

Work the **WeBWorK problems** labeled **bifur**. (These have no written part to include with your written assignment.) Also, you have written HW below, where you analyze a slightly more complex fishing model from the ones in lecture.

1. In class we examined basic models for fishing with constant limits and proportional harvesting. Below we outline a model for the population of fish with a Holling's Type II term for the harvesting of fish. Suppose that a population of fish, $F(t)$ (in thousands), is given by the following model

$$\frac{dF}{dt} = 0.2 F \left(1 - \frac{F}{100}\right) - \frac{hF}{1 + 0.02F},$$

where h is the harvesting term from fishing.

a. Give a modeling description of each term in the equation above. Compare this model to the two models studied in lecture with constant limits and proportional fishing.

b. Assume there is no fishing ($h = 0$). Find all equilibria for this model. Sketch a graph of the right hand side of the model, then draw the phase portrait. Determine the stability of all equilibria.

c. Let the fishing intensity parameter, h , vary with $0 \leq h \leq 0.25$. Find all bifurcations that occur for these values of h , and determine the type of bifurcation. Create a bifurcation diagram for h over this range of values. Draw representative phase portraits (including a graph of the right hand side of the differential equation) for each value of h at a bifurcation point and for values of h on either side of the bifurcation values, showing clearly the stability of all equilibria.

d. Using your bifurcation diagram, write a brief discussion of what the model is saying about the fish population for the different levels of fishing. What level of fishing (value of h) results in the fish going extinct? What type of bifurcation occurs at this value of h ?