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SPECIAL GRAPH REPRESENTATION AND VISUALIZATION OF SEMANTIC NETWORKS

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Abstract. A visibility representation of graphs in which each vertex is mapped to a horizontal segment was originally proposed in 1980s in the context of the VLSI layout construction problem. In this paper, we present an up-to-date survey on this representation and propose a way of using it in visualization of semantic networks.

1. Special Graph Representation

Graph drawing admits different geometrical representations. We consider a class of drawings in which vertices are represented by horizontal segments and edges are represented by vertical segments (Fig. 1). The horizontal segments are called *vertex segments*, while the vertical segments are called *edge segments*. The representation Γ of graph $G = (V, E)$ should satisfy the following conditions:

- vertex segments do not overlap;
- endpoints of the edge segment $\Gamma(u, v)$ belong to the vertex segments $\Gamma(u)$ and $\Gamma(v)$ correspondingly;
- the edge segment $\Gamma(u, v)$ does not intersect any vertex segments except $\Gamma(u)$ and $\Gamma(v)$.

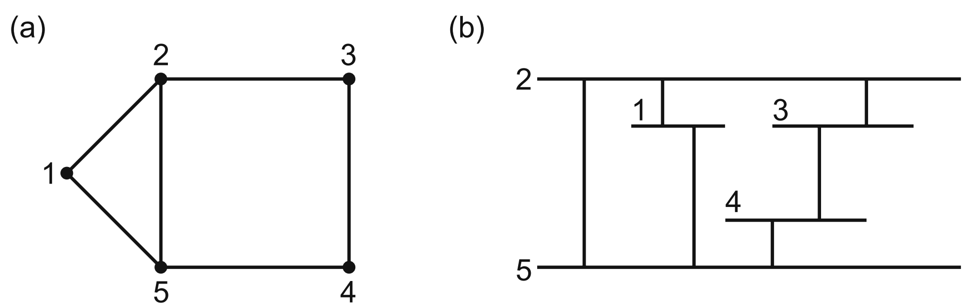

Fig. 1. Different geometrical representations of the graph: (a) vertices are represented by points, edges are represented by curves; (b) vertices are represented by horizontal segments, edges are represented by vertical segments.

This representation was introduced in the context of VLSI layout construction problem [27]. Graph vertices corresponded to scheme components while edges corresponded to connections between them. The geometrical representation Γ can be easily transformed to a planar embedding of the classical graph representation in which vertices are drawn as points and edges are drawn as curves. Thus, the geometrical representation Γ can exist only for planar graphs. A linear time construction algorithm suitable for planar biconnected graphs can be found in the paper [27]. Later in [10] it was shown that the representation Γ exists for general planar graphs. A linear time algorithm for the construction of Γ can be found in [28,30].

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Most of the following papers introduce the representation Γ by means of a visibility concept. Two horizontal segments from a given set are called *visible* if they can be connected by a vertical segment that does not intersect any other horizontal segments of the set. Two horizontal segments from a given set are called *ε-visible* if they can be connected by a vertical stripe of nonzero width that does not intersect any other horizontal segments of the set. All pairs of horizontal segments corresponding to the adjacent vertices in the representation Γ should be visible. In [19], the representation Γ is called a *visibility representation*. According to [30], there are three basic types of visibility representations: a weak visibility representation, an ε-visibility representation, and a strong visibility representation (Fig. 2).

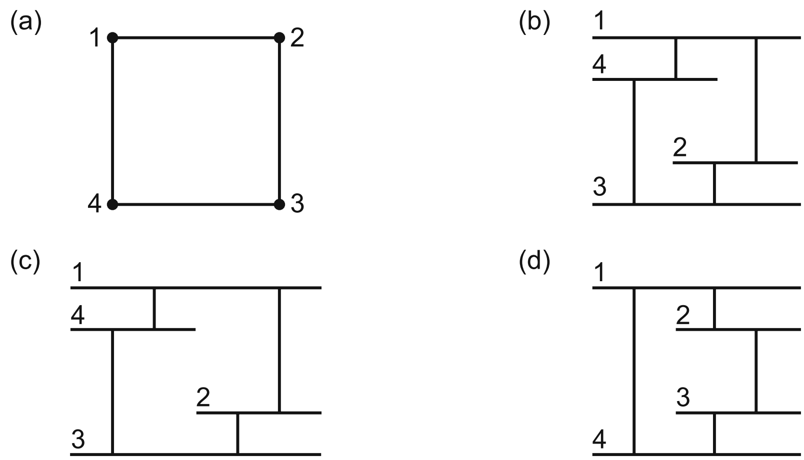

Fig. 2. Visibility representations: (a) classical graph representation; (b) weak visibility representation; (c) ε-visibility representation; (d) strong visibility representation.

Definition 1. A *weak visibility representation* (*w-visibility representation*) of a graph $G = (V, E)$ is a pair of mappings: a mapping of the vertex set V to the set of horizontal segments and a mapping of the edge set E to the set of vertical segments. For every edge $(u, v) \in E$ the ends of corresponding edge segment belong to the vertex segments of u and v and the edge segment does not intersect any other vertex segments.

This definition is in line with the representation Γ concept that is considered in [27]. Vertex segments that correspond to adjacent vertices must be visible but visible vertex segments do not necessarily correspond to adjacent vertices.

Definition 2. An *ε-visibility representation* of a graph G is a w-visibility representation in which two vertex segments are ε-visible if and only if the corresponding graph vertices are adjacent.

This representation was introduced in early 1990s [24]. In [33], it was shown that ε-visibility representation can be constructed for triply connected planar graphs. Later, in [30] and [37] it was proved that an ε-visibility representation of a graph G exists if and only if there exists a planar embedding of G such that all cutpoints belong to the same face. The same articles provide linear time algorithms for the verification of this condition and construction of an ε-visibility representation.

Definition 3. A *strong visibility representation* (*s-visibility representation*) of a graph G is a w-visibility representation in which two vertex segments are visible if and only if the corresponding graph vertices are adjacent.

For the first time s-visibility representation was considered in [22]. The description of graphs that admit s-visibility representation in which all endpoints of vertex segments have different x coordinates can be found in [23]. The description of graphs that admit general s-visibility representation is given in [30].

The same paper contains a polynomial algorithms for the construction of an s-visibility representation for quadruply-connected planar graphs and maximal planar graphs. A planar graph is called *maximal planar* if the addition of any edge between nonadjacent vertices results in a nonplanar graph. As it was shown in [1] the problem of verifying the existence of an s-visibility representation is NP-complete in the general case.

There are several basic research directions concerned with visibility representations:

- the construction of compact visibility representations [6, 11, 12, 16–18, 21, 26, 35, 38];
- the usage of a visibility representation as an intermediate step for the construction of other geometrical representations. This approach is used for the construction of polyline drawings [2,3], in which vertices are represented by points and edges are represented by polylines. Another use of this approach can be found in [31], where a visibility representation is used for the construction of orthogonal drawings. In orthogonal drawings each edge is represented by a polyline that consists of horizontal and vertical segments;
- the study of visibility representation varieties: vertices can be represented by rectangles [4, 29], unit length segments [9, 36], or arcs of concentric circles [13];
- the generalization of the visibility concept [8]. Two horizontal segments of a given set are *k-visible* if they can be connected by a vertical segment that intersects no more than k other segments of the set. The extreme case of the k-visibility concept is reproduced in *interval graphs*, in which vertices are represented by horizontal segments and are adjacent if and only if the projections of the corresponding horizontal segments to the axis Ox intersect;
- the construction of a visibility representation for oriented graphs [34];
- the study of visibility representations on different surfaces [5, 7, 25, 32].

2. Visualization of Semantic Networks

Semantic networks provide a natural representation of information about relations between objects. Formally a semantic network can be considered as an attributed graph that contains labels on vertices and edges. The vertices of this graph correspond to the objects of a knowledge domain, while the edges can be treated as the relations between them. The labels on vertices and edges specify the descriptions for the corresponding objects and relations. The visualization of subnetworks induced by a given set of objects is considered. The input data for the problem is

- a (possibly directed) graph $G_0 = \langle V_0, E_0 \rangle$, where V_0 is a set of vertices and E_0 is a set of edges. There are no selfloops in G, but it may contain multiple edges;
- vertex and edge labels specified with the dimensions of bounding rectangles: $w(v)$, $h(v)$ for $v \in V_0$ and $w(e)$, $h(e)$ for $e \in E_0$, where w is the width and h is the height of rectangle;
- the set of selected vertices $V' \subseteq V_0$ corresponding to the set of selected objects.

The object of multistripe layout visualization is a subnetwork specified by a subgraph $G = \langle V, E \rangle$ of the graph G_0 where

$$E = \{e \in E_0 \mid \text{the edge } e \text{ is incident to some vertex } u \in V'\},$$
$$V = \{v \in V_0 \mid \text{the vertex } v \text{ is incident to some edge } e \in E\}.$$

The graph G contains the selected vertices from V' and the vertices directly adjacent to them. Let us call the vertices from $V \setminus V'$ *secondary* ones. There are no edges between the secondary vertices in G as each edge $e \in E$ is incident to some vertex $u \in V'$. Thus, each edge of the graph G connects either a pair u_1, u_2 of selected vertices from V' or a selected vertex $u \in V'$ and a secondary vertex $v \in V \setminus V'$.

In [20], it was proposed to use a *multistripe layout* for visualization of G. The vertices from the selected set V' are represented by horizontal segments. The place between these horizontal segments is used for layout of labels of secondary vertices and edges (Fig. 3). The horizontal segments corresponding to the vertices from the selected set have the same x coordinates of the endpoints. In some cases this results in

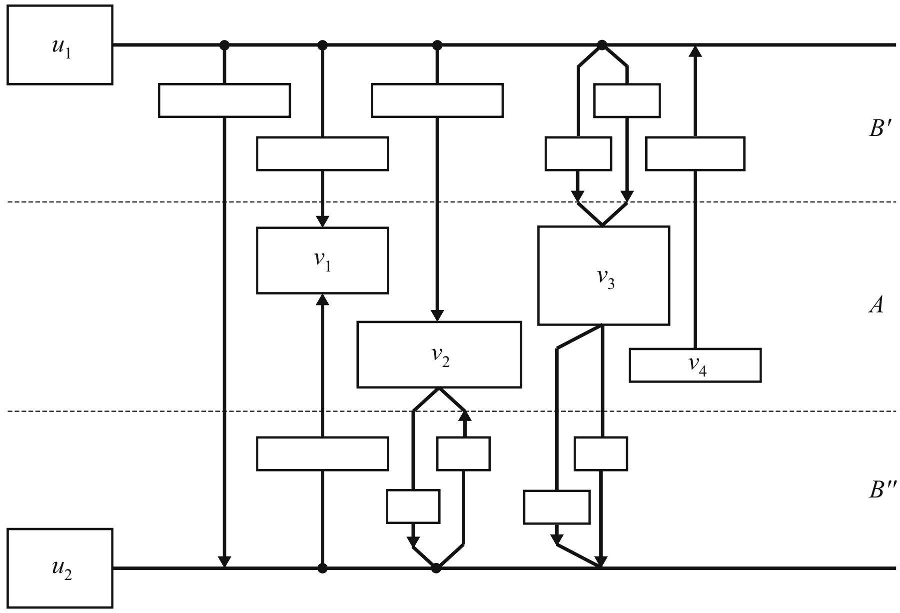

Fig. 3. Multistripe layout fragment: u_1, u_2 are selected vertices, v_1, v_2, v_3, v_4 are secondary vertices.

poorly readable drawings (Fig. 4(a)). There are two configurations that lead to this situation because of the intersections between connection lines and horizontal segments corresponding to the selected vertices:

- the edge connecting selected vertices subject to a situation where corresponding horizontal segments are not neighboring;
- the secondary vertex adjacent to the selected vertices subject to a situation where corresponding horizontal segments are not neighboring.

To address the problem we propose to use a w-visibility representation as a skeleton for the drawing. This helps to avoid the intersections between connection lines and vertex segments in the case of planar graphs (Fig. 4(b), (c)). If the graph is nonplanar, then preliminary planarization can be performed [14,15].

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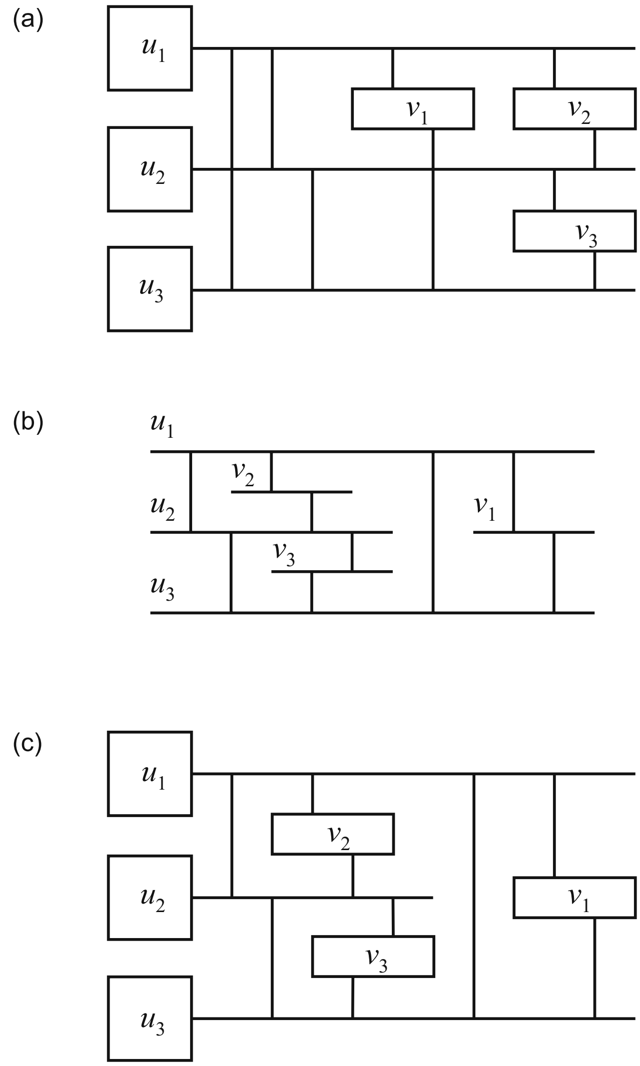

Fig. 4. The usage of a w-visibility representation for the construction of a semantic network drawing: (a) configurations that result in poorer readability of drawings; (b) the w-visibility representation; (c) the drawing constructed on the base of the w-visibility representation.

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