Mathematics-3
Tutorial-1
Discussion on Friday, 9th August

1. Prove that the formula for the probability of the union of two events can be generalized to three events as follows

$$\Pr(A \cup B \cup C) = \Pr(A) + \Pr(B) + \Pr(C)$$
$$- \Pr(A \cap B) - \Pr(A \cap C) - \Pr(B \cap C) + \Pr(A \cap B \cap C).$$

2. We are given a number of darts. When we throw a dart at a target, we have a probability of 1/4 of hitting the target. What is the probability of obtaining at least one hit if three darts are thrown?

3. We have four boxes. Box 1 contains 2000 components of which 5% are defective. Box 2 contains 500 components of which 40% are defective. Box 3 and 4 contain 1000 each with 10% defective. We select at random of the boxes and we remove at random a single component.
   a) What is the probability that the selected component is defective?
   b) We saw that the selected component is defective. What is the probability that it came from Box 2?

4. An experiment consists of selecting two integers (n, k) such that $0 \leq n < 5$ and $0 \leq k < 10$. How many outcomes are in the sample space?
   a) 66, b) 55, c) 50, d) 44

5. John, a biochemist, designs a test for a latent disease. If a subject has the disease, the probability that the test results turn out positive is 0.95. Similarly, if a subject does not have the disease, the probability that the test results come up negative is 0.95. Suppose that one percent of the population is infected by the disease. We wish to find the probability that a person who tested positive has the disease.

6. Two dice are rolled at the same time, a red die and a blue die.
Let A be the event that the number on the red die is odd. Let B be the event that the number on the red die is either two, three or four. Also, let C be the event that the product of the two dice is twelve. Are these events pair-wise independent? What about their independence in general?

7.

Consider the switching networks shown in Fig. 1-5. Let A_1, A_2, and A_3 denote the events that the switches s_1, s_2, and s_3 are closed, respectively. Let A_{ab} denote the event that there is a closed path between terminals a and b. Express A_{ab} in terms of A_1, A_2, and A_3 for each of the networks shown.

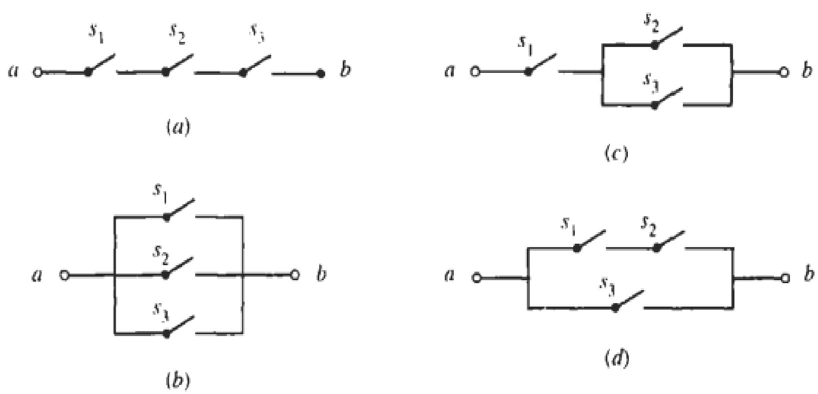

Fig. 1-5

8.

A company producing electric relays has three manufacturing plants producing 50, 30, and 20 percent, respectively, of its product. Suppose that the probabilities that a relay manufactured by these plants is defective are 0.02, 0.05, and 0.01, respectively.

(*a*) If a relay is selected at random from the output of the company, what is the probability that it is defective?

(*b*) If a relay selected at random is found to be defective, what is the probability that it was manufactured by plant 2?

9. An urn contains two black balls and three white balls. Two balls are selected at random from the urn without replacement and the sequence of colors is noted. Find the probability that both balls are black.

10.

(a) Suppose that the lifetime of a computer memory chip is measured, and we find that “the proportion of chips whose lifetime exceeds *t* decreases exponentially at a rate α” Find an appropriate probability law.

(b) A manufacturing process produces a mix of “good” memory chips and “bad” memory chips. The lifetime of good chips follows the exponential law with a rate of failure α . The lifetime of bad chips also follows the exponential law, but the rate of failure is 1000α. Suppose that the fraction of good chips is (1-p), and of bad chips is (*p)*. Find the probability that a randomly selected chip is still functioning after *t* seconds.

(c) A fraction *p* of the chips are bad and tend to fail much more quickly than good chips. Suppose that in order to “weed out” the bad chips, every chip is tested for *t* seconds prior to leaving the factory. The chips that fail are discarded and the remaining chips are sent out to customers. Find the value of *t* for which 99% of the chips sent out to customers are good.