# DAA Problem Set 6

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January 2025

## 1 Dynamic Programming on Graphs

1. Suppose you are given a $2 \times n$ grid graph $G$ with vertex weights $w(v) \geq 0$. The goal is to choose an independent set $S$ of nodes of the grid, so that the sum of the weights of the nodes in $S$ is as large as possible. Provide a dynamic programming based algorithm to find this set $S$.

2. In the same setting as Q1, provide a dynamic programming algorithm to find the minimum weighted dominating set.

3. Consider the following variant for the minimum dominating set problem in trees - in the set $S$, for every node, either the node itself or one or more of its neighbors are present in $S$ (in the previous setting, both a node and its neighbor could be present in the dominating set). Hence this dominating set is also an independent set.

   Given a weighted tree, provide an algorithm to either find such an independent dominating set of minimum weight.

4. Consider the following variant for the minimum dominating set problem in trees - in the set $S$, for every node, either the node itself or *exactly one* of its neighbors is present in $S$ (if the node is present, then none of its neighbors should be present, it may be possible that there does not exist any such set $S$ in the graph).

   Given a weighted tree, provide an algorithm to either find the dominating set of minimum weight which satisfies the above condition, or output that such a set does not exist.

5. A vertex cover of a graph is a set of vertices that includes at least one endpoint of every edge of the graph. Mathematically, for a graph $G(V, E)$ a set $S \subseteq V$ is a vertex cover of $G$ if $\forall (u, v) \in E$, either $u \in S$ or $v \in S$, or both.

   Given a tree with undirected edges, with each node $v$ having a weight $w(v) \geq 0$, find the vertex cover of minimum weight.