INS/GPS NAVIGATION DATA FUSION USING FUZZY ADAPTIVE KALMAN FILTERING

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Abstract: In this paper, a method based on Adaptive Fuzzy Kalman Filtering has been applied to fuse position signals from the Global Positioning System (GPS) and Inertial Navigation System (INS) for the navigation of autonomous mobile vehicles. The presented method is of particular importance for guidance, navigation, and control of flying vehicles and has been validated in 3-D environments. The Extended Kalman Filter (EKF) and the noise characteristic are modified using Fuzzy Logic Adaptive System and compared with the performance of a regular EKF. It has been demonstrated that the Fuzzy Adaptive Kalman Filter gives better results in terms of accuracy than the EKF. *Copyright ©2000 IFAC*

Keywords: Sensor fusion, GPS/INS fusion. , navigation, fuzzy logic, Kalman filtering

1. INTRODUCTION

Kalman filtering is a form of optimal estimation characterized by recursive evaluation, where an internal model of the dynamics of the system is estimated. The dynamic weighting of incoming evidence with ongoing expectation produces state estimates of the observed system (Abide *et al.*, 1992). An extended Kalman filter (EKF) can be used to fuse measurements from Global Positioning System (GPS) and Inertial Navigation System (INS). In this EKF, the INS data are used as a reference trajectory, and GPS data are applied to update and estimate the error states of this trajectory. Global Positioning System (GPS) is a satellite-based navigation system equipped with the receiver that provides the user with appropriate and accurate positioning information anywhere on the globe. (Brown and Hwang, 1992). However, several errors are associated with the GPS measurement. It has superior long-term error performance, but poor short-term accuracy. For many vehicle navigation systems, GPS is insufficient as a stand-alone positioning system. The integration of GPS and INS is ideal for vehicle navigation systems. In general, the short-term accuracy of INS is good, whereas the long-term accuracy is poor. The disadvantages of GPS/INS are ideally cancelled. If the signal of GPS is interrupted, the INS enables the navigation system to coast along until GPS signal is reestablished (Brown and Hwang, 1992). The requirements for accuracy, availability, and robustness are therefore achieved.

The Kalman filter requires that all the plant dynamics and noise processes are exactly known and the noise processes are zero mean white noise. If the theoretical behavior of a filter and its actual behavior do not agree, divergence problems will occur. There are two kinds of divergence: *apparent divergence* and *true divergence* (Gelb, 1974; Lewis, 1986). In the apparent divergence, the actual estimate error covariance remains bounded, but it approaches a larger bound than does predicted error covariance. In true divergence, the actual estimation covariance eventually becomes infinite. The divergence due to modeling errors is critical in Kalman filter application. If the Kalman filter is provided with information that the process behaves one way, whereas, in fact, it behaves another way, the filter will try to continually fit a wrong process signal. When the measurement situation does not provide enough information to estimate all the state variables of the system, in other words, the computed estimation error matrix becomes unrealistically small, and the filter disregards the measurement, then the problem becomes particularly severe. Thus, in order to solve the divergence due to modeling errors, we can estimate unmodeled states, but it adds complexity to the filter and one can never be sure that all the suspected unstable states are indeed model states (Gelb, 1974). Another possibility is to add process noise, which makes sure that the Kalman filter is driven by white noise, and prevents the filter from disregarding new measurement. In this paper, a fuzzy logic adaptive controller (FLAC) is used to adjust the exponential weighting of a weighted EKF and prevent the Kalman filter from divergence. The fuzzy logic adaptive controller (FLAC) will continually adjust the noise strengths in the filter's internal model, and tune the filter as well as possible. The FLAC performance is evaluated by simulating the fuzzy adaptive extended Kalman filtering scheme from Figure 1 and compared with the fuzzy logic adaptive Kalman filter (FLAKF) investigated by Jetto *et al.* (1999).

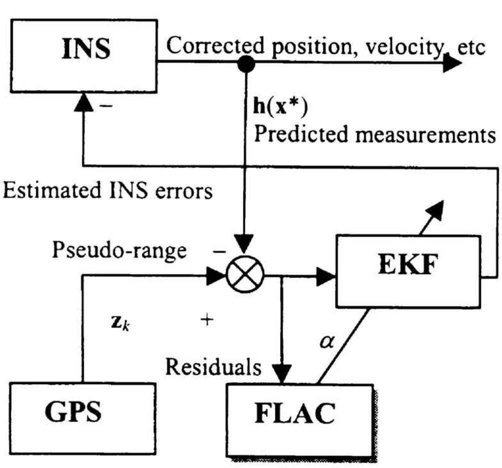

Fig. 1. Fuzzy adaptive extended Kalman filter

2. ERRORS IN INS AND GPS

Both the GPS and INS measurement processes are nonlinear, therefore, linearization is necessary. An extended Kalman filter is used to fuse the measurements from GPS and INS. To prevent divergence by keeping the filter from discounting measurements for large k, the exponential data weighting (Lewis, 1986) is used.

The models and implementation equations for the weighted extended Kalman filter are:

Nonlinear dynamic model:

$$\mathbf{x}_{k+1} = f(\mathbf{x}_k, k) + \mathbf{w}_k \qquad (1)$$
$$\mathbf{w}_k \sim N(0, \mathbf{Q});$$

where $\mathbf{w}_k$ is process noise (Gaussian white noise), $\mathbf{Q}$ is process noise covariance matrix,

Nonlinear measurement model:

$$\mathbf{z}_k = h(\mathbf{x}_k, k) + \mathbf{v}_k \qquad (2)$$
$$\mathbf{v}_k \sim N(0, \mathbf{R});$$

where $\mathbf{v}_k$ is addictive white measurement noise, $\mathbf{R}$ is measurement noise covariance matrix,

Let us set the model covariance matrices as

$$\mathbf{R}_k = \mathbf{R}\alpha^{-2(k+1)} \qquad (3)$$

$$\mathbf{Q}_k = \mathbf{Q}\alpha^{-2(k+1)} \qquad (4)$$

where, $\alpha \geq 1$, and matrices $\mathbf{Q}$ and $\mathbf{R}$ are constant. For $\alpha > 1$, as time k increases, $\mathbf{R}$ and $\mathbf{Q}$ decrease, so that the most recent measurement is given higher weighting. If $\alpha = 1$, we obtain a regular EKF.

By defining the weighted covariance

$$\mathbf{P}_k^{\alpha-} = \mathbf{P}_k^{-}\alpha^{2k} \qquad (5)$$

The Kalman gain can be computed as:

$$\mathbf{K}_k = \mathbf{P}_k^{-}\mathbf{H}_k^T(\mathbf{H}_k\mathbf{P}_k^{-}\mathbf{H}_k^T + \mathbf{R}\alpha^{-2(k+1)})^{-1}$$
$$= \mathbf{P}_k^{\alpha-}\mathbf{H}_k^T\left(\mathbf{H}_k\mathbf{P}_k^{\alpha-}\mathbf{H}_k^T + \frac{\mathbf{R}}{\alpha^2}\right)^{-1} \qquad (6)$$

The predicted state estimate is:

$$\hat{\mathbf{x}}_{k+1}^{-} = f(\hat{\mathbf{x}}_k^{-}, k) \qquad (7)$$

The predicted measurement is:

$$\hat{\mathbf{z}}_k = h(\hat{\mathbf{x}}_k^{-}, k) \qquad (8)$$

The updated estimation on the states can be computed as:

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_k^{-} + \mathbf{K}_k(\mathbf{z}_k - \hat{\mathbf{z}}_k) \qquad (9)$$

Computing the *a priori* covariance matrix:

$$\mathbf{P}_{k+1}^{-} = \Phi_k\mathbf{P}_k\Phi_k^T + \mathbf{Q}\alpha^{-2(k+1)} \qquad (10)$$

$$\text{where } \Phi_k \approx \left.\frac{\partial f(x,k)}{\partial x}\right|_{x=\hat{x}_k^-} \qquad (11)$$

By re-writing (10) we obtain:

$$\mathbf{P}_{k+1}^{\alpha-} = \alpha^2\Phi_k\mathbf{P}_k^{\alpha}\Phi^T + \mathbf{Q} \qquad (12)$$

Computing the *a posteriori* covariance matrix gives

$$\mathbf{P}_k^{\alpha} = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\mathbf{P}_k^{\alpha-} \qquad (13)$$

$$\text{where } \mathbf{H}_k \approx \left.\frac{\partial h(x,k)}{\partial x}\right|_{x=\hat{x}_k^-} \qquad (14)$$

The initial condition is:

$$\mathbf{P}_0^{\alpha -} = \mathbf{P}_0$$

In equation (9), the term $\mathbf{z}_k - \hat{\mathbf{z}}_k = r$ is called residuals or innovations. It reflects the degree to which the model fits the data.

3. FUZZY ADAPTIVE SYSTEM

The fuzzy logic is a knowledge-based system. The advantages of fuzzy logic with respect to more traditional adaptation schemes are the simplicity of the approach and application of the knowledge about the controlled system. In (Abdelnour *et al.*, 1993), fuzzy logic is used to on-line detection and correction of divergence in a single state Kalman filter. There were three inputs and two outputs to fuzzy logic controller (FLC), and 24 rules were used. The purpose of our fuzzy logic adaptive controller (FLAC) in this paper is to detect the bias in the measurements and prevent divergence of the extended Kalman filter. The control is applied in three axes — East (x), North (y), and Altitude (z) individually.

It is assumed that both the process noise $\mathbf{w}_k$ and the measurement noise $\mathbf{v}_k$ are zero-mean white sequences with known covariances **Q** and **R** in the Kalman filter. If a filter is performing optimally, the innovation is therefore a zero-mean white noise process. In general, no *a priori* reliable information of **Q** is available for the Kalman filter. In fact, **Q** is often used to represent the uncertainty in the process model. If the Kalman filter is based on a complete and perfectly tuned model, the residuals should be a zero-mean white noise process. If the residuals are not white noise, there is something wrong with the design and the filter is not performing optimally (Gelb, 1974). The Kalman filters will diverge or coverage to a large bound. In practice, parameters may be known only with some uncertainty or in order to reduce computation, we have to ignore some errors. Sometimes those unmodeled errors will become significant. One can readjust the assumed noise strengths in the filter's internal model based on information obtained in real time, as soon as the measurements are becoming available, so the filter is continually tuned as well as possible. One good way to verify whether the Kalman filter is performing as designed is to monitor the residual or innovation sequence. In fact, the residuals are the differences between actual measurements and best measurement predictions based on the filter's internal model. A well-tuned filter is the one where 95% of the autocorrelation function of innovation series should fall within the $\pm\, 2\sigma$ boundary (Cooper and Durrant-Whyte, 1994). The residuals can be used to adapt the filter. If the filter diverges, the residuals will not be zero mean and become larger. The theoretical covariance of residuals $\mathbf{P}_r$ relates to **Q** and **R**. The covariance of the residual is given by:

$$\mathbf{P}_r = \mathbf{H}_k(\Phi \mathbf{P}_{k-1}\Phi + \mathbf{Q})\mathbf{H}_k^T + \mathbf{R} \qquad (15)$$

As an input to FLAC, the actual covariance of the residuals and mean values of residuals are used to decide the degree of divergence. By choosing n to provide statistical smoothing, the actual smoothing mean and covariance of residuals can be

$$\bar{r} = \frac{1}{n}\sum_{j=i-n}^{i} r_j \qquad (16)$$

$$\hat{\mathbf{P}}_r = \frac{1}{n}\sum_{j=i-n+1}^{i} r_j r_j^T \qquad (17)$$

The actual $\mathbf{P}_r$ can be compared with its theoretical value $\mathbf{P}_r$ calculated from the EKF. Generally, when the actual covariance $\hat{\mathbf{P}}_r$ is becoming larger when compared with theoretical value $\mathbf{P}_r$, and mean value $\bar{r}$ is moving away from zero, the Kalman filter is becoming unstable. In this case, a large α will be applied. A large α means that process noises are added and we are giving more credibility to the recent data by decreasing the noise covariance. It can ensure that in the model all states are sufficiently excited by the process noise. Generally, **R** has more impact to the covariance of residuals. When the covariance is extremely large and the mean is far away from zero, there are some problems with the GPS measurements, so the filter cannot depend on these measurements anymore, and a smaller α will be used. By selecting appropriate α, the fuzzy logic controller will adapt the Kalman filter optimally and try to keep the innovation sequence acting as zero-mean white noise.

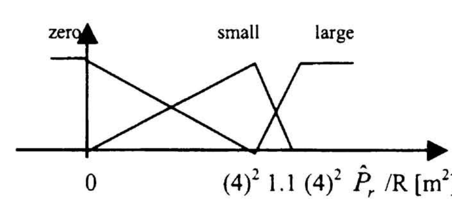

Fig. 2. Covariance Membership Functions

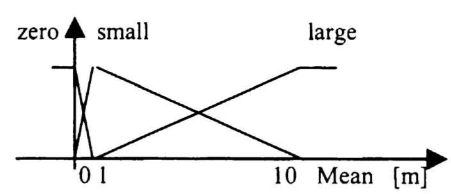

Fig. 3. Mean Value Membership Functions

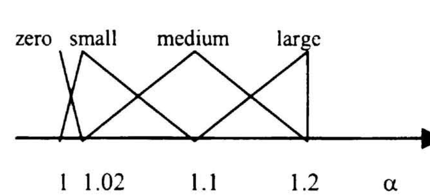

Fig. 4. α-Membership functions

The fuzzy logic controller uses 9 rules, such as:

If the covariance of residuals is large ***and*** *the mean values are zero* ***Then*** *α is large.*
If the covariance of residuals is zero ***and*** *the mean values are large* ***Then*** *α is zero.*

Table 1. Rule table for FLAC

α		**Mean Value** Z	S	L
$\hat{\mathbf{P}}_r$	Z	S	Z	Z
	S	Z	L	M
	L	L	M	Z

S---Small; **M**---Medium; **L**---Large; **Z**---Zero

Fuzzy adaptive Kalman filtering has been used for guidance and navigation of mobile robots, especially for 3-D environment. The navigation of flying robots requires fast and accurate on-line control algorithms. The regular Extended Kalman Filter requires high number of states for accurate navigation and positioning and is unable to monitor the parameters changes. The FLAC requires smaller number of states for the same accuracy, and, therefore, it needs smaller computational effort. Alternatively, the same number of states (as in the regular filter) would allow for more accurate navigation.
In (Jetto *et al*, 1999), FLAKF monitors changes in the innovation sequence, and then adapts the $\mathbf{Q}(k)$ dynamically. If the innovation sequence is neither too near nor too far from zero, then $\mathbf{Q}(k)$ is left almost unchanged; if it is very near zero, then $\mathbf{Q}(k)$ is reduced; if it is very far from zero, then $\mathbf{Q}(k)$ is increased. The input to fuzzy control in (Jetto *et al*, 1999) is the rate of actual innovation and the theoretical one. The output is the scaling factor β, which is used to scale $\mathbf{Q}$.

4. SIMULATIONS

The state variables used in simulation are:

$$\mathbf{x}_k = [x_k, \dot{x}_k, y_k, \dot{y}_k, z_k, \dot{z}_k, c\Delta t, c\dot{\Delta t}] \qquad (18)$$

The states are position and velocity errors of the INS East, North, Altitude, and GPS range bias and range drift. The designed covariance of GPS measurement $\mathbf{R}$ is 5 [m^2]. The sample period is 1 second.
It is assumed that the process noise $\mathbf{w}_k$ is zero mean white noise for Kalman filtering, but in actual situation, sometime the process noise could be not perfect white. In the simulations (Figures 5 and 6), we assume the process noise is non-white noise and the model is inaccuracies. The 3**Q** in the simulation means that the real- time parameters are 3 times as large as the designed **Q**. In the first several steps, the FLAC is off. After the filter is almost stable, the FLAC is turned on and starts tuning the filter on-line. The figures present the corrected position errors. The corrected error is the current INS error minus the estimated INS error. The top dot line (red) is EKF, solid line (blue) is FLAC and dashed line (black) is FLAKF. The simulations show that the both FLAC and FLAKF adjust the filter well. Although the INS measurements experience certain problems, (erroneous model, unmodeled states, INS fail, etc.), both FLAC and FLAKE significantly reduce the corrected position error.

5. CONCLUSIONS

In this paper, a fuzzy adaptive extended Kalman filter has been developed to detect and prevent the EKF from divergence. By monitoring the residual sequences, the FLAC can evaluate the performance of an EKF. If the filter does not perform well, it would apply an appropriate weighting factor α to improve the accuracy of an EKF. The simulation showed that both algorithms improve the performance of EKF when **Q** is not accuracies and the process noise is non-white. By changing α, we also can adjust the EKF well when **R** is uncertainty (Sasiadek and Wang, 1999). This is especially important for a GPS/INS navigation system, because some errors, such as multi-path and selective availability in GPS also produce changes in R in different situations
In the fuzzy logic controller, 9 rules have been used. Therefore, little computational time is needed. When a designer lacks sufficient information to develop complete models or the parameters will slowly change with time, the fuzzy controller can be used to adjust the performance of EKF on-line, and it will remain sensitive to parameter variations by remembering most recent N data samples. It can be used to navigate and guide autonomous vehicles or robots and achieve a relatively accurate performance.

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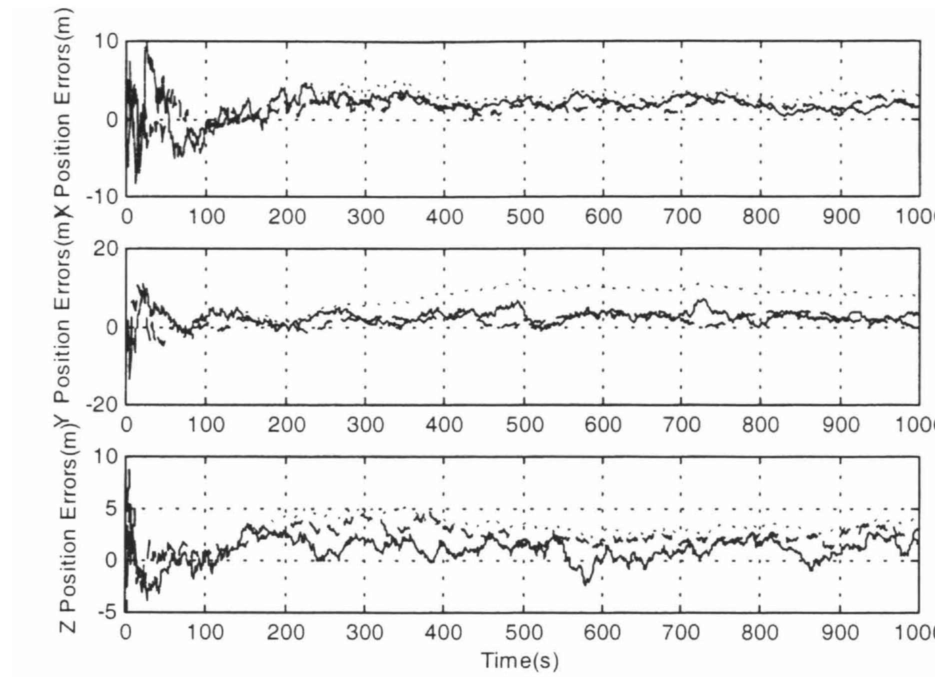

Fig. 5. Position error with process noise mean=0.001, **Q**=3**Q**.

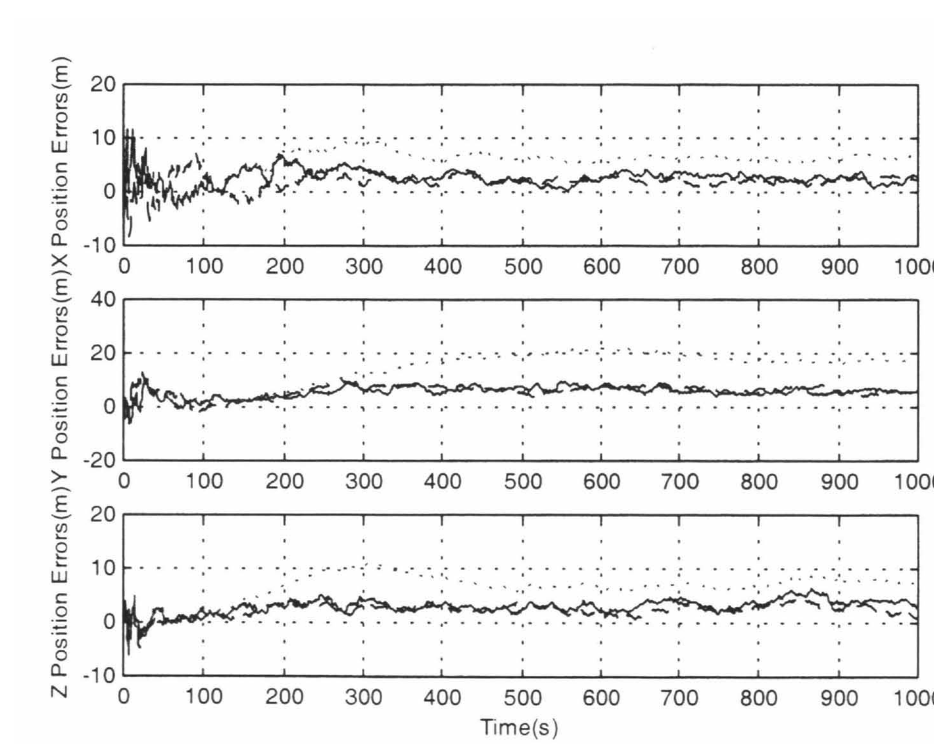

Fig. 6. Position error with process noise mean=0.002, **Q**=**Q**