

Standard notations for Deep Learning

This document has the purpose of discussing a new standard for deep learning mathematical notations.

1 Neural Networks Notations.

General comments:

- superscript (i) will denote the i^{th} training example while superscript [l] will denote the l^{th} layer

Sizes:

- m : number of examples in the dataset
- n_x : input size
- n_y : output size (or number of classes)
- $n_h^{[l]}$: number of hidden units of the l^{th} layer

In a for loop, it is possible to denote $n_x = n_h^{[0]}$ and $n_y = n_h^{[\text{number of layers} + 1]}$.

- L : number of layers in the network.

Objects:

- $X \in \mathbb{R}^{n_x \times m}$ is the input matrix
- $x^{(i)} \in \mathbb{R}^{n_x}$ is the i^{th} example represented as a column vector

- $Y \in \mathbb{R}^{n_y \times m}$ is the label matrix

- $y^{(i)} \in \mathbb{R}^{n_y}$ is the output label for the i^{th} example

- $W^{[l]} \in \mathbb{R}^{\text{number of units in next layer} \times \text{number of units in the previous layer}}$ is the weight matrix, superscript [l] indicates the layer

- $b^{[l]} \in \mathbb{R}^{\text{number of units in next layer}}$ is the bias vector in the l^{th} layer

- $\hat{y} \in \mathbb{R}^{n_y}$ is the predicted output vector. It can also be denoted $a^{[L]}$ where L is the number of layers in the network.

Common forward propagation equation examples:

- $a = g^{[l]}(W_x x^{(i)} + b_1) = g^{[l]}(z_1)$ where $g^{[l]}$ denotes the l^{th} layer activation function

- $\hat{y}^{(i)} = \text{softmax}(W_h h + b_2)$

- General Activation Formula: $a_j^{[l]} = g^{[l]}(\sum_k w_{jk}^{[l]} a_k^{[l-1]} + b_j^{[l]}) = g^{[l]}(z_j^{[l]})$

- $J(x, W, b, y)$ or $J(\hat{y}, y)$ denote the cost function.

Examples of cost function:

- $J_{CE}(\hat{y}, y) = - \sum_{i=0}^m y^{(i)} \log \hat{y}^{(i)}$

Cross-Entropy Loss (classification):
 - 1. single class (0/1, sigmoid activation)
 - 2. multi-class (one-hot encoding, softmax activation)
- $J_1(\hat{y}, y) = \sum_{i=0}^m |y^{(i)} - \hat{y}^{(i)}|$

Mean squared error (regression)

2 Deep Learning representations

For representations:

- nodes represent inputs, activations or outputs
- edges represent weights or biases

Here are several examples of Standard deep learning representations

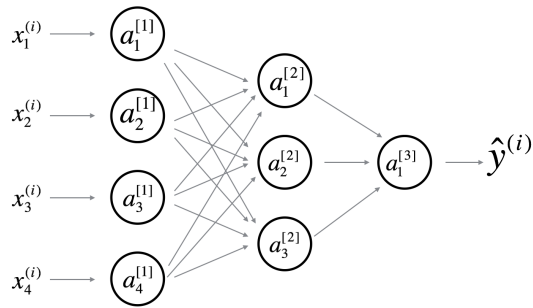


Figure 1: Comprehensive Network: representation commonly used for Neural Networks. For better aesthetic, we omitted the details on the parameters ($w_{ij}^{[l]}$ and $b_i^{[l]}$ etc...) that should appear on the edges

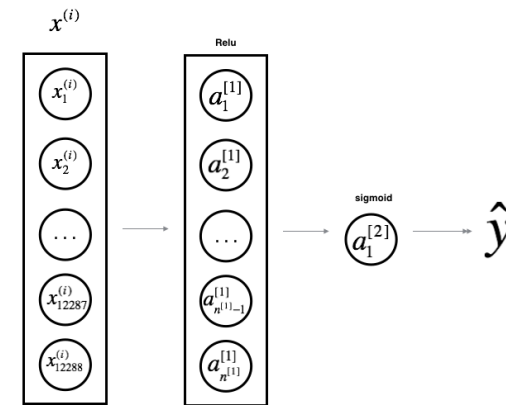


Figure 2: Simplified Network: a simpler representation of a two layer neural network, both are equivalent.