DAA Problem Set 3

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1 Greedy Algorithms

1. Design a **greedy algorithm** to determine the **minimum number of platforms required** at a railway station, given two arrays: $arr[]$ (arrival times) and $dep[]$ (departure times) of trains arriving on the same day. The algorithm should ensure that no train is kept waiting by allocating platforms efficiently, considering that a platform cannot be used by two trains at the same time. Analyse the algorithm.

2. There are m houses in a street extending n kilometers along a straight line. It is needed to build transmitters such that for every house in the street, there should exist a transmitter within 500 m of it. Give an algorithm to find the optimal locations for transmitters such that minimum number of transmitters are built?

3. Given a schedule of lectures on a given day from 9 AM to 5 PM, find a subset of lectures D of minimum cardinality such that, for every lecture of the day, either the lecture belongs to D or it overlaps with at least one lecture in D.

4. Given a graph G(V, E) the maximum independent set problem is to find a maximum cardinality subset of pairwise independent (non-adjacent) vertices. Show that the following greedy algorithm to solve the maximum independent set problem in a graph G(V, E) is not optimal.

   Repeat till G is empty

   Select the vertex v with the minimum degree, delete the neighbors of v

5. Given a graph $G(V, E)$, a dominating set of G is a subset of vertices $D \subseteq V$ such that for every vertex $v \in V$, either v is in D or v has a neighbor in D. The Minimum Dominating Set problem is to find a dominating set of minimum cardinality. Prove that the following greedy strategy does not always return a minimum dominating set, by providing counter example for which the algorithm does not provide the correct output.

   (a) Initially all vertices are unmarked. Select the vertex v with the maximum number of unmarked neighbors. Add v to the dominating set and mark the neighbors of v as dominated. Delete v from G and repeat if there are vertices that are not marked dominated.

6. There are n kids of different ages. Give a greedy algorithm that group the kids into minimum possible number of groups such that the age difference between the eldest and youngest kid in any group is 2 years.

7. Given a graph $G = (V, E)$, a vertex cover is a subset of C of vertices such that for every edge $(u, v) \in E$, $u \in C$ or $v \in C$ or both. A greedy algorithm to find a vertex cover of minimum size for a given graph would be to greedily select the vertex with the maximum degree and add it to the vertex cover set. Remove the node and all its edges from the graph and keep on repeating the same until there are no edges left.

   Provide a counter example to show that this greedy approach will not always yield the optimum answer.