A Fast Quasi-Linear Heuristic for the Close-Enough Traveling Salesman Problem

1 Introduction

The *Close-Enough Traveling Salesman Problem* (CETSP) is a continuous generalization of the classical Traveling Salesman Problem (TSP), where each target is associated with a circular neighborhood that the salesman must intersect rather than a precise point. The CETSP thus combines the discrete optimization of the visiting sequence with the continuous optimization of visiting positions, and it naturally models path-planning tasks such as radio meter reading, drone-based inspection, and robotic welding (Lei and Hao 2024; Di Placido et al. 2022).

Despite recent progress, most high-performing CETSP algorithms rely on population-based or local-improvement metaheuristics that achieve strong solution quality at the expense of high computational cost. In this work, we present a new heuristic for the CETSP that is inspired by the quasi-linear *pair-center algorithm* for the Euclidean TSP proposed by Formella (2024). While originally designed for point-based TSP instances, the pair-center concept extends naturally to the CETSP once disk geometry and intersection logic are handled properly. Our resulting method runs in expected $O(n \operatorname{polylog} n)$ time, making it markedly faster and more scalable than current state-of-the-art CETSP solvers. Although it does not seek to outperform metaheuristic approaches in final tour length, its runtime efficiency allows application to extremely large or real-time instances that remain intractable for existing techniques.

2 Related Work

The CETSP has attracted sustained research interest over the past two decades, with methods broadly classified into exact and metaheuristic approaches. Early constructive heuristics such as those of Gulczynski, Golden, and Wasil (2006) introduced the notion of *Steiner zones*—nonempty intersections of disk neighborhoods—to reduce the continuous space to a discrete set of candidate visitation points. Mennell, Smith, and Behdani (2009) and Mennell and Smith (2011) refined this idea into a three-phase Steiner-zone heuristic incorporating second-order cone programming (SOCP) to optimize positions given a fixed sequence. These pioneering studies established the general "discretize-tour-improve" pattern still used today.

Exact algorithms such as Behdani and Smith (2014) and Coutinho et al. (2016) can guarantee optimality for modest-sized instances but scale poorly, as both depend on discretization granularity. To improve scalability, Carrabs, Cerrone, and Cerulli (2017a), Carrabs, Cerrone, and Cerulli (2017b), and Carrabs, Cerrone, Cerulli, and Di Placido (2020) proposed adaptive internal discretization schemes that convert the CETSP to a generalized TSP, sometimes hybridized with carousel greedy selection. These methods yield high-quality solutions but remain limited by the discretization size and local-search overhead.

The next wave of work focused on hybrid and metaheuristic algorithms. Yang, Sun, and Wu (2018) combined particle swarm optimization (PSO) and genetic algorithms (GA) in a double-loop framework for the more general TSP with arbitrary neighborhoods. Wang, Zhiyong Zhang, and Zhiming Zhang (2019) developed the *Steiner-Zone Variable Neighborhood Search* (SZVNS), a three-phase heuristic integrating multiple insertion and reconstruction operators; while effective, their insertion operators were computationally expensive. Di Placido et al. (2022) later proposed a GA augmented with SOCP-based local refinement and a three-operator improvement scheme (2-opt, SOCP, and 3Alg). Their approach produced numerous new best-known solutions on the

62 standard benchmark instances but incurred substantial runtime due to repeated position optimization.

The current state-of-the-art is the memetic algorithm MA-CETSP of Lei and Hao (2024), which integrates an MSX crossover tailored to CETSP geometry, a VND-based local search, and joint sequence-position optimization. Their method improved or matched 62 benchmark upper bounds and provided publicly available code, but the authors themselves noted that several algorithmic variants (e.g., without approximation or with sparse strategies) became *prohibitively long*. Like most population-based methods, MA-CETSP and related heuristics offer no worst-case runtime guarantees and depend on extensive parameter tuning and stochastic search.

Overall, the literature demonstrates a consistent trade-off between solution quality and runtime: metaheuristics achieve excellent empirical results but come without theoretical efficiency bounds and are practically slow. This motivates the exploration of fast, geometry-driven heuristics, such as the present quasi-linear approach, that deliberately prioritize runtime scalability for large-scale or real-time CETSP applications.

3 Algorithm

(Description forthcoming)

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