Distributed Power Control Method for Ultra-dense D2D Communications in 5G/6G Networks

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May 7, 2023

Abstract

Ultra-dense networks are networks with high cell density. They enable quality communication in base-station-dense, user-dense, or interference-dense networks through proper interference and energy management.(Banez et al., 2021) This paper models energy management and interference dynamics in ultra-dense networks as a mean field game problem, and reformulates this problem as coupled partial differential equations. It then trains a neural operator to solve the coupled partial differential equations computationally. Ultimately, this paper develops an optimal distributed power control strategy in D2D scenarios.

1 Introduction

With the advancement of telecommunication technologies, wireless networking (e.g., 5G, 6G, and Beyond) has become ubiquitous owing to the great demand of pervasive mobile applications. The convergence of computing, communications, and media will allow users to communicate with each other and access any content at anytime, anywhere. A critical issue is devising distributed and dynamic algorithms for ensuring a robust network operation over time-varying and heterogeneous environments. Therefore, in order to support tomorrow's wireless services, it is essential to develop efficient mechanisms that provide an optimal cost-resource-performance tradeoff and that constitute the basis for next generation ubiquitous and autonomic wireless networks.(Yang et al., 2018) Device-to-device (D2D) communications provide significant performance enhancement in terms of spectrum and energy efficiency by proximity and frequency reuse. However, such performance enhancement is largely limited by mutual interference and energy availability, in particular, in ultra-dense D2D networks. In our work, we consider both interference dynamics and available energy of the generic device, and then we formulate an MFG theoretic framework with the interference mean-field approximation. We formulate the cost function by combining both the performance and cost for transmit power.Within the MFG framework, we derive the related Hamilton-Jacobi-Bellman (HJB) and Fokker-Planck-Kolmogorov (FPK) equations.(Goswami et al., 2022)

Recently, mean field game (MFG) theory has been introduced into complex Cyber Physical Systems (CPS) to study the strategic decisions of a large number of symmetric, indistinguishable, and negligible interacting agents.(De Paola et al., 2019) Building on statistical mechanics principles and infusing them into the study of strategic decision making, MFGs investigate the dynamics of a large population of interacting agents seen as particles in a thermodynamic gas. Simply speaking, MFGs consist of (i) a Fokker- Planck-Kolmogorov(FPK)(one PDE) that describes the dynamics of the aggregate distribution of agents, which is coupled to (ii) a Hamilton–Jacobi–Bellman equation (HJB)(another PDE) prescribing the optimal control of an individual.(Lasry and Lions, 2006)

Since the MFG problem is represented by a coupled PDE, we introduce neural operators to solve the MFG.Neural operators focus on learning the mapping between infinite-dimensional spaces of functions.(Li et al., 2020b) The critical feature for neural operators is to model the integral operator namely the Green's function through various neural network architectures. (Li et al., 2020a) The multi-wavelet neural operators(Gupta et al., 2021) utilize the non-standard form of the multi-wavelets to represent the integral operator through 4 neural networks in the Wavelet space. The neural operators are completely data-driven and resolution independent by learning the mapping between the functions directly, which can achieve the state-of-the-art performance on solving PDEs and initial value problems (IVPs).(Xiao et al., 2023) To deal with coupled PDEs in the coupled system and be data-efficient, we aim to decode the various interaction scenarios inside the neural operators.(Lu et al., 2022)

1.1 Novel Contributions

Firstly, in our project, we present a rational power control policy design that can help save energy and mitigate interference. On the one hand, more rational power use means battery life can last longer. On the other hand, all players implement power control with the impact of interference on other players taken into consideration. This is a win-win situation in which the wireless environment has lower interference. We present the system model and formulate the MFG problem for ultra-dense D2D power control.

Secondly, for coupled differential equations like MFG problem, we propose a coupled neural operator(CMWNO) learning scheme to solve it. Utilizing multi-wavelet transform, CMWNO can deal with the interactions between the kernels of coupled differential equations in the Wavelet space. Specifically, we first yield the representation of coupled information during the decomposition process of multi-wavelet transform. Then, the decoupled representation can interact separately to help the operators' reconstruction process. In addition, we propose a dice strategy to mimic the information interaction during the training process.

2 System Model and Problem Formulation

2.1 System Model Setting

We consider an ultra-dense D2D communications network where the D2D communication pairs share uplink resources with some existing macrocell user equipments (MUEs). We assume that there are N D2D pairs sharing the same channel with the cellular uplink, as shown in Fig. 1. We consider the effects of both the interference of a generic D2D transmitter introduced to others, and all others' interference cause to a generic D2D receiver. There are termed as intra-tier and inter-tier interference, respectively. For instance, the D2D transmitter $D2D_1^T$ communicating with its receiver introduces intra-tier interference to the other D2D receivers as shown in Fig. 1. Meanwhile, due to the

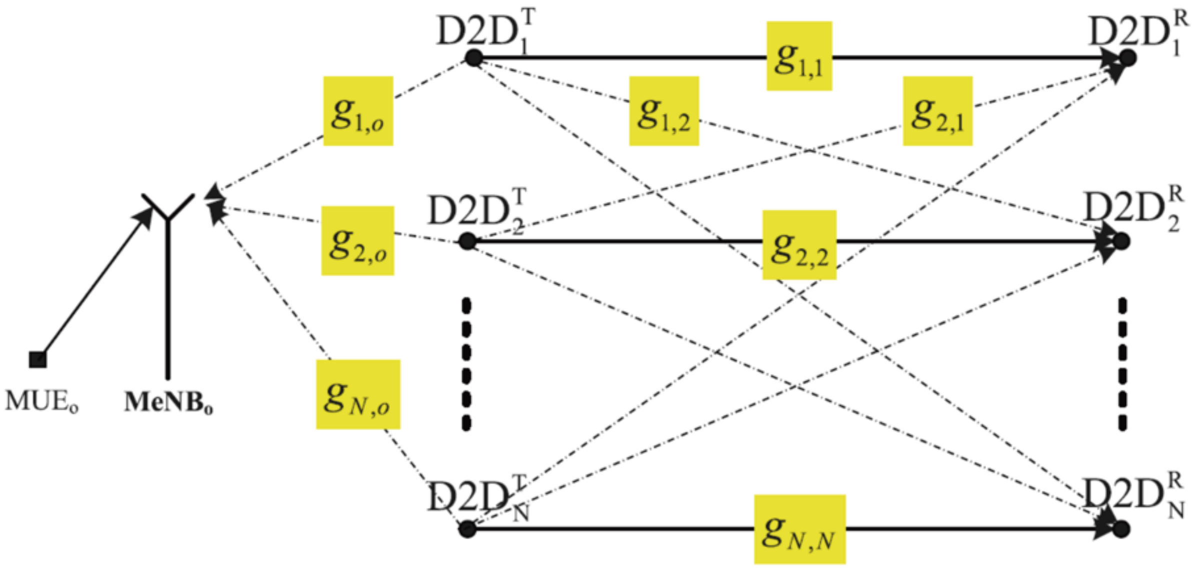

Figure 1: A D2D network with a large number of D2D links.

full frequency reuse, $D2D_1^T$ causes inter-tier interference link to the MUE. Here, we define the interference introduced by player i, to others j, $j \neq i$ at time t as

$$I_{i\rightarrow}\left(t\right) = \sum_{j=1, j\neq i}^{N} p_i\left(t\right) g_{i,j}\left(t\right) \tag{1}$$

where $p_i\left(t\right)$ is the transmit power corresponding to D2D pair i, and $g_{i,j}\left(t\right)$ defines the channel gain from the D2D pair i's transmitter to the D2D pair j 's receiver, $j \neq i$. Therefore, (1) gives the interference introduced by player i, to all other D2D receivers at time t.

At the same time, the transmission of player i also introduces interference to player o, where we define the only existing uplink MUE to Macrocell evolved node B (MeNB) pair as player o. We define the inter-tier interference introduced by player i to player o as

$$I_{i\rightarrow o}\left(t\right) = p_i\left(t\right) g_{i,o}\left(t\right) \tag{2}$$

where $g_{i,o}\left(t\right)$ is channel gain between D2D pair i to macrocell link o. Finally, the interference perceived by the D2D pair i at time t , which is the interference introduced by other D2D links to the generic D2D link i, is given as

$$I_{\rightarrow i}\left(t\right) = \sum_{j=1, j\neq i}^{N} p_j\left(t\right) g_{j,i}\left(t\right) \tag{3}$$

Here, we assume that orthogonal channels are used for different MUEs, and we do not consider any power control policy at the macrocell layer. Our focus is on the power control policy for the D2D transmitters.

The achieved signal-to-interference-plus-noise ratio ($SINR$) at the receiver of D2D pair i at time t is

$$\gamma_i\left(t\right) = \frac{p_j\left(t\right) g_{i,i}\left(t\right)}{I_{i\rightarrow}\left(t\right) + \sigma^2} \tag{4}$$

where σ^2 is the thermal noise power. With the above definition of $SINR$, the power control problem can be summarized as follows: each player i will determine the optimal power control policy $\mathcal{Q}^{\star}{}_i\,(0 \rightarrow T)$ with the interference $I_{i\rightarrow}$ introduced to others, the interference introduced by others $I_{\rightarrow i}$, and remaining energy. The D2D transmitters adapt their transmit power during the time interval of $t \in [0, T]$. This power control problem can be formulated as a differential game due to the interference dynamics and the energy dynamics. In this differential game, the interference from other D2D links to the generic D2D receiver will be considered in the cost function while the interference introduced by the generic D2D transmitter to other D2D links will be regarded as one of the two state variables. The other state variable will be the remaining energy at the generic D2D transmitter.

2.2 Differential Game Model for Power Control

The differential game model for power control in the D2D network described above is defined as follows:

The D2D differential power control game G_s for D2D transmitters is defined by a 5-tuple: $G_s = (\mathcal{N}, \{\mathcal{P}_i\}_{i\in\mathcal{N}}, \{\mathcal{S}_i\}_{i\in\mathcal{N}}, \{\mathcal{Q}_i\}_{i\in\mathcal{N}}, \{c_i\}_{i\in\mathcal{N}})$, where

- **Player set** $\mathcal{N}$: $\mathcal{N} = \{1, ..., N\}$ represents the player set of densely-deployed D2D communication pairs. They are rational policy makers in the D2D game, where the number N of the D2D links is huge, and even goes to infinity.
- **Set of actions** $\{\mathcal{P}_i\}_{i\in\mathcal{N}}$: This is the set of possible transmit powers. Each transmitter determines the power $p_i\,(t) \in \{\mathcal{P}_i\}$ at any time $t \in [0, T]$.
- **State space** $\{\mathcal{S}_i\}_{i\in\mathcal{N}}$: We define the state of player i as the combination of the interference introduced by the D2D transmitter i to other D2D links and the remaining energy at this D2D transmitter. The state space is composed of all possible states.
- **Control policy** $\{\mathcal{Q}_i\}_{i\in\mathcal{N}}$: A full power control policy is denoted by $\{\mathcal{Q}_i\}$, with $t \in [0, T]$ to minimize the average cost over the time interval T with two dimensional states.
- **Cost function** $\{c_i\}_{i\in\mathcal{N}}$: we will define a novel cost function, where we consider both the achieved performance, e.g., the $SINR$ and the transmit power.

To determine the control policy $\{\mathcal{Q}_i\}_{i\in\mathcal{N}}$, we need to define the state space $\{\mathcal{S}_i\}_{i\in\mathcal{N}}$ and the cost function $\{c_i\}_{i\in\mathcal{N}}$.

2.2.1 Definition of the State Space

The power control policy $\{\mathcal{Q}_i\}$, for $t \in [0, T]$ of player i, $i \in \mathcal{N}$ is determined to find the optimal control policy which minimizes the cost subject to given state dynamics. The state space is defined based on the intra-tier and inter-tier interferences in (1) and (2), respectively, and the energy usage dynamics.

Energy Usage Dynamics The remaining energy state $E_i\,(t)$ at time t equals to the amount of available energy. Without loss of generality, we define the evolution law of the remaining energy in the battery as

$$dE_i(t) = -p_i(t)\,dt \tag{5}$$

which means that energy $E_i(t)$ of the battery decreases with the transmit power consumption $p_i(t)$.

Interference Dynamics With intra-tier and inter-tier interference defined in (1) and (2), respectively, we first define the interference function that describes the interference caused by the generic D2D transmitter to others as

$$\mu_i(t) = I_{i\rightarrow}(t) + I_{i\rightarrow o}(t) \tag{6}$$

where (6) describes all the interference impacts introduced by player i to other D2D pairs $j \in \mathcal{N}$, $j \neq i$ and the only MUE o. According to definitions in (1) and (2), we have

$$\mu_i(t) = \sum_{j=1,j\neq i}^{N} p_i(t)\,g_{i,j}(t) + p_i(t)\,g_{i,o}(t) \tag{7}$$

To simplify the notation, we further represent (8) as

$$\mu_i(t) = p_i(t)\,\varepsilon_i(t) \tag{8}$$

where $\varepsilon_i(t) = \sum_{j=1,j\neq i}^{N} g_{i,j}(t) + g_{i,o}(t)$. From (8), the total interference at time t to others depends on $p_i(t)$ and $\varepsilon_i(t)$ at time t . Therefore, we can define the interference state as

$$d\mu_i(t) = \varepsilon_i(t)\,dp_i(t) + p_i(t)\,\partial_t\varepsilon_i(t) \tag{9}$$

So, we define the following state space for player i:

$$s_i(t) = [E_i(t), \mu_i(t)],\, i \in \mathcal{N} \tag{10}$$

where the interference caused by the generic D2D transmitter to other D2D links is regarded as one of the state variables. Also, the other state variable is the remaining energy as given in (5).

Cost Function With the above definition of state space $s_i(t)$, each D2D transmitter i will determine the optimal power control policy $\mathcal{Q}^{\star}{}_i$, with $t \in [0, T]$ to minimize the cost. Generally, the communication performance is related to the $SINR$ definition $\gamma_i(t)$ in (5), and we also introduce the identical $SINR$ threshold γ_{th} . Here, γ_{th} is predefined to meet the communication requirements. Therefore, the cost function is given by

$$c_i(t) = (\gamma_i(t) - \gamma_{th}(t))^2 + \lambda p_i(t) \tag{11}$$

where λ is introduced to balance the units of the achieved $SINR$ difference and the consumed power. Here, it is clear that the player i will minimize the $SINR$ difference from the threshold and the power consumption at any time t . It is easy to prove that the cost function $c_i(t)$, given by (11) is convex with respect to $p_i(t)$.

2.3 Optimal Control Problem and Mean Field Equilibrium

2.3.1 Deriving the HJB function

In the previous subsections, we formulate an MFG theoretic framework for ultra-dense D2D networks, where we assume that the number of D2D links can approach infinity. In this framework, we jointly consider the remaining energy at the D2D transmitters and the interference as the state space and obtain an optimal distributed power control policy.

We consider the problem that each D2D pair i will determine the optimal power control policy $\mathcal{Q}^{\star}{}_i$, with $t \in [0, T]$ to minimize the cost function $c_i\left(t\right)$, given by (11), during a finite time horizon $[0, T]$. The general optimal control problem can be stated as follows:

$$\mathcal{Q}^{\star}{}_i\left(t\right) = arg \min_{p_i(t)} E\left[\int_0^T c_i\left(t\right) dt + c_i\left(T\right)\right] \tag{12}$$

where $c_i\left(T\right)$ is the cost at time T. At this time, we define the value function as follows:

$$u_i\left(t, s_i\left(t\right)\right) = \min_{p_i(t)} E\left[\int_t^T c_i\left(t\right) dt + u_i\left(T, s_i\left(t\right)\right)\right], t \in [0, T] \tag{13}$$

where $u_i\left(t, s_i\left(t\right)\right)$ is a value at the final state $s_i\left(T\right)$ at time T.

None of the players can have a lower cost by unilaterally deviating from the current power control policy. The Nash equilibrium of the above power control differential game can be obtained by solving the HJB equation associated with each player in the optimal control theory. According to optimal control theory followed by Bellman's optimality principle, the value function in (13) should satisfy a partial differential equation which is a HJB equation. The solution of the HJB equation is the value function, which gives the minimum cost for a given dynamic system with an associated cost function. Here, we have the HJB equation in (14)

$$-\partial_t u_i\left(t, s_i\left(t\right)\right) = \min_{p_i(t)} \left[c_i\left(t, s_i\left(t\right), p_i\left(t\right)\right) + \partial_t s_i\left(t\right) \cdot \nabla u_i\left(t, s_i\left(t\right)\right)\right] \tag{14}$$

Obtaining the equilibrium for game G_s for a system with N players involves solving N partial differential equations simultaneously. However, for a dense D2D network, it is not possible to obtain the Nash equilibrium in this manner due to the large number of simultaneous partial differential equations. Therefore, for modeling and analysis of a dense D2D network, a mean field game will be introduced where the system can be defined solely by two coupled equations. In the next section, we show the extension of game G_s to a mean field game.

2.3.2 Deriving the FPK function

The power control MFG in D2D networks is a special form of a differential game described before when the number of D2D links approaches infinity. The power control MFG can be expressed as a coupled system of two equations of HJB and FPK. On one hand, an FPK type equation evolves forward in time that governs the evolution of the density function of the agents. On the other hand, an HJB type equation evolves backward in time that governs the computation of the optimal path for each agent.

For the effects of both the interference of the generic D2D transmitter introduced to others, and all others' interference introduced to the generic link, we first introduce the mean field concept, and then propose the mean-field approximation

- **Mean Field**: This is a critical concept in the defined power control MFG, which is a statistical distribution of the defined two-dimensional states. Given the state space $s_i(t) = [E_i(t), \mu_i(t)]$ in the defined power control MFG, we define the mean field $m(t,s)$ as
$$m(t,s) = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} 1_{\{s_i(t)=s\}} \tag{15}$$
where 1 function denotes an indicator function which returns one if the given condition is true and zero, otherwise. For a given time instant, the mean field is the probability distribution of the states over the set of players.

- **FPK Equation** Based on the above definition of mean field, we derive the FPK equation. The FPK equation of the defined MFG is given by
$$\partial_t m(t,s) + \nabla(m(t,s) \cdot \partial_t s(t)) = 0 \tag{16}$$
where s is the state.

The FPK equation describes the evolution of the defined mean field with respect to time and space. At this time, with the derived HJB(14) and FPK(16) equations, we formulate the D2D MFG problem. Their interactions with each other are shown in Fig. 2. The HJB equation governs the computation of the optimal control path of the player, while the FPK equation governs the evolution of the mean field function of players. Here, the HJB and FPK equations are termed as the backward and forward functions, respectively. Backward means that the final value of the function is known, and we determine the value of $u(t)$ at time $[0,T]$. Therefore, the HJB equation is always solved backwards in time, starting from $t = T$, and ending at $t = 0$. When solved over the the entire state space, the HJB equation is a necessary and sufficient condition for the optimum. The FPK equation evolves forward with time. The interactive evolution finally leads to the mean field equilibrium.

3 Coupled Multiwavelets Neural Operator Learing For MFG Problem

Coupled partial differential equations (PDEs) are key tasks in modeling the complex dynamics of many physical processes. MFG is a typical system of Coupled PDEs. Recently, neural operators have shown the ability to solve PDEs by learning the integral kernel directly in Fourier/Wavelet space, so the difficulty for solving the coupled PDEs depends on dealing with the coupled mappings between the functions. Towards this end, we propose a coupled multiwavelets neural operator (CMWNO) learning scheme by decoupling the coupled integral kernels during the multiwavelet decomposition and reconstruction procedures in the Wavelet space.

To solve a coupled control system characterized by coupled state equations in control

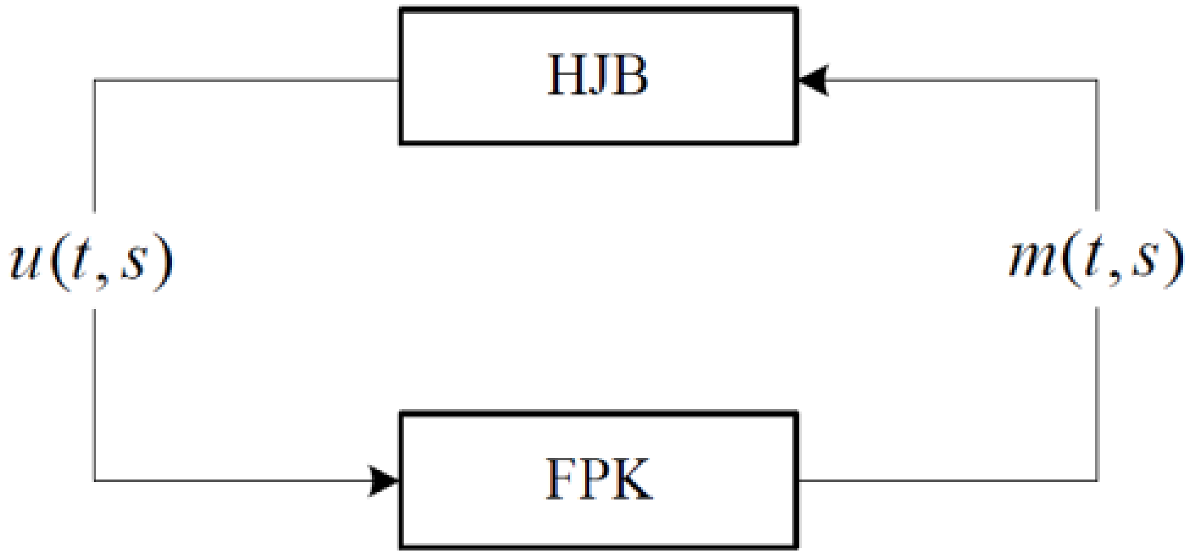

Figure 2: D2D mean field game with HJB and FPK equations.

theory, a popular way is to use the Laplace operator s to represent differential and integral operators. Therefore, the coupled high-order differential equations can be transformed into the first-order differential equations in the Laplace space which will reduce the decoupling difficulty. Inspired by the use of the Laplace operator and the properties of the multiwavelets, we assume that the interactions between kernels can be used to approximate the coupled information by reducing the degree of high-order operators in multiwavelet bases. With this assumption, we are able to build the coupled multiwavelet neural operators (CMWNO) learning scheme, which utilizes decomposition representation from the operator and mimic the interaction via a dice strategy.

3.1 Coupled Differential Equations

To provide a simple example of the coupled kernels, κ_1 and κ_2, let us consider a general coupled system with 2 coupled variables $u(x,t)$ and $v(x,t)$ with the given initial conditions $u_0(x)$ and $v_0(x)$. Given $\mathcal{A}$ and $\mathcal{U}$ as two Sobolev spaces $\mathcal{H}^{s,p}$ with $s > 0, p = 2$, let T denote a generic operator such that $T : \mathcal{A} \to \mathcal{U}$. Without the knowledge of how these two variables are coupled, to solve for $u(x,\tau)$ and $v(x,\tau)$, we need two operators T_1 and T_2 such that $T_1 u_0(x) = u(x,\tau)$ and $T_2 v_0(x) = v(x,\tau)$. The coupled kernels termed as Green's function can be written as follows:

$$\begin{aligned} T_1 u_0(x) &= \int_D \kappa_1(x, y, u_0(x), u_0(y), v_0(x), v_0(y), \kappa_2) u_0(y) dy, \\ T_2 v_0(x) &= \int_D \kappa_2(x, y, u_0(x), u_0(y), v_0(x), v_0(y), \kappa_1) v_0(y) dy, \\ & u(x,0) = u_0(x); \quad v(x,0) = v_0(x), \quad x \in D, \end{aligned} \tag{17}$$

where $D \subset \mathbb{R}^d$ is a bounded domain. The interacted kernels cannot be directly solved without considering the interference from the other kernel, and our idea is to simplify the kernels first and deal with the interactions in the multiwavelet domain.

3.2 Coupled Multiwavelets Model

This section introduces a coupled multiwavelets model to provide a general solution on coupled differential equations. First, we make a mild assumption to decouple two coupled operators given in Section 3.1. To simplify eq. 17, without loss of generality, we assume that we can build two operators T_u and T_v to approximate $u(x,\tau)$ and $v(x,\tau)$, where T_u and T_v are decoupled and do not carry any interference from each other. In other words, we can write $T_u u_0(x) = u'(x,\tau)$; $T_v v_0(x) = v'(x,\tau)$, where $u'(x,\tau)$ and $v'(x,\tau)$ are the approximations of $u(x,\tau)$ and $v(x,\tau)$ without coupling. The assumption is mild and easy to get satisfied in the Wavelet space since the operators can be represented by the first-order multiwavelet coefficients. According to this assumption, we can derive the following relations:

$$\begin{aligned} u(x,\tau) &= T_u u(x,0) + \epsilon_1(T_v), \quad x \in D \\ v(x,\tau) &= T_v v(x,0) + \epsilon_2(T_u), \quad x \in D \end{aligned} \tag{18}$$

where $\epsilon_1(T_u)$ quantifies the interference from operator T_v to solve $u(x,\tau)$ and $\epsilon(T_v)$ represents the measurable interaction from operator T_u. Therefore, the integral operators can be written as:

$$T_u u_0(x) = \int_D \kappa_u(x,y) u_0(y) dy; \quad T_v v_0(x) = \int_D \kappa_v(x,y) v_0(y) dy, \tag{19}$$

the kernels κ_u and κ_v termed as Green's functions can be learned through neural operators, where κ_u can be learned using the data of u while the kernel κ_v is learned from v. To model $\epsilon_1(T_u)$ and $\epsilon_2(T_v)$, we transform the operators into multiwavelet coefficients in the Wavelet space and embed it through simple linear combination after the decomposition steps.

Based on the concept of multiwavelets , here we simply explain the decomposition step and reconstruction step of multiwavelets in our coupled system. Since $\mathbf{V}_n^k = \mathbf{V}_{n-1}^k \bigoplus \mathbf{W}_{n-1}^k$, the bases of V_n^k can be written as a linear combination of the scaling functions φ_i^{n-1} and the wavelet functions ψ_i^{n-1}. The linear coefficients $(H^{(0)}, H^{(1)}, G^{(0)}, G^{(1)})$ are termed as multiwavelet decomposition filters, transforming representation between subspaces $\mathbf{V}_{n-1}^k$, $\mathbf{W}_{n-1}^k$, and $\mathbf{V}_n^k$. For a given function $f(x)$, the scaling/wavelet coefficients s_{jl}^n/d_{jl}^n of scaling/wavelet functions $\varphi_{jl}^n/\ \psi_{jl}^n$ are computed as:

$$s_{jl}^n = \int_{2^{-n}l}^{2^{-n}(l+1)} f(x)\varphi_{jl}^n(x) dx; \quad d_{jl}^n = \int_{2^{-n}l}^{2^{-n}(l+1)} f(x)\psi_{jl}^n(x) dx. \tag{20}$$

Using the multiwavelet decomposition filters, the relations between the coefficients on two consecutive levels n and $n+1$ are computed as (decomposition step):

$$\mathbf{s}_l^n = H^{(0)}\mathbf{s}_{2l}^{n+1} + H^{(1)}\mathbf{s}_{2l+1}^{n+1}; \quad \mathbf{d}_l^n = G^{(0)}\mathbf{s}_{2l}^{n+1} + G^{(1)}\mathbf{s}_{2l+1}^{n+1}. \tag{21}$$

Therefore, starting with the coefficients s_l^n, we repeatedly apply the decomposition step in eq. 21 to compute the scaling/wavelet coefficients on coarser levels. Similarly, the reconstruction step can be represented as:

$$\mathbf{s}_{2l}^{n+1} = H^{(0)\,T}\mathbf{s}_l^n + G^{(0)\,T}\mathbf{d}_l^n, \quad \mathbf{s}_{2l+1}^{n+1} = H^{(1)\,T}\mathbf{s}_l^n + G^{(1)\,T}\mathbf{d}_l^n. \tag{22}$$

Repeatedly applying the reconstruction step, we can compute the coefficients s_l^n from s_l^0 and $d_l^i, i = 0, \ldots, n$. In general, the function can be parameterized as the scaling/wavelet coefficients in the Wavelet space after the decomposition steps, and the coefficients can be mapped to the function after reconstruction steps. In our work, to model the

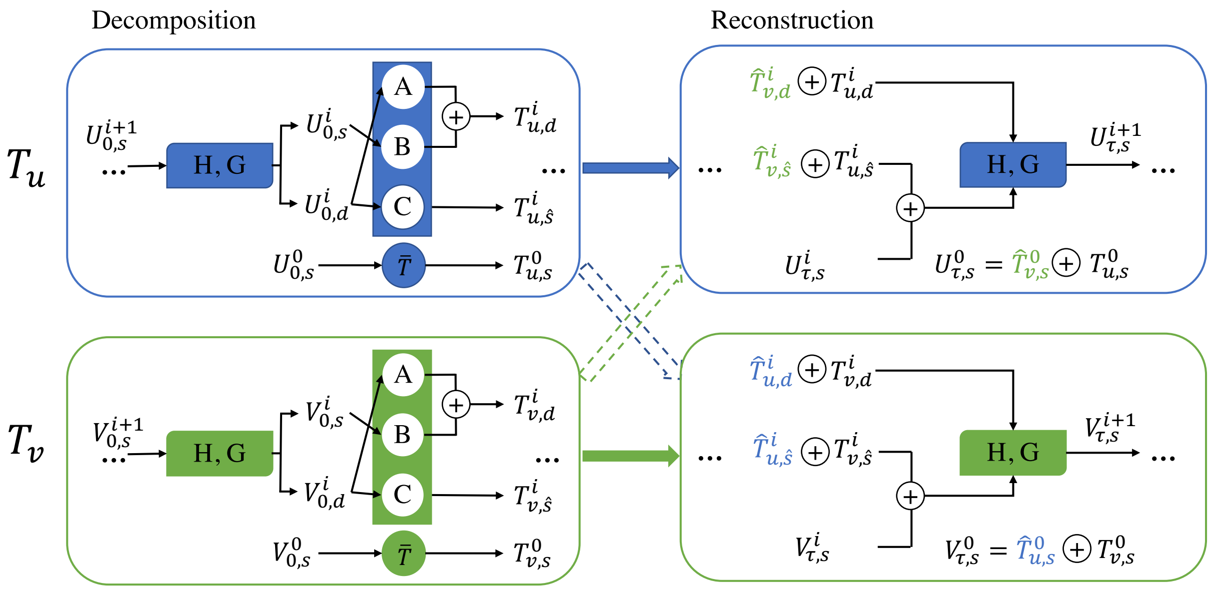

Figure 3: Architecture of CMWNO. Note that there are two coupled operators, T_u and T_v, in our system, which aligns the number of coupled variables. The network $\bar{T}$ is only applied for the coarsest scale L (0 in this system). The dashed arrows correspond to the auxiliary information from the unused operator without gradient during training process. For the interaction between operators, when we update the operator T_u, the decomposed ingredients from T_v will be equipped into the reconstruction module of T_u in the Wavelet domain, vice versa.

interference $\epsilon_1(T_u)$ and $\epsilon_2(T_v)$, we obtain the multiwavelets coefficients of each kernel during the decomposition steps and embed them into the other kernel in the reconstruction step.

The architecture of the CMWNO is shown in Fig. 3, which illustrates the mapping process inside the wavelet space of layer n. The operations inside the wavelet space can be matched by the order of layers in the models, which means the decomposition operations for different resolutions are done independently. After decomposing s^n via eq. 21, we can get the transferred information of input where each component will be used to reconstruct the original input at the layer n.

3.3 Dice strategy

Inspired by scheduled sampling , which is designed to gently bridge the discrepancy between training and inference samples, we propose rolling the dice to randomly decide the interaction order between each neural operators, which is named dice strategy. Specifically, we roll the dice for every sample to decide which path to use, which can effectively mitigate the imbalance update problem for each kernel caused by the fixed training order. As illustrated in Fig. 4, when the dice tells the model to use path 1 (upper path) , we will update operator T_u by equipping the coupled information from the other operator T_v first. Note that, T_v is learned by previous samples and have not updated yet. Then we use the updated operator T_u to decompose the initial state u_0, which can be used to update T_v. Inside the Wavelet space with well-defined basis, where we are able to utilize vary orthogonal information from each initial state jointly.

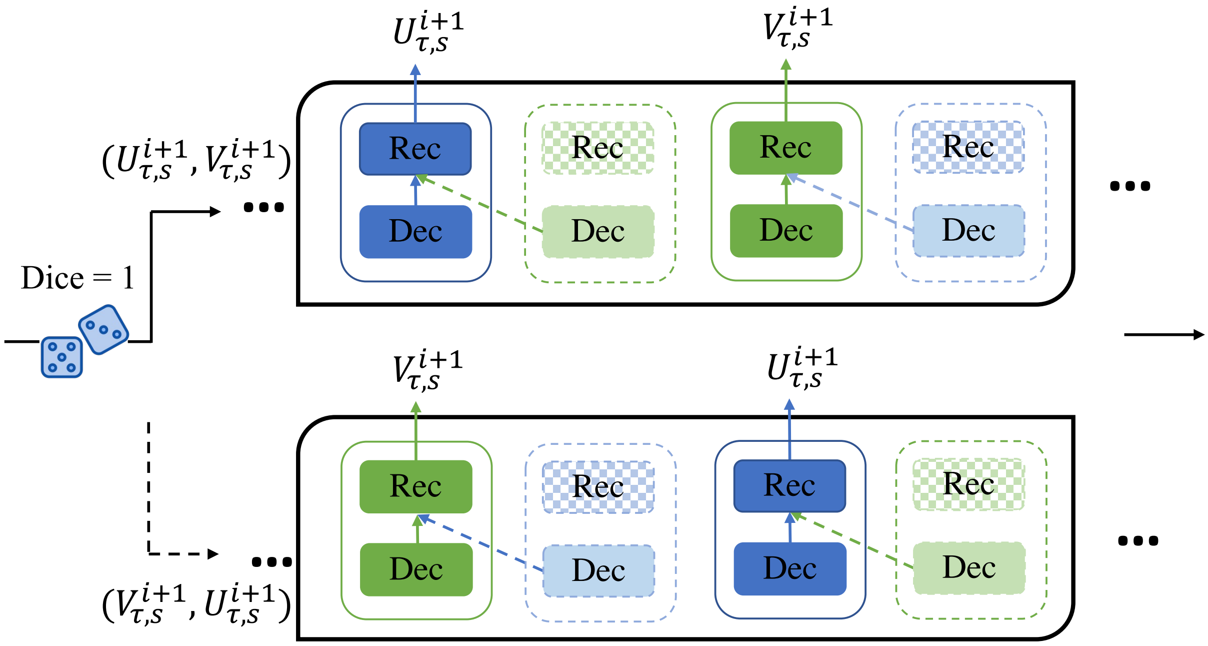

Figure 4: Dice strategy. For each sample, one only needs to go through a specific path (round diagonal corner rectangle).

4 Implementation and Experimental Results

4.1 Experiment Setups

Based on the ultra-dense D2D network constructed earlier, we model it as a mean-field game theory problem. In MFG, we construct two equations, FPK and HJB. In this section, we propose a simulation experiment to validate the application of the CMWNO model in the MFG problem. Firstly, we generated the necessary data for the experiment, including the state space $s_i(t) = [E_i(t), \mu_i(t)], i \in \mathcal{N}$ and mean-field parameters $m(t,s) = \lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} 1_{\{s_i(t)=s\}}$.

To set up the experiment, we generate data for the learning scheme of the coupled partial differential equations. Firstly, we need to determine the partial differential equations and their initial and boundary conditions. The continuous time and space variables are discretized into discrete grid points by using discretization methods such as finite difference, finite element, and spectral methods. Then, numerical methods are used to solve the discretized partial differential equations to obtain numerical solutions. At the discretized grid points, we sample at certain time intervals to obtain a set of discretized data. Finally, we preprocess and process the data, such as removing noise, normalization, etc., to obtain data suitable for model training. Here, we define the duration of one LTE radio frame is $10ms$. We also set up some constraints for $E_{max} = 0.5J$, $\mu_{max} = 5.8 \times 10^{-6}W$. The generated data is shown in Figure 4.

4.2 Software Implementation

To ease the usage of the model and speed up inference, we've developed a multithreaded C++ application backed by this model. The application takes (n, 256, 3) tensors as input, and output (n, 256, 3) tensors, where n is the number of data points. The application

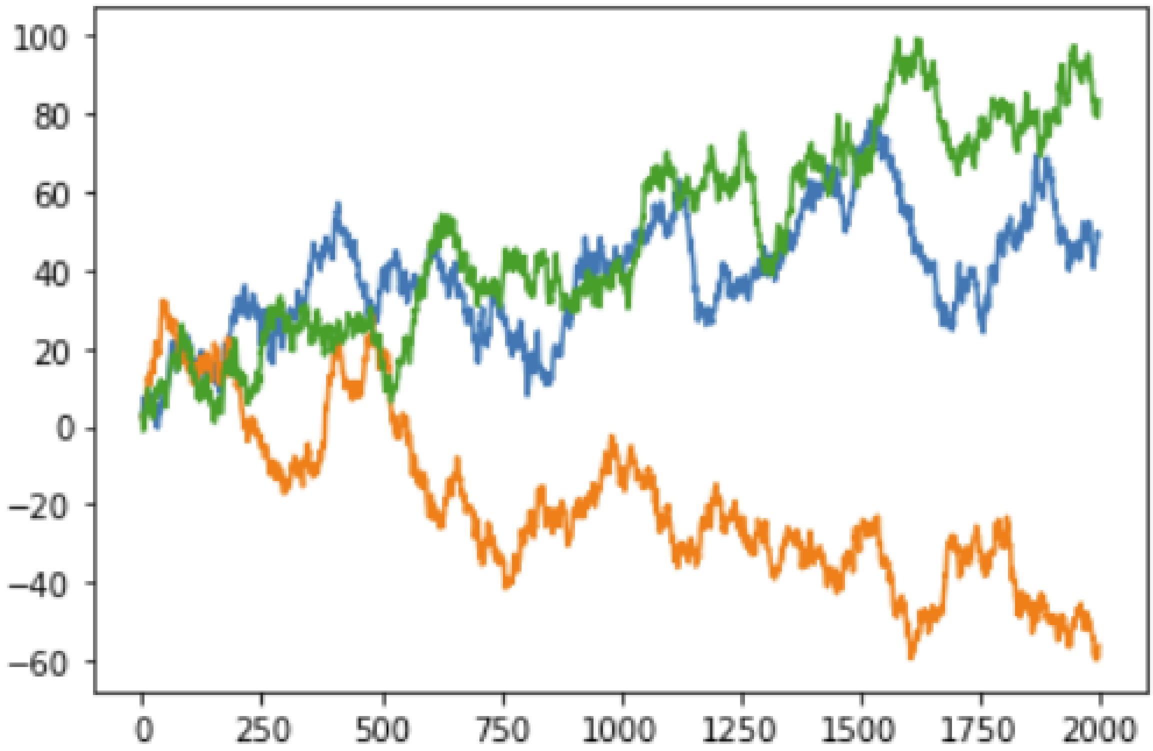

Figure 5: Prediction results for CMWNO model of $E(t)$.

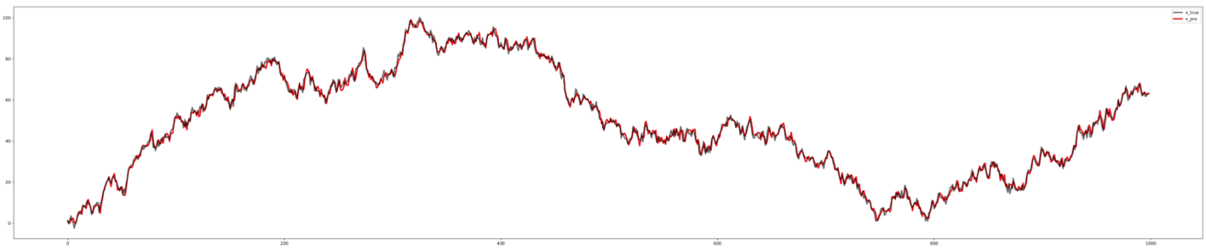

Figure 6: Prediction results for CMWNO model of $E(t)$.

uses LibTorch, the C++ distribution of Pytorch, as its inference framework. To load the original Pytorch model to LibTorch, we serializes the original model(Pytorch). We rely on the pthread library to achieve multithreading. Each thread uses the same input, output, and model, but works with different parts of input and output. The application gets synchronized after the workers finish their inference job. The application utilizes polymorphism in terms of how it load model and input so that we can extend the application to different models in future.

4.3 Experiment Result

We perform CMWNO and FNO models working for synthetic coupled datasets mentioned above to solve this MFG coupled PDEs. We generate the state variables of the players as a time series representation and construct them as a time trajectory for processing. We use training data to train our model and use the trained model for prediction to validate our experimental results. The results of prediction of the state space and the mean feild are shown in Fig.6-8 with the time flows.

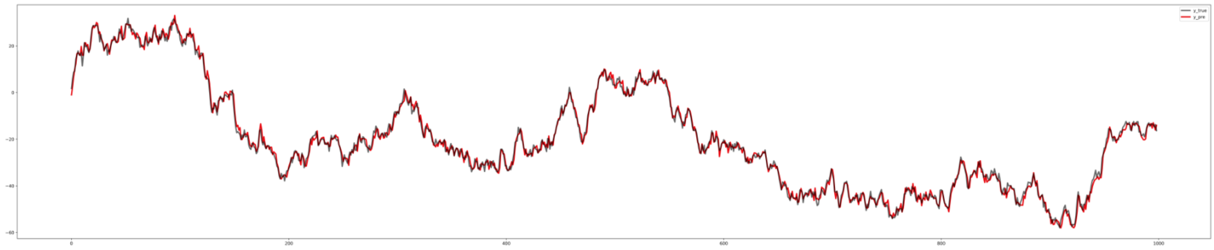

Figure 7: Prediction results for CMWNO model of $\mu(t)$.

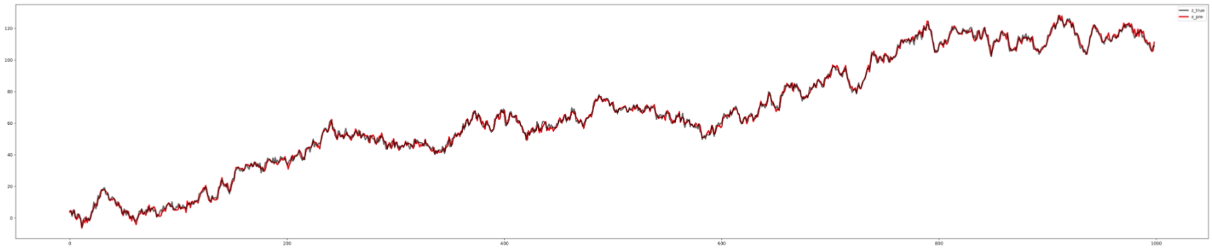

Figure 8: Prediction results for CMWNO model of $m(t)$.

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