Signals

EGC 113

Source : https://www.princeton.edu/~cuff/ele301/files/lecture1_2.pdf

What are Signals?

- A signal is a pattern of variation of values of a quantity w.r.t. an independent variable such as time, space.
- Signals are variables that carry information
  - Voltages and currents in a circuit
  - Acoustic pressure (sound) over time
  - Velocity of a car over time
  - Intensity level of a pixel (camera, video) over time

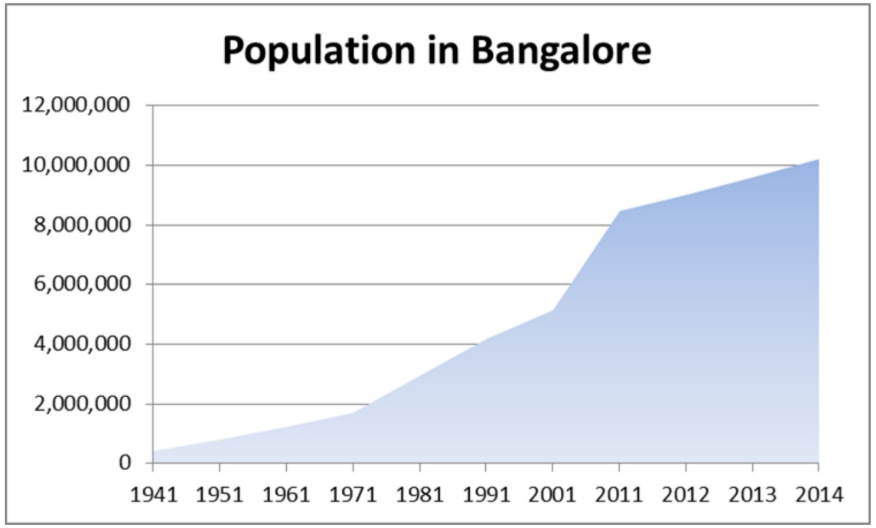
What are Signals?

- Typical thinking of signals in terms of communication and information
  - radio signal
  - broadcast or cable TV
  - Audio
  - Electric voltage or current in a circuit

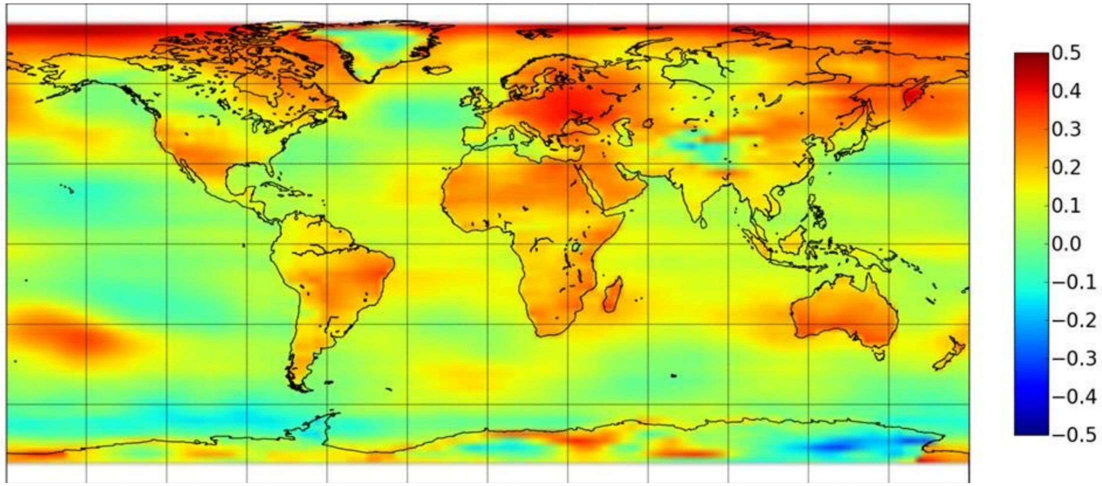
- More generally, any physical or abstract quantity that can be measured, or influences one that can be measured, can be thought of as a signal.
  - Tension on bike brake cable
  - Roll rate of a spacecraft
  - Concentration of an enzyme in a cell
  - The price of dollars in euros
  - The federal deficit
- Very general concept.

Signal ?

- Is it a ….
- Function, Sequence of numbers



- E.g.
  - Avg. Bangalore population plotted each year
  - Temperature at every spatial location in the room or temperature across the globe



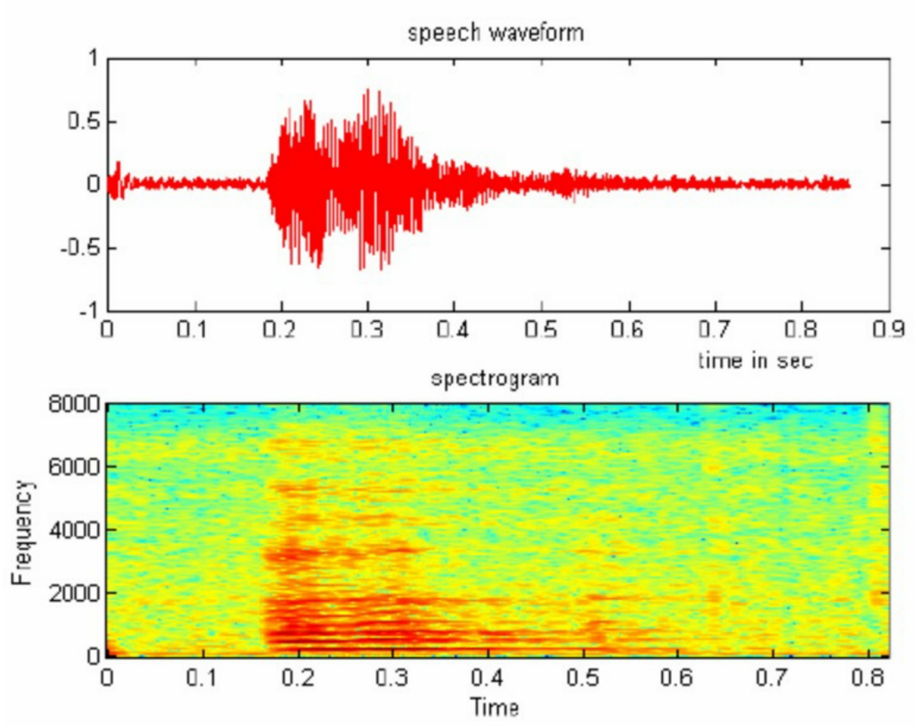
Signal

- Signal Emerges from a Physical Phenomenon..

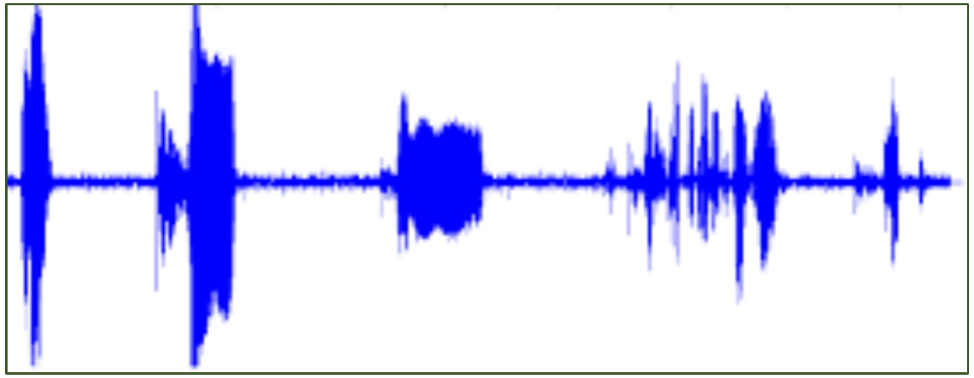
- Representation : Function, Number series…

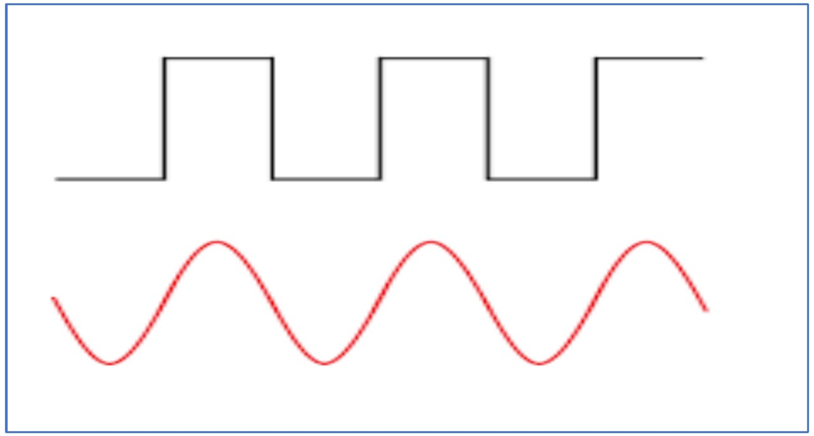
- Could be 1-D, 2-D, 3-D, 4-D…

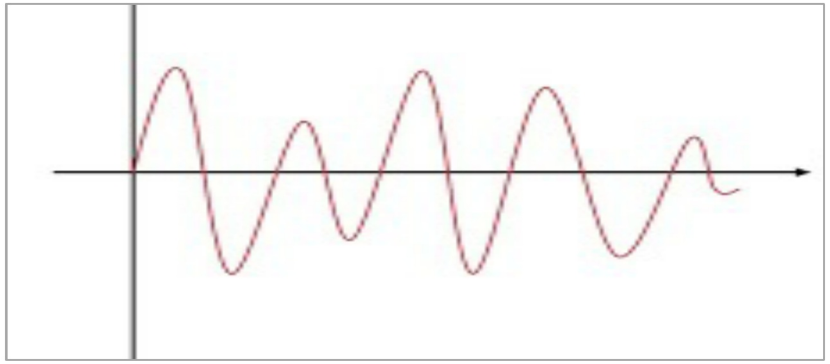
- Example : speech = s(t); Here s denotes the amplitude (intensity); it is the dependent variable and ‘t’ is the independent variable.



Examples







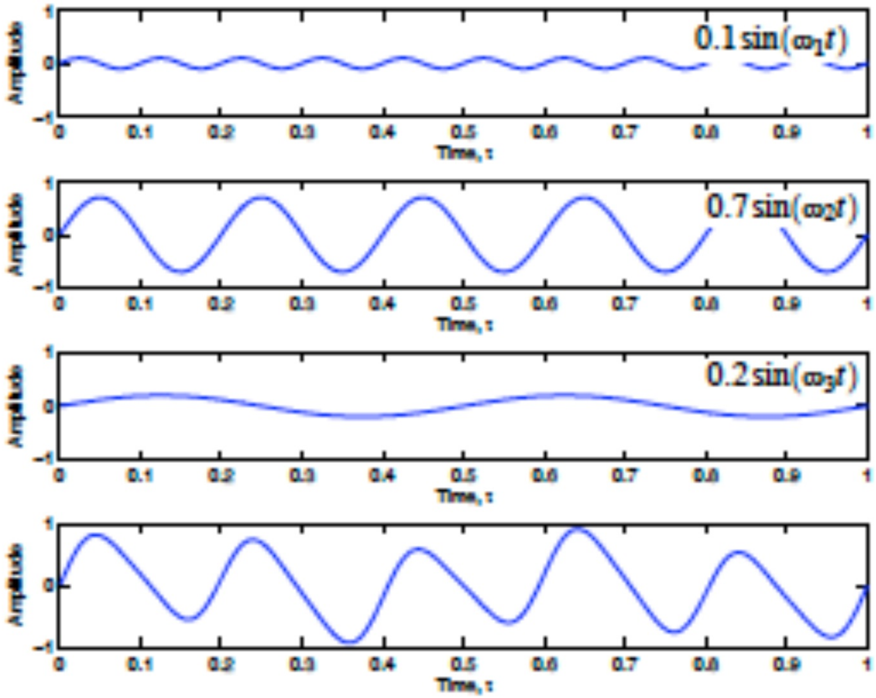
Time Series

- Time-Series Representation of Signals
  Typically think of a signal as a "time series", or a sequence of values in time

Useful for saying what is happening at a particular time
Not so useful for capturing the overall characteristics of the signal.

Frequency Representation

- Represent signal as a combination of sinusoids



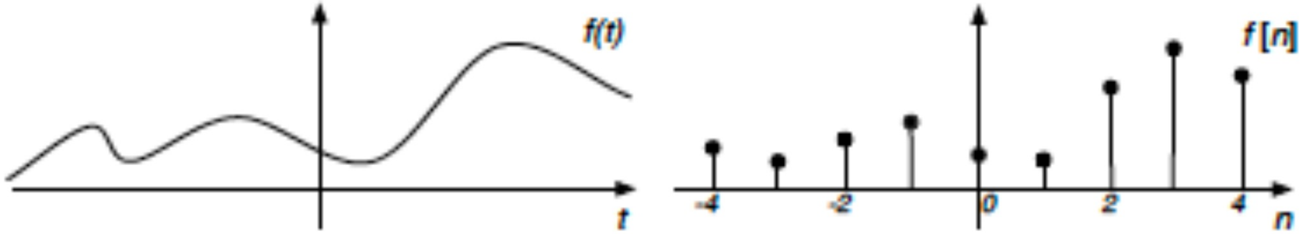
$$f(t) = 0.1\sin(\omega_1 t) + 0.7\sin(\omega_2 t) + 0.2\sin(\omega_3 t)$$

Frequency Representation

- This example is mostly a sinusoid at frequency ω_2, with small contributions from sinusoids at frequencies ω_1 and ω_3.
    - Very simple representation (for this case).
    - Not immediately obvious what the value is at any particular time.
- Why use frequency domain representation?
    - Simpler for many types of signals (AM radio signal, for example)
    - Many systems are easier to analyze from this perspective (Linear Systems).
    - Reveals the fundamental characteristics of a system.
- *Rapidly becomes an alternate way of thinking about the world.*

Continuous Signals

- Most of the signals we will talk about are functions of time.
- There are many ways to classify signals. This class is organized according to whether the signals are continuous in time, or discrete.
- A *continuous-time* signal has values for all points in time in some (possibly infinite) interval.
- A *discrete time* signal has values for only discrete points in time.

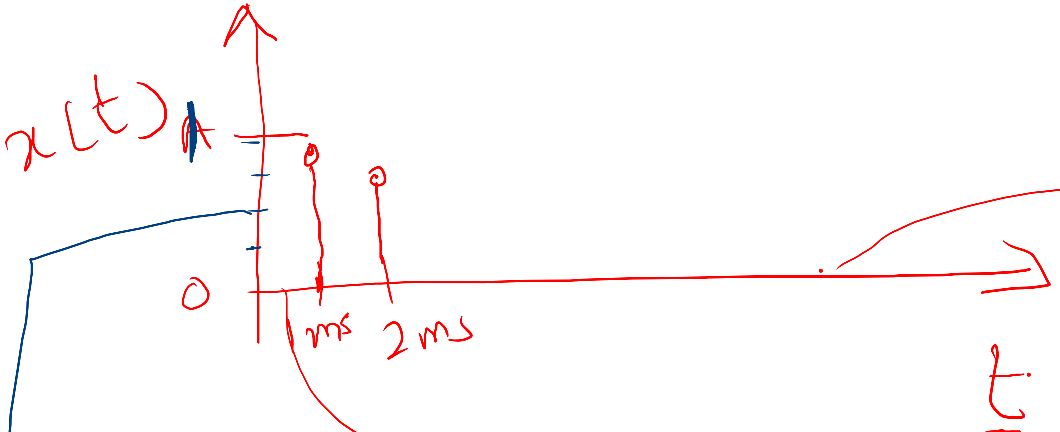


- Signals can also be a function of space (images) or of space and time (video), and may be continuous or discrete in each dimension.

Continuous /

analog signal

all values



Sampling along 't'

Sampling along 'y' axis

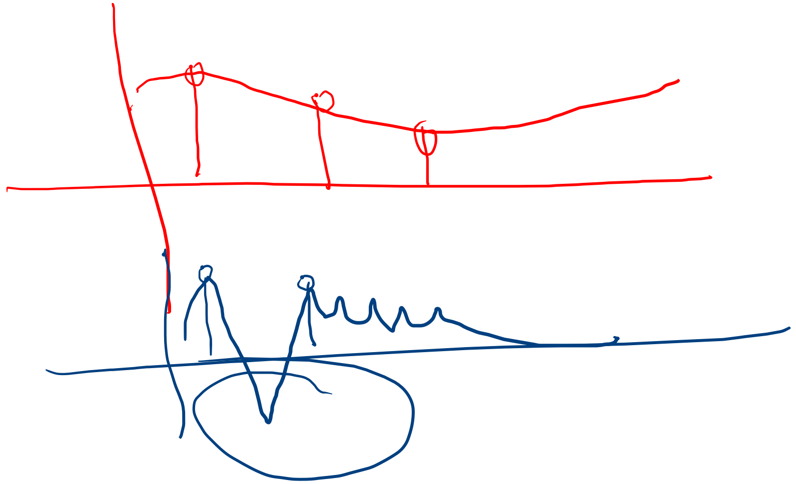
discrete time signal

T = 1 ms

4 bits

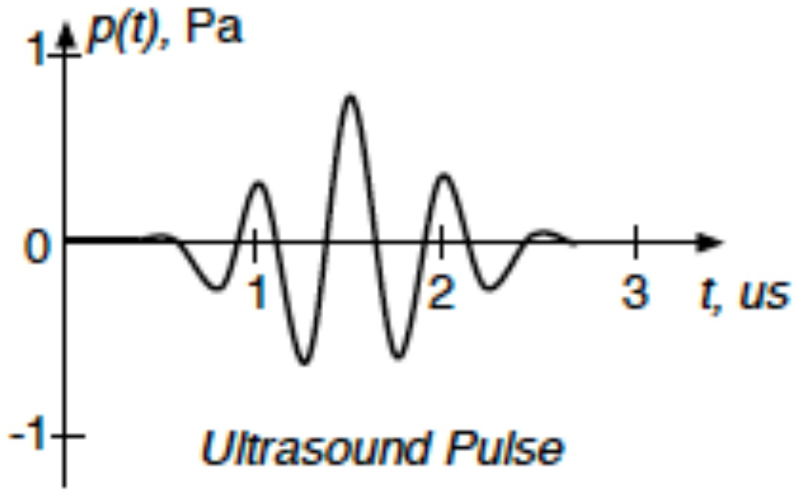
16

$\frac{1}{16}$



Continuous Time Signals

- Function of a time variable, something like t, τ, t_1.
- The entire signal is denoted as v, $v(.)$, or $v(t)$, where t is a dummy variable.
- The value of the signal at a particular time is $v(1.2)$, or $v(t), t = 2$.



Discrete Time Signals

- Fundamentally, a discrete-time signal is sequence of samples, written

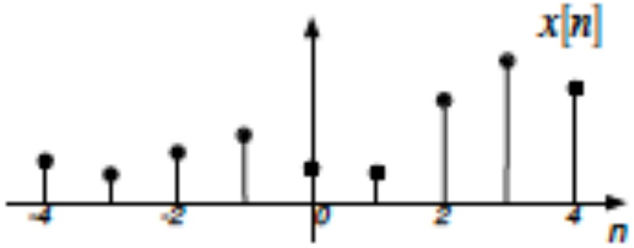
$$x[n]$$

where n is an integer over some (possibly infinite) interval.

- Often, at least conceptually, samples of a continuous time signal

$$x[n] = x(nT)$$

where n is an integer, and T is the *sampling period*.



- Discrete time signals may not represent uniform time samples (NYSE closes, for example)

Continuous Signals

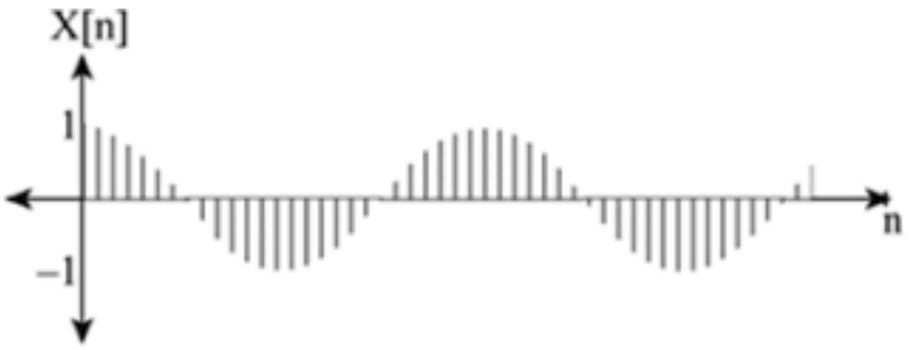
- Can a machine plot a Continuous time signals (CTS) ?
- E.g. s(t) = sin(t)
- What are the values that ‘t’ can take ? It can take infinite values, the range as well as the resolution is infinite.
- What are the values that ‘s’ can take ? Again, Infinite

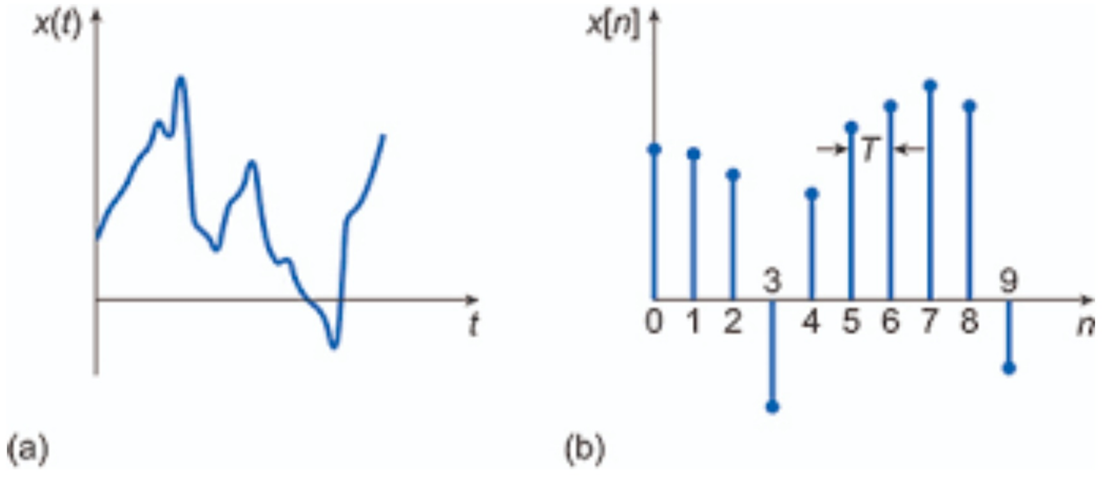
Continuous Signals

In practice, it's impossible to work with Continuous signals !!

Then, why are Continuous signals important ??

Continuous-time vs Discrete-time signals





Examples of Discrete time signals

- Daily Average Bangalore temperature
- Stock market Hourly index

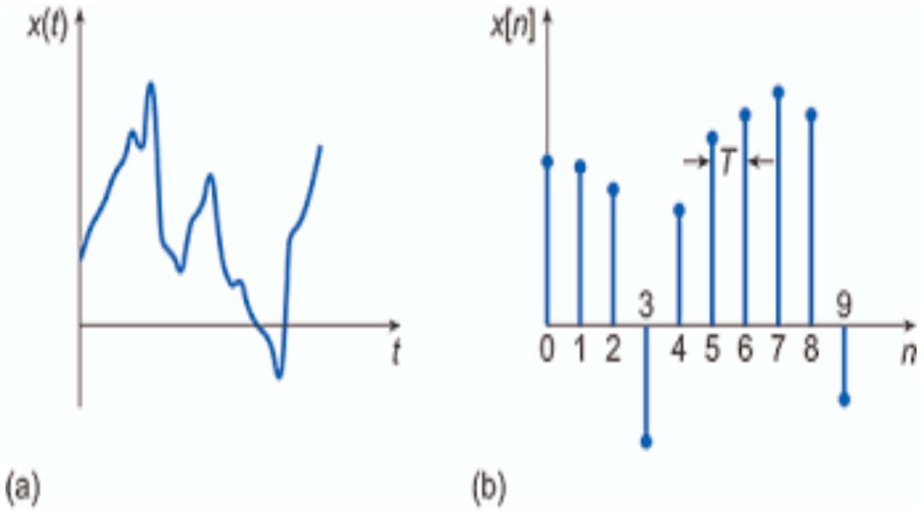
- Can a machine plot a Discrete-time signal ?

(1) x(n) = sin(2*pi*n); Plot over 2 complete cycles

(2) x(n) = sin(2*pi*2*n); Plot over 2 complete cycles

Continuous-time Vs Discrete-time

- Convention :



- x(t) is used for Cts-time signals
- t Ɛ R
- Plotted with solid curves

- x[n] is used for Discrete-time signals
- n Ɛ N
- Plotted with spikes at values takes by n

Continuous vs. Discreet Signals

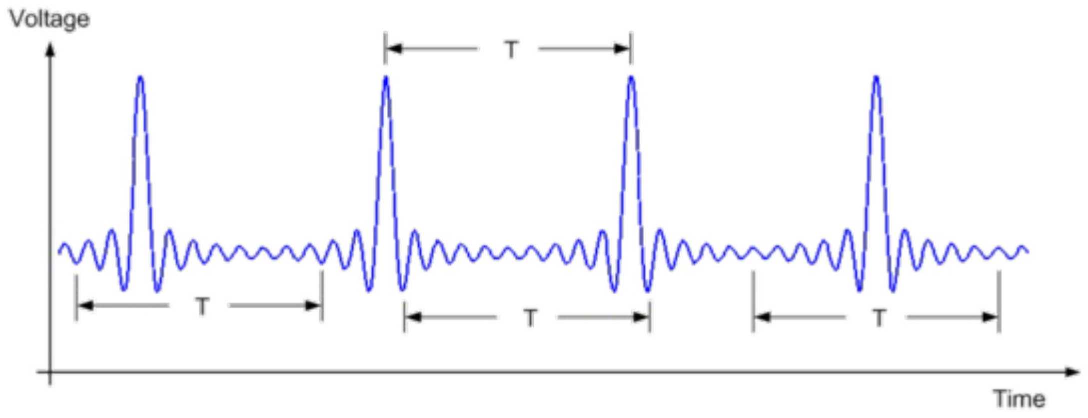
exists for $\forall\ t$

exists only at nT $\quad n \to$ integer

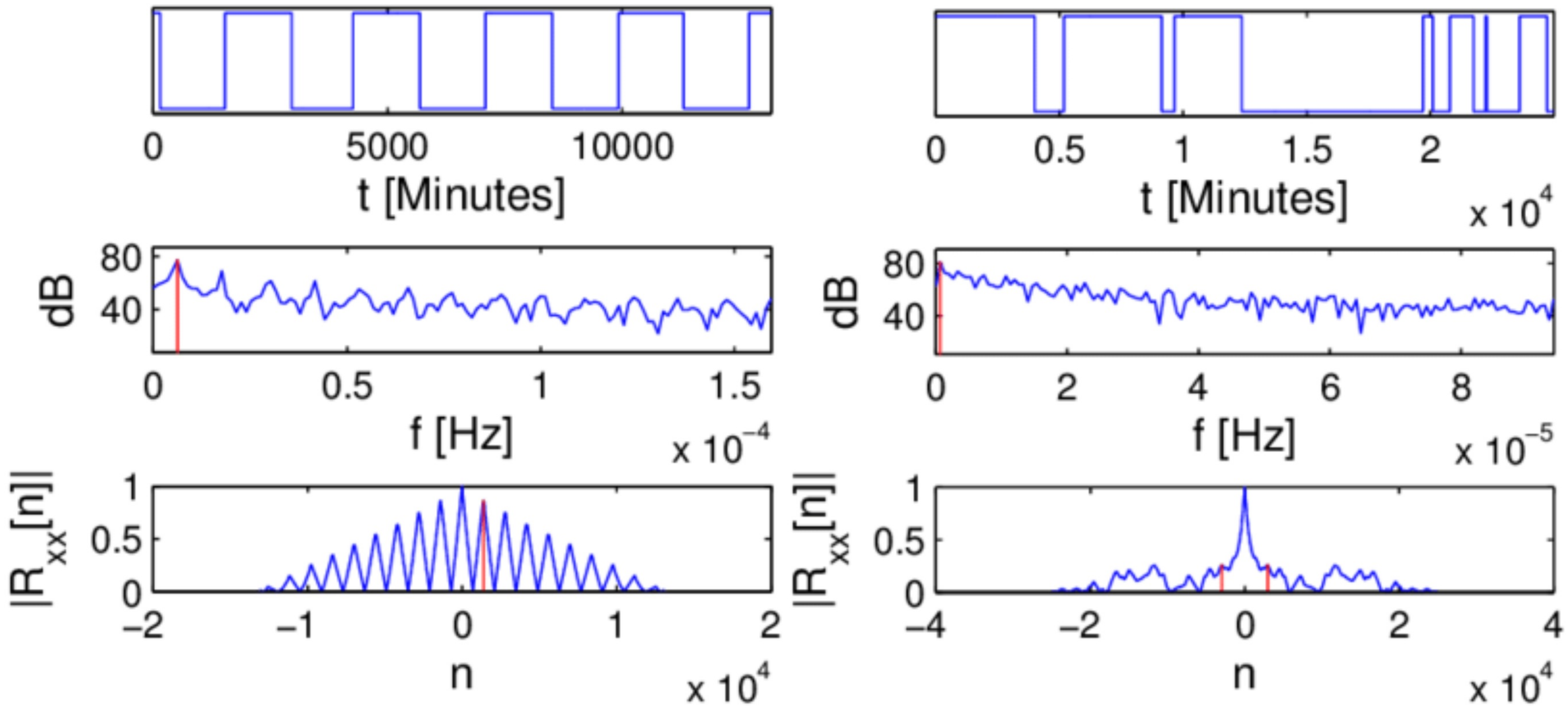
$T \to$ sampling period.

Signal Attributes

- Periodicity :
- For a cts-time signal to be periodic with period 'T' ($T > 0$), it has to satisfy, $x(t) = x(t + T)$, for all values of t
- i.e. $x(t) = x(t + mT)$, $m \in Z$
- Fundamental period ?



Periodic and aperiodic signals



Periodicity

$\sin(2\pi f t)$

$\sin(\omega t)$

$f \rightarrow Hz \rightarrow 1/s$

$T = \frac{1}{f}$ seconds

$\omega = 2\pi f$ rad/s

- What is the span of a Periodic signal ?

- Find the period of :

  (a) sin(t/2) $\rightarrow$ $2\pi f = \frac{1}{2}$ $\quad f = \frac{1}{4\pi}$ Hz

  $T = 4\pi$ seconds

  (b) sin(2t) $\rightarrow$ $f = \frac{1}{\pi}$ Hz $\quad T = \pi$ seconds

  (c) sin(3t) + sin(t/6) $\rightarrow$ $\cancel{F_1} = 2\pi f_1 = 3$ $\quad f_1 = \frac{3}{2\pi}$

  $x_1(t)$ $\quad x_2(t)$

  $T_1 = \frac{2\pi}{3}$

  $2\pi f_2 = \frac{1}{6}$ $\quad f_2 = \frac{1}{12\pi}$ $\quad T_2 = 12\pi$

$T = \frac{12\pi}{2\pi/3}$?

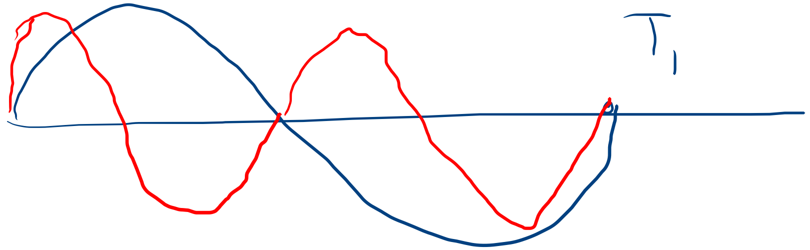
$f_1 = 10\text{ Hz}$ $\longrightarrow$ $T_1 = 100\text{ ms}$

$f_2 = 20\text{ Hz}$ $T_2 = 50\text{ ms}$

$f_3 = 7\text{ Hz}$

$T_3 = \frac{1}{7}$ seconds

143 ms



$T =$ 1 1400 ms

2 700 ms

1s $\longrightarrow$ 10 cycles

20 "

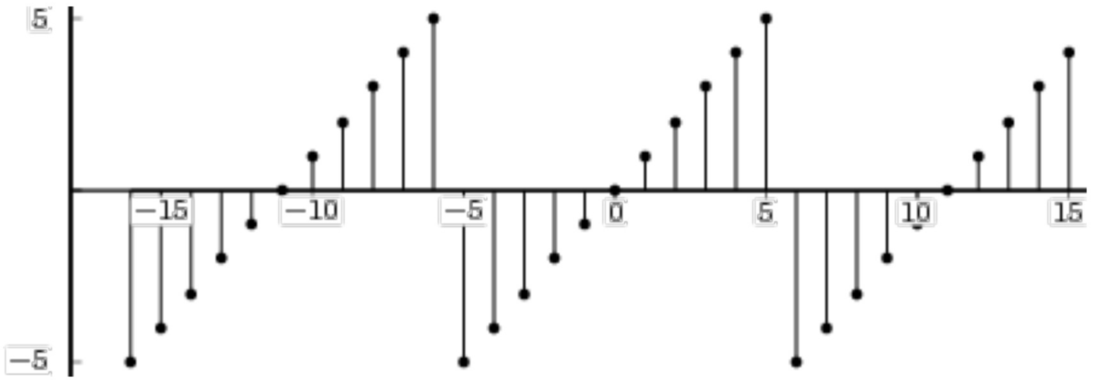
7 "

Signal Attributes

- Periodicity :
- For a Discrete-time signal to be periodic with period ‘N’ it has to satisfy, x[n] = x[n+N], for all integer values of n

- i.e x [n] = x[n + mN], m Ɛ Z

- Fundamental period ?



Periodicity

- Find whether the given signal is periodic and find fundamental period :

1. $x(t) = sin^2(4\pi t)$
2. $x(t) = \sin(6\pi t) + \cos(5\pi t)$
3. $x[n] = e^{j2n}$
4. $x[n] = \cos(\frac{3\pi}{4}n)$
5. $x[n] = \sin(\frac{3\pi}{4}n) + \cos(\frac{5\pi}{7}n)$

$T_1 = \frac{1}{3} \quad T_2 = \frac{2}{5}$

$2 \quad 5$

$\cos(2n) + j\sin(2n)$

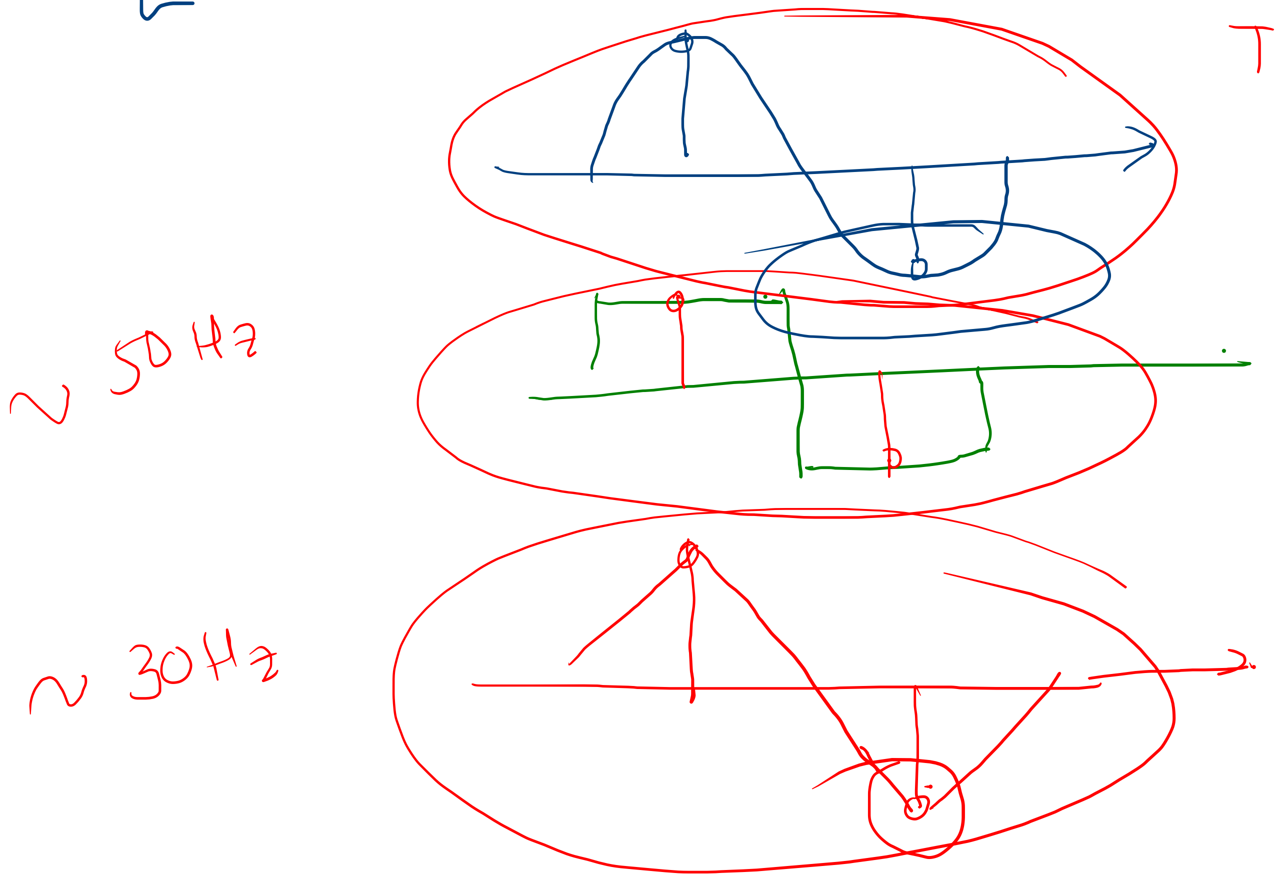
$2\pi f n = 2n$

$f = \frac{1}{\pi}$

$$x[n] = \cos\left[\frac{3\pi}{4} n\right]$$

$f = 10\,Hz$

$T = 100\,ms$



$$x[n] = \cos\left(\frac{3\pi}{4} n\right)$$

$$x(t) = \cos(2\pi f t) \rightarrow t = nT_s$$

$$x[nT_s] = \cos(2\pi (f T_s) n)$$

$\rightarrow$ digital frequency

10 Hz   20 Hz   30 Hz   50 Hz

$f_s = 100$ Hz

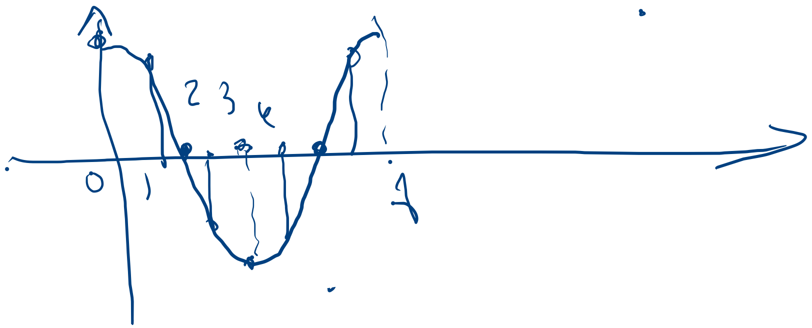
$T_s = 10$ms

$10 \times \frac{1}{100} = 0.1$ 0.2 0.3

$f \rightarrow 0 - 1/2$

$\omega \rightarrow 0 - \pi$

$$x[n] = \cos\left(\frac{3\pi}{4} n\right)$$



$$2\pi f n = \frac{3\pi}{4} n.$$

$$f = \frac{3\pi}{4} \times \frac{1}{2\pi} = \frac{3}{8}$$

$$N = \underline{8}$$

$$x(n) = x(n+N)$$

$x(0) = \cos(0) = 1$ $\quad x(1) = \cos\left(\frac{3\pi}{4}\right)$ $\quad x(2) = \cos\left(\frac{3\pi}{2}\right)$

$x(3) = \cos\left(\frac{9\pi}{4}\right)$ $\quad x(4) = \underline{\cos(3\pi) =}$ $\quad x(5) = \cos\left(\frac{15\pi}{4}\right)$

$x(6) = \cos\left(\frac{18\pi}{4}\right)$ $\quad x(7) = \cos\left(\frac{21\pi}{4}\right)$ $\quad x(8) = \cos(6\pi)$

$= 1 = x(0)$

$$x(t) = \sin^2(4\pi t)$$

$$= \frac{1-\cos(8\pi t)}{2} = \frac{1}{2} - \frac{\cos(8\pi t)}{2}$$

$$8\pi = 2\pi f$$

$$f = 4.$$

$$T = 0.25\,s$$

$$x[n] = \underline{\sin\left(\frac{3\pi}{4} n\right)} + \cos\left(\frac{5\pi}{7} n\right)$$

$$f_1 = \frac{3}{8} \quad N_1 = 8$$

$$f_2 = \frac{5}{14} \quad N_2 = 14$$

$$N = \mathrm{LCM}(N_1, N_2)$$

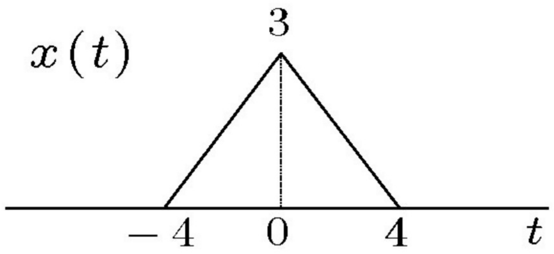
$$= \underline{56}$$

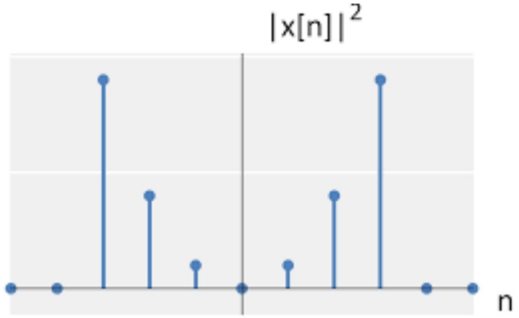
Energy

- Signal with finite energy is Energy signal

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, \mathrm{d}t \qquad\qquad E_\infty = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

Typically, signal with finite energy should be non-zero over a compact interval



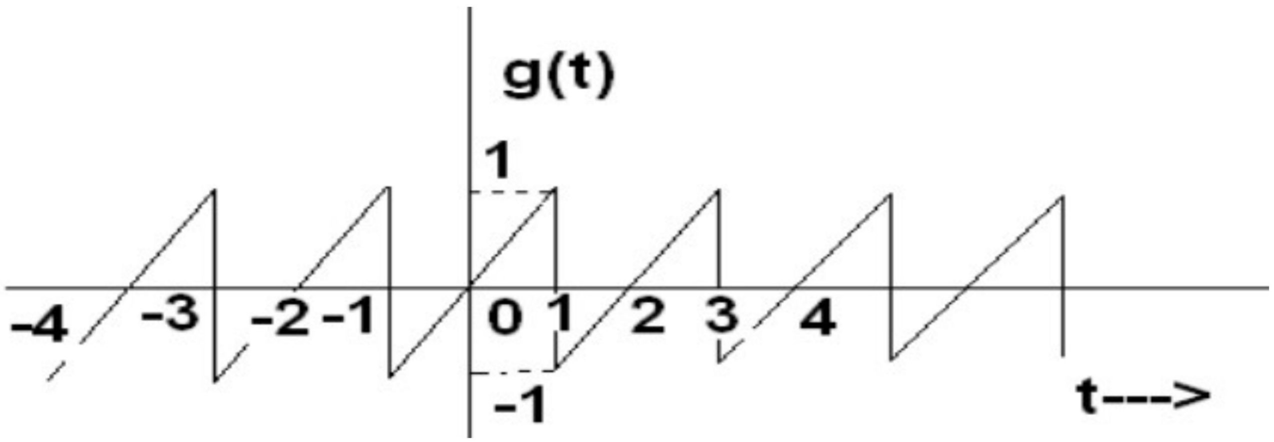


Power

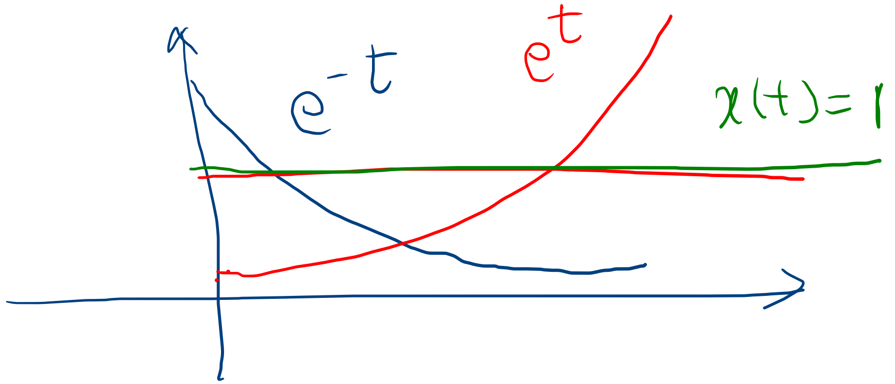
- A signal with finite Power is called Power signal

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, \mathrm{d}t$$

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$



Power-Energy signals



- A signal is called an **energy signal** if $E_\infty < \infty$
- A signal is called a **power signal** if $0 < P_\infty < \infty$

Power-Energy signals

- A signal can not be both an energy signal and a power signal

What is the power of an Energy signal ?

Sample problems

Find the energy of the following signal.

$$g(t) = \begin{cases} e^{t/2} & 0 \le t \\ 0 & t < 0 \end{cases}$$

Find the power of the following signal.

$$g(t) = \begin{cases} t & \frac{-T}{2} \le t < \frac{T}{2} \\ 0 & o.w. \end{cases}$$

What is the period of the following signals?

a. $\cos\left(\frac{\pi}{2}t\right)$

b. $\sin(t) + \cos\left(\frac{t}{2}\right)$

Determine values of P_∞ and E_∞

1. $x(t) = e^{j(2t+\pi/4)}$
2. $x(t) = \cos(t)$
3. $x[n] = e^{j(\frac{\pi n}{2}+\frac{\pi}{8})}$
4. $x[n] = \cos(\frac{\pi}{4}n)$

Problems

1) $x[n] = 1/n$, for $n ‡ 0$; Else 0

Find Energy

$E = 2 \times \Sigma(1/n^2) = \pi/3$

Hence Energy signal

What do you observe in the signal ?

Problems

2) $x[n] = (-1)^n$

What is the Energy of this signal ?

Problems

2) $x[n] = (-1)^n$

What is the Energy of this signal ?

Infinity..why ??

What is the Power of this signal ?

P = 1 → This is a Power signal

$$P = \lim_{n\to\infty} \frac{1}{(2n+1)} * \sum_{m=-n}^{n} 1$$

Different versions of a signal

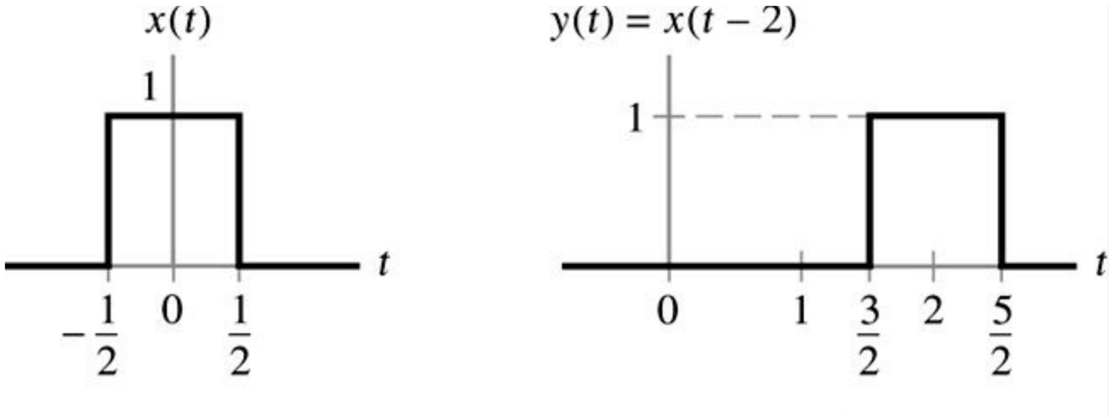
- What are possible Transformations of the Independent Variable ??

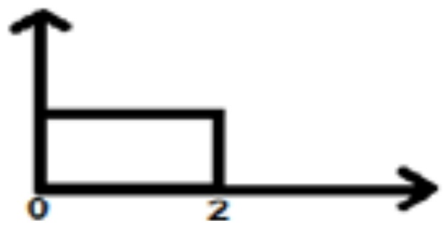
Transformations of the Independent Variable

- Time shift

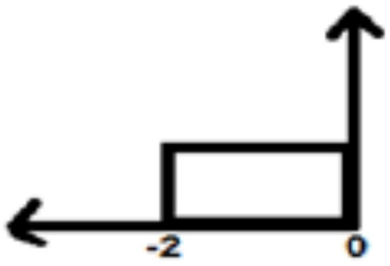
a) For Cts-time signal, x(t), time-shift makes it $x(t-t_0)$

Time-shift preserves the shape of the signal

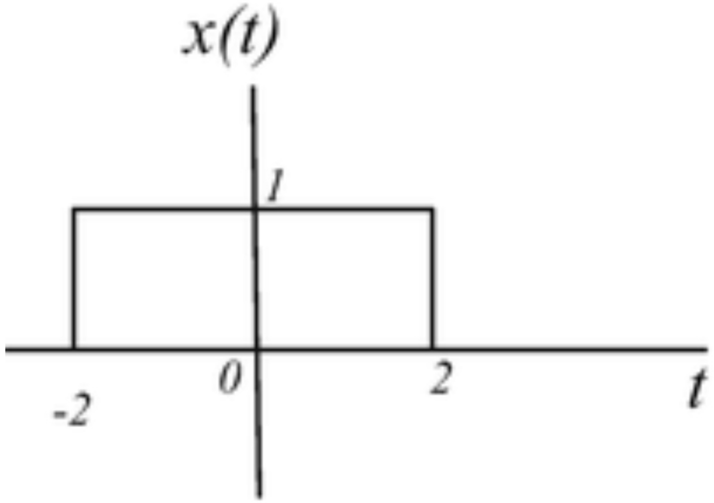




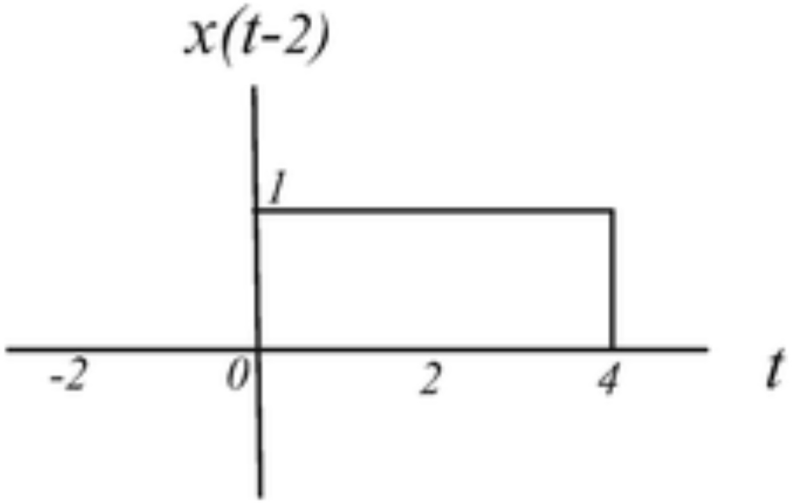
x(t)



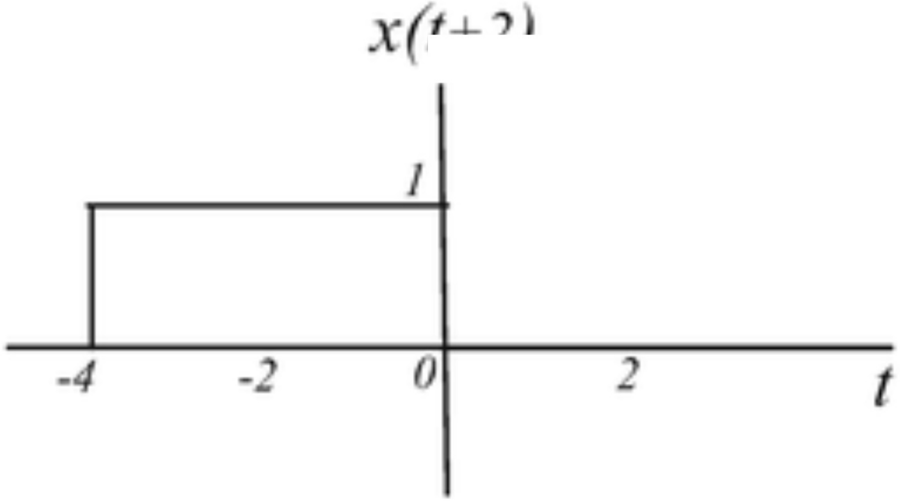
x(t+2)



(a)



(b)

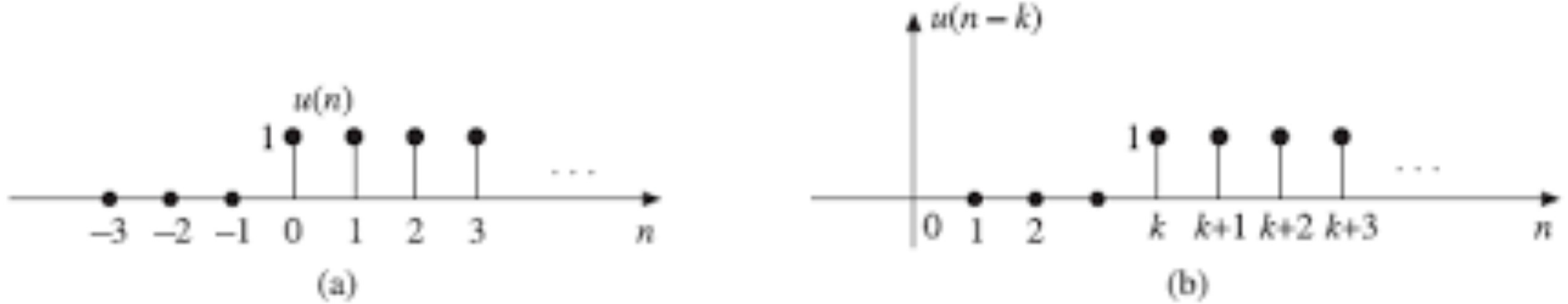


(c)

Time-shift for Discrete-time signals

For Discrete-time signal, x[n], time-shift makes it $x[n-n_0]$

Time-shift preserves the shape of the signal

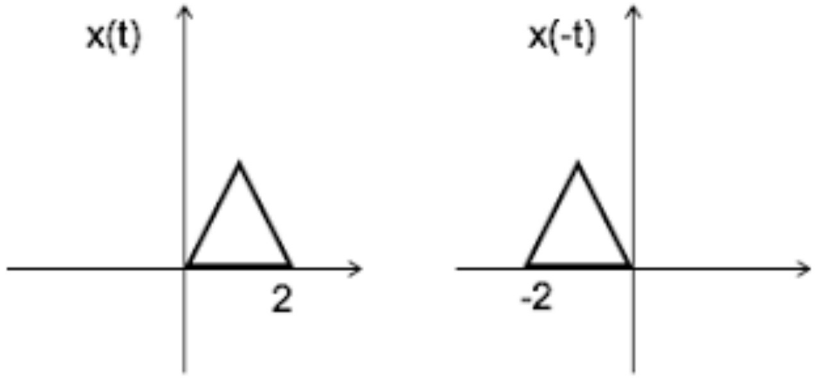


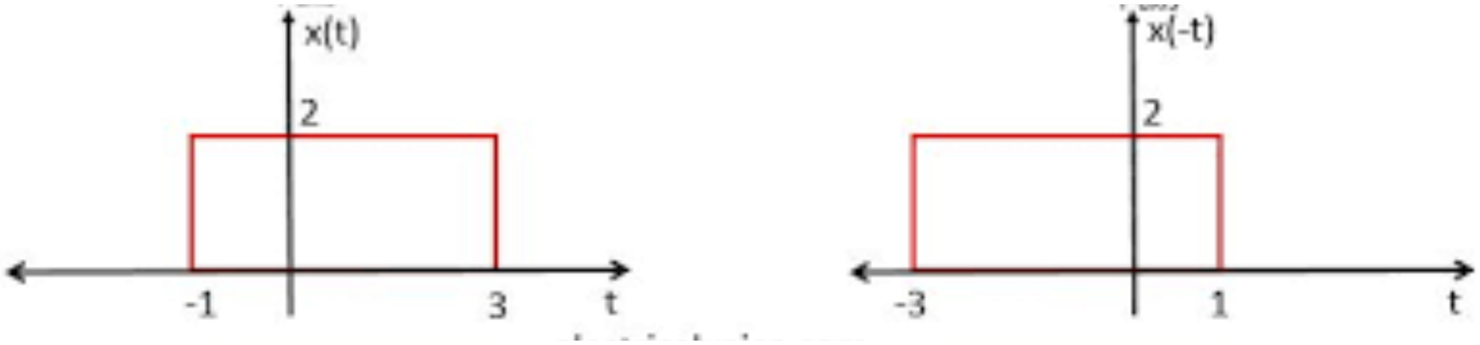
Reflection

- x(t) When time-reversed gives x(-t)
- X[n] when time-reversed gives x[-n]

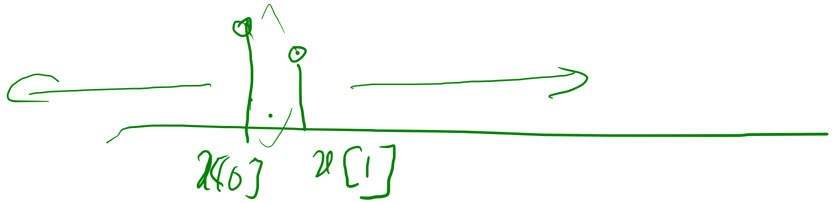
- Real-life examples :
  - Mirror Reflection
  - Playing audio/video tape in reverse

Examples



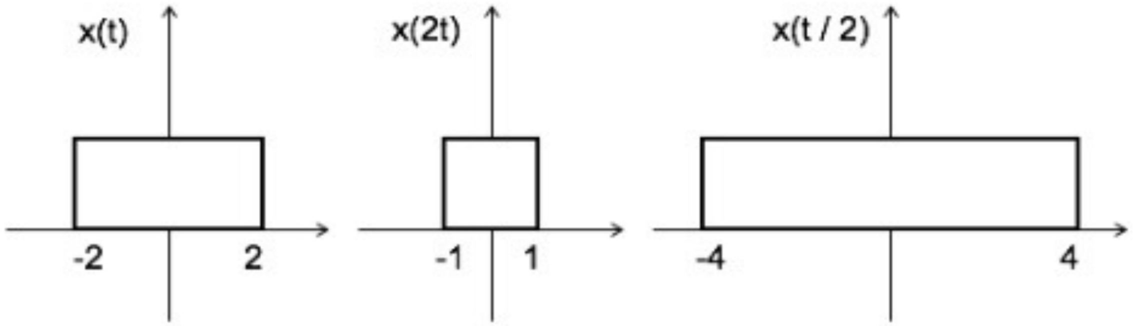


Scaling



- x(t) When time-axis is scaled by "A" gives x(At)

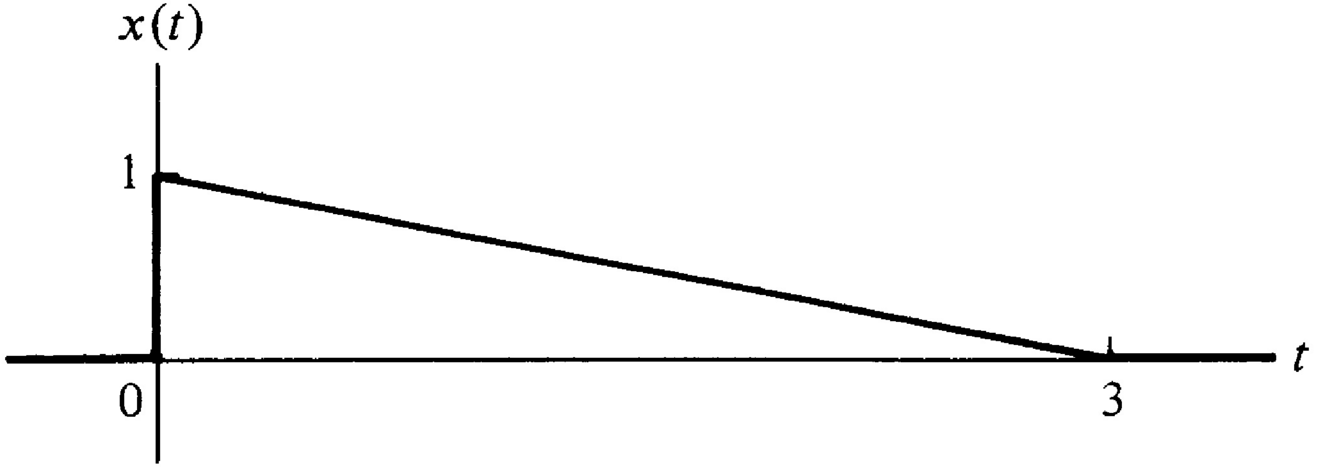
$|A| > 1 \rightarrow$ Compression of the signal
$|A| < 1 \rightarrow$ Expansion of the signal

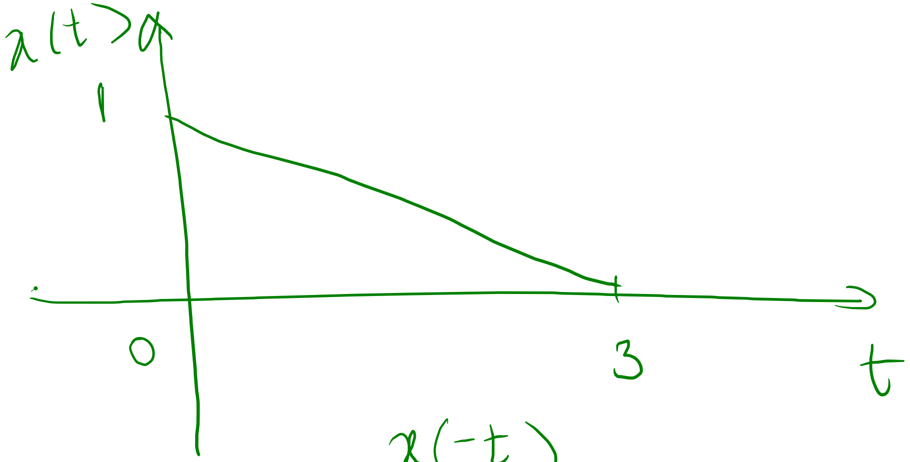


Problem

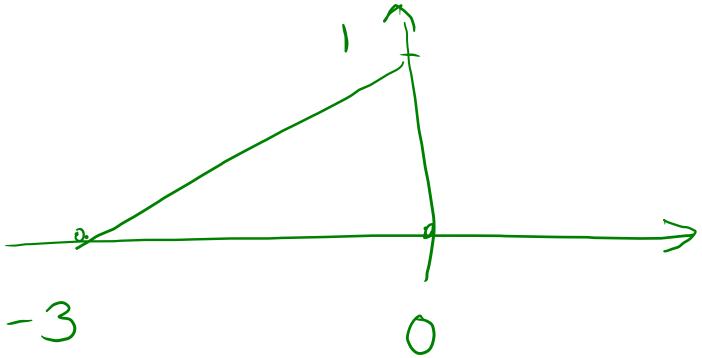
For x(t) indicated in Figure, sketch the following:

1. x(-t)
2. x(t+2)
3. x(2t+2)
4. x(1-t)





$x(-t)$



$x(-t)$

$$x_1(t) = x(-t)$$

$$x_1(0) = x(0)$$

$$x_1(1) = x(-1) = 0$$

$$x(-1) = x(1) =$$

Plot the following signals

1. Sketch the following signals:

a)

$$x(t) = \begin{cases} 0 & \text{if} \quad t < -4 \\ t+2 & \text{if} \quad -4 \le t < 3 \\ t-2 & \text{if} \quad 3 \le t \end{cases}$$

b) y(t) = x(t-1) where x(t) is defined in part a)

c)

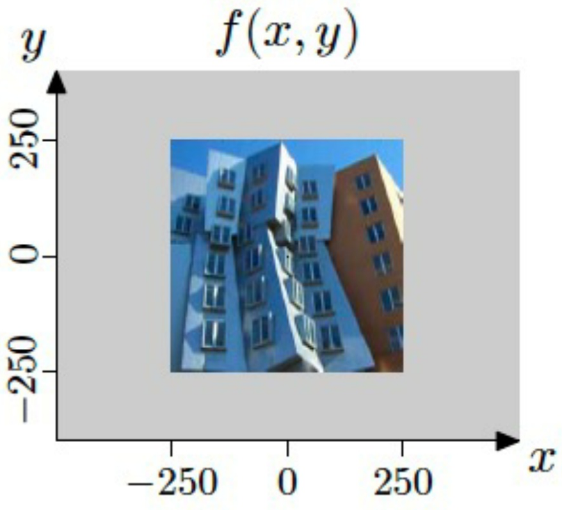
$$x[n] = \begin{cases} 0 & \text{if} \quad n < 2 \\ 2n-4 & \text{if} \quad 2 \le n < 4 \\ 4-n & \text{if} \quad 4 \le n \end{cases}$$

d) y[n] = x[n+1] where x[n] is defined in part c)

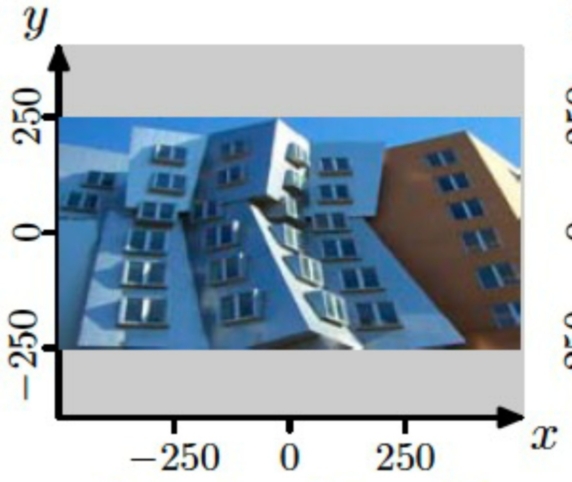
Assignment

- Download any small speech/music file. Make sure it is in .wav format and not longer than 5 seconds in duration. Let this signal be f(t).

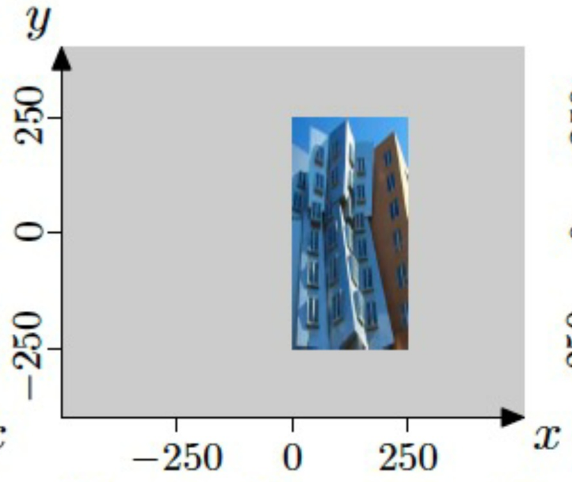
- Listen to
- $f1(t) = 2*f(t)$
- $f2(t) = f(2t)$
- $f3(t) = f(t)/3$
- $f4(t) = -f(t)$



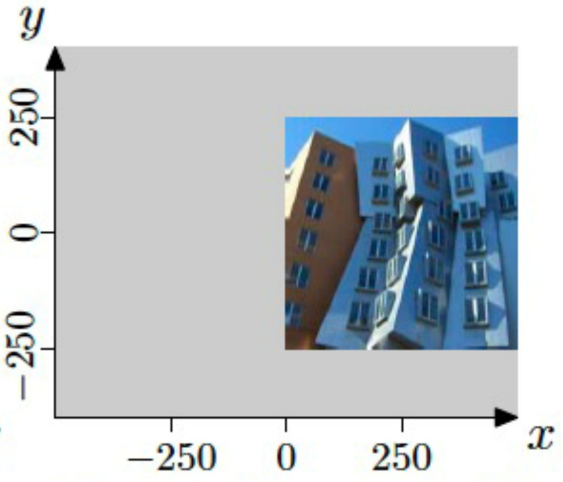
How many images match the expressions beneath them?



$f_1(x,y) = f(2x,y)$?



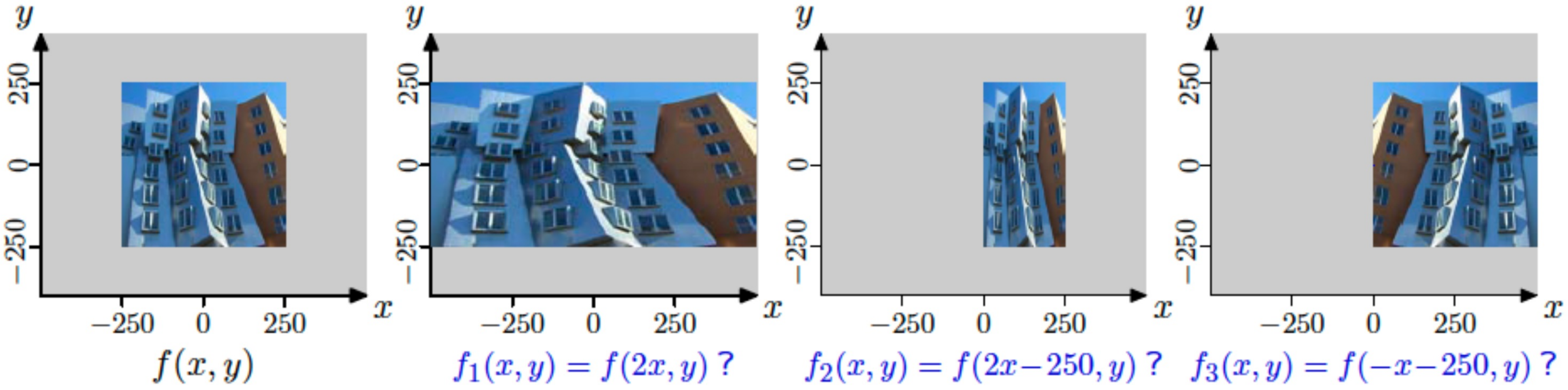
$f_2(x,y) = f(2x-250,y)$?



$f_3(x,y) = f(-x-250,y)$?

Source: MIT, OCW, Lecture notes by Dr. Freeman

Check Yourself



$$x = 0 \quad \rightarrow f_1(0,y) = f(0,y) \qquad \checkmark$$
$$x = 250 \quad \rightarrow f_1(250,y) = f(500,y) \qquad \times$$

$$x = 0 \quad \rightarrow f_2(0,y) = f(-250,y) \qquad \checkmark$$
$$x = 250 \quad \rightarrow f_2(250,y) = f(250,y) \qquad \checkmark$$

$$x = 0 \quad \rightarrow f_3(0,y) = f(-250,y) \qquad \times$$
$$x = 250 \quad \rightarrow f_3(250,y) = f(-500,y) \qquad \times$$