Network Induced Multi-Neuronal Spike Train Pseudometric

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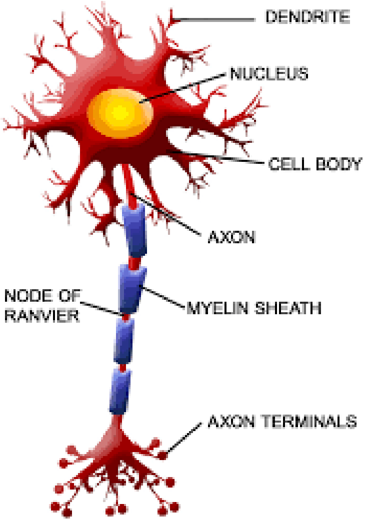
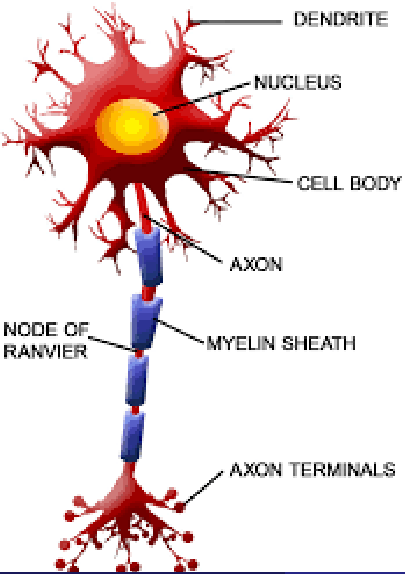
Introduction

- Looking at behavior of subnetworks in spiking neural networks under various learning algorithms.
- Current metrics for multiple neurons treat each neuron as independent of one another.
- Wanted to create a metric which would take the network structure into account.
- Looked then at signal propagation rather than signal generation.

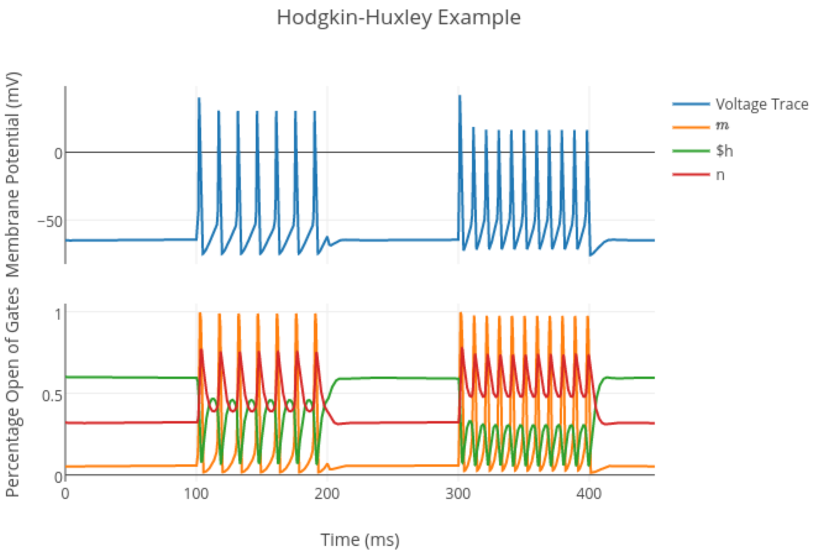
Definitions



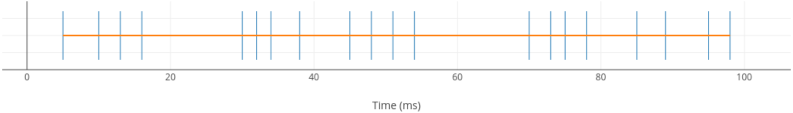
Definitions



- Receives input generally through the dendrites.
- Soma (Cell Body) integrates incoming signals and then potentially generates action potential, or *spike*.
- Spike travels down axon, activating synapses on the axon terminals as it goes.
- Action potential is a short sharp increase in membrane potential, followed by a fast decrease.


Hodgkin-Huxley Example
Membrane Potential (mV)
Percentage Open of Gates
Time (ms)
Voltage Trace
m
$h
n
0
−50
1
0.5
0
0
100
200
300
400

Spike Train



A *spike train* is a listing of times that one particular neuron has fired, either during an experiment or a numerical simulation. We will denote a spike train by S and numerate the spike times in S by t_i^f.

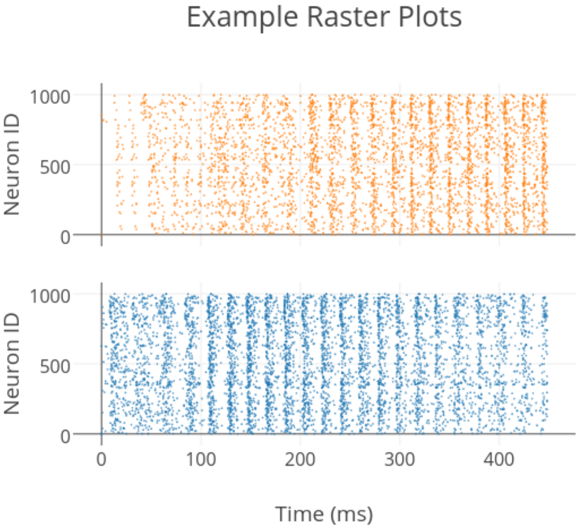
Network of Neurons

Let $N = (V, E, \omega, \Delta)$ be a directed graph where the vertices represent neurons and with two weighing functions,

- $\omega : E \to [w_{I\,\max}, w_{E\,\max}]$, defines the synaptic weight of the connection from one neuron to the next, and
- $\Delta : E \to (0, \Delta_{\max}]$, defines the transmission delay from one neuron to the next.

As we simulate the network of neurons each neuron generates a spike train, and we denote the vector of spike trains, or *spike trainyard*, as η.

Raster Plots



Existing Metrics

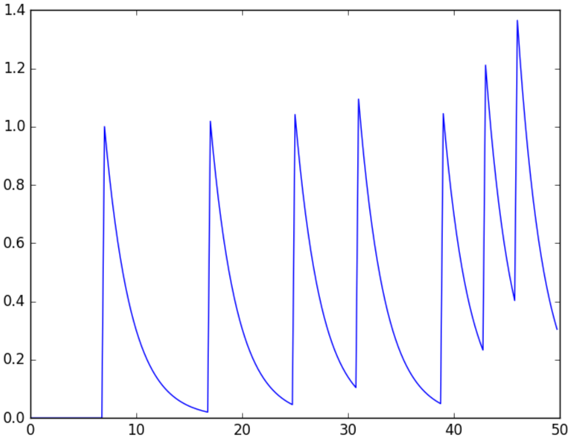
Let S be a spike train, then we define the profile operator,

$$F[S] = \sum_{t^f \in S} H(t - t^f) \exp\left[-(t - t^f)/\delta_f\right] \tag{1}$$

then the distance between spike trains is the L^2 distance between both the operator acting on both spike trains.

$$d(S_1, S_2) = \left[\frac{1}{\tau_f} \int_X (F[S_1] - F[S_2])^2 \, dt\right]^{1/2} \tag{2}$$

Profile Operator



Population Extension

The metric can be extended out to a population of neurons as follows,

$$d(\eta_1, \eta_2) = \left[\sum_{i=1}^{N} \left(d(\eta_1^i, \eta_2^i) \right)^2 \right]^{1/2} \tag{3}$$

, This is a special case of the Population van Rossum metric for multiple spike trains.

Where From Here?

- We see that the previous metric looks at the generation of spikes,
- Population van Rossum treats each neuron as independent, which is necessary due to potentially insufficient data in biological experiments.
- For synthetic networks, where all of the data can be known, can we construct a metric that incorporates the network structure in the calculation?

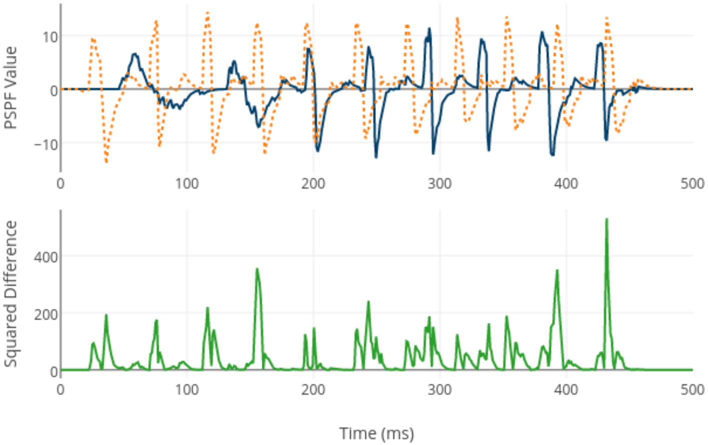
Post-Synaptic Potential Function (PSPF)

Let $N = (V, E, \omega, \Delta)$ be a network, F be a profile operator, and η be a spike train vector. Let $\Delta_M = \max\{\Delta(e) | e \in E\}$, where $\Delta_M \ll T$. Then for $t \in [\Delta_M, T]$ the PSPF for neuron j is given by

$$p_{N,F,j}(\eta, t) = \frac{1}{w_{\max}} \sum_{i=1}^{|V|} w_{i,j} F[\eta_i(t - \Delta_{i,j})] \tag{4}$$

We see that this function is measuring the input into neuron j.

PSPFs and Squared Difference



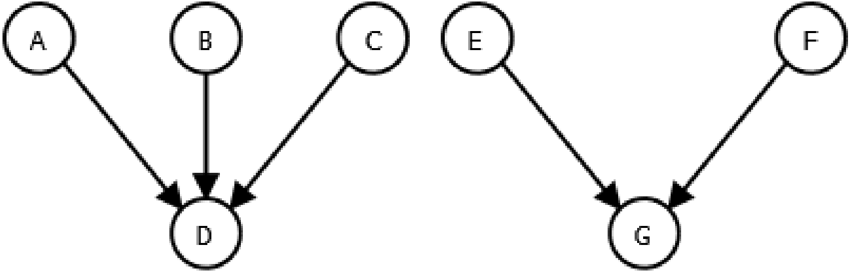
Network Induced Spike Train yard Pseudometric (NISTy)

Let $N = (V, E, \omega, \Delta)$ and η^1, η^2 be two spike train vectors of N on the interval $[0, T]$, F be a profile operator, τ_F be a time constant, and $\Delta_M \ll T$, then NISTy $P_{N,F} :_N \times_N \rightarrow^{|V|} \rightarrow [0, \infty]$ is defined by

$$P_{N,F}(\eta^1, \eta^2) = \left[\frac{1}{\tau_f} \sum_{j=1}^{|V|} \left\| p^1_{N,F,j} - p^2_{N,F,j} \right\|_2^2 \right]^{1/2} \tag{5}$$

where the norm is taken over the time interval $[\Delta_M, T]$.

Two Examples of Pseudometric



(a) Simulation One.　　(b) Simulation Two.

Figure: Network configurations for (a) simulation one, and (b) simulation two.

Example One

For the network defined by Figure 2a let all edges have the same weight.

We set the delays as follows, $\Delta_{D,A} = 5, \Delta_{D,B} = 10, \Delta_{D,C} = 15$.

Let η^1 be a vector of spike trains where A fires at time $t = 15$ and B fires at time $t = 10$, η^2 have firing times for B at $t = 10$ and C at $t = 5$.

Due to the equal weights and the temporal delays we get that $P_{N,F}(\eta^1, \eta^2) = 0$, while population van Rossum would show these as different vectors of spike trains. This is significant as NISTy can be used to successfully identify that output neuron D will behave identically under both η_1 and η_2.

Example Two

For the network defined by Figure 2b let all edges have the same temporal delay of $\Delta = 5$, with $\omega_{G,E} = w > 0$ and $\omega_{G,F} = -w$.

For η^1 let E fire at $t = 5$ and F fire at $t = 5$, and let η^2 be an vector of empty spike trains, then $P_{N,F}(\eta^1, \eta^2) = 0$.

Once again, NISTy can be used to identify identical behavior on an output neuron.

NISTy vs Population van Rossum

- We showed that NISTy displays different behavior from Population van Rossum.
- Due to investigating signal propagation rather than generation we see cases where different spike train vectors lead to equilvalent inputs into neurons.
- How does the behavior of NISTy compare to Population van Rossum in general?

Comparison with Existing Methods

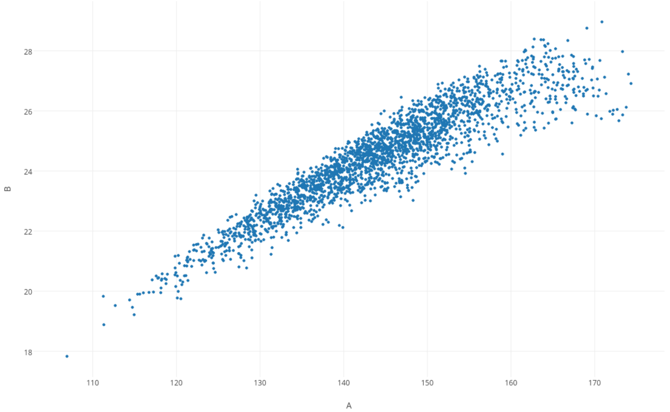


Figure: Horizontal: NISTy, Vertical: Population van Rossum

Conclusions

- Investigates signal propagation instead of generation,
- Suitable for synthetic systems as opposed to biological,
- With proper coding and foreknowledge doesn't significantly increase the computation time of simulations.

Future Work and Applications

- Relationship Between Metric Spaces,
- Applications to studying learning algorithms, and
- Better understanding of the behavior of subnetworks.
- For subnetworks use NISTy for inputs and Population van Rossum for generation of activity.

Thank You.