FIN 514: Project 3

DONG, Yisong (yisongd2)
GUO, Gengyu (gengyug2)
LIU, Ziqin (ziqinl2)

October 15, 2017

0 Road Map

We use Python-based Monte Carlo Simulation to do valuation on the product "Enhanced Buffered Jump Securities Based on the Value of the Worst Performing of the S&P 500®Index and the Russell 2000®Index due July 12, 2022." In this document, we would introduce the parameters and methods we employ. After showing results, we would also do sensitivity analysis and evaluate the model.

1 Product Payment Structure

This product is a promise depending on S&P500 and Russell 2000 Index. It is priced on April 13, 2017, sold on April 19, 2017, and going to mature on July 22, 2017.

Suppose the original prices of the two indices are respectively SPX_0 and RTY_0, and final values, SPX_F and RTY_F, the payment on maturity would be:

$$\begin{cases} PMT = 14.31, SPX_F \geq .8SPX_0 \& RTY_F \geq .8RTY_0 \\ PMT = 10 \times (min(\frac{SPX_F}{SPX_0}, \frac{RTY_F}{RTY_0}) + .2), SPX_F < .8SPX_0 || RTY_F < .8RTY_0 \end{cases} \quad (1)$$

While the final values of the indices is the arithmetic average of the prices of the indices from April 8, 2022 to July 7, 2016, inclusively.

2 Data and Parameters Estimation

2.1 Choose Risk-Free Rate

Use interpolation to find the 6-day and 1926-day risk-free rates on April 13, 2017. Data are from Bloomberg's SWDF curve. 6-day rf is about .944% and 1926-day, 1.916%.

2.2 Estimate the Covariance Matrix

To request historical, we turn to Python package '*pandas-datareader*'. This package would help us collect adjusted closing price within given tickers and date intervals. We are interested in adjusted closing price, which would incorporate all the splits and dividends data to reflect real returns on the security (actually, both indices have no split/dividend actions during the sample period). For each index, the daily log return is:

$$R_{i,t} = \ln \frac{Adj_{i,t}}{Adj_{i,t-1}}$$

We use 5 years data ended April 13, 2017 to estimate the covariance matrix, as well as the correlation.

$$COV = E[(R_{SPX} - Mean_{SPX})(R_{RTY} - Mean_{RTY})]$$

$$\rho = \frac{COV}{\sigma_{SPX}\sigma_{RTY}}$$

Correlation coefficient is about .8652.

2.3 Choose Step Number

Considering the averaging feature about the final value of each index, we decide to make every step lies on the end of a natural day. So we choose to employ 1926 steps within $T = 5.275$.

3 Monte Carlo Simulation

Use Python's random number generator to generate two independent groups of random numbers, X_1 and X_3, from standard normal distribution.

Consider that SPX and RTY follow:

$$dS_1 = \mu S_1 dt + \sigma_1 S_1 dX_1$$

$$dS_2 = \mu S_2 dt + \sigma_2 S_2 dX_2$$

With given correlation coefficient, rewrite X_2:

$$X_2 = \rho X_1 + \sqrt{1-\rho^2} X_3$$

And for each step:

$$S_{1,t} = S_{1,t-1} e^{(\mu - .5\sigma_1^2)dt + \sigma_1 \sqrt{dt}\phi_1}$$

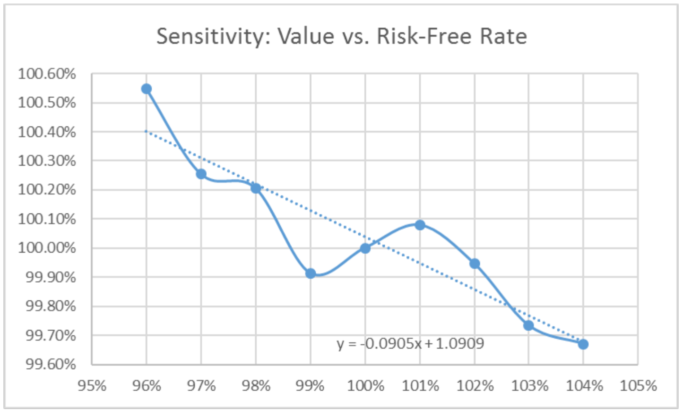
$$S_{2,t} = S_{2,t-1} e^{(\mu - .5\sigma_2^2)dt + \sigma_2 \rho \sqrt{dt}\phi_1 + \sigma_2 \sqrt{1-\rho^2}\sqrt{dt}\phi_2}$$

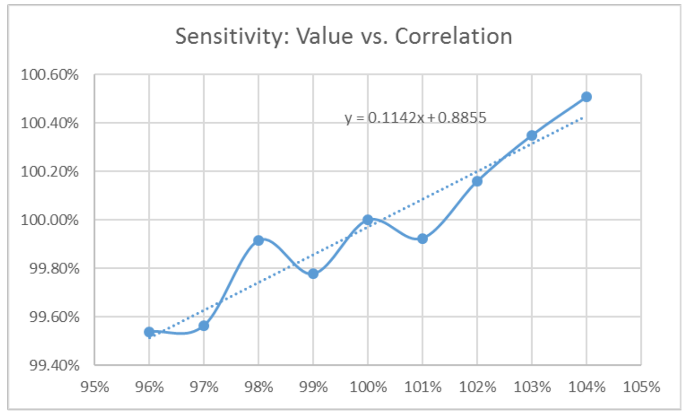
For each trail there would be two series of prices, and one Value contingent on the prices. For each time to simulate, we do 10,000 trails and find the average of the 10,000 values. And we made totally 12 simulation processes and find the average of them.

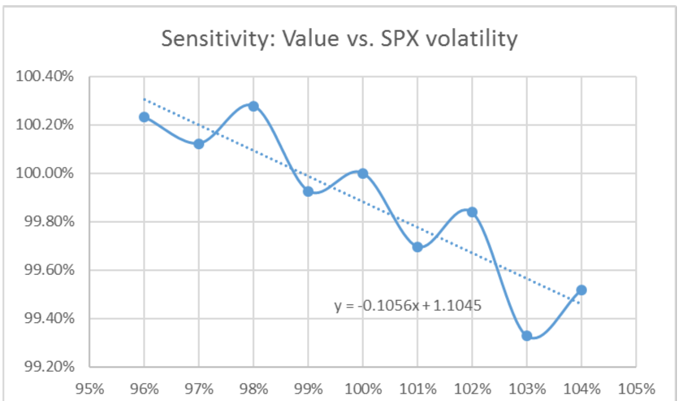
When find the V_T, discount it to pricing date using 1926-day rf and compound it to settlement date using 6-day rf. Finally, our valuation gives a result of 11.5054 USD.

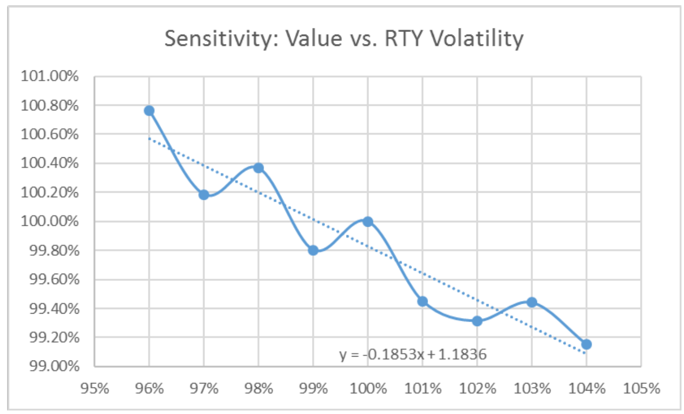
4 Sensitivity Analysis

We do sensitivity analysis to test the sensitivity of the result when risk-free rate, correlation coefficient, or volatilities of each index varies. Setting the outcomes in percentage of the original values, we draw out the analysis graphs.









From the graphs could we see, the valuation result is positively related with risk-free rate and indices volatilities, while negatively related with correlation coefficient. The fluctuation (not a consistent curve) of the curve comes because of the mechanism the random numbers are generated. From the fitting result could we see the valuation result is sensitive to all the four factors we choose.

5 Limitations

We find that our value is far from the recommended price printed on the product filing. However, we believe Monte Carlo could give the most accurate valuation result, overcoming the non-linear errors in other models. The limitation of the model might come from the following sources.

Firstly, the estimation of the covariance and correlation matrices would be biased, if these factors also vary along with time. Possible improvements could be correcting these factors to stochastic ones or specifying a more proper date interval to sample.

Secondly, the simulation is very costly. We only use $12 \times 10000 \times 1926$ iterations but spend great time on it, still not eliminating the sawtoothing shape in the sensitivity analysis and giving an ambiguous result (even the 12 experiments fluctuate a lot). Once permitted, there could be a larger-sized simulation.

Thirdly, the steps we choose is for the convenience to calculate final value, however, during the averaging date interval not every dates has a stock price. So our model also faces up to some realistic limitations.

λ