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% Homework 3, Problem 6.3.10
% Geneva Porter March 20, 2020
% Math 693B: Numerical Partial Differential Equations
%
% This is from Strikewerda exercise number 6.3.10. I compare the analytic
% solution to the 1D Laplace equation with the Finite Difference solution
% using the Crank-Nicolson scheme. Various time steps and spatial density
% combinations were used. Here, h is the spatial density and k is the time-
% step, with mu = k/(h^2). One issue that I was unable to resolve was the
% noise present in the numeric solution. The functions cranknicolson.m,
% plotfigures.m, errors.m, and exact.m are needed to run this script.

% Finding exact solutions

h = [1/10, 1/20, 1/40];
k = [1/10, 1/20, 1/40, 1/160];

e1 = exact(k(1), h(1)); % h=1/10, lambda=1 AND mu=10
e2 = exact(k(2), h(2)); % h=1/20, lambda=1
e3 = exact(k(3), h(2)); %h=1/20, mu=10
e4 = exact(k(3), h(3)); %h=1/40, lambda=1
e5 = exact(k(4), h(3)); %h=1/40, mu=10

% Finding numeric solutions and printing errors

s1 = cranknicolson(k(1), h(1), e1);
figure (1);
plotfigures(k(1), h(1), s1, 'h = 1/10, \lambda = 1 and \mu = 10');
figure(2)
plotfigures(k(1), h(1), e1, 'Analytic Solution');
figure(3)
errors(s1, k(1), h(1), 'h = 1/10, \lambda = 1 and \mu = 10',3);

s2 = cranknicolson(k(2), h(2), e2);
figure (5);
plotfigures(k(2), h(2), s2, 'h = 1/20, \lambda = 1');
figure(6)
plotfigures(k(2), h(2), e2, 'Analytic Solution');
figure(7)
errors(s2, k(2), h(2), 'h = 1/20, \lambda = 1',7);

s3 = cranknicolson(k(3), h(2), e3);
figure (9);
plotfigures(k(3), h(2), s3, 'h = 1/20, \mu = 10');
figure(10)
plotfigures(k(3), h(2), e3, 'Analytic Solution');
figure(11)
errors(s3, k(3), h(2), 'h = 1/20, \mu = 10', 11);

s4 = cranknicolson(k(3), h(3), e4);
figure (13);
plotfigures(k(3), h(3), s4, 'h = 1/40, \lambda = 1');
figure(14)
plotfigures(k(3), h(3), e4, 'Analytic Solution');
figure(15)
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errors(s4, k(3), h(3), 'h = 1/40, \lambda = 1', 15);

s5 = cranknicolson(k(4), h(3), e5);
figure (17);
plotfigures(k(4), h(3), s5, 'h = 1/40, \mu = 10');
figure(18)
plotfigures(k(4), h(3), e5, 'Analytic Solution');
figure(19)
errors(s5, k(4), h(3), 'h = 1/40, \mu = 10', 19);


% Comments and comparison:
%
% It is clear, not only from the parameter values but also from the graphs,
% that mu=10 provides a finer mesh and thus a more accurate solution than
% the solution found when the parameters yield lambda=1 (the exception is
% when h=1/10, where lambda=1 and mu=10).
%
% The error for the supremum norm and the L2 norm are printed below. I
% wasn't sure what the question was asking about the supremum norm and the
% L2 norm decreasing, as they both seem to be converging towards zero.
%
%
%
%
%
%
%
%
%
%
%
%
%
%
%
%
%
%
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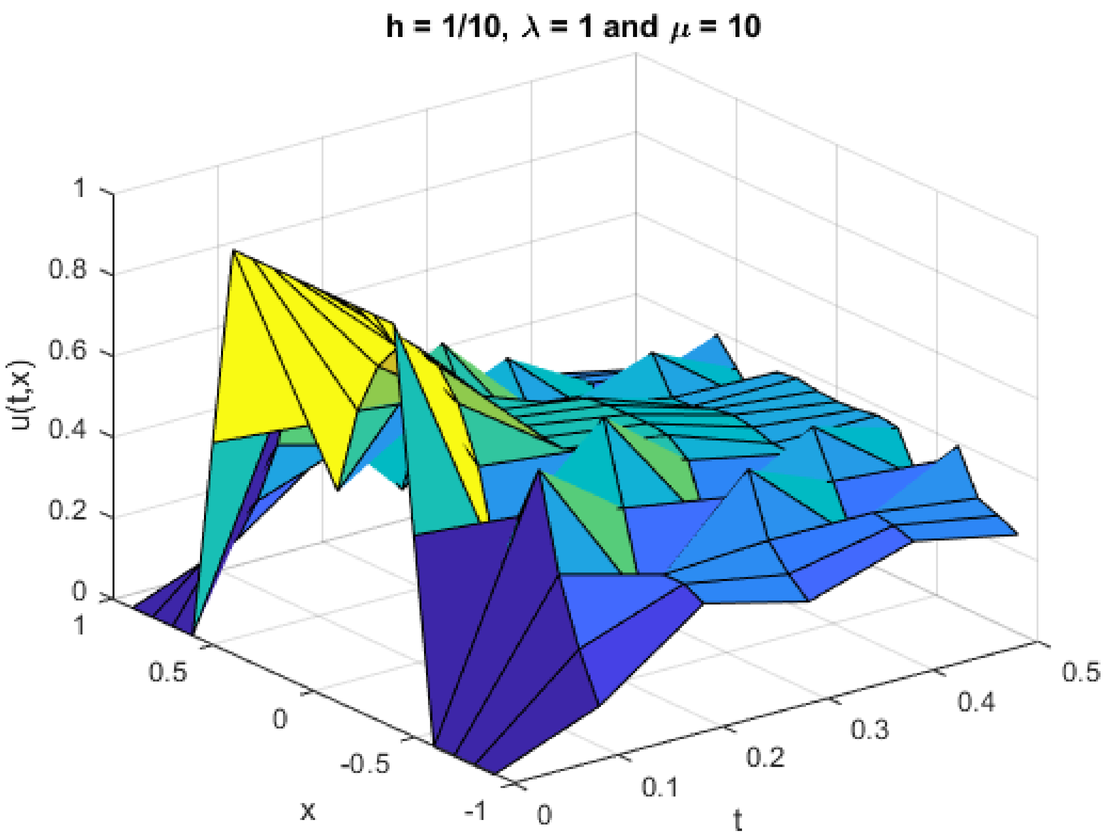
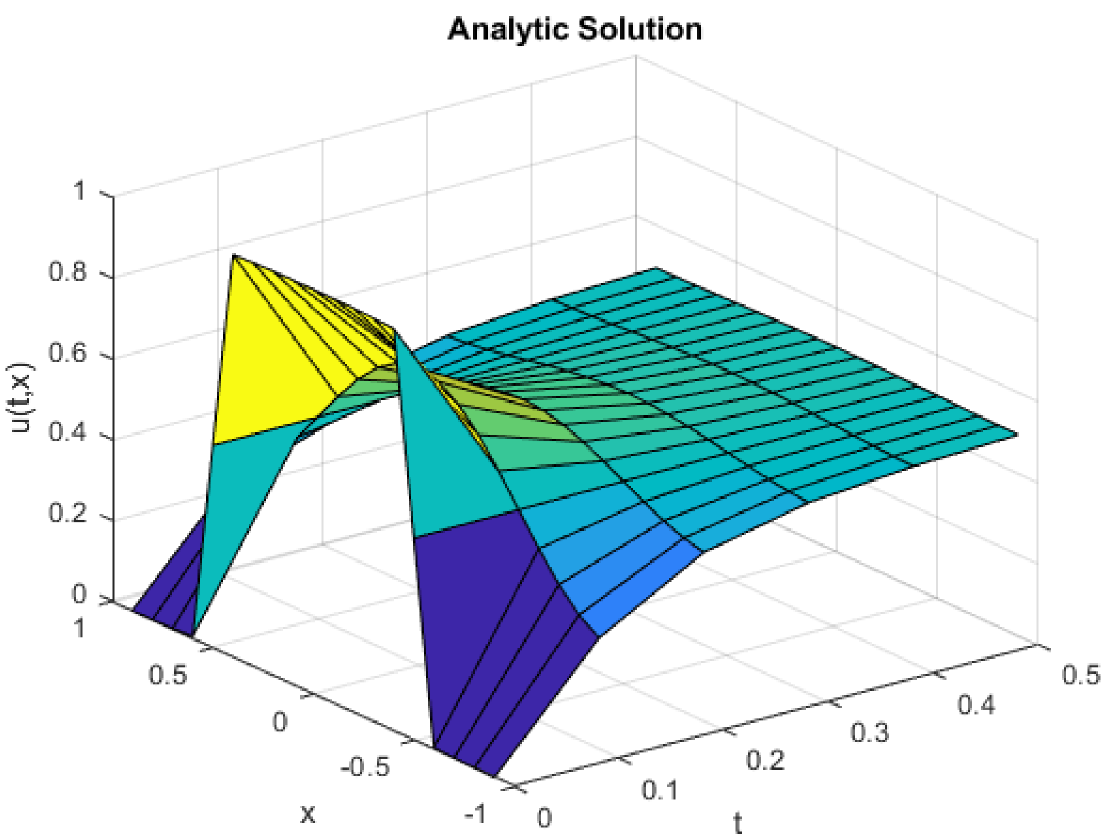
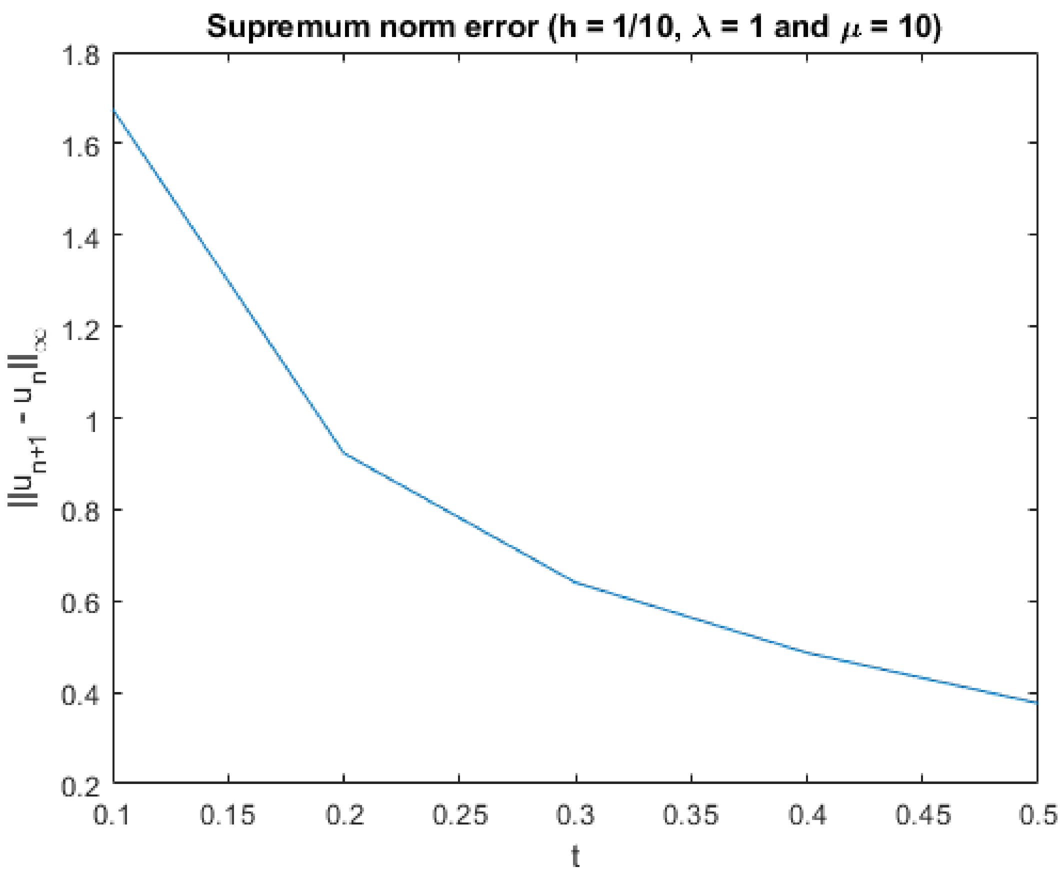
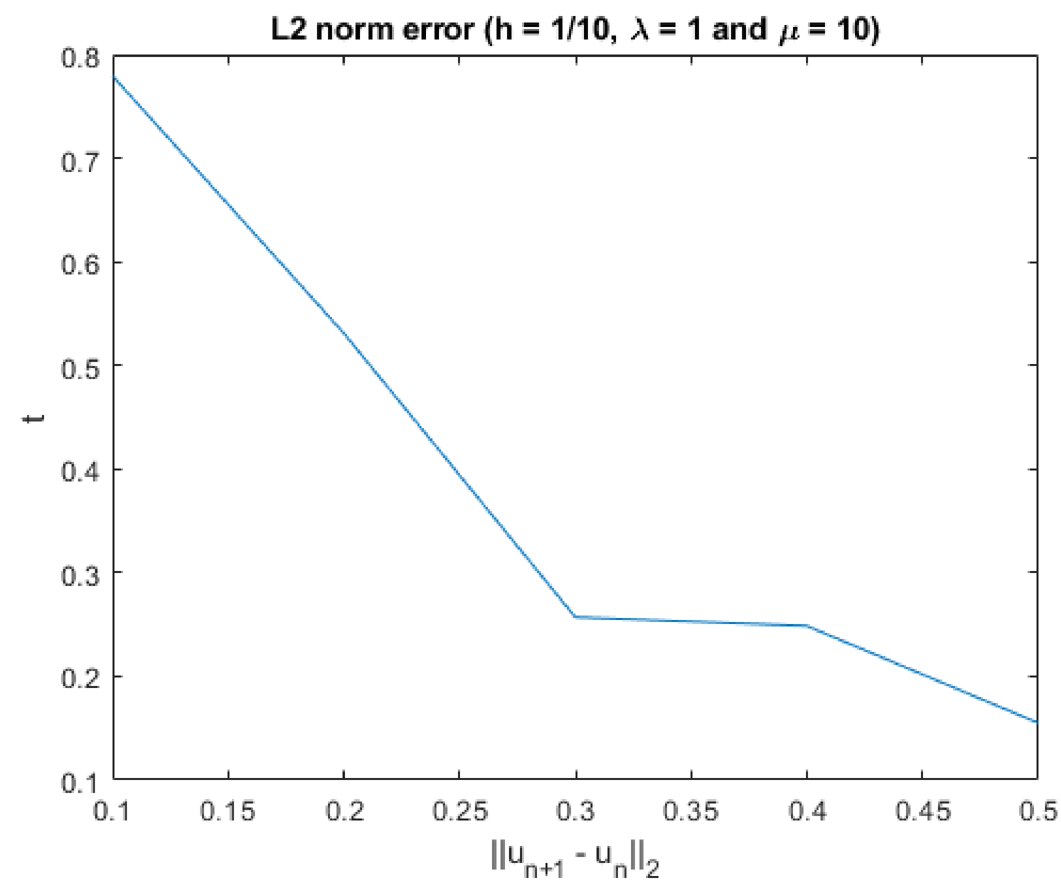
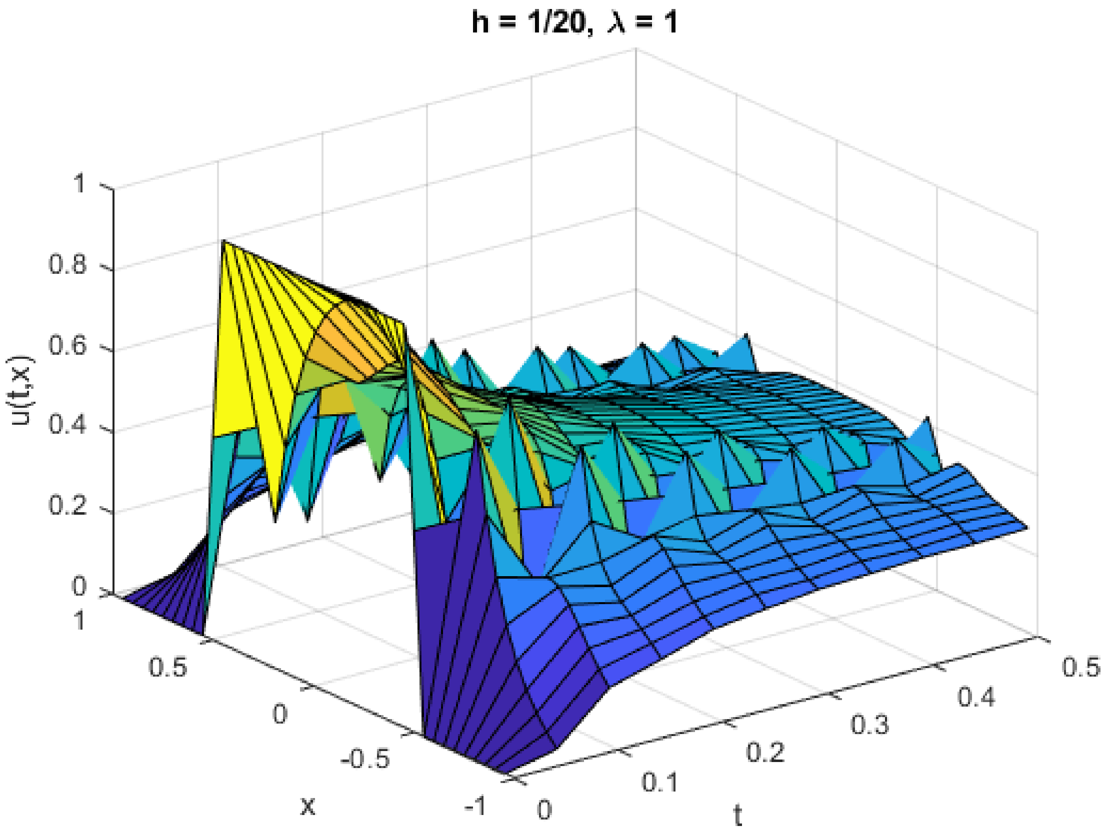

h = 1/10, λ = 1 and μ = 10
u(t,x)
x
t
0
0.2
0.4
0.6
0.8
1
-1
-0.5
0
0.5
1
0
0.1
0.2
0.3
0.4
0.5

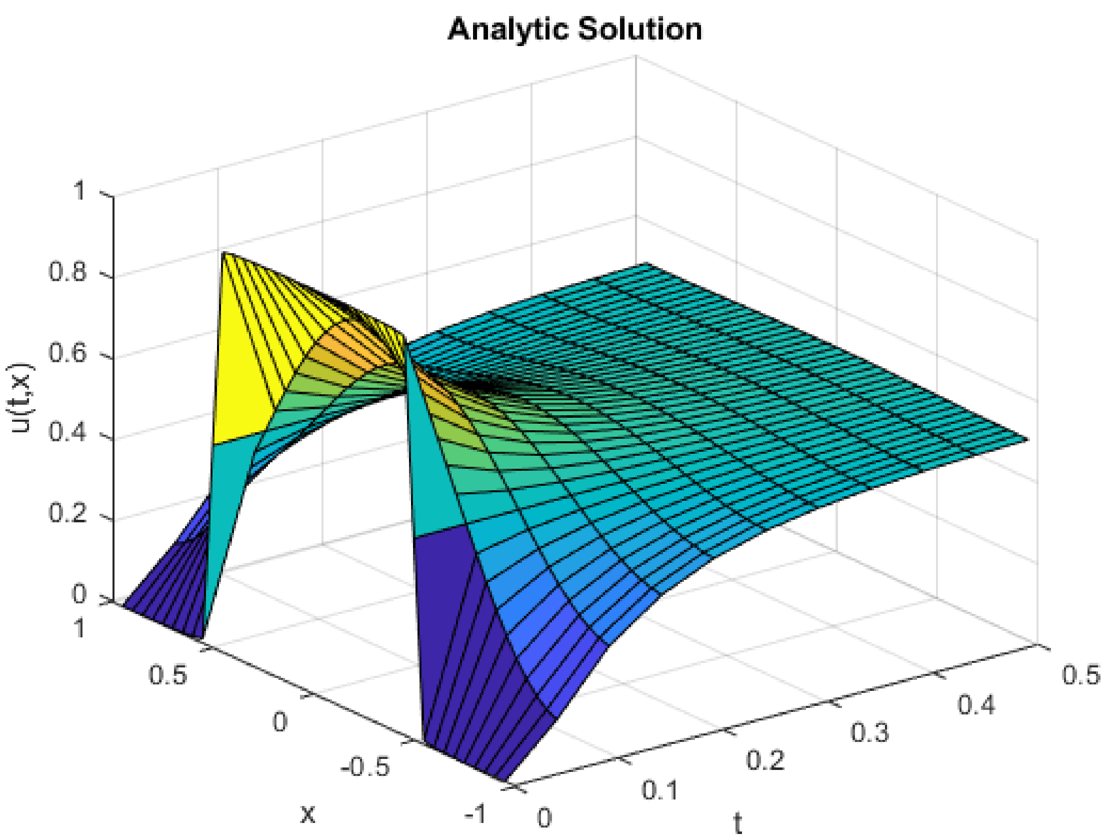

Analytic Solution
u(t,x)
x
t
0
0.2
0.4
0.6
0.8
1
-1
-0.5
0
0.5
1
0
0.1
0.2
0.3
0.4
0.5


Supremum norm error (h = 1/10, λ = 1 and μ = 10)
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
t
||u_{n+1} - u_n||_∞

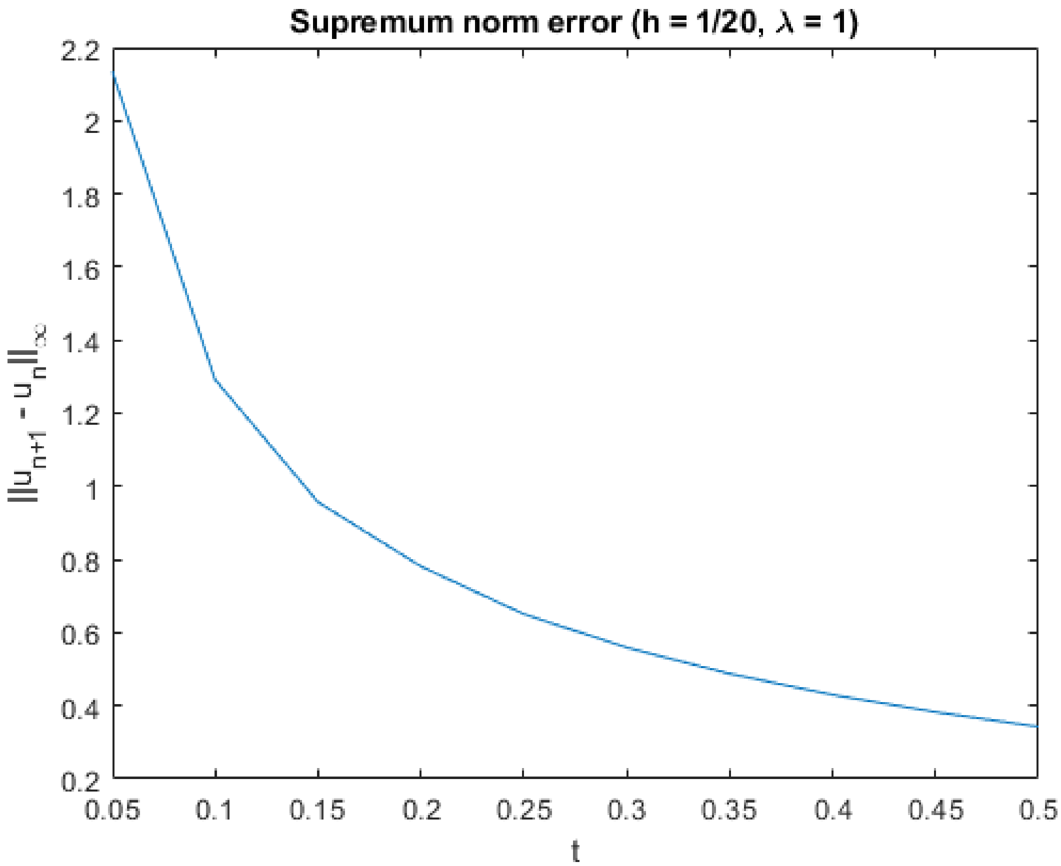

L2 norm error (h = 1/10, λ = 1 and μ = 10)
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
t
||u_{n+1} - u_n||_2



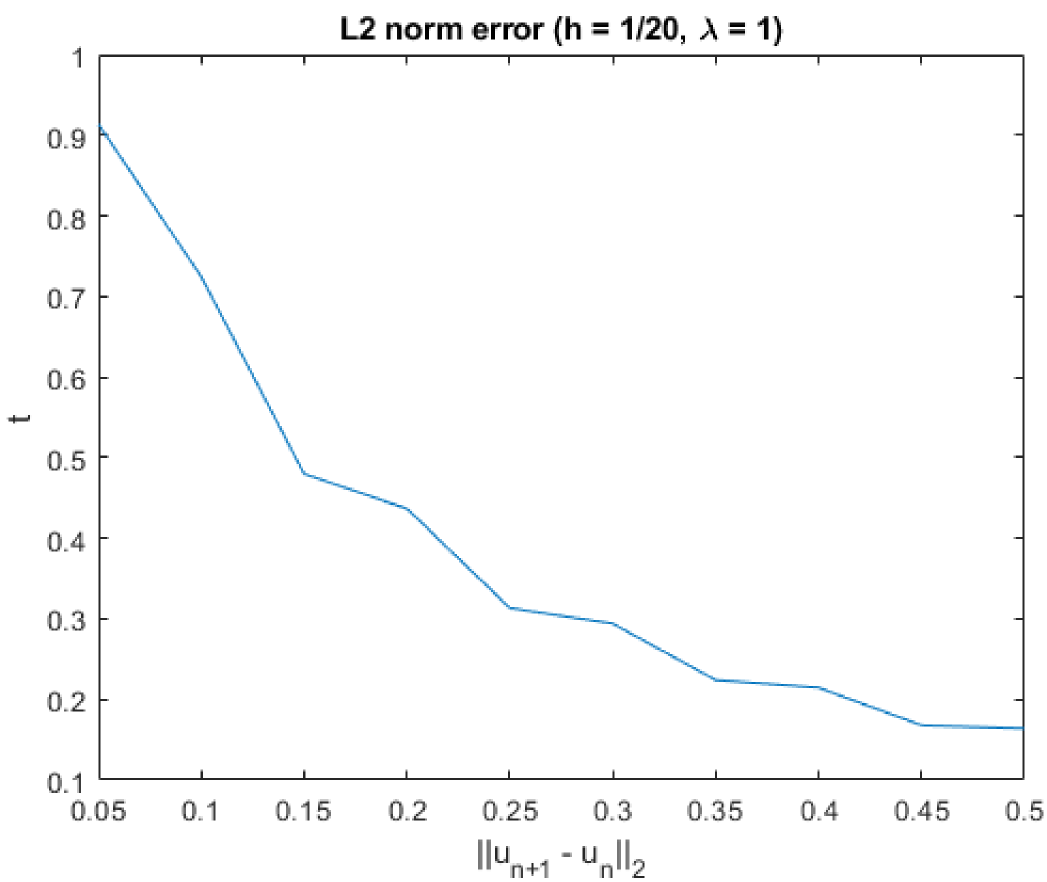
h = 1/20, λ = 1
u(t,x)
1
0.8
0.6
0.4
0.2
0
1
0.5
0
-0.5
-1
x
0
0.1
0.2
0.3
0.4
0.5
t



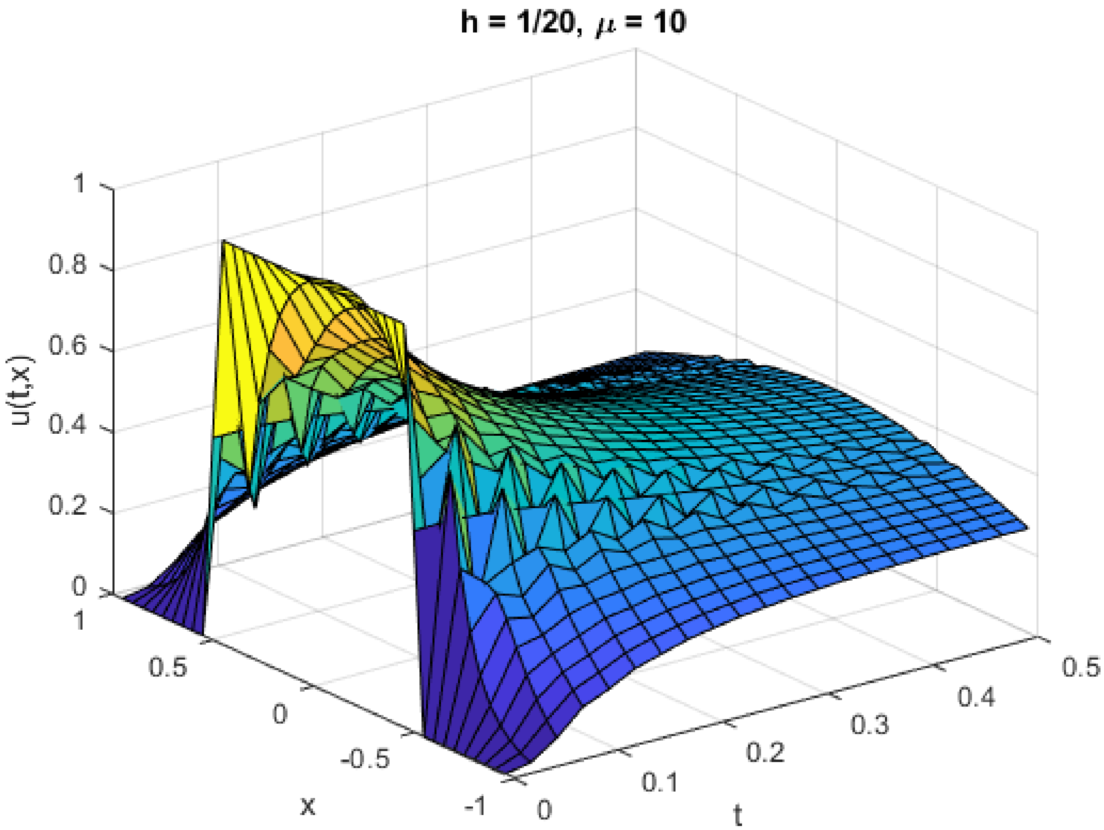
Analytic Solution
u(t,x)
1
0.8
0.6
0.4
0.2
0
1
0.5
0
-0.5
-1
x
0
0.1
0.2
0.3
0.4
0.5
t

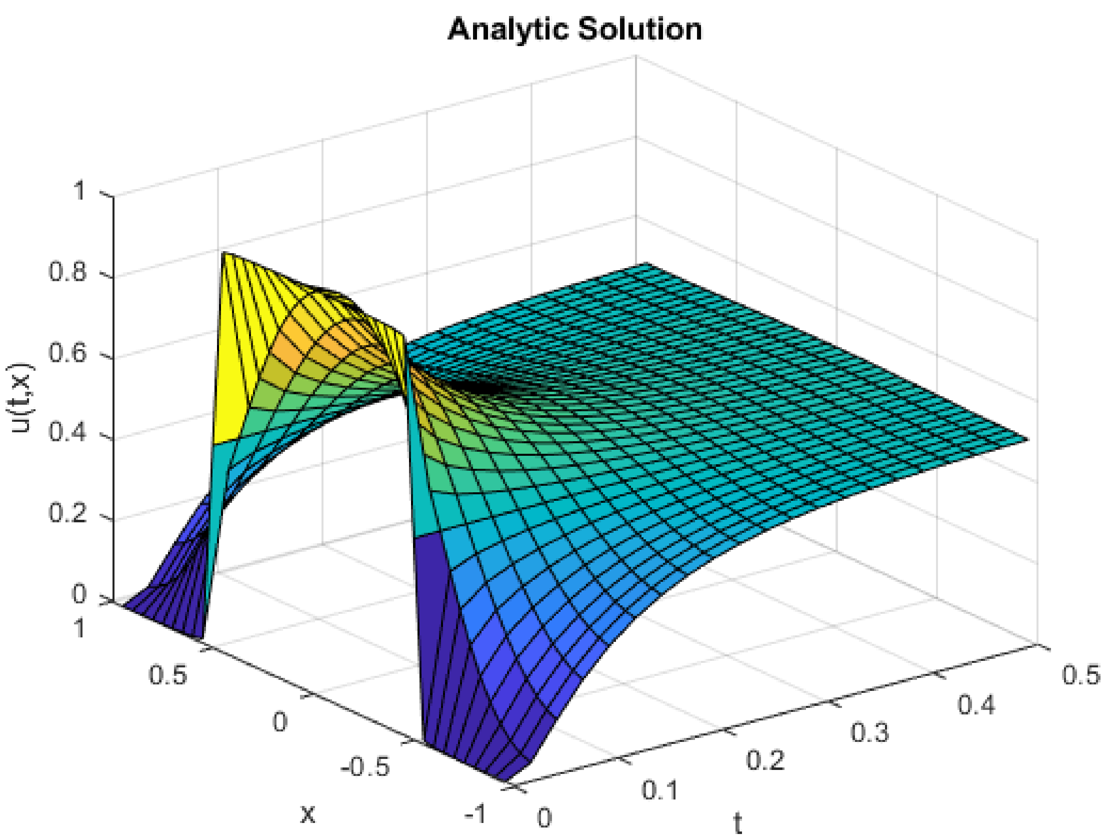


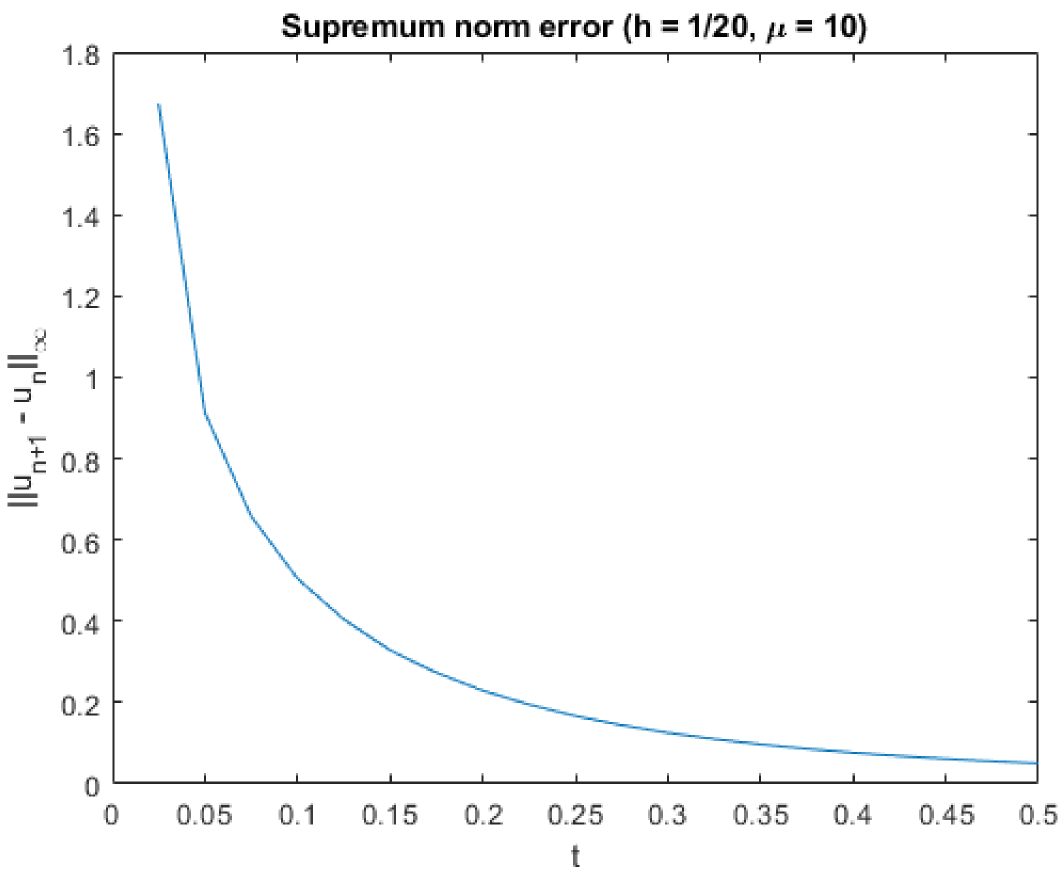
Supremum norm error (h = 1/20, λ = 1)
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
||u_{n+1} - u_n||_∞
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
t



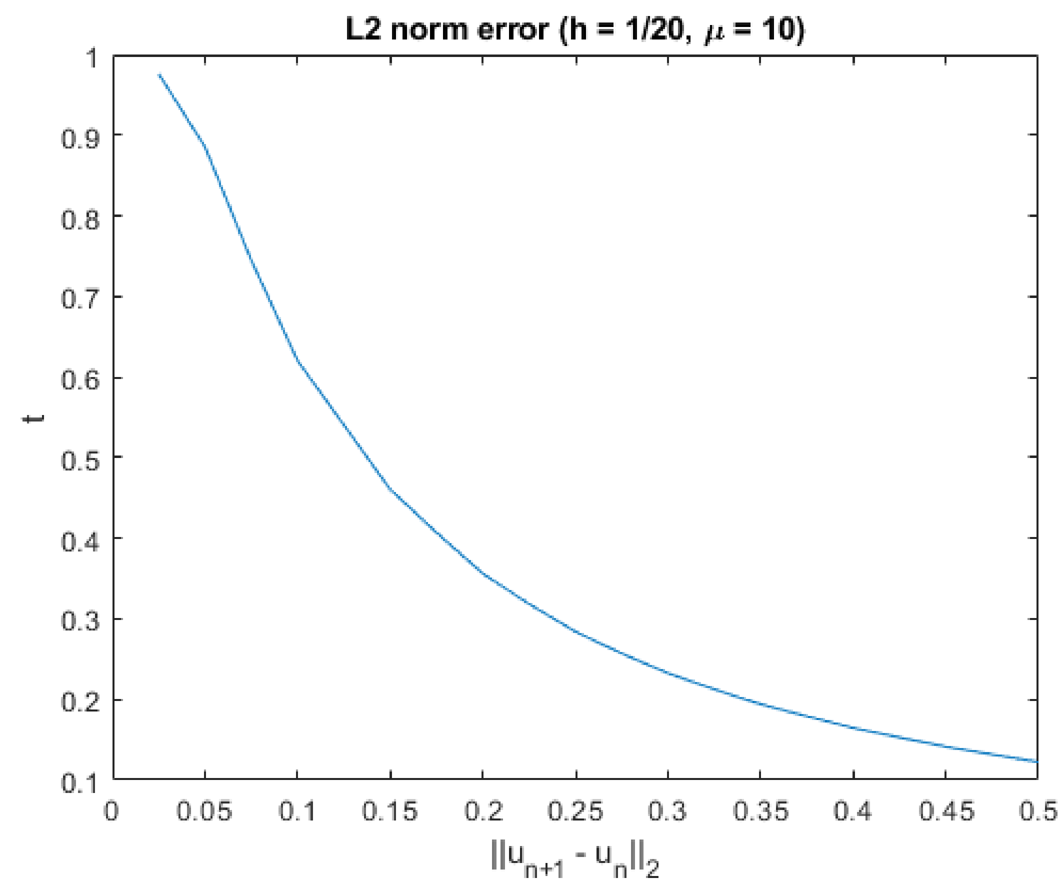
L2 norm error (h = 1/20, λ = 1)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
t
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
||u_{n+1} - u_n||_2


h = 1/20, μ = 10
u(t,x)
x
t


Analytic Solution
u(t,x)
x
t

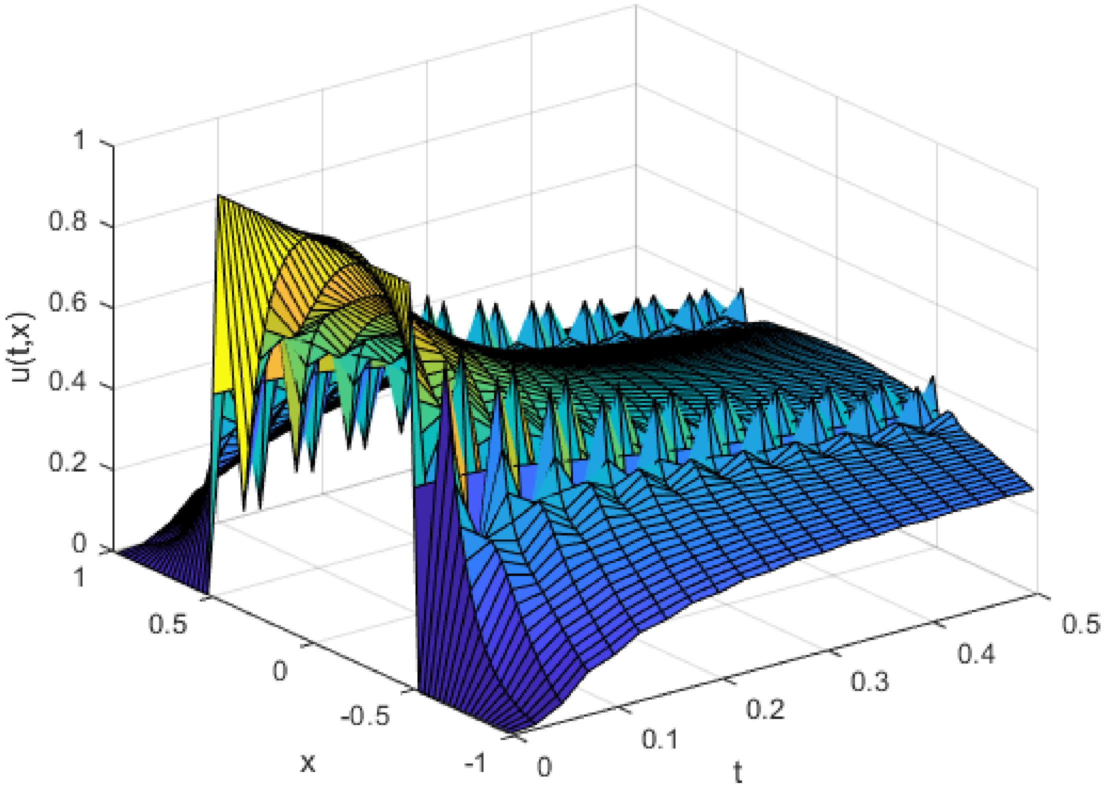


Supremum norm error (h = 1/20, μ = 10)
$||u_{n+1} - u_n||_\infty$
t
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5

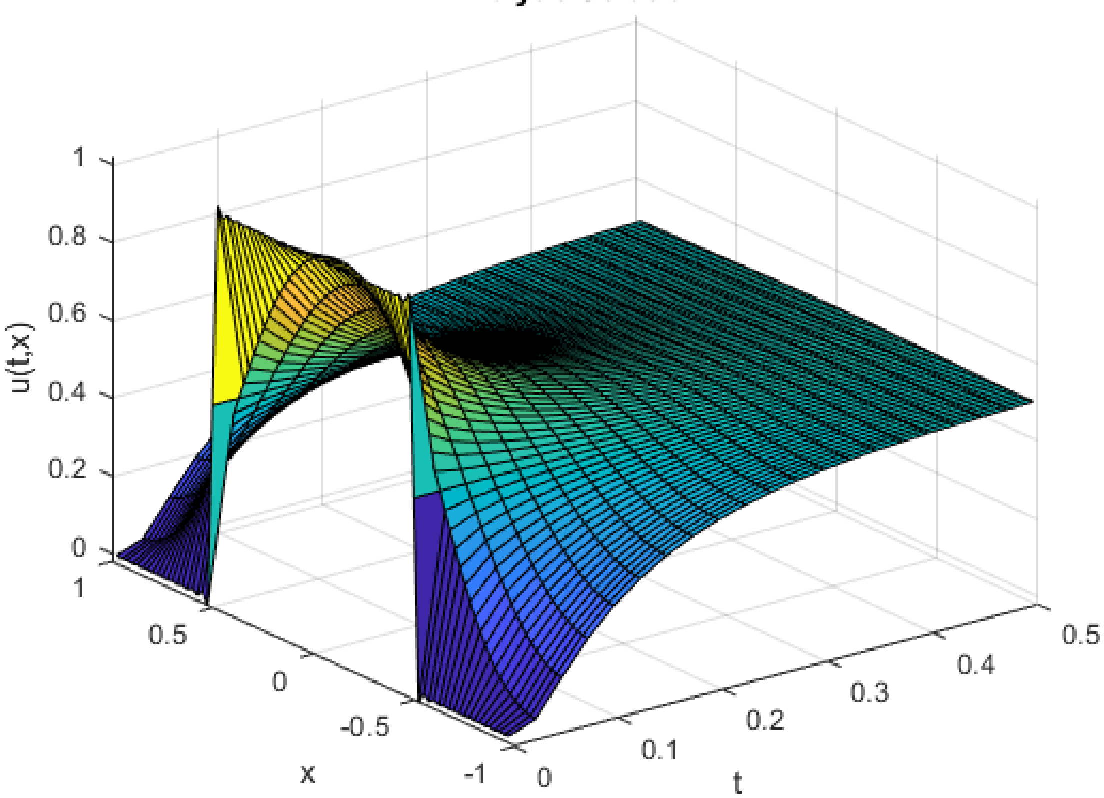


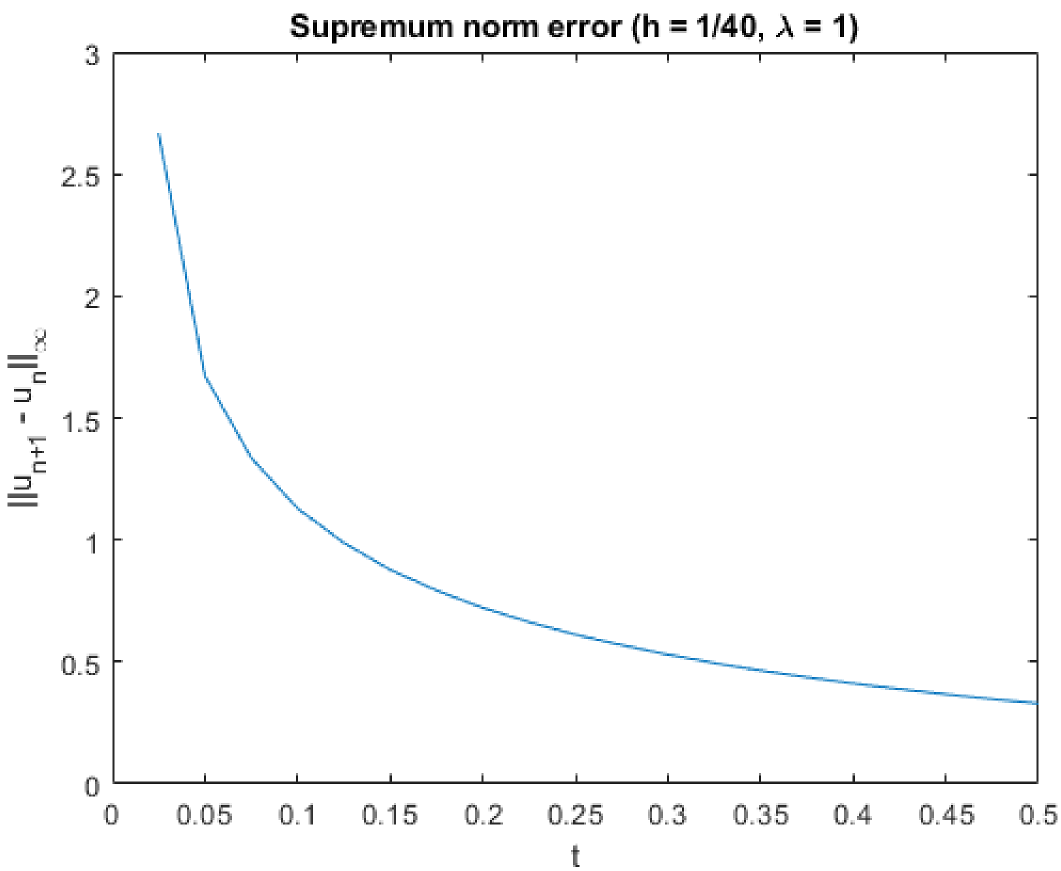
L2 norm error (h = 1/20, μ = 10)
t
$||u_{n+1} - u_n||_2$
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5

h = 1/40, λ = 1

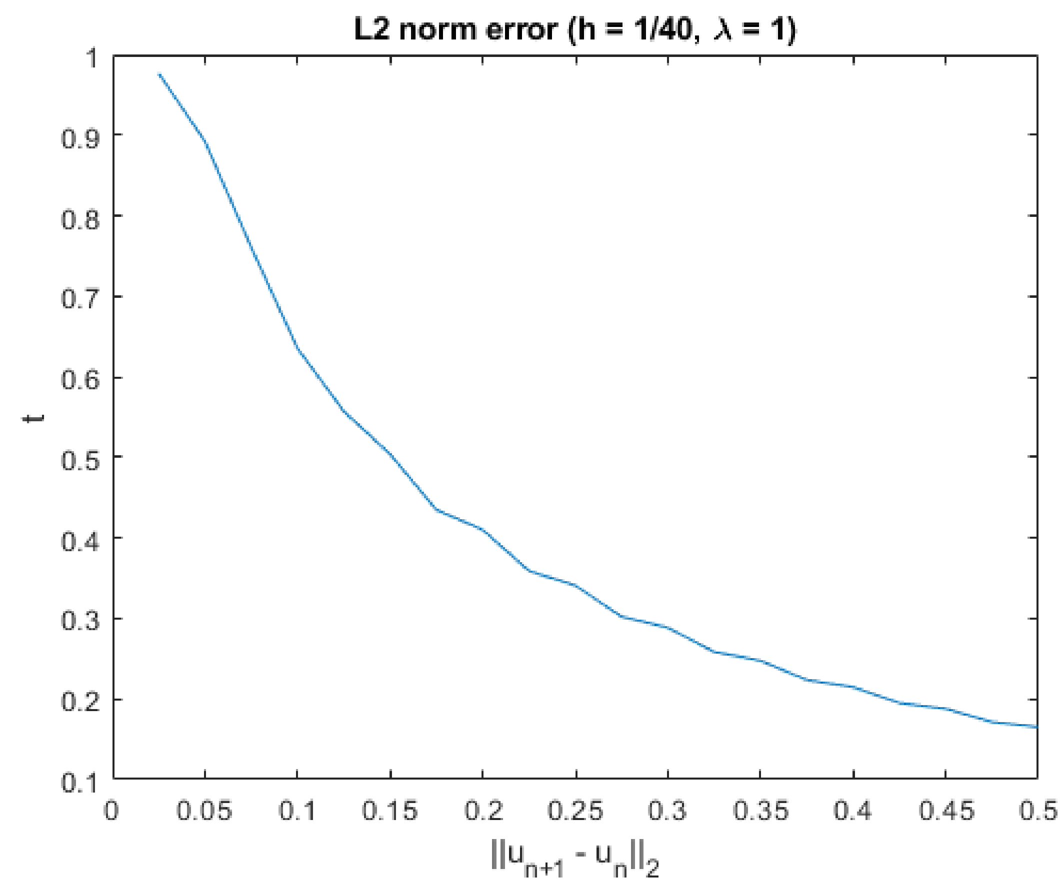


Analytic Solution



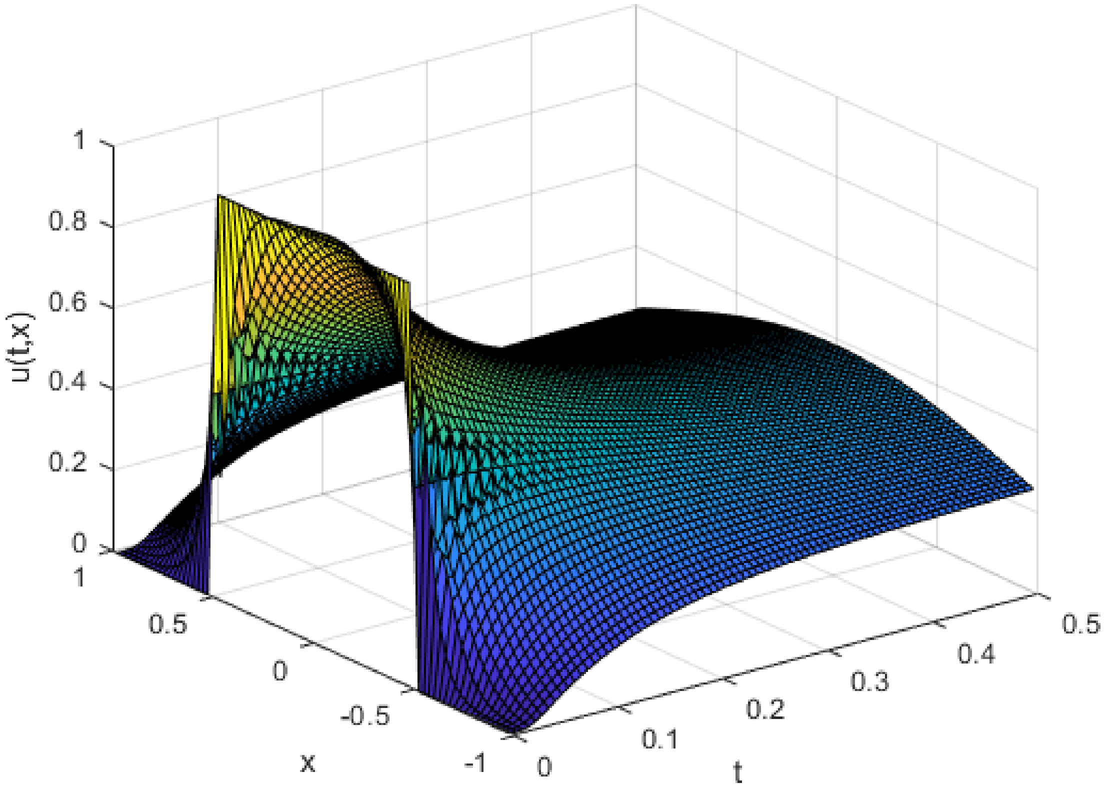


Supremum norm error (h = 1/40, λ = 1)
3
2.5
2
1.5
1
0.5
0
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
t
||u_{n+1} - u_n||_∞

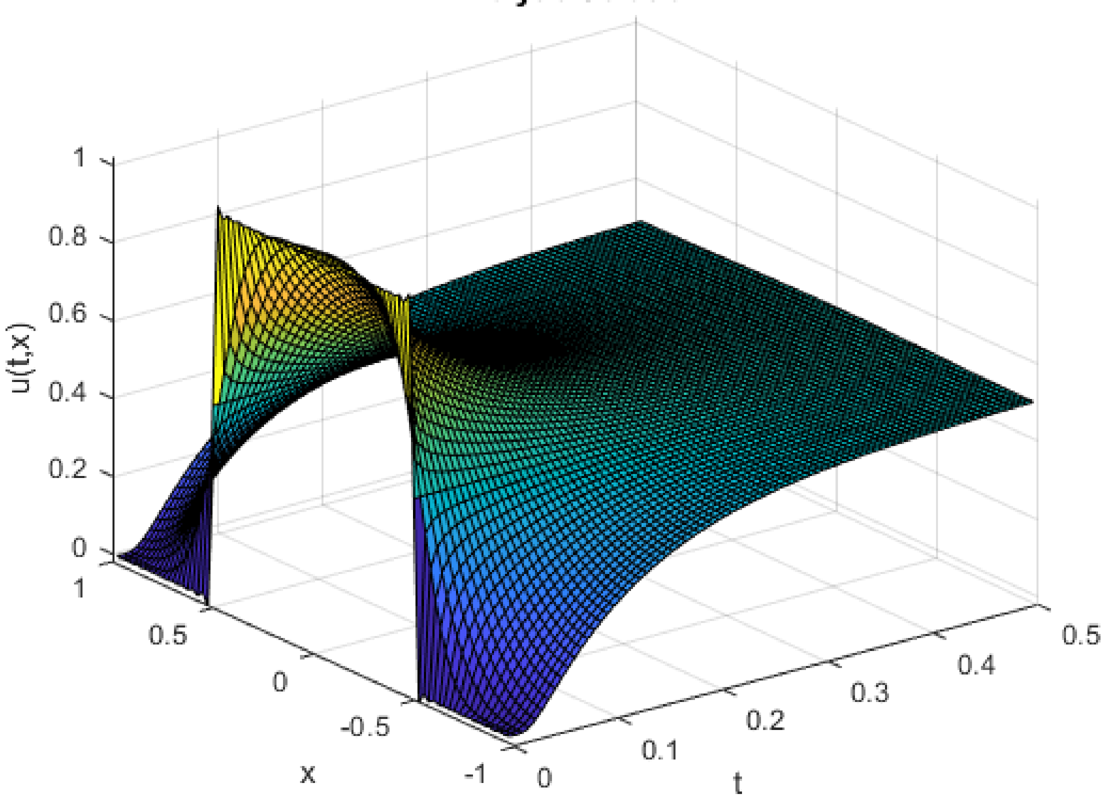


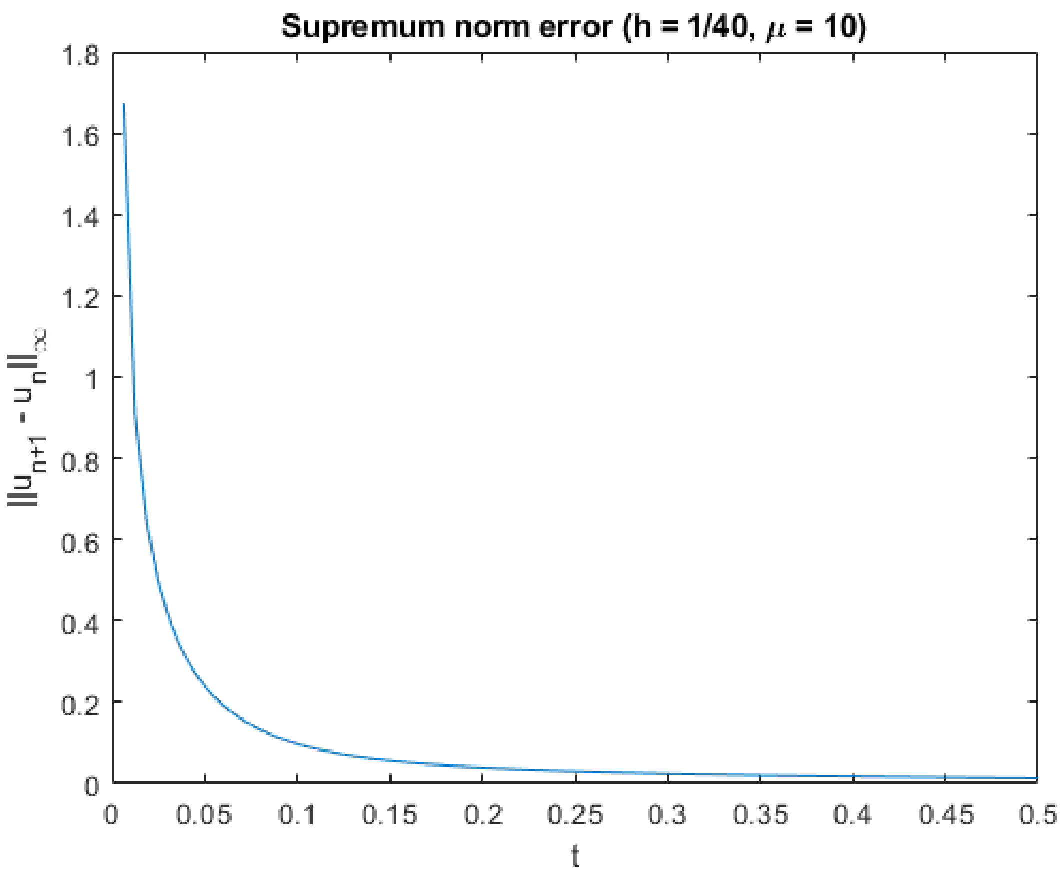
L2 norm error (h = 1/40, λ = 1)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
t
||u_{n+1} - u_n||_2

h = 1/40, μ = 10

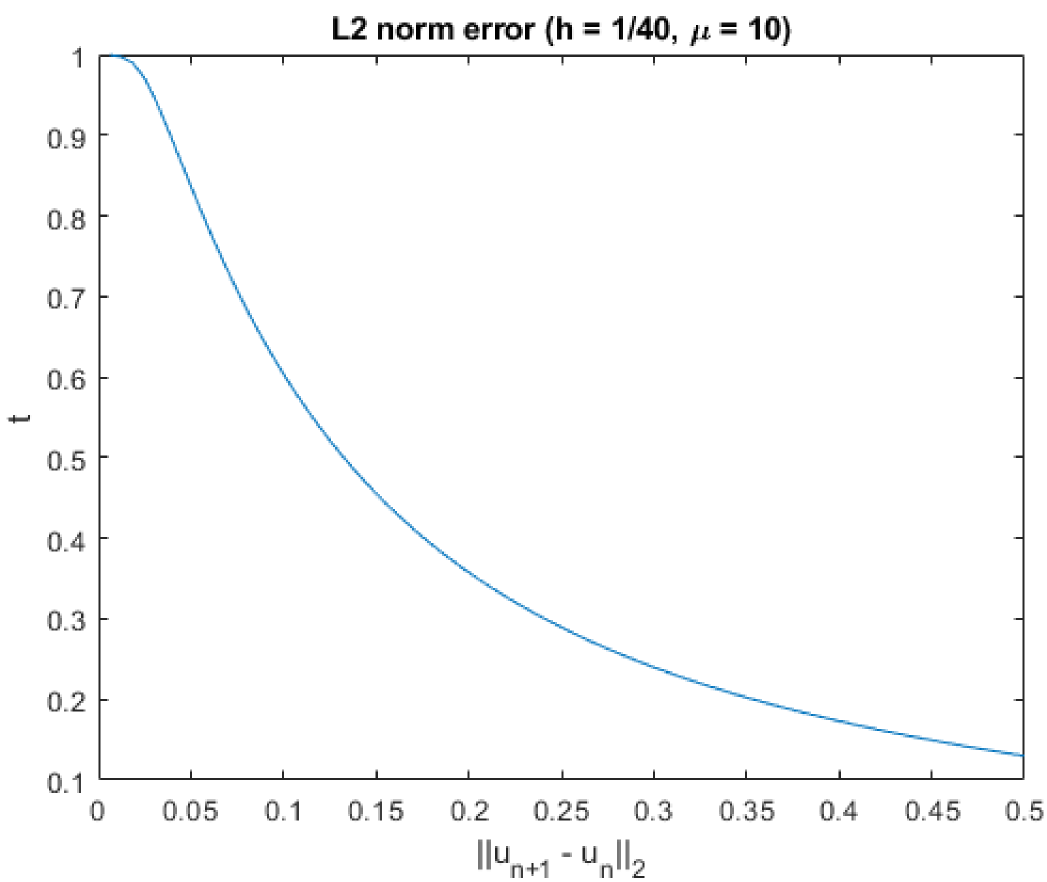


Analytic Solution





Supremum norm error (h = 1/40, μ = 10)
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
t
||u_{n+1} - u_n||_∞



L2 norm error (h = 1/40, μ = 10)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
||u_{n+1} - u_n||_2
t