EE 595 Project Phase 3

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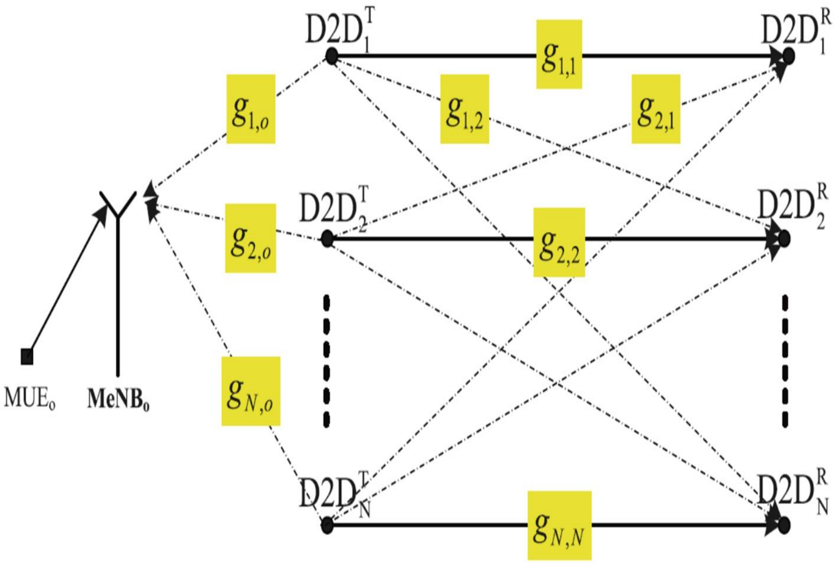
Problem Description



State Space

The state space is defined based on the intra-tier and inter-tier interferences and the energy usage dynamics

- The energy usage dynamics: $dE_i(t) = -p_i(t)dt$

  which means that energy $E_i(t)$ of the battery decreases with the transmit power consumption $p_i(t)$.

- The interference dynamics:

$$\mu_i(t) = I_{i\to}(t) + I_{i\to o}(t)$$

$$\mu_i(t) = \sum_{j=1, j\neq i}^{N} p_i(t) g_{i,j}(t) + p_i(t) g_{i,o}(t)$$

$$\varepsilon_i(t) = \sum_{j=1, j\neq i}^{N} g_{i,j}(t) + g_{i,o}(t)$$

$$\mu_i(t) = p_i(t)\varepsilon_i(t)$$

$$d\mu_i(t) = \varepsilon_i(t) dp_i(t) + p_i(t)\partial_t \varepsilon_i(t)$$

So, we define the state space for player i: $s_i(t) = [E_i(t), \mu_i(t)], \quad i \in \mathcal{N}$

Cost function

Generally, the communication performance is related to the SINR $\gamma_i(t)$

Introduce identical SINR threshold $\gamma_{th}(t)$ (predefined to meet the communication requirements)

The cost function is given by

$$c_i(t) = (\gamma_i(\mathrm{t}) - \gamma_{\mathrm{th}}(\mathrm{t}))^2 + \lambda p_i(t)$$

Here, the player i will minimize the SINR difference from the threshold and the power consumption at any time t .

We consider the problem that each D2D pair i will determine the optimal power control policy to minimize the cost function

$$\mathcal{Q}^{\star}{}_{i}(t) = arg \min_{p_i(t)} E\left[\int_0^T c_i(t)\,dt + c_i(T)\right]$$

None of the players can have a lower cost by unilaterally deviating from the current power control policy. The Nash equilibrium of the above power control differential game can be obtained by solving the HJB equation associated with each player in the optimal control theory.

$$-\partial_t u_i(t, s_i(t)) = \min_{p_i(t)} \left[c_i(t, s_i(t), p_i(t)) + \partial_t s_i(t) \cdot \nabla u_i(t, s_i(t))\right]$$ the HJB equation

Here, we define the value function as follows:

$$u_i(t, s_i(t)) = \min_{p_i(t)} E\left[\int_t^T c_i(t)\,dt + u_i(T, s_i(t))\right], t \in [0, T]$$

The value function gives the minimum cost for a given dynamic system with an associated cost function

Introduce the mean field game for a dense D2D network system

On one hand, an FPK type equation evolves forward in time that governs the evolution of the density function of the agents.
On the other hand, an HJB type equation evolves backward in time that governs the computation of the optimal path for each agent.

- **Mean Field**: This is a critical concept in the defined power control MFG, which is a statistical distribution of the defined two-dimensional states. Given the state space $s_i(t) = [E_i(t), \mu_i(t)]$ in the defined power control MFG, we define the mean field $m(t, s)$ as

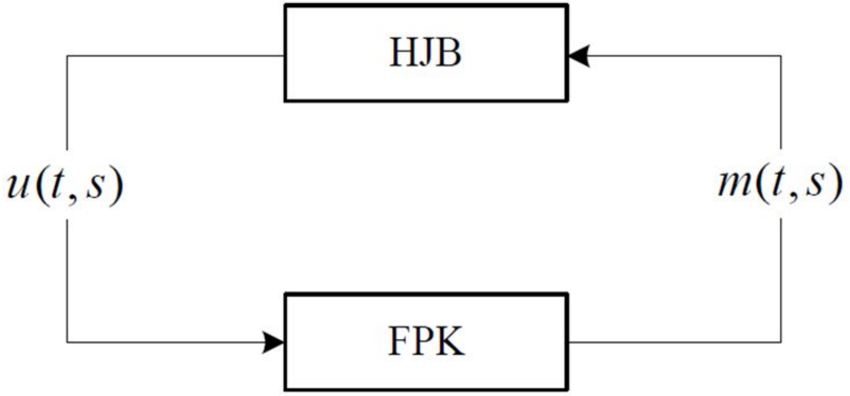
$$m(t, s) = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} 1_{\{s_i(t)=s\}}$$

where 1 function denotes an indicator function which returns one if the given condition is true and zero, otherwise. For a given time instant, the mean field is the probability distribution of the states over the set of players.

- **FPK Equation** Based on the above definition of mean field, we derive the FPK equation. The FPK equation of the defined MFG is given by

$$\partial_t m(t, s) + \nabla (m(t, s) \cdot \partial_t s(t)) = 0$$

where s is the state.



Data Simulation

The duration of one LTE radio frame is 10 ms, and for 500 frames, $t = 5s$.

We also pick, $E_{\max}$ to be 0.5 J.

The tolerable interference level of each player $\mu_{\max}$ is assumed to be $5.8 \times 10^{-6}\ W$.

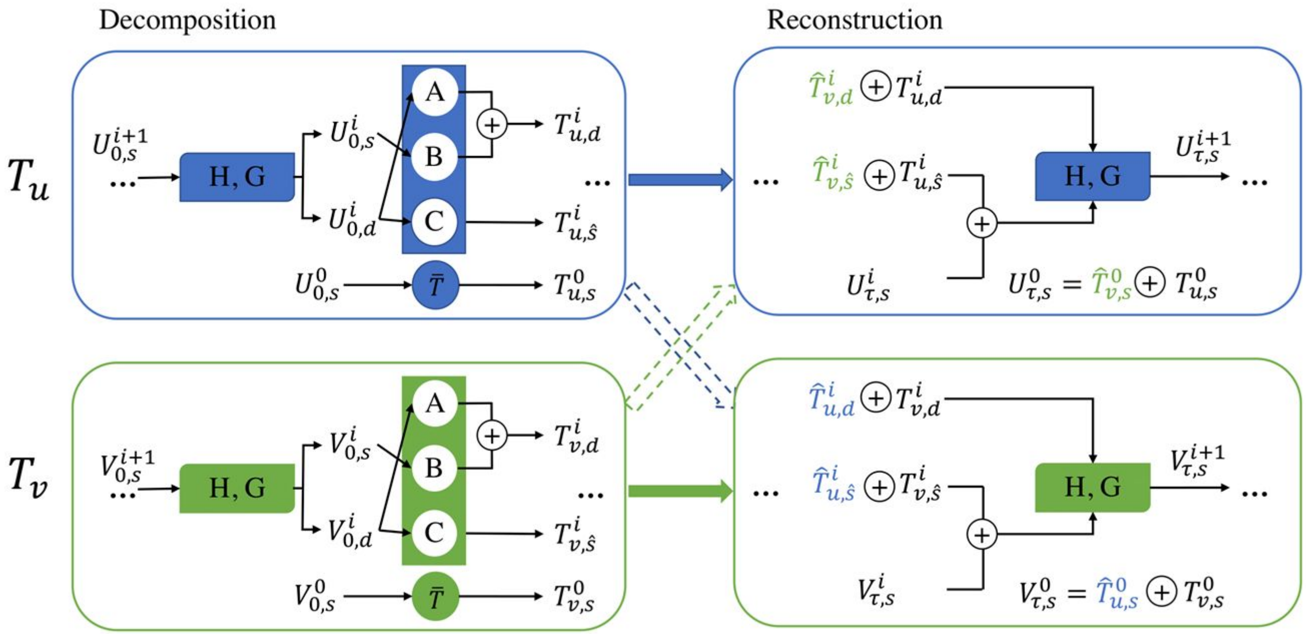
The HJB equation governs the computation of the optimal control path of the player, while the FPK equation governs the evolution of the mean field function of players. Here, the HJB and FPK equations are termed as the backward and forward functions, respectively

$$\partial_t m(t,s) + \nabla\left(m(t,s)\cdot \partial_t s(t)\right) = 0 \qquad \textbf{FPK equation}$$

$$-\partial_t u(t,s(t)) = \min_{p(t)} \left[c(t,s(t),p(t)) + \partial_t s(t)\cdot \nabla u(t,s(t))\right] \qquad \textbf{HJB equation}$$

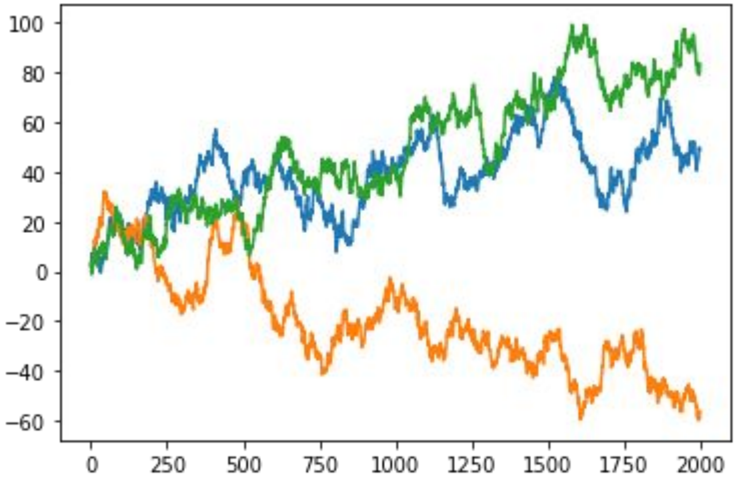
Coupled Multiwavelet Neural Operator

Project the functions into multiwavelet space through multiwavelet transform first and deal with the scaling/ wavelet coefficients from two coupled equations in the wavelet domain.



Experiment Setting

Generate data for the experimental simulation, containing the state space of one player i and the mean field of the power control MFG problem.



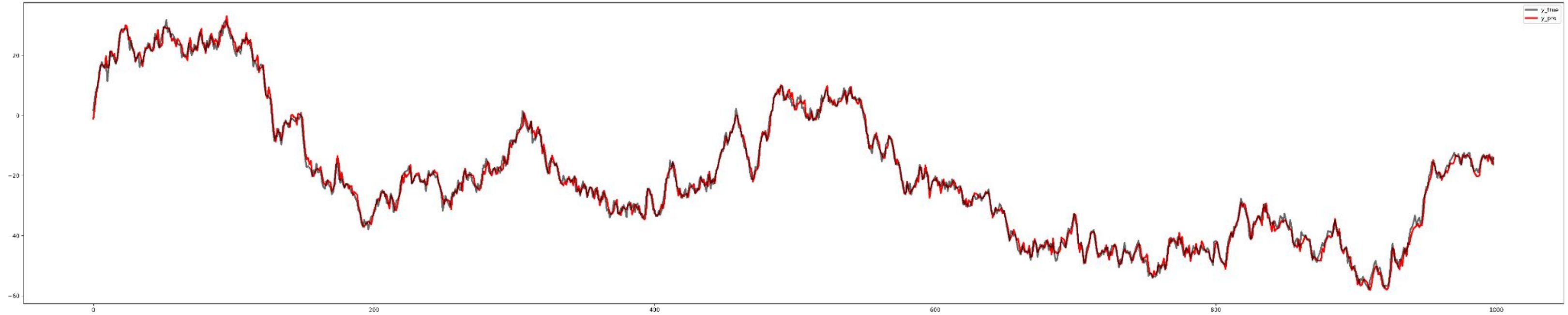
state space of one D2D link

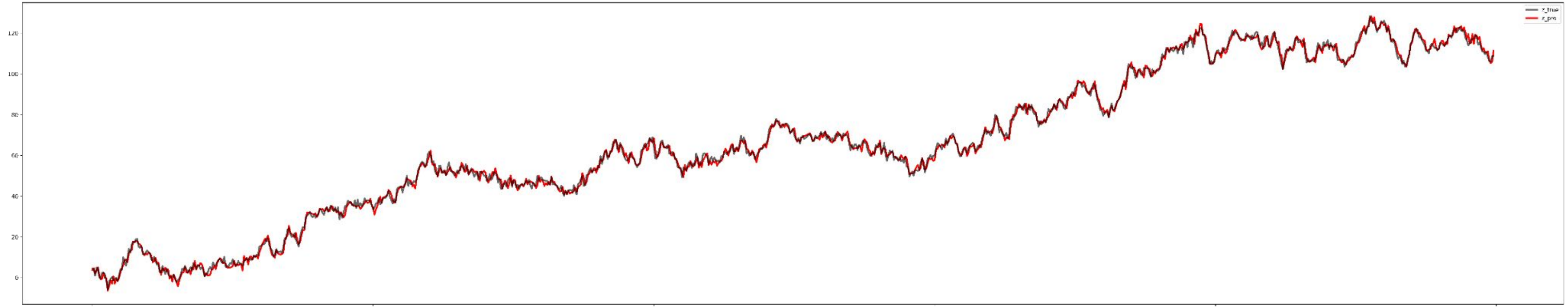
$$s_i(t) = [E_i(t), \mu_i(t)]$$

Mean field parameter

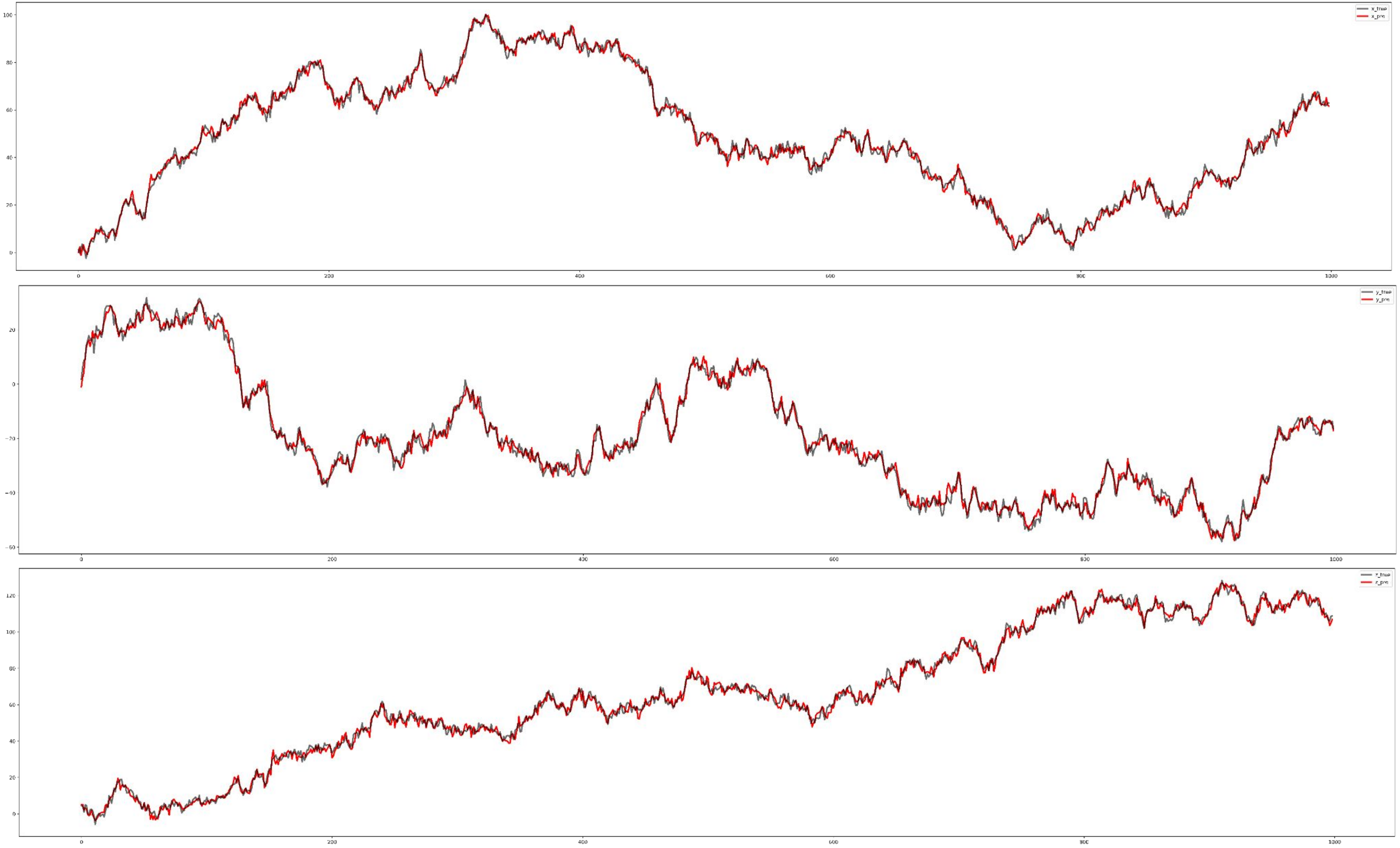
$$m(t, s) = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} 1_{\{s_i(t)=s\}}$$

Experiment Results of FNO model





Experiment Results of CMWNO model



Inference with C++

- Serialize Pytorch model
- Multithreaded C++ inference application depending on Pytorch C++ distribution
  - All the workers share the same input, output and model
  - Each worker works on different parts of the input and output
  - Synchronize all workers after the inferences are done
  - Polymorphism so that the application can be extended to support other models