Graph Traversal - II

Depth- First Search

- edges are explored out of the most recently discovered vertex

Depth- First Search

- $\pi[v]$ - predecessor of v
- d[v] - time when v is discovered (i.e., v turns gray)
- f[v] - when all neighbours of v are visited (v turns black)

Depth- First Search

DFS(G)

for each vertex $u \in V[G]$

color[u] = white

$\pi[u] = NIL$

time = 0

for each $u \in V[G]$

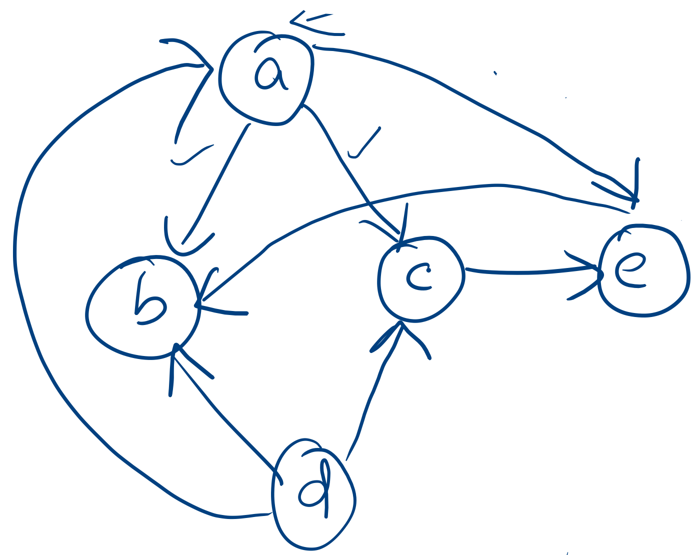
if color[u]=white

DFS-VISIT(u)

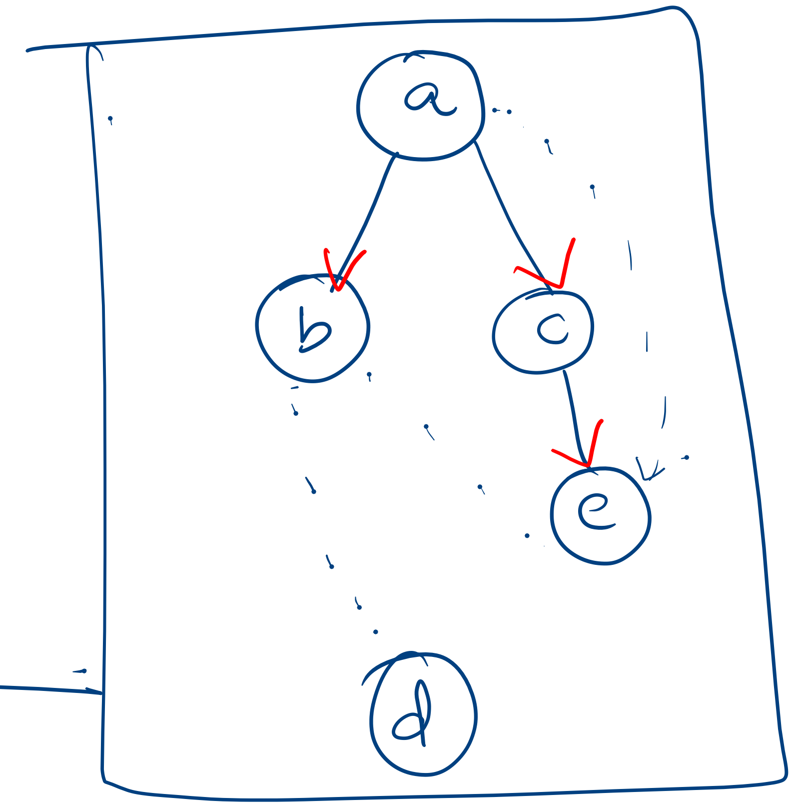
Depth- First Search

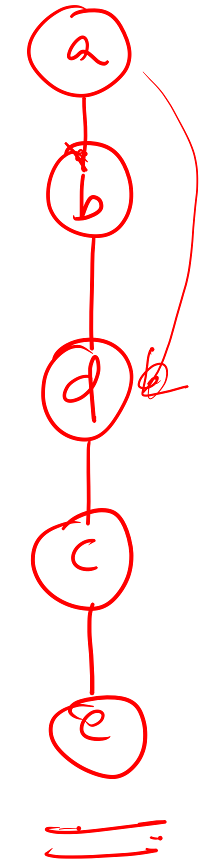
DFS-VISIT(u)

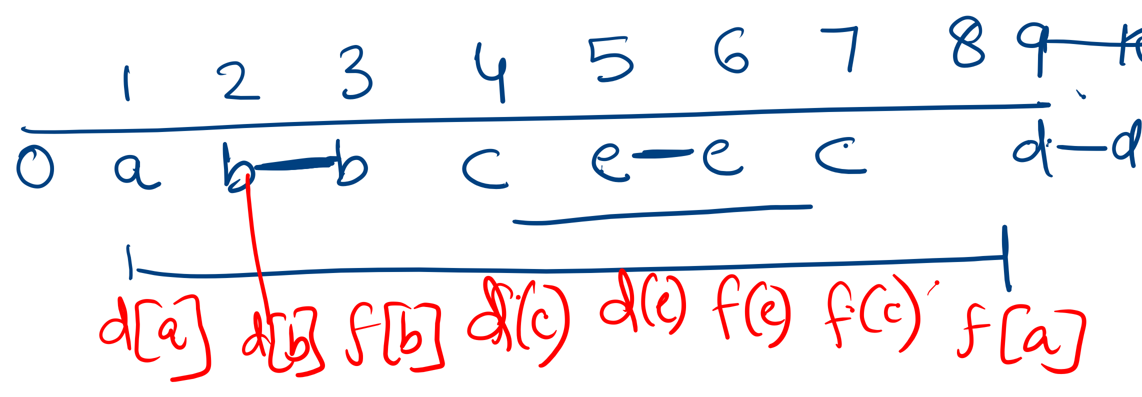
```
color[u]=gray
time = time+1
d[u] = time
for each v ∈ Adj[u]
    if color[v] = white
        π[v] = u
        DFS-VISIT(v)
color[u] = black
f[u]= time
time = time+1
```



	d[]	f[]
a	1	8
b	2	3
c	4	7
e	5	6
d	9	10







Depth- First Search

Properties of DFS :

- The predecessor graph form a forest
- $u = \pi[v]$ if and only if DFS-VISIT(v) was called during a search of u'-s adjacency list.

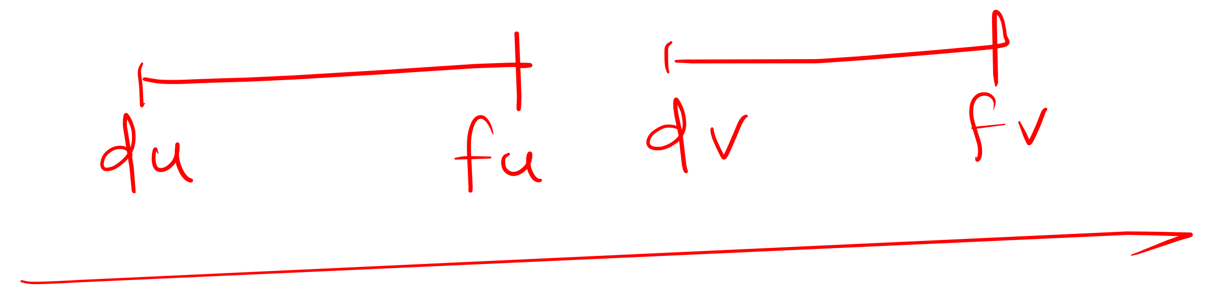
Depth- First Search

Properties of DFS :

Parenthesis Structure :

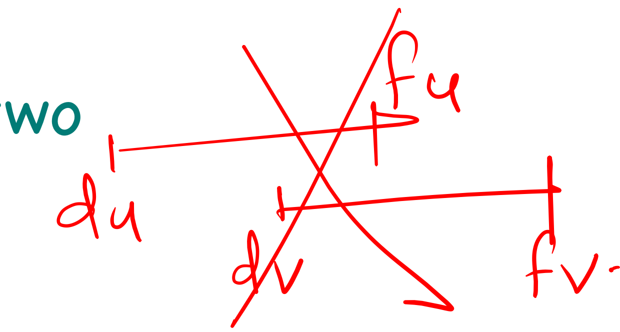
Depth- First Search

Properties of DFS :

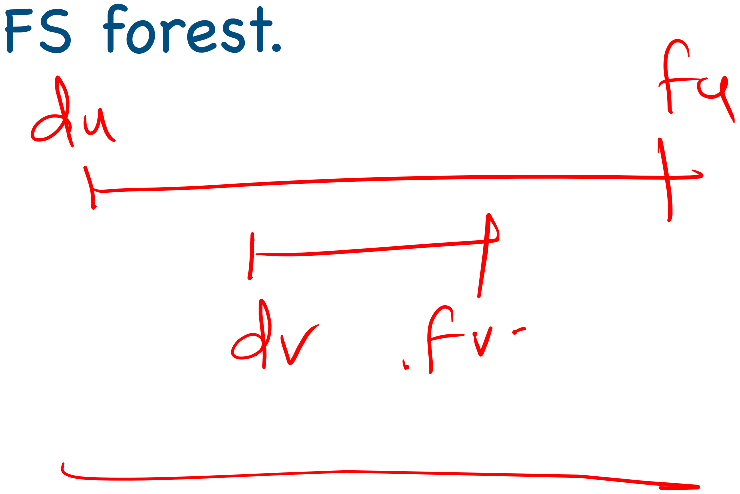


Parenthesis theorem

In any DFS of a directed or undirected graph G = (V,E), for any two vertices u and v, exactly one of the following 3 conditions holds.



1. The intervals [d[u],f[u]] and [[d[v],f[v]] are entirely disjoint. Neither u nor v is a descendant of the other in the DFS forest.



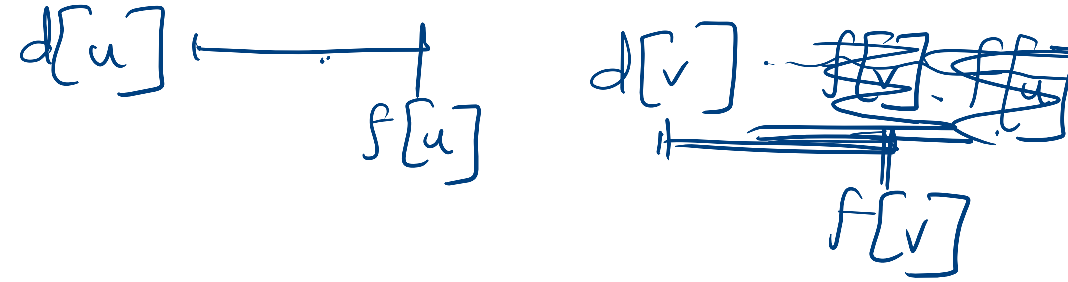
2. [d[u],f[u]] is entirely contained in [[d[v],f[v]] and u is a descendant of v in a DFS tree

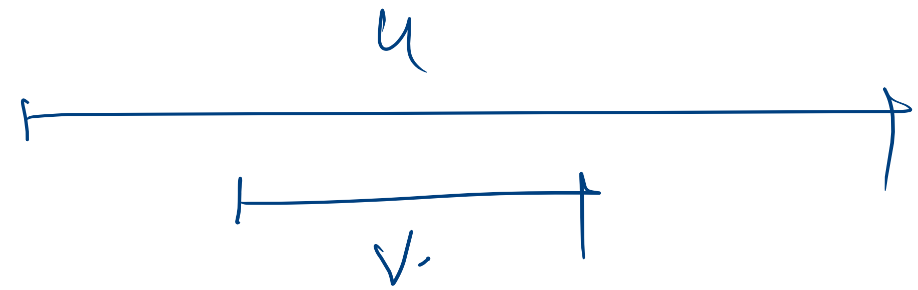
3. [[d[v],f[v]] is entirely contained in [d[u],f[u]] and v is a descendant of u in a DFS tree

Assume, $d[u] < d[v]$

1) $d[v] < f[u]$.

2) $\cancel{d}\, f[u] < d[v]$.



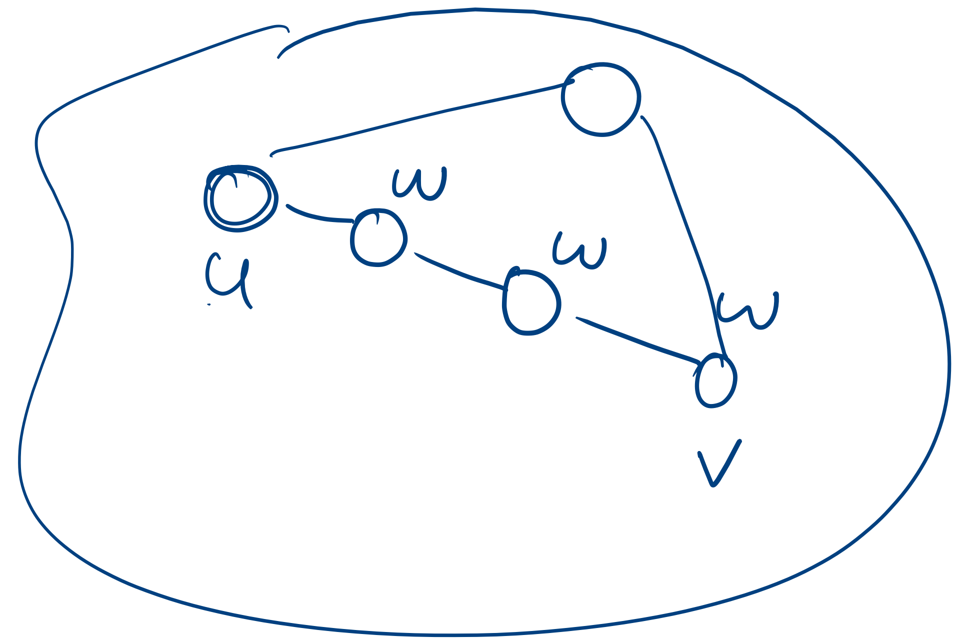


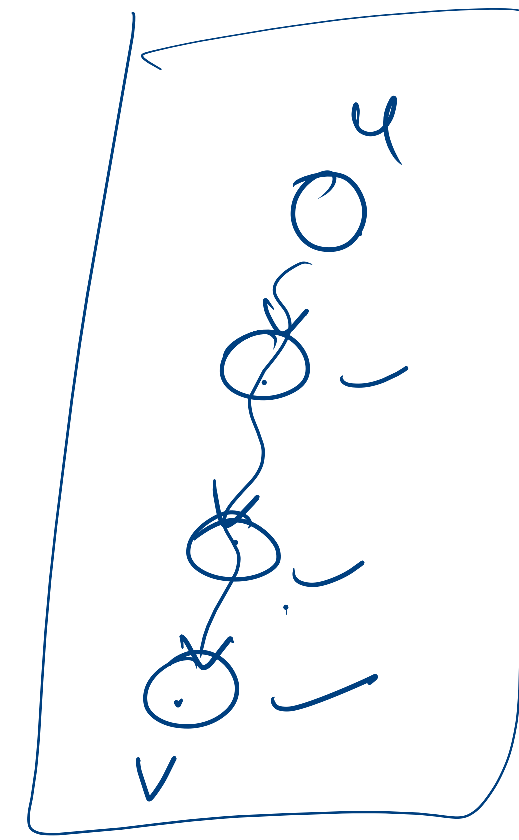
Depth- First Search

Properties of DFS :

If v is a proper descendant of u if and only if $d[u] < d[v] < f[v] < f[u]$.

Depth- First Search





Properties of DFS :

White-Path theorem

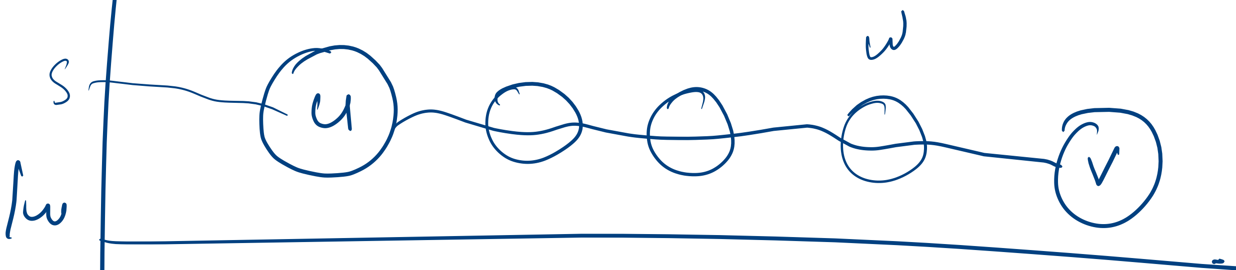
v is a descendant of u if and only if at the time d[u], v can be reached from u along a path consisting entirely of white vertices.

⇒. Let v be a descendant of u.

All vertices in the path b/w u and v in the tree, are white at $d[u]$.

⇐ $\exists$ white $u-v$ path in G at $d[u]$, P

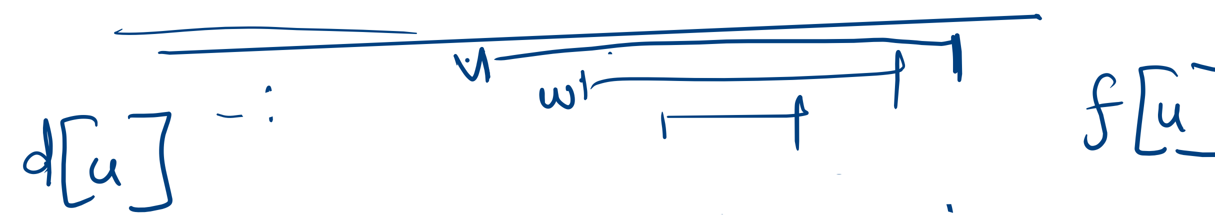
Assume, for contradiction, v is not a descendant of v but every other vertex in P is a descendant of u.

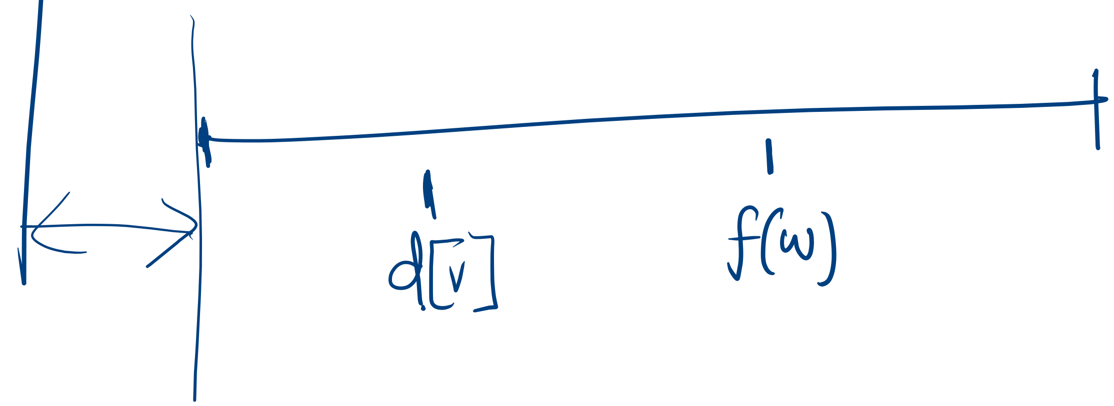


w be the previous vertex to v in P.

w is a descendant of u.

$f(w) \leq f(u)$





Depth- First Search

Properties of DFS :

Classification of edges :

1. Tree Edges :

2. Back Edges :

3. Forward Edges :

4. Cross Edges :

Depth- First Search

Properties of DFS :

Classification of edges :

1. Tree Edges : (u,v) is a tree edge if v was first discovered by exploring edge (u,v)

2. Back Edges :(u,v) where v is an ancestor of u

3. Forward Edges : non-tree edges (u,v) connecting u to a descendant v

4. Cross Edges : Edges between different DFS trees or edges between vertices in the same tree, when they are not ascendant/descendant of each other.