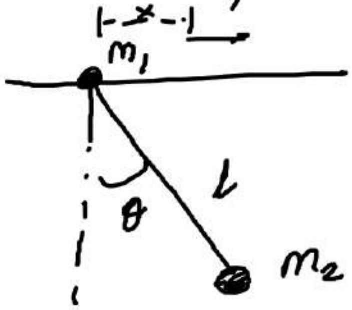
Examples of E-L eqns.



Eqns. of motion ?

$L = ?$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$$

For $m_1 \longrightarrow$ KE : $\frac{1}{2} m_1 \dot{x}^2$

For m_2 — x coord : $x + l\sin\theta$

y coord : $-l\cos\theta$

$$\frac{1}{2} m_2 \left((\dot{x} + l\cos\theta\,\dot{\theta})^2 + l^2 \sin^2\theta\,\dot{\theta}^2 \right)$$

Find the E-L eqns (a) of Assignment.

$$L = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 \Big(\dot{x}^2 + l^2\cos^2\theta\,\dot{\theta}^2 + 2\dot{x}\dot{\theta}\,l\cos\theta + l^2\sin^2\theta\,\dot{\theta}^2 \Big) - (-mgl\cos\theta)$$

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(b) L, E-L eqns.

Tensors

Tensor of rank 0

A is a no. — a scalar — no direction

$\vec{A}$ it is a vector — ~~mag.~~ mag. & direction.

A_i $\quad$ A_{ij} — Tensor of rank 2

1st rank tensor

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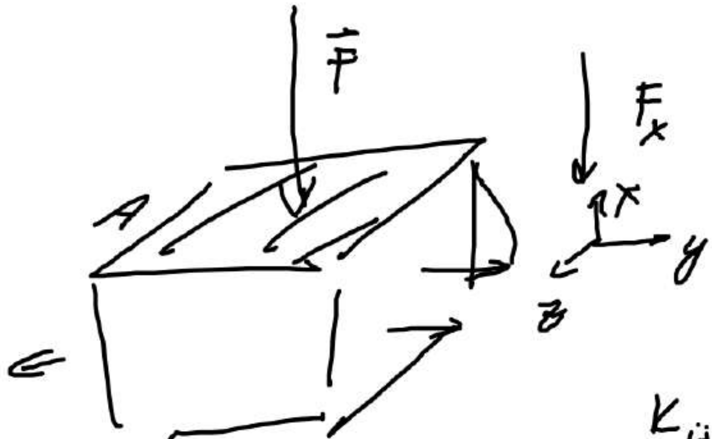

Stress tensor

σ_{ij}

σ_{xy}

K_{ij}

A_{ij}

$$\vec{C} \equiv \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$$

$$\vec{P} \equiv \left(\vec{A} \times (\vec{B} \times \vec{C})\right) \times \vec{G} = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}.$$

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$\delta_{ij} = 0$ if $i \neq j$

$\quad\;\; = 1$ if $i = j$

Kronecker delta fn.

$\vec{A} \cdot \vec{B} = A_1 B_1 + A_2 B_2 + A_3 B_3$ (3D)

$A_i B_i \qquad A_i B_j \delta_{ij} = A_i B_i = A_j B_j$

Einstein summation convention

If an index is repeated it implies summation.

$$\vec{A} \cdot \vec{B} \quad A_i B_i = \sum_{i=1}^{3} A_i B_i = A_1 B_1 + A_2 B_2 + A_3 B_3$$

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$$\vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$$

$$= C_1 \hat{i} + C_2 \hat{j} + C_3 \hat{k}$$

$x \rightarrow y \rightarrow z$

$1 \rightarrow 2 \rightarrow 3$

$\epsilon_{123} = 1$

$\epsilon_{213} = -1$

$C_1 = A_2 B_3 - A_3 B_2$

$C_1 = A_2 B_3 - A_3 B_2$

$C_i = \epsilon_{ijk} A_j B_k$

For $i = 1$

$C_1 = \epsilon_{1jk} A_j B_k \quad \left(= \sum_{j,k} \epsilon_{1jk} A_j B_k \right)$

$C_1 = \sum_k \left(\cancel{\epsilon_{11k} A_1 B_k} + \epsilon_{12k} A_2 B_k + \epsilon_{13k} A_3 B_k \right)$

0

$= \epsilon_{123} A_2 B_3 + \cancel{\epsilon_{121} A_2 B_1} + \cancel{\epsilon_{122} A_2 B_2} + \cancel{\epsilon_{131} A_3 B_1} + \epsilon_{132} A_3 B_2 + \cancel{\epsilon_{133} A_3 B_3}$

Levi-Civita antisymmetric tensor

$$\epsilon_{ijk} = \begin{cases} 1 & \text{if } i,j,k \text{ are symmetric } \text{~~cyclic~~} \\ -1 & \text{if } i,j,k \text{ anti-symmetric} \\ 0 & \text{if any 2 indices are repeated} \end{cases}$$

$\rightarrow \quad \epsilon_{ijk}\,\epsilon_{klm} = \delta_{il}\,\delta_{jm} - \delta_{im}\,\delta_{jl}$

Any index that is summed up — is a "dummy variable"

$\delta_{ij}\,, \quad \epsilon_{ijk}. \quad \vec{D} = \vec{A} \times (\vec{B} \times \vec{C}) \quad \vec{G}$

$\vec{D} = \vec{A} \times \vec{G}$

$D_i = \epsilon_{ijk}\, A_j\, \boxed{\epsilon_{klm}\, B_l\, C_m}$

$= \epsilon_{ijk}\, A_j\, G_k$

$\vec{U} = \vec{V} \times \vec{W}$

$U_p = \epsilon_{plm}\, V_l\, W_m$

$\vec{B} = \vec{A} \times (\vec{B} \times \vec{C})$

$G_k = \epsilon_{klm}\, B_l\, C_m$

$D_i = \epsilon_{ijk}\, A_j\, \epsilon_{klm}\, B_l\, C_m$

$= \epsilon_{ijk}\, \epsilon_{klm}\, A_j\, B_l\, C_m$

$= (\delta_{il}\,\delta_{jm} - \delta_{im}\,\delta_{jl})\, A_j\, B_l\, C_m$

$= A_m\, B_i\, C_m - A_j\, B_j\, C_i$

$\vec{D} = (\vec{A} \cdot \vec{C})\,\vec{B} - (\vec{A} \cdot \vec{B})\,\vec{C}$

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$D_{jkl} \equiv A_{ij} B_{kli}$

$ds^2 = dx^2 + dy^2 + dz^2$

$ds = \sqrt{(\qquad)}$

$ds^2 = \underbrace{g_{ij}}_{\text{metric tensor}} dx^i dx^j$

$dx_i \, dx_j$

$\cancel{g_{ij}} \; \underline{\underline{g}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $g \; \delta$

$\vec{A} \quad \underline{\underline{A}}$

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$$\vec{E} = \left(\vec{A} \times (\vec{B} \times \vec{C})\right) \times \vec{D}$$

$$E_i = \epsilon_{ijk} \left[\epsilon_{jlm} A_l \epsilon_{mpq} B_p C_q\right] D_k$$

$$\vec{F} = \vec{A} \times (\vec{B} \times \vec{C})$$

$$\vec{E} = \vec{F} \times \vec{D}$$

$$E_i = \epsilon_{ijk} F_j D_k$$

$$= \epsilon_{ijk} \epsilon_{jlm} A_l \epsilon_{mpq} B_p C_q D_k$$

$$F_j = \left(\vec{A} \times (\vec{B} \times \vec{C})\right)_j$$

$$= \epsilon_{jlm} A_l \epsilon_m B_p C_q$$

$$= \epsilon_{ijk} \epsilon_{jlm} \epsilon_{mpq} A_l B_p C_q D_k$$

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$$E_i = \underbrace{\epsilon_{ijk}\ \epsilon_{jlm}}\ \epsilon_{mpq} A_l B_p C_q D_k$$

$$= (\epsilon_{kij}\ \epsilon_{jlm})\ \epsilon_{mpq} A_l B_p C_q D_k$$

$$= (\delta_{kl}\delta_{im} - \delta_{km}\delta_{il})\ \epsilon_{mpq} A_l B_p C_q D_k$$

$$= \epsilon_{ipq} A_k B_p C_q D_k - \epsilon_{kpq} A_i B_p C_q D_k$$

$$= (\epsilon_{ipq} B_p C_q) A_k D_k - (\epsilon_{kpq} B_p C_q) D_k A_i$$

$$= (\vec{B} \times \vec{C})(\vec{A} \cdot \vec{D}) - ((\vec{B} \times \vec{C}) \cdot \vec{D})\vec{A}$$

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29 Oct Tue Quiz 1

19 Nov Tue. Quiz 2

Assignment : due next week

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