

**Mathematics-3**  
**Tutorial-2**  
**Discussion on Friday, 16<sup>th</sup> August**  
**Topic: Independent Events and Counting**

1. Show that if three events  $A$ ,  $B$ , and  $C$  are independent, then  $A$  and  $(B \cup C)$  are independent.
2. Let  $A$  and  $B$  be events in a sample space  $S$ . Show that if  $A$  and  $B$  are independent, then so are (a)  $A$  and  $\bar{B}$ , (b)  $\bar{A}$  and  $B$ , and (c)  $\bar{A}$  and  $\bar{B}$ .
3. Let  $A$  and  $B$  be events defined in a sample space  $S$ . Show that if both  $P(A)$  and  $P(B)$  are nonzero, then events  $A$  and  $B$  cannot be both mutually exclusive and independent.
4. In the experiment of throwing two fair dice, let  $A$  be the event that the first die is odd,  $B$  be the event that the second die is odd, and  $C$  be the event that the sum is odd. Show that events  $A$ ,  $B$ , and  $C$  are pairwise independent, but  $A$ ,  $B$ , and  $C$  are not independent.
5. Let  $k$  be the number of active (nonsilent) speakers in a group of eight noninteracting (i.e., independent) speakers. Suppose that a speaker is active with probability  $1/3$ . Find the probability that the number of active speakers is greater than six.
6. A communication system transmits binary information over a channel that introduces random bit errors with probability  $\varepsilon = 10^{-3}$ . The transmitter transmits each information bit three times, and a decoder takes a majority vote of the received bits to decide on what the transmitted bit was. Find the probability that the receiver will make an incorrect decision.
7. A dart is thrown nine times at a target consisting of three areas. Each throw has a probability of .2, .3, and .5 of landing in areas 1, 2, and 3, respectively. Find the probability that the dart lands exactly three times in each of the areas.
8. Computer  $A$  sends a message to computer  $B$  over an unreliable radio link. The message is encoded so that  $B$  can detect when errors have been introduced into the message during transmission. If  $B$  detects an error, it requests  $A$  to retransmit it. If the probability of a message transmission error is  $q = .1$ , what is the probability that a message needs to be transmitted more than two times?