IIIT-Bangalore
Course: Probability and Statistics
Problem Set 3

(Compound Experiment)

1. The probability of hitting a target is $\frac{1}{5}$. If 10 shots are fired, find the probability of at least two hits. Find also the minimum number of shots to be fired such that the probability of hitting the target (at least once) exceeds $\frac{1}{2}$. (Ans. $\left[\frac{\log 2}{\log 5 - \log 4}\right] + 1$)

2. When a defective die is thrown 10 times the probability that an even face occurs 5 times is twice the same event occurs 4 times. Find the probability that an even face never occurs in four throws of the same die. (Ans. $(\frac{3}{8})^4$)

3. If a die is thrown n times show that the problem of an even number of sixes is $\frac{1}{2}\left\{1 + (\frac{2}{3})^n\right\}$.

4. Show that the most probable number of heads in $2n$ throws of a coin is n and that the corresponding maximum probability lies between $\frac{1}{2\sqrt{n}}$ and $\frac{1}{\sqrt{2n+1}}$.

5. A class has only 3 students A, B, C who attend the class independently. The probabilities of their attendance in any day being $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$. respectively. Find the probability that the total number of attendances in 2 consecutive days is exactly 3. (Ans. $\frac{1}{4}$)

6. If a die is thrown n times, find the probability that (i) the greatest and (ii) the least number obtained will have a given number r. (Ans. (i) $\frac{r^n - (r-1)^n}{6^n}$, (ii) $\frac{(7-r)^n - (6-r)^n}{6^n}$)

7. What is the probability that in a company of 500 people only one person will have his birth day on a new year day? (Ans. $\frac{100}{73}e^{\frac{100}{73}}$, approx.)

8. A card is drawn from a pack 260 times (with replacement each time). Find the probability of Queen of Hearts 4 times. (Ans. $\frac{5^4 e^{-5}}{4!}$, approx.)

9. Three coins having probability of heads $\frac{1}{2}$, $\frac{2}{5}$, $\frac{3}{7}$ respectively are thrown. Find the probability of obtaining exactly one head. (Ans. $\frac{32}{70}$)

10. What is the probability that the faces 1, 3, 5 turn up 2, 3, 3 times in 8 throws of a die? (Ans. $\frac{8!}{2!3!3!}(\frac{1}{6})^8$)