FIN 514: Report on Project 1

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This document would focus on the following Financial Product.

12% Callable STock Return Income DEbt Securities^{SM}, The

CUSIP #: 59022W463

Issuer Company: Merrill Lynch & Co., Inc.

Pricing Date: August 27, 2007

Settlement Date: September 4, 2007

Maturity Date: September 4, 2009

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0 Road Map

To value the 12% Callable STock Return Income DEbt Securities[SM] (the Callable STRIDES), we would use 50-to-1000 binomial tree model to simulate the possible prices of the underlying asset (Apple Inc. Common Stock), and the contingent prices (for a specific term, it would be the minimum of exercise payoff, expectation of current value, and the current ceiling price). Using backwards method to find the value of the product at term 0.

First, we use interpolation method to find the proper continuous risk-free rate and implied volatility should be used in the model. Then find all the parameters determining price on every single node in the binomial tree. We expand the process and make 951 (50 to 1,000) single processes to improve the accuracy of the model, with the help of MS Excel VBA. These trees give us a value between $23.46 and $23.47, on Pricing Date.

Second, we specifically illustrate the whole tree of the product prices when the quantity of the steps, N, equals to 200. We choose it because the limit of the column in MS Excel is 256 thus we cannot use 1000-step tree as an example. In other hand, we need a visualized tree to test the validity of our model. The final value estimated by 200-step model is $23.4695, on Pricing Date.

Third, we consider that in most cases the node is not exactly lying on a round date (in days), we particularly specify the process when N = 739. We add more details about coupon day, real number of days during the actual existed month, and real-world trading days and business days interval. We call it 739-Step Semi-Real-World Binomial Tree Model. This more specific model give us a value of $23.6322, on Pricing Date.

In this report, we would discuss how we construct the model, the results and discounting-adjusted results, what brings differences among models, possible errors, and possible approaches to improve the model.

1 Data and Parameters Estimation

1.1 Data Offered by the Official Introduction

In the official introduction of the Callable STRIDES, we find some key parameters. Here is a simple table to illustrate all these parameters.

Parameter	*Value*	*Explanation*
S_0	*133.3278*	*The weighted average price of a common stock of Apple.*
$Face$	*25*	*The face value of a Callable STRIDES*
$Coupon$	*3%*	*Quarterly coupon rate*
#days	*739*	*Time from Pricing Date to Maturity Date, in days.*
T	*2.0247*	*Time from Pricing Date to Maturity Date, in years.*
Barrier	*1*	*Critical share price to redeem.*
M	*.187508*	*A multiplier calculated by* $Face/S_0$.
discount	*20.10%*	*An internal rate to calculate ceiling price.*

Discussion: We use time between Pricing Date and Maturity Date, rather than between Settlement Date and Maturity Date, because the latter one uses risk-free rate and volatility that cannot be implemented between Pricing Date and Settlement Date, during which there definitely exists fluctuation of share price.

1.2 Risk-free Rate and Volatility

For the specific time interval, we use the interpolation method to estimate the proper risk-free rate r and implied volatility σ.

Fortunately, we find from the Project 1 folder on compass the very data to interpolate.

Considering the Callable STRIDE has similar features as call options of apple stock with strike of 133.3278, we use call option price on AUG 27, 2007, with maturity-strike pairs of "20090117, 130", "20100116, 130", "20090117, 140", and "20100116, 140", to do linear interpolation. The simple process is as follows:

$$\sigma_{2,p} = \frac{(T_2 - T_1) * \sigma_3 + (T_3 - T_2) * \sigma_1}{T_3 - T_1}$$

$$\sigma_{T,2} = \frac{\sigma_3 - \sigma_1}{P_3 - P_1} * (P_2 - P_1) + \sigma_1$$

For a proper risk-free rate used during the 739 days we use on AUG 27, 2007 the rate of ZCB with a DTM of 660 and 739. Similarly, we have:

$$r_2 = \frac{r_3 - r_1}{T_3 - T_1} * T_2 - T_1 + r_1$$

With the method above we find the proper risk-free rate is 4.8311% and σ is 42.2665%. The complete process can be found in the attached XLS file.

1.3 Business Day Estimation

In document, there are two details involving calculations with business day. First one is the exercise of the call, where the issuer must inform the holder about the call decision five days in advance. Second one is that when redemption event is triggered (share price is less than $1), the redemption would take place three business days later.

At the very start we use natural days, which is obviously not precise. However, it is also not feasible that to calculate the actual date for most trees during 50 to 1000 steps. So we must find a proper estimate.

Then we think that, there are 365 natural days, while 252 trading days (business days, on average) during a year. So we treat one business day as $\frac{365}{252}$ natural days.

But there are still defects inside this assumption. In most cases, five business days means seven natural days, and three business days can mean three or five natural days, letting along the national holidays.

So we calculate, for each date, the actual numbers of interval after three or five days, and use the mean of each column. Considering the three-day event may take place all through SEPT 5, 2007 to SEPT 4, 2009, while the five-day event may only take place after SEPT 4, 2008, the inputs of average function would be adjusted.

Finally, for 50-to-1000-step tree, we treat three business days as .0113 years, while five, .0189 years.

Specifically, we use actual business-day-or-not data to treat the 739-step binomial tree in Part 4.2.

2 Discussion About the Specific Features

2.1 When Would the Issuer Call?

The issuer would call the bond five days after announcement of exercise decision. But why would they decide to call after five days? What is the their logic to predict and make decisions?

The only signal can indicate the expectation is the current share price, so the logic is that the issuer uses the risk-neutral expectation of share price five days later, and the exercise value with the expectation on that day, to make decision. If the five business days are represented by $b5d$, this value, discounted to current date, would be:

$$E(Call) = e^{-r*b5d} * (S_t e^{(r*b5d)} * M + AccruedCoupon)$$

Here, we assume that, if this value is smaller than the expectation using probability by nodes in previous term (both not exceed ceiling price), a decision would be made.

When calculate value during Day 366 and Day 732, this rule should be applied.

2.2 Is There a Ceiling Price in Every Term?

We notice there is a discount factor of 20.10% in the document to limit the payoff of the holders. After reading the ANNEX A, we find the logic to calculate the ceiling price in each term. For ceiling price in term j we have:

$$ceiling_j = \frac{25 - PV(AllFutureCoupon)}{(1 + discount)^{j*\Delta t}} * (1 + discount)^{j*\Delta t} + AccruedCoupon$$

For example, the ceiling price for the last step is \$29.72.

2.3 What Would Happen When Stock Price is Less than \$1?

When the event takes place, three business days later, the Callable STRIDES would be redeemed. The only difference here with the normal call exercise is that, when redeeming, the issuer should additionally pay the present (current, 3 days after term j) value of all future coupon payments.

Also, the expectation of the stock price three days later should be a risk-neutral prediction, that is, $S_j e^{r*b3d}$.

2.4 Special Dates Treatment

Special Dates refer to:

a) From AUG 27 (Pricing Date) to SEPT 4 (Settlement Date), 2007. During this interval, no barrier should be considered. Actually, during this interval, the product is not settled.

b) From AUG 27 to AUG 28 (the last day issuer has no right to call), 2009. During this interval, no exercise decision could be made.

c) From AUG 28 (day after last day issuer can announce to call) to SEPT 4, 2009. During this interval, no exercise decision could be made.

d) From SEPT 1 to SEPT 4, 2009. During this interval, no Redemption Event could happen.

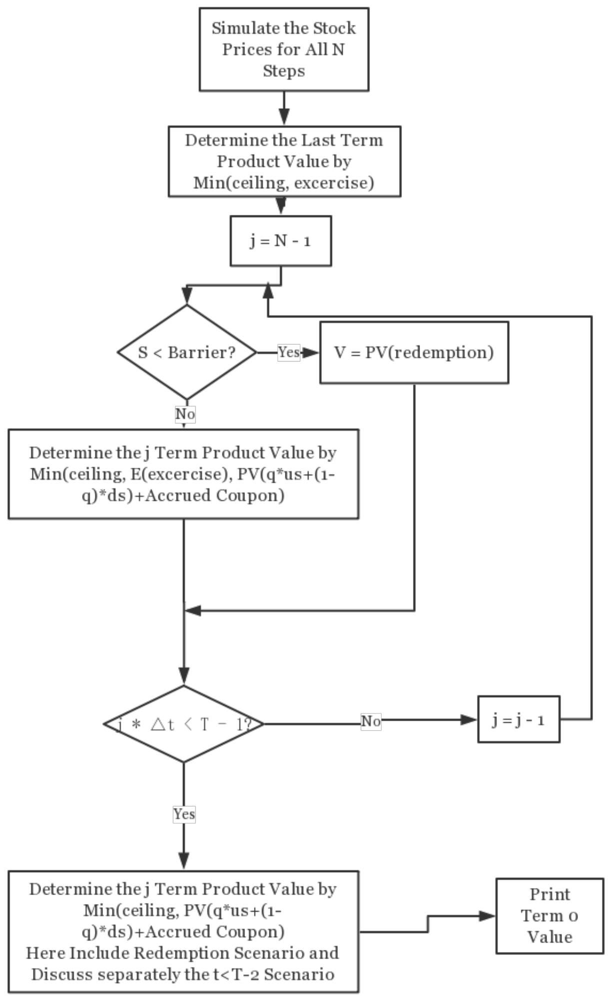
So included in the Min function would be less parameters when dealing with these dates.

3 Algorithm Illustration

Most of the valuation modeling is completed by VBA. The code could be found at the attached XLS file. To read the code, a permission to enable MACRO might be required.[1]

The road map of the algorithm used here is presented as follows:

[1] Apologize for any inconvenience that we did not put notes on the code scripts, but we are willing to answer any questions about our code. If you have questions with our code, please contact gengyug2@illinois.edu.

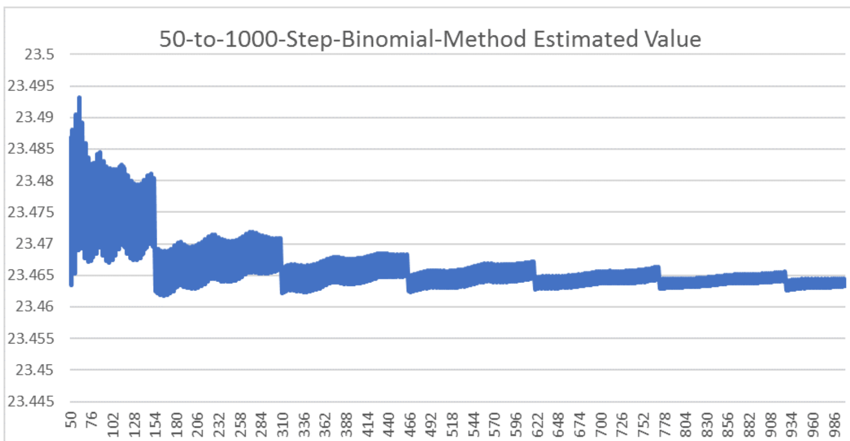


Notes: a) All time variables are measured in years. b) There are j + 1 nodes at Term j.

4 Results

4.1 50-to-1000-Step Binomial Tree Model

Use the algorithm above to construct 50-to-1000 binomial tree, and find that with the increase of N, the difference between N result and N-1 result becomes smaller, and humps here are very obvious. Graph below shows the trend.



To observe a specific tree when N = 200, we find that the ceiling and barrier mechanism works. Below shows part of the 200-step tree. It is not complete because many nodes (both rows and columns) are hidden intentionally. The complete tree would be found at the attached XLS file. To read the hidden nodes, an operation to unhide might be required.

						29.02031	29.074167	29.12
					28.966553	29.02031	29.074167	29.12
				28.912895	28.966553	29.02031	29.074167	29.12
			28.859336	28.912895	28.966553	29.02031	29.074167	29.12
		28.805877	28.859336	28.912895	28.966553	29.02031	29.074167	29.12
	28.752517	28.805877	28.859336	28.912895	28.966553	29.02031	29.074167	29.12
5	28.752517	28.805877	28.859336	28.912895	28.966553	29.02031	29.074167	29.12
5	28.752517	28.805877	28.859336	28.912895	28.966553	29.02031	29.074167	29.12
6	0.7606643	0.7605113	0.7603555	0.7601957	0.7600309	0.75986	0.759682	0.75
9	0.7594177	0.7593167	0.7592106	0.7590985	0.7589794	0.7588523	0.7587162	0.758
3	0.7582729	0.7582195	0.7581591	0.7580908	0.7580136	0.7579267	0.7578292	0.757
3	0.7572213	0.7572117	0.7571933	0.7571652	0.7571265	0.7570766	0.7570145	0.756
3	0.7562555	0.7562861	0.7563062	0.7563151	0.7563118	0.7562957	0.7562662	0.756

The result of value is between \$23.46 and \$23.47, on Pricing Date.

Considering the product is priced on Pricing Date, but the payment occurs on Settlement Date, the real costs on Pricing Date is \$24.97, thus the issuer profits about \$1.01, apart from the underwriting discount of \$.5 .

4.2 739-Step Semi-Real-World Binomial Tree Model

To make every node exactly at the end of a complete day, we use the same algorithm with N = 739. The new features of this model are:

a) Use real-world whether-business-day-or-not data to determine whether the issuer is able to announce exercise or redemption (They cannot announce on non-business day).

b) The accrued coupon would no longer be calculated by universal .25 years, but by real number of days during a quarter. This rule also applies in risk-free discounting.

Although the new model has real-world feature, but it cannot simulate real-world events completely, because in real world, non-business day would has no stock prices, but the assumption of the tree require it to have. That's why the model is called Semi-Real-World.

A complete list of whether-business-day-or-not and accrued part data for each day can be found at the attached XLS file. To read the data, an operation to unhidden might be required.

The final value determined by this model is \$23.6322, which is a little far from the previous model. We would discuss the difference in the following part.

5 Conclusion and Discussion

Through two models, the final value of the 12% Callable STock Return Income DEbt Securities^{SM} (the Callable STRIDES) is either a value around \$23.465, or a value around \$23.63. Apart from underwriting cost, the final profit, on Pricing Date, to Merrill Lynch & Co., Inc. is either around \$1.01, or around \$.87.

The large difference between the results from the two models might be caused by reduced power to call in the Semi-Real-World model. If the actual business days are applied, the number of days reduces sharply, on which issuer is able to announce.

A 739-step model might reflect more real-world information, but it is only a single tree, and somehow loses accuracy because 739 is an odd number (maybe we can use 1478-, 2956-, or 5912- steps). On the otherwise, 50-to-1000 model, although performs as gradually closing the real value, it exaggerate the right to call of the issuer.

One interesting thing we find when adding barrier mechanism into the model is that it influences so little on determining the final value, that we almost conclude the mechanism fails. This influence even less than the estimate of the proxy of a business day. But when we print the column N-1 pre- and post-adding the mechanism we find it really works. However, we still spent a lot of time on the algorithm here to improve the accuracy.

Possible errors may occur in following scenarios:

a) Assumption we made about when the issuer would announce to call fails. That might be due to the calculation of expectation of future share prices. Can we simply uses b5d and b3d to predict or imply future or past stock prices? The question to this answer should be a help to the updated valuation model.

b) The errors in simulating stock prices continuously, ignoring the non-trading days. This would ignore the humps pattern of the stock prices (Assuming that each non-trading days has a price of last trading day), while this pattern, would be highly related with the implied volatility. The solution to this problem would help improve the Semi-Real-World model, and, even, make it a complete-real-world model.

References

1. Dr. Martin Widdicks, *"FIN 514 Financial Engineering II: Lecture Notes,"* Finance Department, College of Business, University of Illinois at Urbana-Champaign.

2. "424B3 1 d424b3.htm TERM SHEET, $100,000,000 12% Callable STock Return Income Debt Securities Due September 4, 2009 Payable on the stated maturity date with Apple Inc. common stock. Term Sheet No. 2835," (2007, August 27). Retrieved February 20, 2017, from SEC website:
http://www.sec.gov/Archives/edgar/data/65100/000119312507194251/d424b3.htm