01-08-2024

a.out <input.txt >output.txt

Errors

Uncertanties in measurement.
→ human instr.
→ physical environment
→ Random errors.
→ Istrument Drift: errors with overtime use.

1) Incomplete definition
→ you didn't define your problem accurately.

2) Failure to account for all factors.
→ Ex- You didn't consider change in temp.

3) Environmental Factors.

4) Resolution

5) Calibration

6) Zero ~~Effect~~ offset

7) Random

8) Parallax
→ Looking at the reading properly & not from diff. angles.

9) Instrument drift

10) Hysteris
→ Instrument has memory of prev. reading, whatever you do will be incorrect, it can overlap with the current one.

11) Personal Errors

How to represent these errors?

Measurement = (Value ± Standard uncertainty) units

Way to calculate

1) Take a lot of readings

→ When you perform an exp. ~~repitively~~ repeatedly, the mean lies within the probable reality.

→ Take standard deviation
- From mean
- From ground-truth.
- When you have a large amount of data, within the data you can find S.D & if you have ground-truth, you can find another SD.

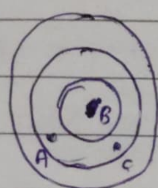
→ For limited no. of measurements & a possible ground-truth, compare with the ground truth & find the relative error.

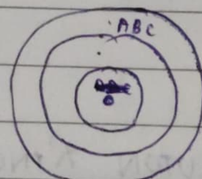
Then calculate Rydberg's Const.

For RLC → do Relative Error Calc. (Since less no. of measurements)
For Newton Rings → do Standard deviation
For Magnetic Field → Since each current will give unique M.F (so even if you have 10 readings, do relative Error Calc.)

2) Representation of Measurement

- → Multiplication (Sig Figuries)
- → Precision (last digit that instrument is able to give you).
- → Accuracy (how close it is to the ground truth).





B is precise & accurate

A, C are neither

A, B, C are all precise (all are around 8.68, 8.69, 8.90)

But the actual value i.e 10 is far from them, so not accurate.

Addition / Multiplication of Errors

Adding or subtraction absolute uncertainties (RMS) of individual is the sum of square of individual RMS.

Multiplication or division relative errors are multiplied.

02-08-2024

Experiment ① (BOUNCING A BALL)

(Observation Time)

$$t_1 = t_n^* \left(\frac{\sqrt{e} - 1}{(\sqrt{e})^2 - 1}\right)$$

$$g = \frac{8h_0}{(t_n^*)^2} \left(\frac{(\sqrt{e})^n - 1}{\sqrt{e} - 1}\right)^2 e$$

Experiment ② (NEWTON RINGS)

Introduction to Electrodynamics
3rd Edition David J. Gri

ELECTROMAGNETICS

05-08-2024

Vector Analysis

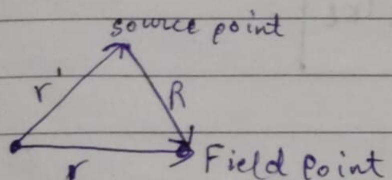
Scalar Triple Product

$$A\cdot(B\times C) = B\cdot(C\times A) = C\cdot($$

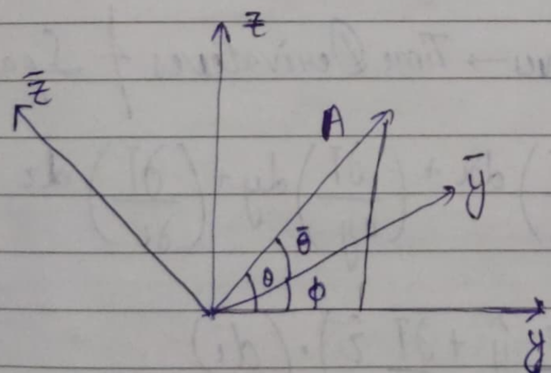
$$r = x\,\hat{x} + y\,\hat{y} + z\,\hat{z}$$

$$|r| = \sqrt{x^2+y^2+z^2} \qquad \hat{r} = \frac{r}{|r|}$$

$$dl = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$$



$$\boxed{R = r - r'}$$



$$\begin{pmatrix} \bar{A}_y \\ \bar{A}_z \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} A_y \\ A_z \end{pmatrix}$$

R_{yx} → what happens in x dir^{n} if press. applied in y dir^{n}.

$$\begin{pmatrix} \bar{A}_x \\ \bar{A}_y \\ \bar{A}_z \end{pmatrix} = \begin{pmatrix} R_{xx} & R_{xy} & R_{xz} \\ R_{yx} & R_{yy} & R_{yz} \\ R_{zx} & R_{zy} & R_{zz} \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

They tell you what happens in other dir^{n}s. if you apply some force / Stress / Pressu. in one dir^{n}.

These are called tensors

$$\bar{T}_{xx} = R_{xx}(R_{xx}T_{xx} + R_{xy}T_{xy} + R_{xz}T_{xz}) + R_{xy}(R_{xx}T_{yx} + R_{xy}T_{yy} + R_{xz}T_{yz}) + R_{xz}(R_{xx}T_{zx} + R_{xy}T_{zy} + R_{xz}T_{zz})$$

$$\begin{pmatrix} T_x \\ T_y \\ T_z \end{pmatrix}$$

$$\boxed{\bar{T}_{ij} = \sum_{k=1}^{3}\sum_{l=1}^{3} R_{ik}R_{jl}R_{kl}}$$

$$\begin{pmatrix} \bar{T}_{xx} & \bar{T}_{xy} & \bar{T}_{xz} \\ \bar{T}_{yx} & \bar{T}_{yy} & \bar{T}_{yz} \\ \bar{T}_{zx} & \bar{T}_{zy} & \bar{T}_{zz} \end{pmatrix} = (R_{matrix}) \begin{pmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{pmatrix}$$

$$\boxed{\bar{T}_{ij} = \sum_{k=1}^{3}\sum_{l=1}^{3} R_{ik}R_{jl}T_{kl}}$$

Derivatives

Two types of derivatives → Time Derivatives & Space Derivatives

Gradient → $dT = \left(\frac{\partial T}{\partial x}\right)dx + \left(\frac{\partial T}{\partial y}\right)dy + \left(\frac{\partial T}{\partial z}\right)dz$

$$dT = \underbrace{\left(\frac{\partial T}{\partial x}\hat{x} + \frac{\partial T}{\partial y}\hat{y} + \frac{\partial T}{\partial z}\hat{z}\right)}_{(\nabla T)(dl)} \cdot (dl)$$

→ Dot product

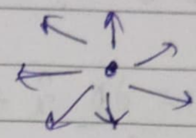
Direction of the slope of two vectors

$dT =$

Divergence Operator

$$\nabla F = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right) \cdot F(x,y,z)$$

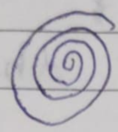
Scalar Field



away is +ve

coming is -ve

~~$\nabla \times T$~~ $\nabla \times T \Rightarrow$ curl Rotation of a given vector $\left\{ \begin{array}{l} \text{ACW} \\ +\text{ve} \end{array} \right.$ & CW -ve



Aim

Apparatus Reqd.

Imp. Formulae

Tabular Form

Precaution

Gradient :- $\underbrace{\left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right)}_{\text{gradient}} \cdot T(x,y,z)$

$\cdot (dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z})$

↳ If you do dot with dI dir^{n} vector

$\nabla T \cdot (dI) \rightarrow$ ~~Direction of slope of vector~~.

$\rightarrow |\nabla T| \cdot |dI| \cos\theta$

It is the direction of the slope of vector, that you can get by

$$\boxed{\nabla T = \frac{\partial T}{\partial x}\hat{x} + \frac{\partial T}{\partial y}\hat{y} + \frac{\partial T}{\partial z}\hat{z}}$$

$$\boxed{\nabla T \cdot dI = dT}$$

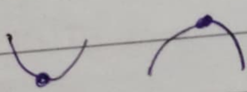
$r = \sqrt{x^2+y^2+z^2} \nearrow \frac{dr}{dx} = \frac{x}{\sqrt{x^2+y^2+z^2}}$

$\nabla r = \frac{\partial r}{\partial x}\hat{x} + \frac{\partial r}{\partial y}\hat{y} + \frac{\partial r}{\partial z}\hat{z}$

$= \frac{x\hat{x} + y\hat{y} + z\hat{z}}{\sqrt{x^2+y^2+z^2}} = \frac{\mathbf{r}}{|r|} = \hat{r}$ the unit vector

if $\nabla T = 0$ at (x, y, z)
$dT = 0$ around that point

Such a point is called stationary point.



Operator : Divergence

$\nabla \cdot \vec{V} \rightarrow$ vector

gradient operator

$\rightarrow$ dot product which gives scalar i.e the divergence

-ve means coming in
+ve means going away

Curl Operator

$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$

$\underbrace{\nabla \times \vec{V}}$

Eg: In fluids
$\nabla \times \vec{V}_{\text{liquid}} =$ Vorticity

$\hookrightarrow$ This gives us the rotation of a particular vector $\vec{V}$.

Product Rules

$$\frac{d(f+g)}{dx} = \frac{df}{dx} + \frac{dg}{dx}$$

$f, g \rightarrow$ scalar fuⁿ

$$\frac{d}{dx}(Kf) = K\frac{df}{dx} \qquad \frac{d}{dx}(fg) = f\frac{dg}{dx} + g\frac{df}{dx}$$

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \cancel{\frac{f\,dg - g\,df}{}}\ \frac{g\,df - f\,dg}{g^2}$$

$\nabla(f+g) = \nabla f + \nabla g$, $\nabla(Kf) = K\nabla f$

$\nabla\cdot(K\vec{A}) = K(\nabla\cdot\vec{A})$, $\nabla\times(KA) = K(\nabla\times\bar{A})$

$\nabla(A+B) = \nabla\cdot A + \nabla\cdot B$, $\nabla\times(A+B) = \nabla\times A + \nabla\times B$

6 Product Rules

1. $\nabla\cdot(fg) = f\nabla g + g\nabla f$
2. $\nabla(A\cdot B) = A\times(\nabla\times B) + B\times(\nabla\times A) + (A\cdot\nabla)B + (B\cdot\nabla)A$

Note: $\underbrace{(V\cdot\nabla)V}_{\text{Advective Derivative}} \longrightarrow$ In Naviour Stokes eqⁿ

↳ in fluids it is the origin of all non-linearities.

3. $\nabla\cdot(\overset{\text{scalar fuⁿ}}{f}\overset{\text{vector}}{A}) = f(\nabla\cdot A) + A(\nabla\cdot f)$
4. $\nabla\cdot(A\times B) = B\cdot(\nabla\times A) - A(\nabla\times B)$
5. $\nabla\times(fA) = f(\nabla\times A) - A\times(\nabla f)$
6. $\nabla\times(A\times B) = (B\cdot\nabla)A - (A\cdot\nabla)B + A(\nabla\cdot B) - B(\nabla\cdot A)$

7. $\nabla(f/g) = \dfrac{g\nabla f - f\nabla g}{g^2}$
8. $\nabla(\bar{A}/g) = \dfrac{g(\nabla\cdot A) - A\cdot(\nabla g)}{g^2}$
9. $\nabla\times(\vec{A}/g) =$

Second Derivatives

1. Divergence of gradient $\nabla \cdot (\nabla T)$
2. Curl of a gradient $\nabla \times (\nabla T)$
3. Gradient of Divergence
4. Divergence of Curl
5. Curl of a curl

1 Divergence of Gradient

$$\nabla \cdot (\nabla T) = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$$

→ Laplacian

↓

Δ or ∇^2

Fourier transform is used to ~~make~~ convert differential eqⁿ to algebraic eqⁿs.

$$\left.\begin{array}{l} f \leftrightarrow t \\ x \leftrightarrow k \end{array}\right\} \text{Fourier Transform}$$

2 Curl of a gradient

$$\nabla \times (\nabla T) = 0$$

↓

$$\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\partial T}{\partial x}\right)$$

Equality of cross derivatives

(3) Gradient of Divergence

$\nabla(\nabla \cdot V)$

(4) Divergence of a Curl

$\nabla \cdot (\nabla \times V) = 0$ $\quad\left(A \cdot (B \times C) = (A \times B) \cdot C\right)$

(5) Curl of a Curl

$$\nabla \times (\nabla \times V) = \underbrace{\nabla(\nabla \cdot V)}_{\text{curl}} - \underbrace{\nabla^2 V}_{\text{Laplacian}}$$

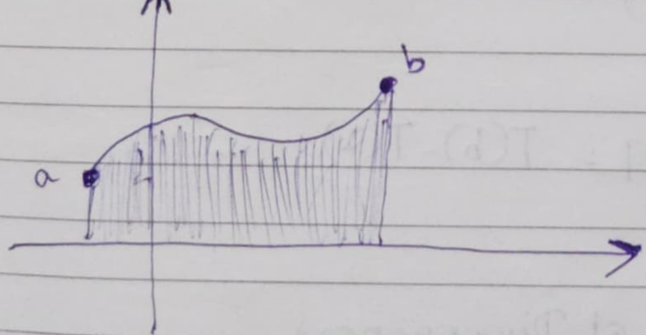
Laplacian → tell spreading out of 'V'

→ tells How the divergence of 'V' changes in space

Integrals

1. Line Integral (Path Integral)
2. Surface Integral
3. Volume Integral

(1)



$\int_{a}^{b} V \cdot dl = V(b) - V(a)$

$\underset{\downarrow}{P}$ Path

$\oint$ Closed Path

$\oint V \cdot dl$

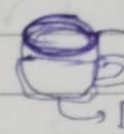
If vector is conservative then it is the diff. b/w the two end pts. of the fn rather than the whole path i.e dirn doesn't matter

$$\int_{a\,P}^{b} V\,dl = V(b) - V(a)$$

For closed path i.e $\oint V\,dl = 0$

② $\cancel{\oint v \cdot da}$ $\int_S v \cdot da$

→ Surface Integrals are not same from diff. dir^{n}s.
→ Line Integrals, sign ~~...~~ change is there but still same
→ Vol. Integrals are same, bcz it covers the entire volume.



Be careful while choosing the surface.
Select the surface which is easy to integrate.

③ $\int_V T \cdot d\tau$ $\qquad d\tau = dx\,dy\,dz$

Fundamental Theorem of Calculus

$$\int_a^b \frac{df}{dx}\,dx = f(b) - f(a)$$

$$\int_{a\,P}^{b} (\nabla T)\cdot dl = \int_{a\,P}^{b} dT = T(b) - T(a)$$

Fundamental Theorem of Divergences

$$\int_V (\nabla \cdot v)\,d\tau = \oint_S v \cdot da$$ → Gauss Theorem

If the given fn or surface is easily differentiable → [go for vol. integral]

i.e $\int_V (\nabla \cdot v) d\tau$ → easy differentiation then vol.

& If (da) is that is (taken wrt. the surface normal) is easier to consider then do surface integral.

$$\oint_S v \cdot da$$

⇓

It might be easy with (x, y, z) but $(\theta, \phi, ..)$ comes, the argument becomes complicated

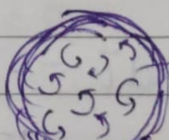
Fundamental Theorem of Curls

$$\int_S (\nabla \times V)\, da = \oint_P V \cdot dl$$

– how much vector field 'V' rotates along the loop

surface   : line

→ STOKES THEOREM



→ Surface Integrals considers all internal rotations but the net will be rotation of the perimeter.

[Bcz some internal rotations will cancel each other]

Corollary: $\int (\nabla \times V)\, da$ depends only on the boundary line not on the particular surface used.

Corollary $\oint (\nabla \times V)\, da = 0$

There is no more curls → nothing to rotate.

Integration By Parts

$$\frac{d}{dx}(fg) = f\left(\frac{dg}{dx}\right) + g\left(\frac{df}{dx}\right)$$

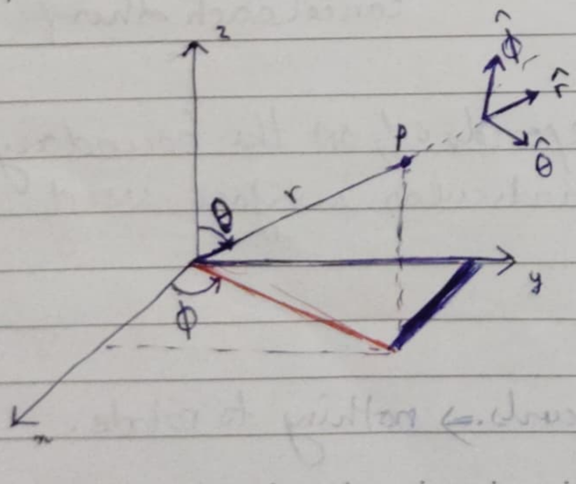
$$\int_a^b \frac{d}{dx}(fg)\,dx = fg\Big|_a^b = \int_a^b f\left(\frac{dg}{dx}\right) + \int_a^b g\left(\frac{df}{dx}\right)$$

$$\nabla\cdot(fA) = f(\nabla\cdot A) + A\cdot(\nabla f)$$

$$\int \nabla\cdot(fA)\,d\tau = \int f(\nabla\cdot A)\,d\tau + \int A\cdot(\nabla f)\,d\tau$$

$$\int_V f(\nabla\cdot A)\,d\tau = -\int_V A\cdot(\nabla f)\,d\tau + \oint_S fA\,da$$

Spherical Co-ordinates



Taking θ cw
& φ Acw is just convention
[Earth moves from W → E, so we feel the opposite way]

$x = r\sin\theta\cos\phi$

$y = y\sin\theta\sin\phi$

$z = r\cos\theta$

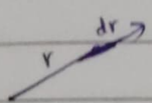
$\hat{r} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}$

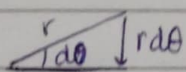
$\hat{\theta} = \cos\theta\cos\phi\,\hat{x} + \cos\theta\sin\phi\,\hat{y} - \sin\theta\,\hat{z}$

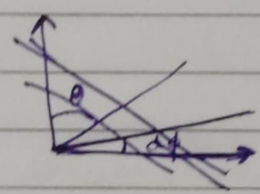
$\hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}$

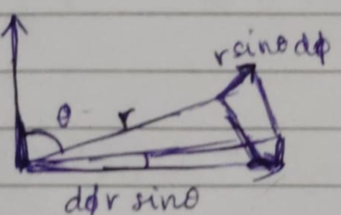
☆ Beware of differentiating the unit vectors.

<u>Unit Displacement Vector</u> $dl = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$









$d\tau = dl_r\, dl_\theta\, dl_\phi$

$= r^2 \sin\theta\, dr\, d\theta\, d\phi$

$da = r^2 \sin\theta\, d\phi\, d\theta\, \hat{r}$

$r\, dr\, d\phi\, \hat{\theta}$

$\theta \to 0$ to π

$\phi \to 0$ to 2π

<u>Vol. of spherical ~~spot~~ surface</u>

Radius

$$V = \int d\tau = \int_0^R \int_0^\pi \int_0^{2\pi} r^2 \sin\theta\, dr\, d\theta\, d\phi = \left(\frac{R^3}{3}\right)(2)(2\pi) = \frac{4\pi R^3}{3}$$

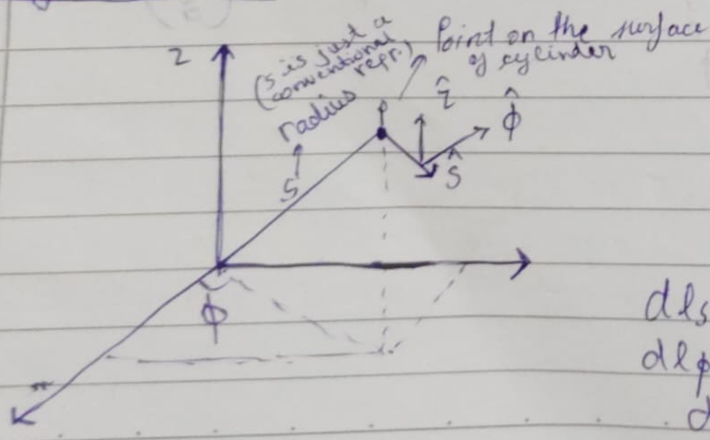
<u>Gradient</u>

6P

Divergence

Curl

Cylindrical



$x = s\cos\phi$
$y = s\sin\phi$
$z = z$

$dl_s = ds$
$dl_\phi = s\,d\phi$
$dl_z = dz$

$\hat{s} = \cos\phi\ \hat{x} +$

URBAN EDGE

$$\nabla T = \frac{\partial T}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial T}{\partial \phi}\hat{\phi} + \frac{\partial T}{\partial z}\hat{z}$$

$$\nabla \cdot V = \frac{1}{s}\frac{\partial}{\partial s}(sV_s) + \frac{1}{s}\frac{\partial V_\phi}{\partial \phi} + \frac{\partial V_z}{\partial z}$$

$$(\nabla \times V) = \left(\frac{1}{s}\frac{\partial V}{\partial \phi} - \frac{\partial V_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial V_s}{\partial z} - \frac{\partial V_z}{\partial s}\right)\hat{\phi}$$

Q. $V = \frac{1}{r^2}\hat{r}$. Verify Gauss Theorem.

$\int(\nabla \cdot V)\,d\tau \rightarrow$ LHS

$\oint V \cdot da \rightarrow$ RHS

LHS $= \nabla \cdot V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \cdot \frac{1}{r^2}\right) = 0$

RHS $= \int\left(\frac{1}{R^2}\hat{r}\right)(R^2 \sin\theta\, d\theta\, d\phi\, \hat{r}) = 4\pi$

This is different

13-08-2024

~~$\int f(x)\,\delta(x) = f(0)$~~

GREEN'S THEOREM

If 'C' is a positively oriented piece wise smooth simple closed curve in a plane & let "D" be the region bounded by 'C'.

If L & M are fu$^{(n)}$s. of (x,y) defined on a open region ~~Define~~ containing 'D' & have continuous partial derivatives then. –

$$\oint_C (L\,dx + M\,dy) = \iint_D \left(\frac{\partial M}{\partial x} - \frac{\partial L}{\partial y}\right) dA$$

This theorem in principle can be reduced to Divergence Theorem & Stokes Theorem by choosing L, M carefully.

Q. Construct vector fu$^{(n)}$s. that has zero div & zero curl everywhere.

Ans. A simple constant but to make it more interesting.

1. $V = y\hat{x} + x\hat{y}$
2. $V = yz\hat{x} + xz\hat{y} + xy\hat{z}$
3. $V = (3x^2 z - z^3)\hat{x} + 3\hat{y} + (x^3 - 3xz^2)\hat{z}$
4. $V = (\sin x)(\cosh y)\hat{x} - (\cos x)(\sinh y)\hat{y}$

1.17 Suppose f is a fu^{n} of two variables y & z. S.T

$\nabla f = \left(\frac{\partial f}{\partial y}\right)\hat{y} + \left(\frac{\partial f}{\partial z}\right)\hat{z}$ transform as a vector under rotations.

Ans. $\bar{y} = y\cos\phi + z\sin\phi$

$\bar{z} = -y\sin\phi + z\cos\phi$

$\bar{y} \times \sin\phi$

$\bar{z} \times \cos\phi$

$\bar{y}\sin\phi + \bar{z}\cos\phi = z$

$\bar{y} \times \cos\phi$

$- \bar{z} \times \sin\phi$

$\bar{y}\cos\phi - \bar{z}\sin\phi = y$

$\frac{\partial y}{\partial \bar{y}} = \cos\phi \qquad \frac{\partial z}{\partial \bar{z}} = \cos\phi$

$\frac{\partial y}{\partial \bar{z}} = -\sin\phi \qquad \frac{\partial z}{\partial \bar{y}} = \sin\phi$

$(\bar{\nabla} f)_y = \frac{\partial f}{\partial \bar{y}} = \frac{\partial f}{\partial y}\frac{\partial y}{\partial \bar{y}} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial \bar{y}}$

$(\bar{\nabla} f)_z = \frac{\partial f}{\partial \bar{z}} = \frac{\partial f}{\partial y}\frac{\partial y}{\partial \bar{z}} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial \bar{z}}$

ST that divergence transform as a scalar under rotation.

$\bar{V}_y = \cos\phi\, V_y + \sin\phi\, V_z$

$\bar{V}_z = -\sin\phi\, V_y + \cos\phi\, V_z$

Fill The Gap

$\exists\ \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$

~~SI divergence transform as a scaler under rotation~~

Q. A & B are vector fu^n.

$$(A\cdot\nabla)B = \left(A_x \frac{\partial B_x}{\partial x} + A_y \frac{\partial B_x}{\partial y} + A_z \frac{\partial B_x}{\partial z}\right)\hat{x}$$
$$+ \left(A_x \frac{\partial B_y}{\partial x} + A_y \frac{\partial B_y}{\partial y} + A_z \frac{\partial B_y}{\partial z}\right)\hat{y}$$
$$+ \left(A_x \frac{\partial B_z}{\partial x} + A_y \frac{\partial B_z}{\partial y} + A_z \frac{\partial B_z}{\partial z}\right)\hat{z}$$

Q. $\hat{r} = \frac{r}{|r|}$ P.T

$$((\hat{r}\cdot\nabla)\hat{r})_x = 0$$

Q. $V_1 = r^2\hat{r}$ Using the vol. of the sphere of Radius R centered at the origin.

$$\nabla\cdot V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2(r^2)\right) = \frac{1}{r^2}4r^3 = 4r$$

$$\int(\nabla\cdot V_1)\,d\tau = \int 4r\,(r^2 \sin\theta\, dr\, d\theta\, d\phi) = 4\pi R^3$$

$$\int V_1\, da = \int (r^2\hat{r})\cdot(r^2\sin\theta\, d\theta\, d\phi)\,\hat{r}$$
$$= r^4 \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\phi = 4\pi R^4$$