Math 269B Instructor: Luminita Vese. Teaching Assistant: Michael Puthawala.
Homework #7 Due on: Friday, March 10, or the following Monday.

[**1**] Consider the equation $u_t+a(x)u_x = 0$ for $x \in [0,1]$, $t \in [0,T]$ with u and a periodic in x with period 1. Assume $a(x) \geq 0$ and is continuous for $x \in [0,1]$. Find the value for λ for which the scheme (forward-time, backward-space)

$$\frac{v_m^{n+1} - v_m^n}{k} = -a_m \frac{v_m^n - v_{m-1}^n}{h}$$

is stable in the infinity norm $\|\cdot\|_\infty$, under the condition $\frac{k}{h} = \lambda$. Here $a_m = a(x_m)$.

[**2**] Consider the equation $u_t + a(x)u_x = 0$ for $x \in [0,1]$, $t \in [0,T]$ with u and a periodic in x with period 1. Assume $a(x)$ is continuous for $x \in [0,1]$. Find the value for λ for which the upwind scheme

$$\frac{v_m^{n+1} - v_m^n}{k} = \begin{cases} -a_m \frac{v_m^n - v_{m-1}^n}{h} & \text{if } a_m \geq 0 \\ -a_m \frac{v_{m+1}^n - v_m^n}{h} & \text{if } a_m < 0 \end{cases}$$

is stable in the infinity norm $\|\cdot\|_\infty$, under the condition $\frac{k}{h} = \lambda$. Here $a_m = a(x_m)$.

[**3**] Give a derivation, based on the Lax-Wendroff idea, of a finite difference method to create approximate solutions of the differential equation

$$u_t + a(x)u_x = 0.$$

What is the leading term of the local truncation error for the scheme you derived ?

[**4**] Consider the modified Lax-Friedrichs scheme for the one-way wave equation $u_t + au_x = f(x,t)$,

$$u_j^{n+1} = \frac{1}{2}(u_{j+1}^n + u_{j-1}^n) - \frac{a\lambda}{1+(a\lambda)^2}(u_{j+1}^n - u_{j-1}^n) + \triangle t f_j^n.$$

(a) Analize the consistency of the scheme.

(b) Show that this explict scheme is stable for all values of λ. Discuss the relation of this explicit and unconditionally stable scheme with the Courant-Friedrichs-Lewy Theorem.

CFL Thm: There are no explicit, unconditionally stable, consistent finite difference schemes for hyperbolic systems of partial differential equations.

[**5**] Computational Exercise 6.3.10 from Strikwerda (page 156). You need a routine to solve the implicit equations (see Thomas Algorithm, Section 3.5).