

**Mathematics-3**  
**Tutorial-5**  
**Discussion on Friday, 20-09-2024**  
**Topic: Two Random Variables**

**Problem 1**

Let  $X$  and  $Y$  be jointly continuous random variables with joint PDF

$$f_{X,Y}(x,y) = \begin{cases} cx + 1 & x, y \geq 0, x + y < 1 \\ 0 & \text{otherwise} \end{cases}$$

1. Show the range of  $(X, Y)$ ,  $R_{XY}$ , in the  $x - y$  plane.
2. Find the constant  $c$ .
3. Find the marginal PDFs  $f_X(x)$  and  $f_Y(y)$ .
4. Find  $P(Y < 2X^2)$ .

**Problem 2**

Let  $X$  and  $Y$  be jointly continuous random variables with joint PDF

$$f_{X,Y}(x,y) = \begin{cases} 6e^{-(2x+3y)} & x, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

1. Are  $X$  and  $Y$  independent?
2. Find  $E[Y|X > 2]$ .
3. Find  $P(X > Y)$ .

**Problem 3**

Let  $X$  be a continuous random variable with PDF

$$f_X(x) = \begin{cases} 2x & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

We know that given  $X = x$ , the random variable  $Y$  is uniformly distributed on  $[-x, x]$ .

1. Find the joint PDF  $f_{XY}(x, y)$ .
2. Find  $f_Y(y)$ .
3. Find  $P(|Y| < X^3)$ .

**Problem 4**

Let  $X$  and  $Y$  be two jointly continuous random variables with joint PDF

$$f_{X,Y}(x, y) = \begin{cases} 6xy & 0 \leq x \leq 1, 0 \leq y \leq \sqrt{x} \\ 0 & \text{otherwise} \end{cases}$$

1. Show  $R_{XY}$  in the  $x - y$  plane.
2. Find  $f_X(x)$  and  $f_Y(y)$ .
3. Are  $X$  and  $Y$  independent?
4. Find the conditional PDF of  $X$  given  $Y = y$ ,  $f_{X|Y}(x|y)$ .
5. Find  $E[X|Y = y]$ , for  $0 \leq y \leq 1$ .
6. Find  $\text{Var}(X|Y = y)$ , for  $0 \leq y \leq 1$ .

**Problem 5**

The joint cumulative distribution function of two random variables  $X$  and  $Y$  is given by

$$F(x, y) = 1 - e^{-x} - e^{-y} + e^{-(x+y)}, \quad x > 0, \quad y > 0.$$

Find the joint probability density function of  $X$  and  $Y$ .

**Problem 6**

$X$  and  $Y$  are two R.V. with joint PDF  $f(x, y) = \frac{1}{8}(6 - x - y)$ ,  $0 < x < 2$ ,  $2 < y < 4$

Find (i)  $P(x < 1, y < 3)$  (ii)  $P(x+y < 3)$ , (iii)  $P(x < 1 \mid y < 3)$

### Problem 7

Two R.V.  $x$  and  $y$  have the joint pdf

$$f(x, y) = \begin{cases} 2 - x - y & 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Show that the correlation coefficient  $\rho(x, y) = -1/11$

### Problem 8

The independent random variables  $X$  and  $Y$  are defined by

$$f(x) = \begin{cases} 4ax & 0 \leq x \leq r \\ 0 & \text{otherwise} \end{cases} \quad f(y) = \begin{cases} 4by & 0 \leq y \leq s \\ 0 & \text{otherwise} \end{cases} \quad \text{find the correlation co-efficient}$$

between  $U = x + y$  and  $V = x - y$ .