

Discrete Mathematics, Tutorial VII

1. A binary tree is called full if every internal vertex has either two children or no children. Let H_n denote the number of full binary trees with $n + 1$ leaves. Derive a recurrence equation for H_n with initial conditions.
2. Consider an $n \times n$ grid, consisting of n^2 square cells. Suppose you want to travel from the lower left corner to the upper right corner, where you are allowed to move exactly one cell at a time, either to the right or to the top. Then derive a formula for the total number of valid paths possible, satisfying the above constraints.
3. How many diagonals does a convex polygon with n sides have? (Recall that a polygon is convex if every line segment connecting two points in the interior or boundary of the polygon lies entirely within this set and that a diagonal of a polygon is a line segment connecting two vertices that are not adjacent.)
4. By triangulation of a polygon, we mean a way of dividing the polygon into triangles by non-intersecting diagonals. For example, there are two ways to triangulate a rectangle.
 - (a) Draw all possible triangulation of pentagon.
 - (b) Derive a recurrence relation for the number of different triangulation of a polygon with n edges.
5. Let D_n denote the number of derangement of n distinct elements. Derive a recurrence relation for D_n .