((\partial_i A_j) B_k + A_j(\partial_i B_k)\right)$$

$$= \underbrace{(\epsilon_{ijk}\,\partial_i A_j)} B_k + (\epsilon_{ijk}(\partial_i B_k)) A_j$$

$$= (\vec{\nabla}\times\vec{A})_k\, B_k - (\epsilon_{ikj}(\partial_i B_k)) A_j$$

$$= (\vec{\nabla}\times\vec{A})\cdot\vec{B} - (\vec{\nabla}\times\vec{B})\cdot\vec{A}$$

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To show

$$\vec{\nabla}(\vec{A}\cdot\vec{B}) = (\vec{B}\cdot\vec{\nabla})\vec{A} + (\vec{A}\cdot\vec{\nabla})\vec{B} + \vec{B}\times(\vec{\nabla}\times\vec{A}) + \vec{A}\times(\vec{\nabla}\times\vec{B})$$

$$\text{LHS} = \partial_p (A_i B_j \delta_{ij}) = \delta_{ij}\left((\partial_p A_i) B_j + A_i(\partial_p B_j)\right)$$

$$= \delta_{ij}\left[\delta_{kp} B_j \partial_k A_i + \delta_{kp} A_i \partial_k B_j\right]$$

$$= (\delta_{ij}\delta_{kp} - \delta_{ip}\delta_{jk}) B_j \partial_k A_i + \delta_{ip}\delta_{jk} B_j \partial_k A_i + \underbrace{\delta_{ij}\delta_{kp} A_i \partial_k B_j}$$

$$= \epsilon_{ikt}\epsilon_{tjp} B_j \partial_k A_i$$

$$= \epsilon_{tjp} B_j \epsilon_{ikt} \partial_k A_i = -\epsilon_{tjp} B_j \epsilon_{tki} \partial_k A_i$$

$$= \epsilon_{pjt} B_j \epsilon_{tki} \partial_k A_i$$

$$\epsilon_{pjt} B_j (\vec{\nabla}\times A)_t = \vec{B}\times(\vec{\nabla}\times\vec{A})$$

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$\vec{B} \cdot \vec{\nabla} A$

$\underbrace{\delta_{ip}\, \delta_{jk}\, B_j\, (\partial_k A_i)}_{B_k \partial_k A_p}$

$(\vec{B} \cdot \vec{\nabla}) \vec{A} \rightarrow \epsilon_{rij}\, B_j\, \partial_k A_i$

$=$

$(B_i\, \partial_j\,) A_k$

$\delta_{ip}\, \delta_{jk}\, B_j\, \partial_k A_i \longrightarrow$

$= \left(\epsilon_{ijr}\, \epsilon_{rpk} + \delta_{ik}\delta_{jp} \right) B_j \partial_k A_i$

$\epsilon_{ijk}\, \epsilon_{kpk}$

$= \underbrace{\epsilon_{ijr}\, \epsilon_{rpk}\, B_j\, \partial_k A_i}_{(\epsilon_{ijr}\, B_j\, \partial_k A_i)\, \epsilon_{rpk}} + \underbrace{\delta_{ik}\, \delta_{jp}\, B_j\, \partial_k A_i}_{B_p (\partial_i A_i)}$

$\epsilon_{ijr}\, \epsilon_{rpk} = \delta_{ip}\, \delta_{jk} - \delta_{ik}\, \delta_{jp}$

$p_i \equiv \frac{\partial L}{\partial \dot{q}_i}$ = "generalized momentum" or "Canonical momentum"

$$L = L(q_i, \dot{q}_i, t) \qquad Q_i' \equiv \frac{\partial L}{\partial q_i} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)$$

$$\frac{dL}{dt} = \frac{\partial L}{\partial t} + \frac{\partial L}{\partial q_i}\dot{q}_i + \frac{\partial L}{\partial \dot{q}_i} q_i$$

$$= \frac{\partial L}{\partial t} + \frac{\partial L}{\partial q_i}\dot{q}_i + \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\dot{q}_i\right) - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)\dot{q}_i$$

$$= \frac{\partial L}{\partial t} + \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\dot{q}_i\right) + Q_i'\dot{q}_i$$

goes to 0 for non-dissipative syst.

What do I mean by explicit time independence?

My L is NOT of the form

$$L = A\dot{q}^2 + Bq^2 + Cqt^2.$$

$\cos(\omega t)$

$\cos\theta$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\dot{q}_i - L\right) = 0$$

Jacobi Integral

ie $\frac{d}{dt}\left(p_i\dot{q}_i - L\right) = 0$

$$p_i\dot{q}_i = \sum_i p_i\dot{q}_i$$

$J \equiv \frac{\partial L}{\partial \dot{q}_i}\dot{q}_i - L$ = "constant of motion" or "Integral of motion"

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L for projectile motion (Emmy Noether) Noether's Theorem

$$L = \frac{1}{2} m v^2 - mgz = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - mgz$$

x, y are cyclic coords

The corresp. generalized momenta are conserved

ie. p_x, p_y " "

$$\frac{dp_i}{dt} = 0 \quad \text{for } q_i \text{ if } q_i \text{ is cyclic}$$

ie. does not appear in the Lagrangian.

If any system or function representing some property of the system does not change under some operation defined on the system, it is said to possess a symmetry w.r.t. that observation.

Every symmetry in the Lagrangian corresponds to a conservation law.

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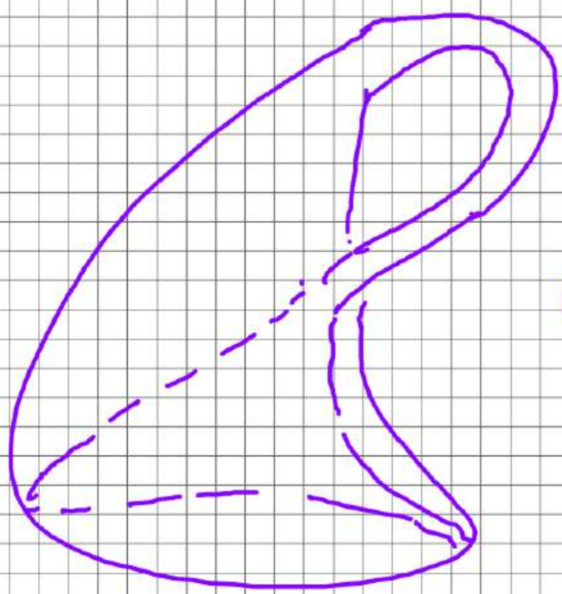
Emmy Noether



Felix Klein

David Hilbert

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$\frac{\partial L}{\partial t} = 0$ — a closed system.

If all possible actions & reactions of a system are totally confined within the system then it is said to be a closed system.

Also $\Rightarrow$ no external force. $\Rightarrow$ potential V is a const. V_0

$L = T - V_0$ — So $\therefore$ L is indep of t as is V_0

$\underbrace{T + V_0}_{T}$ = Energy is const. $\Rightarrow$ T is also a const.

Energy conservation follows for any closed system due to homogeneity of time.