Kleene's Theorem & Properties of Regular Languages

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2025-08-14

Kleene's Theorem

The following are equivalent for a language L:

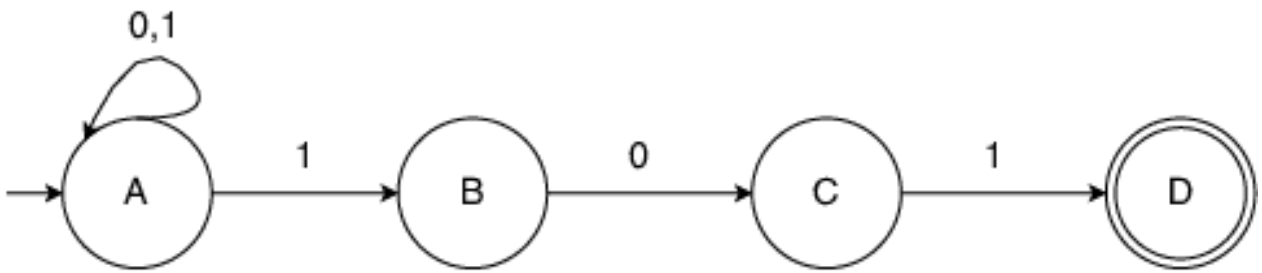
- There is an NFA for L.
- There is a DFA for L.
- There is an RE for L.

Conversion from NFA (without ϵ-transitions) to DFA

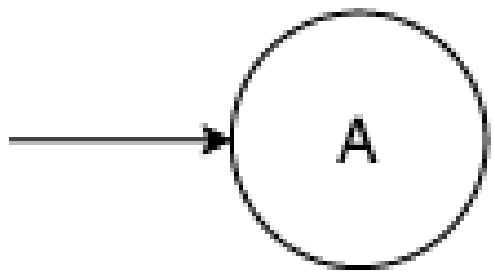
This is known as *subset construction*.

- Each DFA state is given by a set of states from the original NFA.
- The start state of the DFA is labeled $\{q_0\}$ where q_0 was the original start state of the NFA.
- Compute the transitions for the DFA states by combining the possibilities for each state in the NFA.
- Any DFA state that contains at least one NFA accept state is itself an accept state.

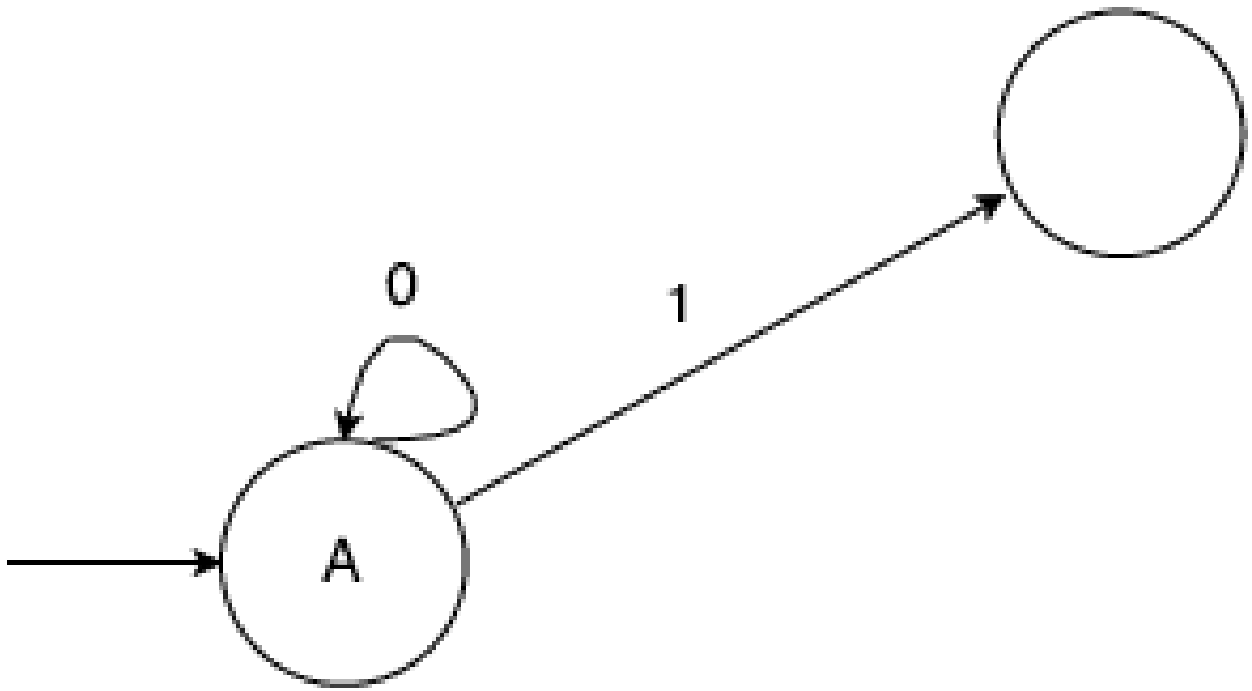
Converting from an NFA (without ϵ-transitions) to a DFA



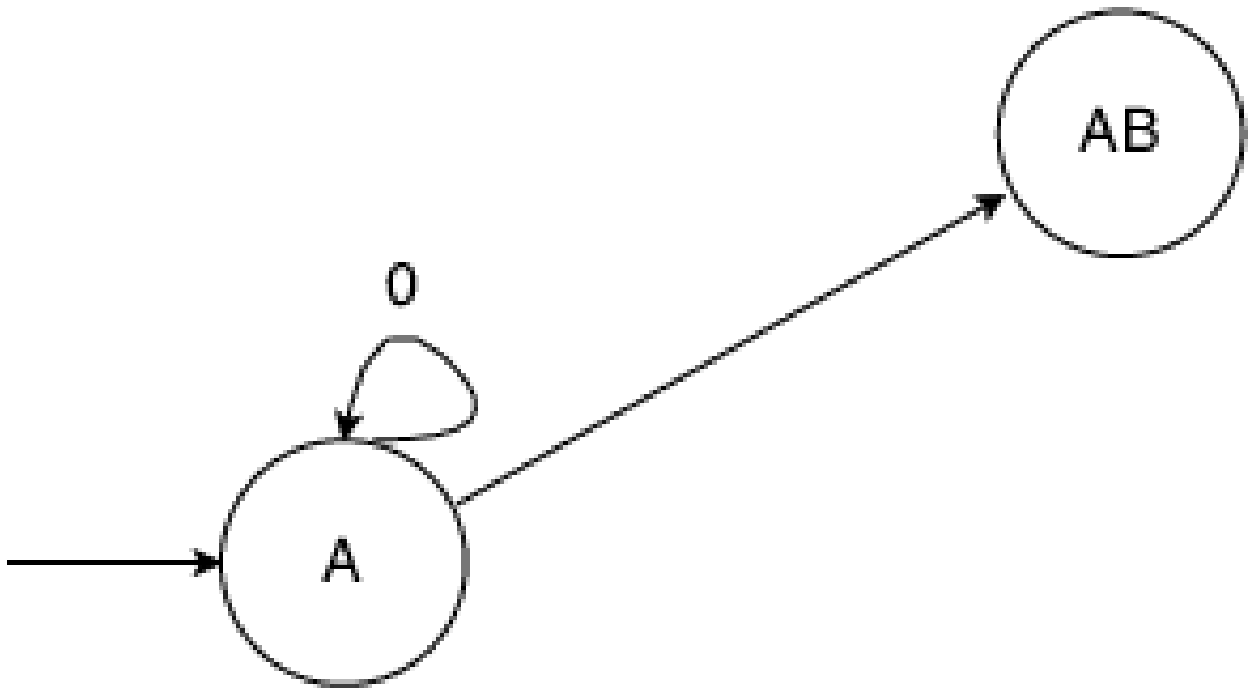
DFA: Step 1



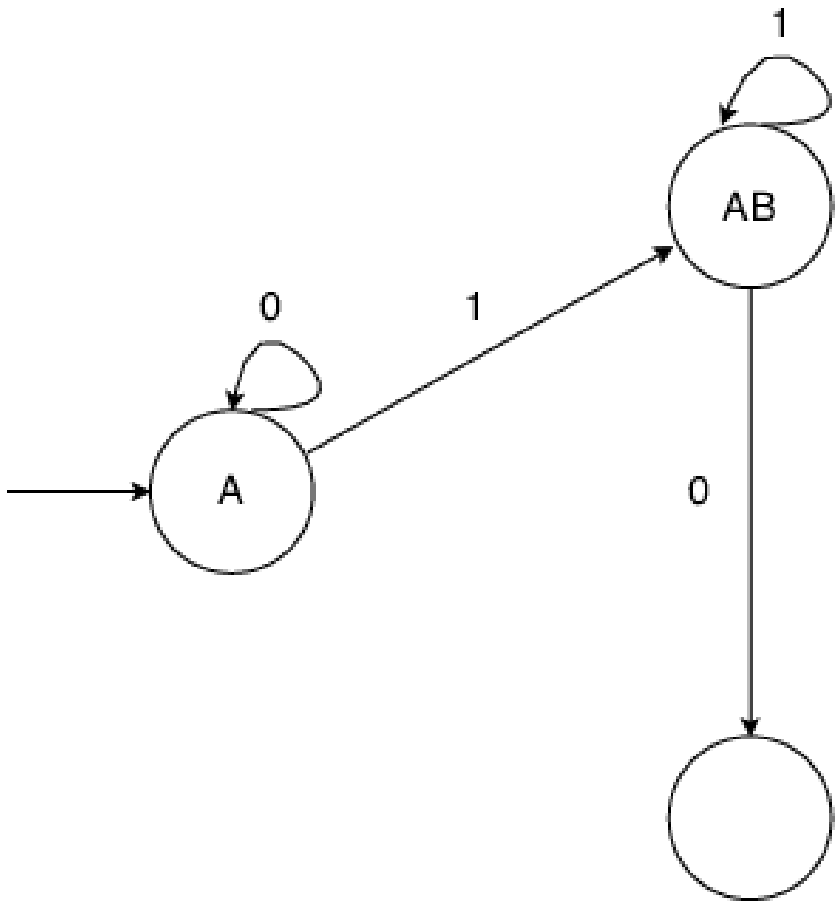
DFA: Step 2



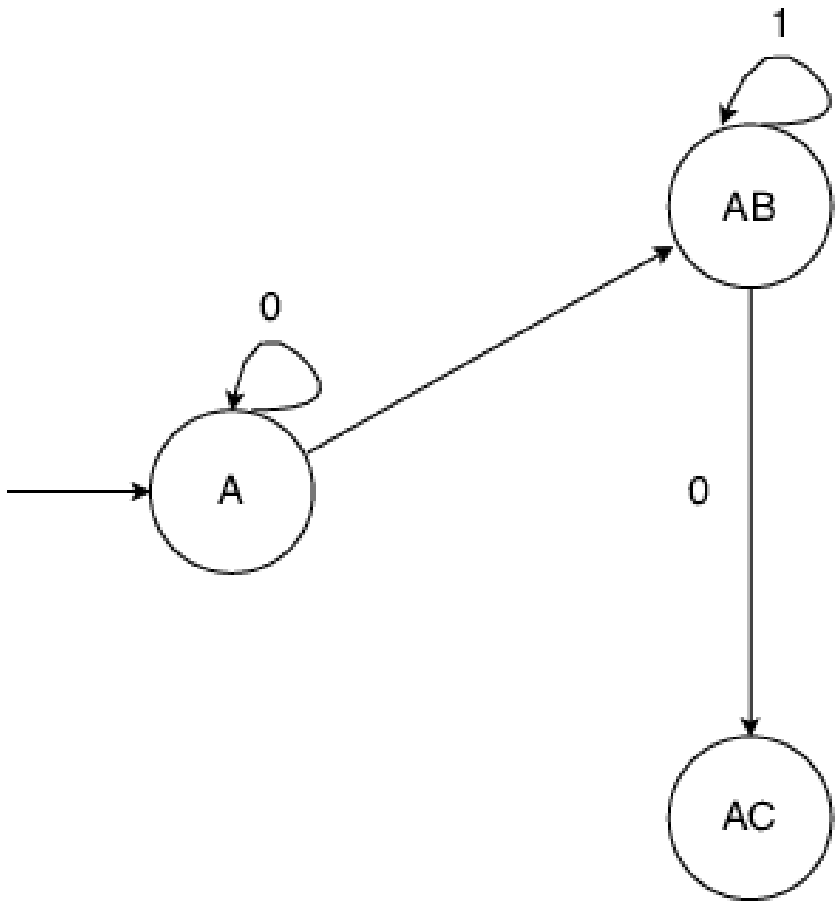
DFA: Step 3



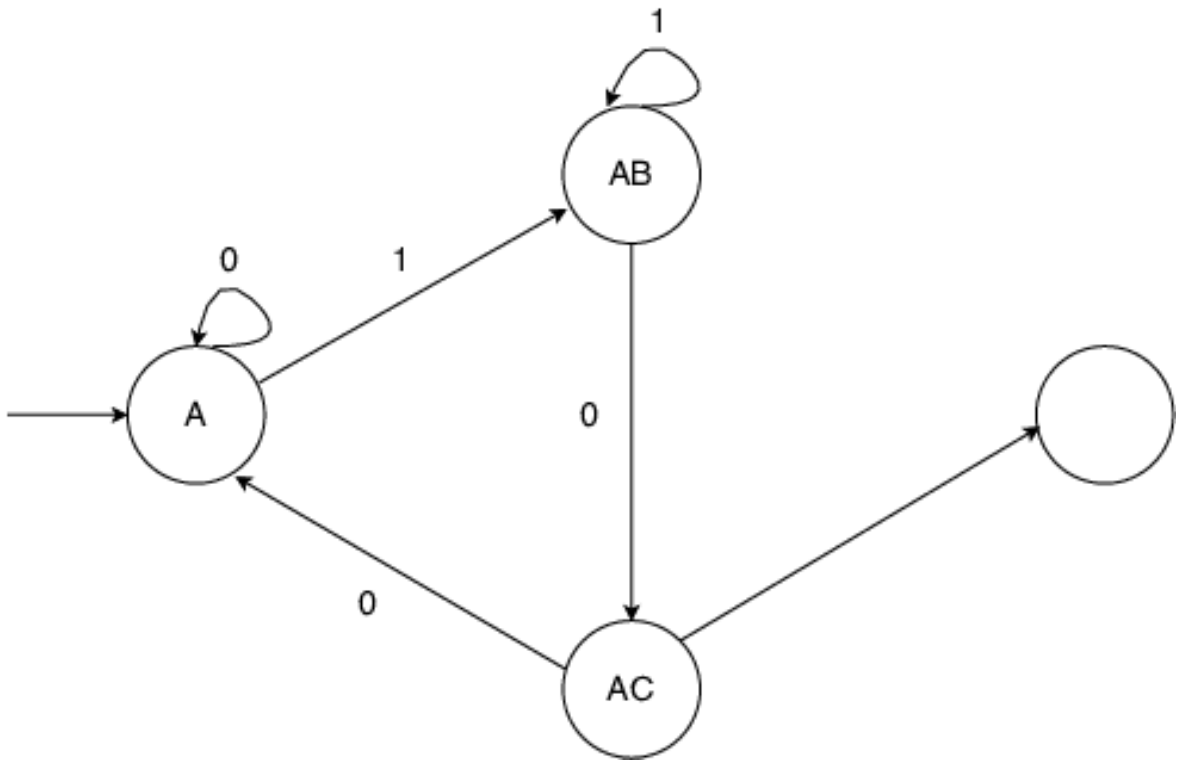
DFA: Step 4



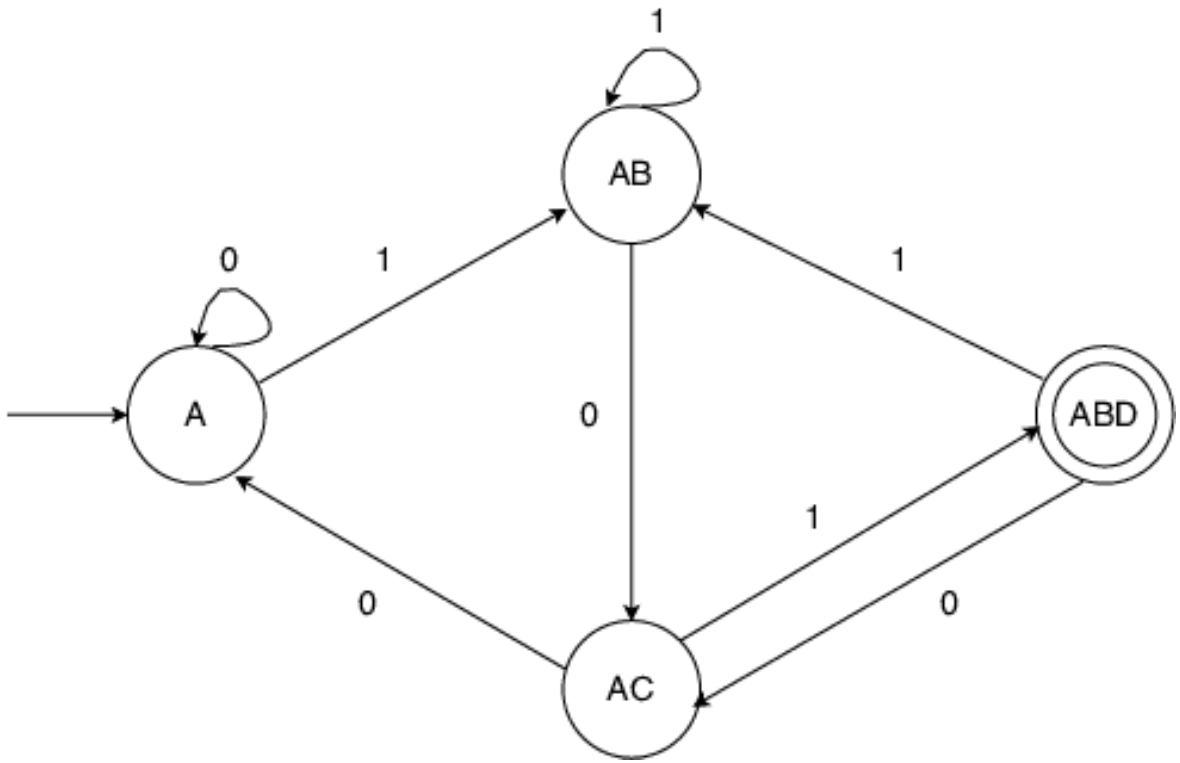
DFA: Step 5



DFA: Step 6



DFA: Completion



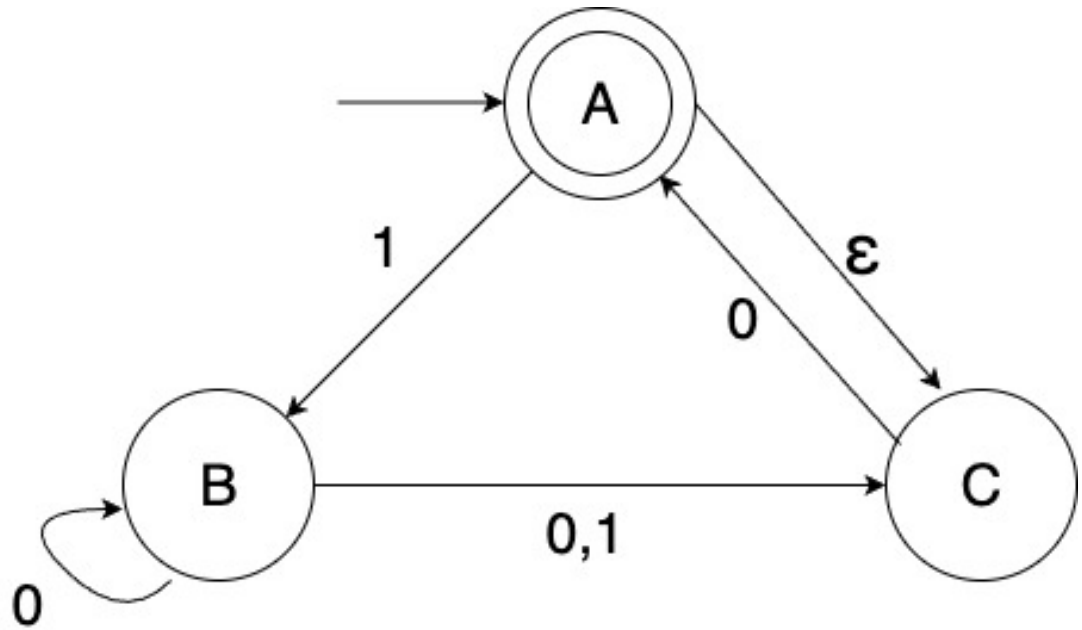
Conversion from NFA (with ϵ-transitions) to DFA

This is the same as before, but also:

- The start state of the DFA is the old start state of the NFA, and every state reachable from it by ϵ-transitions.
- When calculating the states reachable from a state, one includes all states reachable by ϵ-transitions *after* the destination state.

An Exercise

Use subset construction to get a DFA for this NFA.



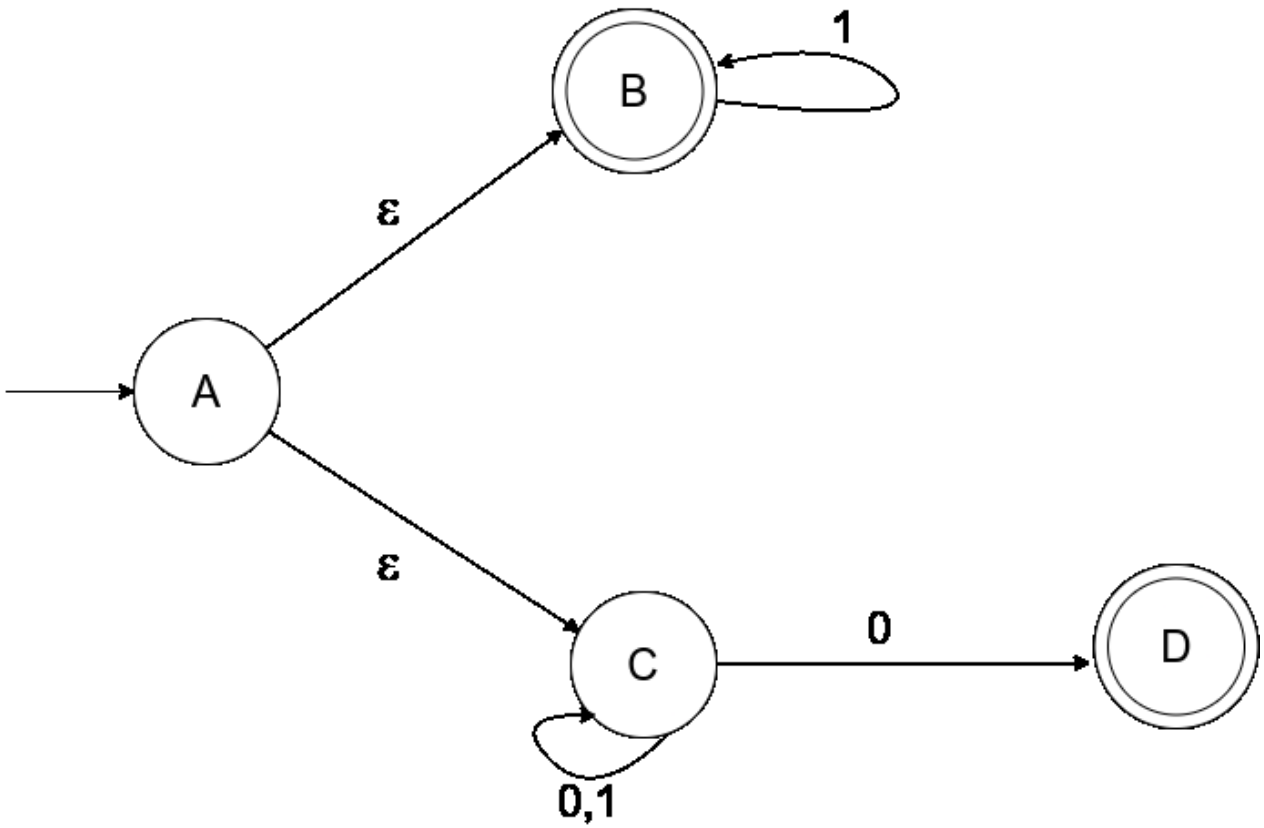
The state set of the DFA

- The state set of the corresponding DFA is:
- $\{\emptyset, \{A\}, \{B\}, \{C\}, \{A,B\}, \{A,C\}, \{B,C\}, \{A,B,C\}\}$.
- Consider the possible transitions in the NFA, and get those for the DFA.

δ	0	1
A	-	-
B	-	-
C	-	-
A,B	-	-
$\vdots$	$\vdots$	$\vdots$

Converting from an NFA to a DFA

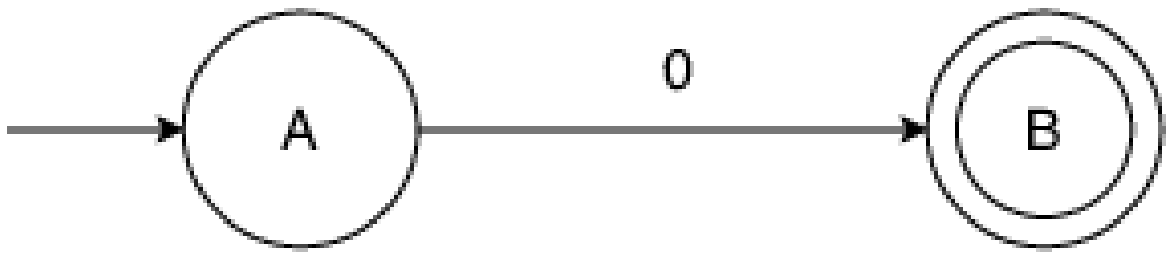
Convert this NFA into a DFA using subset construction.



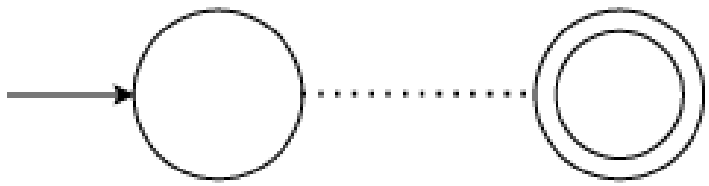
Conversion from RE to NFA

- If the RE is the empty string, then output trivial NFA.
- If the RE is a single symbol, output a simple NFA.
- If the RE has form $X + Y$, combine NFAs for X and Y in parallel.
- If the RE has form XY, then combine the NFAs for X and Y in series.
- If the RE has form X^*, extend the NFA for X.

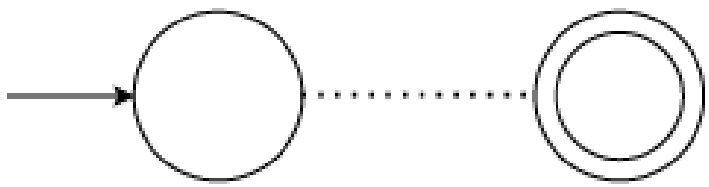
RE to NFA: single symbol



RE to NFA: Combining in parallel for $X + Y$

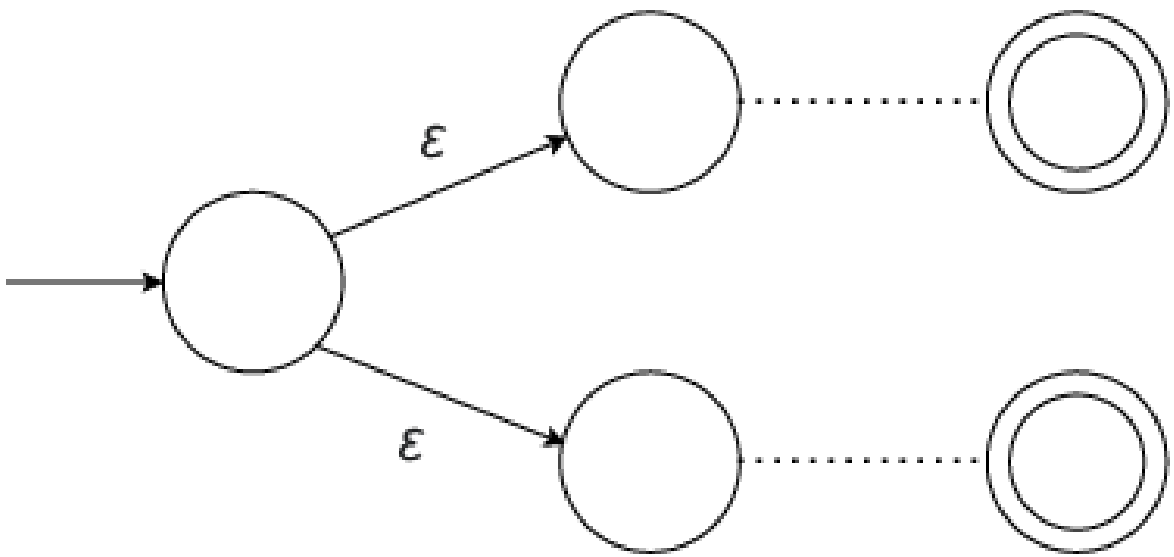


NFA for X

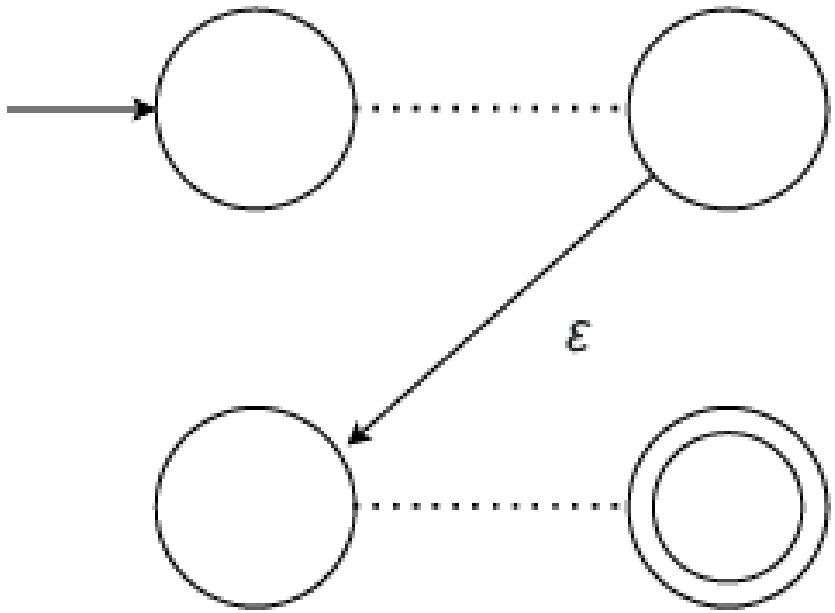


NFA for Y

RE to NFA: combining in parallel for $X + Y$—cont'd



RE to NFA: combining in series for XY



RE to NFA: concatenating NFA for X to get one for X^*