# Homework 2

Geneva Porter
MATH-693B Numerical Partial Differential Equations

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## 1.3.2

Solve the system

$$u_t + \frac{1}{3}(t-2)u_x + \frac{2}{3}(t+1)w_x + \frac{1}{3}u = 0$$
$$w_t + \frac{1}{3}(t+1)u_x + \frac{1}{3}(2t-1)w_x - \frac{1}{3}w = 0$$

by the Lax-Fricdrichs scheme: i.e., each time derivative is approximated as it is for the scalar equation and the spatial derivatives are approximated by central differences. The initial values are

$$u(0,x) = \max(0, 1-|x|)$$
$$w(0,x) = \max(0, 1-2|x|)$$

Consider values of $x$ in [-3,3] and $t$ in $[0,2]$. Take $h$ equal to 1/20 and $\lambda$ equal to 1/2. At each boundary set $u = 0$, and set $w$ equal to the newly computed value one grid point in from the boundary. Describe the solution behavior for $t$ in the range $[1.5, 2]$. You may find it convenient to plot the solution. Solve the system in the form given; do not attempt to diagonalize it.

### Solution

The Lax-Friedrichs Scheme is given by:

$$u_t = \frac{u_m^{n+1} - \frac{1}{2}\left(u_{m+1}^n + u_{m-1}^n\right)}{k}, \qquad u_x = \frac{u_{m+1}^n - u_{m-1}^n}{2h}$$

We will use $k = 1/40$ and $h = 1/20$

## 3.2.4

Using the box scheme (3.2.3), solve the one-way wave equation

$$u_t + u_x = \sin(x - t)$$

on the interval [0,1] for $0 \leq t \leq 1.2$ with $u(0, x) = \sin x$ and with $u(t, 0) = -(1 + t)\sin t$ as the boundary condition.

Demonstrate the second-order accuracy of the solution using $\lambda = 1.2$ and $h = \frac{1}{10}, \frac{1}{20}, \frac{1}{40}$, and $\frac{1}{80}$. Measure the error in the $L^2$ norm (3.1.24) and the maximum norm. To implement the box scheme note that $v_0^{n+1}$ is given by the boundary data, and then each value of $v_{m+1}^{n+1}$ can be determined from $v_m^{n+1}$ and the other values.