FOUNDATION OF SENSOR SIGNAL PROCESSING (II)

FEATURE EXTRACTION IN TIME-FREQUENCY DOMAIN

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Agenda

- Feature extraction in time-frequency domain for signal processing
- *<Morning Break>*
- Feature extraction in time-frequency domain for signal processing (cont’d)
- *<Lunch Break>*
- Statistical signal processing
- *<Afternoon Break>*
- Workshop on feature extraction for signal processing

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Module objective

Module: Time-frequency feature extraction for signal processing

Knowledge and understanding

- Understand the fundamentals of time-frequency domain signal representation, transformation, feature extraction, such as Fourier transformation and wavelet transformation

Key skills

- Design, build, implement and evaluate time-frequency feature extraction methods for signal processing

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Major reference

- [Introduction] Steven W. Smith, ***The Scientist and Engineer's Guide to Digital Signal Processing***, available at http://www.dspguide.com
- [Practical] J. Unpingco, ***Python for Signal Processing: Featuring IPython Notebooks***, 2014, https://github.com/unpingco/Python-for-Signal-Processing
- [Practical] A. B. Downey, ***Think DSP: Digital Signal Processing in Python***, https://github.com/AllenDowney/ThinkDSP

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Topics

- Introduction to feature extraction in time-frequency domain of sensor signal
- Fourier transformation
- Wavelet transformation

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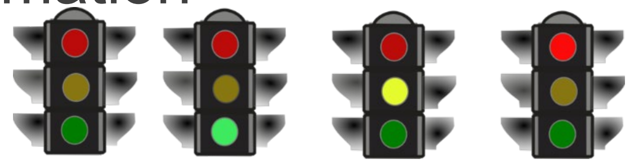
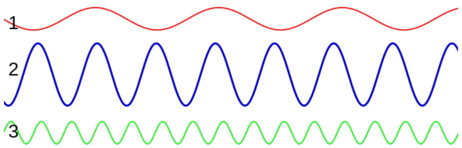
Recap: Signal

- A mechanism for conveying information
  - Gestures, traffic lights..



- Electrical engineering: Currents, voltages



- Digital signals: Ordered collections of numbers that convey information, about a real world phenomenon, such as sounds, images

Source:
1. http://www.publicdomainfiles.com/show_file.php?id=13945761016913
2. https://commons.wikimedia.org/wiki/File:CPT-sound-pitchvolume.svg
3. https://commons.wikimedia.org/wiki/File:A)_Imagen_de_Lenna_en_escala_de_grises;_b)_Imagen_de_Lenna_con_el_filtro_de_Gauss_aplicado..jpg

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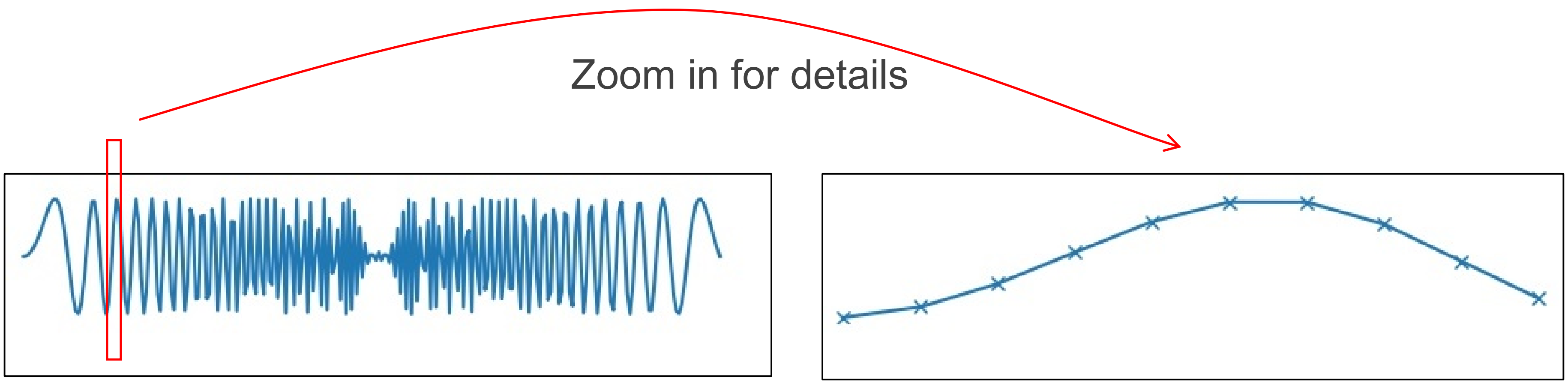
Signal: Audio

- A sequence of numbers
  - The *order* in which the numbers occur is important
  - Represent a perceivable sound



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Signal: Image

- A rectangular arrangement (matrix) of numbers
  - sets of numbers (for color images)
- Each pixel represents a visual representation of one of these numbers
  - E.g., 0 is minimum / black, 1 is maximum / white
  - Position / order is important

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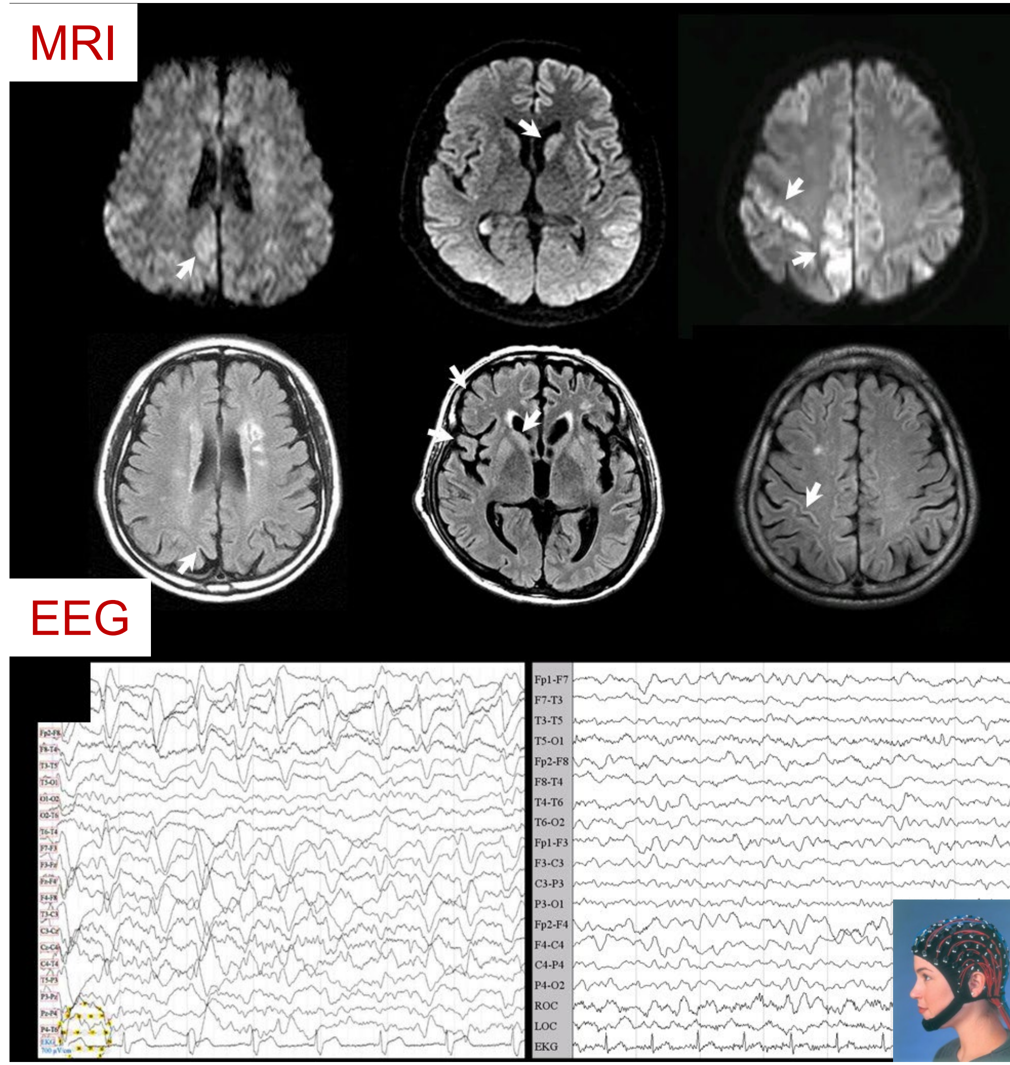
Signal: Biosignal

- MRI: Image diagnosis
- EEG: Many channels of brain electrical activity
- ECG: Cardiac activity
- OCT, Ultrasound: Echo-based imaging



Reference: https://commons.wikimedia.org/wiki/File:CJD_profiles_of_MRI_and_EEG_from_probable_CJD_patient.jpg

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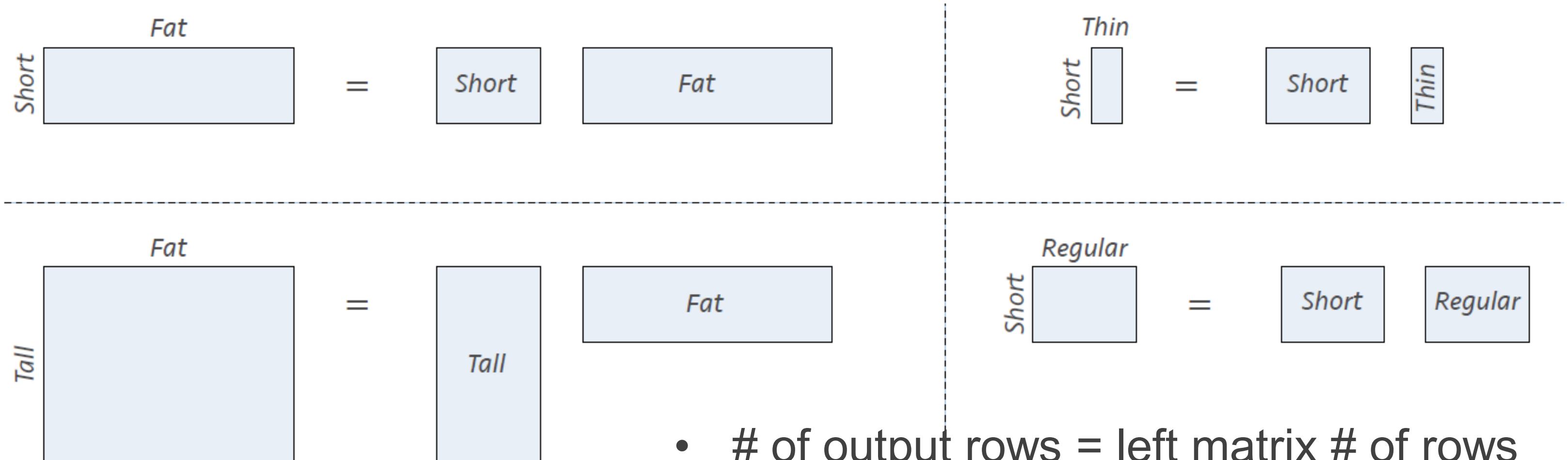
Warm-up: How do we look at signal

- 1D signal (e.g. sound) will be vector
- 2D signal (e.g. image) will be matrix



- # of output rows = left matrix # of rows
- # of output columns = right matrix # of columns

$$\begin{bmatrix} \$3 & \$4 & \$2 \end{bmatrix} \times \begin{bmatrix} 13 & 9 & 7 & 15 \\ 8 & 7 & 4 & 6 \\ 6 & 4 & 0 & 3 \end{bmatrix} = \begin{bmatrix} \$83 & \$63 & \$37 & \$75 \end{bmatrix}$$

$$\$3 \times 13 + \$4 \times 8 + \$2 \times 6$$

Source: https://www.mathsisfun.com/algebra/matrix-multiplying.html

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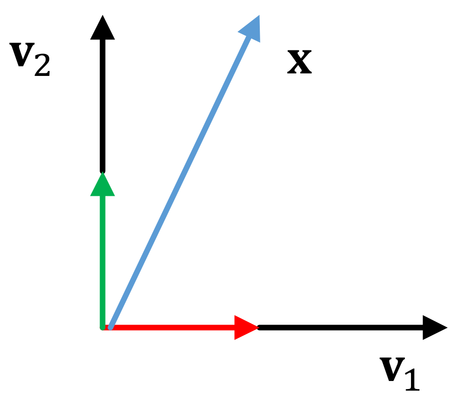
Warm-up: Basis vectors

- A given vector value is represented with respect to a *coordinate system*.
- A coordinate system is defined by a set of linearly independent vectors forming the system *basis*.
- Any vector value is represented as a linear sum of the basis vectors.



Signal	$\mathbf{x} = (1, 2)$
Basis vectors	$\mathbf{v}_1 = (1, 0), \mathbf{v}_2 = (0, 1)$
Signal representation coefficients	$\mathbf{w} = (1, 2)$
Justification	$\mathbf{x} = 1 \times (1, 0) + 2 \times (0, 1)$

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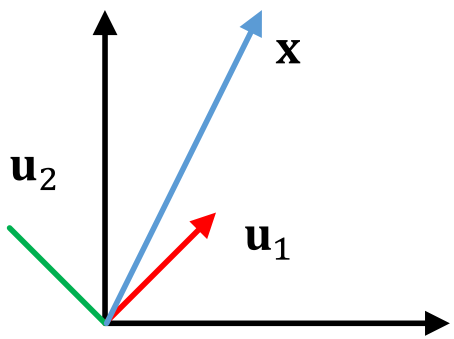
Warm-up: Change of Basis vectors

- **Question**: Given a vector $\mathbf{x}$, represented in an orthonormal basis vectors $\mathbf{v}_1, \mathbf{v}_2$, what is the representation of $\mathbf{x}$ in a different orthonormal basis vectors $\mathbf{u}_1, \mathbf{u}_2$?



Signal	$\mathbf{x} = (1, 2)$
Basis vectors	$\mathbf{u}_1 = \left(\sqrt{2}/_2, \sqrt{2}/_2\right), \mathbf{u}_2 = \left(-\sqrt{2}/_2, \sqrt{2}/_2\right)$
Signal representation coefficients	$\mathbf{w} = \left(3\sqrt{2}/_2, \sqrt{2}/_2\right)$
Justification	$\mathbf{x} = 3\sqrt{2}/_2 \times \left(\sqrt{2}/_2, \sqrt{2}/_2\right) + \sqrt{2}/_2 \times \left(-\sqrt{2}/_2, \sqrt{2}/_2\right)$

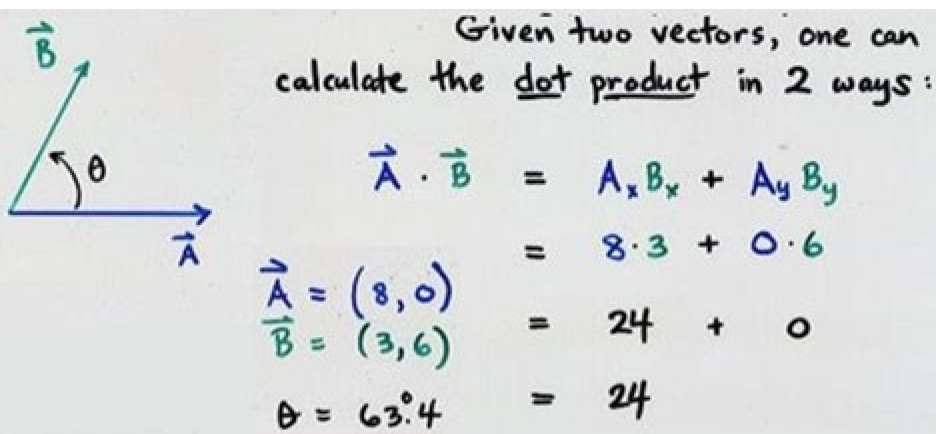


Photo: http://spiff.rit.edu/classes/phys311.old/lectures/dot/dot.html

$$w_i = \langle \mathbf{x}, \mathbf{u}_i \rangle = \mathbf{x}^T \mathbf{u}_i = \sum_j x(j) u_i(j)$$

where <·> is the dot product of two vectors

$$\mathbf{x} = \sum_i w_i \times \mathbf{u}_i$$

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Topics

- Introduction to feature extraction in time-frequency domain of sensor signal
- Fourier transformation
- Wavelet transformation

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Time-frequency analysis of signal

One of top 10 algorithms in 20^{th} century!

1. Metropolis algorithm for Monte Carlo
2. Simplex method for linear programming
3. Krylov subspace iteration
4. Decomposition approach to matrix computation (Singular value)
5. The Fortran compiler
6. QR algorithm for eigenvalues
7. Quick sort
8. Fast Fourier transform
9. Integer relation detection
10. Fast multipole

Source: https://www.computer.org/csdl/mags/cs/2000/01/c1022.html

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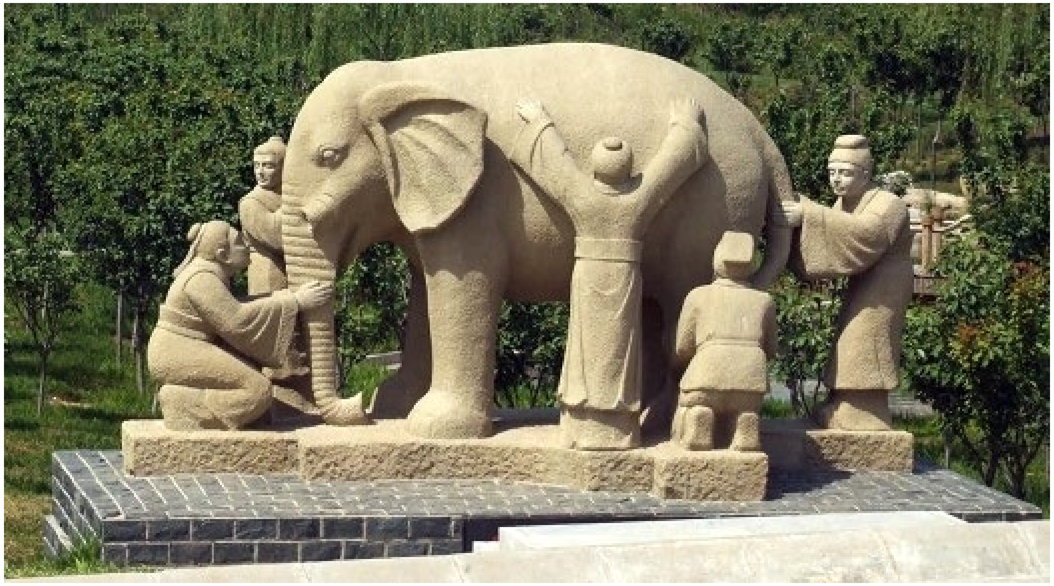
Signal representation

- Aim: Find a function as a weighted summation of basis functions $\mathbf{x} = \sum_i w_i \mathbf{u}_i$
- What is a good set of basis functions?
  - Signal transformation
- How to determine the weights?
  - Signal decomposition



Intuition: Describe this image so that a listener can visualize what you are describing.

- Pixel-based descriptions are uninformative
- Content-based descriptions are infeasible in the general case

Source: https://cellcode.us/quotes/and-parable-blind-men-elephant.html

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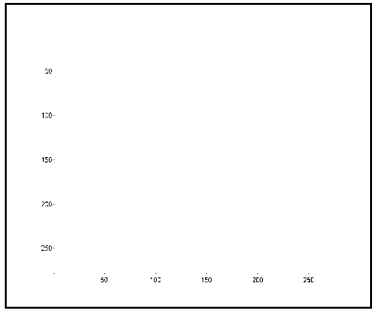
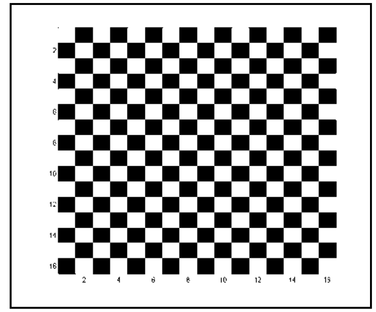
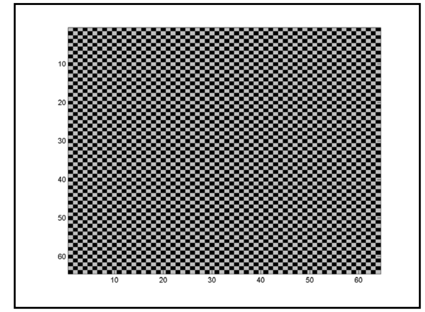
Image: Checkerboard basis



$\mathbf{u}_1$

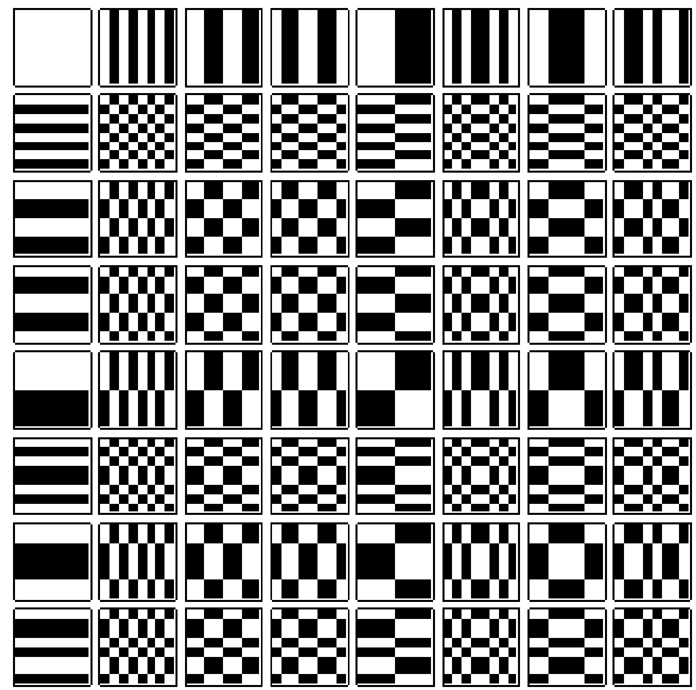


$\mathbf{u}_2$



$$\text{Image} \approx w_1\mathbf{u}_1 + w_2\mathbf{u}_2$$

- Images have some fast varying regions.
  - A first picture with constant color.
  - A second picture that has very fast changes
- How about more checkerboard?



Reference: Hamamard basis image, https://en.wikipedia.org/wiki/Hadamard_transform

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Image: Checkerboard basis

Signal at standard basis: $\mathbf{x} = \begin{bmatrix} 2 & 1 \\ 6 & 1 \end{bmatrix}$

$$\begin{bmatrix} 2 & 1 \\ 6 & 1 \end{bmatrix} = 2 \times \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + 1 \times \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + 6 \times \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + 1 \times \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

New basis

$$\mathbf{u}_1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} / 2 \qquad \mathbf{u}_2 = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} / 2$$

$$\mathbf{u}_3 = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} / 2 \qquad \mathbf{u}_4 = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} / 2$$

Signal at new basis: $\mathbf{x} = \begin{bmatrix} 5 & -2 \\ 2 & -3 \end{bmatrix}$

Recall the formula

$$w_i = \langle \mathbf{x}, \mathbf{u}_i \rangle = \mathbf{x}^T \mathbf{u}_i = \sum_j x(j) u_i(j)$$

where <·> is the dot product of two vectors

$$\mathbf{x} = \sum_i w_i \times \mathbf{u}_i$$

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Summary: Signal representation

- Identify a set of standard structures, such as checkerboards, we will call these "*basis*".
- Express data as a weighted combination of basis $\mathbf{x} = \sum_i w_i \times \mathbf{u}_i$
- Chose weights $\{w_i\}$ for the best representation of $\mathbf{x}$
- The error between $\mathbf{x}$ and $\sum_i w_i \times \mathbf{u}_i$ is minimized.
- The weights $\{w_i\}$ fully specify the data, since the bases are known beforehand, knowing the weights is sufficient to reconstruct the data

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Basis requirements

- Non-redundancy
  - Each basis must represent information *not* already represented by other basis
  - Mathematically, bases must be orthogonal
    - $\langle \mathbf{u}_i, \mathbf{u}_j \rangle = 0, \text{for } i \neq j$
- Compactness
  - Must be able to represent most of the signal with fewest basis
  - For D-dimensional data, need no more than D basis

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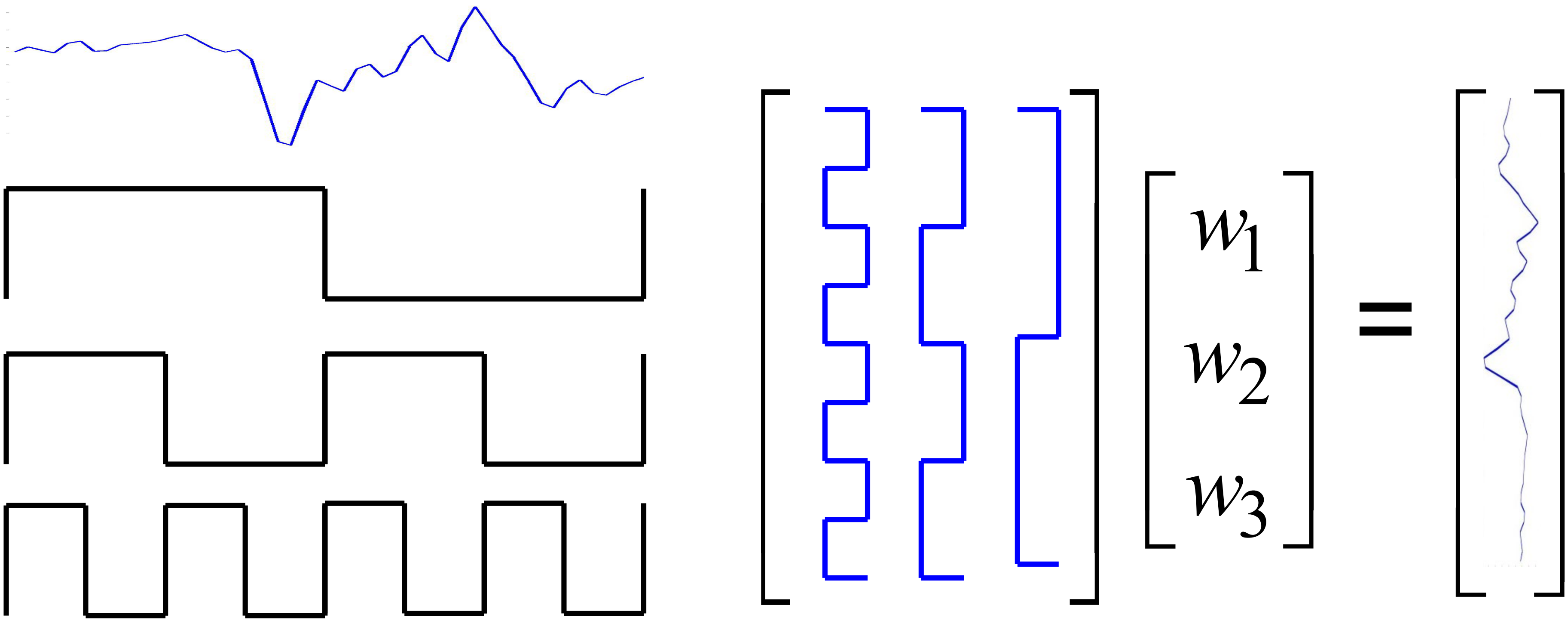
Sound: Wave basis



- Square wave equivalents of checker boards

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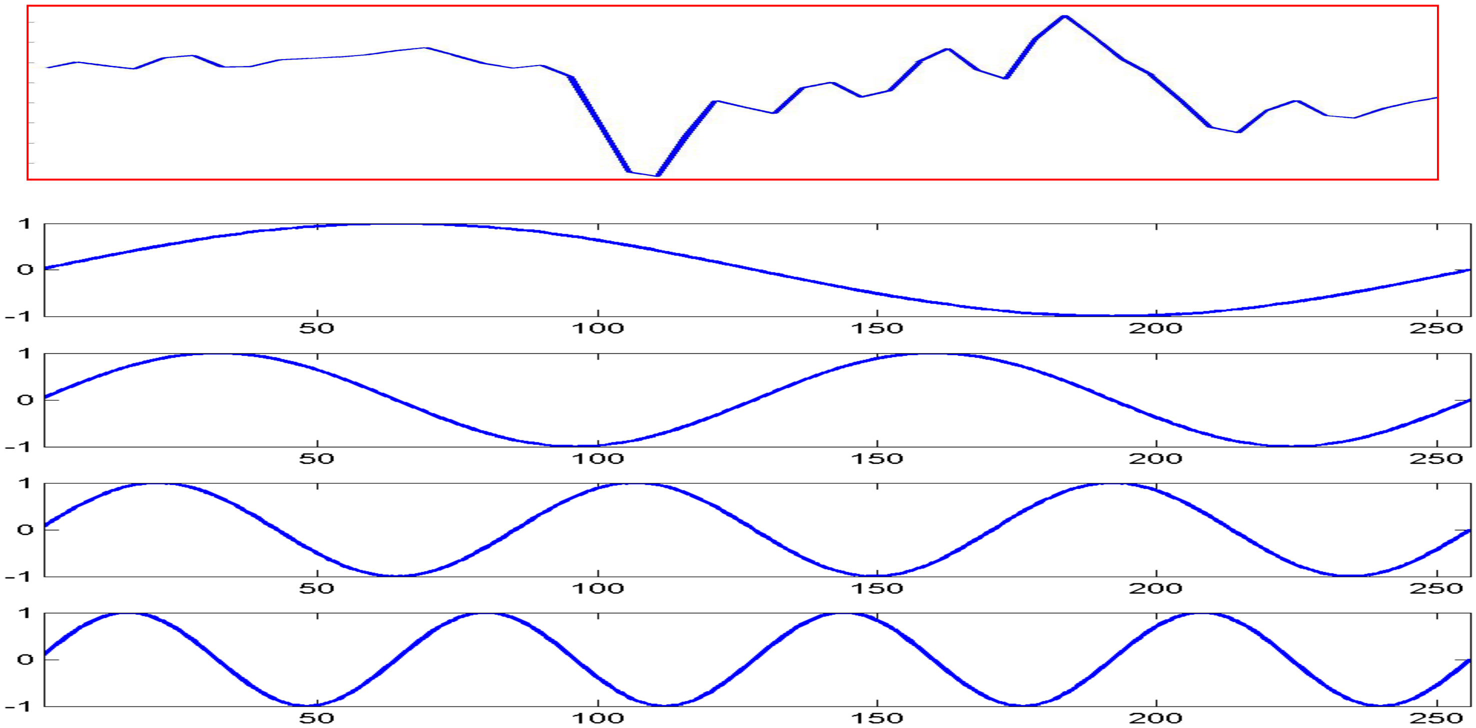
Sinusoids basis



- They are orthogonal
- They can represent rounded shapes nicely
    - Unfortunately, they cannot represent sharp corners

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Sine and Cosine functions

- Periodic functions
- General form of sine and cosine functions:

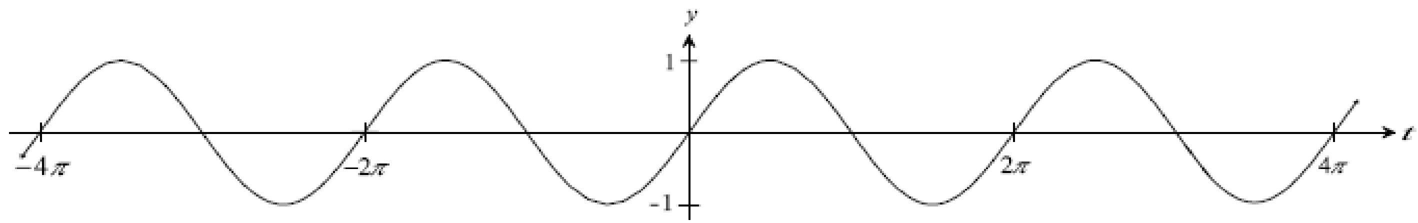
$|A|$ amplitude

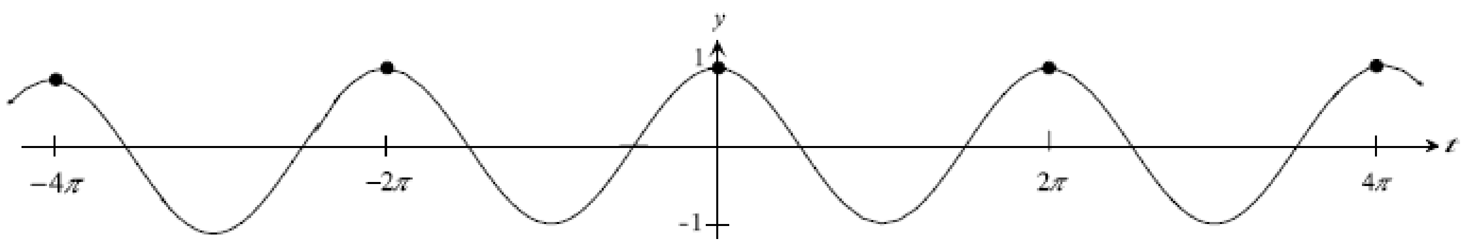
$\dfrac{2\pi}{|\alpha|}$ period

b phase shift

$$y(t) = A\sin(\alpha t + b) \qquad y(t) = A\cos(\alpha t + b)$$

<u>Example</u>: $A = 1, b = 0, \alpha = 1$ period $= 2\pi$





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Fourier analysis

Decompose a signal into sinusoids of different frequencies, by transforming the view of the signal from time domain to frequency domain.

$$\text{Forward DFT: } F(u) = \sum_{x=0}^{N-1} f(x) e^{\frac{-j2\pi ux}{N}}, \text{where } u = 0,1,\cdots,N-1$$

$$\text{Inverse DFT: } f(x) = \frac{1}{N} \sum_{u=0}^{N-1} F(u) e^{\frac{j2\pi ux}{N}}, \text{where } x = 0,1,\cdots,N-1$$

Example

- Signal

$f(x) = [2, 3, 4, 4]$

- Fourier coefficients of signal

$\mathrm{F}(u) = [13, (-2+i), -1, (-2-i)]$

where i is the imaginary unit

$$F(0) = \sum_{x=0}^{3} f(x) e^{\frac{-j2\pi 0x}{4}} = 2+3+4+4 = 13$$

$$F(1) = \sum_{x=0}^{3} f(x) e^{\frac{-j2\pi x}{4}} = 2e^{0} + 3e^{-i\pi/2} + 4e^{-i\pi} + 4e^{-i3\pi/2} = -2+i$$

$$F(2) = \sum_{x=0}^{3} f(x) e^{\frac{-j4\pi x}{4}} = 2e^{0} + 3e^{-i\pi} + 4e^{-i2\pi} + 4e^{-i3\pi} = -1$$

$$F(3) = \sum_{x=0}^{3} f(x) e^{\frac{-j6\pi x}{4}} = 2e^{0} + 3e^{-i3\pi/2} + 4e^{-i3\pi} + 4e^{-i9\pi/2} = -2-i$$

Demo: http://www.falstad.com/fourier/

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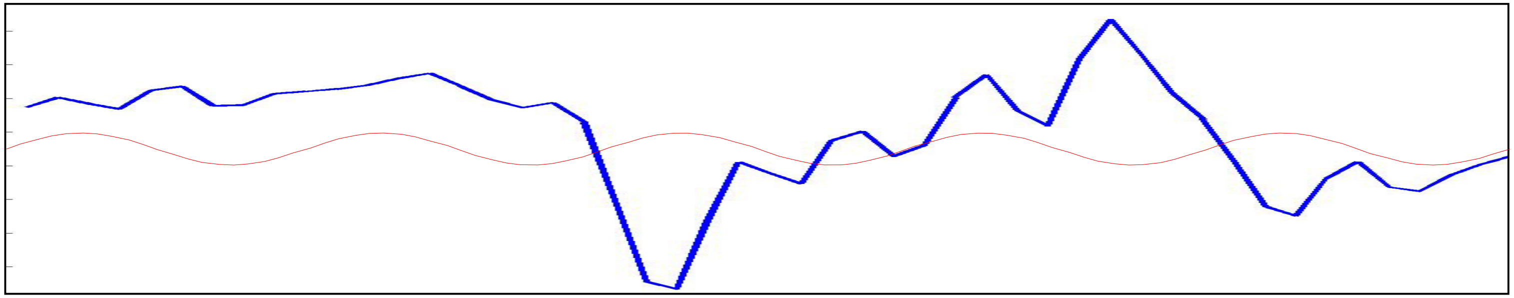
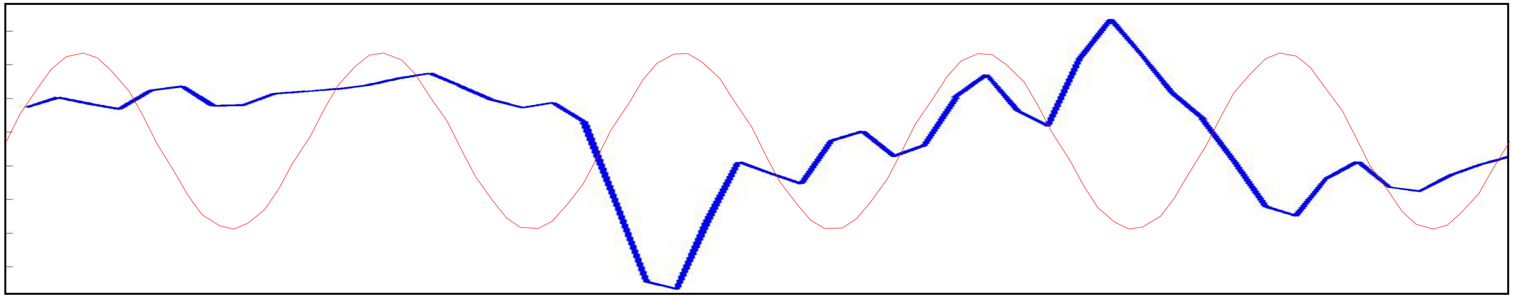
Composition as optimization





- Idea 1: Each sinusoid's amplitude is <u>adjusted</u> until it gives the smallest error. The amplitude is the weight of the sinusoid.

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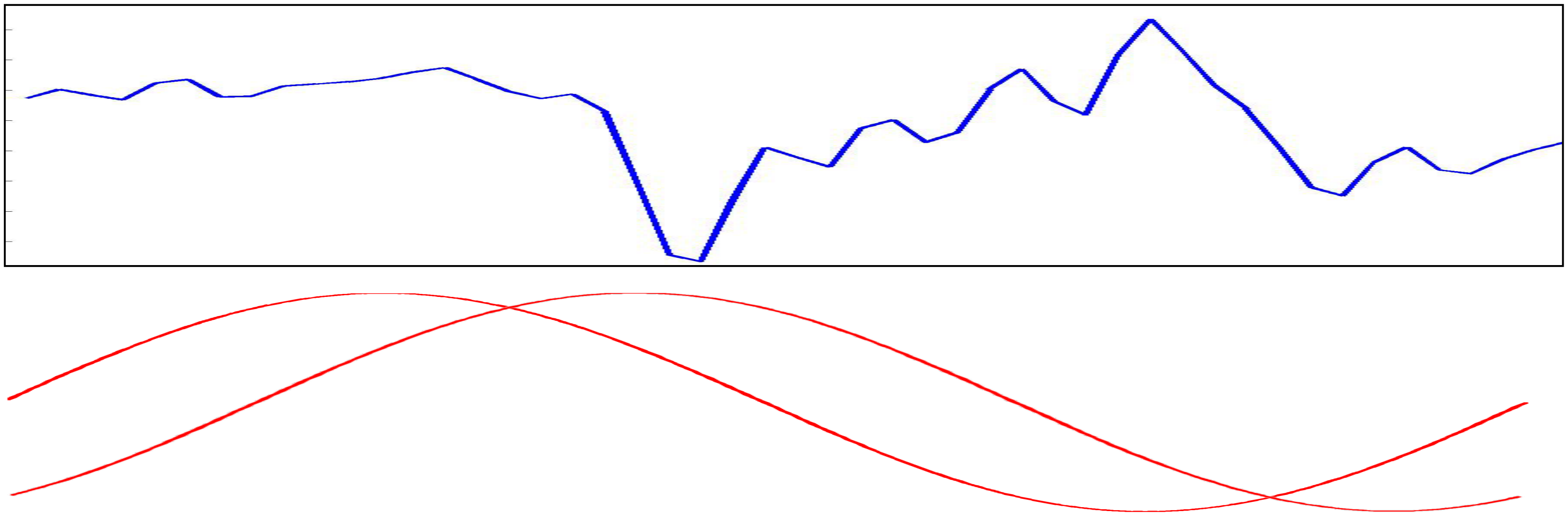
Composition as optimization



- Idea 2: Move the sinusoid left/right, and at each shift, try all amplitudes. Find the combination of amplitude and phase that results in the smallest error.

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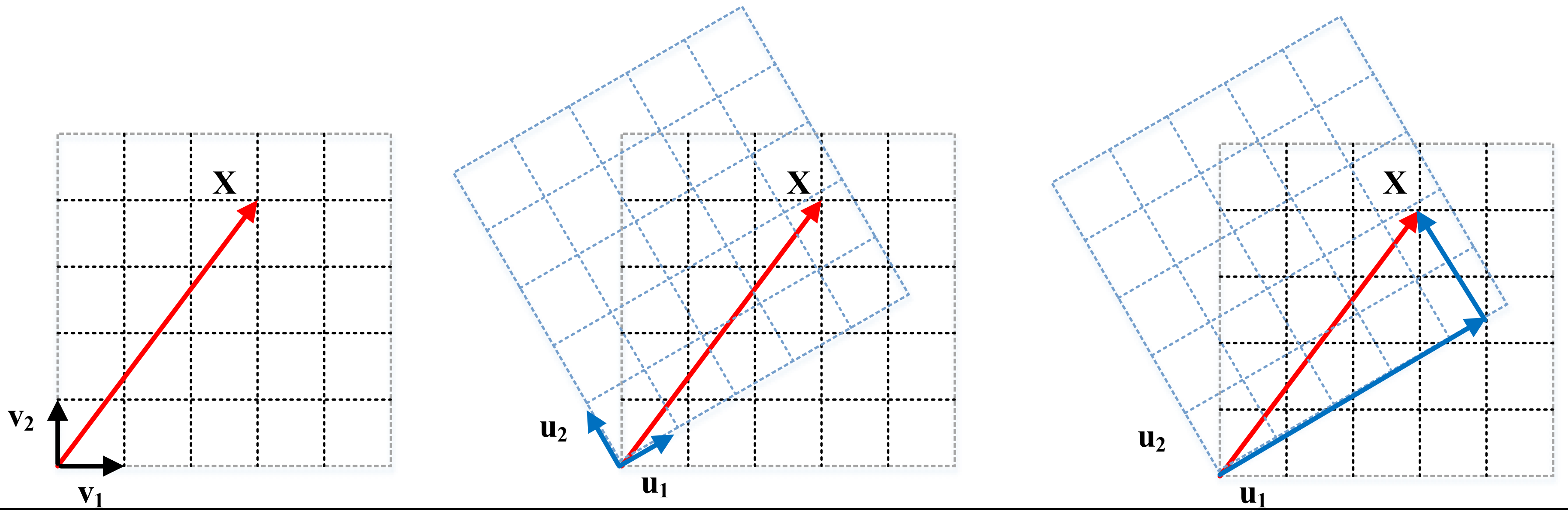
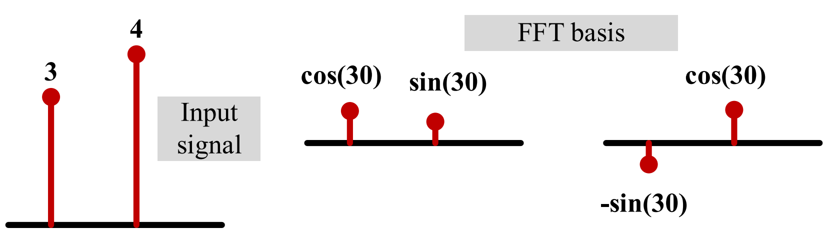
Composition as projection



Signal	$\mathbf{x} = (3,4)$
Basis vectors	$\mathbf{u}_1 = (\cos(30), \sin(30)) = \left(\sqrt{3}/_2, {}^1/_2\right), \mathbf{u}_2 = (-\sin(30),\ \cos(30)) = \left(-{}^1/_2, \sqrt{3}/_2\right)$
Signal representation coefficients	$\mathbf{w} = \left({}^{3\sqrt{3}}/_2 + 2, 2\sqrt{3} - {}^3/_2\right)$
Justification	$\mathbf{x} = \left({}^{3\sqrt{3}}/_2 + 2\right) \times \left(\sqrt{3}/_2, {}^1/_2\right) + \left(2\sqrt{3} - {}^3/_2\right) \times \left(-{}^1/_2, \sqrt{3}/_2\right)$



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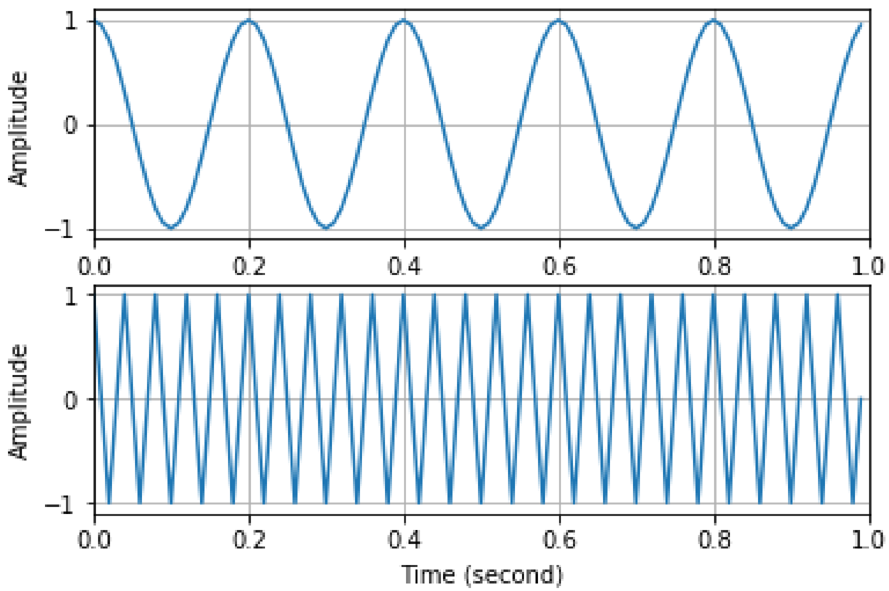
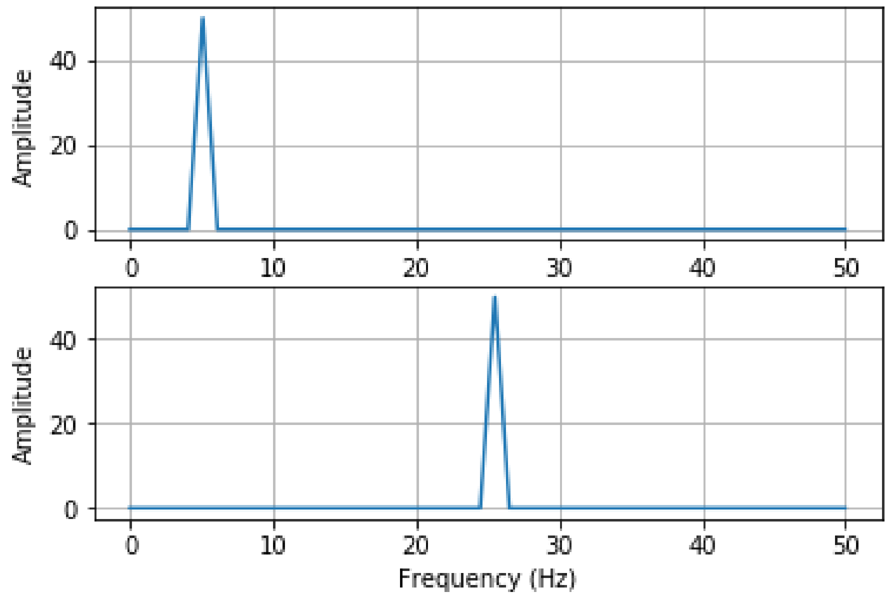
Fourier transform

$$f(t) = \cos(2\pi \cdot 5 \cdot t)$$

Original signal in time domain



Fourier domain



$$f(t) = \cos(2\pi \cdot 25 \cdot t)$$

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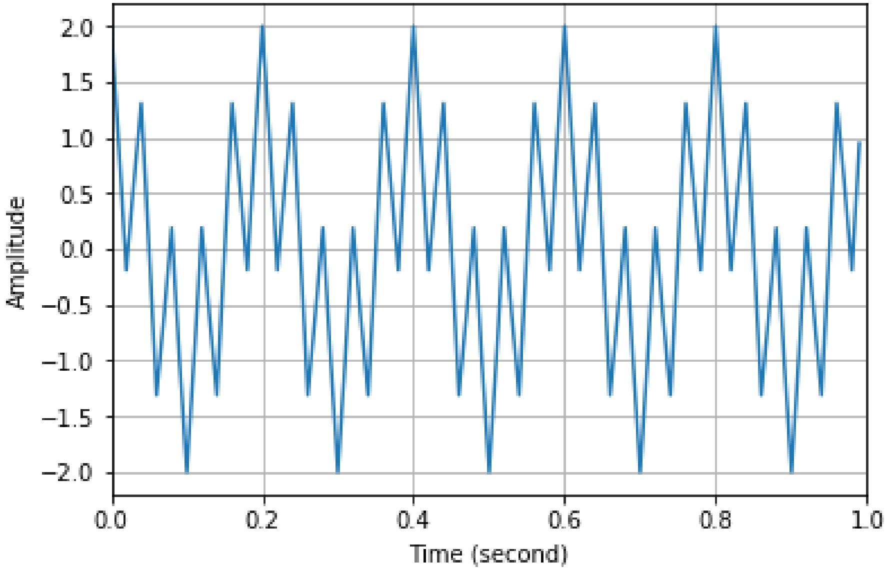
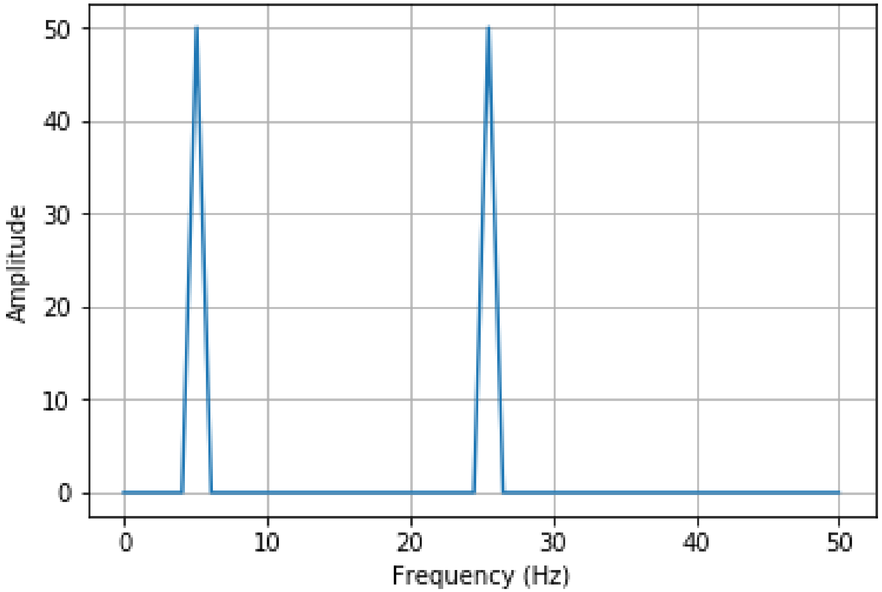
Fourier transform

Challenge 1: Cannot not provide simultaneous time and frequency localization. Provides excellent localization in the frequency domain but poor localization in the time domain.





$$f(t) = \cos(2\pi \cdot 5 \cdot t) + \cos(2\pi \cdot 25 \cdot t)$$

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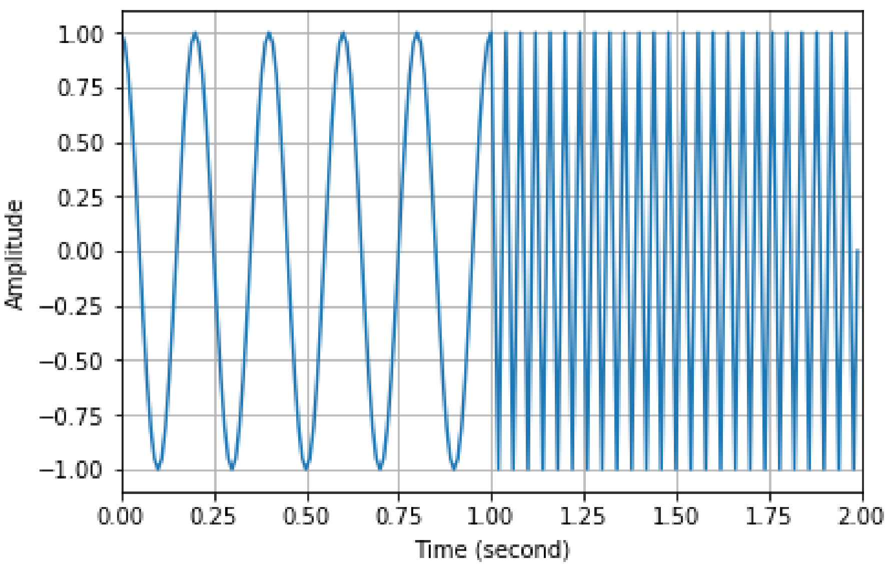
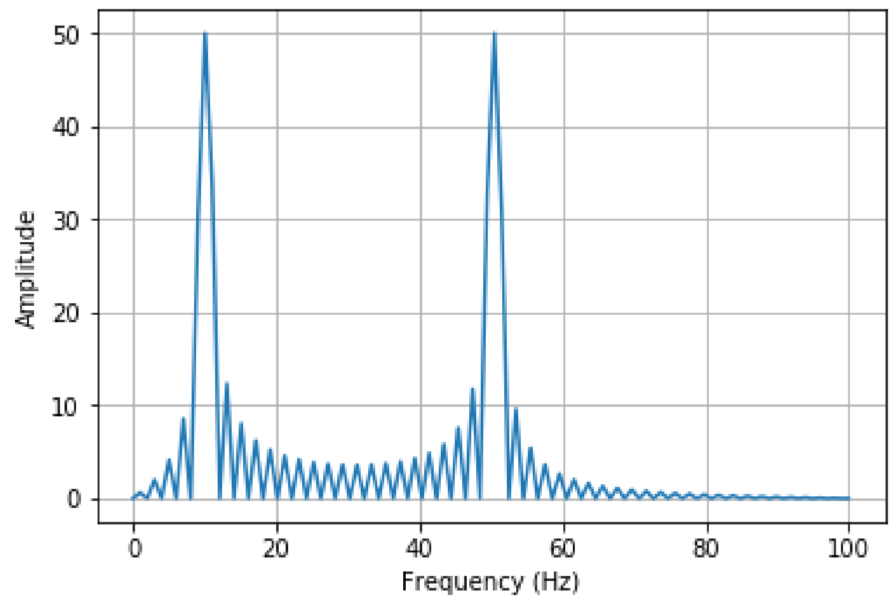
Fourier transform

Challenge 2: Has knowledge of what frequencies exist, but no information about where these frequencies are located in time!





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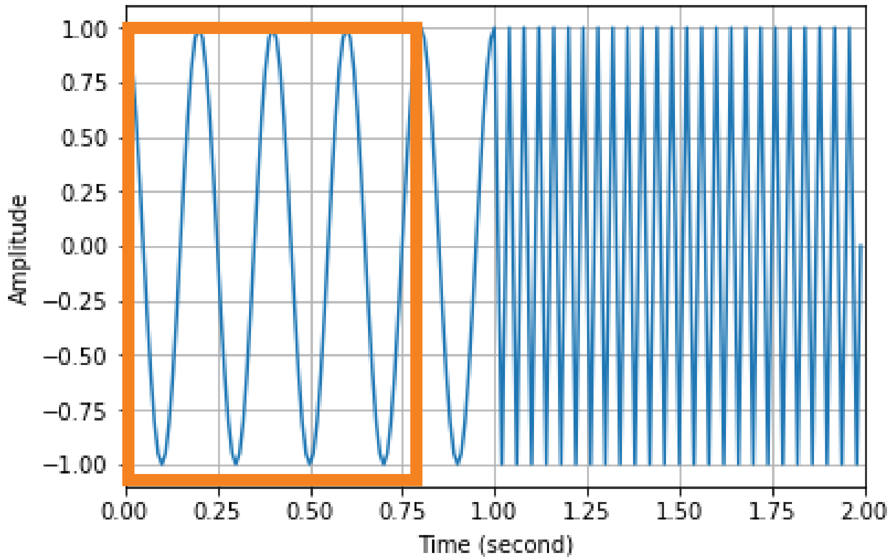
Short time Fourier transform

- Segment the signal into narrow time intervals (i.e., narrow enough to be considered stationary) and take the Fourier transform of each segment.
- Each Fourier transform provides the spectral information of a separate time-slice of the signal, providing simultaneous time and frequency information.



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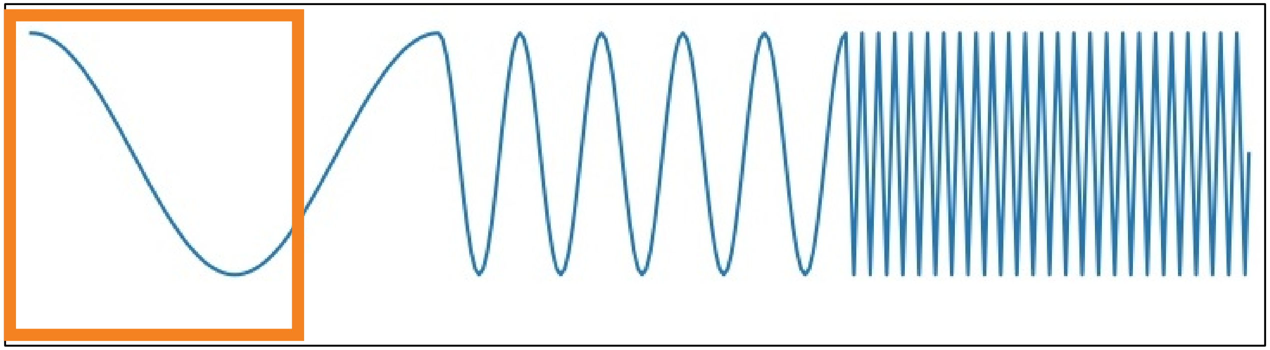
Short time Fourier transform

- Choose a window function of finite length
- Place the window on start of the signal at $t = 0$
- Truncate the signal using this window
- Compute Fourier transform of the truncated signal
- Incrementally slide the window to the right
- Repeat until window reaches the end of the signal



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Short time Fourier transform

- A compromise between time-based and frequency-based views of a signal.
- Both time and frequency are represented in limited precision.
- The precision is determined by the size of the window.
  - Wide window: good frequency resolution, poor time resolution.
  - Narrow window: good time resolution, poor frequency resolution.

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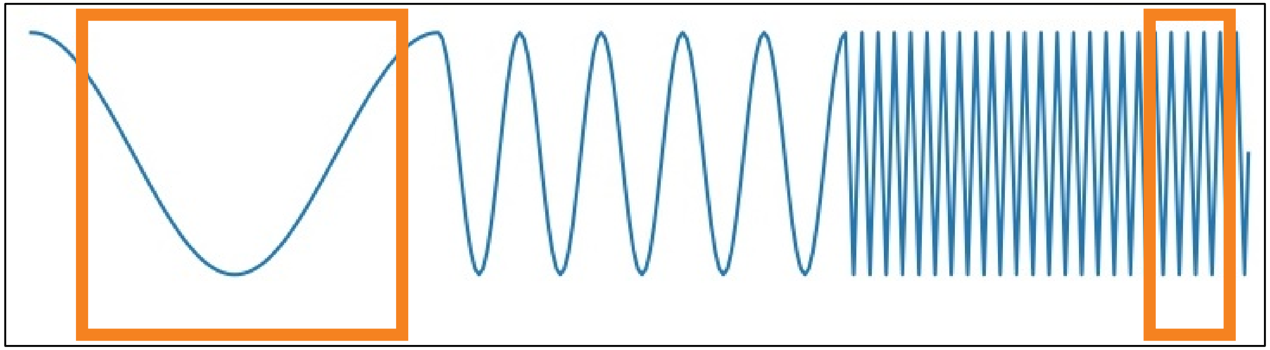
Short time Fourier transform

- Uses a variable length window
  - Narrower windows are more appropriate at high frequencies
  - Wider windows are more appropriate at low frequencies



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Topics

- Introduction to feature extraction in time-frequency domain of sensor signal
- Fourier transformation
- Wavelet transformation

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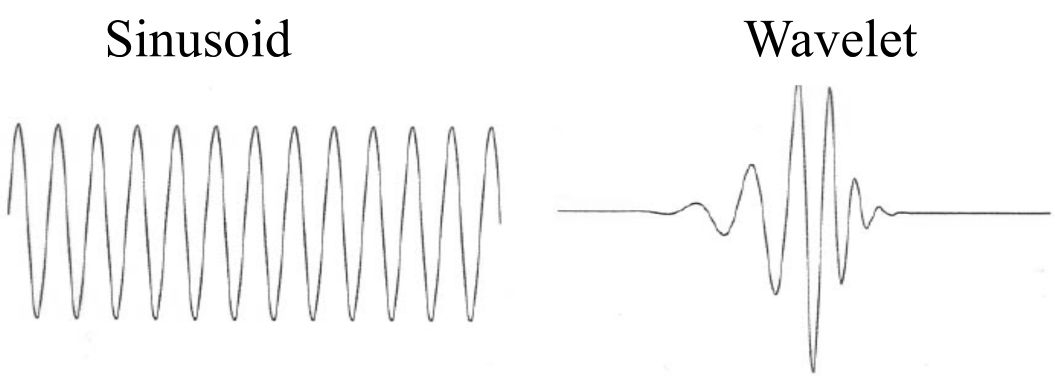
What is a wavelet?

- A function that “waves” above and below the x-axis with the following properties
  - Varying frequency
  - Limited duration
  - Zero average value
- This is in contrast to sinusoids, used by Fourier transform, which have infinite duration and constant frequency.



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Example 1: Decomposition

- Decompose signal by averaging and subsampling the signal together (pairwise) to get new approximation coefficients and detail coefficients.

[9 7 3 5]

low-pass, down-sampling

high-pass, down-sampling

(9+7)/2, (3+5)/2

(9-7)/2, (3-5)/2

[8 4]

[1 −1]

low-pass, down-sampling

high-pass, down-sampling

[6]

[2]

(8+4)/2

(8-4)/2

Result: Wavelet coefficients

[6 2 1 −1]

Note: Haar wavelet is used in this example

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Example 1: Reconstruction

- The original signal can be reconstructed by adding or subtracting the detail coefficients from the approximation coefficients.

Given the wavelet coefficients (obtained in previous slide)

$$[6 \quad 2 \quad 1 \quad -1]$$

$$[6] \xrightarrow{2} [8 \quad 4] \xrightarrow{1\ -1} [9 \quad 7 \quad 3 \quad 5]$$

$8 = (6+2)$, $4 = (6-2)$

$9 = (8+1)$, $7 = (8-1)$, $3 = (4+(-1))$, $5 = (4-(-1))$

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Example 2: Signal decomposition

- The detail layer is produced by convolving with [-½ ½] (then subsampling)
- The approximate layer is produced by convolving with [½ ½] (then subsampling)

Input signal I^3

9	7	3	5	6	10	2	6

Approximation I^2

8	4	8	4

Detail D^2

1	-1	-2	-2

Approximation I^1

6	6

Detail D^1

2	2

Approximation I^0

6

Detail D^0

0

For the j-th level and i-th index

$$I_i^j = \frac{1}{2}\left(I_{2i}^{j+1} + I_{2i+1}^{j+1}\right)$$

$$D_i^j = \frac{1}{2}\left(I_{2i}^{j+1} - I_{2i+1}^{j+1}\right)$$

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Example 2: Matrix formulation

Input signal I^3							
9	7	3	5	6	10	2	6
6	0	2	2	1	-1	-2	-2
Approximation I^0	Detail D^0	Detail D^1		Detail D^2			

$$\begin{array}{c} I^0 \\ D^0 \\ D^1 \\ \\ D^2 \\ \\ \\ \\ \end{array} \begin{bmatrix} 6 \\ 0 \\ 2 \\ 2 \\ 1 \\ -1 \\ -2 \\ -2 \end{bmatrix} = \begin{array}{c} 1/8 \\ 1/8 \\ 1/4 \\ \\ 1/2 \\ \\ \\ \\ \end{array} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 9 \\ 7 \\ 3 \\ 5 \\ 6 \\ 10 \\ 2 \\ 6 \end{bmatrix}$$

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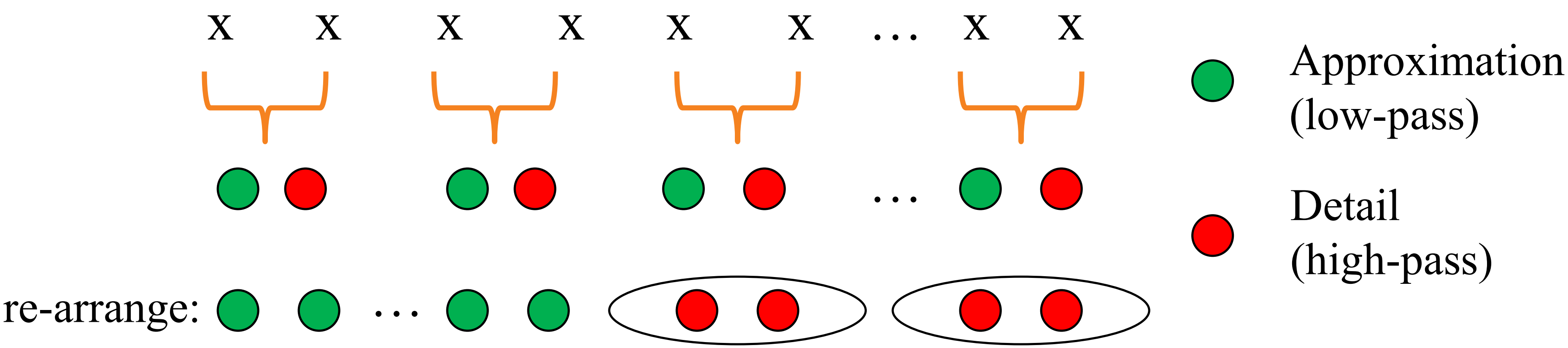
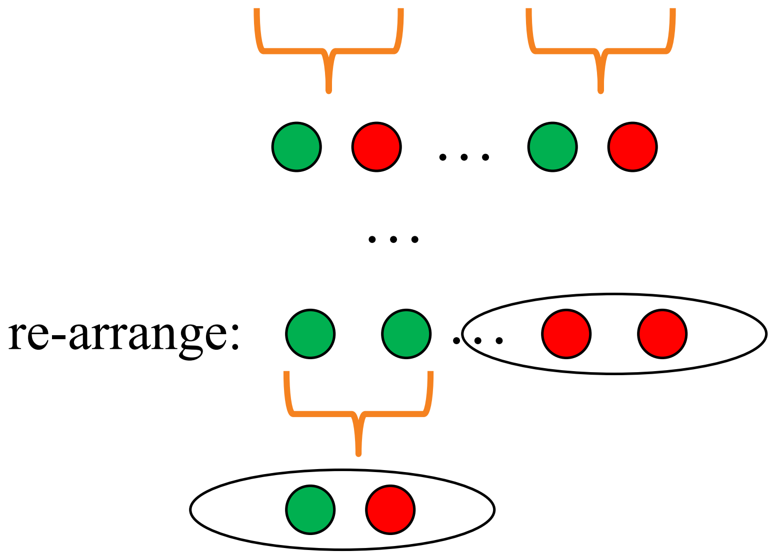
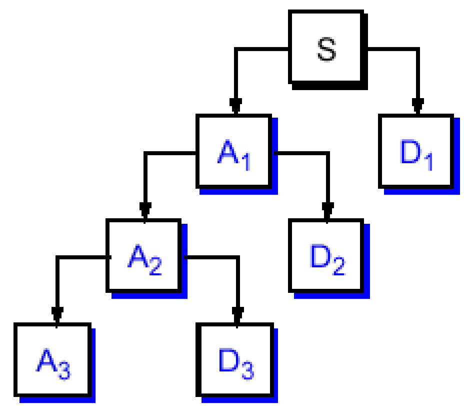
Summary: Decomposition







$$S = A_1 + D_1$$
$$= A_2 + D_2 + D_1$$
$$= A_3 + D_3 + D_2 + D_1$$

- S: Signal
- A: Approximation coefficient
- D: Detail coefficient

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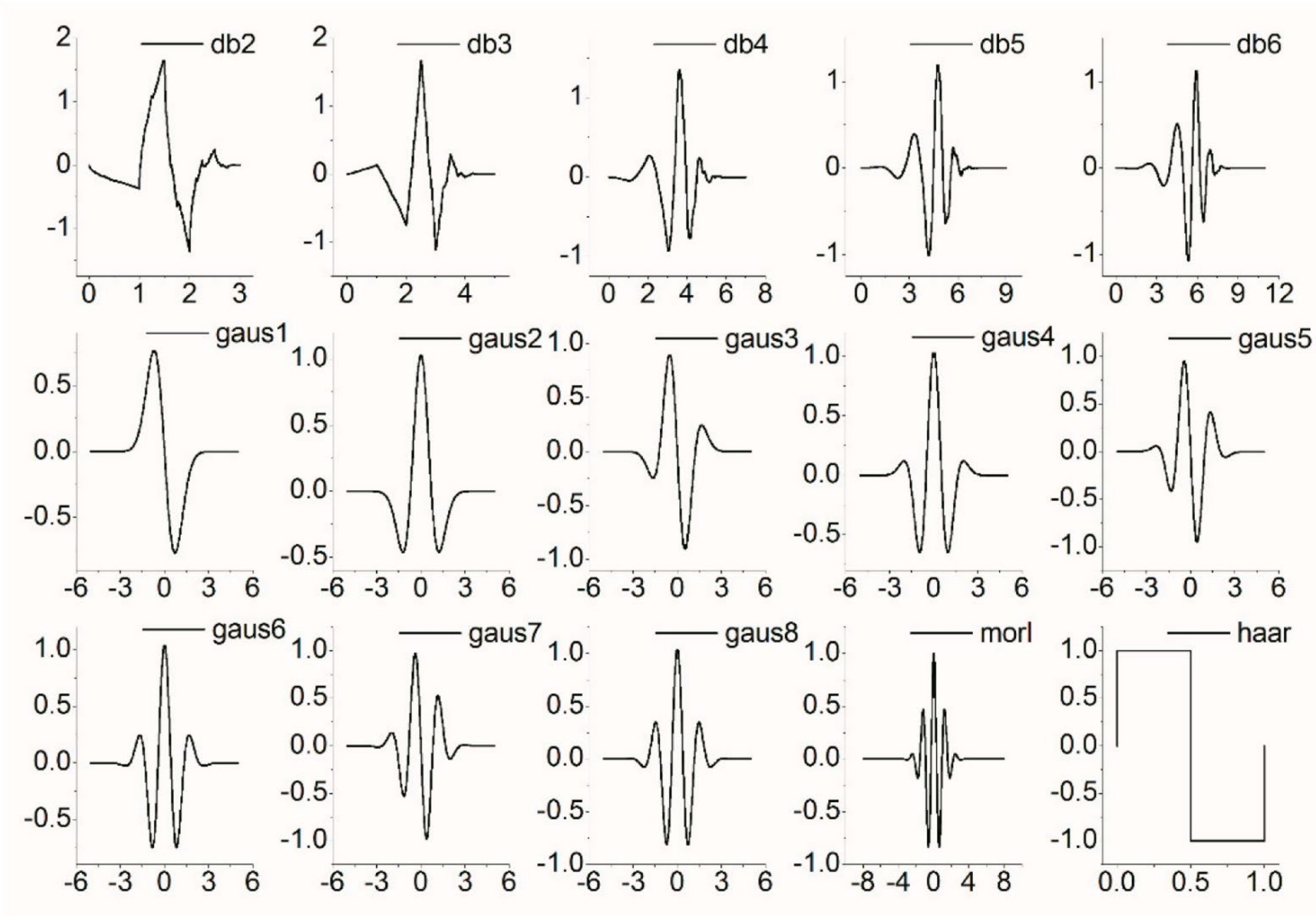
Wavelet: Other choice of filters

The approximation coefficients (lower-resolution) can be calculated from the detail coefficients (higher-resolution) by a tree-structured algorithm (i.e., filter bank).



Source: C. Xia and C. Liu, "Identification and Representation of Multi-Pulse Near-Fault Strong Ground Motion Using Adaptive Wavelet Transform," Applied Sciences, Vol. 9, No. 2, pp. 259, 2019 , https://www.mdpi.com/2076-3417/9/2/259

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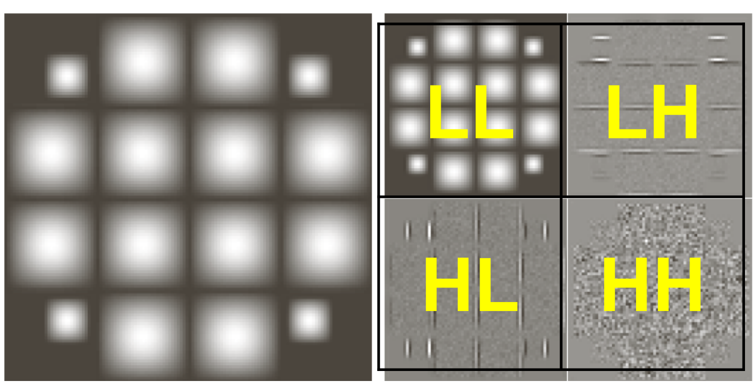
2D wavelet transformation

- LL: The upper left quadrant is filtered by the analysis low-pass filter along the rows and then filtered along the corresponding columns with the analysis low-pass filter. It represents the approximated version of the original at half the resolution.
- HL/LH: The lower left and the upper right blocks are filtered along the rows and columns with low-pass filter and high-pass filter, alternatively. The LH block contains vertical edges. In contrast, the HL blocks shows horizontal edges.
- HH: The lower right quadrant is derived analogously to the upper left quadrant but with the use of the analysis high pass filter, where we find edges of the original image in diagonal direction.



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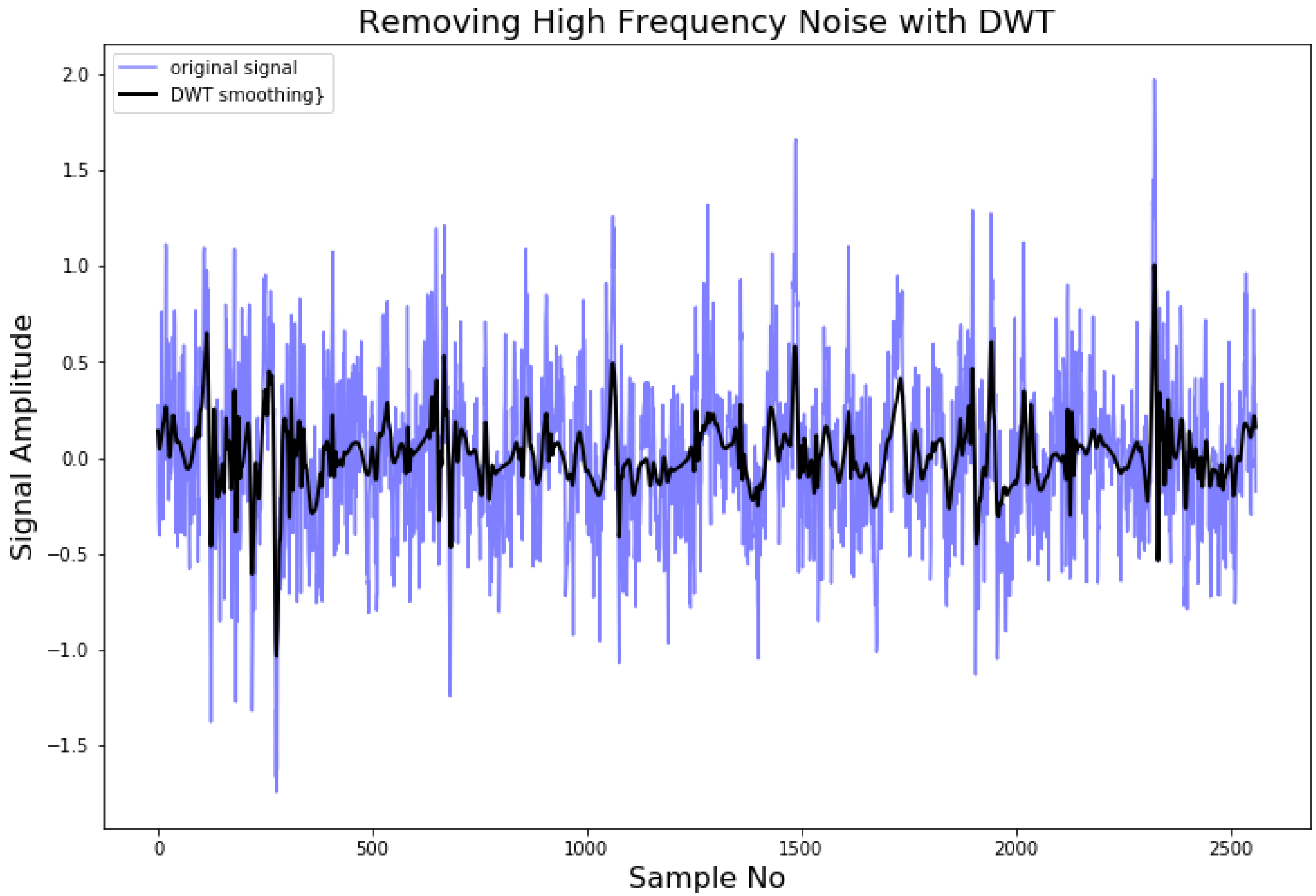
Wavelet application: Denoising

- Signal denoising



Reference: http://ataspinar.com/2018/12/21/a-guide-for-using-the-wavelet-transform-in-machine-learning/

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Wavelet application: Feature extraction

- Perform wavelet transformation on signal
- Extract statistical features
  - Auto-regressive model coefficient values
  - (Shannon) Entropy values; entropy values can be taken as a measure of complexity of the signal.
  - Statistical features like variance, mean median, zero crossing rate
- Perform signal classification or other machine learning tasks

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Workshop

Objective

- Perform wavelet decomposition on signal
- Perform wavelet-based signal denoising
- Extract statistical features from wavelet coefficients from wearable sensor data, and then perform classification for human activity classification

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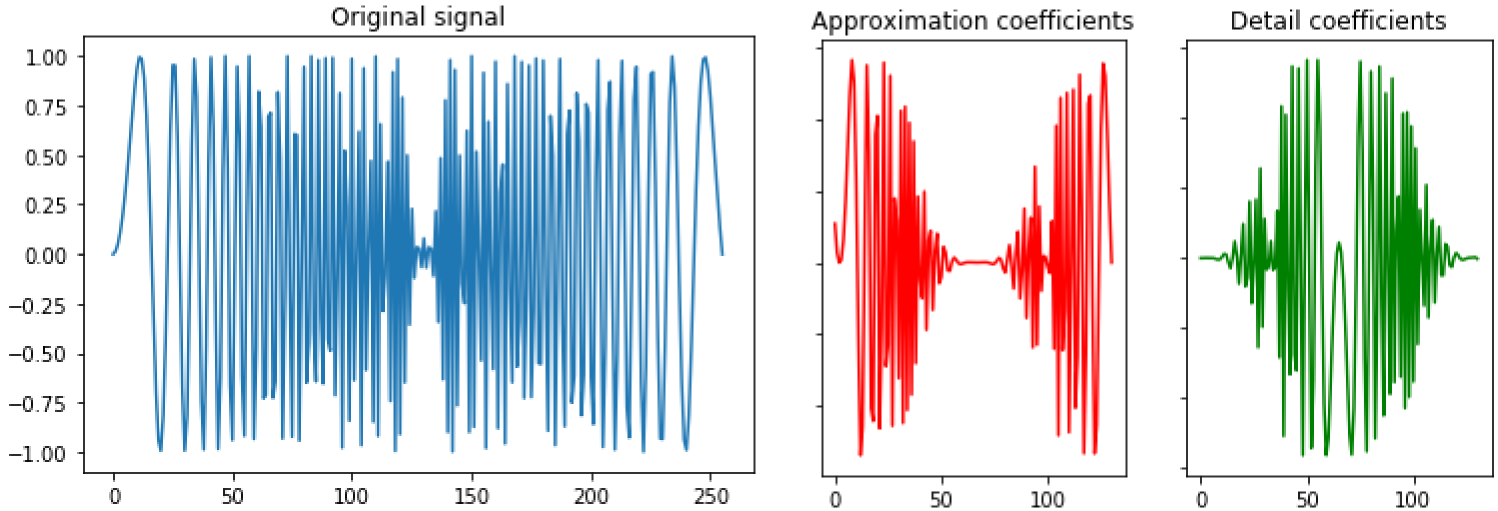
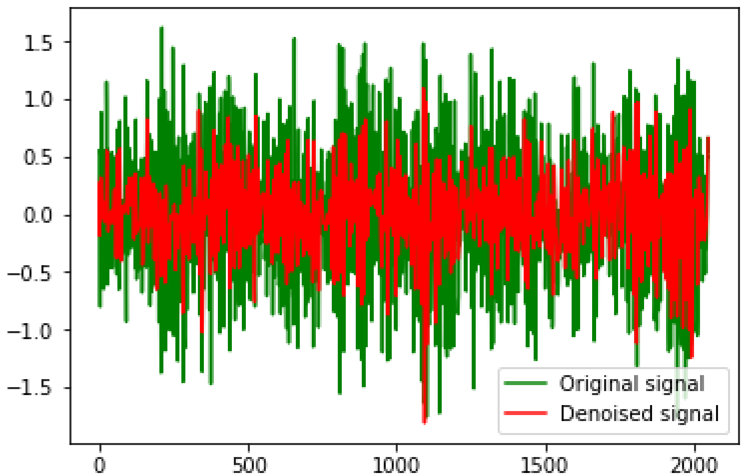
Workshop





Human Activity Recognition Using Smartphones Data Set

- Each person performed six activities (WALKING, WALKING_UPSTAIRS, WALKING_DOWNSTAIRS, SITTING, STANDING, LAYING) wearing a smartphone (Samsung Galaxy S II) on the waist. Using its embedded accelerometer and gyroscope, we captured 3-axial linear acceleration and 3-axial angular velocity at a constant rate of 50Hz.
- Reference: https://archive.ics.uci.edu/ml/datasets/Human+Activity+Recognition+Using+Smartphones

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Summary

- Introduction to feature extraction in time-frequency domain of sensor signal
- Fourier transformation
- Wavelet transformation

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Thank you!

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