As in the case of em waves, the wave and particle aspects of moving bodies can never be observed at the same time.

Which set of properties is most conspicuous depends on how its de Broglie wavelength compares with its dimensions and the dimensions of whatever it interacts with.

The wave function itself, however, has no direct physical significance. There is a simple reason why cannot by interpreted in terms of an experiment. The probabil- ity that something be in a certain place at a given time must lie between 0 (the object is definitely not there) and 1 (the object is definitely there).

But the ampli- tude of a wave can be negative as well as positive, and a negative probability, say 0.2, is meaningless. Hence by itself cannot be an observable quantity.

This objection does not apply to ^{2}, the square of the absolute value of the wave function, which is known as **probability density:**

The probability of experimentally finding the body described by the wave function at the point ***x*, *y*, *z*, at the time *t* is proportional to the value of psi^2 there at *t*.**

While the wavelength of the de Broglie waves associated with a moving body is given by the simple formula *hm*, to find their amplitude as a function of position and time is often difficult.

How fast do de Broglie waves travel? Since we associate a de Broglie wave with a moving body, we expect that this wave has the same velocity as that of the body.

Because the particle velocity must be less than the velocity of light *c*, the de Broglie waves always travel faster than light! In order to understand this unexpected result, we must look into the distinction between **phase velocity** and **group velocity.** (Phase ve- locity is what we have been calling wave velocity.)

In three dimensions *k* becomes a vector **k** normal to the wave fronts and *x* is re- placed by the radius vector **r**. The scalar product **k r** is then used instead of *kx* in Eq. (3.9).

A group of waves need not have the same velocity as

the waves themselves

The amplitude of the de Broglie waves that correspond to a moving body reflects the probability that it will be found at a particular place at a particular time.

If the velocities of the waves are the same, the velocity with which the wave group travels is the common phase velocity. However, if the phase velocity varies with wavelength, the different individual waves do not proceed together. This situation is called **dispersion**

It is not hard to find the velocity v_g with which a wave group travels. Let us suppose that the wave group arises from the combination of two waves that have the same amplitude A but differ by an amount $\Delta\omega$ in angular frequency and an amount Δk in wave number. We may represent the original waves by the formulas

$$y_1 = A \cos(\omega t - kx)$$
$$y_2 = A \cos[(\omega + \Delta\omega)t - (k + \Delta k)x]$$

The resultant displacement y at any time t and any position x is the sum of y_1 and y_2. With the help of the identity

$$\cos\alpha + \cos\beta = 2\cos\tfrac{1}{2}(\alpha+\beta)\cos\tfrac{1}{2}(\alpha-\beta)$$

and the relation

$$\cos(-\theta) = \cos\theta$$

we find that

$$\begin{aligned} y &= y_1 + y_2 \\ &= 2A\cos\tfrac{1}{2}[(2\omega + \Delta\omega)t - (2k + \Delta k)x]\cos\tfrac{1}{2}(\Delta\omega\, t - \Delta k\, x) \end{aligned}$$

Since $\Delta\omega$ and Δk are small compared with ω and k respectively,

$$2\omega + \Delta\omega \approx 2\omega$$
$$2k + \Delta k \approx 2k$$

and so

Beats

$$y = 2A\cos(\omega t - kx)\cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right) \tag{3.10}$$

the probability density 2 is a maximum in the middle of the group, so it is most likely to be found there

At a certain time *t*, the wave group (*x*) can be represented by the **Fourier integral**

Strictly speaking, the wave numbers needed to represent a wave group extend from

k 0 to *k* , but for a group whose length *x* is finite, the waves whose ampli- tudes *g*(*k*) are appreciable have wave numbers that lie within a finite interval *k*.

the narrower the group, the broader the range of wave numbers needed to describe it, and vice versa.

The relationship between the distance *x* and the wave-number spread *k* depends upon the shape of the wave group and upon how *x* and *k* are defined. The minimum value of the product *x k* occurs when the envelope of the group has the familiar bell shape of a Gaussian function. In this case the Fourier transform happens to be a Gaussian function also. If *x* and *k* are taken as the standard deviations of the

respective functions (*x*) and *g*(*k*), then this minimum value is *x k* 1. Because

2

wave groups in general do not have Gaussian forms, it is more realistic to express the

relationship between *x* and *k* as

=

+ +

+ .. .

Figure 3.13 An isolated wave group is the result of superposing an infinite number of waves with dif- ferent wavelengths. The narrower the wave group, the greater the range of wavelengths involved. A narrow de Broglie wave group thus means a well-defined position (*x* smaller) but a poorly defined wavelength and a large uncertainty *p* in the momentum of the particle the group represents. A wide wave group means a more precise momentum but a less precise position.

The longer the wavelength of the observing photon, the smaller the uncertainty in the electron's momentum.

It is worth keeping in mind that the lower limit of 2 for *x p* is rarely attained. More usually *x p* , or even (aswejustsaw) *x ph*.