$\nabla \times E = 0$ $\quad\quad E = -\nabla V$ (scalar potential)

Reference Point

$$V(r) = -\int_0^r E \cdot dl$$

a b

$$V(b) - V(a) = -\int_0^b E \cdot dl - \left(-\int_0^a E \cdot dl\right)$$

$$= \int_0^a E \cdot dl - \int_0^b E \cdot dl$$

We always bring from ∞.

$E = -\nabla V$

$\nabla \cdot E = \frac{\rho}{\epsilon_0}$ $\quad\quad \boxed{\nabla^2 V = -\frac{\rho}{\epsilon_0}}$

↳ Poisson Eq^{n}

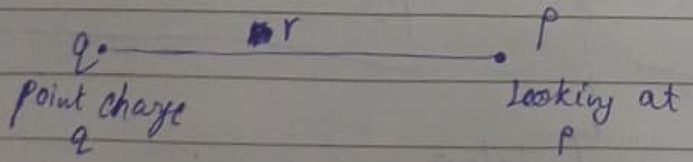
If RHS $= 0$

$\boxed{-\nabla^2 V = 0}$

↳ This is Laplacian

Potential of a Localised Charged Distribution

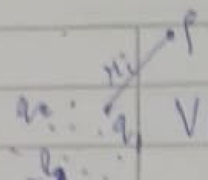
Reference point $\to \infty$



$$V(r) = \frac{-1}{4\pi\epsilon_0} \int_\infty^r \frac{q}{r'^2}\, dr'$$

We are just using another set of variables, that's not a diff. coordinate system.

$$= \frac{1}{4\pi\epsilon_0} \left.\frac{q}{r'}\right|_\infty^r = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$



$V(r) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i}$

If there are n charges

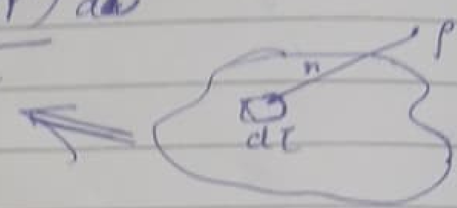
r_i ↓ actual dist b/w q_i & P

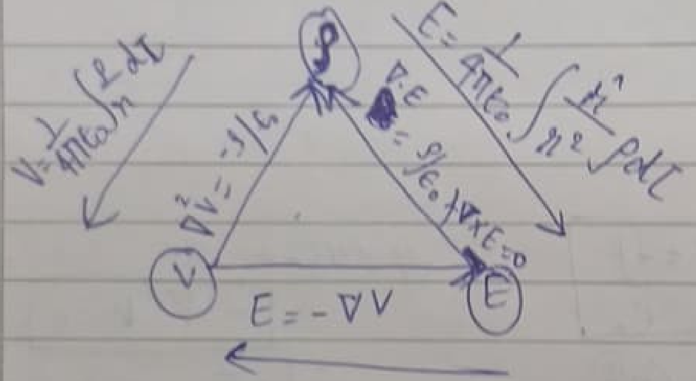
$V(r) = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$

⇓

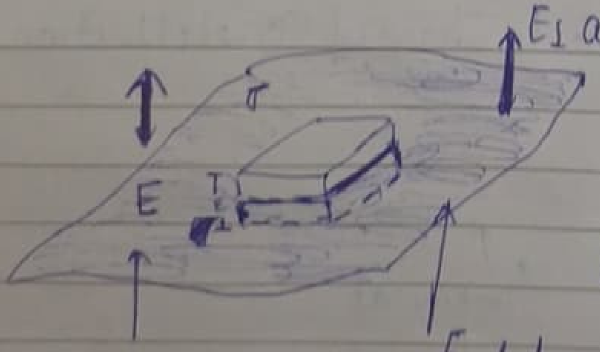
For Continuous Distribution

For vol. charge $V(r) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(r')\, d\tau}{r}$

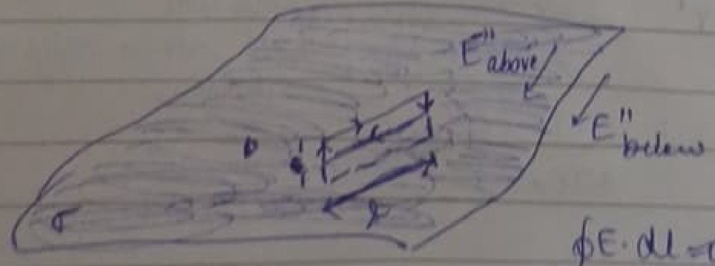




$V = -\int E \cdot dl$ Differential Eq^n.



$$\boxed{\oint E \cdot da = \frac{1}{\epsilon_0} q_{enc} = \frac{\sigma A}{\epsilon_0}}$$



E'' got cancelled

Only thing that remains is the $E^\perp$

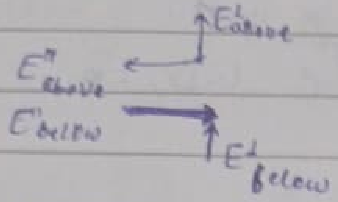
$\oint E \cdot dl = 0$

$E''_{above} = E''_{below}$

$$\boxed{E^{\perp}_{above} - E^{\perp}_{below} = \frac{\sigma}{\epsilon_0}\hat{n}}$$

→ if $\boxed{\lim \varepsilon \to 0}$ vector form

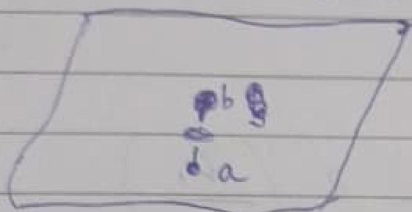
ε → Thickness of plate



$$E^{\perp}_{above} \neq E^{\perp}_{below}$$

[Since there is an uncertainty]

[The obj has a thickness ε]



$$V_{above} - V_{below} = -\int_a^b E \cdot dl = 0$$

as path length shrinks to zero, so does the integral so $V_{above} = V_{below}$

$$E = -\nabla V$$

$$\nabla V_{above} - \nabla V_{below} = -\frac{1}{\epsilon_0}\sigma\hat{n}$$

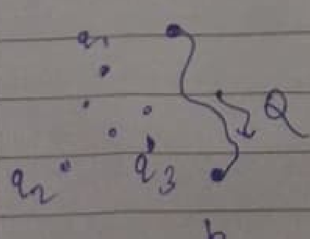
$$\frac{\partial V_{above}}{\partial n} - \frac{\partial V_{below}}{\partial n} = -\frac{1}{\epsilon_0}\sigma$$

$$\boxed{\frac{\partial V}{\partial n} = \nabla V \cdot \hat{n}}$$ → Normal Derivative

Work And Energy In Electrostatics

$$\boxed{F = QE}$$

If we do the work, we put a '−' sign bcz. we'll do the work against the E.F



$$W = \int_a^b F \cdot dl = -Q\int_a^b E \cdot dl$$

$$W = Q(V(b) - V(a)) \Rightarrow \boxed{V(b) - V(a) = \frac{W}{Q}}$$

$$\boxed{W = QV(r)}$$

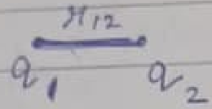
Energy of a point charge

WD to bring $q_1 = 0$

i.e $WD_1 = 0$

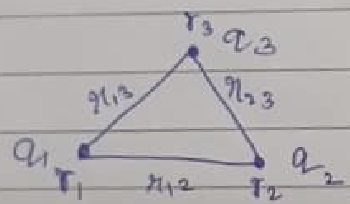
q_1

Now bring q_2



$$W_2 = \frac{1}{4\pi\epsilon_0} q_2 \left(\frac{q_1}{r_{12}}\right)$$

Now bring q_3



$$W_3 = \frac{1}{4\pi\epsilon_0} q_3 \left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right)$$

$$W = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n} \sum_{j>i}^{n} \frac{q_i q_j}{r_{ij}}$$

$$W = \frac{1}{2} \sum_{i=1}^{n} q_i \left(\sum_{j\neq i}^{n} \frac{1}{4\pi\epsilon_0} \frac{q_j}{r_{ij}} \right)$$

$$\boxed{W = \frac{1}{2} \sum_{i=1}^{n} q_i V(r_i)}$$

Continuous Charge Distribution

$$W = \frac{1}{2}\int \rho V d\tau$$

$$\rho = \epsilon_0 \nabla\cdot E \Rightarrow W = \frac{\epsilon_0}{2}\int (\nabla\cdot E) V d\tau$$

$$W = \frac{\epsilon_0}{2}\left[-\int E\cdot(\underbrace{\nabla V}_{E}) d\tau + \oint V E da\right]$$

$$W = \frac{\epsilon_0}{2}\left(+\int_V E^2 d\tau + \oint_S V E da\right)$$

for all space

$$\boxed{W = \frac{\epsilon_0}{2}\int E^2 d\tau}$$

Q What is the Energy of the point charge?

Ans ∞ (This is the inconsistency)

$$W = \frac{\epsilon_0}{2(4\pi\epsilon_0)^2}\int\left(\frac{q^2}{r^4}\right)(r^2\sin\theta\, dr\, d\theta\, d\phi) = \frac{q^2}{8\pi\epsilon_0}\int_0^\infty \frac{1}{r^2} dr = \infty$$

Difference

$$\boxed{W = \frac{1}{2}\sum_{i=1}^{n} q_i V(r_i)}, \quad \boxed{W = \frac{\epsilon_0}{2}\int E^2 d\tau}$$

→ Discrete Charges

→ Assumes the FIRST point charge came somehow & rest others came under that influence.

→ Continuous Distribution

Doesn't matter where the FIRST charge came & any other charge came, we are directly integrating over all space

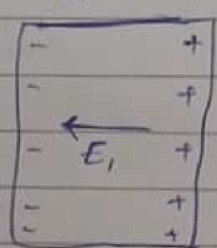
Q. Where does the energy lie in Electrostatics?

Ans: For all practical purposes, the energy is on the charge but as yo as you go into higher physics, energy is said to be on the field.

Conductors

1. $E = 0$ inside a conductor

If there were free charges inside the conductor, they wo uld sort any given field & the idea of electrostatics co

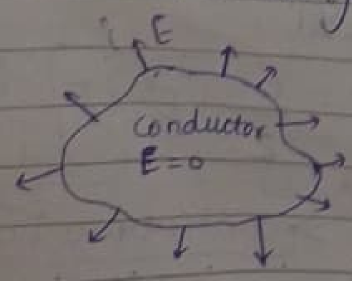


2. $\rho = 0$ inside conductor $\left(\nabla E = \frac{\rho}{\epsilon_0}\right)$ If $E=0$, then $\rho = 0$

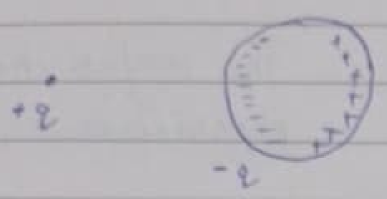
3. Any net charge is on the surface

4. Conductor is an equipotential surface

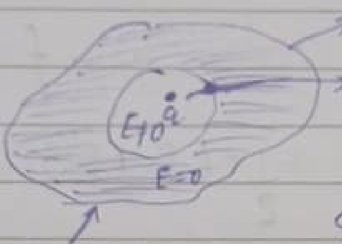
5. E is $\perp^r$ to the surface, just outside a conductor



Induced Charges



We can find the induced charge



$\oint E \cdot da = 0$ → conductor

$$q_{enc.} = q + q_{induced}$$

Since conductor ka $\oint E \cdot da = 0$

$\Rightarrow Q_{enc}$ should be zero

$\Rightarrow q_{enc} = 0 \Rightarrow q + q_{ind.} = 0$

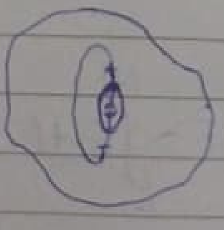
$$\Rightarrow \boxed{q_{ind.} = -q}$$

There are 3 E.Fs E_q, $E_{induced}$, $E_{leftover}$

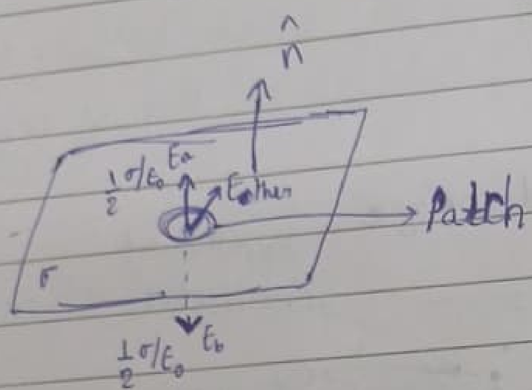
$$\boxed{E_q + E_{induced} + E_{leftover} = 0} \rightarrow \text{inside the conductor}$$

We calculated only these two

This will automatically ~~zero~~ be zero



Surface Charge & Force on a Conductor



The surface charge will experience a force (f)

f - force per unit area

$$f = \sigma E$$

$$f = \sigma E_{average} = \frac{1}{2}\sigma(E_a + E_b)$$

Total E of the surface $(E) = E_{patch} + E_{other}$

$$E_{above} = E_{other} + \frac{\sigma}{2\epsilon_0}\hat{n}$$

$$E_{below} = E_{other} - \frac{\sigma}{2\epsilon_0}\hat{n}$$

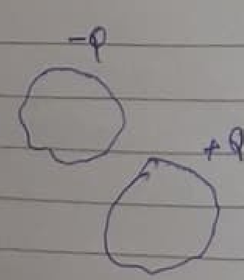
$$E_{other} = \frac{1}{2}(E_a + E_b) = E_{avg}.$$

Net force per unit area $= \sigma E_{avg} = \sigma \cdot \frac{\sigma}{2\epsilon_0} = \frac{1}{2}\frac{\sigma^2}{\epsilon_0}\hat{n}$

$$\boxed{f = \frac{\epsilon_0}{2}E^2}$$

Capacitors

$$V = V_+ - V_- = -\int_{(-)}^{(+)} E \cdot dl$$



$$E = \frac{1}{4\pi\epsilon_0}\int \frac{\rho}{r^2}\hat{r}\, d\tau$$

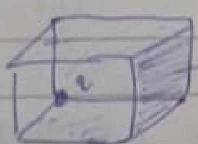
$E \propto Q$

$V \propto Q$ $\boxed{C = \frac{Q}{V}}$

$$dW = \left(\frac{q}{c}\right) dq$$

$$W = \int_0^Q \left(\frac{q}{c}\right) dq \Rightarrow \boxed{W = \frac{Q^2}{2c}}$$

Q. A charge q ~~sitting~~ sits on the back corner of cube



flux here

A corner is shared by 8 cubes

So for this cube $\phi = \frac{q}{8\epsilon_0}$

Now in this cube, only 3 surfaces will have flux (the light shades surfaces will have 0 ∵ EF will be ∥el to those surface)

$\Rightarrow \boxed{\phi = \frac{q}{24\epsilon_0}}$ for required surface

Q. $E(r) = \frac{A\hat{r} + B\sin\theta\cos\phi\,\hat{\phi}}{r}$ $\rho = ?$ where A & B are constants

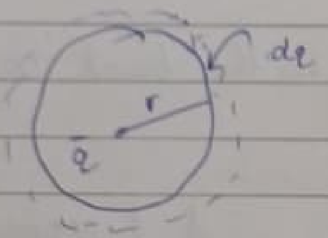
$$\rho = \epsilon_0 (\nabla \cdot E) = \epsilon_0 \left\{ \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{A}{r}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\left(\frac{B\sin\theta\cos\phi}{r}\right) \right.$$

$$= \frac{\epsilon_0}{r^2}(A - B\sin\phi)$$

q. Energy of uniformly charged sphere

Assemble the sphere layer by layer. Each time bringing an infinitesimal charge dq & smearing it uniformly across over the surface. ↑ing the radius.

(i) WD - ? (radius - dr)
(ii) Total work - ? (radius - R)



$$dw = d\bar{q}\, V$$
$$= d\bar{q}\left(\frac{1}{4\pi\epsilon_0}\right)\frac{\bar{q}}{r}$$

$$\bar{q} = \frac{4\pi r^3}{3}\rho = \frac{q r^3}{R^3}$$ (q is total charge on the sphere)

$$d\bar{q} = 4\pi r^2 dr\, \rho$$
$$= \frac{4\pi r^2}{\frac{4}{3}\pi R^3} q\, dr = \frac{3q}{R^3} r^2 dr$$

$$\Rightarrow dw = \frac{3q}{R^3} r^2 dr \left(\frac{1}{4\pi\epsilon_0}\right)\cdot\frac{1}{r}\cdot\frac{q r^3}{R^3}$$
$$= \frac{1}{4\pi\epsilon_0}\frac{3q^2}{R^6}(r^4 dr) \Rightarrow \frac{3q^2 R^5}{20\pi\epsilon_0 R^6} = \frac{3}{5}\frac{kq^2}{R}$$

q.

$$F = \frac{-Gm_1 m_2}{r^2}\hat{r} \quad , \quad F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$$

$$\frac{1}{4\pi\epsilon_0} \longrightarrow G \qquad q \rightarrow m$$

$$W_{grav.} = \left(\frac{3}{5}\underbrace{\left(\frac{1}{4\pi\epsilon_0}\right)}_{G}\right)\left(\frac{\cancel{q^2}\, m^2}{R}\right) = \frac{3}{5}\frac{GM^2}{R}$$

$G = 6.67\times10^{-11}\ Nm^2/kg^2$ $\quad m_{sun} = 1.99\times10^{30}\ kg$
$R = 6.96\times10^{8}\ m$

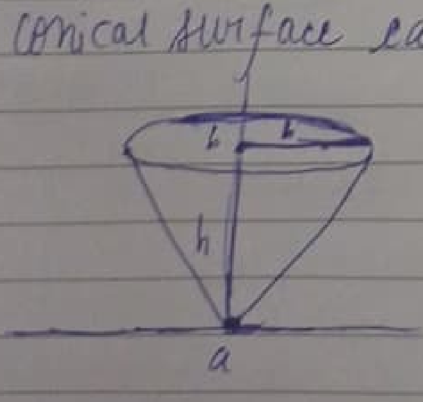
$$W = 2.28\times10^{41}\ J$$

$$t = \frac{W}{P} = \frac{2.28\times10^{41}\ \text{Joules}}{3.86\times10^{26}\ \text{Watts}} = \underbrace{1.87\times10^{7}}_{\text{Very small}}\ \text{years}$$

We can clearly say that Sun's energy is not gravitional in nature (If this had been the case we would n't be here)

$\boxed{\text{Actual process} \rightarrow \text{Nuclear Fission}}$

Q. A conical surface carries a uniform charge density.



Find $V_a - V_b$?

Ans. $V_a - V_b = \frac{\sigma h}{2\epsilon_0}\left(1 - \ln(1+\sqrt{2})\right)$

27,28/2 43

Q. Find EF at a dist r from the center of the sphere of surface radius R which carries a uniform surface charge density σ.

27-08-2024

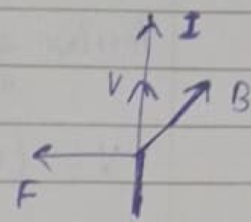
MAGNETOSTATICS

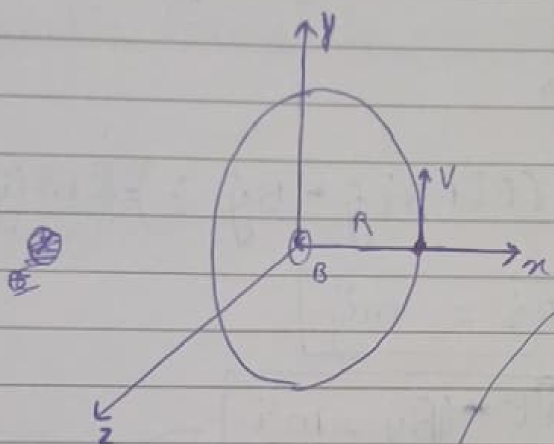
URBAN EDGE

$F_{mag} = Q(V \times B)$

$F = QE + Q(V \times B)$

$= Q(E + V \times B)$ Lorentz Force





$QvB = \frac{mv^2}{R}$ → centripetal

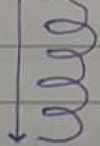
$\boxed{p = mv = QBR}$

↳ This will sustain the circular motion.

Also called $\boxed{\text{cyclotron formula}}$

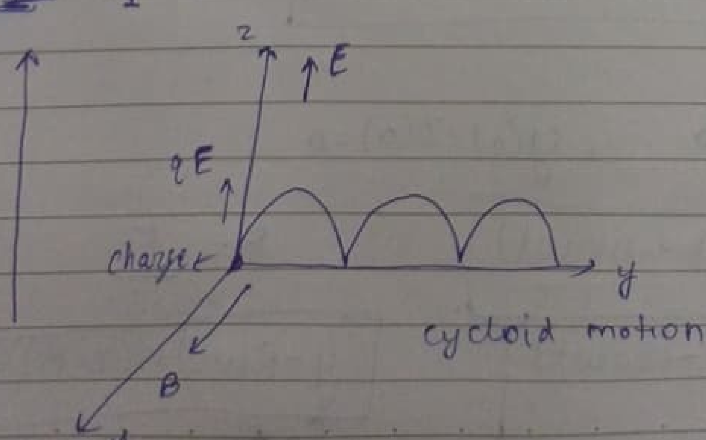
① If the charge moves in a plane ⊥ʳ to B

② It starts at some $V_{\parallel}$



Radius $= R = \frac{V}{QB}$

$V_{\perp} \perp B$



$F = qE$ in z dir
so vel. in z dir
& B in x dir
$\Rightarrow \hat{F}_B = \vec{v} \times \vec{B} = \hat{y}$

Position of particle $(0, y(t), z(t))$

$V = (0, \dot{y}, \dot{z})$

$$\vec{V} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & \dot{y} & \dot{z} \\ B & 0 & 0 \end{vmatrix} = B\dot{z}\,\hat{y} - B\dot{y}\,\hat{z}$$

$$F = Q(E + V \times B) = Q(E\hat{z} + B\dot{z}\hat{y} - B\dot{y}\hat{z}) = m(\ddot{y}\hat{y} + \ddot{z}\hat{z})$$

$$\Rightarrow \boxed{QB\dot{z} = m\ddot{y}}$$

$$\boxed{QE - QB\dot{y} = m\ddot{z}}$$

$\frac{QB}{m}$

$(\because V = R\omega)$

$\omega = \frac{QB}{m}$ cyclotron freq.

$$\ddot{y} = \omega\dot{z} \qquad \ddot{z} = \omega\left(\frac{E}{B} - \dot{y}\right)$$

$$\boxed{y(t) = c_1\cos\omega t + c_2\sin\omega t + \overset{E/B}{K}t + c_3}$$

$$\boxed{z(t) = c_2\cos\omega t - c_1\sin\omega t + c_4}$$

$\dot{y}(0) = \dot{z}(0) = 0$, $y(0) = z(0) = 0$

$$\boxed{y(t) = \frac{E}{\omega B}(\omega t - \sin\omega t)}$$

$$\boxed{z(t) = \frac{E}{\omega B}(1 - \cos\omega t)}$$

$R = \frac{E}{\omega B}$

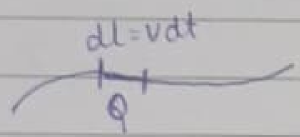
$$\boxed{(y - R\omega t)^2 + (z - R)^2 = R^2}$$

Eq^n of cycloid

$$V = \omega R = \frac{E}{B}$$

Circular Center $(0, R\omega t, R)$

Magnetic Forces do not work



$$dW_{mag} = F_{mag} \cdot dl$$
$$= Q(V \times B) \cdot v\,dt$$
$$= 0$$

Current

Change per unit time $\quad 1A = 1\ C/sec.$

$I = \lambda v \qquad \lambda \to$ line charge

$\qquad\qquad\quad v \to$ vel.

$$F_{mag} = \int (V \times B)\, dq = \int (V \times B) \lambda\, dl$$

$$F_{mag} = \int \underbrace{I}_{\text{scalar}} (\underbrace{dl}_{\text{disp. vector}} \times B)$$

I & dl are in the same dir^{n}

$$= I \int (dl \times B)$$

Q. A rectangular loop of wire supporting a mass m vertically with one end in the uniform MF B which point into the page as shown. For what current I in the loop will balance the gravitational force downward.

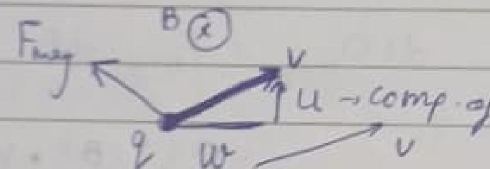
$$F_{mag} = IBa = Mg$$

$$\Rightarrow I = \frac{Mg}{Ba}$$

What happens if you increase the current?

<u>Ans</u> Mass will rise

~~What I~~



due to u → $\overset{quB}{\longleftrightarrow}$

due to w → $\uparrow quB$

The horizontal components of current due to w
$$I = \lambda w$$

Vertical component of Force $= qwB = \overbrace{\lambda a}\, wB = IBa$

Horizontal component of Force $= quB$

↳ opposing the current

Push the charges against the backdrop of existing F_{mag}

horizontal force on the top $\Rightarrow F_{hor} = \lambda a u B$

If dt is time take to move wdt

Then $WD = \lambda a B \int u w dt = IBa\underset{\hookrightarrow}{h}$ height by which the loop rises

Q. Who did the work?
Ans- Some external source

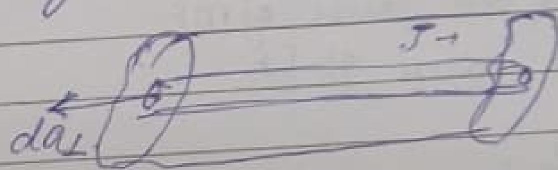
$K \rightarrow$ surface charge density

$$K = \frac{dI}{dl_\perp} \qquad F_{mag} = \int (v \times B)\sigma \, da$$

$$K = \sigma v$$

Vol. current density J

$$J = \frac{dI}{da_\perp}$$



$$J = \rho v \qquad F_{mag} =$$

$$J = \frac{dI}{da_\perp}$$

$$I = \int_S J \cdot da_\perp = \int_V (\nabla \cdot J)\, d\tau$$

dot product

$$\int_V (\nabla \cdot J)\, d\tau = -\frac{d}{dt}\int_V \rho \, d\tau = -\int_V \left(\frac{\partial \rho}{\partial t}\right) d\tau$$

$$\boxed{\nabla \cdot J = -\frac{\partial \rho}{\partial t}}$$ Continuity Eq.

What is the use of Conitnuity Eq(n)?

$$(\nabla \cdot J) = -\frac{\partial \rho}{\partial t}$$

Steady Current → ρ is constant (charge is not moving)
↳ Magndo static condition

$$\boxed{\nabla \cdot J = 0}$$

Q. How do you define M.F?

Ans Moving Charges are cause of MF
f Charges are cause of EF

Biot Savart's Law

$$B(r) = \frac{\mu_0}{4\pi}\int \frac{I \times \hat{r}}{r^2}\, dl' = \frac{\mu_0}{4\pi} I \int \frac{dl' \times \hat{r}}{r^2}$$

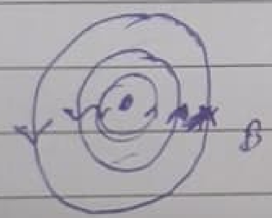
$$\mu_0 = 4\pi \times 10^{-7}\ N/A^2$$

SI unit of B = T(Tesla)
Practical unit (CGS) → Gauss

$$B_{r} = \frac{\mu_0}{4\pi}\int \frac{K(r') \times \hat{r}}{r^2}\, da'$$

$$B(r) = \frac{\mu_0}{4\pi}\int \frac{J(r') \times \hat{r}}{r^2}\, d\tau'$$

Div + Curl of B



$$\oint B\cdot dl = \oint \frac{\mu I}{2\pi s}\, dl$$

where s - radius of any ~~of~~ form circle

$$\oint B\cdot dl = \frac{\mu_0 I}{2\pi s}\oint dl = \frac{\mu_0 I}{2\pi s}\, 2\pi s = \mu_0 I$$