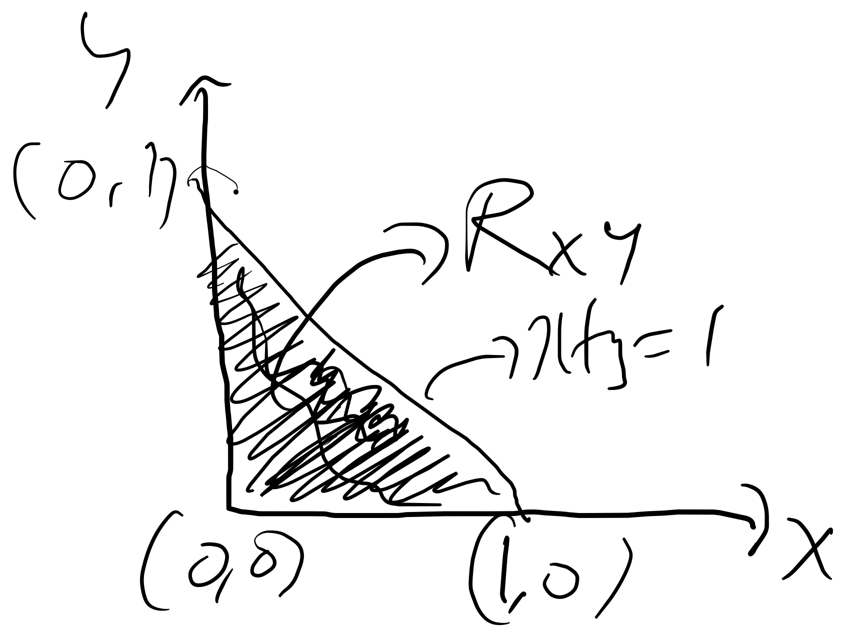
$$f_{X,Y}(x,y) = \begin{cases} c(x+1) & x,y \geq 0 \quad x+y < 1 \\ 0 & \text{obe} \end{cases}$$



a) $\longrightarrow$

b) $$\int_{x=0}^{1}\int_{y=0}^{1-x} (x+1)\,dy\,dx$$

$$= \int_{x=0}^{1} (x+1)(1-x)\,dx$$

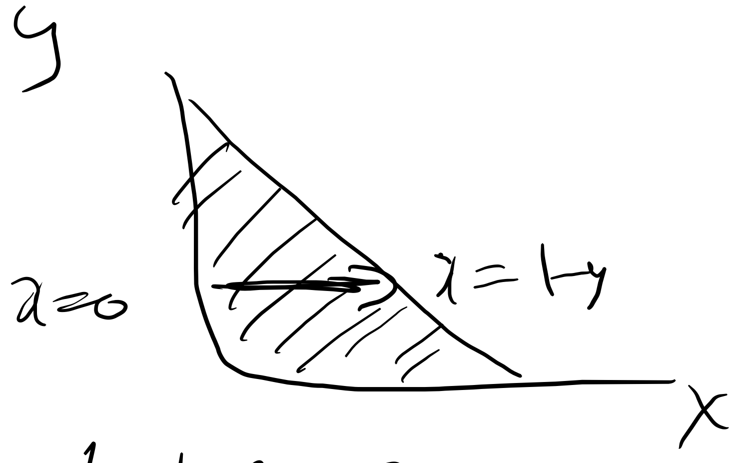
$$\Rightarrow \frac{c}{6} = \frac{1}{2} \Rightarrow \boxed{c=3}$$

c) $$f_X(x) = \int_{y=0}^{1-x} f_{X,Y}(x,y)\,dy = \begin{cases} (3x+1)(1-x) & 0 \leq x \leq 1 \\ 0 & \text{obe} \end{cases}$$

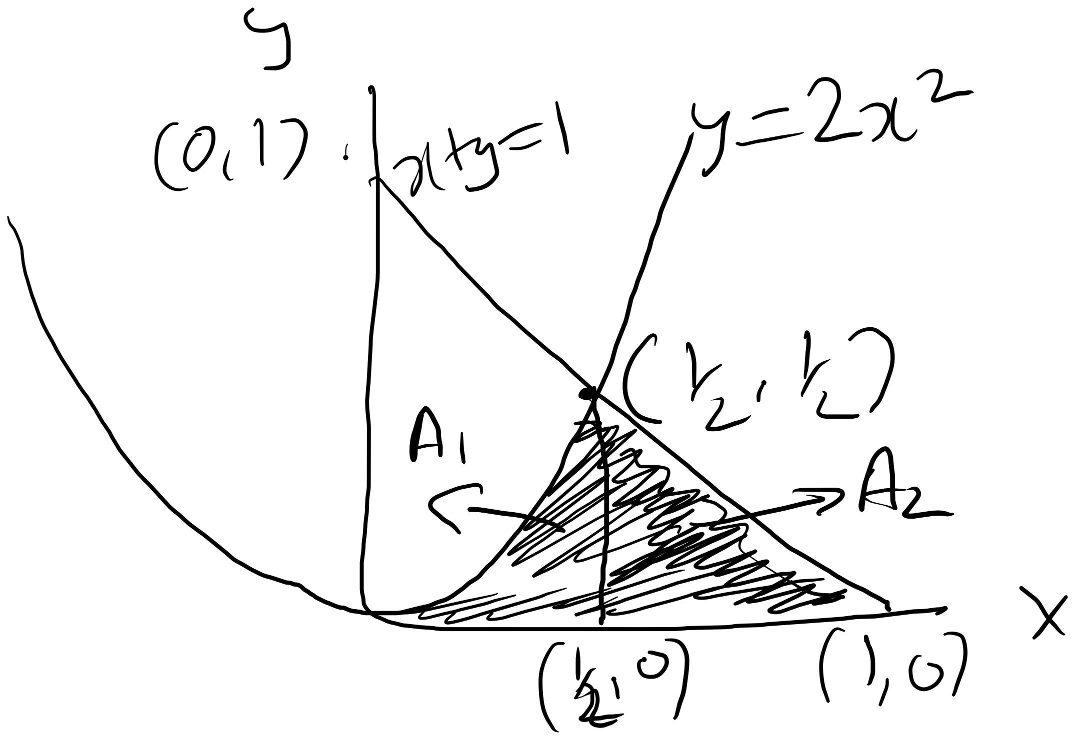
$$f_Y(y) = \int_{x=0}^{1-y} f_{X,Y}(x,y)\,dx$$

$$= \int_{x=0}^{1-y} (3x+1)\,dx$$

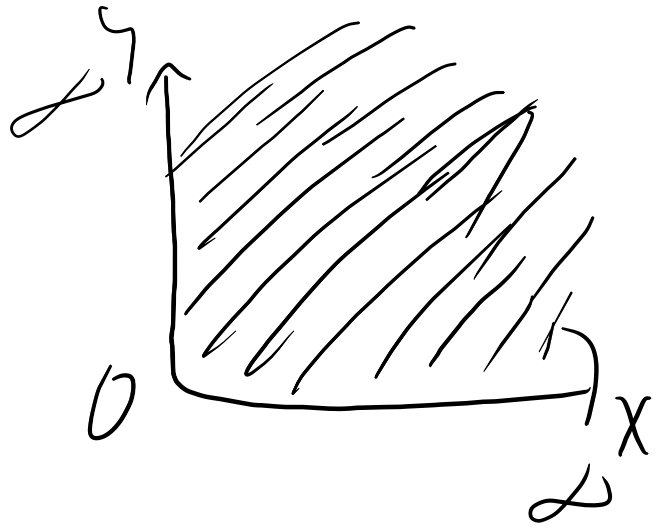
$$= \begin{cases} \dfrac{3(1-y)^2}{2} + (1-y) & 0 \le y \le 1 \\ 0 & \text{dbe} \end{cases}$$



d) $$\int_{x=0}^{1/2} \int_{y=0}^{2x^2} (3x+1)\,dy\,dx + \int_{x=1/2}^{1} \int_{y=0}^{1-x} (3x+1)\,dy\,dx = \frac{53}{96}$$



2)
$$f_{X,Y}(x,y) = \begin{cases} 6e^{-(2x+3y)} & x,y \geq 0 \\ 0 & \text{else} \end{cases}$$



i) X, Y independent?

$$f_X(x) = \int_{y=0}^{\infty} f_{XY}(x,y)\,dy$$

$$= 6e^{-2x}\int_{y=0}^{\infty} e^{-3y}\,dy = 2e^{-2x}$$

$$\therefore\ f_X(x) = \begin{cases} 2e^{-2x} & x \geq 0 \\ 0 & \text{else} \end{cases}$$

$$f_Y(y) = \int_{x=0}^{\infty} f_{XY}\,dx = \begin{cases} 3e^{-3y} & y \geq 0 \\ 0 & \text{else} \end{cases}$$

$$f_X(x)f_Y(y) = \begin{cases} 6e^{-2x}e^{-3y} & x,y \geq 0 \\ 0 & \text{else} \end{cases}$$

$$= f_{X,Y}(x,y)$$

$\therefore$ Independent

b) $E(\cancel{X}Y \mid X>2) \longrightarrow E(Y)$ as they are independent

$$E(Y) = \int_{y=0}^{\infty} y\, f_Y(y)\,dy = \int y\, 3e^{-3y}\,dy = 3\int_{y=0}^{\infty} y\, e^{-3y}\,dy$$

MV

$$= 3 \times \frac{1}{9} = \frac{1}{3}$$

$f_{Y|X>2}(y|x>2)$

c) $P(X>Y) = \int_{x=0}^{\infty}\int_{y=0}^{x} f_{XY}\,dy\,dx$

$$= \frac{3}{5}$$

Problem 3

Let X be a continuous random variable with PDF

$$f_X(x) = \begin{cases} 2x & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

We know that given $X = x$, the random variable Y is uniformly distributed on $[-x, x]$.

1. Find the joint PDF $f_{XY}(x, y)$.
2. Find $f_Y(y)$.
3. Find $P(|Y| < X^3)$.

$$f_{Y|X=x}(y|x) = \begin{cases} \frac{1}{2x} & y \in [-x, x] \\ 0 & \text{else} \end{cases}$$

$$f_{\cancel{X}|X=x}(y|x) = \frac{f_{XY}(x,y)}{f_X(x)}$$

$$f_{XY}(x,y) = \begin{cases} 1 & 0 \le x \le 1 \\ & -x \le y \le x \\ 0 & \text{else} \end{cases}$$

$$\Downarrow$$

$$|y| \le x \le 1$$

$$f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x,y) \cdot dx$$

$$= \int_{|y|}^{1} 1 \cdot dx = 1 - |y|$$

$$\underbrace{P\left(|Y| < X^3\right)}_{f_{|Y|<}} = \int_0^1 \boxed{P\left(|Y| < X^3 \mid X = x\right)} \cdot f_X(x) \cdot dx$$

$$= \int_0^1 2x \cdot \frac{1}{\cancel{2x}} \cdot \cancel{2x} \cdot dx$$

$$= \frac{1}{2}$$

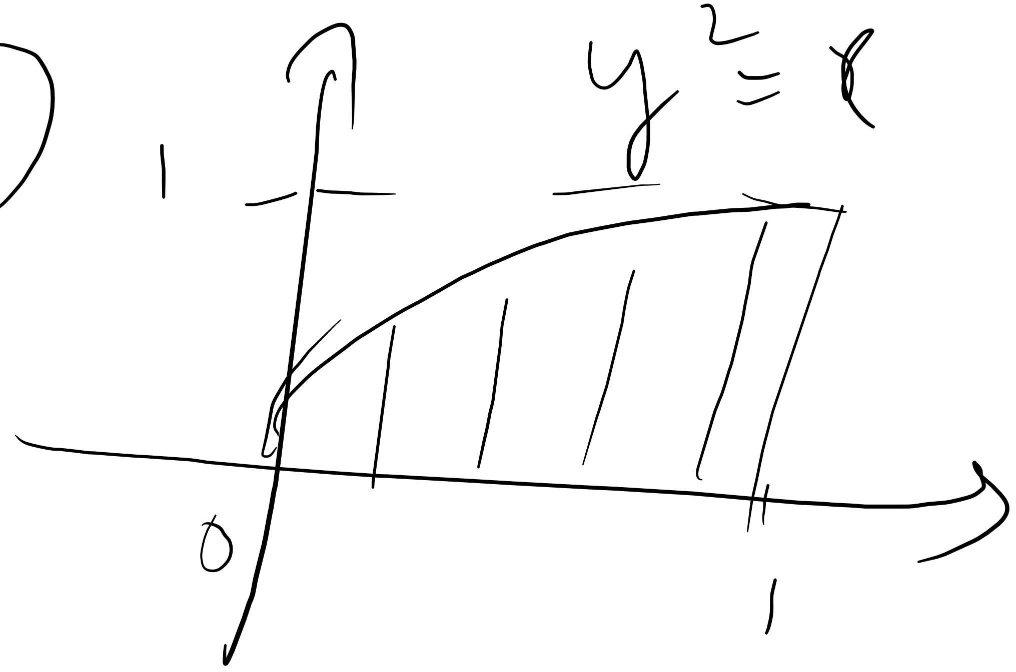
Problem 4

Let X and Y be two jointly continuous random variables with joint PDF

$$f_{X,Y}(x,y) = \begin{cases} 6xy & 0 \le x \le 1, 0 \le y \le \sqrt{x} \\ \\ 0 & \text{otherwise} \end{cases}$$

1. Show R_{XY} in the $x-y$ plane.
2. Find $f_X(x)$ and $f_Y(y)$.
3. Are X and Y independent?
4. Find the conditional PDF of X given $Y=y$, $f_{X|Y}(x|y)$.
5. Find $E[X|Y=y]$, for $0 \le y \le 1$.
6. Find $\mathrm{Var}(X|Y=y)$, for $0 \le y \le 1$.

① ,



$0 \le x \le 1 \quad 0 \le y \le \sqrt{x}$

2

$f_X(x) \qquad f_Y(y)$

$$f_X(x) = \begin{cases} 3x^2 & 0 \le x \le 1 \\ 0 & \text{else} \end{cases}$$

$$f_Y(y) = \begin{cases} 3y(1-y^4) & 0 \le y \le 1 \\ 0 & \text{else} \end{cases}$$

$$f_{X,Y}(x,y) \neq f_X(x) \cdot f_Y(y)$$

(g) $f_{X|Y=y}(x|y) = \dfrac{f_{XY}(x,y)}{\underbrace{f_Y(y)}}$

$$= \begin{cases} \dfrac{2x}{1-y^4} & y^2 \le x \le 1 \\ 0 & \text{else} \end{cases}$$

(3) $E[X \mid Y = y] = \int_{-\infty}^{\infty} x \cdot f_{X \mid Y = y}(x \mid y) \cdot dx$

$= \int_{y^2}^{1} x \cdot \left(\frac{2x}{1 - y^4} \right) dx$

$= \frac{2(1 - y^6)}{1 - y^4}$

$$\boxed{Var = E(X^2) - (E(X))^2}$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 \cdot f_{X|Y=y}(x|y) \cdot dx$$

X and Y are two R.V. with joint PDF $f(x,y) = \frac{1}{8}(6-x-y), 0<x<2, 2<y<4$

Find (i) $P(x<1, y<3)$ (ii) $P(x+y<3)$, (iii) $P(x<1 \mid y<3)$

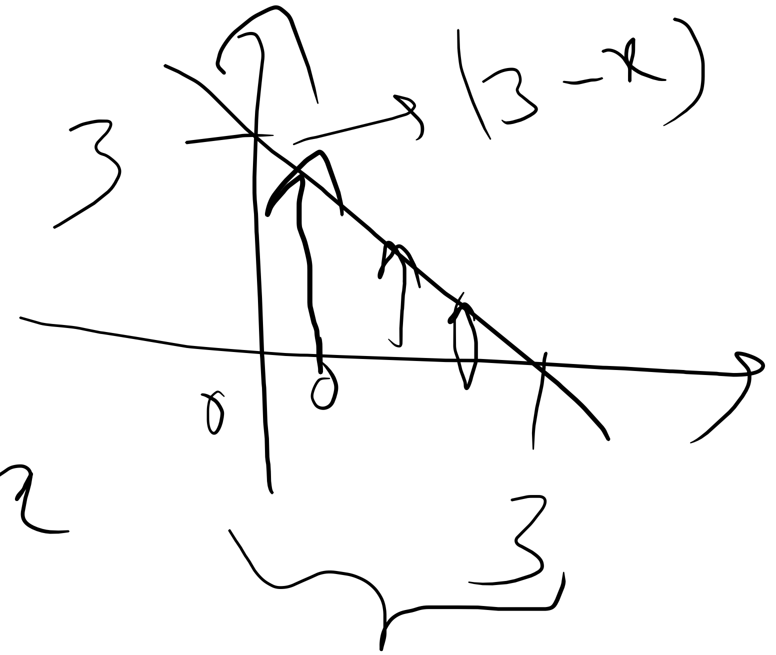
$$f(x,y) = \frac{1}{8}(6-x-y) \qquad \begin{matrix} 0 < x < 2 \\ 2 < y < 4 \end{matrix}$$

(i) $$P(X<1, y<3) = \iint f_{XY}(x,y)\, dx\, dy$$

$$= \int_0^1 \int_2^3 \frac{1}{8}(6-x-y)\, dx\, dy$$

(ii) $P(x + y < 3)$

$$\int_0^3 \int_0^{3-x} \left(f_{XY}(x, y)\, dy \right) dx$$



(iii) $P(x < 1 \mid y < 3) = \dfrac{\boxed{P(x \leq 1, y \leq 3)}}{\boxed{P(y < 3)}}$

$$P(y < 3) = \int_0^{3} \int \left(f_{X,Y}(x, y)\, dy \right) dx$$

5) $F_{XY}(x,y) = 1 - e^{-x} - e^{-y} + e^{-(x+y)} \quad x,y \geq 0$

$f_{XY}(x,y) = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(F_{XY}(x,y)\right)\right)$

$= \frac{\partial}{\partial x}\left(e^{-y} - e^{-y}e^{-x}\right)$

$= e^{-x}e^{-y} \quad // \quad x \geq 0, y > 0$

$0 \quad \text{dde}$