TECHNICAL CHANGE AND THE COMMONS

Dale Squires and Niels Vestergaard*

Abstract—This paper addresses normative exploitation of common renewable resources with changes in technology and technical, allocative, and scale efficiency that exacerbate the commons problem and externality. Their impact depends on the rate and nature of change, investment, and state of property rights. An augmented fundamental equation of renewable resources with a modified marginal stock effect and a new marginal technology effect account for changes in disembodied and embodied technology and technical efficiency. Neglecting these changes generates misleading policy advice and dynamic inefficiency with overaccumulation of physical and natural capital and sizable foregone rents. An empirical application illustrates.

I. Introduction

T ECHNICAL progress and gains in economic efficiency are generally viewed as favorably contributing to economic growth and welfare, but does this normative conclusion hold for industries exploiting common renewable resources? What are their effects on the paradox between the individual firm's private economic efficiency and society's economic efficiency and costs with common renewable resources (Gordon, 1954)? In short, what is the relationship between technical change and a broad notion of efficiency with the tragedy of the commons and the optimum exploitation of common renewable resources?

The normative economics literature on common renewable resources has largely overlooked technical change, instead focusing on steady-state levels of effort or capital stock, resource stock, yield, and their dynamic approaches under constant technology.[1] Smith (1972), an exception, found that an unpriced common resource induces technical change in favor of increased utilization of the unpriced

resource and that the competitive pressures of the race for fish can compel firms to adopt innovations and provides an endogenous source of growth. Smith and Krutilla (1982) observed that technical change under open access accelerates the dissipation of resource rent and depletes resource stocks that are already overexploited. Murray (2007) introduced exogenous disembodied technical progress into a static Gordon-Schaefer model of a fishery to show that overlooking technological change can overestimate the natural growth of the resource stock and that inputs must be removed from the fishery at the rate of technological change to sustain the harvest target. Clark, Clarke, and Munro (CCM) (1979) examined investment but overlooked technical change that is disembodied or embodied in investment.

In short, the normative literature has yet to formally analyze the impact of changes in disembodied and embodied technical progress and technical efficiency on optimum exploitation of common renewable resources. This literature has considered nonautonomous dynamic models to capture exogenous price shocks and disembodied technical change specified as costs a function of time (Clark & Munro, 1975; Clark, 2010). This literature has similarly overlooked the broad Debreu (1951)–Farrell (1957) notion of economic (technical, allocative, and scale) efficiency, instead concentrating solely on the efficient dynamic scale of production, that is, on the optimum level of nominal effort and yield.[2]

In contrast, the exhaustible resource, sustainable growth, and climate change literatures have paid considerably more attention to technical progress and to substitution possibilities between the resource stock and inputs in discussions of natural resource scarcity, limits to growth, and backstop technologies (Arrow et al., 2004). Farzin (1995) and d'Autume and Schubert (2008) summarized the literature on technical progress and exhaustible resources and focused on the impact of technical change on measures of resource scarcity. Climate change and the need to reduce fossil fuel and carbon emissions are focusing considerable attention on directed and induced technical change (Acemoglu, 2002; Pizer & Popp, 2008).

This paper contributes to the literature on technical change, economic efficiency, and optimum exploitation of renewable resources by introducing output-oriented Debreu-Farrell technical, allocative, and scale inefficiency and

[1] See, for example, the following representative classics of renewable resource economics: Gordon (1954), Clark and Munro (1975), Clark, Clarke, and Munro (1979), Dasgupta and Heal (1979), Dasgupta (1982), Clark (2010), and Hannesson (1993), in which the state of technology is not usually mentioned and optimal control models are nonautonomous. Dasgupta (1982) and Brown (2000) explicitly hold technology constant. The literature on trade, renewable resources, economic growth, and technical change, reviewed by McAusland (2005), who allows the intrinsic growth rate r to be a function of technical change, is not germane to normative utilization of renewable resources.

[2] Debreu (1951)–Farrell (1957) economic efficiency includes technical, allocative, and scale efficiency. Scale efficiency arises when price equals marginal cost. In the standard bioeconomic model, homothetic separability between inputs and outputs and joint production is implicitly assumed to form an aggregate output and an aggregate input (Hannesson, 1983; Squires, 1987). Profit or rent efficiency in the renewable resource economics literature—the traditional maximum economic yield—reduces to only (dynamic) scale efficiency, and technical efficiency and allocative efficiency for multiple inputs and for multiple outputs are overlooked as sources of economic efficiency.

disembodied and embodied technical progress into a dynamic model of an industry exploiting a common renewable resource and examining the economic and policy ramifications. The paper clarifies that renewable resource models have previously focused solely on dynamic scale efficiency of nominal effort and overlooked technical and allocative efficiency.

The most important contribution is an augmented fundamental equation of renewable resources, or Golden Rule, that incorporates exogenous changes in disembodied and embodied technology and technical inefficiency into a new term, the *marginal technology effect,* and modifies the existing marginal stock effect. Technical progress, by lowering harvest costs, limits the incentives to accumulate natural capital to lower costs, but not necessarily physical capital embodied with technology. Dynamic inefficiency from overaccumulating natural and physical capital can occur when technical progress is overlooked. The balanced growth path asymptotically approaches the growth limit imposed by the marginal productivity of natural capital and social discount rate. Optimum resource stock levels can decline below the (nogrowth) steady-state equilibrium level of static technology and even below maximum sustainable yield.

Overlooking technical progress and broader notions of economic efficiency in a normative framework and modeling in a nonautonomous framework have profound consequences through exacerbating the commons problem under open access or potentially generating misleading policy advice. In fishing industries, for example, once fish could no longer hide from more technologically advanced vessels, the stage was set for the current biological overfishing crisis in many of the world's fisheries.[3] In fact, perhaps the single greatest pressure on global fisheries is technical change, but policy advice focuses primarily on reducing vessel capital stocks, nominal fishing effort, subsidies, and accumulating

natural capital to reduce costs and mistakenly identifies the region of dynamic scale inefficiency with overaccumulated natural capital. Improvements in technology have been the neglected driving force behind overfishing and overcapacity. Accounting for changes in technology also raises the question of whether global fisheries are as economically overexploited as believed when optimizing economic rents. The risk of extinction under both open access and the economic rent optimum also rises with changes in technology and more comprehensive notions of efficiency, and higher intrinsic population growth rates than considered by Clark (1973) are required to preclude optimal extinction.

The model is applied to a fishery using the classic Gordon-Schaefer specification (Gordon, 1954; Clark & Munro, 1975; CCM, 1979), which lies at the heart of this literature and allows direct and analytical development of the Golden Rule.[4] The technical change specified is exogenous when disembodied and endogenous when embodied in the sense of endogenous investments in exogenous technical change embodied in equipment.[5] The disembodied shifts the production frontier and includes learning by doing and using, and the embodied increases physical capital's technical efficiency.[6]

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[3] Rapid advancements in fishing technologies led to increased fishing pressure on all fish stocks in the twentieth century, calling into question the late-nineteenth- and early-twentieth-century belief of an inexhaustible bounty from the ocean (Huxley, 1883). Mechanical power for vessels replaced sail power in the 1880s, which allowed the development of new types of gear (for example, the otter trawl in 1892) and substantially larger vessels and gear and the exploitation of fish stocks in previously inaccessible ocean locations and depths and at substantially higher levels of productivity. Synthetic materials for gear and the power block in the 1950s were also critical. Small-scale traditional fisheries are now motorized. Vessel electronics, such as sonar, LIDAR, chromoscopes, satellite and thermal imaging, cell phones, broad-swathe mapping of the seabed, bird radar, and GPS all help communications, navigation, locating fish, monitoring gear performance while fishing, and developing markets. Technology-based knowledge of modern electronic equipment, communications, and satellites increasingly supersedes local traditional and craft knowledge, leading to price-induced substitution of the firm's management and other labor services, also raising their marginal products and thereby increasing their services (Jorgenson, 2005), but also allows expansion in output through fishing in new areas and depths (for example) and raising the overall skill level of average and below-average skippers. The quality of electronics has steadily improved and unit costs fallen, increasing the rate of adoption and diffusion, a form of price-induced technical change (Jorgenson, 2005). Old and new vintages of related technologies may coexist, such as radio and Internet access or Loran C and GPS. Rates of diffusion differ by fishery.

[4] A referee suggested other models that examine free entry and exit of capital into the fishery under open access (Smith, 1968; Berck & Perloff, 1984; Homans & Wilen, 1997), but these do not analyze the normative economic optimum and develop a Golden Rule, our central objective. We instead examine both perfectly and imperfectly malleable capital and economic optimum natural and physical capital stock dating to Clark and Munro (1975) and to CCM (1979) when there is investment in imperfectly malleable physical capital, using the classic and explicit Hamiltonian approach to optimal control and yielding the Golden Rule in the tradition of texts in note 1. Clark and Munro (1975) and CCM (1979) are at the heart of policy formation and literature.

[5] Much technical change is autonomous—"off the shelf"—rather than endogenous because of limited formal firm R&D activities and military antecedents, although regulatory induced can be important (but beyond the scope of this paper), and process innovations are not independent of economic incentives such as floating aggregator devices. Learning by doing gives continuous improvement, the unintended by-product of any experience gained during the harvesting of fish on a trip, and informally occurs (as opposed to formal training and formal R&D, for example). Undoubtedly the process of learning is social (Conley & Udry, 2010). A second source is an increase in a firm's net investment that leads to a parallel increase in its stock of knowledge as the firm learns how to fish more efficiently. Knowledge creation is a side product of investment.

[6] Marcoul and Weninger (2008) provide a recent fishery learning by doing (unit costs decrease with cumulative production), positive (but not normative) analysis related to search according to catch rates and resource stock densities. Our paper remains within the conventional learning-by-doing framework of the standard technical change and economic growth literatures. A referee suggested a model focused explicitly on stochastic learning by doing, with, for example, an additional state equation for fishing skill and human capital where the sole owner invests in learning the distribution of the fish across space, but there is a cost to learning (returns to fishing that are forgone to learn the density of fish in a particular area); however, this would introduce a prohibitive third state equation. Building off Sanchirico and Wilen (1999) to explicitly consider spatially linked distinct populations or substocks (that is, metapopulations) would take us away from the classical normative Gordon-Schaefer framework and Golden Rule, which is central to conventional policy formation, control rules, texts in note 1, and empirical studies such as the Sunken Billions (World Bank/FAO, 2009). Moreover, the Sanchirico and Wilen (1999) metapopulation approach simply does not apply to a highly migratory species such as North Pacific albacore that migrate on the surface from waters off Japan into inshore U.S. waters and end in the Western Pacific Ocean (see Block et al., 2011), which broadcast spawn eggs on the sur-

We focus on investment in information technology and equipment for an already established industrial fleet rather than the relatively infrequent new entry of vessels or large-scale accumulation of physical capital.[7] Most major industrial fisheries are fully (if not over-) capitalized with vessels subject to limited entry, and industrial vessel numbers are largely stable (FAO, 2010), although their numbers would decline in an optimally managed fishery. Nonetheless, the framework readily applies to large-scale changes in physical capital such as vessels, with only the size of fixed and variable costs changing.[8]

The balance of the paper is organized as follows. Section II develops stochastic stock flow production frontiers incorporating technical change and technical and allocative inefficiency. Section III develops Gordon-Schaefer cost and rent frontiers. Section IV develops three simple dynamic models of the economic optimum accounting for economic inefficiency, disembodied and embodied technical change, and investment corresponding to CCM (1979). Section V considers optimum policies. Section VI introduces the empirical application. Section VII provides illustrating empirical results, and section VIII concludes. All derivations are available in the appendix.

II. Fishery Production Frontier

The Graham-Schaefer stock flow production function in time t relates catch, Y_t, to the fish stock, S_t, and fishing effort, E_t: $Y_t = f(q, E_t, S_t) = qE_tS_t$.[9] Catch is the output or flow from S_t, and S_t and E_t are specified as aggregate inputs. Effort can be considered as the first stage in a two-stage production process and implicitly assumes strong Leontief-Sono separability of E_t from S_t and input-output separability with Y_t to form a composite index of inputs (Hannesson, 1983; Squires, 1987). Inputs are combined in an allocatively efficient manner in the separable input aggregator function so that each level of effort lies on the input expansion path. The catchability coefficient, $q > 0$, is the probability that 1 unit of effort taken at random will catch 1 unit of the population taken at random.[10]

Introducing time-varying output-oriented technical efficiency gives $Y_t = qE_tS_te^{-\mu(t,z)}$, where $-\mu(t,Z)$ denotes a nonpositive, half-sided error term that introduces deviations from the best-practice frontier or technical inefficiency. Z defines a vector of explanatory variables associated with technical inefficiency. The measure for technical efficiency (TE) is $\exp\{-\mu(t,Z)\} \leq 1$.[11] Technical inefficiency arises when TE < 1 and grows with increases in $\mu(t,Z)$. Technical inefficiency accounts for the time it may take for new technologies to diffuse, different technologies embodied in different vintages of capital that give different average embodied technical efficiencies among vessels, poorly designed regulation, the firm's managerial failure, skipper skill, and other efficiency differences in harvesting units (Kirkley, Squires, & Strand, 1998).[12]

A. Disembodied Technical Change

Technical change, specified as exogenous, disembodied, and captured by a linear time trend t, can be introduced into the Graham-Schaefer surplus production frontier,

$$Y_t = qE_tS_te^{\lambda t-\mu(t,z)}, \tag{1a}$$

where $E_t = f(X_{1t}, X_{2t})$, X_{1t} denotes a vector of variable inputs and X_{2t} denotes physical capital stock. The parameter $\lambda > (<)0$ measures the rate of technical progress (regress), shifting the best practice frontier. Technical change in equa-

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face, do not have spatially distinct but linked populations and, moreover, are juveniles for our vessels and data. The Marcoul and Weninger (2008) approach is inappropriate for albacore, a schooling species found near temperature breaks or oceanic fronts with schools up to 31 km wide, since they examine patchy and sessile clams found in the benthic habitat on the ocean floor in spatially distinct beds with site fidelity. In contrast, migrating albacore swimming at speeds up to 80 km per hour are found near dynamically evolving oceanic fronts and temperature breaks using temperature-sensing devices, satellite data, GPS, and acoustic devices rather than essentially repeated sampling with tows of dredges at familiar sites. We would have to link unavailable fishery data with oceanographic variables in a level of detail that even population biologists cannot (they do not have or use vessel numbers data). Finally, the detail of available Canadian data simply precludes following Sanchirico and Wilen (1999) or Marcoul and Weninger (2008).

[7] Many of the recent mechanical process innovations have been incremental and low cost, such as cort nozzles that give engine thrust, and innovations such as these can be readily accommodated in our model. Gear process innovations, such as floating aggregator devices, cost a pittance, even when they are outfitted with information technology (radio buoys, sonar, GPS, and transmitters) to track them (and are endogenous). Even many mechanical conservation regulatory-induced process innovations to reduce finfish bycatch and incidental takes of sea birds and sea turtles (Tori lines, turtle excluder devices, sorting grids) cost a relative pittance and heavily rely on learning by doing and using. All electronics are exogenous (often with military antecedents). In short, most process innovations are incremental, embedded in equipment, are remarkably low cost, are not part of the hull as a platform or structure, form quality ladders of sequences of improvements, require learning by doing and using, are largely nonrival, and are readily accommodated by our model, although we focus on information technology.

[8] The depreciation rate γ also changes. γ is comparatively high for electronics equipment due to obsolescence and for gear due to physical deterioration, but it is very low for the vessel itself, since vessels are continually maintained and repaired, including occasional dry-docking.

[9] The usual regularity conditions for a production function are assumed (Dasgupta, 1982). We also assume market structure that is consistent with competitive behavior.

[10] We are grateful to Pat Tomlinson for this definition. q converts, in a Hick's-neutral manner, the level of E_t to the proportion of S_t removed and captures changes in technology, technical efficiency, the environment, and other factors not captured by E_t.

[11] With an aggregate production technology, this could reflect average departure from the production frontier by individual production units. In the empirical study, it represents potential departures by the Canadian fleet from the best-practice frontier established by the U.S. fleet.

[12] The sign of the partial derivative $\partial\mu(t,Z)/\partial t$ depends on how the industry adjusts to the introduction of innovations or, in the case of no technical progress, how the industry is catching up relative to the peers. In the large case, the sign is expected to be positive, while in the first case, the sign has to be empirically estimated.

tion (1) is constant and Hick's neutral with a single input, and is output augmenting (and equivalently input augmenting with a composite input, E_t, and linear homogeneity of the effort aggregator function).[13]

Disembodied technical change in equation (1) can in part be viewed as learning by doing and using once exogenous process innovations and investments have been introduced.[14] Usually no formal R&D is devoted to technological advances except perhaps in all but the largest, most capital-intensive, and concentrated fishing industries. Some types of technical change (often regulatory induced) can mitigate the negative environmental externality of incorrectly sized fish or undesirable joint products (for example, a bycatch of turtles), but this paper focuses on technical change enabling production of desirable outputs at lower cost. The focus here is on continuous and gradual process innovations and subsequent learning related to finding and catching fish, especially by the firm's manager (skipper), or learning by doing, and for discovery of new and unanticipated uses of the process innovations, or learning by using, by the manager and labor force (crew). Since industries exploiting renewable resources are largely mature, minimal depreciation of the largely tacit knowledge acquired through learning is anticipated (although little is known about the process by which this tacit knowledge is acquired). Normally there is no opportunity cost other than current cost of production with learning by doing (without opportunity costs such as crowding out from R&D or knowledge market failures). There may also be learning benefit spillovers from one firm to the next, but the lion's share of the learning benefits accrues to firms engaged in the learning, especially to skippers in finding fish and to crews in handling gear and equipment, although learning differences arise among firms. Strong incentives exist for learning due to the share system of remuneration in which the crew, skipper, and owner reap some of the increased profits.

B. Embodied Technical Change

Growth in information technology embodied in electronics and other equipment investment affects catch through an increase in efficiency and capital deepening. Since electronics and most other equipment are comparatively inexpensive and by value are an exceptionally small cost share of the total capital stock (typically 1% to 3%, and even less as a share of total cost), the contribution to capital deepening is largely through an increase in effective physical capital or effort by increasing the average embodied efficiency of equipment. Since labor and skippers now work with better equipment that enables finding fish, substituting for semi-craft traditional skills learned and honed over many years, this investment increases the productivity of the vessel, labor, and skipper. The investment-specific technical progress also affects disembodied technical progress through learning by doing and using. Information technology has little impact on the quality of fish caught. Technical change embodied in the capital stock related to quality of catch after capture, such as icing and refrigeration, has long been adopted, so that output quality has largely remained constant and need not be explicitly modeled.

Successive vintages of investment embody differences in technology. Technical progress in equipment is linked to improvements in new equipment design, and later-vintage electronics is more efficient at producing output (finding and catching fish) than earlier vintages, even if there is no physical loss in capacity. Embodied technical change leading to "better" capital is equivalent to "more" capital (Fisher, 1965).

Let I_t denote gross investment in period t measured in number of natural physical capital units. Each vintage of investment is converted into new equipment equivalents by depreciation γ, so X_{2t} is the sum of all vintage investments in all previous periods measured as total number of machines and vessels in natural physical units in new machine equivalents. Investment in technical efficiency units is $H_t = \Phi_t I_t$, where the index of technical efficiency Φ_t denotes the best-practice level of technology at time t (Hulten, 1992). Changes in Φ_t capture quality differentials between successive vintages, that is differences in technical design. The rate of change in Φ_t, ϕ_t, is associated with the rate of embodied technical change. Total capital at time t measured in efficiency units—Solow's jelly capital—is $J_t = \Psi_t X_{2t}$, where Ψ_t is the weighted average level of best–practice efficiency associated with each past vintage of investment: $\Psi_t = \frac{I_t}{X_{2t}} \Phi_t + \frac{(1-\gamma)I_{t-1}}{X_{2t}} \Phi_{t-1} + \frac{(1-\gamma)^2 I_{t-2}}{X_{2t}} \Phi_{t-2} + \dots$.[15] The growth rate of Ψ_t, ψ_t, is not a fully exogenous parameter like λ and depends on the rate of capital formation.

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[13] Technological change $A(t) = q\lambda(t) = qe^{\lambda t}$ as process innovations affecting E_t in $Y_t = A(t)F(E_t, S_t)$ is both Hick's neutral and output augmenting. The strong Leontief-Sono separability for E_t and linear homogeneity of the input aggregator function required for a consistent composite input index E_t in equation (1) preclude biased technical change among the input pairs comprising E_t in the first stage of production. That is, technical change is implicit Hick's neutral among all input pairs comprising E_t, and changes in the scale of production do not affect input ratios. Blackorby et al. (1978) call this "extended Hick's-neutral technological change" because it is expansion path preserving (remaining on the expansion path ensures allocative efficiency).

[14] We develop this discussion heuristically rather than through a formal model and within the traditional approach of the technical change and economic growth literatures. Learning by doing is disembodied in that it arises from increases in the stock of knowledge, independent of the characteristics of inputs used but explains differences across vessels in the productivity of the same levels and types of inputs (Bahk & Gort, 1993). It can be both firm specific and industrywide and captures routinization of tasks, organizational learning such as matching tasks with individuals, managerial learning, experience gained with electronics, finding fish, navigation, gear handling, and knowledge of the environment and resource conditions, such as currents, weather conditions, water temperature breaks, and resource stock densities.

[15] The parameter Ψ_t differs from capital-augmenting technical change in that Ψ_t depends on underlying efficiency parameters and the age structure of the capital stock, whereas the capital-augmentation parameter depends only on time (Hulten, 1992).

The general aggregate production frontier with both disembodied and embodied technical change can be written as $Y_t = q\hat{E}_t S_t e^{\lambda t - \mu(t,Z)}$, where $\hat{E}_t = f(X_{1t}, \Psi_t X_{2t}) = f(X_{1t}, J_t)$, X_{2t} and J_t are themselves aggregates in full static equilibrium in each time period, and we do not distinguish in these aggregates between structures (hull, engine, and so on) and equipment, given the complexity of the control problem plus lack of data. As the function f is a linear homogeneous function, $\Psi_t \bar{E}_t = \Psi_t f\left(\frac{1}{\Psi_t} X_{1t}, X_{2t}\right)$, it can be combined with disembodied technical change to give another definition of effective effort: $\hat{\bar{E}}_t e^{\lambda t - \mu(t,z)} = \Psi_t \bar{E}_t e^{\lambda t - \mu(t,z)}$. Assuming a constant rate of embodied technical change ψ and constant capital share of income M_2 implies $\Psi_t = e^{M_2 \psi_t}$ (Hulten, 1992; Gort & Wall, 1998). Effective effort is $\bar{E}_t e^{(\lambda + M_2 \psi)t - \mu(t,z)}$ with corresponding harvest frontier:

$$Y_t = q\bar{E}_t S_t e^{(\lambda + M_2 \psi)t - \mu(t,z)}. \tag{1b}$$

When investment is modeled following CCM (1979), effort is formed through Leontief aggregation and separability or Harrod-Domar conditions with embodied technical change occurring through investment. Embodied technical change still occurs inside the linear homogeneous aggregator function: $\tilde{E}_t = \min\{AJ_t, BX_{1t}\} = \min\{A\Psi_t X_{2t}, BX_{1t}\}$, where A and B are fixed coefficients (Aghion & Howitt 2009).[16] This gives the third and fourth definitions of effective effort: $\min\{AJ_t, BX_{1t}\}e^{\lambda t - \mu(t,z)} = \min\{A\Psi_t X_{2t}, BX_{1t}\} e^{\lambda t - \mu(t,z)}$ in the general case, and when capital is strictly the limiting factor, the effective effort is $AJ_t e^{\lambda t - \mu(t,z)} = A\Psi_t X_{2t} e^{\lambda t - \mu(t,z)} = AX_{2t} e^{(\lambda + M_2 \psi)t - \mu(t,z)}$. The Leontief-CCM specification of effort with embodied and disembodied technical change gives the following harvest frontier when capital is the limiting factor and $\lambda, \psi, X_{2t}, M_2$ are constant (A could be absorbed into q):

$$\begin{aligned} Y_t &= q\tilde{E}_t S_t e^{\lambda t - \mu(t,z)} = qAJ_t S_t e^{\lambda t - \mu(t,z)} \\ &= qAX_{2t} S_t e^{(\lambda + M_2 \psi)t - \mu(t,Z)} \end{aligned}. \tag{1c}$$

The catch growth rate with both embodied and disembodied technical change in a standard growth accounting framework is (Hulten, 1992; Squires, 1992) $\dot{Y} = (1 - M_{2t})\dot{X}_{1t} + M_{2t}\dot{X}_{2t} + \dot{S}_t + M_{2t}\psi_t + \lambda$, where the dots denote proportional rates of growth. Rearranging gives the growth rate in total factor productivity or the Solow residual (here assuming constant returns to scale in effort, Hicks neutrality, full capacity utilization, full technical efficiency for rea-

sons other than embodiment of technology in capital, and no changes in output quality): $\dot{T} = M_{2t}\psi_t + \lambda$.

III. The Gordon-Schaefer Cost and Rent Frontiers

The change in S_t with harvesting is the net growth rate of the biomass specified as a simple differential equation: $dS_t/dt = F(S_t) - Y_t$, where $F(S_t)$ denotes a general growth function of S_t measured in biomass. Allowing for disembodied and embodied technical change and substituting equation (1b) and the logistic or Pearl-Verhulst growth function $F(S_t) = rS_t(1 - S_t/K)$ into $dS_t/dt = F(S_t) - Y_t$, gives $dS_t/dt = rS_t(1 - S_t/K) - q\bar{E}_t S_t e^{(\lambda + M_2 \psi)t - \mu(t,z)}$, where r is the maximum intrinsic growth rate of S_t, that is $r = \lim_{t \to 0}[(dS_t/dt)/S_t]$, and K is the environmental carrying capacity, that is, $\lim_{t \to \infty} S_t = K$ with zero harvest.

The cost frontier (and dS_t/dt) specification depends on the specifications of effective effort and the catch frontier. Here we illustrate for $\bar{E}_t$ by solving the production frontier for $\bar{E}_t = \left[Y_t \big/ \left(qS_t e^{(\lambda + M_2 \psi)t - \mu(t,Z)}\right)\right]$ and substituting into the standard Gordon total cost (TC) equation $TC_t = c\bar{E}_t$ to give the cost frontier: $TC[Y_t, S_t, q, \Psi_t, \lambda, -\mu(t,Z)]_t = \frac{c}{e^{(\lambda + M_2 \psi)t - \mu(t,Z)}} \frac{Y_t}{qS_t}$, where $c = $ constant (marginal and average) cost per unit of effort, which allows all costs and inputs to be variable and in full static equilibrium in a given time period. Disembodied and embodied technical progress lead to cost diminution, where the TC function can now be interpreted in terms of input price diminishment or effective input price, given by $\frac{c}{e^{(\lambda + M_2 \psi)t - \mu(t,z)}}$, and where $e^{(\lambda + M_2 \psi)t - \mu(t,Z)}$ is the augmentation coefficient. Technical change is not confounded by scale economies because of the linear homogeneity of the catch frontier in $\bar{E}_t$. With a single composite input $\bar{E}_t$, cost-reducing technical change can be viewed as cost neutral and is equivalent to rent-neutral and Hick's-neutral technical change with the homogeneous Gordon-Schaefer rent frontier. The rent frontier using the cost frontier is $\pi_t[P, c, Y_t, S_t, q, \Psi_t, \lambda, -\mu(t,Z)] = PY_t - \frac{c}{e^{(\lambda + M_2 \psi)t - \mu(t,Z)}}$ $\frac{Y_t}{qS_t} = \left[P - \frac{c}{qS_t e^{(\lambda + M_2 \psi)t - \mu(t,z)}}\right]Y_t$. P denotes the constant price of catch and could reflect nonmarket amenity values when it is an accounting price.

IV. Dynamic Models

To distinguish their individual effects, we develop three cases for the Gordon-Schaefer model with (1) disembodied technical change and perfectly malleable capital, (2) both disembodied and embodied technical change and perfectly malleable capital and (3) both disembodied and embodied technical change and imperfectly malleable capital. The first two models do not specify investment explicitly, but CCM (1979) show that the standard dynamic Gordon-Schaefer model implicitly assumes perfectly malleable capital (and hence full static equilibrium). There is no need

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[16] Because J_t is a stock rather than a flow, we assume J_t to be fully utilized, so that J_t yields a proportional flow of services that increase with investment (raising the marginal product). This composite effort eliminates endogenous growth due to substitution among inputs or changes in capital utilization and capacity utilization. The vessel "structure" has a rental rather than fixed cost, but the new investment has a fixed cost c_f.

to specify a state equation for physical capital dynamics, only the state equation for resource stock dynamics.

A. Disembodied Technical Change

This is the standard dynamic Gordon-Schaefer model of Clark and Munro (1975) with disembodied but not embodied technical change. Nominal effort is $E_t = f(X_{1t}, X_{2t})$, and the harvest frontier is equation (1a): $Y_t = qE_t S_t e^{\lambda t - \mu(t,Z)}$.

The sole owner or social planner maximizes the present value of net benefits derived from exploiting the resource.[17] The cost diminution in our cost-neutral cost frontier and rent-neutral rent frontier exactly measures technical progress because of the linear homogeneity in E_t without confounding by economies or diseconomies of scale. This linear homogeneity in E_t also yields costs linear in the control variable, Y_t, and the solution is bang-bang, or the most rapid approach to the optimal stock level.

If $\delta > 0$ is a constant denoting the continuous social rate of discount, the objective is $PV(\pi) = \int_0^\infty \pi[Y_t, S_t] e^{-\delta t} dt$ subject to $dS_t/dt = F(S_t) - Y_t$ and $S_o = S(0)$, where $\pi_t[P, c, Y_t, S_t, q, \lambda, -\mu(t,z)] = PY_t - \frac{c}{e^{\lambda t - \mu(t,z)}} \frac{Y_t}{qS_t} = \left[P - \frac{c}{qS_t e^{\lambda t - \mu(t,Z)}}\right] Y_t$. The inclusion of time leads to a nonautonomous dynamic model.

The present value Hamiltonian with technical efficiency and technical change is

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\begin{aligned}
H(Y_t S_t, \alpha(t)) &= e^{-\delta t} \pi[Y_t, S_t] + \alpha(t)(F(S_t) - Y_t) \\
&= e^{-\delta t}\left[P - \frac{c}{qS_t e^{\lambda t - \mu(t,z)}}\right] Y_t + \alpha(t)(F(S_t) - Y_t),
\end{aligned}
\tag{2}
$$

where $\alpha(t)$ is the present value multiplier. Solving equation (2) gives an augmented Golden Rule, or fundamental equation of renewable resources, incorporating disembodied technical change and technical inefficiency:

$$
\frac{\partial F}{\partial S_t} + \frac{cF(S_t)}{S_t(PqS_t e^{\lambda t - \mu(t,Z)} - c)} + \frac{c(\lambda - \partial \mu(t,Z)/\partial t)}{(PqS_t e^{\lambda t - \mu(t,Z)} - c)} = \delta.
\tag{3}
$$

Compared to the traditional rule, there is now a new term added beyond the marginal productivity of the resource $(\partial F/\partial S_t = r(1 - 2S_t/K))$, and the marginal stock effect $[cF(S_t)]/[S_t(PqS_t e^{\lambda t - \mu(t,Z)} - c)]$, namely, the last term on the left-hand side, the marginal technology effect. The marginal technology effect captures the effect of technical progress and increases in technical efficiency on cost. The mar-

ginal stock effect is itself augmented by disembodied technical change and technical inefficiency.

The singular solution S_t^*, E_t^*, Y_t^* of equation (3) is given in the appendix and S_t^* is

$$
\begin{aligned}
S_t^* = \frac{K}{4} &\left[\left[\frac{c}{PqKe^{\lambda t - \mu(t,Z)}} + 1 - \frac{\delta}{r}\right] \right. \\
&\left. + \sqrt{\left[\frac{c}{PqKe^{\lambda t - \mu(t,Z)}} + 1 - \frac{\delta}{r}\right]^2 + \frac{8c(\delta + \lambda - \partial\mu(t,Z)/\partial t)}{PqKre^{\lambda t - \mu(t,Z)}}}\right].
\end{aligned}
\tag{4}
$$

Equation (4) clearly indicates that there is not a classic (no-growth) steady-state solution to the problem, but instead a balanced growth path that is eventually limited by the resource stock's productivity. With ongoing exogenous disembodied technical change, the optimal level of the stock declines over time, because $PqKe^{\lambda t - \mu(t,Z)}$ increases due to this technical progress. However, the short-run effects of introducing technical change are an increase in S_t^*. The new marginal technology effect term is positive with technical progress, so that, all things equal, S_t^* is higher compared to the situation without technical progress beyond the marginal stock effect. This can also been seen from the last term in equation (4). In the traditional model, λ is not included, and therefore the immediate effect of technical progress is higher S_t^* compared to the traditional model. It is now profitable to reduce harvest today because of future technical progress.

However, over time, exogenous disembodied technical progress will lead to lower optimal stock levels compared to the initial optimal stock level because the unit profit of harvest, $PqS_t e^{\lambda t - \mu(t,Z)} - c$, increases due to technical progress, so that the effect of this term declines over time. Notably, the marginal stock effect in the modified Golden Rule, equation (3), declines in importance under continued technical progress. Density-dependent harvest costs that increase with declining stock size can be more than balanced by harvest cost diminution through technical progress.[18] Over time, the marginal stock and technology effects decline, requiring continuing increases in the own

---

[17] This is the standard textbook model with the classic Gordon-Schaefer specification (Clark & Munro, 1976; CCM, 1979; Clark, 1990; Dasgupta, 1982; Dasgupta & Heal, 1979; Hannesson, 1993). The model could be augmented by separable nonmarket amenity value (Van Kooten & Bulte, 2000), which would add an extra term in our augmented Golden Rule.

[18] The specification of a linear cost function, $TC_t = cE_t$, in this standard, classic textbook specification gives a cost function $TC_t = cY_t/(e^{\lambda t - \mu(t,Z)} qS_t)$, that in turn leads to a Hamiltonian linear in the control variable Y_t, which in part affects the interaction between the marginal stock effect and the marginal technology effect. A nonlinear specification for E_t would allow for increasing marginal costs in Y_t through diminishing returns with diminished S_t through growing importance of the marginal stock effect relative to the marginal technology effect at lower levels of S_t. Analytical solutions are not possible for the fundamental equation of renewable resources with nonlinear E_t, so that consistency with the traditional literature of note 1 is lost. Moreover, our empirical results fail to reject linear E_t. The main results remain unchanged, but with a slower rate of S_t^{***} decline and a somewhat higher S_t^{***} over an infinite time horizon in the limit case with diminishing returns. Similarly, linearity in E_t increases the benefits from the marginal stock effect and larger S_t^{***}.

rate of return to the resource stock, $\partial F/\partial S_t$, that follow with declining stock size, given constant social discount rate, δ. The own rate of return is increasingly likely to be positive rather than negative. Higher rates of social discount or technical progress hasten the resource stock decline.

B. Disembodied and Embodied Technical Change with Perfectly Malleable Capital

This model corresponds to Clark and Munro (1975) with both disembodied and embodied technical change. The dynamic model is specified:

$$PV(\pi) = \int_0^\infty \left\{ \pi\left[\hat{E}_t, S_t\right] \right\} e^{-\delta t} dt$$
$$= \int_0^\infty \left[\left\{ pqS_t\overline{E}_t e^{(\lambda+M_2\psi)t-\mu(t,z)} - c\overline{E}_t \right\} e^{-\delta t} dt$$

subject to

$$dS_t/dt = F(S_t) - Y_t.$$

The harvest function $Y_t = q\overline{E}_t S_t e^{(\lambda+M_2\psi)t-\mu(t,Z)}$, equation (1b), can be solved for $\overline{E}_t$, giving $\overline{E}_t = \left[Y_t / \left(qS_t e^{(\lambda+M_2\psi)t-\mu(t,z)}\right)\right]$, and substituted into the objective function. The present value Hamiltonian is

$$H(Y_t, S_t, \alpha(t)) = e^{-\delta t}\pi[Y_t, S_t] + \alpha(t)(F(S_t) - Y_t)$$
$$= e^{-\delta t}\left[P - \frac{c}{qS_t e^{(\lambda+M_2\psi)t-\mu(t,Z)}}\right]Y_t$$
$$+ \alpha(t)(F(S_t) - Y_t).$$

This expression can be solved to provide another version of the Golden Rule:

$$\frac{\partial F}{\partial S_t} + \frac{cF(S_t)}{(pqS_t e^{(\lambda+M_2\psi)t-\mu(t,Z)} - c)}$$
$$+ \frac{c(\lambda + M_2\psi - \partial\mu(t,Z)/\partial t)}{(pqS_t e^{(\lambda+M_2\psi)t-\mu(t,Z)} - c)} = \delta. \quad (5)$$

This augmented rule now includes ψ in both the marginal stock and the marginal technology effects. The balanced growth path differs from case 1 when there is embodied technical change, because in the short run, the size of S_t will be higher and Y_t lower. As time progresses, Y_t increases, but eventually Y_t declines when S_t falls below a certain threshold, where the rate of decline accelerates compared to only disembodied technical change. The singular solution $S_t^{**}, E_t^{**}, Y_t^{**}$ of equation (5) is given in the appendix, and S_t^{**} is

$$S_t^{**} = \frac{K}{4}$$
$$\times \left[\frac{\left[\frac{c}{PqKe^{(\lambda+M_2\psi)t-\mu(t,Z)}} + 1 - \frac{\delta}{r}\right]}{+\sqrt{\left[\frac{c}{PqKe^{(\lambda+M_2\psi)t-\mu(t,Z)}} + 1 - \frac{\delta}{r}\right]^2 + \left[\frac{8c(\delta+\lambda+M_2\psi-\partial\mu(t,Z)/\partial t)}{PqKre^{(\lambda+M_2\psi)t-\mu(t,Z)}}\right]}} \right].$$
$$(6)$$

The difference between equations (6) and (4) is $M_2\psi$.

C. Technical Change, Imperfectly Malleable Capital, and Investment

Case 3 is the Gordon-Schaefer model with imperfectly malleable capital, investment, and both embodied and disembodied technical change along with technical inefficiency. Investment in electronics is in practice nonmalleable. Vessel owners purchase electronics at a small cost relative to their total capital stock cost and use the electronics until they are technologically obsolete rather than selling them. There is a limited market for electronics that are obsolete or close to technological obsolescence. Hence, $I_t \geq 0$. Further CCM conditions on $\tilde{E}_t$ are $0 \leq A\Psi_t X_{2t} \leq \Psi_t X_{2t}$. The dynamic model becomes

$$PV(\pi) = \int_0^\infty \left\{ \pi\left[\tilde{E}_t, S_t\right] - c_f I_t \right\} e^{-\delta t} dt$$
$$= \int_0^\infty \left\{ pq\phi_t A X_{2t} S_t e^{(\lambda+M_2\psi)t-\mu(t,Z)} \right.$$
$$\left. - c_v\phi_t A X_{2t} - c_f I_t \right\} e^{-\delta t} dt$$

subject to

$$dS_t/dt = F(S_t) - Y_t = F(S_t) - q\phi_t A X_{2t} S_t e^{(\lambda+M_2\psi)t-\mu(t,Z)}$$
$$dX_{2t}/dt = I_t - \gamma X_{2t},$$

where $0 \leq \phi_t \leq 1$, c_v denotes costs for variable inputs, and let unit investment cost be c_f. We follow the CCM approach, so ϕ_t and I_t are the control variables and S_t and X_{2t} are state variables. ϕ_t replaces the effort variable because $\tilde{E}_t = \phi_t A X_{2t}$. The Hamiltonian is

$$H(\phi_t, I_t, S_t, X_{2t}, \alpha(t), \beta(t))$$
$$= \left\{ pq\phi_t A X_{2t} S_t e^{(\lambda+M_2\psi)t-\mu(t,Z)} - c_v\phi_t A X_{2t} - c_f I_t \right\} e^{-\delta t}$$
$$+ \alpha(t)\left\{ F(S_t) - q\phi_t A X_{2t} S_t e^{(\lambda+M_2\psi)t-\mu(t,Z)} \right\}$$
$$+ \beta(t)\{I_t - \gamma X_{2t}\}$$
$$= \left\{ e^{-\delta t}\left(pqA X_{2t} S_t e^{(\lambda+M_2\psi)t-\mu(t,Z)} - c_v A X_{2t} \right) \right.$$
$$\left. - \alpha(t)qA X_{2t} S_t e^{\left((\lambda+M_2\psi)t-\mu(t,Z)\right)} \right\}\phi_t$$
$$+ \left\{ \beta(t) - c_f e^{-\delta t} \right\}I_t + \alpha(t)F(S_t) - \beta(t)\gamma X_{2t},$$

where $\beta(t)$ is a present value multiplier The Hamiltonian is linear in ϕ_t and I_t, and hence the optimal solution is a set of bang-bang and singular controls.[19] The interesting case here is where $\phi_t = 1$ and I_t is singular. The other cases can either be rejected on intuitive reasons or they are not relevant in the longer run (see CCM). This leads to the following expression for the Golden Rule:

$$\frac{\partial F}{\partial S_t} + \frac{\left(c_v A + c_f(\gamma + \delta)\right)F(S_t)}{\left(PqAS_t e^{(\lambda+M_2\psi)t-\mu(t,Z)} - (c_v A + c_f(\gamma + \delta))\right)S_t} + \frac{(c_v A + c_f(\gamma + \delta))(\lambda + M_2\psi - \partial\mu(t,Z)/\partial t)}{PqAS_t e^{(\lambda+M_2\psi)t-\mu(t,Z)} - (c_v A + c_f(\gamma + \delta))} = \delta. \quad (7)$$

This rule modifies both the marginal stock effect and the marginal technology effect. Compared to case 2, the cost parameter c_f is new, so that the approach to this equilibrium differs because of the last term and the new cost parameter.

The singular solution $S_t^{***}, E_t^{***}, Y_t^{***}$ of equation (7) is given in the appendix and $\hat{S}_t^{***}$ is

$$S_t^{***} = \frac{K}{4}$$
$$\times \frac{\left[\frac{c_v A + c_f(\gamma+\delta)}{PqAKe^{(\lambda+M_2\psi)t-\mu(t,Z)}} + 1 - \frac{\delta}{r}\right] +}{\sqrt{\left[\frac{(c_v A + c_f(\gamma+\delta))}{PqAKe^{(\lambda+M_2\psi)t-\mu(t,Z)}} + 1\right] - \frac{\delta^2}{r} + \left[\frac{8(c_v A + c_f(\gamma+\delta))(\delta+\lambda+M_2\psi-\partial\mu(t,Z)/\partial t)}{PqAKre^{(\lambda+M_2\psi)t-\mu(t,Z)}}\right]}}. \quad (8)$$

To the extent investment costs are a smaller proportion of total costs, equation (7) should be similar to equation (5) and equation (8) to (6) and further differ from equations (3) and (4) due to $M_2\psi$. Investment's biggest impact is through embodied technical change, giving higher rates of change in optimum yield, effort, and natural capital.[20]

A comparison of the marginal stock and technology effects shows that $\left[c_v A + c_f(\gamma + \delta)F(S_t)\right]/\left[S_t\left(PqAS_t e^{(\lambda+\right.\right.}$

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[19] One reviewer suggested adding adjustment cost of capital explicitly into the model, which can be done by adding a term $C(I)$, where this function implies that it is costly for a firm to increase or decrease its capital stock and that the marginal adjustment cost is increasing in the size of investment. While this will lead to a more general formulation that eliminates bang-bang solutions and leads to smooth adjustment paths, it will also reduce the analytical tractability of the model by making it nonlinear without explicit analytical solutions (see Boyce, 1995). But as shown, the relevant solution is when investment is a singular solution. The optimal stock biomass level might be lower because the cost of investments is higher. We believe that imperfectly malleable capital is the most important case related to adjustment cost.

[20] Another difference is the Leontief aggregator function in which there are no longer endogenous sources of growth through input substitution. In addition, effort is now the stock of physical capital, requiring an assumption of full capacity utilization in order to obtain a flow measure of effort (in contrast to the Leontief-Sono form of separability for the other effort aggregator function). This is also a loss in a source of endogenous growth, but there is a gain in endogenous growth through investment.

$M_2\psi)t-\mu(t,Z) - \left(c_v A + c_f(\gamma + \delta)F(S_t))\right)]>(<)[(c_v A + c_f(\gamma + \delta)F(S_t))(\lambda + M_2\psi)t - \delta\mu(t,Z)/\delta t] / [PqAS_t e^{(\lambda+M_2\psi)t-\mu(t,Z)} - (c_v A + c_f(\gamma + \delta)F(S_t))]$ as $[F(S_t)/S_t] > (<)[(\lambda + M_2\psi)t - \delta\mu(t,Z)/\delta t]$, that is, the marginal stock effect exceeds (falls short of) the marginal technology effect when the relative growth of the resource stock exceeds (falls short of) the rate of change of technology and technical efficiency. With the logistic growth function, this is $r(1 - S/K) > (<) ((\lambda + M_2\psi) - \partial\mu(t,Z)/\partial t)$, so that higher r or K or lower S balance higher $\lambda + M_2\psi$. As stock levels decline over time in the face of technical progress, the marginal costs rise to increasingly counterbalance a constant rate of technical progress. With varying rates of technical change, including technical regress, this relationship becomes more complex. The marginal stock effect is comparatively more important in the Golden Rule for relatively lower rates of λ and $M_2\psi$. The difference for cases 1, 2, and 3 is simply $M_2\psi$ and, in case 3, the cost parameter c_f.

The effect of increases in technical efficiency $\mu(t,z)$ on the optimal stock level for a given state of technology (no further technical progress) eventually ends when the best-practice frontier defined by current technology is reached. This is the case after full diffusion of a technology or all skippers and crew fish on the best practice frontier.

D. Limit Case and Economic Optimum under Technical Change

Terms involving $PqKe^{(\lambda+M_2\psi)t-\mu(t,Z)}$ in equation (8) approach 0 in the limit as t approaches infinity giving (the same result holds for all three cases—equations (4), (6), (8)):

$$\lim_{t\to\infty} S_t^{***} = \frac{K}{4}\left[\left[1 - \frac{\delta}{r}\right] + \sqrt{\left[1 - \frac{\delta}{r}\right]^2}\right] = \frac{K}{2}\left[1 - \frac{\delta}{r}\right]. \quad (9)$$

Because the sum of the terms in the brackets is less than or equal to 2, $\lim_{t\to\infty} S_t^{***} \leq S_{MSY}$, which contrasts with results showing that S^*, the dynamic economic optimum under static technology, generally exceeds S_{MSY} (Grafton, Kompas, & Hilbourn, 2007).[21] Essentially, over an infinite time horizon, technical progress erodes costs close to 0, and S_t^{***} is determined solely by δ and biological parameters. In the static case, where $\delta = 0$, $\lim_{t\to\infty} S_t^{***} = K/2 = S_{MSY}$ under continuous technical progress over a long time frame (because costs eventually approach 0); this gives an optimal harvest rate of maximum sustainable yield (MSY), the standard biological case without costs and prices. When $\delta \geq r$, $\lim_{t\to\infty} S_t^{***} = 0$ or extinction is optimal under continuous

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[21] In the traditional model, it depends on the relative size of the discount rate and the marginal stock effect. If the discount rate is lower (higher) than the marginal stock effect, the optimal stock level is higher (lower) than MSY-level.

technical progress, where technical progress should hasten extinction compared to the case without it. In the intermediate and realistic case, $0 < \delta < r$ and $0 < \lim_{t\to\infty} S_t^{***} < S_{MSY}$, which contrast with the traditional dynamic model without technical progress, in which $S^* > S_{MSY}$ for most reasonable levels of costs and in which stock-dependent costs play a more important role. S_t^{***} given by equation (9) is exactly the same as the stock size in $\partial F/\partial S_t = \delta$, from equation (8). The optimum fishery with static technology is dynamically inefficient from oversaving through foregone yield (underconsumption) and overinvesting in natural capital.

Differentiating the singular solution S_t^{***}, equation (8), with respect to time gives (see appendix A)

$$\frac{\partial S_t^{***}}{\partial t} = -[\lambda - \partial\mu(t,Z)/\partial t]A_3(t)\frac{K}{4}C_3(t), \qquad (10)$$

where $A_3(t) = \left(c_v A + c_f(\gamma + \delta)\right) / \left(PqAKe^{(\lambda+M_2\psi)t-\mu(t,Z)}\right)$, $C_3(t) = \left[1 + \frac{1}{2}B_3(t)^{-\frac{1}{2}}\left[2\left[A_3(t)+1-\frac{\delta}{r}\right]+\frac{8(\delta+\lambda+M_2\psi-\partial\mu(t,Z)/\partial t)}{r}\right]\right]$, and $B_3(t)$ is the terms inside the square root in equation (8). When technical inefficiency is constant, then $\partial S_t^{***}/\partial t < 0$: the resource stock declines with technical progress. However, the rate of decline slows, $\partial^2 S_t^{***}/\partial t^2 > 0$, as shown by

$$\frac{\partial^2 S_t^{***}}{\partial t^2} = \lambda A_3(t)\frac{K}{4}\left[\lambda C_3(t) - \frac{\partial C_3(t)}{\partial t}\right]. \qquad (11)$$

The declining stock levels out for a given rate of continuous technical progress and technical inefficiency for the balanced growth path over an infinite time horizon. Hence, with continuous technical progress, the scale and technically efficient stock declines at a slower rate toward a stock level for which $\partial F/\partial S_t = \delta$.

The singular solution for yield Y_t^{***} can be found from the growth equation as $Y_t^{***} = F\left(S_t^{***}\right) - \partial S_t^{***}/\partial t$. Differentiating optimum yield with respect to time gives

$$\frac{\partial Y_t^{***}}{\partial t} = r\frac{\partial S_t^{***}}{\partial t}\left(1 - \frac{S_t^{***}}{S_{MSY}}\right) - \frac{\partial^2 S_t^{***}}{\partial t^2}. \qquad (12)$$

The sign of $\partial Y_t^{***}/\partial t$ can be either negative or positive depending on the optimal stock size. For a given stock level greater than S_{MSY}, the sign of $\partial Y_t^{***}/\partial t$ is positive. For stock levels lower than the given stock level, the sign is negative, and the optimal yield from a given point in time will decline. In more detail, if $S_t^{***} < S_{MSY}$, the sign of the sum of the terms in the brackets in equation (12) is positive and, together with the negative sign of the term outside the brackets, results in $\partial Y_t^{***}/\partial t < 0$. If $S_t^{***} > S_{MSY}$, the sign of the sum of the terms depends on the relative size between S_t^{***}/S_{MSY} and the rate of technical progress. The sign of

FIGURE 1.—EXAMPLES OF OPTIMAL YIELD PATHS
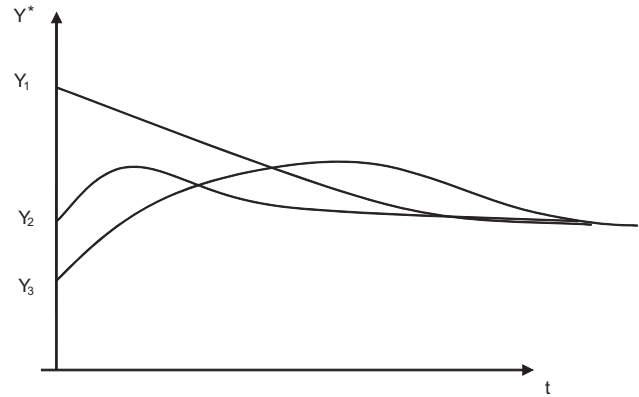
$\partial Y_t^{***}/\partial t$ is positive for sufficiently high stock levels. For a given stock level higher than MSY, $\partial Y_t^{***}/\partial t$ is 0. In contrast to the traditional dynamic model, Y_t^{***} is now conditional on the states of technology and technical inefficiency as well as the entire set of traditional bioeconomic parameters.

Figure 1 illustrates examples of frontier balanced growth paths for the optimal yield frontier with full technical efficiency. The path beginning in Y_1, a high-yield level, declines over time, and hence the optimal stock level at the initial time is less than the stock level in which $\partial Y_t^{***}/\partial t$ is 0. The path beginning in Y_2 is first increasing, indicating that the initial optimal stock level is above the stock level in which $\partial Y_t^{***}/\partial t$ is 0. Since the optimal stock level decreases, the yield inevitably will begin to fall. Finally, the path beginning in Y_3 has an even higher initial optimal stock level, so that the period with increasing yield is longer. To sum up, the optimal frontier yield path is more complicated than the constant equilibrium path obtained in the traditional dynamic model. The time period with the complicated course is followed by a period where the balanced yield path asymptotically approaches the level where $F'(S^\infty) = \delta$, given by $Y^\infty = \frac{rK}{4}\left[1 - \frac{\delta}{r}\right]\left[1 + \frac{\delta}{r}\right]$.[22] Technically inefficient vessels or fleets lie below the best practice optimum yield frontier time path in any given time period, and their position relative to the shifting frontier can change by period, catching up or falling back.

The singular solution for E_t^{***} is found from the production frontier as

$$E_t^{***} = \frac{Y_t^{***}}{qS_t^{***}e^{(\lambda+M_2\psi)t-\mu(t,Z)}}$$
$$= \frac{rS_t^{***}\left(1 - \frac{S_t^{***}}{K}\right) + \lambda A_3(t)\frac{K}{4}C_3(t)}{qS_t^{***}e^{(\lambda+M_2\psi)t-\mu(t,Z)}}. \qquad (13)$$

---

[22] Both S^∞ and Y^∞ indicate the level of stock and yield, respectively, when time goes to infinity.

The sign of the time derivative of optimum effective effort, $\partial E_t^{***}/\partial t$, can be shown to be negative (positive) below (above) a certain level of the stock that is higher than S_{MSY}. Hence, given high initial optimal stock levels, in the short and medium run, the optimal effort level increases, but over time, with technical progress, the optimal level of effort starts to decline because the optimal stock level declines. Similarly, for stock levels below those where $\partial E_t^{***}/\partial t = 0$, gains in technical efficiency for a given state of technology lower E_t^{***} until the Debreu-Farrell best practice frontier is reached.

Technical change can be biased toward using more or less E_t or S_t, in which the direction of bias can be induced by the state of property rights (Smith, 1972). Resource-using technical change allows more effective use of the entire existing resource stock, such as allowing exploitation of formerly unreachable and unexploited fishing grounds or detection of formerly unknown stocks or harvesting different age or size classes of fish formerly unavailable.[23] This bias does not increase the overall stock size or give more catch biomass from existing stock biomass, because man-made inputs simply cannot substitute for fish in a stock flow production process. Specifying autonomous technical change as input augmenting by the use of efficiency units for both E_t and S_t does not allow identifying the individual efficiency gains for E_t and S_t. Thus, for only disembodied technical change and $\alpha = \beta = 1$ with the Schaefer model, $Y_t = q\left(e^{\lambda_1 t}E_t\right)^{\alpha}\left(e^{\lambda_2 t}S_t\right)^{\beta} = qe^{\alpha\lambda_1 t}e^{\beta\lambda_2 t}E_t^{\alpha}S_t^{\beta} = qe^{(\alpha\lambda_2+\beta\lambda_2)t}E_t^{\alpha}S_t^{\beta}$. An alternative specification allowing identification of technical change is $Y_t = qE_t^{(\beta_2+\lambda_3 t)}S_t^{(\beta_2+\lambda_4 t)}e^{(\lambda_1 t+\lambda_2 t^2)}$, where $\lambda_3 > (<)0$ indicates technological change that is E_t-using (saving) and $\lambda_4 > (<)0$ indicates S_t-using (saving).

V.   Renewable Resource Policy

Taxes can induce the social optimum when there is technical change and technical inefficiency and initially open access that leads to equilibrium with dissipated rents (Gordon, 1954). From the rent frontier under open access, price equals unit cost, that is, $P/\left(qe^{(\lambda+M_2\psi)t-\mu(t,Z)}S_t\right)$. The optimum tax, or equivalently, optimum value of a transferable catch share, is

$$P - \frac{c_v A + c_f(\gamma + \delta)}{qAS_t e^{(\lambda+M_2\psi)t-\mu(t,Z)}} = e^{\delta t}\alpha(t) = v(t), \qquad (14)$$

where $v(t)$ is the optimum tax equal to the current value costate variable or the marginal user cost, which now varies over time with the state of technology and technical efficiency. The tax rate can be found as a function of the model

parameters only by substituting the expression for the optimal stock level into equation (14).

A tax equal to $v(t)$ can align the private incentives with the optimal solution (Clark, 2010). Because the proportional rate of change in optimal stock with respect to time is numerically less than the rate of technical progress, the unit cost decreases over time.[24] Hence, with constant biological and economic parameters, the tax rate increases over time. From a practical viewpoint, however, the regulator has to regularly recalculate the tax rate, because, without steady-state equilibrium, the optimal level of stock and yield changes due to technical progress and can also change as economic and biological parameters change. Rates of technical change or changes in technical efficiency that vary by time period or even regress further complicate the picture. Without constant returns to scale in E_t, the tax rate has to adjust to productivity growth, not just technical change.

The shadow price of natural capital or the common renewable resource stock reflects direct use value, representing the marginal user cost along the optimal trajectory of the resource in the face of technical progress and changes in technical efficiency. This does not include the nonmarket indirect use or amenity values from the public goods of the broader ecosystem and its services in which the resource stock is embedded and biodiversity.

Under regulation by transferable use rights on catch held by individuals or groups, the regulator adjusts the total quota so that the equilibrium price in the quota market equals the shadow value of the stock. The TAC has to be recalculated on an ongoing basis to account for technical progress (Murray, 2007). Excess capacity can be expected to persist for even long periods of time after transferable catch shares are introduced, and other measures may be required to reduce excess capacity (Vestergaard, Jensen, & Jørgensen, 2005).

VI.   Empirical Example: The U.S.–Canada Albacore Fleet

The empirical example applies the concepts of technical efficiency and technical change to the single-species U.S. and Canadian troll fleets fishing for North Pacific albacore (*Thunnus alalunga*) over 1981 to 2007 to examine the optimal stock and yield time paths. The data are for the U.S. and Canadian fleets, but the results should closely match the entire North Pacific albacore fishery of troll, pole-and-line, and other surface gear for Taiwan, Japan, Korea, the United States, and Mexico, since catch per unit effort indices for these other surface fisheries closely track those of the U.S.–Canadian troll fishery. The unregulated industry implies no regulatory-induced technical change.

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[23] Growth in global capture marine fisheries landings is driven by resource using technical change through expansion into previously unexploited ocean areas (southward, both poles, high seas, deeper in the water column) and new species (Swartz et al., 2010).

[24] Differentiating the unit cost with respect to time gives an expression whose sign depends on the relative size of $\dot{S}/S$ and λ. From equations (8) and (10), it can be seen that $\dot{S}/S < \lambda$.

The U.S. and Canadian vessels are family owned, relatively small, with U.S. vessel length averaging about 13 meters, and harvest albacore from about $150°E$ eastward. The average U.S. vessel construction year of 1976 suggests a fleet with vessels that are relatively old and stable, well established, and "mom-and-pop." Innovation adoption is only incompletely diffused and varies by innovation (Squires et al., 2003).

The empirical analysis employs the catch and days fished data used in the international stock assessments by the population biologists of the fishery's representative countries (ISC Albacore Working Group, 2011). The intrinsic growth rate, $r = 0.18$, and the environmental carrying capacity, $K = 250$ metric tons (mt), are provided by an albacore population biologist and are developed from life history studies, population assessments, and other biological research and can be treated as exogenous.[25] The catchability coefficient, $q = 0.00526169$, comes from international stock assessments and is the weighted average of age-specific values of q for 2-, 3-, 4,- and 5-year-old year classes (one-third weights each for 3- and 4-year-olds and one-sixth weights for 2- and 5-year-olds). International stock assessments also provide exogenous estimates of resource stock biomass S_t (here age 1+ fish). The ex-vessel albacore price and cost per vessel per day (US$2,001) are set at the 1981–2007 means, giving $P = \$2,021.49/mt$, $c_v = \$1,162.59/vessel-day$, where these variable costs include the operating costs of fuel, oil, food, gear, and labor, and capital costs as a capital services price (the interest rate is Moody's long-term bond rate for Baa investments, and the depreciation rate is 7%), $c_f = \$3,507.66/year$, $c_f = \$46.45/vessel-day$ for 16 pieces of electronic equipment, $c_v A + c_v(\gamma + \delta) = \$1,209.04/vessel-day$, $\gamma = 12.5\%$, $\delta = 2.5\%$, and A is normalized to unity.[26]

To obtain values of the rate of technical change λ and the change in technical efficiency $\partial\mu(t,Z)/\partial t$, the following Cobb-Douglas production function was estimated using the unbalanced panel data for the U.S. and Canadian fleets' catch and effort:

$$\ln Y_{it} = \alpha + \beta_1 \ln E_{it} + \beta_2 \ln S_{it} + \lambda t + \alpha_i + v_{it}, \qquad (15)$$

where i indexes the individual country, α_i captures the fixed effect (and is allowed to be correlated with E_{it}, S_{it} in an unknown correlation structure), in this case Canadian effort, and v_{it} is an i.i.d. stochastic disturbance term with a zero mean, finite variance, and normal distribution. Technical inefficiency $\hat{u}_{it}$ can be measured semiparametrically by (Schmidt & Sickles 1984) $\hat{u}_{it} = \max_j(\hat{\alpha}_j) - (\hat{\alpha}_i)$, which reduces to the relative technical inefficiency between two countries. The limited number of countries in our data set precludes an explicit time-varying semiparametric specification of technical inefficiency—parameterized as a function of time and even parameters that vary over firms—and still identifying technical progress. The limited number of observations also precludes evaluating biased technical change or adding a squared time trend term to equation (15) to allow for a varying rate of Hick's-neutral technical change.

The unobserved composite input E_t, comprising both days fished and number of vessels, is the first stage in a two-stage separable decision-making process (Hannesson, 1983; Squires, 1987). The composite input E_t can be used an instrumental variable predetermined in the effort model that avoids endogeneity issues and can reduce multicollinearity when estimating the second stage (Fuss, 1977).[27] Unavailable vessel and equipment vintages, a time series of investment decisions, and skipper years-of-experience information, preclude their inclusion to measure embodied technical change and learning by doing and skill embodied in the stock of labor (see Bahk & Gort, 1993).

The linearly homogeneous aggregator function for unobserved and Leontief-Sono separable E_t was created, following Fuss (1977) and Squires (1987) and retrieves the actual values of the aggregator function. Consistent with the separability inflexibility of the Cobb-Douglas (with Schaefer restrictions) catch frontier with Leontief-Sono input-output separability (Blackorby, Primont, & Russell, 1978), a linearly homogeneous translog effort aggregator function was specified, which gives a superlative effort index equivalent to the Tornqvist and input allocative efficiency. It is also consistent with Fisher's weak factor reversal test.

The translog aggregator function for unobserved effort is written as

$$\begin{aligned} \ln E_{it} = \alpha &+ \alpha_1 \ln X_{1it} + \alpha_2 \ln X_{2it} + \alpha_{12} \ln X_{1it} \ln X_{2it} \\ &+ \alpha_{11} \ln X_{1it}^2 + \alpha_{22} \ln X_{2it}^2 + a_i + a_{i1} X_{1it} \\ &+ \alpha_{i2} X_{2it} + \varepsilon_{it}, \end{aligned} \qquad (16)$$

where X_{1it} denotes days fished for country i in time t and measures variable input use, X_{2it} denotes the number of vessels for country i in time t and measures the capital stock, and symmetry is imposed by $\alpha_{12} = \alpha_{12}$. Assuming competi-

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[25] The relative biomass estimates and, hence, the annual changes in biomass used for econometric estimation in this paper are widely considered far more reliable than absolute measures of biomass. Different absolute measures of biomass would not change the overall relative pattern and qualitative results of the simulation, although they would change the absolute values of the benchmarks. We retain earlier absolute benchmark values recommended by an albacore population biologist, but we use the later relative biomass estimates in our econometric estimation.

[26] The parameter A could also be absorbed into q. Little is known about depreciation for electronics and other equipment, where obsolescence rather than physical deterioration is the more important factor. A reasonable number of eight years over the sixteen pieces of electronic equipment in the panel data set is selected, giving 12.5% per annum with straight-line depreciation. We assumed that electronic innovation occurred at a constant rate. A capital services price corresponds to all inputs as variable and in full static equilibrium in the effort aggregator function.

[27] The resource stock is treated as exogenous in the production frontier because it was estimated by a Stock Synthesis 3 model using Pacific-wide data on multiple gear types, so that considerable exogenous information (e.g., age structure of the population, gender, length-weight relationships, recruitment, considerable information from other nations, and so on) and exogenous assumptions about the population dynamics were the basis of the stock estimates. This most recent estimate now uses actual catch at age data instead of a presumed Shur growth curve, but does not use fishery-independent data to set the absolute (versus relative) stock size.

TABLE 1.—SUMMARY STATISTICS OF THE DATA

Category	Mean	SD	Minimum	Maximum
U.S. catch (mt)	8,986.36	4,146.74	1,845.00	16,938.35
U.S. days fished	24,608.20	9,815.27	9,000.00	45,710.00
U.S. vessel numbers	735.17	350.31	172.00	1,837.00
U.S. composite effort tornqvist index	0.4452	0.1924	0.1285	1.00
U.S. variable input (days fished) cost share	0.5377	0.1884	0.2872	0.9439
U.S. capital cost share	0.4623	0.1884	0.0561	0.7128
Canada catch (mt)	3,762.18	2,380.18	139.00	7,856.00
Canada days fished	867.14	842.20	273.67	3,371.29
Canada vessel numbers	180.70	70.37	45.00	292.00
Canada composite effort Tornqvist index	2.35	0.7000	0.6777	3.1812
Canada variable input (days fished) cost share	0.6240	0.1625	0.4321	0.9439
Canada capital cost share	0.3760	0.1625	0.0561	0.5680
Price of albacore U.S. vessels (US$/mt)	2,021.49	413.24	1,402.33	3,355.76
Price of albacore Canadian vessels (US$/mt)	2,118.71	484.64	1,213.94	3,355.76
Cost per day (US$/day) (without investment)	1,162.59	279.41	546.31	1,869.53
Price of fuel (US$/gallon)	1.2664	0.5157	0.6653	2.5370

Monetary values in 2001 U.S. dollars.

tive markets (a reasonable assumption in this case because of a global tuna market and substitute species), differentiating equation (16) with respect to $\ln X_{1it}$ and $\ln X_{2it}$ yields the cost share equations:

$$\partial \ln E_{it} / \partial \ln X_{1it} = M_{1t} = \alpha_1 + \alpha_{it} + \alpha_{11} \ln X_{1it} + \alpha_{12} \ln X_{2it},$$
(17a)

$$\partial \ln E_{it} / \partial \ln X_{2it} = M_{2t} = \alpha_2 + \alpha_{it} + \alpha_{12} \ln X_{1it} + \alpha_{22} \ln X_{2it},$$
(17b)

where M_{1t}, M_{2t} are the cost shares for variable inputs (days fished) and vessel numbers. Annual variable input cost shares correspond to the costs of fuel, food, gear, and labor, and annual capital cost shares correspond to a rental or capital services price. Linear homogeneity is imposed by $\alpha_1 + \alpha_2 = 1, \alpha_{i1} + \alpha_{i2} = 0, \alpha_{11} + \alpha_{12} + \alpha_{22} = 0$.

Fitted values from the estimated effort aggregator function, equation (16), can provide an estimate of unobserved E_t (up to the arbitrary scaling factor $\alpha + \alpha_i$ in equation [16]) for unobserved (log of) endogenous E_t in the catch frontier, a equation (15), giving exogenous E_t in the second stage of decision making.[28] Equation (16) can also be directly inserted into the production frontier, equation (15), and equation (15) then estimated by nonlinear least squares, with instrumental variables if necessary.[29] The fixed-effects

specification of the aggregator function can in principle be counted on to address potential selection (entry and exit in the fishery) and input endogeneity.[30] Over the time period, Canadian vessels consistently fished in U.S. waters, and given their longer distances traveled to reach fishing grounds and offloading fish in U.S. ports, or alternatively, the fewer fish found in Canadian waters, along with virtually identical vessels, a constant and different Canadian fixed effects is a reasonable specification that can be tested by the data.

The analysis covered 1981 to 2007 for U.S. and 1991 to 2007 for Canadian vessels to match availability of vessel data. Table 1 summarizes the unbalanced panel data, which is summarized in an Online appendix, Squires et al. (2003), and Squires and Vestergaard (2009).

VII.   Empirical Results

The production frontier with constant Hick's-neutral disembodied technical change equation (15) and substituted equation (16) (allowing for the base period normalization)

[28] The index is subject to a base period normalization, such as all effort component quantities equal to unity (standard for quantity indices). Since the effort index is unique only up to a scalar multiple—$\alpha + a_i$ cannot be identified from equation (16)—$\alpha + a_i$ in equation (16) is also set equal to unity (the case with index numbers, such as the Tornqvist or Fisher). The effort function equals equation (16) without $\alpha + a_i$, which is the normalized form of equation (16) (Fuss 1977).

[29] Initial separate estimation of the share equation, applying a Durbin-Wu-Hausman test for endogenous regressors (chi square with two degrees of freedom of 58.41), and subsequent estimation by nonlinear least squares with a correction for first-order serial correlation gave exogenous parameter values for the generated regressor (Murphy & Topel, 1985) for unobserved effort that provided unreasonable results in subsequent estimation of the catch frontier (for example, exceptionally high parameter values and standard errors); too much was asked of 45 observations and a single cost share equation. We turned to substituting equation (16) into (15).

[30] Such an approach requires the fixed effect in either the effort aggregator function, equation (16), or the catch frontier, equation (15), to remain constant over the entire time period, which in this case is not unreasonable given that it principally captures immutable biophysical differences between the United States and Canada. Nonetheless, we apply Durbin-Wu-Hausman tests for endogeneity of days and vessel numbers, and correct by instrumental variables (see note 31). An alternative approach follows Olley and Pakes (1996) to use investment or Levinsohn and Petrin (2003) to use intermediate inputs to address productivity shocks, but these assumptions are either not applicable (they were developed for manufacturing and more traditional production practices) or require data that are unavailable (such as a long time series of investment or intermediate inputs). Dynamic panel data models (Arellano-Bover, 1995; Arellano-Bond, 1991; Blundell-Bond, 2000 are another possibility but require more observations to implement (because of extensive differencing and relatively long time lags of variables for instruments) than we have available. Markets for ex-vessel albacore and fuel prices are global and extremely competitive, and variation exists along the long coastline. Finally, fixed effects are preferred to random effects since with countries, there is not a random sample from a much larger universe of countries, and if the individual effect represents omitted variables, it is likely that these country-specific characteristics are correlated with the other regressors.

TABLE 2.—PARAMETER ESTIMATES OF TRANSLOG EFFORT AGGREGATOR FUNCTION

Parameter	NLSQ IV No Restrictions			NLSQ IV Effort Coefficient = 1			NLSQ IV Effort and Stock Coefficients = 1		
	Estimate	SE	*t*-statistic	Estimate	SE	*t*-statistic	Estimate	SE	*t*-statistic
Constant	2.4493	0.4835	5.0155	1.1852	7.1567	0.1656	−0.4303	1.7157	−0.2508
Dummy Canada	−0.7695	1.0397	−0.7402	−5.6292	11.6539	−0.4803	−0.0784	1.1183	−0.0701
Days fished	−0.0561	0.1741	−0.3224	0.8348	2.0138	0.4145	−0.0422	0.2304	−0.1833
Number of vessels	−0.1475	0.3378	−0.4367	−1.5852	4.0991	−0.3867	0.1860	0.5251	0.3541

Corresponds to equation (17a). Heteroskedastic-consistent (robust) standard errors. Effort aggregator function, equation (16), is found using symmetry and linear homogeneity constraints, subject to base period normalization.

TABLE 3.—PARAMETER ESTIMATES OF CATCH FRONTIER

Parameter	NLSQ IV No Restrictions			NLSQ IV Effort Coefficient = 1			NLSQ IV Effort and Stock Coefficients = 1		
	Estimate	SE	*t*-statistic	Estimate	SE	*t*-statistic	Estimate	SE	*t*-statistic
Constant	−43.0601	103.973	−0.4141	45.8278	125.1240	0.3663	−11.8332	4.1918	−2.8230
Dummy Canada	22.6875	26.4920	0.8563	20.6766	45.1228	0.4582	0.4994	3.7977	0.1315
Effort	8.0290	5.8299	1.3772	1			1		
Biomass	−1.2182	7.3926	−0.1648	−4.4485	11.7588	−0.3783	1		
Trend	0.03845	0.0512	0.7512	0.0659	0.0782	0.8422	0.3572	0.0139	2.5647
R^2	0.56			0.19			0.78		
Durbin-Watson	1.9337			2.3799			1.5250		

Number of observations = 43. Nonlinear least squares instrumental variables estimation with effort aggregator function, equation (16), embedded in catch frontier, equation (15). Huber-White heteroskedastic-consistent (robust) standard errors.

was estimated by nonlinear least squares with instrumental variables.[31] Quasi-likelihood ratio tests failed to reject at a 1% level of significance: (1) the classic Gordon-Schaefer specification of constant returns to scale in effort, $\beta_1 = 1 \left(\chi^2_{df=1} = 0.4991 \right)$; (2) $\beta_2 = 1 \left(\chi^2_{df=1} = 0.1606 \right)$; and (3) $\beta_1 = \beta_2 = 1 \left(\chi^2_{df=2} = 6.9212 \right)$.[32] Parameter estimates are reported in table 2 for equation (17a), allowing reconstruction of equation (16) using symmetry and linear homogeneity conditions, and table 3 for equation (15) with embedded equation (16) with $R^2 = 0.78$.[33] The best practice frontier

defined by both Canadian and U.S. vessels expands at a 3.57% annual rate due to disembodied technical progress.[34] Canadian vessels' technology lies on the same expanding frontier (the Canadian dummy variable is statistically insignificant), indicating no differences in technical efficiency.

Full technical progress $\dot{T} = M_2 \psi_t + \lambda$ is calculated using estimated parameter estimates from table 2 with $\beta_1 = \beta_2 = 1$, full technical efficiency, arithmetic mean for M_2, and the annual differences in the predicted natural log of catch, allowing for $\dot{S}_t$ (following Kirkley, et al., 2004).[35]

Given $\dot{T}$ and λ, $\dot{T} - \lambda = M_2 \psi_t$ and $\psi_t = \left[\dot{T} - \lambda \right] / M_2$. Thus, mean annual $\dot{T}$ is 8.64%, the contribution of embodied technical change to $\dot{T} - \lambda = M_2 \psi_t$ is 5.07% when λ is 3.57%, and mean $M_2 = 0.46$. The annual growth rate of average embodied efficiency (Ψ) of $\psi = 11.02\%$ is consistent with estimates for the U.S. economy for other sectors that contain comparatively large amounts of information technology embodied in capital, including office computing equip-

[31] A likelihood ratio test value of 39.08 with df = 2 indicated rejection of the null hypothesis of exogeneity for days and vessel numbers in nonlinear least squares estimation of equation (16) with embedded equation (15) subject to the base period normalization. (Following Woolridge, 2002, this Durbin-Wu-Hausman test was implemented by including residuals from regressing days and vessel numbers on exogenous variables as additional variables in nonlinear least squares estimation of this equation, and the likelihood value was compared to that without these two additional variables. There was no serial correlation indicated by Durbin-Watson statistics in the ordinary least squares estimation to obtain the residuals.) Instruments in subsequent nonlinear least squares estimations are the constant term, Canadian dummy variable, biomass, trend, and lagged values of albacore price, cost per day, days, and vessel numbers.

[32] The upper-tail areas are respectively 0.4799, 0.68858, and 0.0314. Tests for overidentifying restrictions are: (1) F= 0.881046E-02 [.926]; (2) F=0.160623 [.691]; (3) F=0.245932E-02 [.998]. The test is e′P(Z)e/ S2(m−k), where e′P(Z)e is the sum of squared residuals after projection onto the instruments: S2 is the standard error squared.

[33] First-order serial correlation was not an issue in the nonlinear estimation of equation (15) with embedded equation (16) and base period normalization with instrumental variables for all versions (effort and biomass coefficients unrestricted or restricted). In the final version, when effort and biomass coefficients are restricted to 1, the Durbin Watson statistic of 1.5202 fell in the indeterminate range, but a separate test of regressing the catch frontier residual on its lagged residual—without a constant—and evaluating the corresponding coefficient for statistical significance using Huber-White robust standard errors had a *t*-ratio of 1.254.

[34] Industry experts confirmed this comparatively high rate of disembodied technical change as reasonable. The high rate is due to increased understanding of ocean conditions allowing forecasting of fish locations through temperature-sensing devices reinforced by satellites, improvements in interpretation, and GPS, all of which give information about the overall distribution of albacore, dramatically reduce searching, and ease finding schools below the surface. Improved communications allows sharing information sharing among code groups, reducing search time, and increasing catch rates. Acoustic devices such as echo sounders are also increasingly sophisticated. The gear itself (troll line and lures) remained largely static. Improved weather forecasts extend the fishing season. Nonetheless, more refined data accounting for input quality might well pare down the Solow residual.

[35] The full static equilibrium in each time period inherent in the effort aggregator function implies that the Solow residual does not need to be adjusted for variations in capacity utilization.

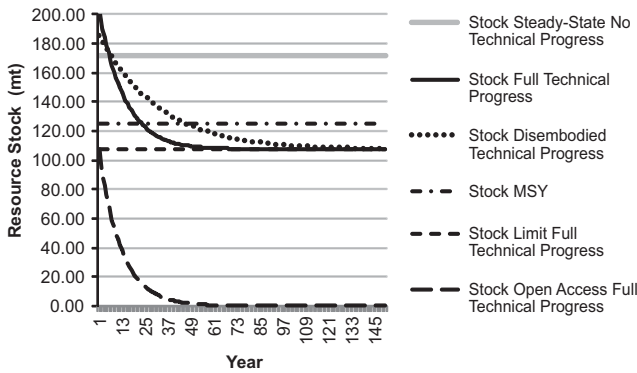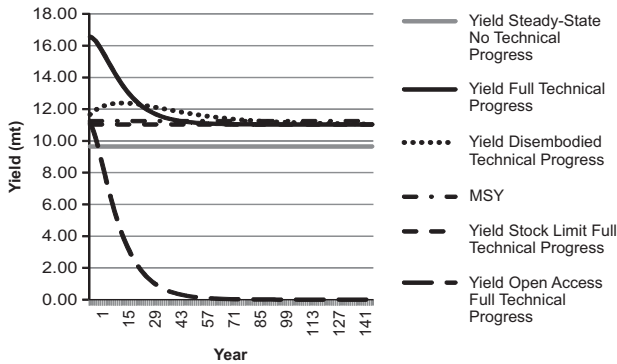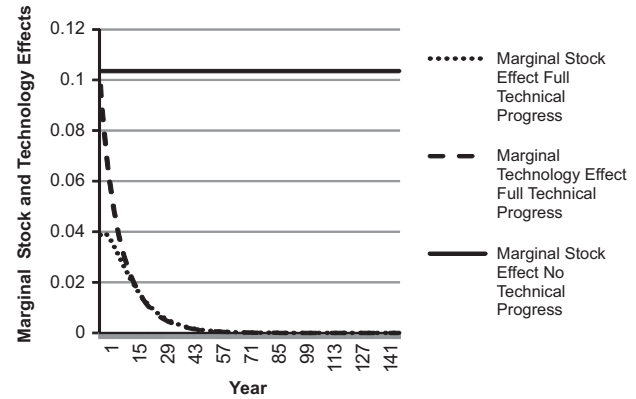FIGURE 2.—OPTIMUM STOCK SIZE OVER A 150-YEAR TIME HORIZON

FIGURE 3.—OPTIMUM YIELD OVER A 150-YEAR TIME HORIZON

FIGURE 4.—MARGINAL STOCK AND TECHNOLOGY EFFECTS IN MODIFIED GOLDEN RULE

ment, communications equipment, and scientific and engineering equipment (Hulten, 1992; Sakellaris & Wilson, 2004).

Figure 2 illustrates northern albacore stocks over a 150-year time horizon for the fundamental equation of renewable resources with static technology S^*; only disembodied technical change with $\lambda = 3.57\%$ and, equation (4), S_t^*; disembodied and embodied technical change, imperfectly malleable capital and investment with $\lambda = 3.57\%$, $M_2\psi = 5.07\%$, $\dot{T} = 8.64\%$, and equation (8), S_t^{***}; $S_{MSY} = K/2 = 125$ mt; $\lim_{t\to\infty} S_t^* = \lim_{t\to\infty} S_t^{***}$, equation (9); and the open-access equilibrium stock with both full (disembodied and embodied) technical progress (the initial value is the open access stock under static technology) $S_\infty^{TE,t} = c/\left[Pqe^{(\lambda+M_2\psi)t-\mu(t,Z)}\right]$ (S_t^{**} from equation [6] and S_t^{***} from equation [8] are almost identical given the small size of c_f, so that only S_t^{***} is presented). The results clearly illustrate the wide divergence between $S^* = 172.21$ mt, S_t^*, $\lim_{t\to\infty} S_t^* = 107.64$ mt, $S_{MSY} = 125$ mt, and $S_\infty^{TE,t}$, which is 107.47 mt in the initial time period and 0.00 mt in the final time period. $S_t^{***} > S^*$ for a finite, number of years, as discussed earlier, before S_t^{***} drops below S_t^*, steadily diverging from S_t^*, and eventually reaches the limit stock. The slowing rate of decline with technical progress reflects

equation (11). Notably, the limit stock lies below not only S_t^* but also S_{MSY} due to the marginal technology effect. $S^* > S_{MSY}$ due to the marginal stock effect and cost savings from keeping fish in the water and static technology.

The effect of technical progress on $S_\infty^{TE,t}$ in figure 2 is striking, demonstrating the importance of property rights or an optimum tax, the dangers of ignoring the rapid stock decline due to technical progress, especially under open access, and the low levels that $S_\infty^{TE,t}$ can reach. The difference $S_t^{***} - S_\infty^{TE,t}$ measures the resource stock externality under full technical change, which increasingly diverges over time due to relentless technical progress. However, although steadily diverging, the externality, which begins at 83.27 mt, levels off as it asymptotically approaches the stock limit $\lim_{t\to\infty} S_t^* = 107.64$ mt and $\lim_{t\to\infty} S_\infty^{TE,t} = 0$ mt, that is $\lim_{t\to\infty} S_t^* - \lim_{t\to\infty} S_\infty^{TE,t} = \lim_{t\to\infty} S_t^*$. Higher rates of technical progress exacerbate the external cost under open access by strengthening the resource stock externality and hastening the decline in stock size and foregone rents.

Figure 3 presents optimum yields over 150 years corresponding to the optimum resource stocks in Figure 2. Y^* is 9.65 mt, and $Y_{MSY} = rK/4$ is 11.25 mt. Optimum yields in the early years of technical progress reflect stock rebuilding and match Y_2 in figure 1. $Y_t^{***} > Y^*$, and the lower costs due to full technical progress lead to net benefits to society of higher harvest rates and consumption than without technical progress in which the only source of lower costs is retaining a higher resource stock in the water. Optimum yields under technical progress are higher than under static technology.

The relationship between the marginal stock effect and the marginal technology effect is illustrated in figure 4 for full (disembodied and embodied) technical change. The marginal stock effect is slightly more important than the marginal technology effect and erodes over time as technological change lowers costs and it becomes less important to leave fish in the water to lower costs. In the albacore fishery, $(F(S_t)/S_t > (\lambda + M_2\psi)t - \partial\mu(t,Z)/\partial t)$ for all but the

FIGURE 5.—DISCOUNTED ANNUAL OPTIMUM ANNUAL RENT
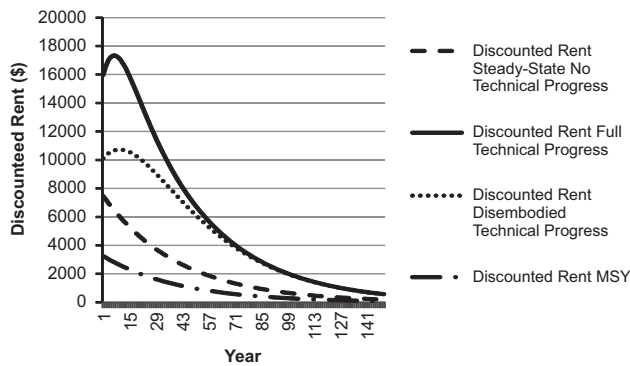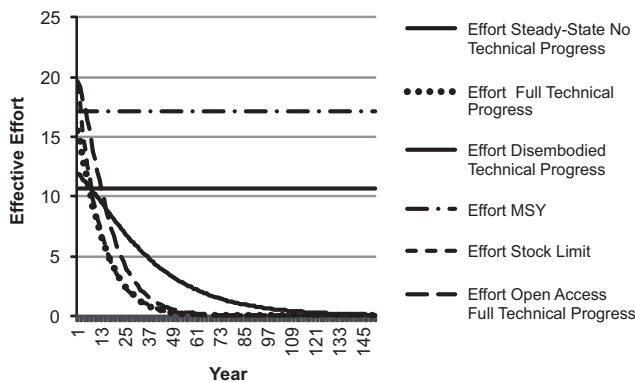

FIGURE 7.—OPTIMUM TEX

FIGURE 6.—OPTIMUM EFFORT

higher and lower stock levels. The marginal stock effect with static technology is much larger than when accounting for full technical progress, clearly demonstrating the importance of accounting for technical change through the marginal stock effect and the new marginal technology effect.

The impact of technical progress on discounted optimum annual economic rents illustrated in figure 5 clearly demonstrates that technical progress leads to higher annual rents in every time period than in the (no-growth) steady state with static technology. S^* lies well above S_{MSY} due to the marginal stock effect and cost savings from keeping fish in the water but represents an opportunity cost of foregone rent with technical progress and dynamic inefficiency, as illustrated by figure 5. Moreover, over the 150-year period, the total discounted rent with full technical progress (and technical, allocative, and scale efficiency) is $874,126, but only 34% of this at $298,146 for S^*, a sizable foregone opportunity cost of $575,980, or 66%, when managing a renewable resource industry while overlooking technical progress and imposing steady-state equilibrium.

Incorporating technical change reorients managing toward effective effort or physical capital and frontier balanced growth paths rather than simply the nominal effort or physical capital and (no-growth) steady-state equilibrium of the static technology approach. The results, as illustrated in figure 6, change dramatically. Steady-state nominal effort
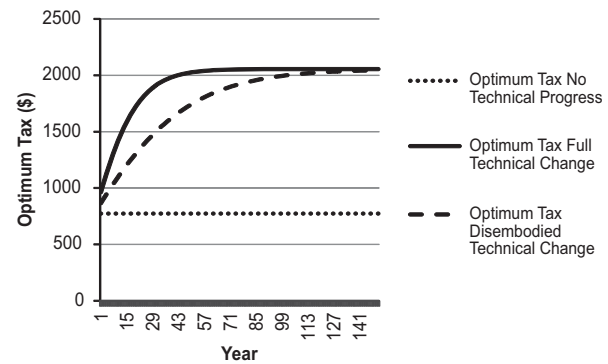
is highest followed by MSY nominal effort. Effective effort allowing for full technical progress and the effective effort corresponding to the stock limit both steadily drop as technology marches on. Overlooking technical progress and managing only for nominal effort or physical capital in spurious (no-growth) steady-state equilibrium raises costs and, when combined with lower optimal yields, lowers economic rents and leads to substantial overaccumulation of physical and natural capital.

The impact of full technical change on the optimum tax (or equivalently, the optimal value of an individual transferable or group quota) is illustrated in figure 7. The optimum tax rises over time to eventually level off. With technical change, the optimum tax must be updated every time period until the externality narrows enough to be de facto negligible. A higher tax is required when accounting for embodied and disembodied technical progress than only under disembodied technical progress. Since the optimum tax addresses the dynamic resource stock externality, this externality is clearly misunderstood and mismeasured when changes in technology and Debreu-Farrell economic efficiency are overlooked.

VIII. Conclusion

Progress in technology and technical efficiency under incomplete property rights exacerbates the commons problem, further widening the wedge between the private and social costs of resource exploitation—the negative resource stock externality. The resource stock declines more rapidly than under static technology, which is hastened under resource using technical change and the more a species schools or concentrates. The risk of extinction under open access as well as optimum extinction, when maximizing rent, can be expected to increase for species with low growth rates. The rapid technical progress over the past century undoubtedly contributed to the decline of most, if not all, global fisheries. Subsidies may have accelerated the adoption of investment-specific technical progress.

Allowing for technical change brings back the standard capital model in which the marginal product of natural capi-

tal eventually equals the discount rate and in which technical change diminishes the marginal stock effect and the value of accumulating natural capital to lower harvest costs. Technical change performs that task without requiring nearly as much help from saving to accumulate natural capital at the cost of foregone resource rent. Embodied technical change accelerates these effects.

The economic optimum can now markedly differ from conventional wisdom (Grafton et al., 2007), in which the dynamic scale-efficient stock under static technology exceeds S_{MSY} due to the marginal stock effect. Instead, the region of dynamic scale efficiency under static technology may be a region of dynamic Debreu-Farrell economic inefficiency and resource misallocation—overaccumulated natural capital—under technical progress. The realistic possibility of $S_t^{***} < S_{MSY}$ suggests that at least some of the approximately 82% of global capture fish stocks that lie at or below MSY (FAO, 2010) may not be economically overfished after all in terms of optimum economic rent and direct use values and that the current crisis in global fisheries may be overstated.[36] The near-universal policy in global capture fisheries of managing for MSY (sometimes modified by a precautionary level) may in a number of instances be economically suboptimal by surprisingly favoring resource stocks too large rather than too small, oversaving and underconsuming through reduced harvests with excessive investment in natural capital, and can create a sizable opportunity cost of forgone rents.

The magnitude of the resource misallocation worsens when accounting for effective rather than nominal effort or physical capital, leading to potentially sizable overaccumulation of not only natural but physical capital: not only are too many fish left in the sea, but far too many vessels are as well. Ironically, while the economic overfishing problem in terms of natural capital may not be as severe as feared, the overaccumulation of steadily more efficient physical capital is worse than imagined and growing with the inexorable march of technology. Managing renewable resources and nominal effort for a nonexistent steady-state economic optimum under static technology, rather than a balanced growth path accounting for changes in technology, Debreu-Farrell economic efficiency, and effective effort, or even (unnecessarily) rebuilding resource stocks mistakenly identified as economically overfished, sets dynamically inefficient targets for yields, effort, or physical capital, investment in physical and natural capital, and resource stocks; creates severe disruptions; and exacerbates the opportunity cost of foregone harvests and rents (Grafton et al., 2007; Munro, 2010; OECD, 2010).

In sum, accounting for changes in technology, technical, allocative, and scale efficiency, and effective effort creates the potential for turning conventional normative economic fisheries models and management on their heads by allowing for the very real possibility of dynamic Debreu-Farrell economically efficient resource stock sizes below S_{MSY} and far fewer but more efficient vessels, and shifting the management focus away from input to output controls. Aiming for (no-growth) steady-state equilibrium and focusing on trajectories of transition to this spurious equilibrium from a nonautonomous dynamic renewable resource framework while ignoring changes in technology and Debreu-Farrell economic efficiency clearly introduces serious suboptimal renewable resource management and imposes unnecessary hardships.

Nonetheless, sound ecological and biodiversity reasons may well argue for larger resource stocks, richer biodiversity, greater ecosystem services, and larger-sized fish (Worm et al., 2006; Anderson et al., 2008), reinforced by uncertainty, nonconvex and nonlinear ecosystems, and nonmarket amenity values. Common resources increasingly require management for public benefits rather than simply as the historic commons problem and direct use value as rent. Direct use values under technological progress can best be fully realized under well-developed individual or group property rights on outputs or marine tenure, but perhaps at lower, sometimes perhaps even much lower, resource stock and vessel levels than previously believed.

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[36] Biologically fully exploited or overexploited means a resource stock at or below maximum sustainable yield (MSY) levels, which in turn means economically overexploited under static technology, no-growth steady-state equilibriums, and nominal effort. The global fisheries' economically optimum species mix with technical change and Debreu-Farrell economic efficiency also undoubtedly differs from that of static technology and no-growth steady-state equilibrium.

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APPENDIX A

Deviation of stock, effort and yield solutions in the three cases

In this appendix optimal levels of stock, effort and yield are derived in each of the three cases in the main text.

Case 1: Disembodied Technical Change and Perfectly Malleable Capital.

From the main text, we have the augmented the Golden Rule, equation (3). Using the logistic growth function $F(S_t) = rS_t(1 - S_t/K)$ and the marginal productivity of the stock $\partial F/\partial S_t = r(1 - 2S_t/K)$, we can rewrite the augmented Golden Rule as

$$r(1 - 2S_t/K) + \frac{crS_t(1 - S_t/K)}{S_t(PqS_t e^{\lambda t - \mu(t,Z)} - c)} + \frac{c(\lambda - \partial\mu(t,Z)/\partial t)}{(PqS_t e^{\lambda t - \mu(t,Z)} - c)} = \delta.$$

This expression is a quadratic equation that can be solved for the relevant solution for stock S_t^* to give equation (4) in the main text:

$$S_t^* = \frac{K}{4}\left[\left[\frac{c}{PqKe^{\lambda t - \mu(t,z)}} + 1 - \frac{\delta}{r}\right] + \sqrt{\left[\frac{c}{PqKe^{\lambda t - \mu(t,Z)}} + 1 - \frac{\delta}{r}\right]^2 + \frac{8c(\delta + \lambda - \partial\mu(t,Z)/\partial t)}{PqKre^{\lambda t - \mu(t,z)}}}\right].$$

The singular solution for yield Y_t^* can be found from the net growth equation as $Y_t^* = F(S_t^*) - \partial S_t^*/\partial t$. Differentiating the singular solution S_t^* with respect to time gives

$$\frac{\partial S_t^*}{\partial t} = -[\lambda - \partial\mu(t,Z)/\partial t]A_1(t)\frac{K}{4}C_1(t), \tag{A1}$$

where $A_1(t) = c/(PqKe^{\lambda t - \mu(t,Z)})$, $C_1(t) = \left[1 + \frac{1}{2}B_1(t)^{-\frac{1}{2}}\left[2[A_1(t) + 1 - \frac{\delta}{r}] + \frac{8(\delta + \lambda - \partial\mu(t,Z)/\partial t)}{r}\right]\right]$, and $B_1(t)$ are the terms inside the square root in equation (4). $\frac{\partial S_t^*}{\partial t}$ is negative, assuming that the rate of technical progress λ is larger than the change in technical inefficiency $\partial\mu(t,Z)/\partial t$ and because $A_1(t)$ and $C_1(t)$ are positive.

The singular solution for yield is then

$$Y_t^* = rS_t^*(1 - S_t^*/K) + [\lambda - \partial\mu(t,Z)/\partial t]A_1(t)\frac{K}{4}C_1(t). \tag{A2}$$

The expression for S_t^* from equation (4) can be inserted to give a solution for yield depending on only the biological and economic parameters:

$$Y_t^* = r\frac{K}{4}\left[\left[A_1(t) + 1 - \frac{\delta}{r}\right] + \sqrt{B_1(t)}\right] \times \left(1 - \frac{1}{4}\left[\left[A_1(t) + 1 - \frac{\delta}{r}\right] + \sqrt{B_1(t)}\right]\right) + [\lambda - \partial\mu(t,Z)/\partial t]A_1(t)\frac{K}{4}C_1(t). \tag{A3}$$

Finally, the singular solution for effort can be found from the Graham-Schaefer surplus production frontier: $Y_t = qE_t S_t e^{\lambda t - \mu(t,Z)}$ as

$$E_t^* = \frac{Y_t^*}{qS_t^* e^{\lambda t - \mu(t,Z)}}, \tag{A4}$$

where the optimal solutions of yield, equations (A2) and (A3), and stock, equation (4), can be inserted to give an expression depending on only the biological and economic parameters:

$$E_t^* = \frac{rS_t^*(1 - S_t^*/K) + [\lambda - \partial\mu(t,z)/\partial t]A_1(t)\frac{K}{4}C_1(t)}{qS_t^* e^{\lambda t - \mu(t,Z)}} = \frac{r\left[[A_1(t) + 1 - \frac{\delta}{r}] + \sqrt{B_1(t)}\right]\left(1 - \frac{1}{4}[[A_1(t) + 1 - \frac{\delta}{r}] + \sqrt{B_1(t)}]\right) + [\lambda - \partial\mu(t,Z)/\partial t]A_1(t)C_1(t)}{q\left[[A_1(t) + 1 - \frac{\delta}{r}] + \sqrt{B_1(t)}\right]e^{\lambda t - \mu(t,Z)}}. \tag{A5}$$

Case 2: Both Disembodied and Embodied Technical Change and Perfectly Malleable Capital

As in case 1, the augmented Golden Rule, equation (5) in the main text, can be solved with respect to the optimal stock level S_t^{**}:

$$S_t^{**} = \frac{K}{4}\left[\left[\frac{c}{PqKe^{(\lambda + M_2\psi)t - \mu(t,Z)}} + 1 - \frac{\delta}{r}\right] + \sqrt{\left[\frac{c}{PqKe^{(\lambda + M_2\psi)t - \mu(t,Z)}} + 1 - \frac{\delta}{r}\right]^2 + \frac{8c(\delta + \lambda + M_2\psi - \partial\mu(t,Z)/\partial t)}{PqKre^{(\lambda + M_2\psi)t - \mu(t,Z)}}}\right]. \tag{A6}$$

The singular solution for yield Y_t^{**} can be found from the net growth equation as $Y_t^{**} = F(S_t^{**}) - \partial S_t^{**}/\partial t$. Differentiating the singular solution S_t^{**}, equation (A6), with respect to time gives

$$\frac{\partial S_t^{**}}{\partial t} = -[\lambda + M_2\psi - \partial\mu(t,Z)/\partial t]A_2(t)\frac{K}{4}C_2(t), \tag{A7}$$

where $A_2(t) = c/(PqKe^{(\lambda + M_2\psi)t - \mu(t,Z)})$, $C_2(t) = \left[1 + \frac{1}{2}B_2(t)^{-\frac{1}{2}}\left[2[A_2(t) + 1 - \frac{\delta}{r}] + \frac{8(\delta + \lambda + M_2\psi - \partial\mu(t,Z)/\partial t)}{r}\right]\right]$, and $B_2(t)$ are the terms inside the square root in equation (A6). $\frac{\partial S_t^{**}}{\partial t}$ is negative, assuming that the total rate of technical progress $\lambda + M_2\psi$ is larger than the change in technical inefficiency $\partial\mu(t,Z)/\partial t$ and because $A_2(t)$ and $C_2(t)$ are positive.

The singular solution for yield is then

$$Y_t^{**} = rS_t^{**}(1 - S_t^{**}/K) + [\lambda + M_2\psi - \partial\mu(t,Z)/\partial t]A_2(t)\frac{K}{4}C_2(t). \tag{A8}$$

The expression for S_t^{**} from equation (A6) can be inserted to give a solution depending on only the biological and economic parameters:

$$Y_t^* = r\frac{K}{4}\left[\left[A_2(t) + 1 - \frac{\delta}{r}\right] + \sqrt{B_2(t)}\right] \times \left(1 - \frac{1}{4}\left[\left[A_2(t) + 1 - \frac{\delta}{r}\right] + \sqrt{B_2(t)}\right]\right) + [\lambda - \partial\mu(t,Z)/\partial t]A_2(t)\frac{K}{4}C_2(t). \tag{A9}$$

Finally, the singular solution for effort can be found from the Graham-Schaefer surplus production frontier, $Y_t = qE_t S_t e^{(\lambda + M_2\psi)t - \mu(t,Z)}$ as

$$E_t^{**} = \frac{Y_t^{**}}{qS_t^{**} e^{(\lambda + M_2\psi)t - \mu(t,z)}}, \tag{A10}$$

where the optimal solutions of yield, equations (A8) and (A9), and stock, equation (A6), can be inserted to give an expression depending on only the biological and economic parameters:

$$E_t^{**} = \frac{rS_t^{**}(1 - S_t^{**}/K) + [\lambda + M_2\psi - \partial\mu(t,z)/\partial t]A_2(t)\frac{K}{4}C_2(t)}{qS_t^{**}e^{(\lambda+M_2\psi)t-\mu(t,Z)}} =$$

$$\frac{r\left[\frac{[A_2(t)+1-\frac{\delta}{r}]}{+\sqrt{B_2(t)}}\right]\left(1-\frac{1}{4}\left[A_2(t)+1-\frac{\delta}{r}+\sqrt{B_2(t)}\right]\right) + [\lambda + M_2\psi - \partial\mu(t,Z)/\partial t]A_2(t)C_2(t)}{q\left[[A_2(t)+1-\frac{\delta}{r}]+\sqrt{B_2(t)}\right]e^{(\lambda+M_2\psi)t-\mu(t,Z)}}.$$

$$(A11)$$

Case 3: Both Disembodied and Embodied Technical Change and Imperfectly Malleable Capital

The augmented Golden Rule is in this case,

$$\frac{\partial F}{\partial S_t} + \frac{(c_vA + c_f(\gamma+\delta))F(S_t)}{(PqAS_te^{(\lambda+M_2\psi)t-\mu(t,Z)} - (c_vA + c_f(\gamma+\delta)))S_t}$$
$$+ \frac{(c_vA + c_f(\gamma+\delta))(\lambda + \psi M_2 - \partial\mu(t,Z)/\partial t)}{PqAS_te^{(\lambda+M_2\psi)t-\mu(t,Z)} - (c_vA + c_f(\gamma+\delta))} = \delta. \quad (A12)$$

As in cases 1 and 2, the augmented Golden Rule can be solved with respect to the optimal stock level S_t^{***}:

$$S_t^{***} = \frac{K}{4}$$
$$\times \left[\begin{array}{c} \left[\frac{c_vA+c_f(\gamma+\delta)}{PqAKe^{(\lambda+M_2\psi)t-\mu(t,Z)}} + 1 - \frac{\delta}{r}\right] \\ +\sqrt{\left[\frac{c_vA+c_f(\gamma+\delta)}{PqAKe^{(\lambda+M_2\psi)t-\mu(t,Z)}} + 1 - \frac{\delta}{r}\right]^2 + \frac{8(c_vA+c_f(\gamma+\delta))(\delta+\lambda+M_2\psi-\partial\mu(t,Z)/\partial t)}{PqAKre^{(\lambda+M_2\psi)t-\mu(t,Z)}}} \end{array} \right].$$

$$(A13)$$

The singular solution for yield Y_t^{***} can be found from the net growth equation as $Y_t^{***} = F(S_t^{***}) - \partial S_t^{***}/\partial t$. Differentiating the singular solution S_t^{***}, equation (A13), with respect to time gives

$$\frac{\partial S_t^{***}}{\partial t} = -[\lambda + M_2\psi - \partial\mu(t,Z)/\partial t]A_3(t)\frac{K}{4}C_3(t), \quad (A14)$$

where $A_3(t) = (c_vA + c_f(\gamma+\delta))/(PqAKe^{(\lambda+M_2\psi)t-\mu(t,Z)})$, $C_3(t) = \left[1 + \frac{1}{2}B_3(t)^{-\frac{1}{2}}\left[2[A_3(t) + 1 - \frac{\delta}{r}] + \frac{8(\delta+\lambda+M_2\psi-\partial\mu(t,Z)/\partial t)}{r}\right]\right]$, and $B_3(t)$ is the terms

inside the square root in equation (A13). $\frac{\partial S_t^{***}}{\partial t}$ is negative, assuming that the total rate of technical progress $\lambda + M_2\psi$ is larger than the change in technical inefficiency $\partial\mu(t,Z)/\partial t$ and because $A_3(t)$ and $C_3(t)$ are positive.

The singular solution for yield is then

$$Y_t^{***} = rS_t^{***}(1 - S_t^{***}/K) + [\lambda + M_2\psi - \partial\mu(t,Z)/\partial t]A_3(t)\frac{K}{4}C_3(t). \quad (A15)$$

The expression for S_t^{***} from equation (A13) can be inserted to give a solution depending only on the biological and economic parameters:

$$Y_t^{***} = r\frac{K}{4}\left[\left[A_3(t) + 1 - \frac{\delta}{r}\right] + \sqrt{B_3(t)}\right]$$
$$\times \left(1 - \frac{1}{4}\left[\left[A_3(t) + 1 - \frac{\delta}{r}\right] + \sqrt{B_3(t)}\right]\right)$$
$$+ [\lambda - \partial\mu(t,Z)/\partial t]A_3(t)\frac{K}{4}C_3(t). \quad (A16)$$

Finally, the singular solution for effort can be found from the Graham-Schaefer surplus production frontier: $Y_t = qE_tS_te^{(\lambda+M_2\psi)t-\mu(t,Z)}$ as

$$E_t^{***} = \frac{Y_t^{***}}{qS_t^{***}e^{(\lambda+M_2\psi)t-\mu(t,Z)}} \quad (A17)$$

where the optimal solutions of yield, equations (A15) and (A16), and stock, equation (A13), can be inserted to give an expression depending on only the biological and economic parameters:

$$E_t^{***} = \frac{rS_t^{***}(1 - S_t^{***}/K) + [\lambda + M_2\psi - \partial\mu(t,z)/\partial t]A_3(t)\frac{K}{4}C_3(t)}{qS_t^{***}e^{(\lambda+M_2\psi)t-\mu(t,Z)}} =$$

$$\frac{r\left[\frac{[A_3(t)+1-\frac{\delta}{r}]}{+\sqrt{B_3(t)}}\right]\left(1-\frac{1}{4}\left[[A_3(t)+1-\frac{\delta}{r}]+\sqrt{B_3(t)}\right]\right) + [\lambda + M_2\psi - \partial\mu(t,Z)/\partial t]A_3(t)C_3(t)}{q\left[[A_3(t)+1-\frac{\delta}{r}]+\sqrt{B_3(t)}\right]e^{(\lambda+M_2\psi)t-\mu(t,Z)}}.$$

$$(A18)$$