Gradient :- $\left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\hat{z}\right) T(x,y,z)$

Vector

$\cdot\left(dx\,\hat{x} + dy\,\hat{j} + dz\,\hat{z}\right)$

$(\nabla T)\cdot(dl) =$

$$\boxed{\nabla T \equiv \frac{\partial T}{\partial x}\hat{x} + \frac{\partial T}{\partial y}\hat{y} + \frac{\partial T}{\partial z}\hat{z}}$$

vector

$|\nabla T|\,|dl|\cos\theta$

Scalar

$$\boxed{r = \sqrt{x^2+y^2+z^2}}$$

$d\hat{l}$

$dx\,\hat{x} + dy\,\hat{y}$
$+ dz\,\hat{z}$

$$\nabla r = \frac{\partial r}{\partial x}\hat{x} + \frac{\partial r}{\partial y}\hat{y} + \frac{\partial r}{\partial z}\hat{z}$$

$$= \frac{x\hat{x} + y\hat{y} + z\hat{z}}{\sqrt{x^2+y^2+z^2}} = \frac{\vec{r}}{r} = \hat{r}$$

the unit vector

if $\nabla T = 0$ at (x, y, z)

$dT = 0$ around that point

Stationary point

---

operator : Diversence

$\nabla \cdot V$ → vector → Scalar

F

gradient

Curl operator $\nabla \times \underline{v}$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$$

$\cancel{x}$

$f\cancel{(x)}$ $\quad yz\,\hat{i} + xz\,\hat{j} + xy\,\hat{k}$

$\nabla \cdot E$

$\frac{\partial}{\partial x}($

Product Rules :-

$$\frac{d}{dx}(f+g) = \frac{df}{dx} + \frac{dg}{dx}$$

$$\frac{d}{dx}(kf) = k\frac{df}{dx}, \quad \frac{d}{dx}(fg) = f\frac{dg}{dx} + g\frac{df}{dx}$$

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{g\frac{df}{dx} - f\frac{dg}{dx}}{g^2}$$

$$\nabla(f+g) = \nabla f + \nabla g$$

$$\nabla(kf) = k\nabla f; \quad \nabla\cdot(k\bar{A}) = k(\nabla\cdot\bar{A})$$

$$\nabla\times(kA) = k(\nabla\times A)$$

$$\nabla\cdot(A+B) = \nabla\cdot A + \nabla\cdot B$$

$$\nabla\times(A+B) = (\nabla\times A) + (\nabla\times B)$$

6 Product Rules :-

(i) $\nabla(fg) = f\nabla g + g\nabla f$

(ii) $\nabla(A \cdot B) = A \times (\nabla \times B) + B \times (\nabla \times A)$

$+ (A \cdot \nabla)B + (B \cdot \nabla)A$

$\underbrace{\left(\partial_t + (v \cdot \nabla)\right)}_{D} v$

$\partial_t v + (v \cdot \nabla) v =$

f, g → Scalar function

vector

$\nabla T \quad \nabla V$

(iii) $\nabla \cdot (fA) = f(\nabla \cdot A) + A \cdot (\nabla f)$

$$\nabla (f/g) = \frac{g\nabla f - f\nabla g}{g^2}$$

(iv) $\nabla \cdot (A \times B) = B \cdot (\nabla \times A) - A \cdot (\nabla \times B)$

$$\nabla \cdot \left(\frac{A}{g}\right) = \frac{g(\nabla \cdot A) - A \cdot (\nabla g)}{g^2}$$

(v) $\nabla \times (fA) = f(\nabla \times A) - A \times (\nabla f)$

(vi) $\nabla \times (A \times B) = (B \cdot \nabla) A - (A \cdot \nabla) B + A(\nabla \cdot B) - B(\nabla \cdot A)$

$$\nabla \times (A/g) = \frac{g(\nabla \times A) + A \times (\nabla g)}{g^2}$$

Second Derivatives

1 ~~2~~ ∇ ∇

3 Gradient of Divergence $\cancel{\nabla T}$ $(\nabla \cdot V)$

4 Diversence of Curl

5 curl of a curl $(\nabla \times V)$

1 Divergence of a gradient. $\nabla \cdot (\nabla T)$

2 Curl of a gradient $\nabla \times (\nabla T)$

1. Divergence of a gradient

$$\nabla \cdot (\nabla T) = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$$

$\Rightarrow$ Laplacian $\cancel{\Delta = \nabla^2}$

Δ or ∇^2

$f \longleftrightarrow t$

$x \longleftrightarrow k$

(2) Curl of a gradient

$$\nabla \times (\nabla T) = 0 \Leftrightarrow$$

$$\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\partial T}{\partial x}\right)$$

Equality of cross derivatives

(3) $\nabla(\nabla\cdot V)$ gradient of the divergence.

(4) Divergence of a curl

$$\nabla\cdot(\nabla\times V)=0$$

$$\cancel{(A\cdot(B\times C)=(A\times B)\cdot C)}$$

$$\frac{\partial}{\partial x}$$

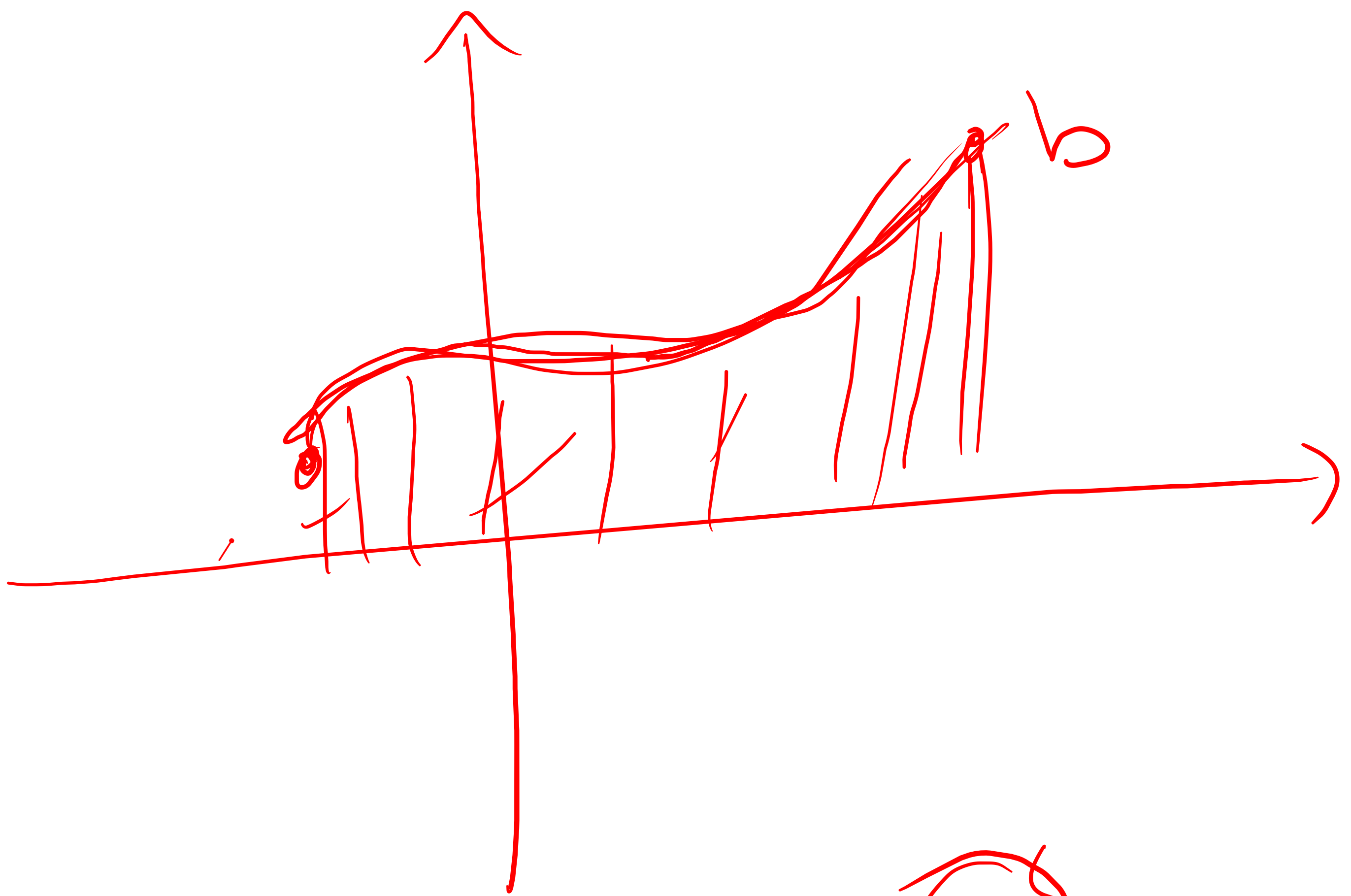
⑤ $\nabla \times (\nabla \times V) = \nabla(\nabla \cdot V) - \underline{\underline{\nabla^2}} V$

Integrals

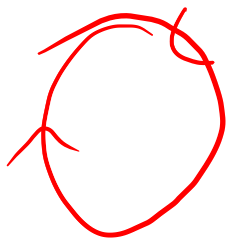
① line Integral — path integral

② Surface Integral

③ Volume Integral.



$$\int_a^b V\cdot dl = V(b) - V(a)$$



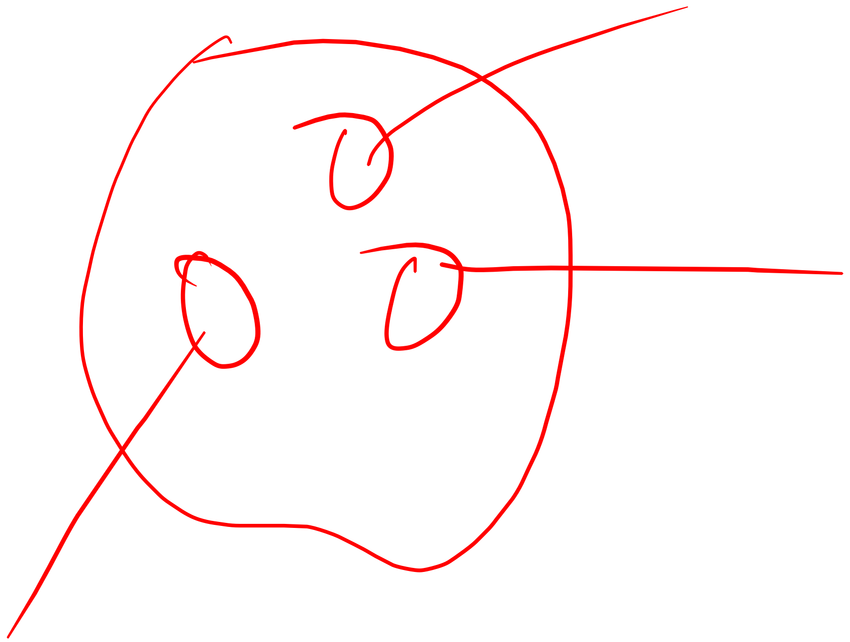
Closed path

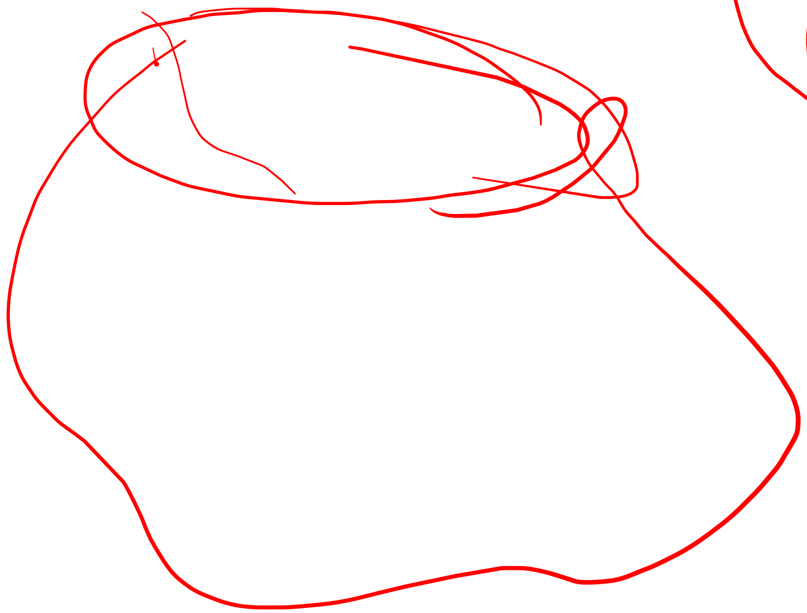
$$\oint V\cdot dl \rightarrow$$

vector is conservative

② $\int_S v \cdot da$

$\oint v \cdot da$





③ $$\int_{\mathcal{V}} T\, d\tau$$

$d\tau = dx\,dy\,dz$

Fundamental Theorem of Calculus

$$\int_a^b \frac{df}{dx}\,dx = f(b) - f(a)$$

$$\int_a^b (\nabla T)\cdot d\mathbf{l} = T(b) - T(a)$$