IIIT-Bangalore
Probability and Statistics
Problem Set 3

(Interval Estimation)

1. Find $100 \times (1 - \epsilon)\%$ confidence interval for m for a normal (m, σ) population considering the cases (i) σ known and (ii) σ unknown.

2. Find $100 \times (1 - \epsilon)\%$ confidence interval for σ for a normal (m, σ) population.

3. Find an approximate $100 \times (1 - \epsilon)\%$ confidence interval for p for a binomial (n, p) population.

4. Find 95% confidence interval for the mean of a normal distribution with $\sigma = 3$, given the sample $(2.3, -0.2, -0.4, -0.9)$. (Given $P(U > 1.96) = 0.025$ where U is a normal $(0, 1)$ variate.

5. In random sample of 400 articles 40 are found to be defective. Obtain 95% confidence interval for the true proportion of defectives in the population of such articles. Given

$$\frac{1}{\sqrt{2\pi}} \int_0^{1.96} e^{-\frac{x^2}{2}} \, dx = 0.4750.$$

6. The probability density function of a population random variable X is given by

$$f(x; \alpha) = \begin{cases} \frac{2}{\alpha^2}(\alpha - x), & 0 < x < \alpha, \\ 0, & \text{elsewhere.} \end{cases}$$

Obtain 95% confidence limits for the parameter α on the basis of a random sample x of unit size from the population of X by using the sampling distribution of the statistic $\frac{\alpha - x}{\alpha}$.

7. The population of scores of 10-year children in a test is known to have a standard deviation 5.2. If a random sample of size 20 shows a mean 16.9, find 95% confidence limits for the mean score of the population assuming that the population is normal.

$$\frac{1}{\sqrt{2\pi}} \int_{1.96}^{\infty} e^{-\frac{x^2}{2}} \, dx = 0.025$$

8. Ten individuals are chosen at random from a normal (m, σ) population and their heights in inches are found to be $63, 66, 63, 67, 68, 69, 70, 71, 72, 71$.

On the basis of the above data, obtain 95% confidence interval (i) for the parameter m when σ is unknown and (ii) for the parameter σ. Given

$$P(\chi^2 > 19.023) = 0.025$$
$$P(\chi^2 > 2.700) = 0.975$$
$$P(t(9) > 2.262) = 0.025 \text{ for 9 degrees of freedom.}$$

9. The weights in gram of 12 items are 7, 13, 22, 15, 12, 14, 18, 8, 21, 23, 10, 17 taken at random from its population which is normal having standard deviation 5. Find 95% confidence interval for the mean of the population.

10. The marks obtained by 17 candidates in an examination have a mean 57 and variance 64. Find 99% confidence limits for the mean of the population of marks, assuming it to be normal.