Mathematics-3
Tutorial-4
Discussion on Friday, 30th August
Topic: Continuous Random Variables

1. For a nonnegative random variable Y,

$$E[Y] = \int_0^{\infty} P\{Y > y\}\, dy$$

2. From the above result, for any function g for which $g(x) \geq 0$, show that

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) f(x)\, dx$$

3. The density function of X is given by

$$f(x) = \begin{cases} a + bx^2 & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

If $E[X] = \frac{3}{5}$, find a and b.

4. **The random variable x is $N(5, 2)$ and $y = 2x + 4$. Find η_y, σ_y, and $f_y(y)$.**

5. **Find $F_y(y)$ and $f_y(y)$ if $y = -4x + 3$ and $f_x(x) = 2e^{-2x}U(x)$.**

Where U(x) =1 if x>=0, otherwise 0

6. Let X be a normal random variable with mean 12 and variance 4. Find the value of c such that $P\{X > c\} = 0.10$

7. The lifetime in hours of an electronic tube is a random variable having a probability density function given by

$$f(x) = xe^{-x} \qquad x \geq 0$$

Compute the expected lifetime of such a tube.

8.

Consider the function

$$f(x) = \begin{cases} C(2x - x^3) & 0 < x < \frac{5}{2} \\ 0 & \text{otherwise} \end{cases}$$

Could f be a probability density function? If so, determine C. Repeat if $f(x)$ were given by

$$f(x) = \begin{cases} C(2x - x^2) & 0 < x < \frac{5}{2} \\ 0 & \text{otherwise} \end{cases}$$

9.

Compute $E[X]$ if X has a density function given by

(a) $f(x) = \begin{cases} \frac{1}{4}xe^{-x/2} & x > 0 \\ 0 & \text{otherwise} \end{cases}$;

(b) $f(x) = \begin{cases} c(1 - x^2) & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$;

(c) $f(x) = \begin{cases} \frac{5}{x^2} & x > 5 \\ 0 & x \leq 5 \end{cases}$.

10.

If X is an exponential random variable with parameter $\lambda = 1$, compute the probability density function of the random variable Y defined by $Y = \log X$.

If X is uniformly distributed over $(0, 1)$, find the density function of $Y = e^X$.