IIIT-Bangalore
Statistics
Problem Set 2

(Point Estimation)

1. Prove that the sample mean is consistent and unbiased estimate of the population mean.

2. Prove that a_k, the k-th order sample moment is a consistent and unbiased estimate of α_k, the k-th order moment of the population, provided that the latter exists.

3. If μ_k exists, prove that m_k is a consistent estimate of μ_k.

4. Prove that the sample variance is a consistent but biased estimate of population variance.

5. Show that $s^2 = \frac{n}{n-1}S^2$ is a consistent and unbiased estimate of σ^2, the population variance.

6. Find the maximum likelihood estimate, say $\hat{p}$, of p of Binomial (N, p) population. Prove that $\hat{p}$ is consistent and unbiased estimate of p.

7. Find the maximum likelihood estimate of m and σ^2 for a normal (m, σ) population. Then find an unbiased maximum likelihood estimate of σ^2.

8. Find the maximum likelihood estimate of μ of Poisson μ population.

9. Find the maximum likelihood estimate of the parameter θ in an exponential population with p.d.f. $f(x) = \frac{1}{\theta}e^{-\frac{x}{\theta}}$ $(x > 0, \theta > 0)$ by drawing a random sample $(x_1, x_2, \ldots, x_n)$ of size n. Show that the estimate is consistent and unbiased.

10. Prove that the maximum likelihood estimate of the parameter α from the population with probability density function $f(x; \alpha) = \frac{2(\alpha - x)}{\alpha^2}$, $(0 < x < \alpha)$ for a sample x_1 of unit size is $2x_1$ and the estimate is biased.

11. Let $(y_1, y_2, \ldots, y_n)$ be a random sample from the population with p.d.f.

$$f(y; \theta) = \frac{2\theta^2}{y^3}, \; 0 < \theta \leq y < \infty.$$

Find the MLE of θ.

12. Find the maximum likelihood estimates of the parameters a, b of the population having uniform distribution given by the density function

$$f(x) = \frac{1}{b-a} \text{ if } a \leq x \leq b, \text{ where } b > a.$$