Interval Estimation

$$\boxed{P(\hat{\theta}-\epsilon<\theta<\hat{\theta}+\epsilon)}$$

Point Estimation: Estimating an unknown population parameter θ by a statistic $\hat{\theta}(x_1, x_2, \dots, x_n)$ $\quad P(\theta-\epsilon<\hat{\theta}<\theta+\epsilon)\approx 1, \forall\theta\in\Theta$

Disadvantage: Gives no information regarding the actual error committed in an estimation.

$P(A<\theta<B)=1-\epsilon$, $0<\epsilon<1$

Interval Estimation: Choose two estimators A an B simultaneously for the unknown parameter θ so that interval (a, b) contains θ with certain probability, where a, b are the corresponding estimates.

confidence coeff.

Confidence Interval : θ : unknown population parameter

$\theta \in \Theta$ (set of admissible values)

We know the functional form of the distribution function of the population r.v. X, i.e. $F_X(x)$ but it contains the unknown population parameter θ.

Let, $(x_1, x_2, \ldots, x_n)$ be a random sample of size n and $0 < \epsilon < 1$ be a given no. Then if there exists two statistics $a = a(x_1, x_2, \ldots, x_n)$ and $b = b(x_1, x_2, \ldots, x_n)$ s.t. $P(A < \theta < B) = 1 - \epsilon$

where A, B are the r.v.^{s} corr. to a and then the interval (a, b) is called a confidence interval of confidence coefficient $(1-\epsilon)$.

<u>Note</u>: 1. (a, b) is also called $(1-\epsilon)\times 100\%$ confidence interval.

2. a, b are called respectively the lower and upper limit of C.I. of θ.

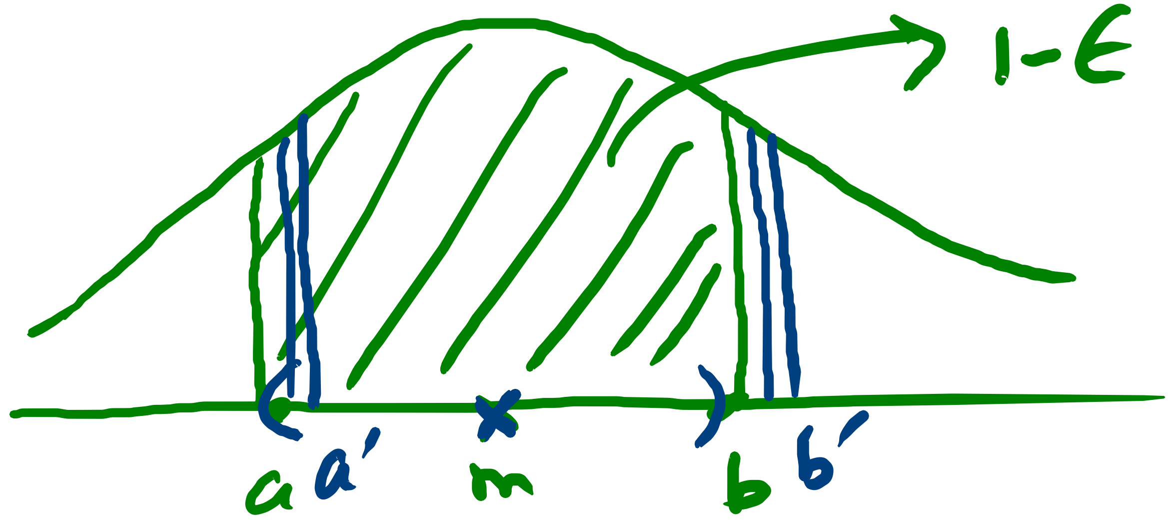
$$P(A < \theta < B) = \underbrace{1-\epsilon}_{\text{fixed}}$$

If $\epsilon = 0.05$

$1-\epsilon = 0.95$

$100 \times 0.95\,\% = 95\%$

C.I.



$$\boxed{P(\underbrace{A<m<B}) = 1-\epsilon}$$

Note: (3) There may be infinite number of confidence intervals with confidence coefficient $1-\epsilon$.

(4) If (a_1, b_1) and (a_2, b_2) be two C.I.s for θ with confidence coefficient $1-\epsilon$ then we choose the one with minimum length, i.e. we choose (a_1, b_1) if $b_1 - a_1 < b_2 - a_2$.

Frequency Interpretation of Confidence Intervals:

$$\boxed{P(A < \theta < B) = 1-\epsilon}$$

Let $(x_1^{(i)}, x_2^{(i)}, \ldots, x_n^{(i)})$ be a random sample of size n

$i = 1, 2, \ldots, N$

Let, $(a_1, b_1), (a_2, b_2), \ldots, (a_N, b_N)$ be the corresponding confidence intervals.

If N be large then θ will belong to approximately $(1-\epsilon)N$ of the computed intervals.

i.e. (a, b) will include θ approx.ly $(1-\epsilon)N$ times out of N times.

Method of Computing C.I. for an Unknown Population Parameter

To find a confidence interval of an unknown population parameter θ with confidence coeff. $1-\epsilon$.

Let, $(x_1, x_2, \ldots, x_n)$ be a random sample of size n.

I. Choose a statistic $t = t(x_1, x_2, \ldots, x_n)$ such that it contains $\boxed{\theta}$ explicitly (in its expression) and it does not contain any other unknown population parameter $\theta_1, \theta_2, \ldots, \theta_k$

II. Sampling distribution of t is known and independent of all parameters $\theta, \theta_1, \theta_2, \ldots, \theta_k$.

$$U = \frac{\bar{X} - m}{\sigma/\sqrt{n}} \sim N(0,1)$$

Let, f_T be the p.d.f. of T (corr. to t).

Now we choose $u = u(\epsilon)$ and $v = v(\epsilon)$ s.t.

(from Statistical Tables for the distribution of T)

$$P(u < T < v) = 1-\epsilon \iff \int_u^v f_T(t)\,dt = 1-\epsilon$$

$$T \downarrow (\theta)$$

(If possible)

$$P(A < \theta < B) = 1-\epsilon$$

then $(a, b) \longrightarrow$ C.I. with confidence coeff. $(1-\epsilon)$.

① Find $100 \times (1-\epsilon)\%$ confindence interval for the parameter m of a normal (m, σ) population.

$\epsilon = 0.05$

$1-\epsilon = 0.95$

$95\% \to$ C.I.

Sol. Case I: σ is known

Choose statistic $U = \dfrac{\bar{x}-m}{\sigma/\sqrt{n}}$ whose sampling distribution is $N(0,1)$. Choose points $\pm u_\epsilon$ (from the table of standard normal distribution) symmetrically about the orgin s.t.

$$P(-u_\epsilon < U < u_\epsilon) = 1-\epsilon$$

$$\Rightarrow P\left(-u_\epsilon < \frac{\bar{X}-m}{\sigma/\sqrt{n}} < u_\epsilon\right) = 1-\epsilon$$

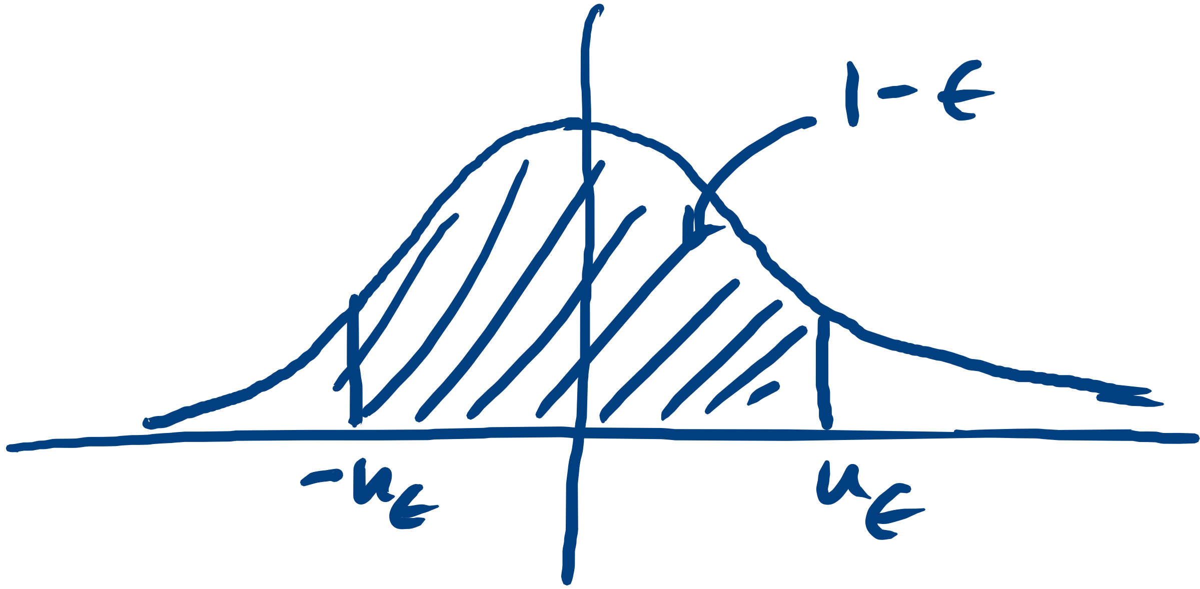
$$\Rightarrow P\left(\bar{X} - \frac{\sigma u_\epsilon}{\sqrt{n}} < m < \bar{X} + \frac{\sigma u_\epsilon}{\sqrt{n}}\right) = 1-\epsilon \,,$$

The C.I. for m is

$$\left(\bar{x} - \frac{\sigma u_\epsilon}{\sqrt{n}},\ \bar{x} + \frac{\sigma u_\epsilon}{\sqrt{n}}\right)$$ when u_ϵ is given

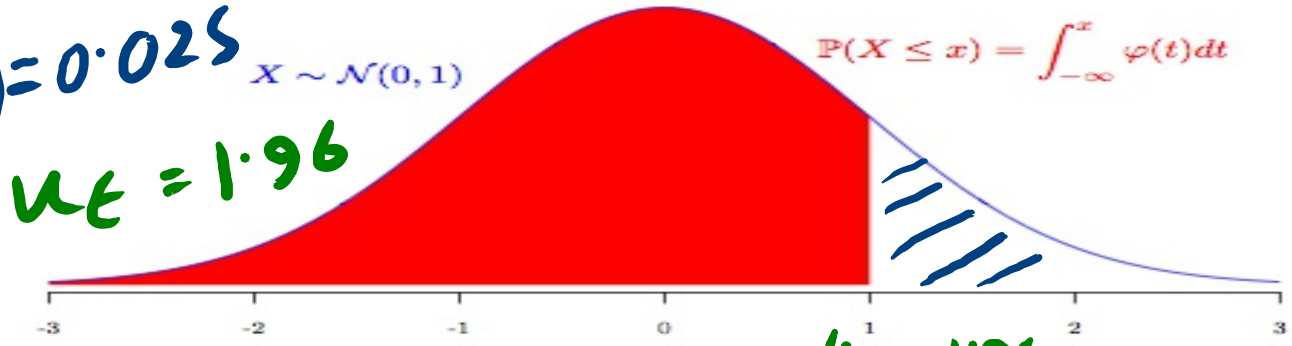
by $$P(|U| < u_\epsilon) = 1-\epsilon$$

$$\Rightarrow P(U > u_\epsilon) = \frac{\epsilon}{2}$$



$P(U > u_t) = 0.025$

$u_t = 1.96$



$1 - 0.025 = 0.975$

$u_t = 1.96$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990