STATS 102C - HW #5

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Problem 1

(a)

```
P <- matrix(c(0.6, 0.6, 0.2, 0.3, 0.3, 0.2, 0.1, 0.1, 0.6), nrow = 3)
P
```

```
##      [,1] [,2] [,3]
## [1,]  0.6  0.3  0.1
## [2,]  0.6  0.3  0.1
## [3,]  0.2  0.2  0.6
```

(b)

```
c(0, 0, 1) %*% (P %*% P)
```

```
##      [,1] [,2] [,3]
## [1,] 0.36 0.24  0.4
```

```
# rainy, cloudy, sunny
```

(c)

Yes the Markov chain is irreducible because all the states communicate with each other. And yes it is aperiodic since each element in the matrix is positive.

The stationary distribution:

```
n <- 8
P2 <- P
for (i in 1:(n-1)) {
  P2 <- P2 %*% P2
}
P2
```

```
##      [,1] [,2] [,3]
## [1,] 0.52 0.28  0.2
## [2,] 0.52 0.28  0.2
## [3,] 0.52 0.28  0.2
```

```
# this is the P^{2^n} = P^256, large number
```

Problem 2

$P(X_{t+1} = k + 1|X_t = k) = \frac{N-k}{N}\frac{k}{N}$
$P(X_{t+1} = k - 1|X_t = k) = \frac{k}{N}\frac{N-k}{N}$
$P(X_{t+1} = k|X_t = k) = \frac{k}{N}\frac{k}{N} + \frac{N-k}{N}\frac{N-k}{N}$

Problem 3

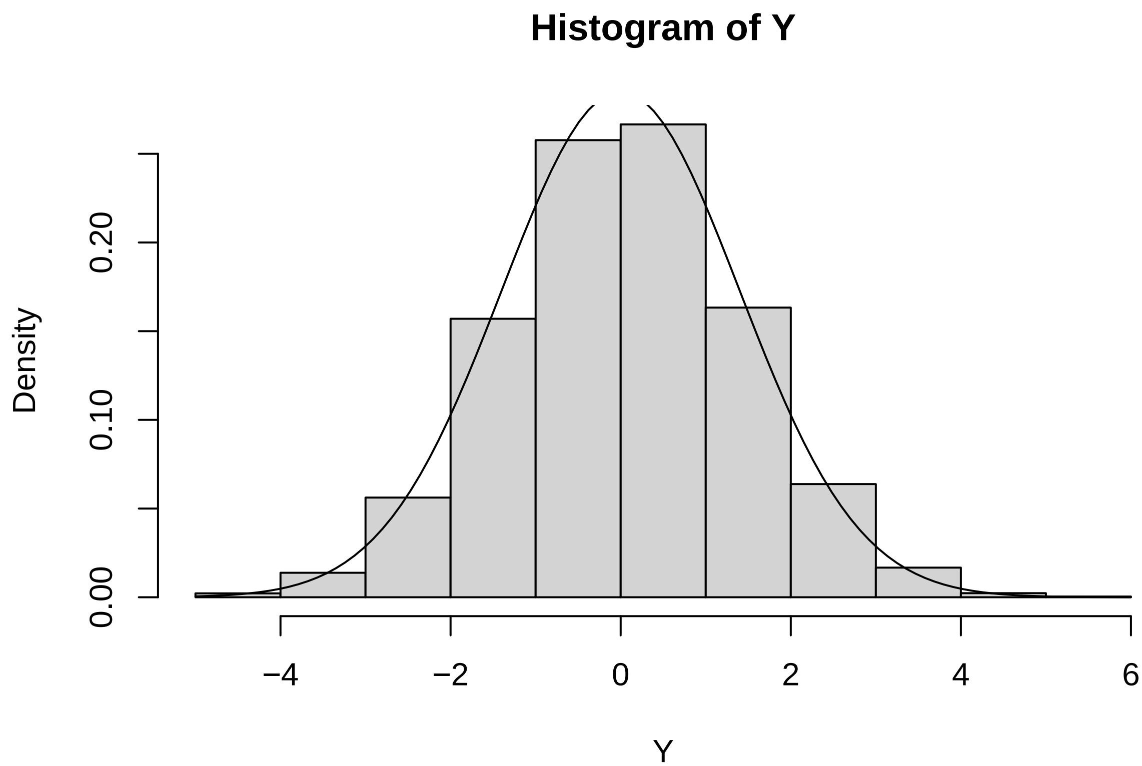
```
alpha <- 0.7
Y <- numeric(10^4)
Y[1] <- rnorm(1)
for(i in 2:10^4) {
  Y[i] <- alpha * Y[i-1] + rnorm(1)
}

mean(Y)
```

```
## [1] 0.04790337
```

```
sqrt(var(Y))
```

```
## [1] 1.411518
```

```
hist(Y, probability = T)
curve(dnorm(x, sd=sqrt(1/(1-alpha^2))), add = T)
```



Problem 4

(a)

$\pi(\lambda|y) = \frac{f(y|\lambda)*\pi(\lambda)}{\int f(y|\lambda)*\pi(\lambda)d\lambda}$

(b)

By (a), $E(\lambda|y) = \frac{\alpha+\sum_i y_i}{\beta+n}$

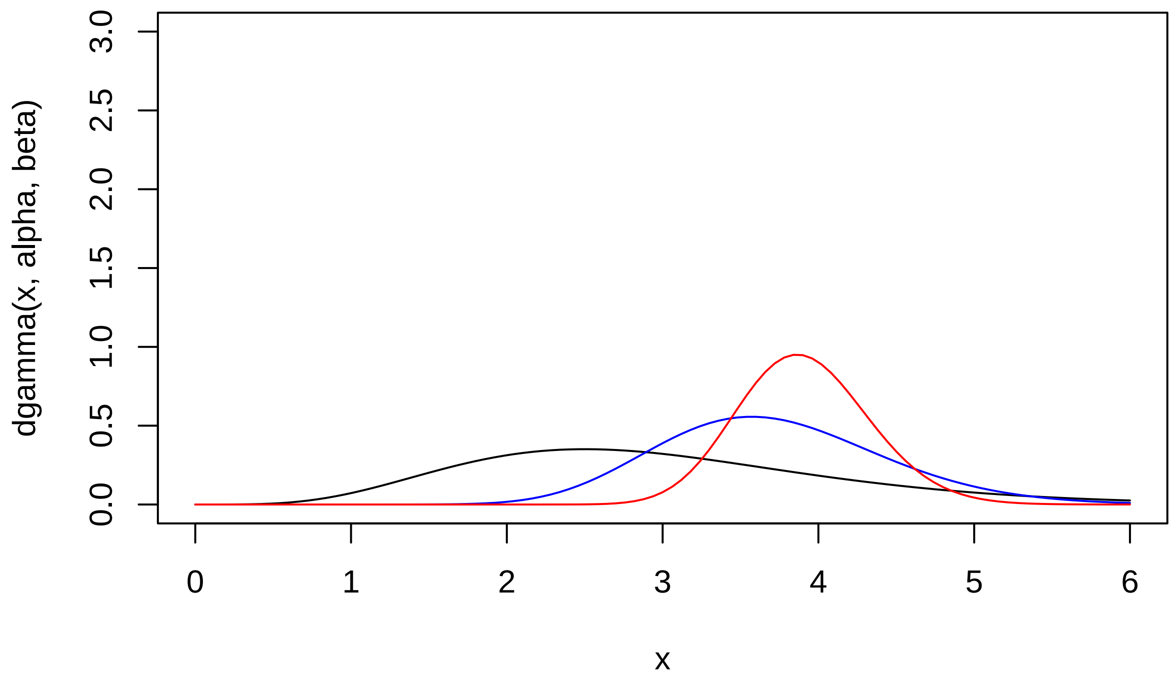
(c)

Take the limit of (b)'s results. When n = ∞, it means we have known exactly $f(y|\lambda)$, so λ is known

(d)

```
alpha <- 6
beta <- 2
E_lambda_0 <- alpha / beta
```

```r
n <- 5
Ts <- 20
alpha_1 <- Ts + alpha
beta_1 <- n + beta

n <- 20
Ts <- 80
alpha_2 <- Ts + alpha
beta_2 <- n + beta

curve(dgamma(x, alpha, beta), xlim = c(0,6), ylim = c(0,3))
curve(dgamma(x, alpha_1, beta_1), add = T, col = "blue")
curve(dgamma(x, alpha_2, beta_2), add = T, col = "red")
```



(e)

```r
c(qgamma(0.025, alpha, beta), qgamma(0.975, alpha, beta))
```

```
## [1] 1.100947 5.834166
```

```r
c(qgamma(0.025, alpha_1, beta_1), qgamma(0.975, alpha_1, beta_1))
```

```
## [1] 2.426295 5.272133
```

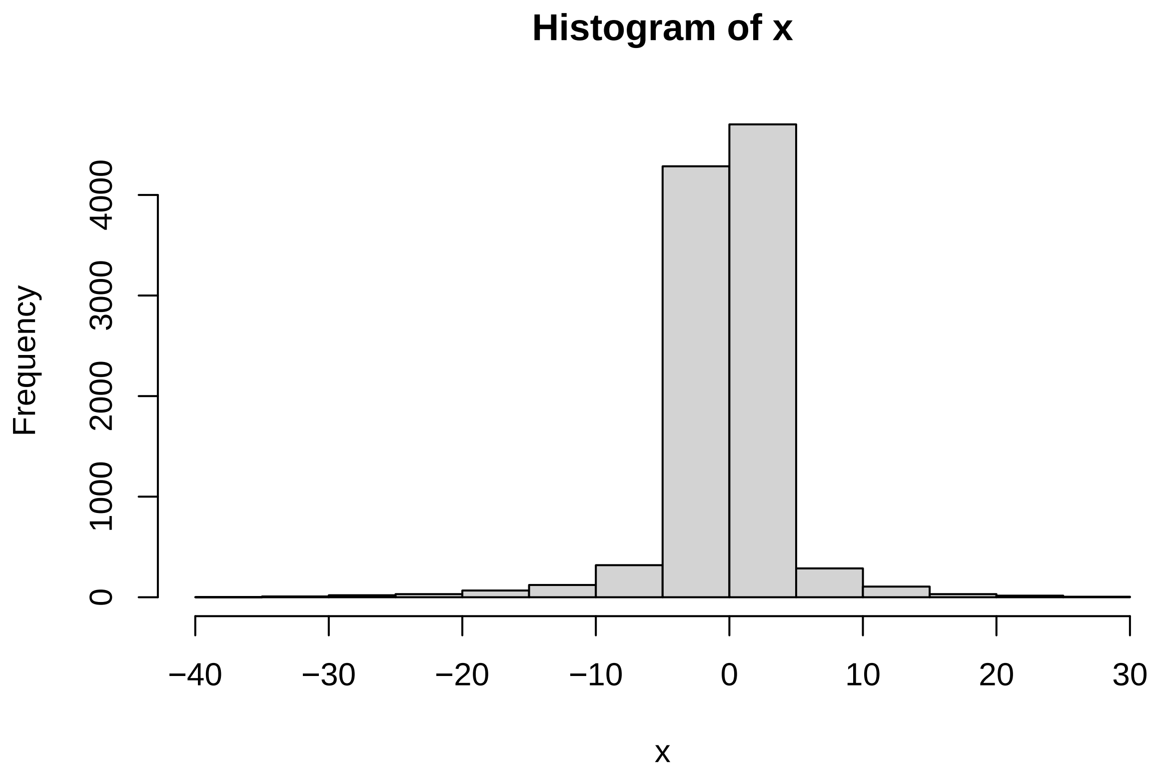
```r
c(qgamma(0.025, alpha_2, beta_2), qgamma(0.975, alpha_2, beta_2))
```

```
## [1] 3.126768 4.777446
```

Problem 5

(a) and (b)

```r
m <- 10000
patholog <- function(x) {
  1 / (pi * (1 + x^2))
}

x <- numeric(m)
sigma <- 3
x[1] <- rnorm(1, 1, sigma)
for(i in 2:m) {
  xt <- x[i-1]
  y <- rnorm(1, mean = xt, sigma)
  r <- patholog(y) / patholog(xt) * dnorm(xt, mean=y, sigma)/ dnorm(y, mean=xt, sigma)
  u <- runif(1)
  if (u <= r) {
    x[i] <- y
  }
  else {
    x[i] <- xt
  }
}
hist(x)
```



(c)

```
p <- seq(0.1, 0.9, 0.1)
burn <- 1000
xb <- x[(burn+1):m]
Q <- quantile(xb, p)
round(rbind(Q, qcauchy(p)), 3)
```

```
##       10%    20%    30%    40%   50%   60%   70%   80%   90%
## Q -2.949 -1.375 -0.712 -0.298 0.039 0.368 0.739 1.275 2.460
##   -3.078 -1.376 -0.727 -0.325 0.000 0.325 0.727 1.376 3.078
```