PHYSICS Chapter 7

Ohm’s Law:
current density J is proportional to the force per unit charge, f:
J = σ f
J = σ (E + v × B)
Ordinarily, the velocity of the charges is sufficiently small that the second term can be ignored:
J = σ E.
V = I R.
For steady currents and uniform conductivity,

$$\nabla \cdot \mathbf{E} = \frac{1}{\sigma}\nabla \cdot \mathbf{J} = 0,$$

P = V I = I^2R.

This is the Joule heating law. With I in amperes and R in ohms, P comes out in watts(joules per second).

$$\mathcal{E} \equiv \oint \mathbf{f} \cdot d\mathbf{l} = \oint \mathbf{f}_s \cdot d\mathbf{l}.$$

E is called the electromotive force, or emf, of the circuit. It’s a lousy
term, since this is not a force at all—it’s the integral of a force per unit charge.

Generators exploit motional emfs, which arise when you move a wire
through a magnetic field.

$$\mathcal{E} = \oint \mathbf{f}_{\text{mag}} \cdot d\mathbf{l} = vBh,$$

$$\boxed{\mathcal{E} = -\frac{d\Phi}{dt}.}$$

the emf generated in the loop is minus the rate of
change of flux through the loop
This is the flux rule for motional emf.

A changing magnetic field induces an electric field.

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{a}.$$

This is Faraday's law, in integral form. We can convert it to differential form by
applying Stokes' theorem:

$$\boxed{\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.}$$

Lenz's law, whose sole purpose is to help you get the directions right:

Nature abhors a change in flux

In Faraday's
first experiment it's the Lorentz force law at work; the emf is magnetic. But in the
other two it's an electric field (induced by the changing magnetic field) that does
the job.

Faraday's law generalizes the electrostatic rule $\nabla \times \mathbf{E} = \mathbf{0}$ to the time-dependent régime. The *divergence* of $\mathbf{E}$ is still given by Gauss's law ($\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho$). If $\mathbf{E}$ is a *pure* Faraday field (due exclusively to a changing $\mathbf{B}$, with $\rho = 0$), then

$$\nabla \cdot \mathbf{E} = 0, \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.$$

This is mathematically identical to magnetostatics,

$$\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J}.$$

Conclusion: Faraday-induced electric fields are determined by $-(\partial \mathbf{B}/\partial t)$ in exactly the same way as magnetostatic fields are determined by $\mu_0 \mathbf{J}$. The analog to Biot-Savart is[13] is

$$\mathbf{E} = -\frac{1}{4\pi}\int \frac{(\partial \mathbf{B}/\partial t) \times \hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}\,d\tau = -\frac{1}{4\pi}\frac{\partial}{\partial t}\int \frac{\mathbf{B} \times \hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}\,d\tau, \tag{7.18}$$

and if symmetry permits, we can use all the tricks associated with Ampère's law in integral form ($\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{enc}}$), only now it's *Faraday's* law in integral form:

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt}. \tag{7.19}$$

1. M21 is a purely geometrical quantity, having to do with the sizes, shapes,
and relative positions of the two loops.

2. The integral in Eq. 7.23 is unchanged if we switch the roles of loops 1 and
2; it follows that
M21 = M12 .

$$M = \mu_0 \pi a^2 n_1 n_2 l.$$

$$\Phi = LI. \tag{7.26}$$

The constant of proportionality L is called the **self inductance** (or simply the **inductance**) of the loop. As with M, it depends on the geometry (size and shape) of the loop. If the current changes, the emf induced in the loop is

$$\mathcal{E} = -L\frac{dI}{dt}. \tag{7.27}$$

Inductance is measured in **henries** (H); a henry is a volt-second per ampere.

Inductance (like capacitance) is an intrinsically positive quantity. Lenz's law,
which is enforced by the minus sign in Eq. 7.27, dictates that the emf is in such
a direction as to oppose any change in current. For this reason, it is called a
back emf.

Electrodynamics Before Maxwell

$$\begin{array}{llll}
\text{(i)} & \nabla \cdot \mathbf{E} = \dfrac{1}{\epsilon_0}\rho & & \text{(Gauss's law)}, \\
\text{(ii)} & \nabla \cdot \mathbf{B} = 0 & & \text{(no name)}, \\
\text{(iii)} & \nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t} & & \text{(Faraday's law)}, \\
\text{(iv)} & \nabla \times \mathbf{B} = \mu_0 \mathbf{J} & & \text{(Ampère's law)}.
\end{array}$$

But when you do the same thing to number (iv), you get
into trouble:
$\nabla \cdot (\nabla \times B) = \mu 0 \, (\nabla \cdot J)$;
the left side must be zero, but the right side, in general, is not. For steady currents,
the divergence of J is zero, but when we go beyond magnetostatics Ampère's law cannot be right.

How Maxwell Fixed Ampère's Law

$$\boxed{\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}.}$$

A changing electric field induces a magnetic field.