Mathematics 3 (SM 211): Probability and Statistics

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Ch. 2: Compound Experiment

Kautuk Raj

Theorem

Let A_i denote the event that there are exactly i successes in a sequence of n Bernoulli trials with probability of success p. Then

$$P(A_i) = \binom{n}{i} p^i q^{n-i},\ 0 < p < 1,\ q = 1 - p.$$

Q. Given $S = A_0 + A_1 + A_2 + \cdots + A_n$
prove that $P(S) = 1$.

$P(A_i) = {}^nC_i\, p^i (1-p)^{n-i}$ (Shrey)

$P(S) = P(A_0) + P(A_1) \cdots + P(A_n)$

$= {}^nC_0\, p^0 (1-p)^n + \cdots\ {}^nC_n\, p^n (1-p)^0$

$P(S) = (p + 1 - p)^n = 1$

Q. Given $S = A_0 + A_1 + A_2 + \cdots + A_n$

prove that $P(S) = 1$.

Proof: $P(S) = \sum_{i=0}^{n} P(A_i)$

$\boxed{\text{why ?}}$

$$= \sum_{i=0}^{n} \binom{n}{i} p^i q^{n-i} = (p+q)^n = (p+1-p)^n$$

$$= 1$$

Q. In a Bernoullian sequence of n trials with probability p, find the probability that the i^{th} success occurs at the n^{th} trail ?

Goutam

$\boxed{(i-1) \text{ success}}$ $\boxed{\text{succ}}$ess

$n-1$ trials $\qquad$ n^{th} trials

$$\binom{n-1}{i-1} p^{(i-1)} q^{[n-1-(i-1)]} \times p = \binom{n-1}{i-1} p^{i} q^{n-i}$$

Q. In a Bernoullian sequence of n trials with probability p, find the probability that the i^{th} success occurs at the n^{th} trial?

answer: $\binom{n-1}{i-1} p^{i} (1-p)^{n-i}$

Poisson Approximation to Binomial Law

coming up soon

Theorem

If $p = \frac{\mu}{n}$, where μ is a positive constant, n is a positive integer and $0 < p < 1$, then

$$\lim_{n \to \infty} \binom{n}{i} p^i (1-p)^{n-i} = e^{-\mu} \frac{\mu^i}{i!}.$$

How to prove this ?

If $p = \frac{\mu}{n}$, where μ is a positive constant, n is a positive integer and $0 < p < 1$, then

$$\lim_{n \to \infty} \binom{n}{i} p^i (1-p)^{n-i} = e^{-\mu} \frac{\mu^i}{i!}.$$

$$P(A_i) = \binom{n}{i} p^i (1-p)^{n-i} =$$

If $p = \frac{\mu}{n}$, where μ is a positive constant, n is a positive integer and $0 < p < 1$, then

$$\lim_{n\to\infty} \binom{n}{i} p^i (1-p)^{n-i} = e^{-\mu} \frac{\mu^i}{i!}.$$

$$P(A_i) = \binom{n}{i} p^i (1-p)^{n-i} = \frac{\underline{\lfloor n}}{\underline{\lfloor n-i}\ \underline{\lfloor i}} p^i (1-p)^{n-i}$$

$$= \frac{n\cdot(n-1)\cdots(n-i+1)}{\underline{\lfloor i}} p^i (1-p)^{n-i}$$

$$=$$

If $p = \frac{\mu}{n}$, where μ is a positive constant, n is a positive integer and $0 < p < 1$, then

$$\lim_{n\to\infty} \binom{n}{i} p^i (1-p)^{n-i} = e^{-\mu} \frac{\mu^i}{i!}.$$

$$P(A_i) = \binom{n}{i} p^i (1-p)^{n-i} = \frac{\lfloor n}{\lfloor n-i \; \lfloor i} p^i (1-p)^{n-i}$$

$$= \frac{\overset{1}{n} \cdot \underbrace{(n-1)}_{n(1-\frac{1}{n})} \cdots \underbrace{(n-i+1)}_{n(1-\frac{i-1}{n})}}{\lfloor i} p^i \underbrace{(1-p)^{n-i}}$$

$$p = \frac{\mu}{n}$$

$$= \frac{\mu^i}{\lfloor i} \left(1 - \frac{1}{n}\right) \cdots \left(1 - \frac{i-1}{n}\right) \underbrace{\left(1 - \frac{\mu}{n}\right)^{n-i}}$$

$$\mu = pn$$

If $p = \frac{\mu}{n}$, where μ is a positive constant, n is a positive integer and $0 < p < 1$, then

$$\lim_{n\to\infty} \binom{n}{i} p^i (1-p)^{n-i} = e^{-\mu} \frac{\mu^i}{i!}.$$

$$P(A_i) = \binom{n}{i} p^i (1-p)^{n-i} = \frac{\lfloor n}{\lfloor n-i \cdot \lfloor i} p^i (1-p)^{n-i}$$

$$= \frac{n\cdot(n-1)\cdots(n-i+1)}{\lfloor i} p^i \left(1-\frac{\mu}{n}\right)^{n-i}$$

$$= \frac{\mu^i}{\lfloor i} \left(1-\frac{1}{n}\right)\cdots\left(1-\frac{i-1}{n}\right)\left(1-\frac{\mu}{n}\right)^n \left(1-\frac{\mu}{n}\right)^{-i} \to 1$$

$$\boxed{\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^n = e^{-\lambda}}$$

take limits now $[n\to\infty]$ $= e^{-\mu} \frac{\mu^i}{\lfloor i}$ as $n \to \infty$

As $p = \frac{\mu}{n}$ and $n \to \infty$, then $p \to 0$

$\therefore$ If the probability of success [p] is small and the number of trials is large such that $\mu = np$ is of moderate magnitude, we have the following approximation :-

$$P(A_i) \simeq e^{-\mu} \frac{\mu^i}{i!} \quad (i = 0, 1, 2, ---)$$

Q. Given $P(A_i) \simeq e^{-\mu} \dfrac{\mu^i}{i!}$ $(i = 0, 1, 2, \ldots \infty)$

and $S^{\infty} = A_0 + A_1 + \ldots\ldots + A_{\infty}$

prove that $P(S^{\infty}) = 1$

Q. Given $P(A_i) \simeq e^{-p} \frac{p^i}{i!}$ $(i = 0, 1, 2, \ldots)$

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prove that $P(S^{\infty}) = 1$

Proof $P(S^{\infty}) = P(A_0) + P(A_1) + \ldots\ldots P(A_{\infty})$

$$= e^{-p} \frac{p^0}{0!} + e^{-p} \frac{p^1}{1!} + \ldots\ldots e^{-p} \frac{p^{\infty}}{\infty!}$$

$$= e^{-p} \sum_{i=0}^{\infty} \frac{p^i}{i!} = e^{-p} \cdot e^{p} = 1$$

expansion of e^p

Q. An urn contains 1 white and 99 black balls, if 1000 draws are made with replacements, what is the probability of 10 white balls?

(1) Binomial Law

(2) Poisson Approximation

(1) $^{1000}C_{10} \left(\frac{1}{100}\right)^{10} \left(\frac{99}{100}\right)^{990} \approx 0.12574$

≈ 0.126

Q. An urn contains 1 white and 99 black balls, if 1000 draws are made with replacements, what is the probability of 10 white balls?

Approach 1 : using binomial law

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Approach 1 : using binomial law

$P(\text{success}) =$

$n =$

$i =$

$P(A_{10}) =$

$= 0.126$

Courtesy WolframAlpha

Q. An urn contains 1 white and 99 black balls, if 1000 draws are made with replacements, what is the probability of 10 white balls ?

Approach 2 : using Poisson approximation

Q. An urn contains 1 white and 99 black balls, if 1000 draws are made with replacements, what is the probability of 10 white balls?

find Q where approximation fails

Approach 2 : using Poisson approximation

Varad

∴ If the probability of success [p] is small and the number of trials is large such that $\mu = np$ is of moderate magnitude, we have the following approximation :-

$$P(A_i) \simeq e^{-\mu} \frac{\mu^i}{i!} \quad (i = 0, 1, 2, \ldots)$$

$$\mu = np = 1000 \times \frac{1}{100} = 10$$

$$P(A_{10}) = e^{-\mu} \frac{\mu^i}{i!} = \frac{e^{-10} (10)^{10}}{10!} \approx 0.125100$$

PS 3

Q7

7. What is the probability that in a company of 500 people only one person will have ~~his~~ their birth day on a new year day? (Ans. $\frac{100}{73}e^{-\frac{100}{73}}$, approx.)

∴ If the probability of success [p] is small and the number of trials is large such that $\mu = np$ is of moderate magnitude, we have the following approximation :- $P(A_i) \simeq e^{-\mu}\frac{\mu^i}{i!}$ $(i = 0, 1, 2, \ldots)$

Akash

$n =$, $p =$

$p = \frac{1}{365}$

$n = 500$

$i = 1$

$\mu = np =$

$\frac{\mu}{n} = \frac{1}{365}$

$\mu = \frac{500}{365}$

$e^{-\frac{500}{365}}\left(\frac{500}{365}\right)$

$\therefore e^{-\mu}\frac{\mu^1}{1!}$

$\therefore \cancel{e^{-\frac{500}{3}}}$

PS 3
Q8

8. A card is drawn from a pack 260 times (with replacement each time). Find the probability of Queen of Hearts 4 times. (Ans. $\frac{5^4 e^{-5}}{4!}$, approx.)

$\therefore$ If the probability of success [p] is small and the number of trials is large such that $\mu = np$ is of moderate magnitude, we have the following approximation :- $P(A_i) \simeq e^{-\mu}\frac{\mu^i}{i!}$ $(i = 0, 1, 2, ---)$

$n = 260$, $p = \frac{1}{52}$, $i = 4$

Vikas, K

$\mu = np = \frac{260}{52} = 5$

$\therefore P(A_4) = e^{-\mu}\frac{\mu^i}{i!} = e^{-5}\frac{(5)^4}{4!}$

Poisson Trials

Repeated independent trials are called Poisson trials if

(i) there are only two outcomes in each trials: ‘success’ and ‘failure’

(ii) the probability of ‘success’ in a trial is not constant throughout the trials

Introduction

A sequence of independent trials of a random experiment, the event space of which contains two points - success and failure, is called a **Poisson sequence of trials** if the probability of success is not constant but varies from one trial to another. It will be rather cumbersome to deduce general formulae for a Poisson sequence of n trials, and let us, for convenience, consider only three trials.

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Let the probabilities of success be p_1, p_2, p_3 in the three trials respectively, and hence the probabilities of failure are respectively $1 - p_1 = q_1, 1 - p_2 = q_2, 1 - p_3 = q_3$.

The event space of the compound experiment will then contain the following 8 points :

$\underline{S/F} \times \underline{S/F} \times \underline{S/F} = 8$

Introduction

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Let the probabilities of success be p_1, p_2, p_3 in the three trials respectively, and hence the probabilities of failure are respectively $1 - p_1 = q_1, 1 - p_2 = q_2, 1 - p_3 = q_3$.
The event space of the compound experiment will then contain the following 8 points :
$U_1 = (S, S, S), \quad U_2 = (S, S, F), \quad U_3 = (S, F, S),$
$U_4 = (F, S, S), \quad U_5 = (F, F, S), \quad U_6 = (F, S, F), U_7 = (S, F, F), \quad U_8 = (F, F, F)$

An Example

Since the trials are independent, the distribution of probabilities in this event space is given by

$$\begin{aligned}
&P(U_1) = p_1 p_2 p_3, \quad P(U_2) = p_1 p_2 q_3, \quad P(U_3) = p_1 q_2 p_3, \\
&P(U_4) = q_1 p_2 p_3, \quad P(U_5) = q_1 q_2 p_3, \quad P(U_6) = q_1 p_2 q_3, \\
&P(U_7) = p_1 q_2 q_3, \quad P(U_8) = q_1 q_2 q_3
\end{aligned}$$

$$\begin{aligned}
&U_1 = (S, S, S), \quad U_2 = (S, S, F), \quad U_3 = (S, F, S), \\
&U_4 = (F, S, S), \quad U_5 = (F, F, S), \quad U_6 = (F, S, F), U_7 = (S, F, F), \quad U_8 = (F, F, F)
\end{aligned}$$

An Example

Since the trials are independent, the distribution of probabilities in this event space is given by

$$\begin{aligned} &P(U_1) = p_1p_2p_3, \quad P(U_2) = p_1p_2q_3, \quad P(U_3) = p_1q_2p_3, \\ &P(U_4) = q_1p_2p_3, \quad P(U_5) = q_1q_2p_3, \quad P(U_6) = q_1p_2q_3, \\ &\qquad\qquad\qquad\quad\; P(U_7) = p_1q_2q_3, \quad P(U_8) = q_1q_2q_3 \end{aligned}$$

If A_i denotes the event 'i successes', then

$$A_0 = U_8, \quad A_1 = U_5 + U_6 + U_7, \quad A_2 = U_2 + U_3 + U_4, \quad A_3 = U_1$$

$$\begin{aligned} &U_1 = (S, S, S), \quad U_2 = (S, S, F), \quad U_3 = (S, F, S), \\ &U_4 = (F, S, S), \quad U_5 = (F, F, S), \quad U_6 = (F, S, F), U_7 = (S, F, F), \quad U_8 = (F, F, F) \end{aligned}$$

An Example

Since the trials are independent, the distribution of probabilities in this event space is given by

$$\begin{array}{lll} P(U_1) = p_1p_2p_3, & P(U_2) = p_1p_2q_3, & P(U_3) = p_1q_2p_3, \\ P(U_4) = q_1p_2p_3, & P(U_5) = q_1q_2p_3, & P(U_6) = q_1p_2q_3, \\ & P(U_7) = p_1q_2q_3, & P(U_8) = q_1q_2q_3 \end{array}$$

If A_i denotes the event 'i successes', then

$$A_0 = U_8, \quad A_1 = U_5 + U_6 + U_7, \quad A_2 = U_2 + U_3 + U_4, \quad A_3 = U_1$$

Hence,

$$\left.\begin{array}{l} P(A_0) = q_1q_2q_3, \quad P(A_1) = q_1q_2p_3 + q_1p_2q_3 + p_1q_2q_3 \\ P(A_2) = p_1p_2q_3 + p_1q_2p_3 + q_1p_2p_3, \quad P(A_3) = p_1p_2p_3 \end{array}\right\}$$

and

$$\sum_{i=0}^{3} P(A_i) = (p_1 + q_1)(p_2 + q_2)(p_3 + q_3) = 1$$

Question 1

$$P(1) = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$$

$$P(2) = \frac{1}{2} \times \frac{1}{3} \times \frac{3}{4} = \frac{1}{8}$$

Three marks~~men~~ person can hit a target with probabilities $1/2, 2/3, 3/4$ respectively. They shoot simultaneously, and two hits are registered. Find the probability that each of the three marks~~men~~ person misses the target.

$$P(3) = \frac{1}{2} \times \frac{2}{3} \times \frac{1}{4} = \frac{1}{12}$$

Goutham

Three ~~marksmen~~ person can hit a target with probabilities $1/2, 2/3, 3/4$ respectively. They shoot simultaneously, and two hits are registered. Find the probability that each of the three ~~marksmen~~ person misses the target.

$$P(A|E_1) = \frac{P(E_1|A) \times P(A)}{P(E_1|A) \times P(A) + \ldots}$$

Kesha✓

Denominator $= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} + \frac{1}{2} \times \frac{1}{3} \times \frac{3}{4} + \frac{1}{2} \times \frac{2}{3} \times \frac{1}{4}$

$= \frac{11}{24}$

P(Marksmen1 misses the target) $= \dfrac{\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}}{\frac{11}{24}} = \frac{1}{4} \times \frac{24}{11} = \frac{6}{11}$

P(mark person 1) & P(mark person 3)

Solution 1

Three marksmen can hit a target with probabilities $1/2, 2/3, 3/4$ respectively. They shoot simultaneously, and two hits are registered. Find the probability that each of the three marksmen misses the target.

Calling 'hitting the target' a success, here we have a Poisson sequence of three trials with $p_1 = 1/2, p_2 = 2/3, p_3 = 3/4$.

Solution 1

Three marksmen can hit a target with probabilities $1/2, 2/3, 3/4$ respectively. They shoot simultaneously, and two hits are registered. Find the probability that each of the three marksmen misses the target.

Calling 'hitting the target' a success, here we have a Poisson sequence of three trials with $p_1 = 1/2, p_2 = 2/3, p_3 = 3/4$. Let A_2- two successes and B_i- failure in ith trial $(i = 1, 2, 3)$. Then $A_2B_1 = (F, S, S), A_2B_2 = (S, F, S), A_2B_3 = (S, S, F)$ and

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$$P(A_2) = \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{3}{4} + \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4}$$

$$P(A_2B_1) = \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4}, \quad P(A_2B_2) = \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{3}{4}, \quad P(A_2B_3) = \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{4}$$

Solution 1

Three marksmen can hit a target with probabilities $1/2, 2/3, 3/4$ respectively. They shoot simultaneously, and two hits are registered. Find the probability that each of the three marksmen misses the target.

Calling 'hitting the target' a success, here we have a Poisson sequence of three trials with $p_1 = 1/2, p_2 = 2/3, p_3 = 3/4$. Let A_2- two successes and B_i- failure in ith trial $(i = 1, 2, 3)$. Then $A_2B_1 = (F, S, S), A_2B_2 = (S, F, S), A_2B_3 = (S, S, F)$ and

$$P(A_2) = \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{3}{4} + \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4}$$

$$P(A_2B_1) = \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4}, \quad P(A_2B_2) = \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{3}{4}, \quad P(A_2B_3) = \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{4}$$

Hence Similarly, $P(B_2/A_2)$ & $P(B_3/A_2)$

$$P(B_1 \mid A_2) = P(A_2B_1) / P(A_2) = 6/11 \text{ and so on.}$$

Question 1

Can the same question be solved using just Bayes' Theorem?

yes.

Three marksmen can hit a target with probabilities $1/2, 2/3, 3/4$ respectively. They shoot simultaneously, and two hits are registered. Find the probability that each of the three marksmen misses the target.

Question 2

A man seeks advice regarding one of two possible courses of action from three advisers who arrive at their recommendations independently. He follows the recommendation of the majority. The probabilities that the individual advisers are wrong are $0.1, 0.05$ and 0.05 respectively. What is the probability that the man takes the incorrect decision?

Solution 2

$P(A_1)$ = one person gives him wrong advice
$P(A_2)$ = two people give him wrong advice
$P(A_3)$ = two people give him wrong advice

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$P(A_2) =$

$P(A_3) =$

Required probability $= P(A_2) + P(A_3)$
$=$

- Let a random experiment E have m outcomes denoted by the elementary events, say $U_1 = \{a_1\}$, $U_1 = \{a_2\}$, ..., $U_m = \{a_m\}$. Also let $P(U_i) = p_i$, $i = 1, 2, \ldots, m$ with $p_1 + p_2 + \ldots + p_m = 1$.

unfair random die

- Let us consider n independent trails of E where the probabilities of $U_1, U_2, \ldots, U_m$ remain constant in each trail of E. In the compound experiment E_n, let $A_{i_1 i_2 \ldots i_m}$ denote the event that U_1 occurs i_1 times, U_2 occurs i_2 times, ..., U_m occurs i_m times, so $i_1 + i_2 + \ldots + i_m = n$. Then we have the following theorem:

what if not?

Why is called 'multinomial law'?

Theorem

$$P(A_{i_1 i_2 \ldots i_m}) = \frac{n!}{i_1! i_2! \ldots i_m!} p_1^{i_1} p_2^{i_2} \ldots p_m^{i_m}.$$

- Let a random experiment E have m outcomes denoted by the elementary events, say $U_1 = \{a_1\}$, $U_1 = \{a_2\}$, ..., $U_m = \{a_m\}$. Also let $P(U_i) = p_i$, $i = 1, 2, \ldots, m$ with $p_1 + p_2 + \ldots + p_m = 1$.

- Let us consider n independent trails of E where the probabilities of $U_1, U_2, \ldots, U_m$ remain constant in each trail of E. In the compound experiment E_n, let $A_{i_1 i_2 \ldots i_m}$ denote the event that U_1 occurs i_1 times, U_2 occurs i_2 times, ..., U_m occurs i_m times, so $i_1 + i_2 + \ldots + i_m = n$. Then we have the following theorem:

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Why is called 'multinomial law'?

Theorem

RHS is the general term in expansion of $(p_1+p_2+\cdots+p_m)^n$

$$P(A_{i_1 i_2 \ldots i_m}) = \underbrace{\frac{n!}{i_1! i_2! \ldots i_m!}}_{\hookrightarrow\ {}^{n}C_{i_1} \cdot {}^{n-i_1}C_{i_2} \cdots {}^{i_m}C_{i_m}} p_1^{i_1} p_2^{i_2} \ldots p_m^{i_m}.$$

Def

Q. A die is thrown 10 times in succession. Find the probability of the occurrence of six 4 times, five 2 times and all other faces once.

$$P(A_{i_1 i_2 \ldots i_m}) = \underbrace{\frac{n!}{i_1! i_2! \ldots i_m!}}_{\hookrightarrow \; {}^{n}C_{i_1} \cdot {}^{n-i_1}C_{i_2} \cdots {}^{i_m}C_{i_m}} p_1^{i_1} p_2^{i_2} \ldots p_m^{i_m}.$$

$n = 10 \quad i_1 = 4, \; i_2 = 2, \; i_3 = 1, \; i_4 = 1, \; i_5 = 1, \; i_6 = 1$

$$P = \frac{10!}{4!\,2!\,1!\,1!\,1!\,1!} \times \left(\frac{1}{6}\right)^{4+2+1+1+1+1}$$

$$= \frac{10!}{4!\,2!} \times \left(\frac{1}{6}\right)^{10}$$

Ketaki

Q. A die is thrown 10 times in succession. Find the probability of the occurrence of six 4 times, five 2 times and all other faces once.

$$P(A_{i_1 i_2 \ldots i_m}) = \frac{n!}{i_1! i_2! \ldots i_m!} p_1^{i_1} p_2^{i_2} \ldots p_m^{i_m}.$$

$n =$,

$i_1 =$, $i_2 =$, $i_3 =$, $i_4 =$, $i_5 =$, $i_6 =$

$P(\quad) =$

Moving Forward,

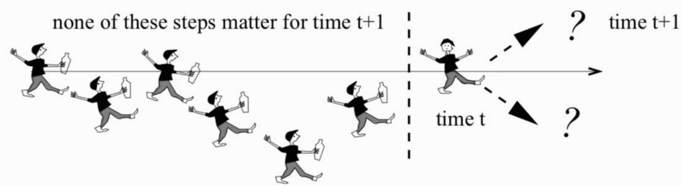
MML

Markov Chain: a set of dependent trails such that the outcome of any trail depends on the outcome of the immediately preceeding trail but not on the outcomes of earlier trials.

X_{t+1} depends only on X_t.

It does not depend upon $X_0, X_1, \ldots, X_{t-1}$.

Example: Random Walk ([redacted])



In a Markov chain, the future depends only upon the present: NOT upon the past.