# FIN 514: Project 2

# Valuation Report


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**Part 0 Product Intro**:

The product we choose is Callable Reserve Exchangeable Notes Linked to the Class A Common Stock of Facebook, Inc., which has following features:

1) This product was priced on June 10 ,2016 and issued on June 15 ,2016. Therefore, the price of the product on pricing date should equal to the 6-day discounted value of the product on issuing date.

2) On interest payment day, investors will receive $5.833 interest. There are 12 interest payment dates which are July 18, 2016, August 17, 2016, September 19, 2016, October 18, 2016, November 18, 2016, December 19, 2016, January 18, 2017, February 17, 2017, March 17, 2017, April 18, 2017, May 17, 2017 and the Maturity Date.

3) On call date, the issuer has the right to redeem the note in whole at the price of ($1000+accrued interest). The redemption will realize if the value of the note is larger than ($1000+accrued interest) on call date. There are two call dates which are December 19, 2016 and March 17, 2017 (also interest payment date).

4) At maturity, there will be two situations:
   (1) Non-trigger payoff:
   If the trigger event has not happened, which means the stock price has not fallen below 65%*$1000=$75.803, **or** the final value (value on review date) is above the initial value, which is $116.62. Then investors will receive $1000 principal and $5.833 interest.
   (2) Trigger payoff
   If the trigger event has happened, which means the stock price has fallen below 65%*$1000=$75.803, **and** the final value (value on review date) is below the initial value, which is $116.62. Then investors will receive fractional principal plus $5.833 interest. The fractional principal equals to $1000*(ST/S0)

## Part 1 Parameters Estimation

1) Risk-free Rate Estimation

   As the pricing date and maturity of the product are June 10, 2016 and June 16, 2017 respectively, we need the risk-free rate in this period. To find an appropriate risk-free rate, we use simplest linear interpolation. This is an estimation of .77%.

$$r_{2017.06.16} = r_{2017.03.15} + \frac{r_{2017.06.21} - r_{2017.03.15}}{98} * 93$$

2) Volatility Estimation

   Observing the payoff of the product, we find it is similar to the portfolio that shorts a put and long a fixed coupon bonds. So here we use the implied volatility data for put options on Pricing Date. Choose moneyness of 100% for the strike is equal to initial stock price. To find a volatility applicable to our maturity, we use simplest linear interpolation. This is an estimation of 30.33%.

$$\sigma_{p,2017.06.16} = \sigma_{2017.06.10} + \frac{\sigma_{2017.10.10} - \sigma_{2017.06.10}}{122} * 6$$

3) 5-business Day

   The redemption policy requires that if issuers will redeem the note before maturity, issuers should deliver the notice at least five business days before the redemption. Therefore, when we estimate the value on the call date, we take the 5-business day discounted value of ($1000+accrued interest) into consideration. Because there are only 2 call dates, therefore we just found that in real world, the 5-business day before call dates equal to 7 natural days.

4) Range of Stock Price

   The we choose 3 times of initial price as the upper level and 0 as the lower level.

5) Steps I , Range of J and A, B, and C.

   We choose 20000 steps, which also means i=20000. Therefore, delta t =T/20000. As there are two boundaries for the choice of j, which are:

$$j > \left|\frac{r-\delta}{\sigma^2}\right|$$

$$j^2 < \frac{1}{\sigma^2 \Delta t}$$

Therefore, the range of j is 40 to 461. As the step i, delta t and range of j has been settled, we can determine the value of A, B and C, which are:

$$A(j) = \left(0.5\sigma^2 j^2 + \frac{r}{2}\right)\Delta t$$

$$B(j) = 1 - r\Delta t - \sigma^2 j^2 \Delta t$$

$$C(j) = \left(0.5\sigma^2 j^2 - \frac{r}{2}\right)\Delta t$$

## Part 2 Methodology

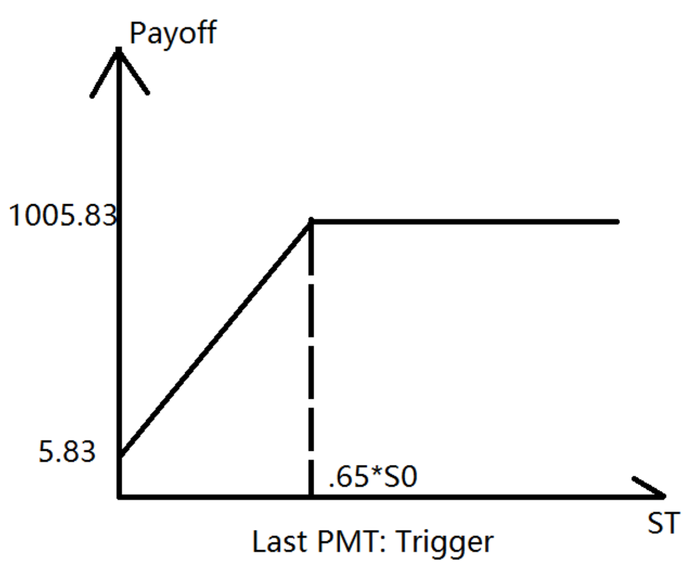
We choose the Explicit Finite Difference Grid method to estimate the value of this product. Here are the detail formulas and calculations for different stage or situation:

1) When Trigger Event Happens:

   (1) Payoff at Maturity

$$When\ \ final\ \ value \geq \$116.62 \quad Payoff = \$1000 + \$5.833$$

$$When\ \ final\ \ value < \$116.62 \quad Payoff = \$1000 * \frac{S_T}{S_0} + \$5.833$$



(2) Lower Boundary

On general day:

$$V(i, j = 0) = e^{-r\Delta t} V(i + 1, j = 0)$$

On interest payment day:

$$V(imax, j = 0) = \$5.833$$

$$V(i, j = 0) = \$5.833 * e^{-r(interest\ \ payment\ \ day - i*\Delta t)} + e^{-r\Delta t} V(i + 1, j = 0)$$

We don't need to discuss specially for redemption because the price at lower boundary won't lead to the redemption.

(3) Upper Boundary

On general day:

$$V(i, jmax) = e^{-r\Delta t} V(i + 1, jmax)$$

On interest payment day:

$$V(imax, jmax) = \$1000 + \$5.833$$

$$V(i, jmax) = e^{-r\Delta t} V(i + 1, jmax) + \$5.833 * e^{-r(interest\ \ payment\ \ day - i*\Delta t)}$$

On call day:

$$V(i, jmax) = \min\{e^{-r\Delta t} V(i + 1, jmax) + \$5.833 * e^{-r(interest\ \ payment\ \ day - i*\Delta t)}, \$1000 + \$5.833 * e^{-r(interest\ \ payment\ \ day - i*\Delta t)}\}$$

(4) General Grid Point:

On general day:

$$V(i, j) = A(j)V(i + 1, j + 1) + B(j)V(i + 1, j) + C(j)V(i + 1, j - 1)$$

On interest payment day:

$$V(i, j) = A(j)V(i + 1, j + 1) + B(j)V(i + 1, j) + C(j)V(i + 1, j - 1) + \$5.833 * e^{-r(interest\ \ payment\ \ day - i*\Delta t)}$$

On call day:

$$V(i, j) = \min\{A(j)V(i + 1, j + 1) + B(j)V(i + 1, j) + C(j)V(i + 1, j - 1) + \$5.833 * e^{-r(interest\ \ payment\ \ day - i*\Delta t)}, \$1000 + \$5.833 * e^{-r(interest\ \ payment\ \ day - i*\Delta t)}\}$$

2) Non-trigger Event

(1) Payoff at Maturity

$$Payoff = \$1000 + \$5.833$$

**(2) Lower Boundary (combine trigger and non-trigger situation)**

After the trigger event has been estimated, we can get the value of V(i ,jb), where i=0,1,2……imax and jb is the j that is close to and under $75.803. We will take this value as the lower boundary now. Input V(i,jb) as lower boundary is applicable in programming.

(3) Upper Boundary

On general day:

$$V(i, jmax) = e^{-r\Delta t} V(i+1, jmax)$$

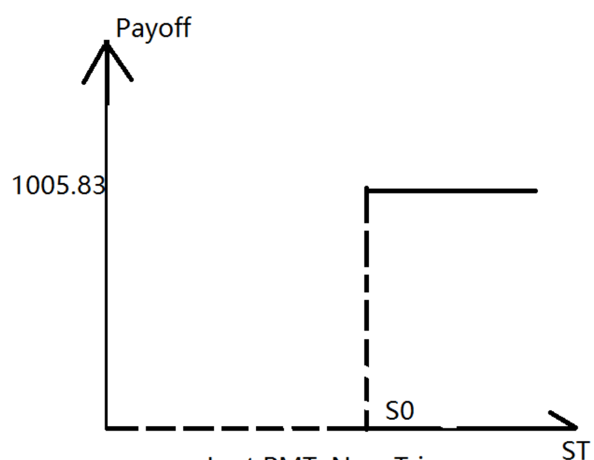
On interest payment day:

$$V(imax, jmax) = \$1000 + \$5.833$$

$$V(i, jmax) = e^{-r\Delta t} V(i+1, jmax) + \$5.833 * e^{-r(interest\ \ payment\ \ day - i*\Delta t)}$$

On call day:

$$V(i, jmax) = \min\{e^{-r\Delta t} V(i+1, jmax) + \$5.833 * e^{-r(interest\ \ payment\ \ day - i*\Delta t)}, \$1000 + \$5.833 * e^{-r(interest\ \ payment\ \ day - i*\Delta t)}\}$$




Last PMT: Non-Trigger

(4) General Grid Point

On general day:

$$V(i, j) = A(j)V(i+1, j+1) + B(j)V(i+1, j) + C(j)V(i+1, j-1)$$

On interest payment day:

$$V(i,j) = A(j)V(i+1,j+1) + B(j)V(i+1,j) + C(j)V(i+1,j-1) + \$5.833 * e^{-r(interest\ \ payment\ \ day - i*\Delta t)}$$

On call day:

$$V(i,j) = \min\{A(j)V(i+1,j+1) + B(j)V(i+1,j) + C(j)V(i+1,j-1) + \$5.833 * e^{-r(interest\ \ payment\ \ day - i*\Delta t)}, \$1000 + \$5.833 * e^{-r(interest\ \ payment\ \ day - i*\Delta t)}\}$$

For all the methods listed above, we use Python to do valuation. Please find our scripts at the attached.

## Part 3 Estimate Result

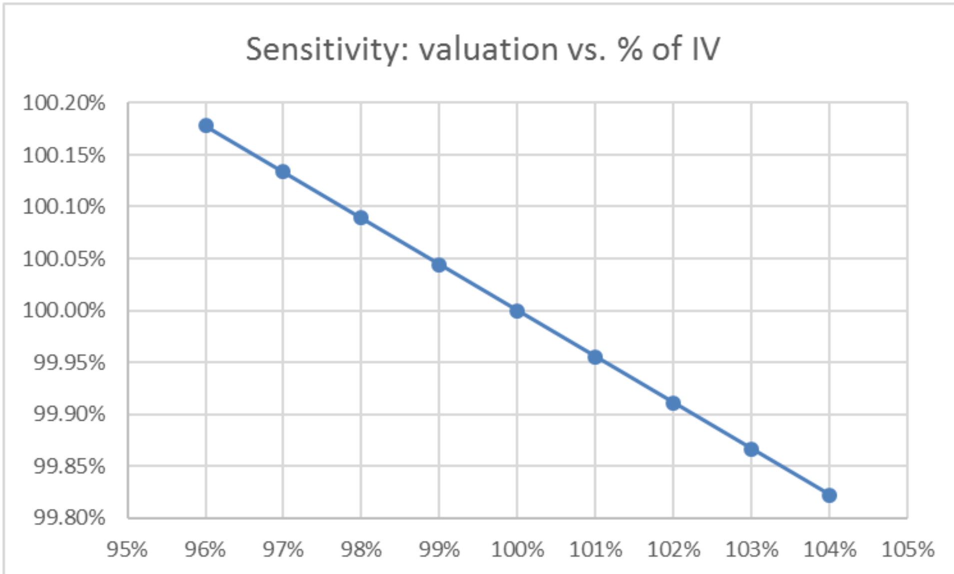
We make the step i settle to 20000 and get a series of estimated value of the product as j changes. Due to we can't use BS formula to estimate the precise value of this product, we need to find the convergence of our estimations and take the lambda (which we will discuss later) and convergence (related to j ) into consideration. **Finally, we choose the value \$956.49 under the condition j=420, lambda=0 and $S^L = 2.4S_0$.**

The 424B2 form of this product told us that the value they estimate is \$969. We are about \$12.51 below their estimation.

## Part 4 Sensitivity Analysis

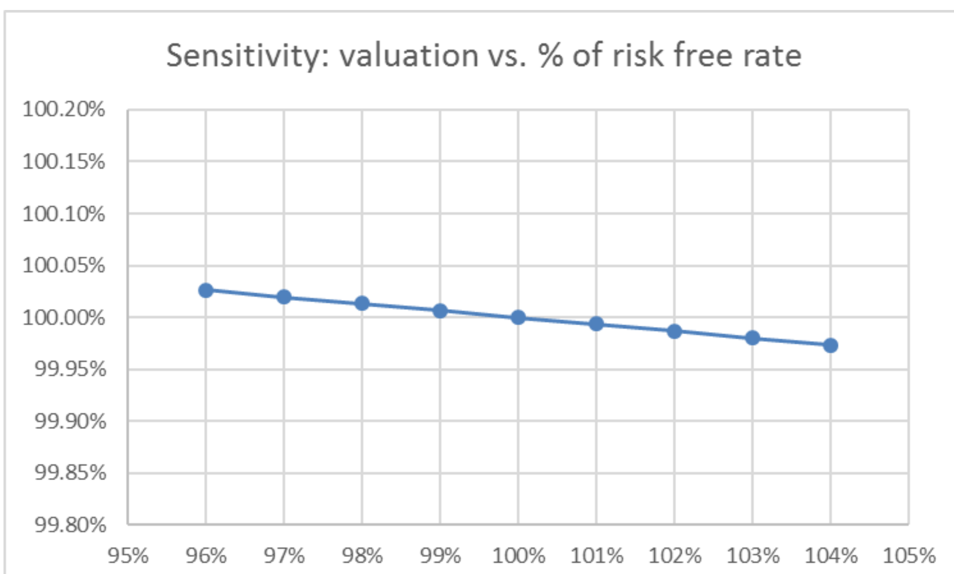
(1) Sensitivity of Implied Volatility

We settle i=20000 and j=420, and changes the implied volatility from $0.96\sigma$ =29.12% to $1.04\sigma$ =31.55%. We found that the estimation result is sensitive to the volatility changes. As implied volatility increases 1%, the estimated value of the product decreases 0.04%. And the relation turns out to be linear.




(2) Sensitivity of Risk-free Rate

We settle i=20000 and j=420, and changes the risk-free rate from to 0.96r=0.743% to 1.04r=0.805%. We found that the estimation result is sensitive to the risk-free rate changes. As risk-free rate increases 1%, the estimated value of the product decreases 0.0066%. And the relation turns out to be linear.



(3) Sensitivity of Delta S (related to j)

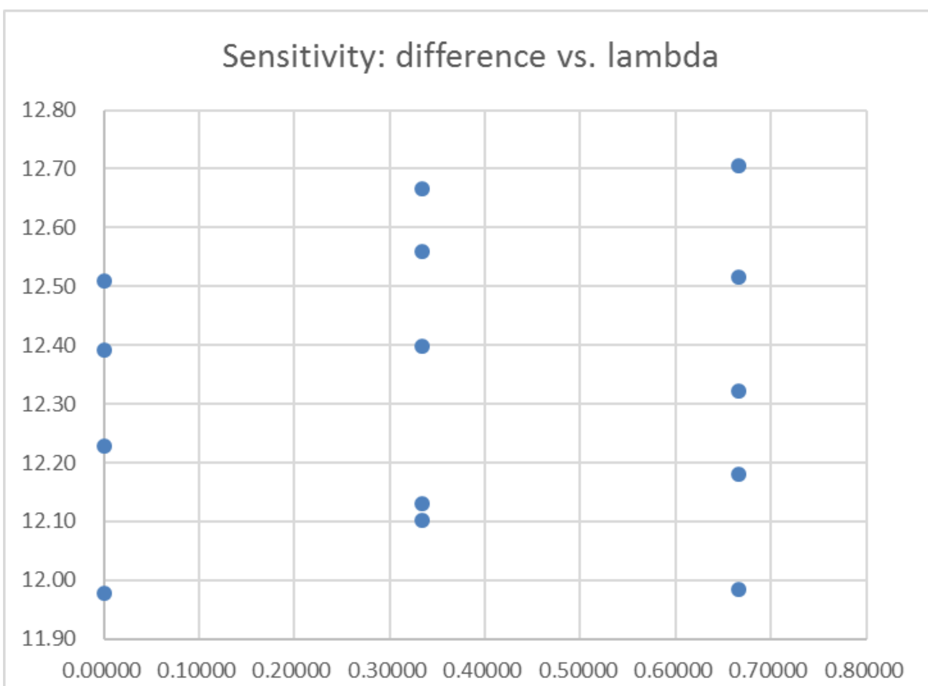
This is related to an important ratio *lambda,* the further discussion is in next part.

## Part 5 Error Discussion

We found that there are 4 sources of error.

Source 1: The Barrier

This product has a trigger value which equals to \$75.803. When steps i and range of stock price are settled, delta S will change as j varies. As delta S varies, the distance of grid which is the closest to and simultaneously under the barrier=\$75.803 will change. We denote the ratio of (distance/delta S) as lambda. We found that the error is the smallest when lambda is very close to 0, which means that the grid point almost falls on the barrier. The error is greatest when lambda is 0.5, which means the barrier falls in the middle of the grid point.




Source 2: The Discontinuous Payoff.

The payoff at maturity of this product is determined by whether trigger event happens. When trigger event happens, there will be discontinuous payoff, which leads to $\frac{\partial(Value\ \ of\ \ product)}{\partial S}$ becomes discontinuous. Therefore, maybe we can use the logic in BD method to eliminate the discontinuous payoff at maturity when trigger event happens.

Source 3: The Callable Feature

There are two call dates for this product. The early call feature will cause error. However, as there are 2 days that have early call boundary, the error caused by this feature is very small. We can ignore it.

Source 4: Choice of j

When steps i has been settled, which leads to delta t settled, then j will be locked in a range. In this range, the choice of j affects the value of A(j), B(j) and C(j). In our model, the range of j is (40<j<461). When j<260, value of B is too large. When j>360, value of B will be too small. Therefore, the choice of j will affect the error of estimation.