Chapter 8

Voting Logic Fusion

Voting logic fusion overcomes many of the drawbacks associated with using single sensors or sensors that recognize signals based on only one signature generation phenomenology to detect targets in a hostile environment. For example, voting logic fusion provides protection against false alarms in high clutter backgrounds and decreases susceptibility to countermeasures that may mask a signature of a valid target or cause a weapon system to fire at a false target. Voting logic may be an appropriate data fusion technique to apply when a multiple sensor system is used to detect, classify, and track objects. Figure 8.1 shows the strengths and weaknesses of combining sensor outputs in parallel, series, and in series/parallel. Generally, the parallel configuration provides good detection of targets with suppressed signatures because only one sensor in the suite is required to detect the target. The series configuration provides good rejection of false targets when the sensors respond to signals generated by different phenomena. The weaknesses of these configurations become apparent by reversing their advantages. The parallel is subject to false target detection and susceptibility to decoys, since one sensor may respond to a strong signal from a nontarget. The series arrangement requires signatures to be generated by all the phenomena encompassed by the sensors. Thus, the series configuration functions poorly when one or more of the expected signature phenomena is absent or weak, such as when a target signature is suppressed.

The series/parallel configuration supports a voting logic fusion process that incorporates the advantages of the parallel and series configurations. These are rejection of signatures from decoys, clutter, and other nontargets and detection of targets that have one or more of their signature domains suppressed. We will show that voting fusion (one of the feature-based inference fusion techniques for object classification) allows the sensors to automatically detect and classify objects to the extent of their knowledge. This process does not require explicit switching of sensors based on the quality of their inputs to the fusion processor or the real-time characteristics of the operating environment. The sensor outputs are always connected to the fusion logic, which is designed to incorporate all anticipated combinations of sensor knowledge. Auxiliary operating modes may be added to the automatic voting process to further optimize sensor system performance under some special conditions that are identified in advance. The special conditions may include countermeasures, inclement weather, or high-clutter backgrounds, although the automatic voting may prove adequate in these

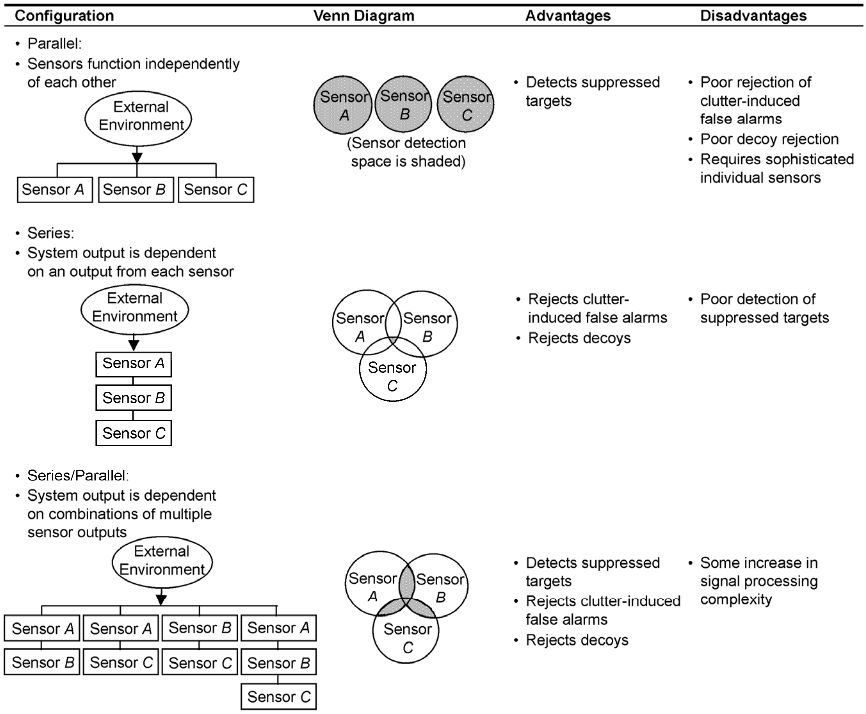

Figure 8.1 Attributes of series and parallel sensor output combinations.

circumstances as well. Testing and simulation of system performance are needed to ascertain whether auxiliary modes are needed to meet performance goals and objectives.

Neyman-Pearson and Bayesian formulations of the distributed sensor detection problem for parallel, serial, and tree data fusion topologies are discussed by Viswanathan and Varshney.[1] Liggins et al. describe Bayesian approaches for the fusion of information in centralized, hierarchical, and distributed sensor architectures used for target tracking.[2]

Voting logic fusion is illustrated in this chapter with a three-sensor system whose detection modes involve two or more sensors. Single sensor detection modes are not implemented in the first examples in order to illustrate how the voting logic process avoids the shortcomings of the parallel sensor output configuration. The last example does address the incorporation of single sensor detection modes into voting logic fusion when the system designer wishes to have these modes available. The sensors are assumed to operate using sensor-level fusion, where fully processed sensor data are sent to the fusion processor as target reports that contain the object detection or classification decision, associated confidence, and object location information.

In general, the fusion algorithm combines the target report data from all the sensors to assess the identity of the potential target, its track, and the immediacy of the threat. In the classification application discussed here, the Boolean algebra-based voting algorithm gives closed-form expressions for the multiple sensor system's estimation of true target detection probability and false alarm probability. In order to correlate confidence levels with detection and false alarm probabilities, the characteristics of the sensor input signals (such as spatial frequency, bandwidth, and amplitude) and the features in the signal processing algorithms used for comparison with those of known targets must be well understood. The procedures for relating confidence levels to detection and false alarm probabilities are described in this chapter through application examples.

8.1 Sensor target reports

Detection information contained in the target reports reflects the degree to which the input signals processed by the sensor conform to or possess characteristics that match predetermined target features. The degree of conformance to target or object features is related to the "confidence" with which the potential target or object of interest has been recognized. Selected features are a function of the target size, sensor operation (active or passive), and sensor design parameters such as center frequency, number and width of spectral bands, spatial resolution, receiver bandwidth, receiver sensitivity, and other parameters that were shown in Table 3.8, as well as the signal processing employed. Time domain processing, for example, may use features such as amplitude, pulse width, amplitude/width ratio, rise and fall times, and pulse repetition frequency. Frequency domain processing may use separation between spectral peaks, widths of spectral features, identification of periodic structures in the signal, and number of scattering centers producing a return signal greater than a clutter-adaptive running-average threshold.[3] Multiple pixel, infrared radiometer or FLIR sensor imagery may employ target discriminants such as image-fill criteria where the number of pixels above some threshold is compared to the total number of pixels within the image boundaries, length/width ratio of the image (unnormalized or normalized to area or edge length), parallel and perpendicular line relationships, presence of arcs or circles or conic shapes in the image, central moments, center of gravity, asymmetry measures, and temperature gradients across object boundaries. Multispectral and hyperspectral sensors operating in the visible and infrared spectral bands may use color coefficients, apparent temperature, presence of specific spectral peaks or lines, and the spatial and time signatures of the detected objects. Target reports also contain information giving target or object location. The target can, of course, be generalized to include the recognition of decoys, jammers, regions of high clutter, and anything of interest that can be ascertained within the design attribute limits of the sensor hardware and signal processing algorithms.

8.2 Sensor detection space

Sensor system detection probability is based on combinations of sensor outputs that represent the number and degree to which the postulated target features are matched by features extracted from individual sensor output data. The sensor combinations that make up the detection space are determined by the number of sensors in the sensor suite, the resolution and algorithms used by the sensors, and the manner in which the sensor outputs are combined. These considerations are discussed below.

8.2.1 Venn diagram representation of detection space

Detection space (or classification space) of a three-sensor system having Sensors *A*, *B*, and *C* is represented by a Venn diagram in Figure 8.2. Regions are labeled to show the space associated with one-sensor, two-sensor, and three-sensor combinations of outputs.

8.2.2 Confidence levels

Sensor detection space is not the same as confidence-level space in general, and a mapping of one into the other must be established. Nonnested or disjoint confidence levels, illustrated in the Venn diagram of Figure 8.3, are defined by any combination of the following:

- Number of preidentified features that are matched to some degree by the input signal to the sensor;
- Degree of matching of the input signal to the features of an ideal target; or
- Signal-to-interference ratio.

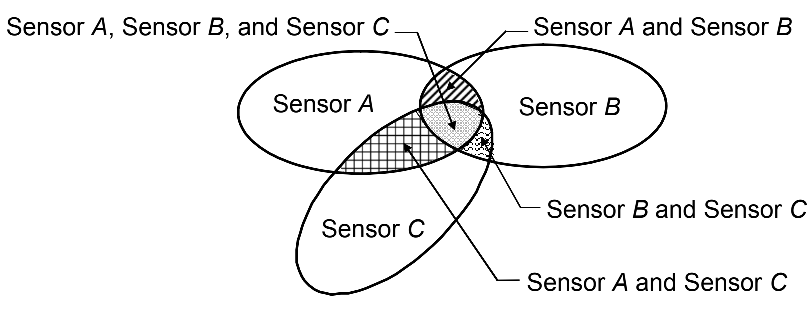

Figure 8.2 Detection modes for a three-sensor system.

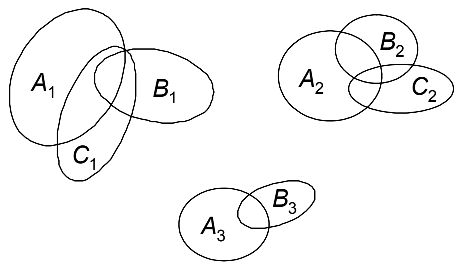

Figure 8.3 Nonnested sensor confidence levels.

Signal processing algorithms or features suitable for defining confidence levels depend on sensor type and operating characteristics (e.g., active, passive, spectral band, resolution, and field of view) and type of signal processing used (e.g., time domain, frequency domain, and multipixel image processing). Representative features, which can potentially be used to assist in defining confidence levels, are listed in Table 3.2 and Section 8.1.

Signal-to-interference ratio is used as a generalization of signal-to-noise ratio so that clutter can be incorporated as the limiting interference when appropriate. Nonnested confidence levels allow optimization of false alarm probabilities for each sensor's confidence levels since the confidence levels have a null-set intersection as described in Section 8.3.1. In the nomenclature used here, A_3 in Sensor A is a higher confidence level than A_2, and A_2 represents a higher confidence than A_1. Similar definitions apply to the confidence levels of Sensors B and C.

The number of confidence levels required of a sensor is a function of the number of sensors in the system and the ease with which it is possible to correlate target recognition features extracted from the sensor data with distinct confidence levels. The more confidence levels that are available, the easier it is to develop combinations of detection modes that meet system detection and false alarm probability requirements under wide-ranging operating conditions. Conversely, as the number of confidence levels is increased, it may become more difficult to define a set of features that unambiguously correlates a detection with a confidence level. For example, processing of radar signals in some instances contains tens of features against which the input signal is compared. Confidence levels, in this case, can reflect the number of feature matches and the degree to which the input signal conforms to the ideal target features.[4]

8.2.3 Detection modes

Combinations of sensor outputs, called detection modes, that are allowed to declare valid targets are based on the ability of the sensor hardware and signal processing discriminants to distinguish between true and false targets or countermeasure effects. Ultimately the permitted sensor confidence combinations are determined by the experience and knowledge of the system designer and analysis of data gathered with the sensor system.

Table 8.1 gives the allowable detection modes for the illustrative three-sensor system. Modes that contain at least two sensors are used to avoid susceptibility to single-sensor false alarm events or countermeasures. The three-sensor mode {ABC} results from a combination of at least low confidence outputs from all sensors. The low confidence suffices because all three sensors participate in the decision. This produces a low likelihood that a false target or countermeasure-induced event will be detected as a true target, especially if the sensors respond to data that are generated from different phenomena.

Three two-sensor detection modes are also shown. The {AC} and {BC} modes use intermediate confidence levels from each of two sensors. The confidence level required has been raised, as compared to the three-sensor mode, since only two sensors are involved in making the detection decision. In mode {AB}, it is assumed that the hardware and algorithms contributing information are not as robust as they are in the other two-sensor modes. Thus, the highest third-level confidence output is required of the *A* and *B* sensors before a detection decision is made using this mode.

Table 8.1 Multiple sensor detection modes that incorporate confidence levels in a three-sensor system.

Mode	Sensor and Confidence Level		
	A	***B***	***C***
ABC	A_1	B_1	C_1
AC	A_2	–	C_2
BC	–	B_2	C_2
AB	A_3	B_3	–

The designer may also decide that certain detection modes should be excluded altogether from the decision matrix. For example, two of the sensors may be known to false alarm on similar types of terrain. Therefore, the detection mode that results from the combination of these two sensors does not give information based on independent signature-generation phenomena and is excluded. However, these sensors, when used with a third sensor, may provide powerful target discriminants and so are retained in the sensor suite.

8.3 System detection probability

The remaining steps for calculating the system detection probability are discussed in this section. These are: derivation of the system detection probability equation based on the confidence-level structure and the selected detection mode's relation of confidence levels to detection and false alarm probabilities, computation of signal-to-noise or signal-to-clutter ratio for each sensor, and identification of the target fluctuation characteristics as observed by each sensor.

8.3.1 Derivation of system detection and false alarm probability for nonnested confidence levels

Once the detection modes are identified, Boolean algebra may be used to derive an expression for the sensor system detection probability and false alarm probability. For the above example containing one three-sensor and three two-sensor detection modes, the system detection probability equation takes the form

$$\text{System } P_d = P_d\{A_1\, B_1\, C_1 \text{ or } A_2\, C_2 \text{ or } B_2\, C_2 \text{ or } A_3\, B_3\}. \tag{8-1}$$

By repeated application of the Boolean algebra expansion given by

$$P\{X \text{ or } Y\} = P\{X\} + P\{Y\} - P\{XY\}, \tag{8-2}$$

Eq. (8-1) can be expanded into a total of fifteen sum and difference terms as

$$\begin{aligned} \text{System } P_d = {} & P_d\{A_1\, B_1\, C_1\} + P_d\{A_2\, C_2\} + P_d\{B_2\, C_2\} + P_d\{A_3\, B_3\} \\ & - P_d\{B_2\, C_2\, A_3\, B_3\} - P_d\{A_2\, C_2\, B_2\} - P_d\{A_2\, C_2\, A_3\, B_3\} \\ & + P_d\{A_2\, C_2\, B_2\, A_3\, B_3\} - P_d\{A_1\, B_1\, C_1\, A_2\, C_2\} \\ & - P_d\{A_1\, B_1\, C_1\, B_2\, C_2\} - P_d\{A_1\, B_1\, C_1\, A_3\, B_3\} \\ & + P_d\{A_1\, B_1\, C_1\, B_2\, C_2\, A_3\, B_3\} + P_d\{A_1\, B_1\, C_1\, A_2\, C_2\, B_2\} \\ & + P_d\{A_1\, B_1\, C_1\, A_2\, C_2\, A_3\, B_3\} \\ & - P_d\{A_1\, B_1\, C_1\, A_2\, B_2\, C_2\, A_3\, B_3\}. \end{aligned} \tag{8-3}$$

Since the confidence levels for each sensor are independent of one another (by the nonnested or disjoint assumption), the applicable union and intersection relations are

$$P_d\{A_1 \cup A_2\} = P_d\{A_1\} + P_d\{A_2\} \tag{8-4}$$

and

$$P_d\{A_1 \cap A_2\} = 0, \tag{8-5}$$

respectively. Analogous statements apply for the other sensors.

The above relations allow Eq. (8-3) to be simplified to

$$\text{System } P_d = P_d\{A_1\, B_1\, C_1\} + P_d\{A_2\, C_2\} + P_d\{B_2\, C_2\} + P_d\{A_3\, B_3\} - P_d\{A_2\, B_2\, C_2\}. \tag{8-6}$$

The four positive terms in Eq. (8-6) correspond to each of the detection modes, while the one negative term eliminates double counting of the $\{A_2\ B_2\ C_2\}$ intersection that occurs in both $\{A_2\ C_2\}$ and $\{B_2\ C_2\}$. The Venn diagrams in Figure 8.4 illustrate the detection modes formed by the allowed combinations of sensor outputs at the defined confidence levels.

If the individual sensors respond to independent signature-generation phenomena (e.g., backscatter of transmitted energy and emission of energy by a warm object) such that the sensor detection probabilities are independent of one another, then the individual sensor probabilities can be multiplied together to give

$$\text{System } P_d = P_d\{A_1\}\, P_d\{B_1\}\, P_d\{C_1\} + P_d\{A_2\}\, P_d\{C_2\} + P_d\{B_2\}\, P_d\{C_2\} + P_d\{A_3\}\, P_d\{B_3\} - P_d\{A_2\}\, P_d\{B_2\}\, P_d\{C_2\}. \tag{8-7}$$

The interpretation of the terms in Eq. (8-7) is explained by referring to the first term $P_d\{A_1\}\ P_d\{B_1\}\ P_d\{C_1\}$. The factors in this term represent the multiplication of the detection probability associated with confidence level 1 of Sensor *A* by the detection probability associated with confidence level 1 of Sensor *B* by the detection probability associated with confidence level 1 of Sensor *C*. Similar explanations may be written for the other four terms.

Equation (8-7) is also used to calculate the false alarm probability of the sensor system by replacing the detection probability by the appropriate false alarm probability for the sensor confidence levels. Thus,

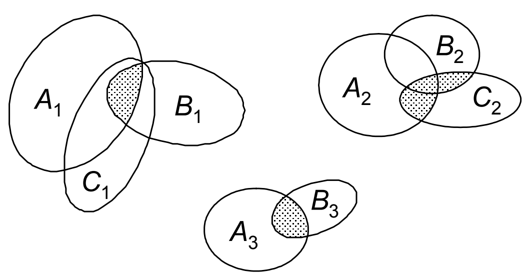

Figure 8.4 Detection modes formed by combinations of allowed sensor outputs.

$$\text{System } P_{fa} = P_{fa}\{A_1\}\, P_{fa}\{B_1\}\, P_{fa}\{C_1\} + P_{fa}\{A_2\}\, P_{fa}\{C_2\} + P_{fa}\{B_2\}\, P_{fa}\{C_2\} + P_{fa}\{A_3\}\, P_{fa}\{B_3\} - P_{fa}\{A_2\}\, P_{fa}\{B_2\}\, P_{fa}\{C_2\}. \quad (8\text{-}8)$$

8.3.2 Relation of confidence levels to detection and false alarm probabilities

Mapping of the confidence-level space into the sensor detection space is accomplished by multiplying the inherent detection probability of the sensor by the conditional probability that a particular confidence level is satisfied given a detection by the sensor. Since the signal-to-interference ratio can differ at each confidence level, the inherent detection probability of the sensor can also be different at each confidence level. Thus the probability $P_d\{A_n\}$ that Sensor A will detect a target with confidence level A_n is

$$P_d\{A_n\} = P_d'\{A_n\}\, P\{A_n/\text{detection}\}, \quad (8\text{-}9)$$

where

$P_d'\{A_n\}$ = inherent detection probability calculated for confidence level n of Sensor A using the applicable signal-to-interference ratio, false alarm probability, target fluctuation characteristics, and number of samples integrated

and

$P\{A_n/\text{detection}\}$ = probability that detection with confidence level A_n occurs given a detection by Sensor A.

Similar definitions apply to the detection probabilities at the confidence levels associated with the other sensors.

Analogous relations allow the false alarm probability to be calculated at each confidence level of the sensors. Thus the probability $P_{fa}\{A_n\}$ that a detection at confidence level A_n in Sensor A represents a false alarm is

$$P_{fa}\{A_n\} = P_{fa}'\{A_n\}\, P\{A_n/\text{detection}\}, \quad (8\text{-}10)$$

where

$P_{fa}'\{A_n\}$ = inherent false alarm probability selected for confidence level n of Sensor A

and

$P\{A_n/\text{detection}\}$ is the same as defined above.

The same value of the conditional probability factor is used to convert from confidence-level space into probability space when calculating both the detection and false alarm probabilities associated with detection by a sensor at a particular confidence level. Other models (such as the nested confidence level example in Appendix F) that incorporate the conditional probability that a false alarm at confidence level A_n occurs, given a false alarm by Sensor A, may also be developed. The false alarm probabilities that characterize the sensor system and the confidence levels are dependent on the thresholds that establish the false alarm probabilities. However, detection probability is not only a function of false alarm probability, but also of signal processing gain, which acts to increase detection probability. Signal processing gain is proportional to how well the signal matches target-like features designed into an algorithm and is related to the conditional probability factor in Eq. (8-9).

8.3.3 Evaluation of conditional probability

Conditional probabilities $P\{A_n/\text{detection}\}$ are evaluated using an off-line experiment to determine the performance of the signal processing algorithm. Target and nontarget data are processed by a trial set of algorithms containing confidence-level definitions based on the criteria discussed in Section 8.2. The number of detections passing each confidence level's criteria is noted and the conditional probabilities are then computed from these results. For example, if 1,000 out of 1,000 detections pass confidence level 1, then the probability is one that detection with confidence level 1 occurs, given detection by the sensor. If 600 out of the 1,000 detections pass confidence level 2, then the probability is 0.6 that detection with confidence level 2 occurs, given detection by the sensor.

Once the conditional probabilities are established, the system detection and false alarm probabilities are computed using Eqs. (8-7–10). The first step in this procedure is to find the probability of a false alarm by the sensor at a particular confidence level using Eq. (8-10). Next, the false alarm probability of the mode is calculated by multiplying together the false alarm probabilities of the sensors at the confidence level at which they operate in the detection mode. Finally, the overall system false alarm probability is found by substituting the modal probabilities and the value for the negatively signed term into Eq. (8-8). If the system false alarm requirement is met, the algorithm contains the proper confidence level discrimination. If the requirement is not satisfied, then another choice of conditional probabilities is selected and the algorithm is adjusted to provide the new level of discrimination. The inherent sensor false alarm probabilities may also be adjusted to meet the system requirement, as explained in the following section.

8.3.4 Establishing false alarm probability

False alarm probabilities corresponding to each sensor's confidence levels can be different from one another because of the null set intersection described by Eq. (8-5). It is this characteristic that also allows the signal-to-interference ratio to differ at each confidence level. The inherent false alarm probabilities $P_{fa}'\{\bullet\}$ at each sensor's confidence levels are selected as large as possible consistent with satisfying the system false alarm requirement. This maximizes the detection probability for each mode. The resulting probability $P_{fa}\{\bullet\}$ that a detection by the sensor represents a false alarm at the given confidence level is also dependent on the algorithm performance through the conditional probability factor in Eq. (8-10).

Two methods may be used to establish the inherent false alarm probability at each sensor's confidence levels. In the first, the inherent false alarm probability is made identical at all confidence levels by using the same detection threshold at all levels of confidence. The inherent detection probabilities are a function of this threshold. Although the threshold is the same at each confidence level, the detection probabilities can have different values at the confidence levels if the signal-to-interference ratios differ. Likewise, when the detection thresholds are the same at each confidence level, the false alarms can be reduced at the higher confidence levels through the subsequent benefits of the signal processing algorithms. This reduction in false alarms is modeled by multiplying the inherent false alarm probability by the conditional probability factor that reflects the signal processing algorithm performance at the confidence level.

In the second method, a different threshold controls the inherent false-alarm probability at each confidence level. Higher confidence levels have higher thresholds and hence lower false alarm probabilities. False alarms are also reduced by subsequent signal processing as above. With this method of false alarm control, the inherent detection probability is a function of the different thresholds and, hence, the different false alarm probabilities that are associated with the confidence levels.

Either method may be employed to control false alarms. The off-line experiment will have to be repeated, however, to find new values for the conditional probabilities if the false alarm control method is changed.

In any detection mode there is a choice in how to distribute the false alarm probabilities among the different sensors. The allocations are based on the ability of the sensor's anticipated signal processing to reject false alarms, and ultimately on the conditional probabilities that relate inherent false alarm probability to the probability that the sensor will false alarm when a detection occurs at the particular confidence level. The tradeoff between conditional probability and low false alarm and detection probabilities becomes obvious from Eq. (8-10). It can be seen that as the conditional probability for any confidence level is decreased to reduce false alarms, the corresponding detection probability also decreases.

8.3.5 Calculating system detection probability

The final steps in calculating the system detection probability require the use of target, background, and sensor models to compute the signal-to-clutter or noise ratios and number of samples integrated. Upon deciding on the fluctuation characteristics that apply to the target, the inherent detection probabilities for each confidence level are calculated or found in a table or figure corresponding to the active (microwave, millimeter-wave, or laser radar) or passive (infrared or millimeter-wave radiometer, FLIR, or IRST) sensor type and the direct (sensor does not contain a mixer to translate the frequency of the received signals) or heterodyne (sensor contains a mixer) detection criterion.[5–8] The probability of detection by the sensor at a particular confidence level is found by multiplying the inherent detection probability by the conditional probability. Then the modal detection probability is obtained by multiplying together the sensor detection probabilities corresponding to the confidence levels in the detection mode. Finally, the overall system detection probability is calculated by substituting the modal detection probabilities and the value for the negatively signed term into Eq. (8-7).

8.3.6 Summary of detection probability computation model

The procedure for computing the sensor system detection probability is shown in Figure 8.5. The steps are summarized below.

1. Determine allowable sensor output combinations (detection modes).

2. Select the inherent false alarm probability for each sensor's confidence levels.

3. Through an off-line experiment, determine the number of detections corresponding to the sensor confidence levels and calculate the conditional probabilities defined in Eq. (8-9).

4. Calculate the probabilities that detections at given confidence levels represent false alarms using Eq. (8-10).

5. Calculate the sensor system false alarm probability using Eq. (8-8) and verify against requirement.

6. Note inherent false alarm probability at the confidence levels of each sensor.

7. Compute the signal-to-clutter, signal-to-noise, or signal-to-clutter plus noise ratios, as appropriate, as well as the number of samples integrated, if applicable.

8. Determine the target fluctuation characteristics that apply, e.g., steady state, slow fluctuation, and fast fluctuation.

9. Calculate the inherent sensor detection probability at each confidence level.

10. Calculate the probabilities for target detection by each sensor at the appropriate confidence levels using Eq. (8-9).

11. Calculate the sensor system detection probability using Eq. (8-7) and verify that the requirement is satisfied.

8.4 Application example without singleton sensor detection modes

Consider the design of a three-sensor system that must achieve a false alarm probability equal to or less than 10^{-6} with a detection probability greater than or equal to 0.8.

In this example, Sensor *A* is assumed to be a millimeter-wave radar system to which the target has Swerling III fluctuation characteristics. Sensor *B* is a laser

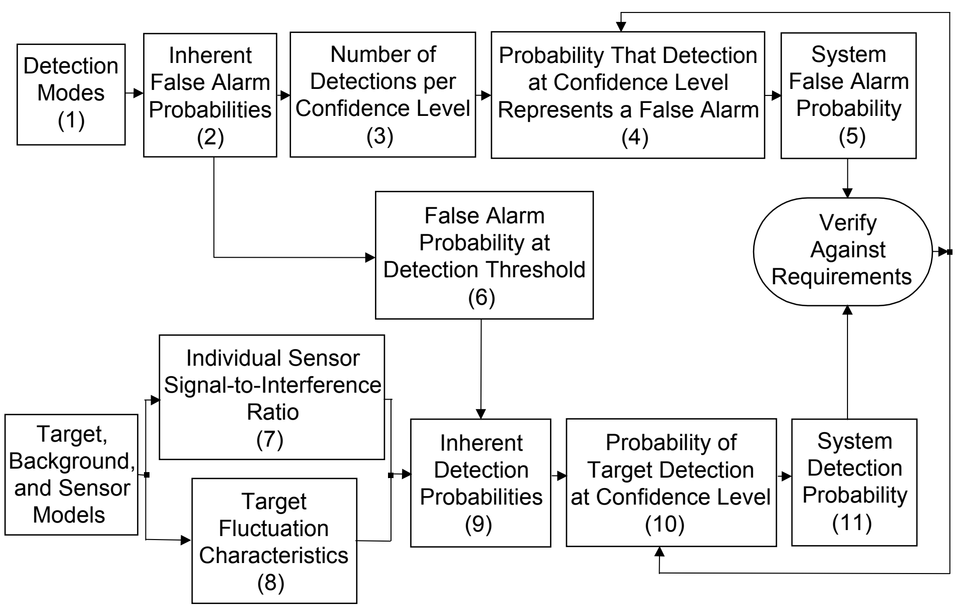

Figure 8.5 Sensor system detection probability computation model.

radar to which the target behaves as a Swerling II fluctuation model. Sensor *C* is an imaging infrared radiometer to which the target appears nonfluctuating. Different thresholds have been assumed at the sensor confidence levels.

Suppose that through an off-line experiment the number of detections corresponding to each sensor's confidence levels is determined as shown in Table 8.2. The number of detections is governed by the threshold settings, signal processing approach, and target discrimination features that are selected for each sensor. For example, based on the signal processing used in Sensor *A*, 600 threshold crossings out of 1,000 were observed to satisfy confidence level A_2 and 400 observed to satisfy confidence level A_3. These data determine the conditional probabilities listed in Table 8.2, which are subsequently used to evaluate Eqs. (8-9) and (8-10).

Table 8.2 Distribution of detections and signal-to-noise ratios among sensor confidence levels.

Sensor	← *A* →			← *B* →			← *C* →	
Confidence level	A_1	A_2	A_3	B_1	B_2	B_3	C_1	C_2
Distribution of detections	1,000	600	400	1,000	500	300	1,000	500
Conditional probability	1.0	0.6	0.4	1.0	0.5	0.3	1.0	0.5
Signal-to-noise ratio (dB)	10	13	16	14	17	20	11	15

8.4.1 Satisfying the false alarm probability requirement

False alarm probabilities are chosen as large as possible, consistent with satisfying the system false alarm requirements, in order to maximize system detection probability. With the selections shown in Table 8.3 for $P_{fa}'\{\bullet\}$ and the conditional probability data from Table 8.2, the system false alarm probability is calculated from Eqs. (8-8) and (8-10) as

$$\begin{aligned}\text{System } P_{fa} &= 2.6 \times 10^{-7} + 2.4 \times 10^{-7} + 2.5 \times 10^{-7} + 2.4 \times 10^{-7} - 2.4 \times 10^{-10} \\ &= 9.9 \times 10^{-7}, \end{aligned} \tag{8-11}$$

which satisfies the requirement of 10^{-6} or less.

8.4.2 Satisfying the detection probability requirement

Sensor detection probability at each confidence level is calculated from the inherent false alarm probability at the confidence level, signal-to-noise ratio, number of samples integrated, and appropriate target fluctuation characteristics. The selected signal-to-noise ratios and corresponding false alarm probabilities require only one sample per integration interval to satisfy the system detection probability requirement in this example. Noise is used as the limiting interference to simplify the calculations. The different signal-to-noise ratios at each sensor's confidence levels, as given in Table 8.2, have been postulated to aid in defining the criteria that denote the confidence levels.

Table 8.3 Inherent and conditional false alarm probabilities at the confidence levels and detection modes of the three-sensor system.

Mode	Sensor A	Sensor B	Sensor C	Mode P_{fa}
$A_1\ B_1\ C_1$	$1.6 \times 10^{-2} \times 1.0 = 1.6 \times 10^{-2}$	$1.6 \times 10^{-2} \times 1.0 = 1.6 \times 10^{-2}$	$1 \times 10^{-3} \times 1.0 = 1.0 \times 10^{-3}$	2.6×10^{-7}
$A_2\ C_2$	$1.6 \times 10^{-3} \times 0.6 = 9.6 \times 10^{-4}$	—	$5 \times 10^{-4} \times 0.5 = 2.5 \times 10^{-4}$	2.4×10^{-7}
$B_2\ C_2$	—	$2.0 \times 10^{-3} \times 0.5 = 1.0 \times 10^{-3}$	$5 \times 10^{-4} \times 0.5 = 2.5 \times 10^{-4}$	2.5×10^{-7}
$A_3\ B_3$	$1.2 \times 10^{-3} \times 0.4 = 4.8 \times 10^{-4}$	$1.7 \times 10^{-3} \times 0.3 = 5.1 \times 10^{-4}$	—	2.4×10^{-7}

The matrix in Table 8.4 gives the resulting detection probabilities. The first entry at each confidence level is the inherent false alarm probability (in parentheses) that establishes the threshold from which the inherent sensor detection probability is found. The second entry shows the results of the detection probability calculation for the confidence level.

The sensor system detection probability is calculated by inserting the individual sensor detection probabilities for the appropriate confidence levels into Eq. (8-7). Thus,

$$\text{System } P_d = 0.39 + 0.24 + 0.21 + 0.11 - 0.11 = 0.84, \tag{8-12}$$

assuming independence of sensor detection probabilities. The first four terms represent the detection probabilities of each of the four detection modes, while the last term represents the detection probability associated with $\{A_2\ B_2\ C_2\}$. As noted earlier, this term is incorporated twice in the sum operations and, therefore, has to be subtracted to get the correct system detection probability.

Therefore, the system detection probability requirement of 0.8 or greater and the false alarm probability requirement of 10^{-6} or less have been satisfied. If the requirements had not been met, another choice of conditional probabilities, inherent sensor false alarm probabilities, or number of samples integrated would be selected. Once this analysis shows that the system false alarm and detection probability requirements are satisfied, the sensor hardware or signal processing algorithms are modified to provide the required levels of discrimination.

Table 8.4 Detection probabilities for the confidence levels and detection modes of the three-sensor system.

Mode	Sensor *A*	Sensor *B*	Sensor *C*	Mode P_d
$A_1\ B_1\ C_1$	$(P_{fa}' = 1.6{\times}10^{-2})$ $0.80 \times 1.0 = 0.80$	$(P_{fa}' = 1.6{\times}10^{-2})$ $0.91 \times 1.0 = 0.91$	$(P_{fa}' = 1.0{\times}10^{-3})$ $0.53 \times 1.0 = 0.53$	0.39
$A_2\ C_2$	$(P_{fa}' = 1.6{\times}10^{-3})$ $0.85 \times 0.60 = 0.51$	—	$(P_{fa}' = 5.0{\times}10^{-4})$ $0.96 \times 0.50 = 0.48$	0.24
$B_2\ C_2$	—	$(P_{fa}' = 2.0{\times}10^{-3})$ $0.88 \times 0.50 = 0.44$	$(P_{fa}' = 5.0{\times}10^{-4})$ $0.96 \times 0.50 = 0.48$	0.21
$A_3\ B_3$	$(P_{fa}' = 1.2{\times}10^{-3})$ $0.95 \times 0.40 = 0.38$	$(P_{fa}' = 1.7{\times}10^{-3})$ $0.94 \times 0.30 = 0.28$	—	0.11

Table entry key: Each cell represents a confidence-level entry. Inherent false alarm probability (in parentheses) is shown on the top line of a cell. Detection probability is shown on the bottom line of a cell.

8.4.3 Observations

The use of multiple confidence levels produces detection modes with different false alarm probabilities, as well as detection probabilities. The relatively large detection probability of the $\{A_1\ B_1\ C_1\}$ mode is achieved with relatively large false alarm probabilities, i.e., 1.6×10^{-2} at confidence level 1 of Sensors *A* and *B*, and 1.0×10^{-3} at confidence level 1 of Sensor *C*. Although the smaller false alarm probabilities of the two-sensor modes reduce their detection probabilities, they do allow these modes to function optimally in the overall fusion process and contribute to the larger system detection probability. If only one confidence level was available for each sensor, the system detection probability would not be as large or the false alarm probability would not be as small.

Another interesting observation is the correspondence of the system detection and false alarm probabilities given by Eqs. (8-7) and (8-8). The fusion process increases the detection probability over that of a single-mode multiple sensor suite (e.g., 0.84 for the fusion system as compared to 0.39 for the $\{A_1\ B_1\ C_1\}$ mode). This is exactly compensated for by an increase in system false alarm probability (9.9×10^{-7} for the fusion system as compared to 2.6×10^{-7} for the $\{A_1\ B_1\ C_1\}$ mode).

8.5 Hardware implementation of voting logic sensor fusion

Figure 8.6 illustrates how AND and OR gates connected to the confidence level output states of each sensor give the required Boolean result for the system detection probability expressed by Eq. (8-7). When each sensor's confidence level is satisfied, a binary bit is set. Then when all the bits for any AND gate are set, the output of the AND gate triggers the OR gate and a validated target command is issued.

For example, the $\{A_1\ B_1\ C_1\}$ mode is implemented by connecting the lowest confidence level output from the three sensors to the same AND gate. The $\{A_2\ C_2\}$ and $\{B_2\ C_2\}$ modes are implemented by connecting the intermediate confidence-level outputs from Sensors A and C and Sensors B and C, respectively, to two other AND gates. Likewise, the $\{A_3\ B_3\}$ mode is implemented by connecting the highest confidence level outputs from Sensors A and B to the last AND gate.

8.6 Application example with singleton sensor detection modes

If it is known that a particular combination of sensors is robust enough to support single sensor detection modes, then another set of equations analogous to Eqs. (8-7) and (8-8) can be derived to model this situation. The detection modes shown in Table 8.5 are an example of this condition.

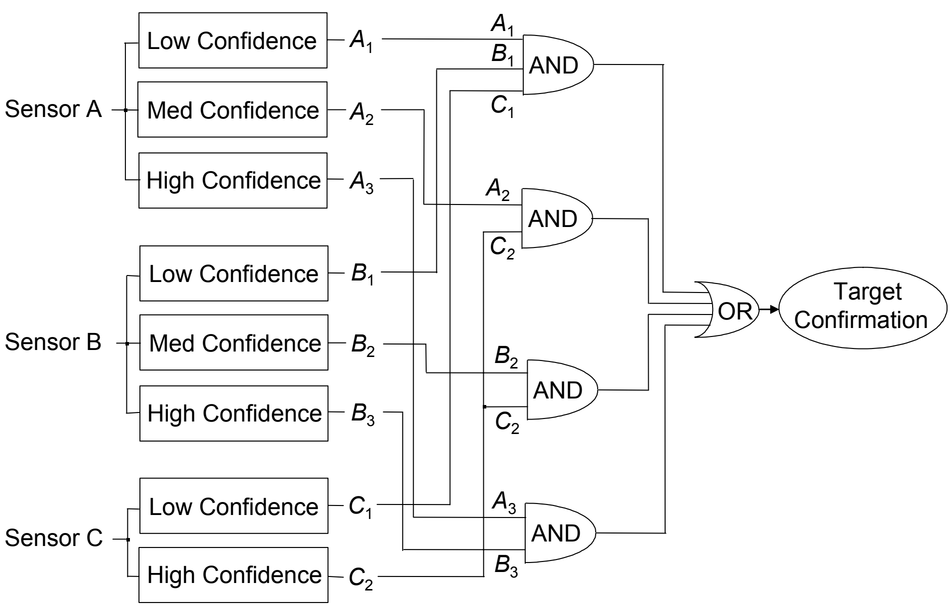

Figure 8.6 Hardware implementation for three-sensor voting logic fusion with multiple sensor detection modes.

Table 8.5 Detection modes that incorporate single sensor outputs and multiple confidence levels in a three-sensor system.

Mode	Sensor and Confidence Level		
	A	***B***	***C***
ABC	A_1	B_1	C_1
AC	A_2	–	C_2
BC	–	B_2	C_2
AB	A_2	B_2	–
A	A_3	–	–
B	–	B_3	–

The system detection probability is now expressed as

$$\text{System } P_d = P_d\{A_1\, B_1\, C_1 \text{ or } A_2\, C_2 \text{ or } B_2\, C_2 \text{ or } A_2\, B_2 \text{ or } A_3 \text{ or } B_3\}. \tag{8-13}$$

Applying the same simplifying union and intersection relations given by Eqs. (8-4) and (8-5) allows Eq. (8-13) to be reduced to

$$\begin{aligned}\text{System } P_d = {} & P_d\{A_1\, B_1\, C_1\} + P_d\{A_2\, C_2\} + P_d\{B_2\, C_2\} + P_d\{A_2\, B_2\} \\ & + P_d\{A_3\} + P_d\{B_3\} - P_d\{A_3\, B_3\} - 2P_d\{A_2\, B_2\, C_2\}.\end{aligned} \tag{8-14}$$

If the individual sensor detection probabilities are independent of each other, then

$$\begin{aligned}\text{System } P_d = {} & P_d\{A_1\}\, P_d\{B_1\}\, P_d\{C_1\} + P_d\{A_2\}\, P_d\{C_2\} + P_d\{B_2\}\, P_d\{C_2\} \\ & + P_d\{A_2\}\, P_d\{B_2\} + P_d\{A_3\} + P_d\{B_3\} - P_d\{A_3\}\, P_d\{B_3\} \\ & - 2P_d\{A_2\}\, P_d\{B_2\}\, P_d\{C_2\}.\end{aligned} \tag{8-15}$$

Similarly, the system false alarm probability is given by

$$\begin{aligned}\text{System } P_{fa} = {} & P_{fa}\{A_1\}\, P_{fa}\{B_1\}\, P_{fa}\{C_1\} + P_{fa}\{A_2\}\, P_{fa}\{C_2\} + P_{fa}\{B_2\}\, P_{fa}\{C_2\} \\ & + P_{fa}\{A_2\}\, P_{fa}\{B_2\} + P_{fa}\{A_3\} + P_{fa}\{B_3\} - P_{fa}\{A_3\}\, P_{fa}\{B_3\} \\ & - 2P_{fa}\{A_2\}\, P_{fa}\{B_2\}\, P_{fa}\{C_2\}.\end{aligned} \tag{8-16}$$

The six positive terms correspond to the six detection modes, while the two negative terms eliminate double counting of intersections that occurs when summing the probabilities of the detection modes.

The combination of AND and OR gates that implements the Boolean logic for this particular combination of sensor outputs is shown in Figure 8.7. The single

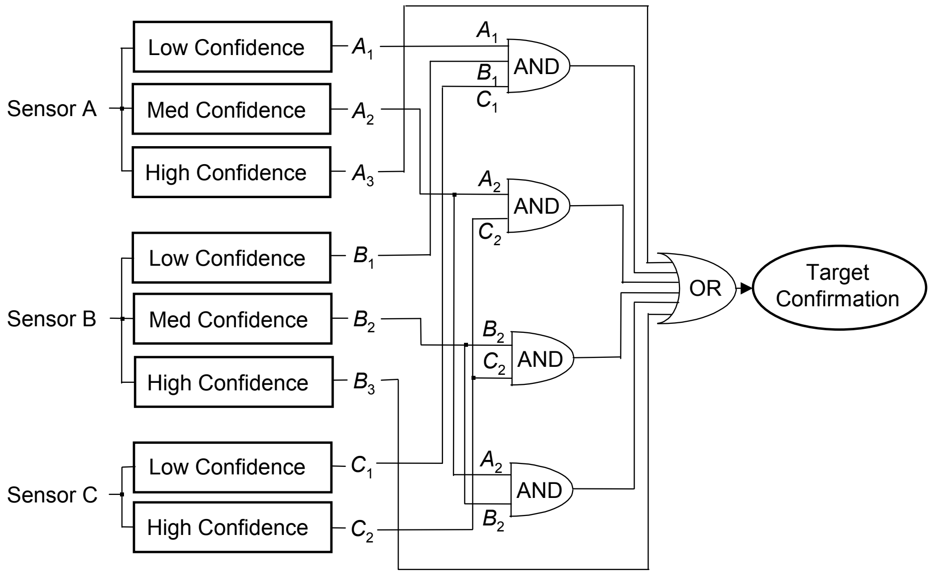

Figure 8.7 Hardware implementation for three-sensor voting logic fusion with single sensor detection modes.

sensor detection modes are connected directly to the OR gate, while the multiple sensor detection modes are connected to the OR gate through AND gates as in the earlier example.

Voting logic fusion has found application to antitank landmine detection using four, six, and eleven detection-mode fusion algorithms.[9] The three sensors supplying data to the fusion process are a forward looking infrared camera, a minimum metal detector, and a ground penetrating radar. The eleven detection-mode algorithm, which allows high-confidence single sensor object confirmations and combinations of low and medium confidence two sensor object confirmations, performs as well as a baseline algorithm with respect to detection and false alarm probabilities. The performance is relatively insensitive to the selected confidence thresholds.

8.7 Comparison of voting logic fusion with Dempster-Shafer evidential theory

In voting logic fusion, individual sensor information is used to compute detection probabilities that are combined according to a Boolean algebra expression. The principle underlying voting fusion is the combining of logical values representing sensor confidence levels, which in turn are based on predetermined detection probabilities for an object. Since weather, terrain, and countermeasures will generally affect sensors that respond to different signature-generation phenomena to varying degrees, the sensors can report different detection probabilities for the same object.

In Dempster-Shafer, sensor information is used to compute the amount of knowledge or probability mass associated with the proposition that an object is, or is not, of a particular type or combination of types. The sensors, in this case, combine compatible knowledge of the object type, using Dempster's rule to compute the probability mass associated with the intersection (or conjunction) of the propositions in the observation space.

The probability mass assignments by the sensors to propositions in Dempster-Shafer are analogous to the confidence level assignments to target declarations in voting fusion. However, whereas voting fusion combines the sensor confidence levels through logic gates, Dempster-Shafer combines the probability masses through Dempster's rule.

Comparisons of the information needed to apply classical inference, Bayesian inference, Dempster-Shafer evidential theory, artificial neural networks, voting logic, and fuzzy logic fusion algorithms to a target identification application are summarized in Chapter 11.

8.8 Summary

Boolean algebra has been applied to derive an expression for the system detection probability of a three-sensor system operating with sensors that are sensitive to independent signature-generation phenomenologies. Detection modes consisting of combinations of two and three sensors have been proposed to provide robust performance in clutter, inclement weather, and countermeasure environments. Sensor detection modes are defined through multiple confidence levels in each sensor. Elimination of single-sensor target-detection modes can be implemented to reduce sensitivity to false targets and countermeasures. The ability to detect targets with more than one combination of sensors increases the likelihood of suppressed signature target detection.

Nonnested confidence levels allow the detection probability to be independently selected and implemented at each sensor confidence level. The false alarm probabilities corresponding to the sensor confidence levels can be established in two ways. The first uses a common threshold to define the inherent false alarm probability at all the confidence levels of a particular sensor. The second allows the detection threshold, and hence inherent false alarm probability, to differ at each confidence level. The transformation of confidence-level space into detection space is accomplished by multiplying two factors. The first factor is the inherent detection probability that characterizes the sensor confidence level. The second factor is the conditional probability that detection with that confidence level occurs given detection by the sensor. Analogous transformations permit the false alarm probability to be calculated at the confidence levels of each sensor. The simple hardware implementation for voting logic fusion follows from the Boolean description of the sensor-level fusion process and leads to a low-cost implementation for the fusion algorithm.

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