Cooperative Control of Multiple Nonholonomic Mobile Agents

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***Abstract*—This paper considers two cooperative control problems for nonholonomic mobile agents. In the first problem, we discuss the design of cooperative control laws such that a group of nonholonomic mobile agents cooperatively converges to some stationary point under various communication scenarios. Dynamic control laws for each agent are proposed with the aid of σ-processes and results from graph theory. In the second problem, we discuss the design of cooperative control laws such that a group of mobile agents converges to and tracks a target point which moves along a desired trajectory under various communication scenarios. By introducing suitable variable transformations, cooperative control laws are proposed. Since communication delay is inevitable in cooperative control, in each of the above cooperative control problems, we analyze the effect of delayed communication on the proposed controllers. As applications of the proposed results, formation control of wheeled mobile robots is discussed. It is shown that our results can be successfully used to solve formation control problem. To show effectiveness of the proposed approach, simulation results are included.**

***Index Terms*—Cooperative control, decentralized control, formation control, mobile robots, nonholonomic system, nonlinear control.**

I. Introduction

Cooperative control of multiple agents has received considerable attention recently due to its challenging features and many applications, e.g., rescue mission, large object moving, troop hunting, formation control, and satellite clustering. Various control strategies have been proposed, such as behavior-based [1]–[3], virtual structure [4]–[6], leader-follower [7]–[10], artificial potentials [11]–[16], and graph theoretical [17]–[20] methods. On the cooperative control of mobile robots, several other methods were proposed in [21]–[26].

The consensus problem is closely related to cooperative control. Cooperative solutions to the consensus problem using nearest neighbor rules were proposed in [27]. The consensus problem for networks of dynamic agents with fixed and switching topologies was discussed in [28], [29]. State agreement for nonholonomic unicycles was discussed in [30], [31].

Most graph theory results related to cooperative control are obtained for linear agents. However, many practical cooperative control applications involve agents that are nonlinear and nonholonomic. Therefore, it is necessary to discuss cooperative control of multiple nonholonomic agents. The papers [23], [24], [26], [31] considered cooperative control of only a portion of the state vector of each mobile robot and their proposed methods were specialized to a specific class of robotic system. To our knowledge, there is no paper which discusses the cooperative control problem for general nonholonomic agents with limited communication capabilities among neighbors. In this paper, for different communication scenarios, we discuss two cooperative control problems. For each problem, we are interested in a group of nonholonomic mobile agents and propose systematic cooperative controller design methods. The first problem addresses the design of cooperative control laws such that the state of each agent converges to some stationary point. This problem is solved herein through the definition of dynamic cooperative control laws with the aid of σ-processes and results of graph theory; if desired, we can make the stationary point be any desired point by introducing a virtual agent. The second problem that we address concerns the design of cooperative control laws such that the state of each agent converges to and tracks a target point which moves along a desired trajectory. By introducing a suitable variable transformation we show that this problem can be solved with the aid of the results on the cooperative control of multiple linear agents under mild assumptions on the desired trajectory, and we propose appropriate cooperative control laws. In both cooperative control problems, communication delay between neighboring agents are inevitable; therefore, we analyze the affects of time-delay on the proposed cooperative control laws. It is shown that our proposed cooperative control laws are robust to small communication delay (i.e., less than a quantifiable parameter τ^*). Finally, we show that the proposed results can be successfully applied to achieve formation control of multiple mobile robots. To verify effectiveness of the proposed cooperative control laws, simulation results are included for applications. Compared with the results in [23], [24], [26], [31], this paper discusses cooperative control of general nonholonomic agents and proposes systematic cooperative control design methods. Furthermore, in this paper the cooperative objectives are achieved for the entire state of each agent, compared to a portion of the state of each agent in the existing literature. In contrast to the results in [20], [32] which give a set of conditions under which the stability of multiple agents can be achieved, in this paper we explicitly propose cooperative control laws for each nonholonomic agent in chained form.

The rest of the paper is organized as follows. In Section II, we formally state the two cooperative control problems that are the focus of this article. In Section III, cooperative control laws are proposed for the first cooperative control problem under various communication scenarios. In Section IV, cooperative control

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laws are proposed for the second cooperative control problem under various communication scenarios. Section V shows how formation control problems with multiple mobile robots can be transformed to fit the problems stated in Section II. Section VI includes simulation results to illustrate the applications of the results presented herein. Section VII states conclusions related to the analysis of this paper.

II. Problem Statement

In Sections II–IV, we consider the kinematics for m mobile agents, indexed by $j \in [1, m]$, after they have been transferred into canonical chained form (see [33] and Section V)

$$\dot{q}_{1j} = u_{1j},\ \dot{q}_{2j} = u_{2j},\ \dot{q}_{ij} = q_{i-1,j}u_{1j},\ \text{for } i \in [3, n] \quad (1)$$

where $q_{*j} = [q_{1j}, \ldots, q_{nj}]^\top$ and $u_{*j} = [u_{1j}, u_{2j}]^\top$ are the state and input of agent j, respectively. The communication between the agents can be described by the edge set $\mathcal{E}$ of the digraph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ where the m mobile agents are represented by the m nodes in $\mathcal{V}$ (basic graph theory and terminology can be found in [34], [35]). The existence of an edge $(l, j) \in \mathcal{E}$ means that the state q_{*l} of agent l is available to agent j for control (i.e., unidirectional communication). Bidirectional communication, if it exists, would be represented by the edge (j, l) also being in the edge set $\mathcal{E}$. The symbol $\mathcal{N}_j$ denotes the neighbors of node j and is the set of indices of agents whose state is available to agent j. The information available to agent j for the controller design is the j–th agent's own state and the state of each agent l for $l \in \mathcal{N}_j$. Due to sensor range limitations and bounded communication bandwidth between agents, $\mathcal{N}_j$ may change with time, which means that the edge set $\mathcal{E}$ may be time-varying and consequently the Laplacian matrix L corresponding to $\mathcal{G}$ may be time-varying.

Two types of cooperative control problems will be addressed in this article. They are stated in the following paragraphs.

Problem 1: Design a control law u_{*j} for agent j using q_{*j} and the relative state information between q_{*j} and q_{*l} for $l \in \mathcal{N}_j$ such that

$$\lim_{t\to\infty} q_{*j}(t) = c, \quad j \in [1, m] \quad (2)$$

where c is a constant vector.

Remark 1: In *Problem 1*, the control law for each agent j is designed based on q_{*j} and the relative state information between agent j and its neighbors. The control laws are required to make the state of each agent converge to some constant vector c which depends on the initial states of the group of agents, the control parameters, and the communication digraph topology.

Remark 2: By Brockett's necessary condition for stabilizing a nonholonomic system [36], there does not exist a feedback law which is a smooth function of its own state and the states of its neighbors such that the state of each agent converges to the same stationary point c. Therefore, the results obtained for cooperative control of multiple linear agents cannot be used to solve *Problem 1* because all linear controllers are smooth functions of the states of agents.

Given a desired trajectory $q^d = [q_1^d, \ldots, q_n^d]^\top$ which is generated by

$$\dot{q}_1^d = w_1,\ \dot{q}_2^d = w_2,\ \dot{q}_i^d = q_{i-1}^d w_1,\ \ i \in [3, n] \quad (3)$$

where w_1 and w_2 are known time-varying functions, the second type of cooperative control problem that will be discussed in this article is defined as follows.

Problem 2: Design a control law u_{*j} for agent j using q_{*j}, the relative state information between q_{*j} and q_{*l} for $l \in \mathcal{N}_j$, and $q^d(t)$ such that

$$\lim_{t\to\infty} (q_{*l}(t) - q_{*j}(t)) = 0 \quad (4)$$

and

$$\lim_{t\to\infty} \left(q_{*j}(t) - q^d(t)\right) = 0. \quad (5)$$

Remark 3: In *Problem 2*, the control law for agent j is designed based on the desired trajectory q^d, the state of agent j, and the relative state information between q_{*j} and q_{*l} for $l \in \mathcal{N}_j$. Equation (4) holds if (5) is satisfied for all $j \in [1, m]$. However, satisfaction of (4) does not imply (5). Equation (4) is listed in *Problem 2* to emphasize that the controllers will be designed in this paper such that the convergence rate of (4) (i.e., *rendezvous rate*) and the convergence rate of (5) (i.e., *tracking rate*) can be adjusted independently by suitably choosing control parameters. For details, please see the remarks after Theorem 7 and Theorem 8.

In this paper, we will use the following terminology. For a bounded signal $s(t)$ and a constant ϱ_1, we will state that $\lim_{t\to\infty} s(t) = \varrho_1$ exponentially if there exists a positive constant ϵ and a constant ϱ_2 such that $\lim_{t\to\infty} e^{\epsilon t}(s(t) - \varrho_1) = \varrho_2$. We will state that $\lim_{t\to\infty} s(t) = \varrho_1$ with exponential rate ϵ if there exists a constant ϱ_2 such that $\lim_{t\to\infty} e^{\epsilon t}(s(t) - \varrho_1) = \varrho_2$.

III. Controllers for Problem 1

Since the communication digraph $\mathcal{G}$ may change with time, we discuss two cases: $\mathcal{G}$ is time-invariant and $\mathcal{G}$ is time-varying.

A. Fixed Communication Graph Case

Related to the communication digraph, we make the following assumption.

Assumption 1: The communication digraph $\mathcal{G}$ has a spanning tree.

For any digraph $\mathcal{G}$ satisfying Assumption 1, its Laplacian matrix L has the following property which is almost a restatement of Theorem 3 in [28]. The proof of this lemma is almost the same as that in [28] and is omitted here.

Lemma 1: If the digraph $\mathcal{G}$ satisfies Assumption 1 and L is the Laplacian matrix for $\mathcal{G}$ with weight matrix $\mathcal{B} = [b_{jl}]$ $(b_{jl} > 0)$, then

$$\lim_{t\to\infty} e^{\mu t}(e^{-Lt} - \mathbf{1}w^\top) = 0$$

for any $\mu \in [0, Re(\lambda_2(L)))$, where λ_2 is the nonzero eigenvalue of L with the smallest real part, w satisfies $w^\top L = 0$ and $w^\top \mathbf{1} = 1$, and $\mathbf{1}$ is a vector with element one.

In Lemma 2, we design u_{1j} for agent j. The design of u_{2j} will be considered in Lemma 4.

Lemma 2: For system (1), under Assumption 1, control laws

$$u_{1j} = -\sum_{l\in\mathcal{N}_j} b_{jl}(q_{1j} - q_{1l}) + \eta_j \quad (6)$$

$$\dot{\eta}_j = -\gamma\eta_j, \quad \eta_j(0) = \beta(\neq 0) \quad (7)$$

for $j \in [1, m]$ make

$$\lim_{t\to\infty} e^{\gamma t}(q_{1j}(t) - q_{1l}(t)) = 0, \quad \forall l \neq j \tag{8}$$

and

$$\lim_{t\to\infty} q_{1j}(t) = w^\top \left(q_{1*}(0) + \frac{\mathbf{1}\beta}{\gamma}\right) =: c_1 \tag{9}$$

where $q_{1*} = [q_{11}, q_{12}, \ldots, q_{1m}]^\top$, $\gamma \in (0, Re(\lambda_2(L)))$, $\beta \neq 0$, $b_{jl} > 0$, the constant vector w is defined in Lemma 1.

Proof: With the control laws in (6)–(7) we have

$$\dot{q}_{1*} = -Lq_{1*} + e^{-\gamma t}\beta\mathbf{1}. \tag{10}$$

The transformed state

$$\xi(t) = [\xi_1(t), \ldots, \xi_m(t)]^\top = q_{1*}(t) + \frac{\beta}{\gamma}e^{-\gamma t}\mathbf{1}$$

has time derivative

$$\dot{\xi} = -L\xi.$$

Therefore

$$\xi(t) = e^{-Lt}\xi(0)$$

and

$$q_{1*}(t) = \xi(t) - \frac{\beta}{\gamma}e^{-\gamma t}\mathbf{1} = e^{-Lt}\xi(0) - \frac{\beta}{\gamma}e^{-\gamma t}\mathbf{1}.$$

So

$$\lim_{t\to\infty} e^{\gamma t}(q_{1j}(t) - q_{1l}(t)) = 0$$

where we have skipped considerable algebra and applied Lemma 1. Also

$$\lim_{t\to\infty} q_{1*}(t) = \lim_{t\to\infty} \xi(t) = \mathbf{1}w^\top\xi(0) = \mathbf{1}w^\top\left[q_{1*}(0) + \frac{\mathbf{1}\beta}{\gamma}\right] = c_1\mathbf{1}.$$

Thus, (8) and (9) are satisfied. ■

Remark 4: The proof of Lemma 2 goes through for any β. The constraint that $\beta \neq 0$ will be required in the proof of Lemma 4 due to the nonholonomy of each agent.

We next design the control laws u_{2j} using the concept of σ-processes [37], for $j \in [1, m]$. For each j, define the state transformation

$$x_{2j} = q_{2j} - \sum_{k=3}^{n}\mu_k x_{kj}, \quad x_{ij} = \frac{\beta^{i-2}q_{ij}}{\eta_j^{i-2}}, \quad i \in [3, n] \tag{11}$$

where the constants μ_k for $k \in [3, n]$ are chosen such that the eigenvalues of the $(n-2) \times (n-2)$ matrix Ω lie in the open left complex plane (i.e., $\lambda(\Omega) \in \mathcal{C}^-$), where

$$\Omega = \begin{bmatrix} \gamma + \beta\mu_3 & \beta\mu_4 & \beta\mu_5 & \cdots & \beta\mu_{n-1} & \beta\mu_n \\ \beta & 2\gamma & 0 & \cdots & 0 & 0 \\ 0 & \beta & 3\gamma & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & (n-3)\gamma & 0 \\ 0 & 0 & 0 & \cdots & \beta & (n-2)\gamma \end{bmatrix}.$$

Remark 5: Since $\beta \neq 0$ and $\gamma \neq 0$, the eigenvalues of Ω can be arbitrarily assigned via μ_k, $k \in [3, n]$. In fact, given any desired eigenvalues λ_i, $i \in [1, n-2]$, we can find μ_k by solving $|\lambda I - \Omega| \equiv \Pi_{i=1}^{n-2}(\lambda - \lambda_i)$.

By (6)–(7) and (11), we have for $j \in [1, m]$

$$\begin{cases} \dot{x}_{2j} = u_{2j} - \sum_{k=3}^{n}(k-2)\gamma\mu_k x_{kj} - \sum_{k=4}^{n}\mu_k x_{k-1,j}\frac{\beta u_{1j}}{\eta_j} \\ \qquad -\mu_3\left(x_{2j} + \sum_{k=3}^{n}\mu_k x_{kj}\right)\frac{\beta u_{1j}}{\eta_j} \\ \dot{x}_{3j} = \gamma x_{3j} + \beta\sum_{k=3}^{n}\mu_k x_{kj} + \beta x_{2j} \\ \qquad -\left(x_{2j} + \sum_{k=3}^{n}\mu_k x_{kj}\right)\frac{\beta}{\eta_j}\sum_{l\in\mathcal{N}_j} b_{jl}(q_{1j} - q_{1l}) \\ \dot{x}_{4j} = 2\gamma x_{4j} + \beta x_{3,j} - x_{3j}\frac{\beta}{\eta_j}\sum_{l\in\mathcal{N}_j} b_{jl}(q_{1j} - q_{1l}) \\ \quad\vdots \\ \dot{x}_{nj} = (n-2)\gamma x_{nj} + \beta x_{n-1,j} \\ \qquad -x_{n-1,j}\frac{\beta}{\eta_j}\sum_{l\in\mathcal{N}_j} b_{jl}(q_{1j} - q_{1l}). \end{cases} \tag{12}$$

Before proposing the control laws u_{2j} for $j \in [1, m]$, we first give the following useful lemma. We do not include a proof, as it is an extension of the result in Section IV.B.II in [38].

Lemma 3: Consider the system

$$\dot{\zeta} = (\Lambda_1 + \Lambda_2(t))\zeta + \Lambda_3(t) \tag{13}$$

where the constant matrix $\Lambda_1 \in \Re^{k\times k}$ is asymptotically stable, and $\Lambda_2(t)$ is bounded and exponentially converges to zero. If $\Lambda_3(t)$ is bounded and exponentially converges to zero, then ζ exponentially converges to zero.

With the control laws in (6)–(7) applied to the dynamics defined in (12), we are still free to define u_{2j}. This is the purpose of the following lemma.

Lemma 4: For system (12), under Assumption 1, the control laws u_{1j} in (6)–(7) and

$$u_{2j} = -\sum_{l\in\mathcal{N}_j} b_{jl}(x_{2j} - x_{2l}) + \sum_{k=3}^{n}(k-2)\gamma\mu_k x_{kj} + \mu_3\left(x_{2j} + \sum_{k=3}^{n}\mu_k x_{kj}\right)\frac{\beta u_{1j}}{\eta_j} + \sum_{k=4}^{n}\mu_k x_{k-1,j}\frac{\beta u_{1j}}{\eta_j} \tag{14}$$

for $j \in [1, m]$ make

$$\lim_{t\to\infty} x_{ij} = d_i, \text{ for } i \in [2, n] \tag{15}$$

where $b_{jl} > 0$, $\beta \neq 0$, $\gamma \in (0, Re(\lambda_2(L)))$, L is the Laplacian matrix of the communication digraph $\mathcal{G}$ with weight $\mathcal{B} = [b_{jl}]$, μ_k for $k \in [3, n]$ are chosen such that $\lambda(\Omega) \in \mathcal{C}^-$, $d_2 = w^\top x_{2*}(0)$, $d_k = -\beta d_2[\Omega^{-1}e_1]_k$, e_1 is a $(n-2)$-vector and $e_1 = [1, 0, \ldots, 0]^\top$, $x_{2*} = [x_{21}, \ldots, x_{2m}]^\top$, $[\Omega^{-1}e_1]_k$ denotes the kth element of vector $\Omega^{-1}e_1$ for $k \in [3, n]$.

Proof: With control law (14) we have

$$\dot{x}_{2*} = -Lx_{2*}. \tag{16}$$

Therefore, $x_{2*}(t) = e^{-Lt}x_{2*}(0)$. By Assumption 1 and Lemma 1, we have $\lim_{t\to\infty} e^{\mu t}(x_{2*}(t) - d_2\mathbf{1}) = 0$ for any $\mu \in [0, Re(\lambda_2(L)))$. Note that d_2 is a scalar.

To show that $\lim_{t\to\infty} x_{kj}(t) = d_k$, let

$$y_j = \overline{x}_{3j} + \Omega^{-1}e_1\beta d_2$$

where $\overline{x}_{3j} = [x_{3j}, x_{4j}, \ldots, x_{nj}]^\top$, then

$$\dot{y}_j = \Omega y_j + A_j(t)y_j + D_j(t) \tag{17}$$

where

$$\begin{aligned} D_j(t) = &- A_j(t)\Omega^{-1}e_1\beta d_2 + e_1\beta(x_{2j} - d_2) \\ &- e_1 x_{2j} e^{\gamma t}\sum_{l\in\mathcal{N}_j} b_{jl}(q_{1j} - q_{1l}) \end{aligned}$$

and

$$A_j = - e^{\gamma t}\sum_{l\in\mathcal{N}_j} b_{jl}(q_{1j} - q_{1l}) \cdot \begin{bmatrix} \mu_3 & \mu_4 & \cdots & \mu_{n-1} & \mu_n \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}.$$

In the definitions of $D_j(t)$ and $A_j(t)$, we replace $\eta_j(t)$ by its representation $\beta e^{-\gamma t}$. By Lemma 2

$$\lim_{t\to\infty} e^{\gamma t}\sum_{l\in\mathcal{N}_j} b_{jl}(q_{1j} - q_{1l}) = 0$$

exponentially. So $A_j(t)$ is bounded and exponentially converges to zero. Since x_{2j} exponentially converges to d_2 and $0 < \gamma < Re(\lambda_2(L))$, $D_j(t)$ is bounded and exponentially converges to zero. With control laws (6)–(7), by Lemma 3 and the fact that Ω is Hurwitz, $\lim_{t\to\infty} y_j(t) = 0$, which implies that $\lim_{t\to\infty} \overline{x}_{3j} = -\beta d_2\Omega^{-1}e_1$. Therefore, $\lim_{t\to\infty} x_{kj} = -\beta d_2[\Omega^{-1}e_1]_k$. ∎

By Lemmas 2 and 4, we have the following theorem.

Theorem 1: For system (1), under Assumption 1, the control laws in (6)–(7) and (14) solve *Problem 1* with $c = [c_1, c_2, 0, \ldots, 0]^\top$, where c_1 is defined in (9),

$$c_2 = w^\top\left(q_{2*}(0) - \sum_{k=3}^{n}\mu_k q_{k*}(0)\right)\left(1 - \sum_{k=3}^{n}\beta\mu_k[\Omega^{-1}e_1]_k\right) \tag{18}$$

$q_{i*} = [q_{i1}, \ldots, q_{im}]^\top$ for $i \in [1, n]$. The control parameters satisfy $b_{ji} > 0, \beta \neq 0, \gamma \in (0, Re(\lambda_2(L)))$; L is the Laplacian matrix of the digraph $\mathcal{G}$ with weight $\mathcal{B} = [b_{jl}]$; and, μ_k $(k \in [3, n])$ are chosen such that $\lambda(\Omega) \in \mathcal{C}^-$.

Proof: By Lemma 2, $\lim_{t\to\infty} q_{1j}(t) = c_1$. By Lemma 4, $\lim_{t\to\infty} x_{ij}(t) = d_i$ for $i \in [2, n]$. Noting (11), $\lim_{t\to\infty} q_{2j}(t) = d_2 + \sum_{k=3}^{n}\mu_k d_k$, and $\lim_{t\to\infty} q_{kj}(t) = 0$ for $k \in [3, n]$. So, $\lim_{t\to\infty} q_{*j}(t) = c$. ∎

Remark 6: In control law (6), the first term is a weighted sum of the relative state information between agent j and its neighbors. The variable η_j is used to adjust the exponential convergence rate of the states q_{1j} to constant c_1. This variable plays an important role in designing u_{2j}. With this variable in the control laws u_{1j} $(j \in [1, m])$, the necessary condition for stabilizing nonholonomic agents is satisfied. In control law (14), the first term is a weighted sum of the relative state information between agent j and its neighbors (i.e., $(x_{2j} - x_{2l}), l \in \mathcal{N}_j$) and the other terms are used to cancel the terms induced by the variable transform. The motion of the system is driven by the relative information between neighbors, which distinguishes our control laws from the stabilizing control laws for single nonholonomic systems in the preexisting literature.

Remark 7: In the control laws, the control parameters are b_{jl}, β, γ and μ_k. The weights of the communication digraph, b_{jl}, can be chosen by the designer and affect the cooperative performance of q_{1j} and q_{2j} $(j \in [1, m])$. The parameter γ is in $(0, Re(\lambda_2(L)))$. Increasing γ increases the convergence rate of q_{1j} to c_1 and decreases the convergence rate of q_{ij} to c_i for $i \in [2, n]$. Therefore, there is a tradeoff when choosing γ. The parameters μ_k $(k \in [3, n])$ are chosen such that $Re(\lambda(\Omega)) \in \mathcal{C}^-$. The location of the eigenvalues of Ω affect the convergence rate of q_{k*}. In the controller design, the σ process is applied. The transformation defined in (11) is the so-called σ-processes [37]. The use of σ-processes introduces singular points when η_j goes through zero. In our control laws, the singularity is avoided by defining the initial condition $\eta_j(0)(= \beta)$ to be non-zero. During control, η_j can be replaced with its expression $\beta e^{-\gamma t}$ in control laws (6) and (14).

Remark 8: The value $\lambda_2(L)$ affects the convergence rate of q_{1*} and q_{2*}. It depends on the topology of the digraph $\mathcal{G}$ and the weights b_{jl}. For a bidirectional graph, $\lambda_2(\mathcal{G})$ is known as the algebraic connectivity [34]. Generally, a dense interconnection of $\mathcal{G}$ means a larger value for $Re(\lambda_2(L))$. Therefore, more interconnections facilitate the cooperative performance. However, increasing the number of interconnections does not necessarily imply a larger value of $Re(\lambda_2(L))$. Under the same topology of the communication digraph $\mathcal{G}$, different weights b_{jl} may lead to different $Re(\lambda_2(L))$. One can choose the weights b_{jl} to maximize $Re(\lambda_2(L))$ using the methods in [39]. Also, one can simply increase each b_{jl} to be ρb_{jl} $(\rho > 1)$ to get a large $Re(\lambda_2)$. However, with the latter method the largest real part of the eigenvalues of L is increased too, which means the control laws tolerate smaller communication delays (see Remark 14 for more details). If the structure of $\mathcal{G}$ is time-varying and the set of link weights b_{jl} to be assigned to the currently active links are predefined before the control, then the smallest value of $Re(\lambda_2(L))$ over all possible digraphs $\mathcal{G}$ can be precomputed. So, γ can be chosen prior to the control. Therefore, β and μ_k can be determined prior to the control too.

In Theorem 1, the constant vector c depends on the initial state of the system, the topology of the communication digraph, and the control parameters. In practical applications, the designer would sometimes prefer to be able to specify the value of c to be a desired value c^*. Without lose of generality, we assume $c^* = \mathbf{0}$ (see Remark 10). To do so, we introduce a simplified virtual agent with state vector $q_{*0} = [q_{10}, x_{20}]^\top \in \Re^2$ and

$$\dot{q}_{10} = u_{10}, \quad \dot{x}_{20} = u_{20} \tag{19}$$

with $q_{10}(0) = -\beta/\gamma$ and $x_{20}(0) = 0$. With the virtual agent, we have a new communication digraph $\mathcal{G}$ with node set $\mathcal{V} =$

$\{0, 1, 2, \ldots, m\}$ where we will abuse the notation slightly by referring to $\mathcal{G}$ as an augmented digraph with $m+1$ nodes.

Assumption 2: The virtual agent has no neighbors, i.e., $\mathcal{N}_0 = \emptyset$.

With the aid of the virtual agent, we have the following result.

Theorem 2: For system (1) and virtual agent (19), under Assumptions 1 and 2 on the augmented digraph $\mathcal{G}$, the control laws in (6)–(7) and (14) for $j \in [1, m]$, and

$$u_{10} = \beta e^{-\gamma t}, \quad u_{20} = 0 \tag{20}$$

make

$$\lim_{t\to\infty} q_{ij} = 0, \text{ for } i \in [1, n], \quad j \in [0, m] \tag{21}$$

where the constant control parameters satisfy $b_{jl} > 0$, $\beta \neq 0$, $\gamma \in (0, Re(\lambda_2(L)))$; L is a Laplacian matrix of the augmented digraph $\mathcal{G}$ with weight $\mathcal{B} = [b_{jl}]$; and, μ_k ($k \in [3, n]$) are chosen such that $\lambda(\Omega) \in \mathcal{C}^-$.

Proof: With the control laws in (6)–(7), (14), and (20), we have (10) and (16) with $q_{1*} = [q_{10}, q_{11}, \ldots, q_{1m}]^\top$ and $x_{2*} = [x_{20}, x_{21}, \ldots, x_{2m}]^\top$. Following the proof of Lemma 2, we can prove that $\lim_{t\to\infty} q_{1j} = c_1$ ($j \in [0, m]$) exponentially and $\lim_{t\to\infty} e^{\gamma t}(q_{1j} - q_{1l}) = 0$ ($\forall j \neq l$). Noting (20), $q_{10}(t) = -e^{-\gamma t}\beta/\gamma$. So, $c_1 = \lim_{t\to\infty} q_{10}(t) = 0$. Following the method for the proof of Lemma 4, we can show that $\lim_{t\to\infty} x_{2j} = d_2$ for $j \in [0, m]$. Noting (20), $x_{20}(t) = 0$. So, $c_2 = \lim_{t\to\infty} x_{20} = 0$. Similarly, we can prove that $\lim_{t\to\infty} x_{ij} = 0$ for $i \in [3, n]$ and $j \in [1, m]$. Noting (11), (21) holds. ■

Remark 9: The virtual agent has two states and is linear. The purpose of introducing it is to make q_{1j} and q_{2j} converge to zero. One can introduce a virtual agent which has the same kinematic as in (1) and plays the same role as the virtual agent (19). But the additional states are not necessary. The virtual agent neither receives information from the other agents nor sends out information to other agents. In control, if the virtual agent is assumed to be known to agent j, we can directly substitute the expressions of $q_{10}(= -e^{-\gamma t}\beta/\gamma)$ and $x_{20}(= 0)$ into the control laws. Therefore, the control laws in Theorem 2 are decentralized.

Remark 10: In Theorem 2, for any initial condition of the system described by (1), the state of each agent will converge to zero. If we require the state of each agent to converge to $c^* = [c_1^*, \ldots, c_n^*]^\top$ ($\neq \mathbf{0}$), we can convert this problem by the transformation in [40] into the form for which Theorem 2 is applicable and for which convergence of the transformed state to zero is equivalent to convergence of the state of the original system to c^*.

B. Switching Communication Graph Case

In practice, the communication digraph $\mathcal{G}$ is time-varying, because the distances between agents and the local surroundings of each agent change with time. Let $\Gamma = \{\mathcal{G}_i, 1 \le i \le s\}$ be the set of all possible communication digraphs during the control. Set Γ is finite because there are finite nodes. On the possible communication digraphs we make the following assumption.

Assumption 3: For any $\mathcal{G} \in \Gamma$, $\mathcal{G}$ with weight $\mathcal{B} = [b_{jl}]$ ($b_{jl} > 0$) is balanced and has a spanning tree.

Based on the results of the last section, we have the following result.

Theorem 3: For system (1), under Assumption 3, the control laws in (6)–(7) and (14) solve *Problem 1* with $c = [c_1, c_2, 0, \ldots, 0]^\top$, where c_1 and c_2 are defined in (9) and (18) with $w = (1/m)\mathbf{1}$ respectively; the control parameters $b_{jl} > 0$, $\beta \neq 0$, μ_k for $k \in [3, n]$ are chosen such that $\lambda(\Omega) \in \mathcal{C}^-$; and, $\gamma \in \cap_{\mathcal{G}\in\Gamma} \left(0, \lambda_2(L(\mathcal{G}) + L^\top(\mathcal{G})/2)\right)$ where L is the Laplacian matrix of digraph $\mathcal{G}(t)$ with weight matrix $\mathcal{B} = [b_{jl}]$.

Proof: For each digraph $\mathcal{G}$, we can obtain (10) and (16) with the aid of the control laws in (6) and (14). Since the digraph $\mathcal{G}$ is balanced, $\mathbf{1}^\top L = 0$. Let

$$e(t) = q_{1*}(t) - \left(\frac{1}{m}\sum_{j=1}^{m} q_{1j}(0) + \frac{\beta}{\gamma} - \frac{\beta}{\gamma}e^{-\gamma t}\right)\mathbf{1}$$

then using the fact that $L\mathbf{1} = 0$

$$\dot{e} = \dot{q}_{1*} - \beta e^{-\gamma t}\mathbf{1} = -Lq_{1*} = -Le. \tag{22}$$

Therefore, $\mathbf{1}^\top \dot{e} = -\mathbf{1}^\top Le = 0$ and

$$\begin{aligned}\mathbf{1}^\top e &= \mathbf{1}^\top e(0)\\ &= \mathbf{1}^\top\left[q_{1*}(0) - \left(\frac{1}{m}\sum_{j=1}^{m} q_{1j}(0) + \frac{\beta}{\gamma} - \frac{\beta}{\gamma}\right)\mathbf{1}\right] = 0.\end{aligned}$$

Define the positive definite Lyapunov function $V = (1/2)e^\top e$. Differentiating V along the solutions of (22), yields

$$\dot{V} = -\frac{1}{2}e^\top\left(L + L^\top\right)e. \tag{23}$$

Let $\tilde{L} = (L + L^\top)/2$, following the proof of Theorem 8 in [28], we have

$$\dot{V} \le -\lambda_2(\tilde{L})e^\top e = -2\lambda_2(\tilde{L})V. \tag{24}$$

Therefore, V converges to zero with exponential rate $2\lambda_2(\tilde{L})$. Furthermore, e converges to zero with exponential rate $\lambda_2(\tilde{L})$, which means q_{1*} converges to $(\sum_{l=1}^{m} q_{1l}(0)/m + \beta/\gamma)\mathbf{1}$ with exponential rate γ.

Similarly, we can prove that x_{2*} exponentially converges to $d_2\mathbf{1}$ where $d_2 = \sum_{l=1}^{m} x_{2l}(0)/m$. By the proof of Lemma 4, it can be proved that $\lim_{t\to\infty} x_{kj}(t) = d_k$, for $k \in [3, n]$ and $j \in [1, m]$. By the proof of Theorem 1, it can be proved that $\lim_{t\to\infty} q_{*j} = c$. ■

Remark 11: The control laws in Theorem 3 are the same as those in Theorem 1. However, in the control laws in Theorem 3, $\mathcal{N}_j$ is time-varying because the digraph $\mathcal{G}$ is time-varying. In Theorem 3, the assumptions are stronger than those in Theorem 1 because the digraph is time-varying. Since the weights b_{jl} are chosen by the designer, it is not hard to make $\mathcal{G}$ balanced. Moreover, $\mathcal{G}$ is always balanced if the communication is bidirectional and b_{ji} is properly chosen. The control parameters in Theorem 3 are also the same as those in Theorem 1. The convergence rate of the state of each agent depends on γ, b_{jl}, the topologies of the digraphs in Γ, and the eigenvalues of Ω. If the weight matrix $\mathcal{B} = [b_{ji}]$ is fixed *a priori*, there is a smallest positive value of $\lambda_2\left((L(\mathcal{G}) + L^\top(\mathcal{G}))/2\right)$ for any balanced digraph [41]. Furthermore, the smallest value of $\lambda_2\left((L(\mathcal{G}) + L^\top(\mathcal{G}))/2\right)$ can be increased by increasing b_{jl} with the same scale. Since the set Γ

is finite, $\cap_{\mathcal{G}\in\Gamma}\left(0, \lambda_2((L(\mathcal{G})+L^\top(\mathcal{G}))/2)\right)$ is the intersection of finite open intervals and is not empty. Therefore, there always exists a suitable $\gamma(>0)$.

To make the state of each agent converge to zero, we introduce the virtual agent 0 described in (19) and have the following result.

Theorem 4: For system (1) and virtual agent (19), if the digraph $\mathcal{G}$ (having m nodes) with weight matrix $\mathcal{B}=[b_{jl}]$ is balanced and the virtual agent is available to each agent, then the control laws in (6)–(7) and (14) for $j\in[1,m]$ based on $(m+1)$ nodes, and

$$u_{10}=-\alpha_{10}q_{10}, \quad u_{20}=-\alpha_{20}x_{20} \tag{25}$$

make

$$\lim_{t\to\infty} q_{ij}=0, \quad i\in[1,n], \quad j\in[0,m] \tag{26}$$

where the constant control parameters satisfy $b_{jl}>0$, $b_{0l}=\alpha_1$ for $l\in[1,m]$, $\alpha_1>\alpha_{10}>\gamma>0$, $0<\alpha_{20}<\alpha_1$, $\beta\neq 0$, and μ_k ($k\in[3,n]$) are chosen such that $\lambda(\Omega)\in\mathcal{C}^-$.

Proof: By (25), we have $q_{10}(t)=q_{10}(0)e^{-\alpha_{10}t}$ and $x_{20}(t)=x_{20}(0)e^{-\alpha_{20}t}$. With the control laws in (6) and (14), we have

$$\dot{q}_{1*}=-(L+\alpha_1 I)q_{1*}+\alpha_1 q_{10}(0)e^{-\alpha_{10}t}\mathbf{1}+\beta e^{-\gamma t}\mathbf{1} \tag{27}$$

$$\dot{x}_{2*}=-(L+\alpha_1 I)x_{2*}+\alpha_1 x_{20}(0)e^{-\alpha_{20}t}\mathbf{1} \tag{28}$$

where $q_{1*}=[q_{11},\ldots,q_{1m}]^\top$, $x_{2*}=[x_{21},\ldots,x_{2m}]^\top$, and L is the Laplacian matrix of the digraph $\mathcal{G}$ with agents $\{1,2,\ldots,m\}$. Define the positive definite Lyapunov function

$$V=\frac{1}{2}q_{1*}^\top q_{1*}.$$

Differentiating it along the solutions of (27) and noting the assumptions made on the agents, we can prove that $\lim_{t\to\infty} q_{1*}(t)=0$ with exponential convergence rate γ and $\lim_{t\to\infty} e^{\gamma t}(q_{1j}(t)-q_{1l}(t))=0$ exponentially for any $j\neq l$. Similarly, we can show that x_{2*} exponentially converges to zero. Furthermore, we can show that x_{ij} converges to zero for $i\in[3,n]$ and $j\in[1,m]$. Moreover, we can show that (26) holds. ■

Remark 12: In Theorem 4, the assumption made on the digraph $\mathcal{G}$ with m nodes is weaker than Assumption 3. This is because the virtual agent is available to each agent. During control, we can substitute the expressions of $q_{10}(=q_{10}(0)e^{-\alpha_{10}t})$ and $x_{20}(=x_{20}(0)e^{-\alpha_{20}t})$ directly into the control laws in (6) and (14). Therefore, the control laws are decentralized.

C. Stability of the Closed-Loop Systems With Communication Delays

In the previous controller design, we did not consider communication delays in the control design and analysis. In practice, there are always delays due to communication and other factors. For simplicity, in this paper we assume that all delays are the same.

For the fixed communication digraph case, we make the following assumption in order to simplify our analysis. For more general cases, we have the similar results which are omitted here.

Assumption 4: The communication digraph $\mathcal{G}$ is fixed and has a spanning tree. Let L be the Laplacian matrix of the digraph G with weight $\mathcal{B}=[b_{jl}]$ and $b_{jl}>0$, the eigenvalues of L are distinct real values.

Theorem 5: For system (1), under Assumption 4, control laws

$$u_{1j}(t)=-\sum_{l\in\mathcal{N}_j} b_{jl}(q_{1j}(t-\tau)-q_{1l}(t-\tau))+\eta_j, \tag{29}$$

$$\begin{aligned}u_{2j}(t)=&-\sum_{l\in\mathcal{N}_j} b_{jl}(x_{2j}(t-\tau)-x_{2l}(t-\tau))\\&+\gamma\sum_{k=3}^{n}(k-2)\mu_k x_{kj}(t)+\sum_{k=4}^{n}\mu_k x_{k-1,j}(t)\frac{\beta u_{1j}(t)}{\eta_j(t)}\\&+\mu_3\left(x_{2j}(t)+\sum_{k=3}^{n}\mu_k x_{kj}(t)\right)\frac{\beta u_{1j}(t)}{\eta_j(t)}\end{aligned} \tag{30}$$

with η_j as defined in (7) for $j\in[1,m]$, make $\lim_{t\to\infty} q_{*j}=c$ (i.e., the control laws in (29) and (30) solve *Problem 1*.) where c is a constant vector. The control parameters $b_{jl}>0$, $\beta\neq 0$, $\gamma\in(0,\lambda_2(L))$, μ_k ($k\in[3,n]$) are chosen such that $\sigma(\Omega)\in\mathcal{C}^-$. The parameter $\tau\in[0,\tau^*)$ where τ^* is the minimal positive solution of the following equation

$$\tau^*=\frac{\arccos\left(\frac{\gamma}{\lambda_m e^{\tau^*\gamma}}\right)}{\sqrt{\lambda_m^2 e^{2\tau^*\gamma}-\gamma^2}} \tag{31}$$

and λ_m is the largest eigenvalue of L.

Proof: With control laws in (29)–(30)

$$\dot{q}_{1*}=-Lq_{1*}(t-\tau)+\beta e^{-\gamma t}\mathbf{1}, \quad \dot{x}_{2*}=-Lx_{2*}(t-\tau).$$

Let $y=[y_1,\ldots,y_m]^\top=Q^{-1}q_{1*}$ where Q is a nonsingular matrix with $\mathbf{1}$ as its first column such that $Q^{-1}LQ=\mathrm{diag}[0,\lambda_2,\ \ldots,\lambda_m]$ where λ_l ($l\in[2,m]$) are the eigenvalues of L and $0<\lambda_2<\lambda_3<\cdots<\lambda_m$. With the control law of (29) we have

$$\dot{y}(t)=-\mathrm{diag}[0,\lambda_2,\ldots,\lambda_m]y(t-\tau)+\Pi e^{-\gamma t} \tag{32}$$

where $\Pi=[\Pi_1,\ldots,\Pi_m]^\top=\beta Q^{-1}\mathbf{1}$. Let $z_l=e^{\gamma t}y_l$ for $l\in[2,m]$, we have

$$\dot{z}_l(t)=\gamma z_l(t)-\lambda_l e^{\gamma\tau}z_l(t-\tau)+\Pi_l. \tag{33}$$

We analyze the stability of (33) using the D-decomposition method [42]. By the Laplace transform

$$Z_l=\frac{z_l(0)}{s-\gamma+\lambda_l e^{-\tau(s-\gamma)}}+\frac{\Pi_l}{s(s-\gamma+\lambda_l e^{-\tau(s-\gamma)})} \tag{34}$$

for $l\in[2,m]$, where Z_l is the Laplace transform of z_l. If each quasi-polynomial $p_l(s,\tau)=s-\gamma+\lambda_l e^{-\tau(s-\gamma)}$ has all zeros in $\mathcal{C}^-$, the systems in (33) are stable. Based on the fact that zeros of $p_l(s,\tau)$ are continuous functions of τ and the systems in (33) are stable when $\tau=0$, it is only needed to find τ such that $p_l(j\omega,\tau)=0$ for some ω. Let τ_l be the minimal positive solution to the following equation:

$$\tau_l=\frac{\arccos\left(\frac{\gamma}{\lambda_l e^{\tau_l\gamma}}\right)}{\sqrt{\lambda_l^2 e^{2\tau_l\gamma}-\gamma^2}} \tag{35}$$

if $\tau<\min\{\tau_2,\ldots,\tau_m\}$, then the zeros of $p_l(s,\tau)$ for $l\in[2,m]$ are in $\mathcal{C}^-$. Therefore, the systems in (33) are stable by the D-decomposition method. Noting the structure of (35), $\tau_2\geq\tau_3\geq\cdots\geq\tau_m\geq\tau^*$ because $\lambda_2<\lambda_3<\cdots<\lambda_m$. Therefore, if

$\tau \in [0, \tau^*)$, the zeros of $p_l(s,\tau)$ ($l \in [2,m]$) are all in $\mathcal{C}^-$ and z_l converges a constant. Furthermore, the y_l ($l \in [2,m]$) exponentially converge to zero with rate γ and q_{1*} exponentially converge to constant vector c with rate γ.

Let $g = [g_1, \ldots, g_m]^\top = Q^{-1}x_{2*}$, we have $\dot{g} = -\mathrm{diag}[0, \lambda_2, \ldots, \lambda_m]g(t-\tau)$. Let G_l ($l \in [2,m]$) be Laplace transforms of g_l, by the properties of Laplace transforms we have $G_l = g_l(0)/(s + \lambda_l e^{-\tau s})$. By the D-composition method, it is easy to show that g_l is asymptotically stable if $\tau \in [0, \pi/2\lambda_m)$. Noting $\tau^* < \pi/2\lambda_m$, g_l is asymptotically stable if $\tau \le \tau^*$. Furthermore, x_{2*} converges to constant vector $d_2\mathbf{1}$. Noting that $e^{\gamma t}\sum_{l\in\mathcal{N}_j} b_{jl}(q_{1j}(t-\tau) - q_{1l}(t-\tau))$ exponentially converges to zero, by the proof of Lemma 4 we can prove that $\lim_{t\to\infty} x_{kj} = d_k$ for $k \in [3,n]$. Therefore, $\lim_{t\to\infty} q_{*j} = c$. ∎

Remark 13: Theorem 5 is the delayed version of Theorem 1. In control laws (29)–(30), only the relative state information has delay. This assumption is reasonable because delays are caused mainly by communication between neighbors. The other terms in (30) (excluding the first term) are introduced to compensate the terms which are induced by the state transform and have nothing to do with the communication. Therefore, in the control laws these terms have no time delays. In Assumption 4, we assume that the Laplacian matrix has different real eigenvalues. In fact, if the eigenvalues are complex, we can obtain a similar result.

Remark 14: By (31), τ^* is small if λ_m is large. Therefore, we should choose b_{jl} such that λ_2 is as large as possible and λ_m is as small as possible. Furthermore, if γ is large, τ^* is small. So, if we want the control laws to tolerate large delays, γ should be small.

Corresponding to Theorem 2, we have the following delayed version result.

Theorem 6: For system (1) and virtual agent (19), under Assumptions 2 and 4 on the augmented digraph $\mathcal{G}$, the control laws in (29)–(30) for $j \in [1,m]$ and (20) make (21) hold where the control parameters $b_{jl} > 0$, $\beta \neq 0$, $0 < \gamma < \lambda_2(L)$, μ_k ($k \in [3,n]$) are chosen such that $\sigma(\Omega) \in \mathcal{C}^-$, $\tau \in [0,\tau^*)$ and τ^* is the minimal positive solution of (31).

In the case that the digraph changes with time and there exist communication delays, additional similar results can be developed. They are omitted here due to space limitations.

IV. Controller Design for Problem 2

In this section, we design controllers for *Problem 2*. To facilitate the controller design, the following assumptions are made on the desired trajectory q^d in (3).

Assumption 5: Trajectories q_i^d for $i \in [2, n-1]$ are bounded.

Assumption 6: Variables $d^l w_1/dt^l$ ($l \in [0, n-3]$; $d^0 w_1/dt^0 = w_1$) are bounded and w_1 satisfies the following condition: the integral $\int_t^{t+T} w_1^{2n-4}(s)ds \ge \delta$ for some $T > 0$, $\delta > 0$, and all $t \ge 0$.

A. A Transformation Based on Backstepping

To simplify the controller design, we introduce new variables based on the following backstepping procedure.

1) Let
$$\begin{cases} \phi_{nj} = 0 \\ e_{nj} = q_{n-1,j} \\ z_{nj} = q_{nj} - q_n^d - \phi_{nj}, \end{cases} \tag{36}$$
we have
$$\dot{z}_{nj} = w_1(q_{n-1,j} - q_{n-1}^d) + (u_{1j} - w_1)e_{nj}.$$

2) Let
$$\begin{cases} \phi_{n-1,j} = -\alpha_n z_{nj} w_1^{2n-5} \\ e_{n-1,j} = q_{n-2,j} - \sum\limits_{l=n-1+1}^{n} \frac{\partial \phi_{n-1,j}}{\partial z_{lj}} e_{lj} \\ z_{n-1,j} = q_{n-1,j} - q_{n-1}^d - \phi_{n-1,j} \end{cases} \tag{37}$$
where constant $\alpha_n > 0$, then
$$\begin{cases} \dot{z}_{nj} = -\alpha_n w_1^{2n-4} z_{nj} + w_1 z_{n-1,j} + (u_{1j} - w_1)e_{n,j} \\ \dot{z}_{n-1,j} = w_1(q_{n-2,j} - q_{n-2}^d) - \dot{\phi}_{n-1,j} \\ \qquad + (u_{1j} - w_1)q_{n-2,j}. \end{cases}$$

3) Let
$$\begin{cases} \phi_{n-2,j} = -\alpha_{n-1} z_{n-1,j} w_1^{2n-5} - z_{nj} + \frac{\partial \phi_{n-1,j}}{\partial w_1}\frac{\dot{w}_1}{w_1} \\ \qquad + \frac{\partial \phi_{n-1,j}}{\partial z_{nj}}\left(-\alpha_n w_1^{2n-5} z_{nj} + z_{n-1,j}\right) \\ e_{n-2,j} = q_{n-3,j} - \sum\limits_{l=n-2+1}^{n} \frac{\partial \phi_{n-2,j}}{\partial z_{lj}} e_{lj} \\ z_{n-2,j} = q_{n-2,j} - q_{n-2}^d - \phi_{n-2,j} \end{cases} \tag{38}$$
where constant $\alpha_{n-1} > 0$, then
$$\begin{cases} \dot{z}_{n-1,j} = -\alpha_{n-1} z_{n-1} w_1^{2n-4} - z_{nj} w_1 + z_{n-2,j} w_1 \\ \qquad + (u_{1j} - w_1)e_{n-1,j} \\ \dot{z}_{n-2,j} = w_1(q_{n-3,j} - q_{n-3}^d) - \dot{\phi}_{n-2,j} \\ \qquad + (u_{1j} - w_1)q_{n-3,j}. \end{cases}$$

4) Let (39), shown at the bottom of the page, where constant $\alpha_{n-2} > 0$, the notation $w_1^{[i]}$ denotes the ith derivative of w_1 (i.e., $w_1^{[i]} = d^i w_1/dt^i$), then
$$\begin{cases} \dot{z}_{n-2,j} = -\alpha_{n-2} z_{n-2} w_1^{2n-4} - z_{n-1,j} w_1 + z_{n-3,j} w_1 \\ \qquad + (u_{1j} - w_1)e_{n-2,j} \\ \dot{z}_{n-3,j} = w_1(q_{n-4,j} - q_{n-4}^d) - \dot{\phi}_{n-3,j} \\ \qquad + (u_{1j} - w_1)q_{n-4,j}. \end{cases}$$

$$\begin{cases} \phi_{n-3,j} = -\alpha_{n-2} z_{n-2,j} w_1^{2n-5} - z_{n-1,j} + \sum\limits_{l=0}^{1} \frac{\partial \phi_{n-2,j}}{\partial w_1^{[l]}} \frac{w_1^{[l+1]}}{w_1} + \frac{\partial \phi_{n-2,j}}{\partial z_{nj}}\left(z_{n-1,j} - \alpha_n w_1^{2n-5} z_{nj}\right) \\ \qquad + \sum\limits_{l=n-1}^{n-1} \frac{\partial \phi_{n-2,j}}{\partial z_{lj}}\left(z_{l-1,j} - \alpha_l w_1^{2n-5} z_{lj} - z_{l+1,j}\right) \\ e_{n-3,j} = q_{n-4,j} - \sum\limits_{l=n-3+1}^{n} \frac{\partial \phi_{n-3,j}}{\partial z_{lj}} e_{lj} \\ z_{n-3,j} = q_{n-3,j} - q_{n-3}^d - \phi_{n-3,j} \end{cases} \tag{39}$$

5) For $i = n-4, n-5, \ldots, 2$, let (40), shown at the bottom of the page, where $\alpha_{i+1} > 0$, then

$$\begin{cases} \dot{z}_{i+1,j} = -\alpha_{i+1} z_{i+1,j} w_1^{2n-4} - z_{i+2,j} w_1 + z_{ij} w_1 \\ \qquad\qquad +(u_{1j} - w_1) e_{i+1,j} \\ \dot{z}_{ij} = w_1(q_{i-1,j} - q_{i-1}^d) - \dot{\phi}_{i-1,j} + (u_{1j} \\ \quad -w_1) q_{i-1,j}, \quad \text{for } i = n-4, n-5, \ldots, 3, \\ \dot{z}_{2j} = u_{2j} - w_2 - \dot{\phi}_{2j}. \end{cases}$$

6) Let

$$\begin{cases} \phi_{1j} & = & 0 \\ z_{1j} & = & q_{1j} - q_1^d - \phi_{1j} \end{cases} \tag{41}$$

we have

$$\dot{z}_{1j} = u_{1j} - w_1.$$

In summary, by introducing the variables

$$z_{ij} = q_{ij} - q_i^d - \phi_{ij}, \quad i \in [1, n], \ \ j \in [1, m] \tag{42}$$

we have a set of differential equations in the following special forms [43] in (43), shown at the bottom of the page.

Lemma 5: For the variables defined in (42), under Assumption 6, if $\lim_{t\to\infty}(z_{kj} - c_k) = 0$ for $k \in [1, n]$ and $j \in [1, m]$, then $\lim_{t\to\infty}(q_{kl} - q_{kj}) = 0$ for $l \in [1, m]$ and $l \neq j$. Furthermore, if

$$\lim_{t\to\infty} z_{kj} = 0 \tag{44}$$

then $\lim_{t\to\infty}(q_{kj} - q_{kl}) = 0$ and $\lim_{t\to\infty}(q_{kj} - q_k^d) = 0$, where c_k are constants or bounded time-varying functions.

Proof: Noting the definitions of ϕ_{ij}, $\phi_{*j} = \Lambda z_{*j}$ where $\phi_{*j} = [\phi_{1j}, \ldots, \phi_{nj}]^\top$, $z_{*j} = [z_{1j}, \ldots, z_{nj}]^\top$, and Λ is a bounded matrix function of $w_1^{[i]}$ for $i \in [0, n-3]$. So, $\lim_{t\to\infty}(q_{*j} - q_{*l}) = \lim_{t\to\infty}(z_{*j} - z_{*l} + \phi_{*j} - \phi_{*l}) = \lim_{t\to\infty}(I - \Lambda)(z_{*j} - z_{*l})$. If $w_1^{[i]}$ ($i \in [0, n-3]$) are bounded, then Λ is bounded. Therefore, $\lim_{t\to\infty}(q_{*j} - q_{*l}) = 0$ if $\lim_{t\to\infty}(z_{kj} - c_k) = 0$. Furthermore, if $\lim_{t\to\infty} z_{*j} = 0$, we have $\lim_{t\to\infty} \phi_{*j} = 0$. So, $\lim_{t\to\infty}(q_{kj} - q_{kl}) = 0$ and $\lim_{t\to\infty}(q_{kj} - q_k^d) = 0$. ■

By Lemma 5, it is possible to solve *Problem 2* by designing control laws such that (44) holds.

Lemma 6: For system (43), under Assumptions 5-6, if both $\lim_{t\to\infty}(u_{1j} - w_1) = 0$ and $\lim_{t\to\infty} z_{2j} = d_2$ exponentially for $j \in [1, m]$, then $\lim_{t\to\infty}(z_{ij} - d_i) = 0$ for $i \in [3, n]$ where d_2 is a constant and d_i are bounded functions. If both $\lim_{t\to\infty}(u_{1j} - w_1) = 0$ and $\lim_{t\to\infty} z_{2j} = 0$ exponentially, then $\lim_{t\to\infty} z_{ij} = 0$ for $i \in [3, n]$.

Proof: First, for $i \in [3, n]$, we prove that z_{ij} are bounded if z_{2j} are bounded and $\lim_{t\to\infty} z_{ij} = 0$ if $\lim_{t\to\infty} z_{2j} = 0$. Define the positive Lyapunov function $V = 1/2 \sum_{i=3}^{n} z_{ij}^2$. Differentiating V along the solution of (43), we have

$$\begin{aligned} \dot{V} = & -\sum_{i=3}^{n} \alpha_i w_1^{2n-4} z_{ij}^2 + (u_{1j} - w_1) \sum_{i=3}^{n} z_{ij} e_{ij} + z_{2j} z_{3j} w_1 \\ \leq & -2\underline{\alpha} w_1^{2n-4} V + 2 f_1 V + 2 f_2 \sqrt{V}. \end{aligned}$$

where $\underline{\alpha} = \min\{\alpha_i, i \in [3, n]\}$, nonnegative function f_1 exponentially converges to zero, and nonnegative function f_2 is bounded and exponentially converges to zero if $\lim_{t\to\infty} z_{2j} = 0$ exponentially. Let $V_1 = \sqrt{V}$, we have $D^+ V_1 \leq \left(-\underline{\alpha} w_1^{2n-4} + f_1\right) V_1 + f_2$ where D^+ denotes the upper right-hand derivative. So

$$\begin{aligned} V_1(t) \leq & \, e^{\int_0^t \left(-\underline{\alpha} w_1^{2n-4}(\tau) + f_1(\tau)\right) d\tau} V_1(0) \\ & + \int_0^t e^{\int_\tau^t \left(-\underline{\alpha} w_1^{2n-4}(s) + f_1(s)\right) ds} f_2(\tau) d\tau. \end{aligned}$$

Noting Assumption 6, we can prove that V_1 is bounded. Therefore, V and z_{ij} are bounded. Furthermore, if $\lim_{t\to\infty} z_{2j} = 0$

$$\begin{cases} \phi_{ij} = -\alpha_{i+1} z_{i+1,j} w_1^{2n-5} - z_{i+2,j} + \sum_{l=0}^{n-i-2} \frac{\partial \phi_{i+1,j}}{\partial w_1^{[l]}} \frac{w_1^{[l+1]}}{w_1} + \frac{\partial \phi_{i+1,j}}{\partial z_{nj}} \left(z_{n-1,j} - \alpha_n w_1^{2n-5} z_{nj}\right) \\ \qquad + \sum_{l=i+2}^{n-1} \frac{\partial \phi_{i+1,j}}{\partial z_{lj}} (z_{l-1,j} - \alpha_l w_1^{2n-5} z_{lj} - z_{l+1,j}) \\ e_{ij} = q_{i-1,j} - \sum_{l=i+1}^{n} \frac{\partial \phi_{ij}}{\partial z_{lj}} e_{lj} \\ z_{ij} = q_{ij} - q_i^d - \phi_{ij} \end{cases} \tag{40}$$

$$\begin{cases} \dot{z}_{1j} = u_{1j} - w_1 \\ \quad \dot{z}_{2j} = u_{2j} - w_2 - \sum_{l=0}^{n-3} \frac{\partial \phi_{2j}}{\partial w_1^{[l]}} w_1^{[l+1]} - \frac{\partial \phi_{2j}}{\partial z_{nj}} \left(w_1 z_{n-1,j} - \alpha_n w_1^{2n-4} z_{nj}\right) \\ -\sum_{l=3}^{n-1} \frac{\partial \phi_{2j}}{\partial z_{lj}} \left(-\alpha_l w_1^{2n-4} z_{lj} - w_1 z_{l+1,j} + w_1 z_{l-1,j}\right) - (u_{1j} - w_1) \sum_{l=3}^{n} \frac{\partial \phi_{2j}}{\partial z_{lj}} e_{lj} \\ \dot{z}_{3j} = -\alpha_3 z_{3j} w_1^{2n-4} - z_{4j} w_1 + z_{2j} w_1 + (u_{1j} - w_1) e_{3j} \\ \vdots \\ \dot{z}_{n-1,j} = -\alpha_{n-1} z_{n-1,j} w_1^{2n-4} - z_{nj} w_1 + z_{n-2,j} w_1 \\ +(u_{1j} - w_1) e_{n-1,j} \\ \dot{z}_{nj} = -\alpha_n w_1^{2n-4} z_{nj} + w_1 z_{n-1,j} + (u_{1j} - w_1) e_{nj}. \end{cases} \tag{43}$$

exponentially, V_1 converges to zero which means that z_{ij} $(i \in [3,n])$ converge to zero.

Next, we will prove that $\lim_{t\to\infty}(z_{kl} - z_{kj}) = 0$. Let $\xi_k = z_{kl} - z_{kj}$, for $k \in [3,n]$ and $l \neq j$, we have

$$\begin{cases} \dot{\xi}_3 &= -\alpha_3 w_1^{2n-4}\xi_3 - \xi_4 w_1 + (z_{2l} - z_{2j})w_1 \\ &\quad +(u_{1l} - w_1)e_{3l} - (u_{1j} - w_1)e_{3j} \\ &\vdots \\ \dot{\xi}_{n-1} &= -\alpha_{n-1} w_1^{2n-4}\xi_{n-1} - \xi_n w_1 + \xi_{n-2} w_1 \\ &\quad +(u_{1l} - w_1)e_{n-1,l} - (u_{1j} - w_1)e_{n-1,j} \\ \dot{\xi}_n &= -\alpha_n w_1^{2n-4}\xi_n + \xi_{n-1}w_1 + (u_{1l} - w_1)e_{nl} \\ &\quad -(u_{1j} - w_1)e_{nj}. \end{cases} \tag{45}$$

Define the positive Lyapunov function as $V_2 = 1/2\sum_{i=3}^{n}\xi_i^2$. Differentiating V_2 along the solution of (45), yields

$$\dot{V}_2 = -\sum_{i=3}^{n}\alpha_i w_1^{2n-4}\xi_i^2 + \sum_{i=3}^{n}\xi_i[(u_{1l} - w_1)e_{il} - (u_{1j} - w_1)e_{ij}] + (z_{2l} - z_{2j})\xi_3 w_1. \tag{46}$$

Noting z_{ij} are bounded and the definitions of e_{il}, we have

$$\dot{V}_2 \leq -2\underline{\alpha} w_1^{2n-4}V_2 + 2f_3\sqrt{V}_2 \tag{47}$$

where nonnegative function f_3 is bounded and exponentially converges to zero. Let $V_3 = \sqrt{V}_2$, we have $D^+V_3 \leq -\underline{\alpha} w_1^{2n-4}V_3 + f_3$, which by direct integration and the comparison lemma [44] yields

$$V_3(t) \leq e^{-\int_0^t \underline{\alpha} w_1^{2n-4}(\tau)d\tau}V_3(0) + \int_0^t e^{-\int_\tau^t \underline{\alpha} w_1^{2n-4}(s)ds} f_3(\tau)d\tau.$$

By Assumption 6 and noting that f_3 exponentially converge to zero, V_3 converges to zero. Therefore, V_2 converges to zero. Furthermore, ξ_k for $k \in [3,n]$ converge to zero. So $\lim_{t\to\infty}(z_{kl} - z_{kj}) = 0$, for $k \in [3,n]$ and $1 \leq l \neq j \leq m$ which means there exist d_l such that $\lim_{t\to\infty}(z_{ij} - d_i) = 0$ for $i \in [3,n]$ and $j \in [1,m]$. ■

Remark 15: By the proof of Lemma 6, Assumption 5 can be relaxed as: $\lim_{t\to\infty}(u_{1j} - w_1)q_{kj}^d = 0$ exponentially for $k \in [2, n-1]$ and $j \in [1,m]$.

With the aid of Lemmas 5 and 6, *Problem 2* can be solved by designing control laws u_{1j} and u_{2j} using the relative state information between neighbors for system (43) such that $\lim_{t\to\infty}(u_{1j} - w_1) = 0$, $\lim_{t\to\infty} z_{1j} = 0$, and $\lim_{t\to\infty} z_{2j} = 0$ exponentially. Next, we design the control laws for different communication scenarios.

B. Fixed Communication Graph Case

In this section, we assume the communication digraph is fixed and satisfies Assumption 1. The fact that the dynamics of z_{1j} and z_{2j} are linear, with the results from the last section, yield the following result.

Lemma 7: For system (43), under Assumption 1, control laws

$$u_{1j} = -\sum_{l\in\mathcal{N}_j} b_{jl}(z_{1j} - z_{1l}) + w_1 \tag{48}$$

$$u_{2j} = -\sum_{l\in\mathcal{N}_j} b_{jl}(z_{2j} - z_{2l}) + w_2 + \sum_{l=0}^{n-3}\frac{\partial\phi_{2j}}{\partial w_1^{[l]}}w_1^{[l+1]} + \frac{\partial\phi_{2j}}{\partial z_{nj}}(-\alpha_n w_1^{2n-4}z_{nj} + w_1 z_{n-1,j}) + \sum_{l=3}^{n-1}\frac{\partial\phi_{2j}}{\partial z_{lj}}(-\alpha_l w_1^{2n-4}z_{lj} - w_1 z_{l+1,j} + w_1 z_{l-1,j}) - \sum_{l\in\mathcal{N}_j} b_{jl}(z_{1j} - z_{1l})\sum_{i=3}^{n}\frac{\partial\phi_{2j}}{\partial z_{ij}}e_{ij} \tag{49}$$

for $j \in [1,m]$ with control parameters satisfying $b_{jl} > 0$ and $\alpha_i > 0$ make

$$\lim_{t\to\infty} z_{1j} = d_1, \ \lim_{t\to\infty} z_{2j} = d_2, \ \lim_{t\to\infty}(u_{1j} - w_1) = 0 \tag{50}$$

exponentially where d_1 and d_2 are constants.

Proof: With control laws (48)–(49), we have $\dot{z}_{1*} = -Lz_{1*}$ and $\dot{z}_{2*} = -Lz_{2*}$ where $z_{1*} = [z_{11}, \ldots, z_{1n}]^\top$, $z_{2*} = [z_{21}, \ldots, z_{2n}]^\top$, and L is the Laplacian matrix associated with the communication digraph $\mathcal{G}$ with weight $\mathcal{B} = [b_{ji}]$. Under Assumption 1, with the aid of Lemma 1, equations in (50) hold exponentially. ■

With the aid of Lemmas 5, 6, and 7, we have the following theorem.

Theorem 7: For system (1), under Assumptions 1, 5, and 6, control laws (48)–(49) with parameters $b_{jl} > 0$ and $\alpha_i > 0$ make (4) hold and

$$\lim_{t\to\infty}(q_{1j} - q_1^d) = c_1, \ \lim_{t\to\infty}(q_{ij} - q_i^d - c_i) = 0 \tag{51}$$

for $i \in [2,n]$ and $j,l \in [1,m]$ where c_1 is a constant, c_i are bounded functions.

Proof: By Lemma 7, $\lim_{t\to\infty} z_{1*} = d_1\mathbf{1}$ and $\lim_{t\to\infty} z_{2*} = d_2\mathbf{1}$ exponentially. Therefore, $\lim_{t\to\infty}(q_{1j} - q_1^d) = \lim_{t\to\infty} z_{1j} = d_1 =: c_1$. By Lemma 6, $\lim_{t\to\infty}(z_{ij} - d_i) = 0$. So, $\lim_{t\to\infty}(q_{ij} - q_i^d - \phi_{ij} - d_i) = 0$. Noting the definitions of ϕ_{ij} and Lemma 6, $\lim_{t\to\infty}(\phi_{ij} - \overline{d}_i) = 0$ $(i \in [1,n], j \in [1,m])$ where $\overline{d}_i$ are bounded functions. Therefore, $\lim_{t\to\infty}(q_{ij} - q_i^d - c_i) = 0, i \in [1,n], j \in [1,m]$ where $c_i = d_i + \overline{d}_i$ are bounded functions. Furthermore, (4) holds. ■

Remark 16: Controllers (48)–(49) make (4) hold and $(q_{ij} - q_i^d)$ converge to bounded functions c_i which are functions of d_2, w_1 and their derivatives. In the control laws, the control parameters are b_{jl} and α_i. In control law (48), the first term is the relative state information between neighbors and the other term is used to compensate for the desired input. In control law (49), the first term is also the relative state information; the other terms are used to compensate for the desired input and for the terms induced by the variable transformation. The motion of the system is driven by the relative state information between neighbors and the desired trajectory. In Theorem 7, the rendezvous rate (see Remark 3) depends on $\lambda_2(L)$ and α_l $(l \in [3,n])$. It can be adjusted by suitably choosing the control parameters b_{jl} $(j \neq l \in [1,m])$ and α_l $(l \in [3,n])$. Equation (5) does not hold with the control laws in Theorem 7, which means that the tracking rate (see Remark 3) is zero. The tracking rate will be further discussed in Theorem 8.

Remark 17: By Remark 15 and the proof of Lemma 7, Assumption 5 in Theorem 7 can be relaxed as follows: $\lim_{t\to\infty} e^{-\epsilon t} q_{ij}^d(t) = 0$ exponentially for any $\epsilon > 0$, $i \in [2, n-1]$ and $j \in [1, m]$.

In Theorem 7, q_{ij} do not asymptotically converge to q_i^d. To make q_{ij} converge to q_i^d, we introduce a simplified virtual agent 0:

$$\dot{z}_{10} = u_{10},\ \dot{z}_{20} = u_{20} \tag{52}$$

with $z_{10}(0) = 0$ and $z_{20}(0) = 0$. Let the augmented digraph $\mathcal{G}$ be the communication digraph with $m+1$ nodes. We have the following result.

Theorem 8: For system (1) and the virtual agent (52), under Assumptions 5–6 and Assumptions 1 and 2 on the augmented digraph $\mathcal{G}$, control laws (48)–(49) for $j \in [1, m]$ and control laws

$$u_{10} = 0 \text{ and } u_{20} = 0 \tag{53}$$

solve *Problem 2*, where the control parameters satisfy $b_{lj} > 0$ and $\alpha_i > 0$.

Proof: With control laws (48)–(49) and (53), we have $\dot{z}_{1*} = -Lz_{1*}$ and $\dot{z}_{2*} = -Lz_{2*}$ where $z_{1*} = [z_{10}, z_{11}, \ldots, z_{1m}]^\top$, $z_{2*} = [z_{20}, z_{21}, \ldots, z_{2m}]^\top$, and L is the Laplacian matrix of the augmented matrix $\mathcal{G}$ with weigh matrix $\mathcal{B} = [b_{ji}]$. So, both $\lim_{t\to\infty} z_{1*} = d_1$ and $\lim_{t\to\infty} z_{2*} = d_2$ exponentially. Noting $z_{10} = 0$ and $z_{20} = 0$, $d_1 = d_2 = 0$. Therefore, both $\lim_{t\to\infty} z_{1*} = 0$ and $\lim_{t\to\infty} z_{2*} = 0$ exponentially. By Lemma 7, $\lim_{t\to\infty} z_{ij} = 0$ for $i \in [3, n]$. Noting the definitions of ϕ_{ij}, $\lim_{t\to\infty} \phi_{ij} = 0$. Therefore, $\lim_{t\to\infty}(q_{ij} - q_i^d) = 0$. Furthermore, *Problem 2* is solved. ■

Remark 18: In control laws (48)–(49) and (53), the control parameters are α_i and b_{jl}. The motion of the closed-loop systems are driven by the relative information between the neighbors and the desired trajectory. The rendezvous rate is determined by the control parameters b_{jl} $(j \neq l \in [1, m])$, α_i $(i \in [3, n])$, and the topology of the communication digraph. The tracking rate can be adjusted by the control parameters b_{j0} $(j \in [1, m])$ and α_i $(i \in [3, n])$. Therefore, with the control laws in Theorem 8 we can adjust both the rendezvous rate and the tracking rate by choosing suitable control parameters. While with the control laws in Theorem 7, we can only adjust the rendezvous rate.

Remark 19: If each agent is aware of the virtual agent and there is no communication between any two agents j and l for $j \neq l \in [1, m]$, the control law in Theorem 8 will become a single agent tracking controller [43], [45], [46]. Therefore, the single agent tracking controller which independently solves the tracking objective of (5) is a specific case of the controllers in Theorem 8.

Remark 20: In Theorem 8, each agent is aware of the desired trajectory q^d. One advantage of the control laws in Theorem 8 is that $\|q_j - q^d\|$ and $\|q_j - q_l\|$ $(j \neq l \in [1, m])$ are bounded even if there is no communication between any two agents.

Remark 21: Assumption 5 can be relaxed as in Remark 17.

Remark 22: In practice, the cooperative control laws to *Problem 2* can be used to solve *Problem 1* practically. For example, let

$$q_1^d(t) = \epsilon_1 \sin(\omega t), \quad q_i^d(t) = 0, \quad i \in [2, n]$$

where constants $\epsilon_1 > 0$ and $\omega > 0$, then $w_1(t) = \epsilon_1 \omega \cos(\omega t)$. Since q^d and w_1 satisfy Assumptions 5 and 6, the control laws in Theorem 8 make $\lim_{t\to\infty}(q_1 - \epsilon_1 \sin(\omega t)) = 0$ and $\lim_{t\to\infty} q_i = 0$ for $i \in [2, n]$. If ϵ_1 is very small, q_1 is close to zero as time tends to infinity, but q_1 does not converge to zero. For this case, we say *Problem 1* is solved practically. It should be noted that *Problem 1* cannot be solved by the control laws proposed for *Problem 2*.

C. Switching Communication Graph Case

Let $\mathcal{G}(t)$ represent the communication digraph at time t. We make the same assumption on the communication digraph as in Section III-B and have the following result.

Lemma 8: For system (43), under Assumption 3, control laws (48)–(49) ensure that the equations in (50) hold exponentially where d_1 and d_2 are constants and the control parameters satisfy $b_{jl} > 0$ and $\alpha_i > 0$.

Proof: By control laws (48)–(49), we have $\dot{z}_{1*} = -Lz_{1*}$ and $\dot{z}_{2*} = -Lz_{2*}$ where z_{1*} and z_{2*} are defined in the proof of Theorem 8, L is a time-varying Laplacian matrix. By the proof of Theorem 3, it can be proved that the equations in (50) hold exponentially. ■

With the aid of Lemmas 5, 6, and 8, we have the following theorem.

Theorem 9: For system (1), under Assumptions 3, 5, and 6, control laws (48)–(49) make (4) hold where c_1 is a constant, c_i $(i \in [2, n])$ are bounded functions, and the control parameters satisfy $b_{jl} > 0$ and $\alpha_i > 0$.

Proof: Following the proof of Theorem 7, this theorem can be proved. ■

Controllers (48)–(49) make $(q_{ij} - q_i^d)$ converge to a bounded function c_i. To make $(q_{ij} - q_i^d)$ converge to zero, we introduce the virtual agent 0 as in (52). With the aid of the virtual agent, we have the following result.

Theorem 10: For system (1) and virtual agent (52), under Assumptions 5-6, if the digraph $\mathcal{G}$ (having m nodes) with weight matrix $\mathcal{B} = [b_{jl}]$ is balanced and the virtual agent is available to each agent, then control laws (48)–(49) for $j \in [1, m]$ based on $(m+1)$ nodes and (53) solve *Problem 2*, where the control parameters satisfying $b_{jl} > 0$, $b_{0l} > 0$, and $\alpha_i > 0$.

Proof: Following the proof of Theorem 4, we can prove that the equations in (50) hold exponentially. By Lemma 6, $\lim_{t\to\infty} z_{ij} = 0$ for $i \in [1, n]$ and $j \in [1, m]$. By lemma 5, *Problem 2* is solved. ■

Remark 23: In control laws (48)–(49), the control parameters are b_{jl} and α_i. The motion of the agents is driven by the desired trajectory q^d and the relative information between neighbors. The control laws make $(q_{*j} - q^d)$ asymptotically converge to zero. The convergence rate of $(q_{*j} - q^d)$ depends on the topologies of the communication digraph at each time and also on the switching pattern of the communication digraphs.

D. Closed-Loop System Stability With Communication Delays

Next, we consider the effects of communication delays on the proposed results in the last subsection. The proofs of the results are omitted because they can be obtained with the aid of the proofs of Theorems 7–10 and the D-decomposition method [42].

Corresponding to Theorem 7 we have the following result.

Theorem 11: For system (1), under Assumptions 4, 5, and 6, the control laws

$$u_{1j}(t) = -\sum_{l\in\mathcal{N}_j} b_{jl}(z_{1j}(t-\tau) - z_{1l}(t-\tau)) + w_1(t) \quad (54)$$

$$\begin{aligned} u_{2j}(t) =& -\sum_{l\in\mathcal{N}_j} b_{jl}(z_{2j}(t-\tau) - z_{2l}(t-\tau)) + w_2(t) \\ &+ \frac{\partial\phi_{2j}(t)}{\partial z_{nj}(t)}\left(w_1(t)z_{n-1,j}(t) - \alpha_n w_1^{2n-4}(t)z_{nj}(t)\right) \\ &+\sum_{l=0}^{n-3}\frac{\partial\phi_{2j}(t)}{\partial w_1^{[l]}(t)}w_1^{[l+1]}(t)+\sum_{l=3}^{n-1}\frac{\partial\phi_{2j}(t)}{\partial z_{lj}(t)}\Big(w_1(t)z_{l-1,j}(t) \\ &- \alpha_l w_1^{2n-4}(t)z_{lj}(t) - w_1(t)z_{l+1,j}(t)\Big) \\ &- \sum_{l\in\mathcal{N}_j} b_{jl}(z_{1j}(t-\tau)-z_{1l}(t-\tau))\sum_{i=3}^{n}\frac{\partial\phi_{2j}(t)}{\partial z_{ij}(t)}e_{ij}(t) \end{aligned} \quad (55)$$

for $j \in [1,m]$ make (51) hold where c_1 is a constant, $c_i(t)$ $(i \in [2,n])$ are bounded functions, the control parameters satisfy $b_{jl} > 0$ and $\alpha_i > 0$, and $\tau \in [0,\tau^*)$ where $\tau^* = \pi/2\lambda_m(L)$.

Remark 24: In the theorem, only the relative information has communication delay. The maximum allowable delay τ^* depends only on the largest eigenvalue of L. So, the designer can carefully choose b_{jl} such that λ_m is as small as possible and λ_2 is as large as possible.

Corresponding to Theorem 8, we have the following delayed state result.

Theorem 12: For system (1) and virtual agent (52), under Assumptions 5-6 and Assumptions 2 and 4 on the augmented digraph $\mathcal{G}$, control laws (53) and (54)–(55) for $j \in [1,m]$ solve *Problem 2*, where the control parameters satisfy $b_{jl} > 0$, $\alpha_i > 0$, and $\tau \in [0,\tau^*)$ where τ^* is defined in Theorem 11.

Results corresponding to Theorems 9 and 10 with time delay can be obtained in a similar way, but are omitted here.

V. Applications

Almost all wheeled mobile robots can be globally or locally converted into the chained form (1). Therefore, the proposed results have numerous practical applications. This section presents two applications of the proposed results.

A. Formation Control of Wheeled Mobile Robots

Consider a set of m wheeled mobile robots which move on a plane (Fig. 1). Throughout this section, without loss of generality, the mobile robots will be indexed by $j \in [1,m]$. The kinematics of each robot are as follows [47]:

$$\dot{x}_j = v_{1j}\cos\theta_j, \quad \dot{y}_j = v_{1j}\sin\theta_j, \quad \dot{\theta}_j = v_{2j} \quad (56)$$

where (x_j, y_j) is the coordinate of the center point of the front wheels of robot j in the fixed coordinate frame O-XY , θ_j is the orientation of robot j with respect to the X-axis of the coordinate frame O-XY, v_{1j} and v_{2j} are the speed and angular rate of robot j, respectively.

Given a desired formation $\mathcal{F}$ for the m robots which is described by constant vectors $[p_{jx}, p_{jy}]^\top$ in the coordinate O-XY,

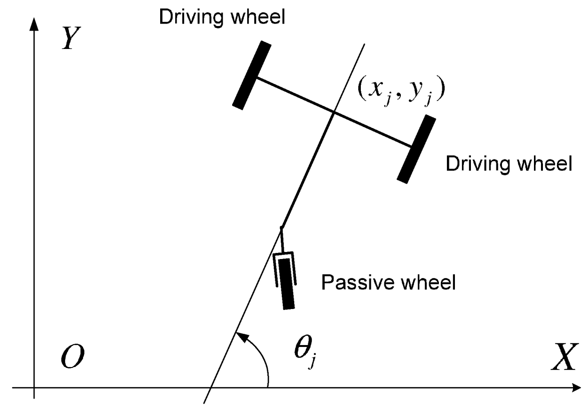

Fig. 1. Simplified wheeled mobile robot.

assume the communication digraph among the m robots is $\mathcal{G}$. The first formation control problem for the m robots can be defined as follows.

Formation Control: Design a controller for each robot, based on its own state and the relative state information between its neighbors, such that the group of robots come into formation $\mathcal{F}$, i.e., design control laws for system (56) such that

$$\lim_{t\to\infty}\begin{bmatrix} x_l - x_j \\ y_l - y_j \end{bmatrix} = \begin{bmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{bmatrix}\begin{bmatrix} p_{lx} - p_{jx} \\ p_{ly} - p_{jy} \end{bmatrix} \quad (57)$$

for $1 \le l \ne j \le m$, where ψ is some constant.

Next, we show how this formation control problem can be converted into *Problem 1*. Let

$$\begin{cases} q_{1j} &=& -\theta_j, \\ q_{2j} &=& (x_j - p_{jx})\cos\theta_j + (y_j - p_{jy})\sin\theta_j, \\ q_{3j} &=& -(x_j - p_{jx})\sin\theta_j + (y_j - p_{jy})\cos\theta_j \end{cases}$$

and

$$u_{1j} = -v_{2j}, \quad u_{2j} = v_{1j} + q_{3j}v_{2j} \quad (58)$$

then (56) can be converted into (1) with $n = 3$. Simple calculation derives the following result.

Lemma 9: If $\lim_{t\to\infty} q_{ij} = c_i$ for $i \in [1,3]$ and $j \in [1,m]$, where c_1, c_2 and c_3 are constants, then the formation control problem is solved, i.e., equations in (57) hold with some constant ψ.

By Lemma 9, solving the formation control problem is equivalent to solving *Problem 1* for the systems described by (1). By the results in Section III, we can obtain controllers for each robot under different communication scenarios. It should be noted that collision avoidance is not guaranteed with the control laws in Section III.

B. Formation Control With a Desired Trajectory

Given a desired formation $\mathcal{F}$ described by constant centroid offset vectors (p_{jx}, p_{jy}) and a desired trajectory (x^d, y^d, θ^d) which is generated by

$$\dot{x}^d = v_1^d\cos\theta^d, \quad \dot{y}^d = v_1^d\sin\theta^d, \quad \dot{\theta}^d = v_2^d. \quad (59)$$

We consider the following problem.

Formation Control With a Desired Trajectory: Design control laws for each robot, based on its own state, the relative state information between its neighbors, and the desired trajectory, such that the group of robots come into formation $\mathcal{F}$ and move along the desired trajectory, i.e., design control laws for system (56) such that

$$\lim_{t\to\infty}\left[\begin{bmatrix} x_l - x_j \\ y_l - y_j \end{bmatrix} - \begin{bmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{bmatrix}\begin{bmatrix} p_{lx} - p_{jx} \\ p_{ly} - p_{jy} \end{bmatrix}\right] = 0, \quad (60)$$

$$\lim_{t\to\infty}\left(\sum_{i=1}^{m}\frac{x_i}{m} - x^d\right) = 0, \ \lim_{t\to\infty}\left(\sum_{i=1}^{m}\frac{y_i}{m} - y^d\right) = 0 \quad (61)$$

for $1 \le l \ne j \le m$, where ψ is some variable.

Unlike the formation control problem in the last subsection, in this subsection, the robots are required not only to come into formation but also to move along the desired trajectory. This problem can be converted to *Problem 2*. Letting

$$\begin{cases} q_{1j} = -\theta_j, \\ q_{2j} = \left(x_j - p_{jx} + \frac{1}{m}\sum_{i=1}^{m} p_{ix}\right)\cos\theta_j \\ \qquad + \left(y_j - p_{jy} + \frac{1}{m}\sum_{i=1}^{m} p_{iy}\right)\sin\theta_j \\ q_{3j} = -\left(x_j - p_{jx} + \frac{1}{m}\sum_{i=1}^{m} p_{ix}\right)\sin\theta_j \\ \qquad + \left(y_j - p_{jy} + \frac{1}{m}\sum_{i=1}^{m} p_{iy}\right)\cos\theta_j \end{cases} \quad (62)$$

with (u_{1j}, u_{2j}) as defined in (58), we have (1) with $n = 3$. Letting

$$\begin{cases} q_1^d = -\theta^d, \\ q_2^d = x^d\cos\theta^d + y^d\sin\theta^d, \\ q_3^d = -x^d\sin\theta^d + y^d\cos\theta^d \\ w_1 = -v_2^d, \\ w_2 = (-x^d\sin\theta^d + y^d\cos\theta^d)v_2^d - v_1^d \end{cases} \quad (63)$$

we have (3) with $n = 3$. Simple calculation derives the following result.

Lemma 10: By the transformations in (58), (62), and (63), under Assumptions 5-6 with $n = 3$, if $\lim_{t\to\infty}(q_{ij} - q_i^d) = 0$ for $i \in [1, 3]$ and $j \in [1, m]$, then (60)–(61) are satisfied.

By Lemma 10, the formation control problem with a desired trajectory can be solved by the controllers proposed for *Problem 2* in Section IV.

VI. Simulations

To verify the effectiveness of the proposed results, we present some simulation results for Section V. Due to space limitation, brief simulations are presented.

A. Formation Control

Consider the examples discussed in Section V. Let $m = 5$ and the initial conditions of the robots be (−10, −5,0.1), (−5,6, −0.2), (5,25,0.3), (15,20, −0.5), and (25, −10, 0). Assume the desired formation $\mathcal{F}$ is described by $(p_{1x}, p_{1y}) = (-6.7, 7.4)$, $(p_{2x}, p_{2y}) = (-9.1, -4.1)$, $(p_{3x}, p_{3y}) = (1.0, -9.9)$, $(p_{4x}, p_{4y}) = (9.8, -2.1)$, and $(p_{5x}, p_{5y}) = (5.0, 8.7)$ (see Fig. 2). By Lemma 9 , the formation control problem for the five robots can be solved with the aid of the results in Section III.

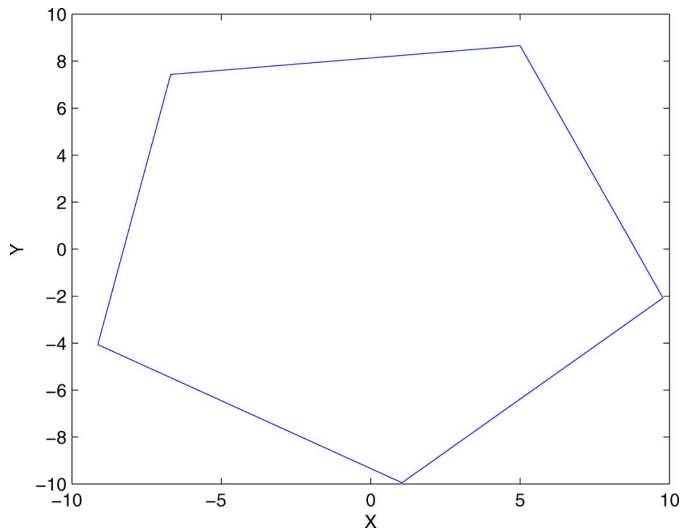

Fig. 2. Desired formation.

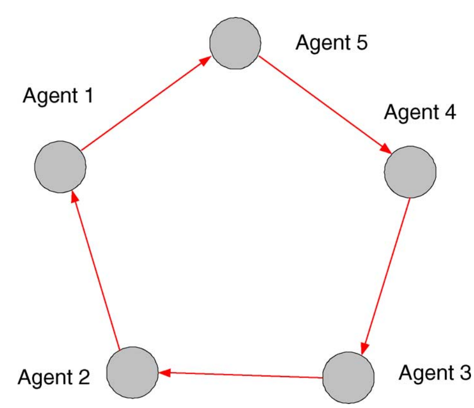

Fig. 3. Digraph $\mathcal{G}$.

If the communication digraph $\mathcal{G}$ is fixed as shown in Fig. 3, the cooperative control laws can be obtained by Theorem 1. We select the control parameters $b_{jl} = 1$, $\gamma = 0.3$, $\mu_3 = -1.3/4$, and $\beta = 1$. Fig. 4 shows the movement of each robot as the platoon of vehicles converges to the desired formation. If there is a small communication delay in the cooperative control laws, according to Theorem 5, the controllers still work. In the simulation, we set the communication delay $\tau = 0.1$ s. Fig. 5 shows the movement of each robot during convergence to the desired formation. This simulation demonstrates that the cooperative control laws in Theorem 5 are robust to small communication delay. The control laws in Theorem 2 and their delayed versions also work well in simulations and will not be given here.

If the communication digraph switches as a function of time according to

$$\mathcal{G} = \begin{cases} \mathcal{G}_1, & \text{if } t - round(t) \ge 0 \\ \mathcal{G}_2, & \text{if } t - round(t) < 0 \end{cases} \quad (64)$$

where $round(t)$ is the nearest integer to t, and $\mathcal{G}_1$ and $\mathcal{G}_2$ are shown in Figs. 6 and 7, respectively. The control laws can be

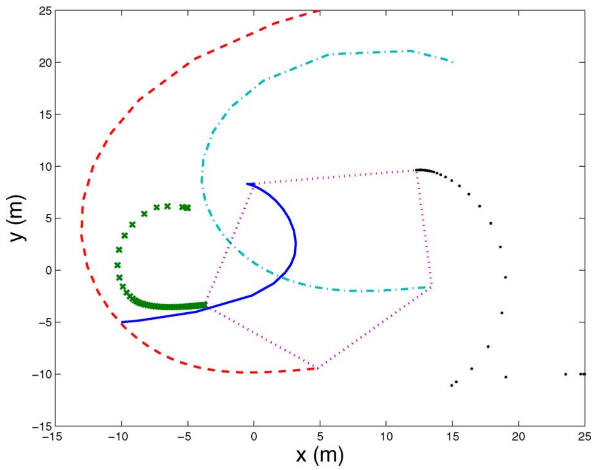

Fig. 4. Path of each robot (solid: robot 1, x-mark: robot 2, dashed: robot 3, dashdot: robot 4, point: robot 5, dotted: desired formation) during formation convergence for the zero delay case.

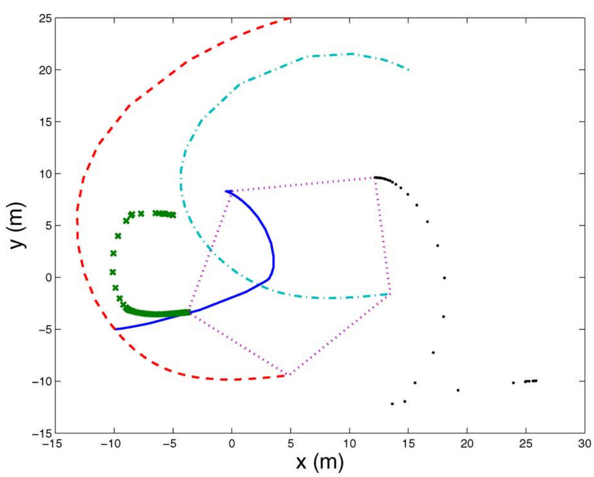

Fig. 5. Path of each robot (solid: robot 1, x-mark: robot 2, dashed: robot 3, dashdot: robot 4, point: robot 5, dotted: desired formation) during formation convergence for the $\tau = 0.1$ s delay case.

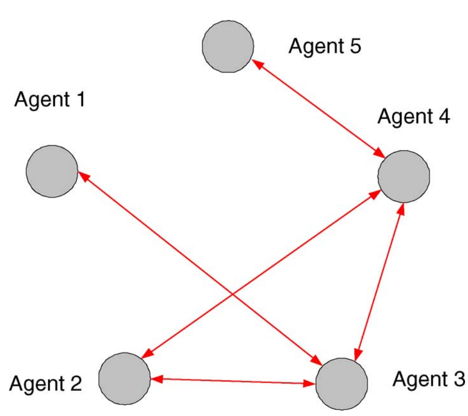

Fig. 6. Digraph $\mathcal{G}_1$.

obtained by Theorem 3, Fig. 8 shows the movement of each robot. It can be seen that the five robots still converge to the desired formation.

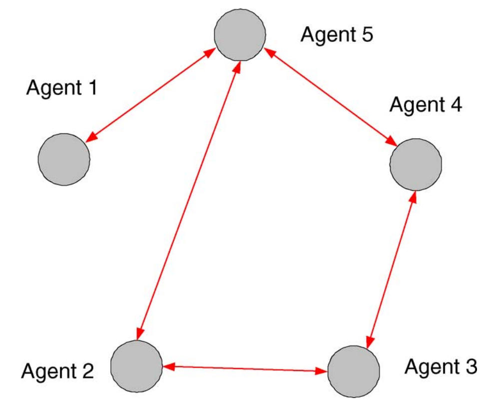

Fig. 7. Digraph $\mathcal{G}_2$.

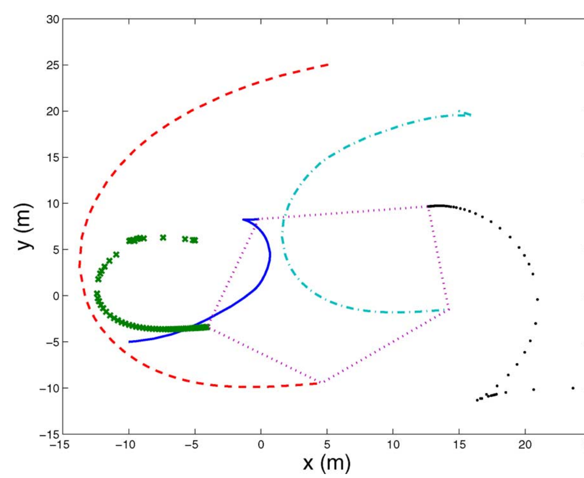

Fig. 8. Path of each robot (solid: robot 1, x-mark: robot 2, dashed: robot 3, dashdot: robot 4, point: robot 5, dotted: desired formation) during formation convergence for the switching digraph case.

B. Formation Control With a Desired Trajectory

Consider the same example as in Subsection A, the desired formation is shown in Fig. 2, the desired trajectory (x^d, y^d, θ^d) is generated by (59) with $v_1^d = \sqrt{2}/2$, $v_2^d = 0.2\sin t$, and the initial conditions as previously stated. The desired trajectory satisfies the assumptions in Section IV (see Remark 17). By Lemma 10, the controllers can be obtained with the aid of the results in Section IV. For the fixed digraph case, the assumed form of the augmented digraph $\mathcal{G}$ is shown in Fig. 9. The cooperative controllers can be obtained by Theorem 8. Fig. 10 shows the path of each robot. It is shown that the five robots come into formation and the center of the group of robots converges to the desired trajectory. Fig. 11 shows the path of each robot with communication delay ($\tau = 0.1$ s) in the control laws. The figure shows that the robots come into formation and the center of the group of robots converges to the desired trajectory too, which confirms the statement in Theorem 12.

If the communication digraph switches with time, the proposed controllers in Theorem 10 work. Assume the communication digraphs switch between the communication digraphs $\mathcal{G}_1$ and $\mathcal{G}_2$ which are shown in Figs. 6 and 7, respectively. The switching pattern is the same as (64). Fig. 12 shows the path

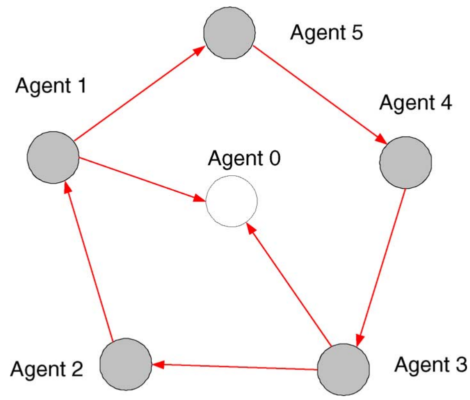

Fig. 9. Augmented digraph $\mathcal{G}$.

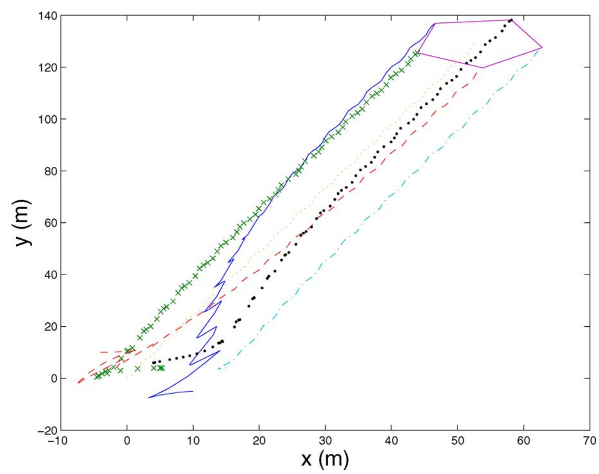

Fig. 10. Path of each robot (solid: robot 1, x-marked: robot 2, dashed: robot 3, dashdot: robot 4, point: robot 5, dotted: desired trajectory of the center of the formation) during formation convergence and trajectory following for the zero delay case.

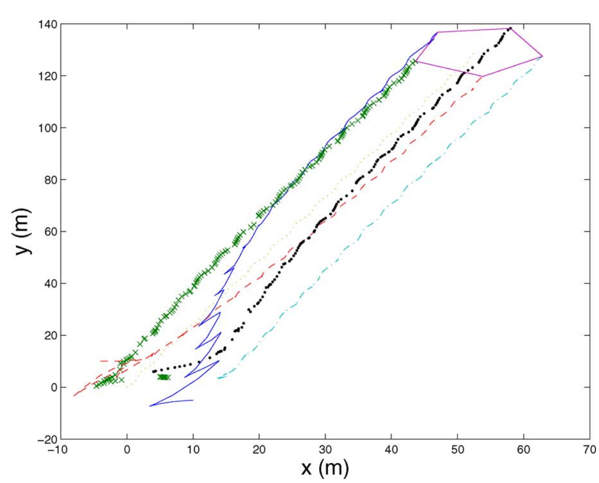

Fig. 11. Path of each robot (solid: robot 1, x-marked: robot 2, dashed: robot 3, dashdot: robot 4, point: robot 5, dotted: desired trajectory of the center of the formation) during formation convergence and trajectory following for the $\tau = 0.1$ s. delay case.

of each robot. It can be seen that the five robots come into formation and the center of the five robots converge to the desired trajectory, which confirm the results in Theorem 10 and Lemma 10.

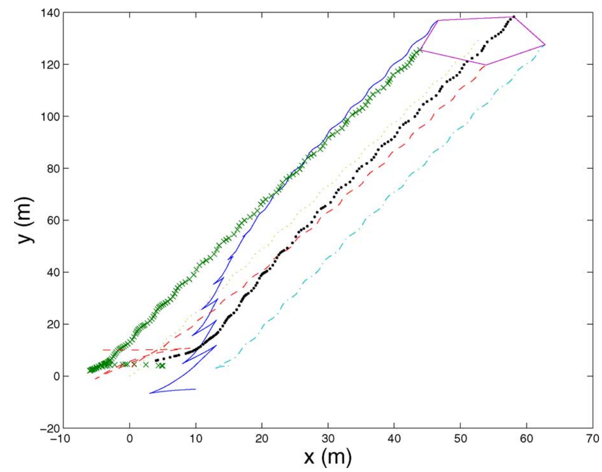

Fig. 12. Path of each robot (solid: robot 1, x-marked: robot 2, dashed: robot 3, dashdot: robot 4, point: robot 5, dotted: desired trajectory of the center of the formation) during formation convergence and trajectory following for digraph switching case.

VII. Conclusion

This paper discusses the cooperative control of multiple nonholonomic agents under different communication scenarios. Cooperative control laws are proposed with the aid of suitable transformations and results from graph theory. Stable control laws robust to communication delays are also developed and studied. Applications of the proposed results to wheeled mobile robots are discussed. Simulation results show the effectiveness of the proposed control laws. The control laws and design methods presented herein apply to kinematic models. With the aid of backstepping techniques, they can be extended to the control of dynamic nonholonomic agents, in multivehicle cooperative control applications.

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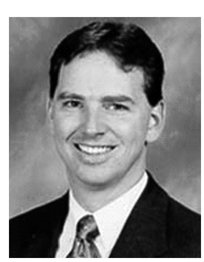

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