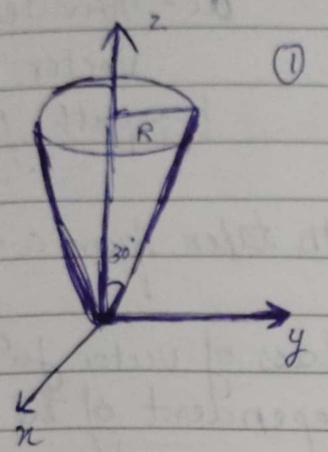
Q. $V = s(2+\sin^2\phi)\,\hat{s} + s\sin\phi\cos\phi\,\hat{\phi} + 3z\,\hat{z} = 8$??

Prove it



Q. ① calc. the Div. of given fig.

② Check the divergence Th. for the fu^n using the vol. of ice cream cone shown

$$V = r^2\sin\theta\,\hat{r} + 4r^2\cos\theta\,\hat{\theta} + r^2\tan\theta\,\hat{\phi}$$

①

② The system has two parts:

ⓐ The ice cream part + ⓑ The cone part

Ice cream ⓐ $r = R$, $\phi \Rightarrow 0 \to 2\pi$, $\theta = 0$ to $\pi/6$, $da = R^2\sin\theta\, d\theta\, d\phi\, \hat{r}$

ⓑ $\theta = \pi/6$, $\phi = 0 \to 2\pi$, $r = 0 \to R$, $da = r\sin\theta\, d\phi\, dr\, \hat{\theta}$

Answer in any case $\to \left(\frac{\pi R^4}{12}\right)(2\pi + 3\sqrt{3})$

17-08-2024

Integral Calculus

(1) **Line Integral** → $\int_{a\,P}^{b} v\,dl$

v → vector fn
dl → infinitesimal disp vector.
P → path from a to b

If $a = b$
then $\oint v \cdot dl$

Line Integral depends on the path taken from a to b.

There is an important special class of vector fn for which the Line integral is independent of the path (Ex → conservative Force).

(2) **Surface Integrals** → $\int_S v \cdot da$

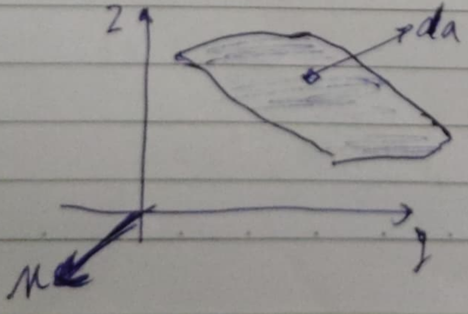
v → vector fn
da → infinitesimal patch of area.

Closed surface (i.e forming a "balloon")

Sign of surface integral is ambiguous as there are two dir $\perp$ to any surface.

$$\oint v \cdot da$$

If v → flow of liquid (mass per unit area per unit time) then $\int v \cdot da$ represents total mass per unit time passing through the surface. → $\boxed{\text{flux}}$



(3) Volume Integrals $\longrightarrow \int T \cdot d\tau$

$T \to$ scalar fun
$d\tau \to$ infinitesimal vol. element

$$d\tau = dx\, dy\, dz$$

If $T \to$ density of substance
then vol. integral would give the total mass.

$$\int \mathbf{v}\, d\tau = \int (v_x \hat{x} + v_y \hat{y} + v_z \hat{z})\, d\tau = \hat{x}\int v_x d\tau + \hat{y}\int v_y d\tau + \hat{z}\int v_z d\tau$$

Fundamental Theorem of ~~Calculus~~ Gradients

$$dT = (\nabla T)\, d\mathbf{l}_1$$

$T(x,y,z) \to$ scalar fun of three variables

[As $df = (df/dx)\cdot dx$ is the infinitesimal small change in f when you go from x to $(x+dx)$

Now we move a little further, by an additional small disp. $d\mathbf{l}_2$, the incremental change in T will be $(\nabla T)\, d\mathbf{l}_2$

Total change in T in going from a to b along the path selected is

$$\boxed{\int_{a\,P}^{b} (\nabla T)\, d\mathbf{l} = T(b) - T(a)}$$

We know that line integrals ordinarily depend on the path taken from a to b. But the right side of G.T.⊗ makes no reference to the path — only to the end points.

→ Gradients have the special property that their line integrals are path independent.

Corollary 1 $\int_a^b (\nabla T) \cdot dI$ is independent of path taken from a to b.

Corollary 2 $\oint (\nabla T) \cdot dI = 0$ since the beginning and end points are identical & hence $\boxed{T(b) - T(a) = 0}$

Fundamental Theorem of Divergence

$$\boxed{\int_V (\nabla \cdot V)\, d\tau = \oint_S V \cdot da}$$

This is called Gauss' Theorem, Green's Theorem & the Divergence Theorem.

→ Integral of the derivative (in this case the divergence) over a region (in this case a volume) is equal to the value of the fn at the boundary (in this case the surface that bounds the vol.).

The boundary term is itself an integral (specifically, a surface integral). This is reasonable: the boundary of a line is just two end points, the boundary of a vol. is a (closed) surface.

Fundamental Theorem for Curls

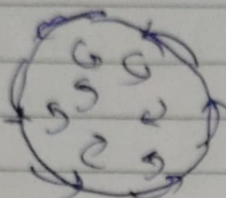
Stoke's Theorem $\longrightarrow$ $\boxed{\int_S (\nabla \times v)\, da = \oint_P v \cdot dl}$

- Boundary of a vol. $\rightarrow$ surface that bounds the vol.
  Boundary of a surface $\rightarrow$ its perimeter.
  Boundary of line $\rightarrow$ end points

Integral of derivative (in this case the curl) over a region (here, patch of surface) is equal to the value of the fuⁿ at the boundary (here, perimeter of the patch).

As in the case of the divergence theorem, the boundary term is itself an integral - a closed line integral.

Integral of the curl over some surface (the flux of the curl through that surface) represents the "total amount of swirl" & we can determine that swirl just as well by going around the edge & finding how much the flow is following the boundary.



Surface integral considers all internal rotations but the net will be rotation of the perimeter. [Bcz. some internal rotations will cancel each other]

Which way are you supposed to go?
CW or ACW
↳ Ans. It doesn't matter (you can fix any)
↳ i.e you should be consistent

Where does da point?
Ans da points in the dirⁿ of outward normal for a closed surface (as in divergence Th.)

Consistency in Stoke's Th. is given by the right-hand thumb rule: If your fingers points in the dirⁿ of the line integral, then your thumb fix the dirⁿ of da.

For Stokes Th. → the surface you choose doesn't matter.

Corollary 1 → $\int (\nabla \times V) \cdot da$ depends only on the boundary line, not on the particular surface used.

Corollary 2 → $\oint (\nabla \times V) da = 0$ for any closed surface, since the boundary line, like the mouth of a balloon, shrinks down to a point & hence the right side vanishes.

Integration By Parts

$$\frac{d}{dx}(fg) = f\frac{dg}{dx} + g\frac{df}{dx}$$

Integrating both sides,

$$\int_a^b \frac{d}{dx}(fg)\,dx = \int_a^b f\left(\frac{dg}{dx}\right)dx + \int_a^b g\left(\frac{df}{dx}\right)dx$$

line ka boundary end point

$$\boxed{\int_a^b f\left(\frac{dg}{dx}\right)dx = -\int_a^b g\left(\frac{df}{dx}\right)dx + fg\Big|_a^b}$$

cost of minus sign

Transfer the derivative from g to f at the cost of minus sign & a boundary term

$$\nabla\cdot(fA) = f(\nabla\cdot A) + A\cdot(\nabla f)$$

$$\int \nabla\cdot(fA)\,d\tau = \int f(\nabla\cdot A)\,d\tau + \int A\cdot(\nabla f)\,d\tau = \oint fA\cdot da$$

$$\boxed{\int_V f(\nabla\cdot A)\,d\tau = -\int A\cdot(\nabla f)\,d\tau + \oint_S fA\cdot da}$$

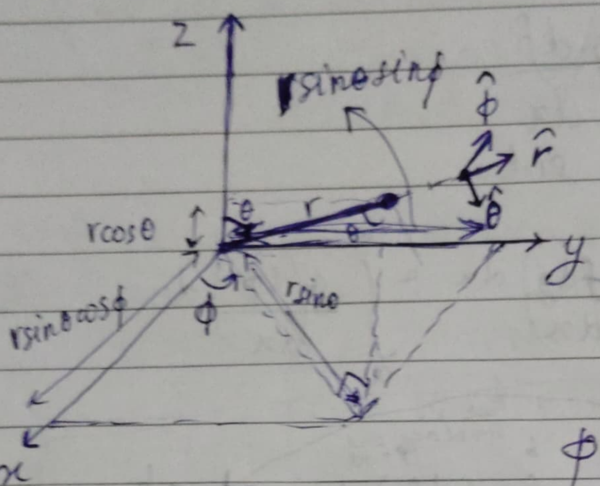
Integral involving the product of one fun & the derivative of another

cost of minus sign

vol. ka boundary surface

Similarly derivative was transferred from A to f at the the cost of minus sign & a boundary term.

Spherical Polar Coordinates



$z = r\cos\theta$

$y = r\sin\theta\sin\phi$

$x = r\sin\theta\cos\phi$

$\theta \rightarrow \angle$ down from the z axis (polar angle)

$\phi \rightarrow$ the angle around from the x axis (azimuthal angle)

⇨ $\hat{r}, \hat{\theta}, \hat{\phi} \rightarrow$ pointing in the dir^n of increase in the corresponding coordinates.

↳ orthogonal

$$A = \underbrace{A_r \hat{r}}_{\text{radial}} + \underbrace{A_\theta \hat{\theta}}_{\text{polar}} + \underbrace{A_\phi \hat{\phi}}_{\text{azimuthal}}$$

$$\hat{r} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}$$

$$\hat{\theta} = \cos\theta\cos\phi\,\hat{x} + \cos\theta\sin\phi\,\hat{y} - \sin\theta\,\hat{z}$$

$$\hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}$$

Note

⇨ $\hat{r}, \hat{\theta}, \hat{\phi}$ are associated with a particular point P, & they change dir^n as P moves around.

For ex → $\hat{r}$ always points radially outward, but "radially outward" can be the x dir^n, the y dir^n, or any other dir r r $\theta = l/r$ ηγr

URBAN EDGE

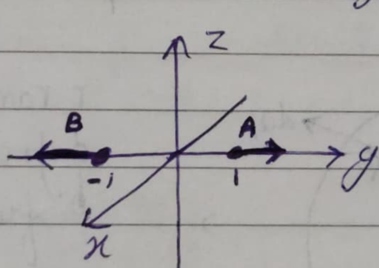
depending on where you are.

$A = \hat{y}$ & $B = -\hat{y}$ & yet both of them would be written as $\hat{r}$ in spherical coordinates.

⇨ Do not combine the spherical components of vectors associated with different points (For ex → $A+B = 0$, $A \cdot B = -1$)

⇨ Beware of differentiating a vector that is expressed in spherical coordinates, since the unit vectors themselves are fu^n of position ($\frac{d\hat{r}}{d\theta} = \hat{\theta}$).

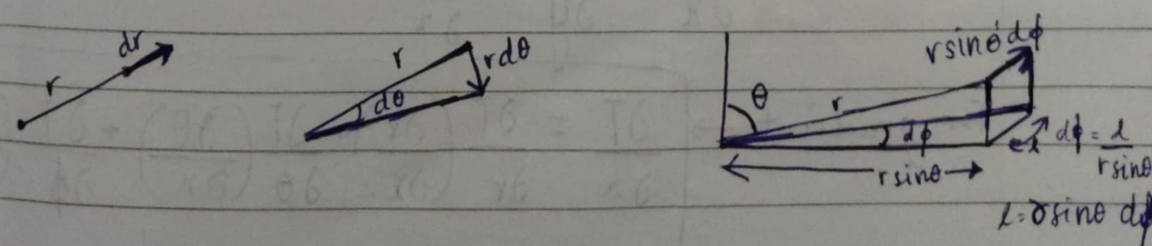
⇨ Do not $\hat{r}, \hat{\theta}, \hat{\phi}$ outside an integral, as we did with $\hat{x}, \hat{y}, \hat{z}$.



An infinitesimal displacement in the

1) $\hat{r}$ dir^n → dr ($dl_r = dr$)
2) $\hat{\theta}$ dir^n → $r\,d\theta$ ($dl_\theta = r\,d\theta$)
3) $\hat{\phi}$ dir^n → $r\sin\theta\,d\phi$ ($dl_\phi = r\sin\theta\,d\phi$)

general infinitesimal disp. $\boxed{dl = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}}$



Infinitesimal vol-element $d\tau$ in spherical coordinates is ~~the product of the~~

$$d\tau = (dl_r)(dl_\theta)(dl_\phi) = r^2 \sin\theta \, dr \, d\theta \, d\phi$$

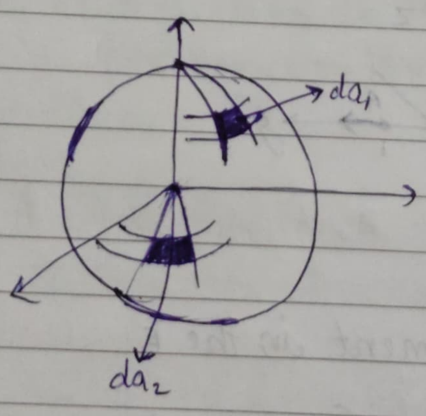
There is no generalisation for surface elements (da) since these depend on the orientation of the surface.

→ For a sphere, r is constant

$$da_1 = dl_\theta \, dl_\phi \, \hat{r} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{r}$$

→ In the surface lies in the x-y plane then θ is constant

$$da_2 = dl_r \, dl_\phi \, \hat{\theta} = r \, dr \, d\phi \, \hat{\theta}$$



r ranges from 0 to ∞
ϕ from 0 to 2π
θ from 0 to π

Translating vector derivatives (gradient, divergence, curl & Laplacian) into r, θ, ϕ notation

$$\nabla T = \frac{\partial T}{\partial x}\hat{x} + \frac{\partial T}{\partial y}\hat{y} + \frac{\partial T}{\partial z}\hat{z}$$

For gradient → $\boxed{\frac{\partial T}{\partial x} = \frac{\partial T}{\partial r}\left(\frac{\partial r}{\partial x}\right) + \frac{\partial T}{\partial \theta}\left(\frac{\partial \theta}{\partial x}\right) + \frac{\partial T}{\partial \phi}\left(\frac{\partial \phi}{\partial x}\right)}$

$\sin\theta = \frac{x}{}$ $r = \frac{x}{\sin\theta\cos\phi}$

$z = r\cos\theta$ $x = r\sin\theta\cos\phi$ $y = r\sin\theta\sin\phi$

$\boxed{\frac{\partial r}{\partial x} = \frac{1}{\sin\theta\cos\phi}}$ $\boxed{\frac{\partial r}{\partial y} = \frac{1}{\sin\theta\sin\phi}}$ $\boxed{\frac{\partial r}{\partial z} = \frac{1}{\cos\theta}}$

~~Similarly you can get~~ $\cancel{\frac{\partial T}{\partial y}, \frac{\partial T}{\partial z}}$

$\cancel{\frac{\partial r}{\partial y}, \frac{\partial \theta}{}}$

$\frac{\partial\theta}{\partial x} \rightarrow \theta = \sin^{-1}\left(\frac{x}{r\cos\phi}\right)$

$$\frac{\partial\theta}{\partial x} = \frac{1}{\sqrt{1-\frac{x^2}{r^2\cos^2\phi}}}\cdot\frac{1}{r\cos\phi} = \frac{1}{\sqrt{r^2\cos^2\phi - x^2}}$$

$\frac{\partial\theta}{\partial y} \rightarrow \theta = \sin^{-1}\left(\frac{y}{r\sin\phi}\right)$ $\frac{\partial\theta}{\partial z} \rightarrow \theta = \cos^{-1}\left(\frac{z}{r}\right)$

$$\frac{\partial\theta}{\partial y} = \frac{1}{\sqrt{r^2\sin^2\phi - y^2}} \qquad \frac{\partial\theta}{\partial z} = -\frac{1}{\sqrt{r^2 - z^2}}$$

$$\frac{\partial\phi}{\partial x} = -\frac{1}{\sqrt{r^2\sin^2\theta - x^2}} \qquad \frac{\partial\phi}{\partial y} = \frac{1}{\sqrt{r^2\sin^2\theta - y^2}} \qquad \frac{\partial\phi}{\partial z} = 0$$

Now substitute $\hat{x}, \hat{y}, \hat{z}$ in terms of $\hat{r}, \hat{\theta}, \hat{\phi}$.

$\boxed{\text{Not a good way}}$

Final Formulas in terms r, θ, ϕ

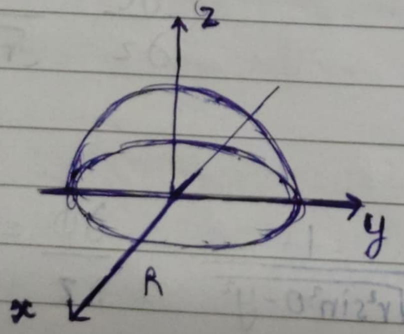
Gradient $$\nabla T = \frac{\partial T}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial T}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial T}{\partial \phi}\hat{\phi}$$

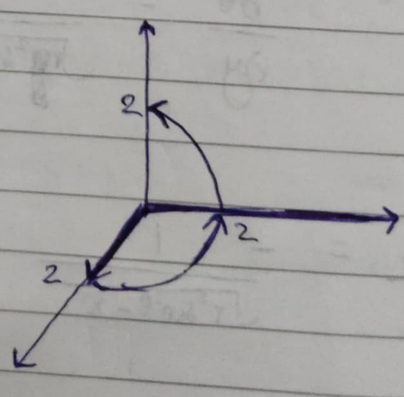
Divergence $$\nabla \cdot V = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 V_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, V_\theta) + \frac{1}{r\sin\theta}\frac{\partial V_\phi}{\partial \phi}$$

Curl $$\nabla \times V = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta\, V_\phi) - \frac{\partial V_\theta}{\partial \phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial V_r}{\partial \phi} - \frac{\partial}{\partial r}(r V_\phi)\right]\hat{\theta}$$
$$+ \frac{1}{r}\left[\frac{\partial}{\partial r}(r V_\theta) - \frac{\partial (V_r)}{\partial \theta}\right]\hat{\phi}$$

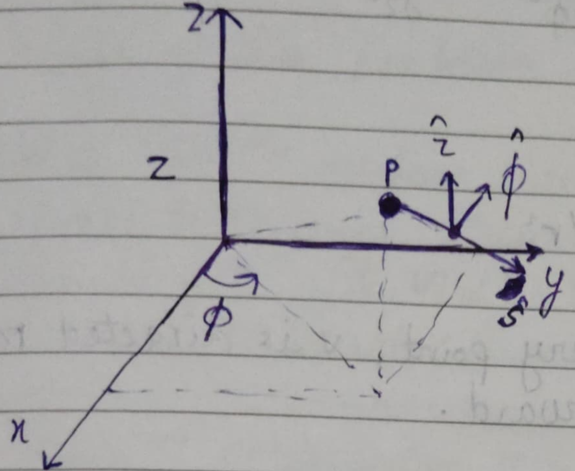
Laplacian $$\nabla^2 T = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}$$

Curvilinear Coordinates





Cylindrical Coordinates



cylindrical coordinates (s, ϕ, z)

$\phi \rightarrow$ ~~same~~ $\angle$ around from the x axis (same as spherical coordinate)

$z \rightarrow$ same as the Cartesian

$s \rightarrow$ distance to P from the z axis.

$$x = s\cos\phi, \quad y = s\sin\phi \quad z = z$$

$$\hat{s} = \cos\phi\,\hat{x} + \sin\phi\,\hat{y}$$
$$\hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}$$
$$\hat{z} = \hat{z}$$

Infinitesimal disp. are $\rightarrow$ $dl_s = ds$, $dl_\phi = s\,d\phi$, $dl_z = dz$.

$$d\mathbf{l} = ds\,\hat{s} + s\,d\phi\,\hat{\phi} + dz\,\hat{z}$$

Infinitesimal vol. element $\rightarrow$
$$d\tau = s\,ds\,d\phi\,dz$$

Range of s is 0 to ∞, ϕ goes from $0 \rightarrow 2\pi$ & z from $-\infty$ to ∞

Gradient
$$\nabla T = \frac{\partial T}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial T}{\partial \phi}\hat{\phi} + \frac{\partial T}{\partial z}\hat{z}$$

Divergence
$$(\nabla \cdot V) = \frac{1}{s}\frac{\partial}{\partial s}(sV_s) + \frac{1}{s}\frac{\partial V_\phi}{\partial \phi} + \frac{\partial V_z}{\partial z}$$

Curl
$$(\nabla \times V) = \left(\frac{1}{s}\frac{\partial V_z}{\partial \phi} - \frac{\partial V_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial V_s}{\partial z} - \frac{\partial V_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left[\frac{\partial (sV_\phi)}{\partial s} - \frac{\partial V_s}{\partial \phi}\right]\hat{z}$$

Laplacian $\nabla^2 T = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial T}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}$

Dirac Delta Function

The Divergence of $\hat{r}/r^2$

$V = \frac{1}{r^2}\hat{r}$ At every point, v is directed radially outward.

If ever there was a fn that ought to have large positive divergence, this is it. And yet, when we actually calculate divergence, we get zero
(From divergence of sph. coordinates)

$$\nabla \cdot V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{1}{r^2}\right) = \frac{1}{r^2}\frac{\partial}{\partial r}(1) = 0$$

On applying divergence theorem. Suppose we integrate over a sphere of radius R, centered at origin, the surface integral is

$$\oint V \cdot da = \int \left(\frac{1}{R^2}\hat{r}\right) \cdot (R^2 \sin\theta \, d\theta \, d\phi \, \hat{r})$$

$$= \left(\int_0^{\pi} \sin\theta \, d\theta\right)\left(\int_0^{2\pi} d\phi\right) = 4\pi$$

But, the vol. integral, $\int (\nabla \cdot V) d\tau = 0$

But acc. to Divergence Th.

$$\boxed{\int (\nabla \cdot V) d\tau = \oint V \cdot da}$$

Q. Whats the catch?

Ans: Source of the problem is $r=0$, where V is N.D

Notice that the surface integral $\oint$ is independent of R, if the divergence theorem is right (and it is) we should get $\int(\nabla \cdot V)\, d\tau = 4\pi$ for any sphere centered at origin, no matter how small.

$\boxed{\text{The entire contribution must be coming from the point } r=0.}$

Thus $(\nabla \cdot V)$ has the bizarre prop. that it vanishes everywhere except at one point, & yet its integral (over any vol. containing that point) is 4π.

-08.2024

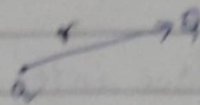
ELECTROSTATICS

$\cdot Q$ Test charge

q Source charge

$$\boxed{F = \frac{1}{4\pi\epsilon_0}\frac{qQ}{r^2}\hat{r}}$$

$\frac{1}{r^2}$



ϵ_0 is called permitivity of free space.

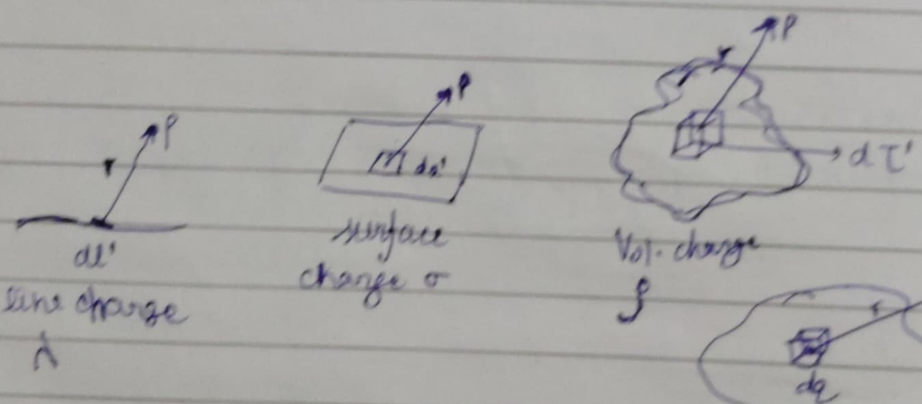
$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$

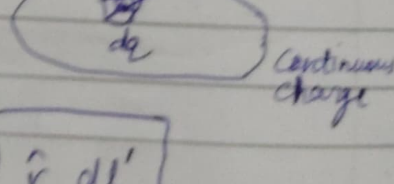
$$F = F_1 + F_2 + \cdots + F_n$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{q_1}{r_1^2}\hat{r}_1 + \frac{q_2}{r_2^2}\hat{r}_2 + \cdots + \frac{q_n}{r_n^2}\hat{r}_n\right)$$

$$= \frac{Q}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i = Q\frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i$$

$$\Rightarrow \boxed{F = QE}$$

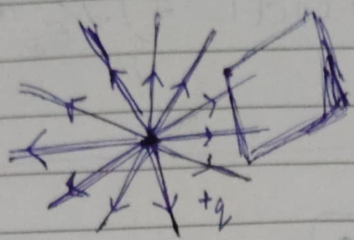


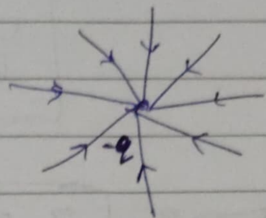


$$\boxed{E(r) = \frac{1}{4\pi\epsilon_0}\int\frac{1}{r^2}\hat{r}\,dq} \quad \boxed{= \frac{1}{4\pi\epsilon_0}\int_P\frac{\lambda(r')}{r^2}\hat{r}\,dl'}$$

$$\boxed{= \frac{1}{4\pi\epsilon_0}\iint_S\frac{\sigma(r')}{r^2}\hat{r}\,da'} \quad \boxed{= \frac{1}{4\pi\epsilon_0}\int_V\frac{\rho(r')}{r^2}r'\,d\tau'}$$

Field Lines





$$\Phi_E = \int_S E \cdot da$$

The area in the plane ⊥ to E

q r (sphere radius)

$$\oint E \cdot da = \int \frac{1}{4\pi\epsilon_0} \left(\frac{q}{r^2}\hat{r}\right)(r^2 \sin\theta \, d\theta \, d\phi \, \hat{r})$$

$$= \frac{q}{\epsilon_0}$$

If there are multiple charges $\rightarrow \sum_{i=1}^{n} \frac{1}{\epsilon_0} q_i = 1/\epsilon_0 \sum_{i=1}^{n} q_i$

$$= \frac{Q_{enclosed}}{\epsilon_0}$$

$$\boxed{\oint_S E \cdot da = \frac{Q_{enc.}}{\epsilon_0}}$$

$$\oint_S E \cdot da = \int_V (\nabla \cdot E) \, d\tau$$ (Acc. to Divergence Th.)

$$\boxed{Q_{enclosed} = \int_V \underbrace{\rho}_{\text{charge per unit vol.}} \, d\tau}$$

$$\boxed{\int_V (\nabla \cdot E)\, d\tau = \int_V \rho/\epsilon_0 \, d\tau}$$

$$\boxed{\nabla \cdot E = \frac{\rho}{\epsilon_0}}$$

Divergence of E

$$E(r) = \frac{1}{4\pi\epsilon_0} \int_{\text{all space}} \frac{\hat{\eta}}{\eta^2} \rho(r')\, d\tau'$$

$$\nabla \cdot E = \frac{1}{4\pi\epsilon_0} \int \underbrace{\nabla \cdot \left(\frac{\hat{\eta}}{\eta^2}\right)}_{\text{Dirac Delta Function}} \rho(r')\, d\tau'$$

$\eta = r - r'$

$\nabla \cdot \left(\frac{\hat{\eta}}{\eta^2}\right) = 4\pi\delta^3(\eta)$

$= 4\pi\delta^3(r - r')$

$$= \frac{1}{4\pi\epsilon_0} \int 4\pi\, \delta^3(r - r')\, \rho(r')\, d\tau'$$

$$\boxed{\nabla \cdot E = \frac{\rho(r)}{\epsilon_0}}$$

LOOK AT THE DIRAC DELTA FUNCTION

Gauss Law can be used for symmetry.

Q. A long cylinder carries a charge density $\rho = ks$ where $s \rightarrow$ radius.

'l' & 's' $\quad \oint E \cdot da = \frac{Q_{enc}}{\epsilon_0}$

$Q_{enc} = \int \rho \, d\tau = \int (ks')(s' ds' \, d\phi \, dz) = 2\pi k l \int_0^s s'^2 ds'$

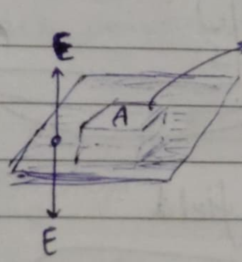
$$\boxed{Q_{enc} = \frac{2\pi k l s^3}{3}}$$

$\int E \, da = \int |E| da = |E| \int da = |E| 2\pi s l$

$\Rightarrow |E| 2\pi s l = \frac{Q_{enc}}{\epsilon_0} \Rightarrow |E| \cancel{2\pi} \cancel{s} \cancel{l} = \frac{\cancel{2\pi} k \cancel{l} s^{\cancel{3} 2}}{3\epsilon_0}$

$$\Rightarrow \boxed{|E| = \frac{2ks^2}{3\epsilon_0}}$$

(i) 'σ'



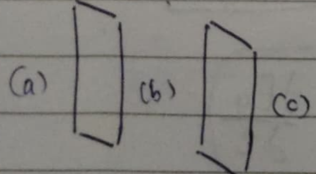
Penetrating the bigger surface

$\oint E da = \frac{Q_{enc}}{\epsilon_0}$

$\int E \cdot da = 2A|E|$

$2\cancel{A}|E| = \frac{1}{\epsilon_0} \sigma \cancel{A} \Rightarrow |E| = \frac{\sigma}{2\epsilon_0} \hat{n}$

normal to the surface

(ii) 

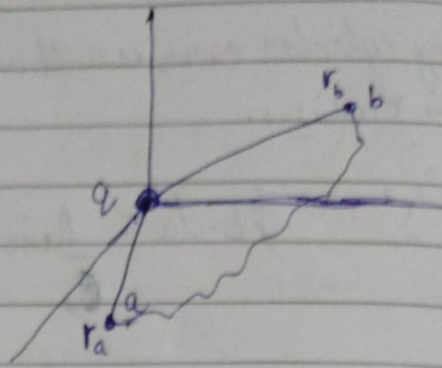
(a) $E = 0$

(b) $E = \frac{\sigma}{\epsilon_0}$

(c) $E = 0$

Curl of E

$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$



$\int_a^b E \cdot dl$

↓ int dx dy dz

bcz we are using **spherical** coordinates

$$dl = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

$$\int E \cdot dl = \int_a^b \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}\,dr \qquad \left[E \cdot dl = \left(\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}\right) \cdot \left(dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}\right) \right.$$

↳ dot product

$$\Rightarrow \boxed{\int E \cdot dl = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r_a} - \frac{q}{r_b}\right)}$$

If the path is closed

$$\boxed{\oint E \cdot dl = 0}$$

$$\boxed{\nabla \times E = 0}$$

From Stokes Law

$$\boxed{\int_s (\nabla \times E)\,da = \oint E \cdot dl}$$

$$\boxed{\begin{array}{c} \nabla \cdot E = \frac{\rho}{\epsilon_0} \\ \hline \nabla \times E = 0 \end{array}}$$

E → any vector field

its curl is always zero

if

$$\frac{\partial E_x}{\partial y} = \frac{\partial E_y}{\partial x} \;\&\; \frac{\partial E_x}{\partial z} = \frac{\partial E_z}{\partial x} \;\&\; \frac{\partial E_y}{\partial y} = \frac{\partial E_y}{\partial z}$$