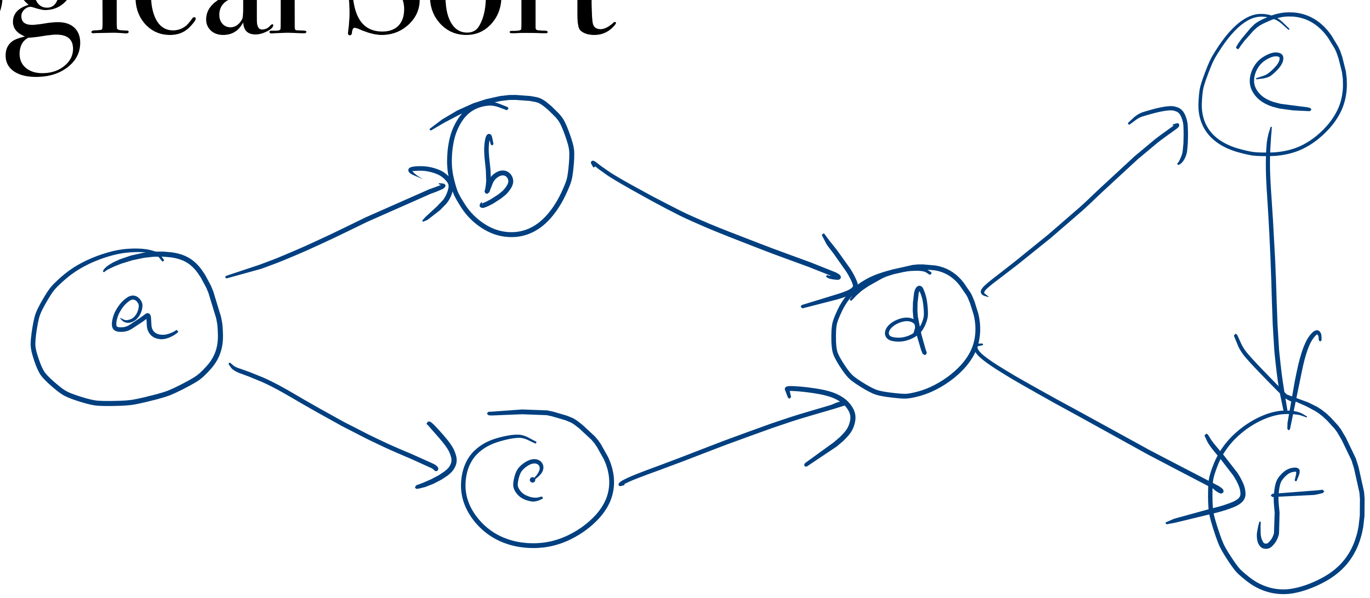
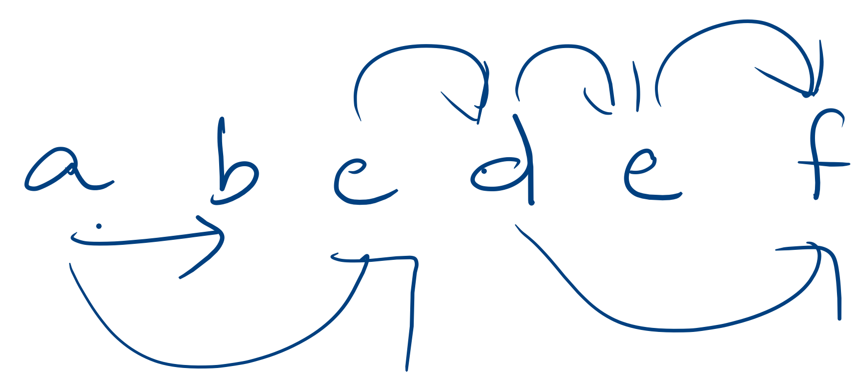
Applications of BFS and DFS

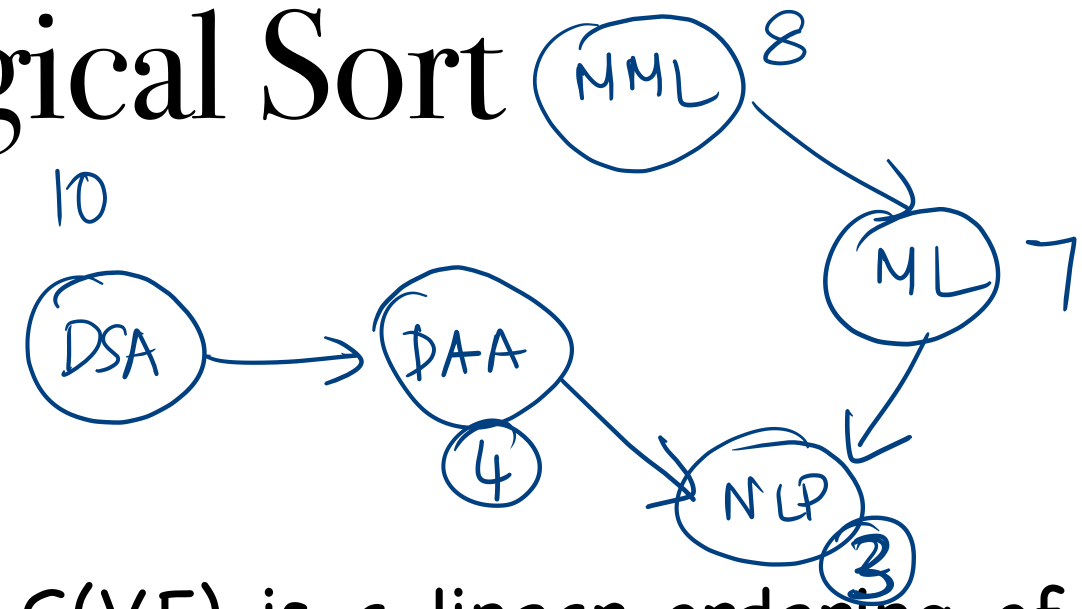
Topological Sort



A topological sort of a DAG G(V,E) is a linear ordering of vertices of G such that if G contains an edge (u,v) then u appears before v in the ordering.



Topological Sort



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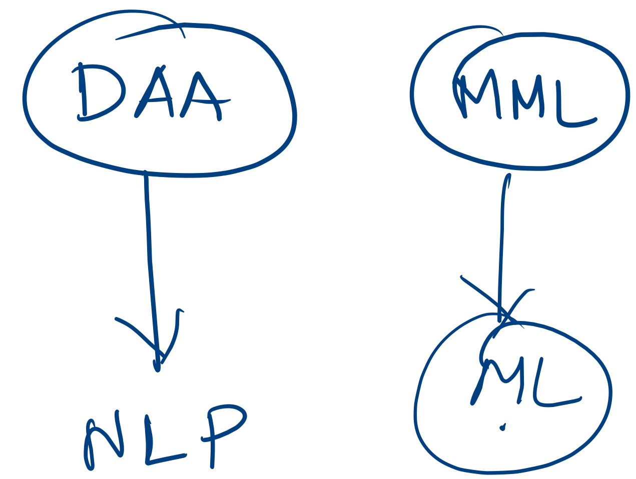


- used to show precedence among events

DSA – MML – ML – DAA – NLP

Topological Sort

$u \rightarrow v$

$\Longleftrightarrow f[u] > f[v]$

- call DFS(G) to compute f[v] for each v
- As each vertex is finished, insert it to the front of a linked list
- return the linked list

Topological Sort

A directed graph G is acyclic if an only if a DFS of G yields no back edges

$\Leftarrow$ Let (u,v) be a back edge. Path from ~~u~~ v to ~~u~~ u in DFS tree with (u,v) forms a cycle.

$\Rightarrow$ $\exists$ C $(v_1 - v_2 - \cdots v_k - v_1)$. Let v_1 be the first vertex in C to be discovered. At $d[v_1]$, $v_2 \ldots v_k$ are all white

$\exists$ $v_1 \rightsquigarrow v_k$ white path $\Rightarrow$ v_k is a descendant of v_1.

$\Rightarrow$ (v_k, v_1) is a back edge.

Topological Sort

Algorithm is correct.

If $(u,v) \in E$, then $f(u) > f(v)$.

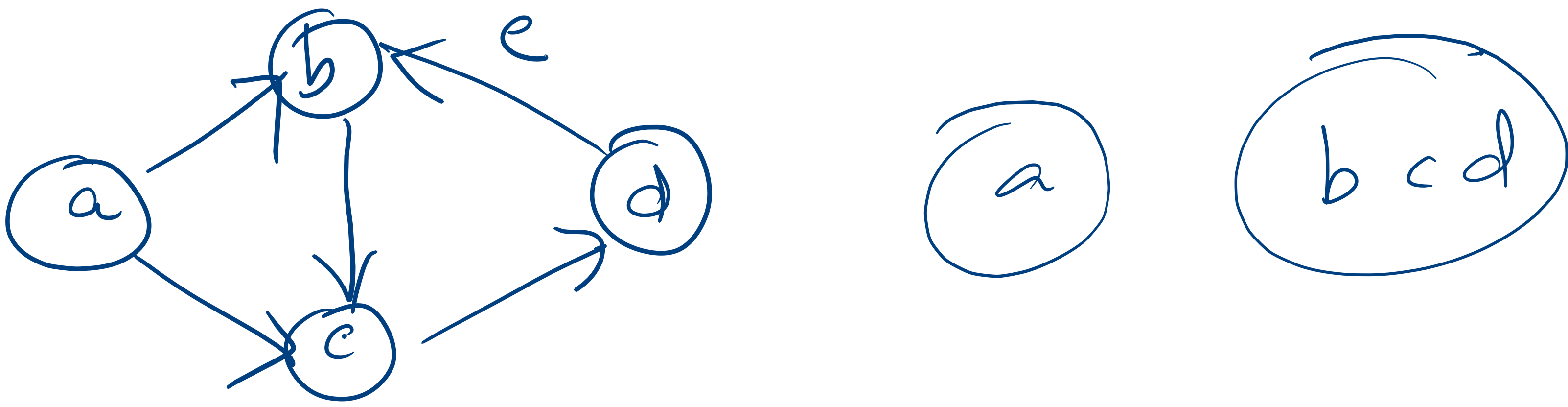
when the edge (u,v) is explored,

Case 1: v is white $\Rightarrow$ v is a descendant of $u \Rightarrow f(u) > f(v)$

Case 2: " gray $\Rightarrow (u,v)$ is a back edge $\rightarrow$ not possible

3: " black $\Rightarrow f(v) < f(u)$

Strongly Connected Components



- Strongly connected component of a directed graph $G(V, E)$ is a maximal set of vertices $C \subseteq V$ such that for every pair of vertices u and v in C, u and v are reachable from each other.

Call DFS(G) to compute $f[u]$ for each vertex u

Compute G^T

call DFS(G^T), consider the vertices in order of decreasing $f[u]$

Output the vertices in each tree in the DFS forest (formed in the previous step) as a separate strongly connected component

Let C, C' be distinct strongly connected components in directed graph $G(V, E)$.

Let $u, v \in C, u', v' \in C'$. Suppose there is a path from u to u'. Then there cannot also be a path from v to v'.

Let C, C' be distinct strongly connected components in directed graph $G(V, E)$.

- Suppose $(u, v) \in E$ such that $u \in C$ and $v \in C'$. Then $f(C) > f(C')$.

Let C, C' be distinct strongly connected components in directed graph $G(V, E)$.

- Suppose $(u, v) \in E$ such that $u \in C$ and $v \in C'$. Then $f(C) > f(C')$.
- Suppose there exists $(u, v) \in E^T, u \in C, v \in C'$. Then $f(C) < f(C')$.