Math-3

9/9/22

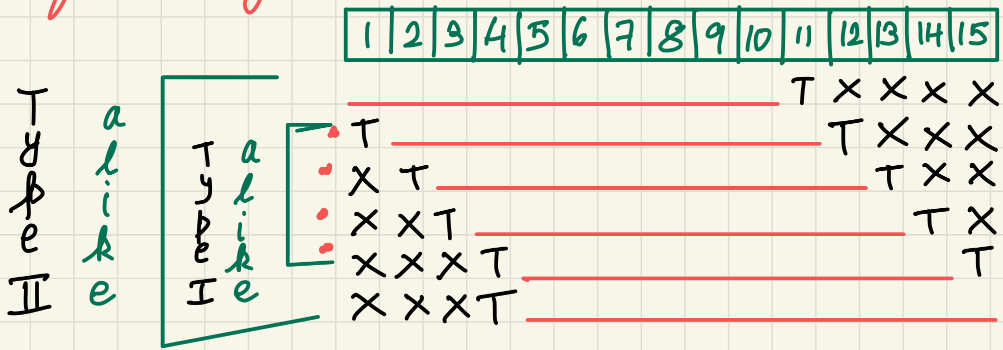
PS 1 & PS 2

discussion

8. A coin is tossed $(m+n)$ times $(m > n)$. Show that (i) the probability of exactly m consecutive heads is $(n+3)/2^{m+2}$ and (ii) the probability of at least m consecutive heads is $(n+2)/2^{m+1}$.

(i) the probability of exactly m consecutive heads

$m = 10$
$n = 5$



Probability of a Type I situation $= \frac{1}{2} \times \frac{1}{2^m} \times \frac{1}{2} = \frac{1}{2^{m+2}}$

Number of times a Type I situation will occur $= n-1$

Probability of a Type II situation $= \frac{1}{2^m} \times \frac{1}{2} = \frac{1}{2^{m+1}}$

Number of times a Type II situation will occur $= 2$

Probability(Type I + Type II) = Probability (Type I) + Probability (Type II)

$$= 2 \times \frac{1}{2^{m+1}} + (n-1) \times \frac{1}{2^{m+2}} = \frac{n+3}{2^{m+2}}$$

8. A coin is tossed $(m+n)$ times $(m>n)$. Show that (i) the probability of exactly m consecutive heads is $(n+3)/2^{m+2}$ and (ii) the probability of at least m consecutive heads is $(n+2)/2^{m+1}$.

(ii) the probability of at least m consecutive heads

Consider a small example :

1	2	3	4	5	6	7	8	9

let $m+n=9$, $m>n$: 6, 3

What is the probability of m consecutive heads from the first throw? What about the next n throws?

$$\underbrace{\frac{1}{2}\times\frac{1}{2}\times\cdots\times\frac{1}{2}}_{m}\times\underbrace{1\times\cdots\times 1}_{n}=\frac{1}{2^m}$$

What is the probability of m consecutive heads from the second throw? What about the next $n-1$ throws?

$$T\quad\underbrace{H\cdots\cdots H}_{m}\quad\underbrace{\text{don't care}}_{n-1}=\frac{1}{2}\times\frac{1}{2^m}=\frac{1}{2^{m+1}}$$

What is the probability of m consecutive heads from the third throw? What about the next $n-2$ throws?

$$X \ T \ \underbrace{H \cdots\cdots H}_{m} \quad \underbrace{\text{don't care}}_{n-1} = \frac{1}{2} \times \frac{1}{2^m} = \frac{1}{2^{m+1}}$$

What are total number of ways to get at least n consecutive heads when we do NOT start at the first position?

3, in the above example

n, in general

$$\therefore \quad \underbrace{\frac{1}{2^m}}_{\text{first}} + n \times \underbrace{\frac{1}{2^{m+1}}}_{\text{do NOT start at first}} = \frac{n+2}{2^{m+1}}$$

9. The integers x and y are chosen at random with replacement from nine natural numbers $1, 2, \ldots, 9$. Find the probability that $|x^2 - y^2|$ is divisible by 2. (Ans. $\frac{41}{81}$)

< solved by a student from the class >

$$\frac{{}^5C_1 \times {}^5C_1 + {}^4C_1 \times {}^4C_1}{{}^9C_1 \times {}^9C_1}$$

odd

even

9. The integers x and y are chosen at random with replacement from nine natural numbers $1, 2, \ldots, 9$. Find the probability that $|x^2 - y^2|$ is divisible by 2. (Ans. $\frac{41}{81}$)

$|x^2 - y^2| = (x+y)(x-y)$ as $1 \leq x, y < 9$

For this quantity to be divisible by 2, following cases arise:

1. $x+y = E$, $x-y = E$
2. $x+y = O$, $x-y = E$
3. $x+y = E$, $x-y = O$

ONLY $x, y \rightarrow$ odd and $x, y \rightarrow$ even satisfies these

$P(x, y \rightarrow \text{odd}) = \frac{5 \times 5}{9^2}$, $P(x, y \rightarrow \text{even}) = \frac{4 \times 4}{9^2}$

$P(\text{total}) = \frac{25}{81} + \frac{16}{81} = \frac{41}{81}$

10. What is the probability that a bridge hand will contain (i) all the aces (ii) at least one ace. (Ans. (i) $\frac{{}^4C_4 \times {}^{48}C_9}{{}^{52}C_{13}}$, (ii) $1 - \frac{{}^{48}C_{13}}{{}^{52}C_{13}}$)

< solved by a student from the class >

What is a bridge hand? a collection of 13 cards

Total number of aces in a pack of cards = 4

$$\frac{{}^4C_4 \times {}^{48}C_9}{{}^{52}C_{13}}$$

(ii) $1 - \frac{{}^{48}C_{13}}{{}^{52}C_{13}}$

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