APM466 Assignment 1

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Fundamental Questions - 25 points

1.
   (a) Governments issue bonds instead of printing money because excessive money printing leads to inflation, devaluing the currency, while bonds raise capital without immediate inflationary pressure by borrowing from investors with a promise of future repayment.
   (b) The long-term part of the yield curve flattens when investors expect an economic slowdown or lower inflation, leading them to buy long-term bonds, which raises their prices, lowers their yields and reduces the difference between short-term and long-term interest rates.
   (c) Quantitative easing (QE) is a monetary policy in which central banks buy long-term securities to inject liquidity and lower interest rates, which the US Federal Reserve implemented during COVID-19 by purchasing trillions in Treasury bonds and mortgage-backed securities to stabilize markets and support economic recovery.

2. Since the bonds are semi-annual coupon bonds, the time to maturity of each bond is strategically picked to have 0.5 year spacing between each other to use bootstrapping. Having the 6-month gap makes the process of calculating the spot rates and the yield curve easier.

Name	**ISIN**	**Issue Date**
CAN 1.25 Mar 25	CA135087K528	2019-10-11
CAN 0.5 Sep 25	CA135087K940	2020-04-03
CAN 0.25 Mar 26	CA135087L518	2020-10-09
CAN 1 Sep 26	CA135087L930	2021-04-16
CAN 1.25 Mar 27	CA135087M847	2021-10-15
CAN 2.75 Sep 27	CA135087N837	2022-05-13
CAN 3.5 Mar 28	CA135087P576	2022-10-21
CAN 3.25 Sep 28	CA135087Q491	2023-04-21
CAN 4 Mar 29	CA135087Q988	2023-10-13
CAN 3.5 Sep 29	CA135087R895	2024-04-08

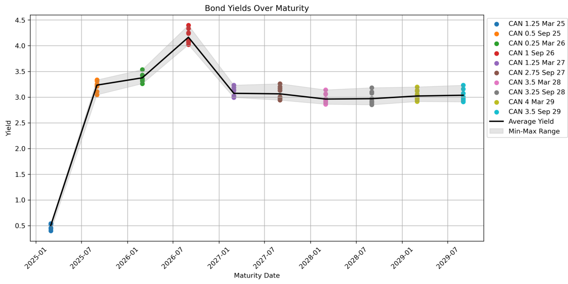
Table 1: 10 Government Bonds Selected

3. Eigenvalues and eigenvectors of the covariance matrix help analyze the variability and structure of stochastic processes along the curve. Eigenvalues measure how much variance exists in each principal direction, while eigenvectors define the directions where the data shows the most variation. Principal Component Analysis (PCA) leverages these concepts to reduce data dimensionality, allowing us to concentrate on the most significant variations while disregarding less impactful ones. This approach reveals correlations within stochastic processes and highlights the key directions that capture the most critical information.

Empirical Questions - 75 points

4.

(a) Before answering the question, a few assumptions were made to make the calculation easier. Assume January 1st 2025 as t = 0, 1 year = 360 days and 1 month = 30 days. The yield to maturity is calculated by using the *irr*(Internal Rate of Return) function in Python after collecting the 10 days of 10 bonds' data. The 10 selected bonds' Yield to Maturity(YTM) curve is shown below.



(b) 1. Calculate the dirty price of each bond by using the equation 1 and save them into a list.

$$\text{Dirty Price} = (\text{Price} + \text{Coupon}) * \frac{(4*30)(today-1)}{360} \tag{1}$$

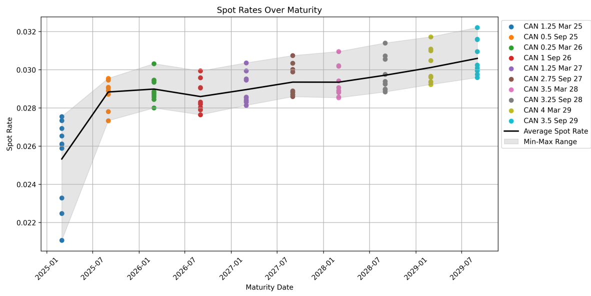
2. Calculate the spot rate by using bootstrapping: Calculate the spot rate of the first bond which has no coupon payment until it matures. Here, solve for r_1 as shown in equation 2.

$$r_1 = -\ln\left(\frac{\text{Dirty Price}}{C+FV}\right) \tag{2}$$

Next, use bootstrapping for all remaining bonds to calculate the spot rates using equation 3.

$$r_N = -\frac{1}{N}\ln\left(\frac{\text{Dirty Price} - \sum_{t=1}^{N-1} Ce^{-r_t t}}{C+FV}\right) \tag{3}$$

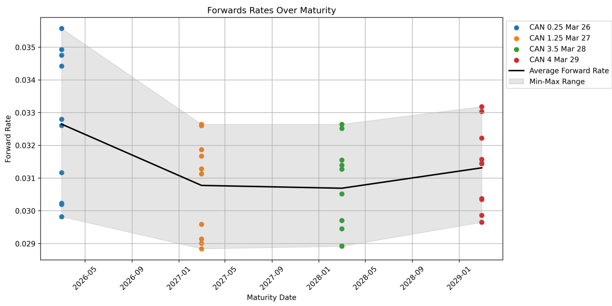
Append each spot rate into a list. The 10 selected bonds' spot rate curve is shown below.



(c) 1. Use the equation 4 and calculate the forward rates: 1yr-1yr, 1yr-2yr, 1yr-3yr, 1yr-4yr. Here, $F_{t,t+n}$ is the forward rate for the period t to t + n and S_t is the spot rate at t. Then, repeat the calculations for each date from Jan 6th to 17th.

$$F_{t,t+n} = \frac{S_{t+n} \cdot (t+n) - S_t \cdot t}{n} \tag{4}$$

The 1-year forward rate curve is shown below.



5. The first table below shows the covariance matrix or the time series of daily log-returns of yield, while the second table shows it for the 1-yr forward rates.

	x_1	x_2	x_3	x_4	x_5
x_1	0.004077	0.000674	0.000712	0.000852	0.000777
x_2	0.000674	0.000559	0.000527	0.000486	0.000520
x_3	0.000712	0.000527	0.000623	0.000550	0.000581
x_4	0.000852	0.000486	0.000550	0.000575	0.000571
x_5	0.000777	0.000520	0.000581	0.000571	0.000604

	x_1	x_2	x_3	x_4
x_1	0.001485	0.001032	0.000723	0.000765
x_2	0.001032	0.000974	0.000688	0.000722
x_3	0.000723	0.000688	0.000638	0.000619
x_4	0.000765	0.000722	0.000619	0.000656

6. The largest eigenvalue represents the largest source of variation in the yield or forward rate changes, and its corresponding eigenvector indicates the direction in which these rates move together. The first table shows the eigenvalues and eigenvectors of the yield covariance matrix, while the second table shows those of the forward rate covariance matrix.

λ	4.9197×10^{-3}	1.3744×10^{-3}	8.7152×10^{-5}	4.2997×10^{-5}	1.3862×10^{-5}
$\vec{v}$	$\begin{bmatrix} -0.8736 \\ -0.2227 \\ -0.2392 \\ -0.2594 \\ -0.2506 \end{bmatrix}$	$\begin{bmatrix} 0.4846 \\ -0.4277 \\ -0.4755 \\ -0.3921 \\ -0.4498 \end{bmatrix}$	$\begin{bmatrix} 0.0263 \\ 0.8358 \\ -0.0758 \\ -0.4649 \\ -0.2810 \end{bmatrix}$	$\begin{bmatrix} 0.0315 \\ -0.2487 \\ 0.8291 \\ -0.4527 \\ -0.2115 \end{bmatrix}$	$\begin{bmatrix} 0.0169 \\ -0.0844 \\ -0.1532 \\ -0.5983 \\ 0.7818 \end{bmatrix}$

λ	3.3133×10^{-3}	3.2607×10^{-4}	8.7995×10^{-5}	2.6262×10^{-5}
$\vec{v}$	$\begin{bmatrix} -0.6272 \\ -0.5229 \\ -0.4003 \\ -0.4159 \end{bmatrix}$	$\begin{bmatrix} -0.7326 \\ 0.1478 \\ 0.4984 \\ 0.4394 \end{bmatrix}$	$\begin{bmatrix} -0.2644 \\ 0.8326 \\ -0.4230 \\ -0.2409 \end{bmatrix}$	$\begin{bmatrix} -0.0035 \\ -0.1078 \\ -0.6422 \\ 0.7589 \end{bmatrix}$

References and GitHub Link to Code

GitHub Link: https://github.com/genie-y/466