A Fast Quasi-Linear Heuristic for the Close-Enough Traveling Salesman Problem

1 Introduction

The *Close-Enough Traveling Salesman Problem* (CETSP) is a continuous generalization of the classical Traveling Salesman Problem (TSP), where each target is associated with a circular neighborhood that the salesman must intersect rather than a precise point. The CETSP thus combines the discrete optimization of the visiting sequence with the continuous optimization of visiting positions, and it naturally models path-planning tasks such as radio meter reading, drone-based inspection, and robotic welding (**LeiHao2024**; Di Placido, Archetti, and Cerrone 2022).

Despite recent progress, most high-performing CETSP algorithms rely on population-based or local-improvement metaheuristics that achieve strong solution quality at the expense of high computational cost. In this work, we present a new heuristic for the CETSP that is inspired by the quasi-linear *pair-center algorithm* for the Euclidean TSP proposed by Formella (2024). While originally designed for point-based TSP instances, the pair-center concept extends naturally to the CETSP once disk geometry and intersection logic are handled properly. Our resulting method runs in expected $O(n \operatorname{polylog} n)$ time, making it markedly faster and more scalable than current state-of-the-art CETSP solvers. Although it does not seek to outperform metaheuristic approaches in final tour length, its runtime efficiency allows application to extremely large or real-time instances that remain intractable for existing techniques.

2 Related Work

3 Algorithm

Our approach follows the overall structure of the quasi-linear *pair-center algorithm* introduced by Formella (2024) for the Euclidean TSP, but extends it to handle circular neighborhoods as in the Close-Enough Traveling Salesman Problem. Conceptually, the method retains the same two-phase organization: a *clustering phase* that builds a hierarchical representation of the instance, and a *construction phase* that incrementally expands a feasible tour from this hierarchy.

In the clustering phase, the algorithm recursively merges the closest pair of geometric objects into a new composite "proxy" object until a single hierarchy remains. In the CETSP setting, these objects are circles rather than points as in Formella (2024). The result is a binary clustering tree whose internal nodes represent proxy circles.

The construction phase then traverses this tree to build a closed tour. As in the original pair-center approach, the tour is dynamically maintained so that it remains feasible at every step. However, the continuous nature of the CETSP introduces two additional challenges: first, multiple circles may correspond to a single effective tour point when their feasible regions overlap; second, each tour point admits continuous local optimization within its circle. To address these, the algorithm performs lightweight dynamic updates—reinserting or locally re-optimizing tour points—while preserving near-linear expected runtime.

Overall, the method retains the speed and structural simplicity of the pair-center algorithm while incorporating the geometric flexibility required for close-enough constraints. Detailed definitions, data structures, and optimization steps are presented in the following sections.

3.1 Clustering Phase

References

Di Placido, Andrea, Claudia Archetti, and Carmine Cerrone (Sept. 2022). “A genetic algorithm for the close-enough traveling salesman problem with application to solar panels diagnostic reconnaissance”. In: *Computers & Operations Research* 145, p. 105831. ISSN: 03050548. DOI: 10.1016/j.cor.2022.105831. URL: https://linkinghub.elsevier.com/retrieve/pii/S0305054822001095 (visited on 06/05/2025).

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