IIIT-Bangalore
Probability and Statistics
Problem Set 8

(Two Dimensional Distribution)

1. Determine the value of k which makes

$$f(x,y) = \begin{cases} kxy, & 0 < x < 1,\ 0 < y < x \\ 0, & \text{elsewhere.} \end{cases}$$

a joint p.d.f. Calculate the marginal p.d.f. s and show that the variates are dependent. (Ans. $k = 8$, $f_X(x) = 4x^3$, $f_Y(y) = 4y(1-y^2)$)

2. If

$$f(x,y) = \begin{cases} 3x^2 - 8xy + 6y^2, & 0 < x < 1,\ 0 < y < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

find $f_X(x|y)$ and $f_Y(y|x)$.

3. The joint p.d.f. of X and Y is given by

$$f(x,y) = \begin{cases} \frac{6-x-y}{8}, & 0 < x < 2,\ 2 < y < 4 \\ 0, & \text{elsewhere.} \end{cases}$$

Calculate (i) $P(X < 1, Y < 3)$, (ii) $P(X + Y < 3)$, (iii) $P(X < 1|Y = 3)$, (iv) $P(X < 1|Y < 3)$. (Ans. (i) $\frac{3}{8}$, (ii) $\frac{5}{24}$, (iii) $\frac{5}{8}$, (iv) $\frac{5}{8}$)

4. The joint p.d.f. of X and Y is given by

$$f(x,y) = \begin{cases} 2, & 0 < x < 1,\ 0 < y < x \\ 0, & \text{elsewhere.} \end{cases}$$

Calculate (i) the marginal and conditional p.d.f.s.
(ii) Compute $P(\frac{1}{4} < X < \frac{3}{4}|Y = \frac{1}{2})$.

5. Two points are independently chosen at random in open interval $(0,1)$. Find the probability that the distance between them is less than a fixed number k, $(0 < k < 1)$.

6. Two numbers are independently chosen at random between 0 and 1. Show that the probability that their product is less than a constant k, $(0 < k < 1)$ is $k(1 - \log k)$.

7. If $f(x,y) = x + y, (0 < x < 1, 0 < y < 1)$ is the joint p.d.f. of (X,Y), find the distribution of $X + Y$.

8. If X and Y are independent variates both uniformly distributed over $(0, 1)$, find the distribution of $X + Y$, $X - Y$ and XY.

9. If the cartesian coordinate of a random point are independent standard normal variates, show that its polar coordinates are also independent and find their distributions.

10. If X_1 and X_2 and independent random variates each having density functions $2xe^{-x^2}$, $(0 < x < \infty)$, find the density function of $\sqrt{X_1^2 + X_2^2}$.

11. Consider the random experiment of throwing a pair of dice. Let X denote the number of sixes and Y denote the number of fives that turn up. Find the joint p.m.f. of the two-dimensional random variable (X, Y) and the marginal p.m.f.s of X and Y.