Mathematics-3
Tutorial-4
Discussion on Friday, 30th August
Topic: Continuous Random Variables

1.

For a nonnegative random variable Y,

$$E[Y] = \int_0^\infty P\{Y > y\}\, dy$$

2. From the above result, for any function g for which $g(x) \geq 0$, show that

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) f(x)\, dx$$

3.

The density function of X is given by

$$f(x) = \begin{cases} a + bx^2 & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

If $E[X] = \frac{3}{5}$, find a and b.

4.

The random variable x is $N(5, 2)$ and $y = 2x + 4$. Find η_y, σ_y, and $f_y(y)$.

5.

Find $F_y(y)$ and $f_y(y)$ if $y = -4x + 3$ and $f_x(x) = 2e^{-2x}U(x)$.

Where U(x) =1 if x>=0, otherwise 0

6.

Let X be a normal random variable with mean 12 and variance 4. Find the value of c such that $P\{X > c\} = 0.10$

7.

The lifetime in hours of an electronic tube is a random variable having a probability density function given by

$$f(x) = xe^{-x} \qquad x \geq 0$$

Compute the expected lifetime of such a tube.

8.

Consider the function

$$f(x) = \begin{cases} C(2x - x^3) & 0 < x < \frac{5}{2} \\ 0 & \text{otherwise} \end{cases}$$

Could f be a probability density function? If so, determine C. Repeat if $f(x)$ were given by

$$f(x) = \begin{cases} C(2x - x^2) & 0 < x < \frac{5}{2} \\ 0 & \text{otherwise} \end{cases}$$

9.

Compute $E[X]$ if X has a density function given by

(a) $f(x) = \begin{cases} \frac{1}{4}xe^{-x/2} & x > 0 \\ 0 & \text{otherwise} \end{cases}$;

(b) $f(x) = \begin{cases} c(1 - x^2) & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$;

(c) $f(x) = \begin{cases} \frac{5}{x^2} & x > 5 \\ 0 & x \le 5 \end{cases}$.

10.

If X is an exponential random variable with parameter $\lambda = 1$, compute the probability density function of the random variable Y defined by $Y = \log X$.

If X is uniformly distributed over $(0, 1)$, find the density function of $Y = e^X$.

11.

Let X be a continuous random variable with the following PDF:

$$f_X(x) = \begin{cases} 2x & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Let also

$$Y = g(X) = \begin{cases} X & 0 \le X \le \frac{1}{2} \\ \frac{1}{2} & X > \frac{1}{2} \end{cases}$$

Find the CDF of Y.

12. (from Q-11)

Let Y be the mixed random variable defined in Example

a. Find $P(\frac{1}{4} \leq Y \leq \frac{3}{8})$.
b. Find $P(Y \geq \frac{1}{4})$.
c. Find EY.

13.

Let X be a $Uniform(-2,2)$ continuous random variable. We define $Y = g(X)$, where the function $g(x)$ is defined as

$$g(x) = \begin{cases} 1 & x > 1 \\ x & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the CDF and PDF of Y.