# Problem Set 2

Solutions

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### Question 5

There are two identical urns containing 4 white and 3 red balls; 3 white and 7 red balls. An urn is chosen at random and a ball is drawn from it. Find the probability that the ball is white. If the ball drawn is white, what is the probability that it is from the first urn?

# Solution 5.1

Let
A: Urn 1 is chosen
B: Urn 2 is chosen
C: White ball is drawn

$$P(C) = P(C|A)P(A) + P(C|B)P(B)$$
$$= \frac{4}{7} \times \frac{1}{2} + \frac{3}{10} \times \frac{1}{2}$$
$$= \frac{61}{140}$$

## Solution 5.2

Let
$A$: Urn 1 is chosen
$B$: Urn 2 is chosen
$C$: White ball is drawn
We are required to find $P(A|C)$:

$$P(A|C) = \frac{P(C|A)P(A)}{P(C)}$$
$$= \frac{\frac{4}{7} \times \frac{1}{2}}{\frac{61}{140}}$$
$$= \frac{40}{61}$$

### Question 6

There are three identical boxes each provided with two drawers. In the first, each drawer contains a gold coin; in the third, each drawer contains a silver coin; in the second, one drawer contains a gold and other a silver coin. A box is selected at random and one of the drawers is opened. If a gold coin is found what is the probability that the box chosen is the second one?

## Solution 6

Let
$E$: Finding a gold coin
$A_i$: Choosing the $i^{th}$ box
We are required to find $P(A_2|E)$:

$$P(A_2|E) = \frac{P(E|A_2)P(A_2)}{\sum_{i=1}^{3} P(E|A_i)P(A_i)}$$

$$= \frac{\frac{1}{2} \times \frac{1}{3}}{\frac{1}{2} \times \frac{1}{3} + 1 \times \frac{1}{3} + 0 \times \frac{1}{3}}$$

$$= \frac{1}{3}$$

## Question 7

A letter is known to have come from either from TATANAGAR or from CALCUTTA. Only two consecutive letters TA are visible on the envelope. What is the probability that the letter is from CALCUTTA?

## Solution 7

Let
$E$: Letters 'T' and 'A' occur consecutively in order
$A_1$: Letters are from "CALCUTTA"
$A_2$: Letters are from "TATANAGAR"

$$P(A_1|E) = \frac{P(E|A_1)P(A_1)}{\sum_{i=1}^{2} P(E|A_i)P(A_i)}$$

$$= \frac{\frac{1}{7} \times \frac{1}{2}}{\frac{1}{7} \times \frac{1}{2} + \frac{2}{8} \times \frac{1}{2}}$$

$$= \frac{4}{11}$$