B. Ashok

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|---|---|---|
| Attendance | 10% | Theory |
| Assignments | 10% | |
| Quizzes | 20% | |
| Endterm | 60% | |

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## 3 laws of Newton.

scalar vectors

$A \neq \vec{A}$ Tensors

**1st law** Inertial systems exist.

In an inertial coord. syst. an obj. that is moving with some constant vel. $\vec{V}$ continues to do so while undisturbed.

**2nd law** The Rate of change of a momentum of a system equals the force acting on it

$$\frac{d}{dt}(m\vec{V}) = \vec{F} \quad \text{or} \quad \vec{F} = m\vec{a} \quad \text{for const. mass}$$

**3rd law** Action & reaction are equal & opposite.

$$\vec{F}_1 = -\vec{F}_2 \qquad \frac{d\vec{p}_1}{dt} = -\frac{d\vec{p}_2}{dt} \quad \text{or} \quad \frac{d}{dt}(\vec{p}_1 + \vec{p}_2) = 0$$

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**Projectile motion.**

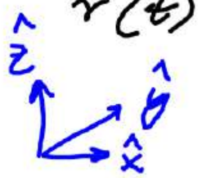
Acceleration $\vec{a}(t) = \frac{d}{dt}\vec{v}$.

Integrate: $\vec{v}(t) = \vec{v}(t_0) + \int_{t_0}^{t} \vec{a}(t)\,dt$

For uniform acceleration

$$\vec{v}(t) = \vec{v}(t_0) + \vec{a}t$$

$\frac{d}{dt}\vec{r} = \vec{v}(t)$ $\qquad \vec{r}(t) = \vec{r}_0 + \int_0^t \vec{v}(t')\,dt'$




$$\vec{r}(t) = \vec{r}_0 + \int_0^t \left(\vec{v}_0 + \vec{a}t'\right) dt'$$

$$= \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

Distance

$$s = |\vec{r}(t) - \vec{r}_0|$$

$$= v_0 t + \frac{1}{2}at^2$$

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In a uniform gravitational field: $\uparrow \hat{z}$

$\vec{a} = -\vec{g}$.

An object is thrown upwards at $t=0$ with some unit. vel. $\vec{v}_0$

$$x = x_0 + v_x t$$

To obtain the trajectory eliminate time 't'

$$y = y_0 + v_y t$$

$$z = z_0 + \int_0^t (v_{0z} - gt')\,dt'$$

$$= z_0 + v_{0z} t - \frac{1}{2} g t^2.$$

$v_{0y} = 0$ $\quad \vec{r}_0 = 0$

for simplicity, consider 2D

$$\begin{cases} x = v_{0x} t \\ z = v_{0z} t - \frac{1}{2} g t^2 \end{cases}$$

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$t = x / v_{0x}$ $\qquad z = v_{0z} t - \frac{1}{2} g t^2$

$$\therefore \quad z = \frac{v_{0z}}{v_{0x}} x - \frac{1}{2} g \frac{x^2}{v_{0x}^2}$$

$$z = ax - bx^2$$

eqn. of a parabola. for free fall projectile motion.

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## Principle of least action or Hamilton's principle.

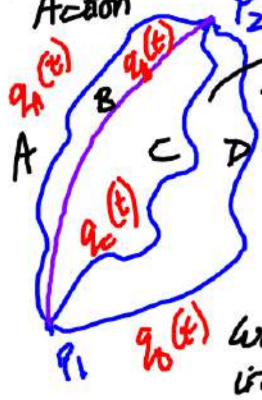
How do we find the shortest path betw. 2 points.
ie. how do you find a geodesic?

$$I = \int_{t_1}^{t_2} \underbrace{L(t, q, \dot{q})}_{\text{"Lagrangian"}} \, dt$$

"Action"

$q \equiv$ generalized coords

$\frac{dq}{dt} = \dot{q} \equiv$ generalized vel.




$q(t)$

Let curve B, Corresp $q = Q(t)$
makes $I$ an extremum

Then $q = Q(t) + \epsilon \eta(t)$ indep of $t$ $\quad t_1 \leq t \leq t_2$

will be any neighbouring curve through the points in space at an instance $t$!

$I = \int_{t_1}^{t_2} L(q, \dot{q}, t)\, dt$ makes $I$ an extremum $\left( L = KE - PE \right)$

Action Lagrangian

$q(t) = Q(t) + \epsilon\, \eta(t)$ $\quad t_1 \leq t \leq t_2$

indep of $t$.


As all the curves coincide at $P_1$ & $P_2$, $\Rightarrow$ $\eta(t_1) = \eta(t_2) = 0$

$I$ is an extremum for $\epsilon = 0$, ie when the curve followed is $Q(t)$.

ie. $\left.\dfrac{dI}{d\epsilon}\right|_{\epsilon=0} = 0$ $\therefore$ $\left.\dfrac{dI}{d\epsilon}\right|_{\epsilon=0} = \displaystyle\int_{t_1}^{t_2} \left( \frac{\partial L}{\partial q} \eta + \frac{\partial L}{\partial \dot{q}} \dot{\eta} \right) dt = 0$

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$$I = \int_{t_1}^{t_2} L(q, \dot{q}, t)\, dt \qquad \begin{aligned} q &= Q(t) + \epsilon\eta(t) \\ \dot{q} &= \dot{Q} + \epsilon\dot{\eta} \end{aligned}$$

$$\frac{dI}{d\epsilon} = \frac{d}{d\epsilon}\int_{t}^{t_2} L(q, \dot{q}, t)\, dt$$

$$= \int^{2} \left( \frac{\partial L}{\partial q}\frac{\partial q}{\partial \epsilon} + \frac{\partial L}{\partial \dot{q}}\frac{\partial \dot{q}}{\partial \epsilon} \right) dt = 0$$

$$= \int_{t_1}^{t_2} \left( \frac{\partial L}{\partial q}\eta + \frac{\partial L}{\partial \dot{q}}\dot{\eta} \right) dt$$

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$$\frac{dA}{d\epsilon} = ?$$

$$A(x_1, x_2)$$

$$\left.\frac{\partial A}{\partial x_1}\frac{\partial x_1}{\partial \epsilon}\right|_{x_2 \text{ fixed}} + \left.\frac{\partial A}{\partial x_2}\frac{\partial x_2}{\partial \epsilon}\right|_{x_1}$$

$$q(t) = Q(t) + \epsilon\eta(t)$$

$$\left.\frac{dI}{d\epsilon}\right|_{\epsilon=0} = 0 = \int_{t_1}^{t_2} \left( \frac{\partial L}{\partial q}\eta + \frac{\partial L}{\partial \dot{q}}\dot{\eta} \right) dt$$

$$= \int_{t_1}^{t_2} \frac{\partial L}{\partial q}\eta + \left.\frac{\partial L}{\partial \dot{q}}\eta\right|_{t_1}^{t_2} - \int_{t_1}^{t_2} \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)\eta \, dt$$

ie. $$\int_{t_1}^{t_2} \eta \left( \frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)\right) + \underbrace{\cancel{\left.\frac{\partial L}{\partial \dot{q}}\eta\right|_{t_1}^{t_2}}}_{=0} = 0$$

ie. $$\int_{t_1}^{t_2} \eta \left( \frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)\right) = 0$$

$\Rightarrow$ Since it should hold for all curves,

$$\frac{\partial L}{\partial q} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)$$

or

$$\boxed{\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = 0}$$

Euler-Lagrange eqns

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