2012 IEEE International Conference on Robotics and Automation
RiverCentre, Saint Paul, Minnesota, USA
May 14-18, 2012

Distributed Formation Control of Unicycle Robots

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Abstract— **In this paper, we consider the problem of distributed formation control for a group of unicycle robots. We propose a control algorithm that solves the formation control problem in that it ensures that robots create a desired time–varying formation shape while the formation as a whole follows a prescribed trajectory. Moreover, we show that it is also possible to obtain coordination of robots in the formation, regardless of the trajectory tracking of the formation. We illustrate the behavior of a group of robots controlled by the formation control algorithm proposed in this paper in a simulation study.**

I. Introduction

Cooperative control of multiple mobile agents receives increasing attention within the control and robotics communities. The reason for this fact is the abundance of potential applications of the cooperative agents, mainly due to the well-known advantages of a multi-agent system, such as higher robustness, accuracy and speed of conducting the required tasks in comparison to application of a single agent, see e.g. [10] for details.

In this paper, we specifically study a formation control problem in which the objective is for a group of robots to maintain a given time–varying formation geometry while following a desired trajectory. This problem has received ample attention in the literature, see e.g. [1]–[3], [7], [11], [13], [16]. In [1] a leader–follower approach with a receding-horizon scheme was studied. It could be debated however that the dependence on the leaders in the leader–follower strategies may prove to be fault-prone. In turn, in [2] and [3] control algorithms were proposed that allow for constant formation shapes only. Arguably, this feature limits the applicability of these controllers which to some degree prompted our study. Furthermore, in [7] a saturated formation control algorithm for time–varying formation shapes utilizing a global communication network was proposed. Besides considering tracking errors of all robots, a coordination error between a pair of robots is also exploited in the formation control law. This type of additional variables, likewise studied in [15], is also taken into account in the control law proposed in this paper. In this sense, our results may be considered as an extension of [7] by introducing a distributed communication between robots with the exception that we do not aim for the saturated controller as in [7]. This is to allow us to focus on advancing the solution to formation control problem as such without the additional technical issues. In contrast with the approach proposed here and in [7], where the coordination error between a pair of interacting agents is explicitly used in the control law, in numerous other results regarding formation control of mobile agents, including [2], [3], [11], [13], [16], coordination of agents is achieved implicitly by extending tracking controllers of these agents with additional terms describing their interactions. Then, by implication, one has that the desired formation shape is achieved along with trajectory tracking.

Similarly to our current results, the control algorithm proposed in [13] also has a distributed character. Furthermore, regarding distributed–local interconnection control schemes for multiple agent groups, an interesting result is presented by Jadbabaie et al [4] in which a control algorithm is proposed based on the nearest neighbor rule. Moreover in [8], the problem of flocking is considered and the so-called geodesic control law is proposed. This scheme relates to coordination and velocity alignment of nonholonomic mobile agents to obtain flocking by means of minimizing a so-called misalignment energy. Note that in comparison to the work communicated in this paper, both [4] and [8] consider the flocking control problem as opposed to the formation control problem.

In this paper it is shown that coordination of agents in the formation is equivalent to the consensus [9] of trajectory tracking errors of the individual agents; hereto these errors must be represented in a common coordinate system. To this end, we express all tracking error variables in the earth–fixed frame to show that when these errors are in consensus, the cooperative agents form a desired formation shape. This is also what further distinguishes this work from existing work, such as e.g. [13], [16], where all tracking error variables are given in local moving frames associated with individual robots. As a consequence, in these works consensus of the error variables does not imply physical coordination of the agents in the formation.

In light of the above, the main contributions of this paper are as follows: (i) introduction of a *distributed formation control algorithm* that explicitly ensures both trajectory tracking and coordination of members of the formation; (ii) analysis of the tracking error variables in the *earth–fixed frame* and therewith determination of the *analogy between the consensus control problem and the formation control problem* for cooperative unicycle robots; (iii) analysis of the *influence of a connected and disconnected communication topology* on the formation behavior; (iv) a solution to the formation control problem for *time-varying formation geometries*.

This paper is organized as follows. In Section II, we define

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the formation control problem that is solved by the control law presented in Section III. We include a simulation study in Section IV and concluding remarks are given in Section V.

II. Problem Formulation

In this section, we revisit the formation control problem briefly mentioned in the Introduction in more detail. We consider a formation consisting of n identical unicycle–type mobile robots. Let $\mathcal{I} = \{1, \ldots, n\}$ denote the set of indices of robots in the formation, let $p_i(t) = \mathrm{col}(x_i(t), y_i(t))$ denote the Cartesian coordinates of robot i with respect to an earth-fixed coordinate frame, and let $\theta_i(t)$ denote the heading angle of this robot. Let $q_i(t) = \mathrm{col}(p_i(t), \theta_i(t))$. We will assume that each robot satisfies the nonholonomic *no-side-slip condition* $\dot{x}_i \sin\theta_i - \dot{y}_i \cos\theta_i = 0$. This gives that the kinematics of the i-th robot are given by

$$\dot{x}_i = v_i \cos\theta_i, \quad \dot{y}_i = v_i \sin\theta_i, \quad \dot{\theta}_i = \omega_i, \tag{1}$$

in which v_i is the forward speed and ω_i is the angular velocity. We also consider a *virtual center*, which is a virtual robot with prescribed Cartesian coordinates $p_{vc}^d(t) = \mathrm{col}(x_{vc}^d(t), y_{vc}^d(t))$ with respect to the same earth-fixed coordinate frame as the robots in the formation. Denote by $\theta_{vc}^d(t)$ the heading angle of the virtual center that satisfies the no-side-slip-condition. Note that $p_{vc}^d(t)$ and $\theta_{vc}^d(t)$ define a local moving coordinate frame with the origin at $p_{vc}^d(t)$ and orientation with respect to the earth-fixed frame given by $\theta_{vc}^d(t)$. Given vectors $l_i^d(t) = \mathrm{col}(l_{ix}^d(t), l_{iy}^d(t))$, $i \in \mathcal{I}$, in principle the control objective may be stated as the requirement for the robots to follow trajectories that are described by $l_i^d(t)$ $(i \in \mathcal{I})$ in the local virtual-center-fixed coordinate frame. In other words, the robots are to follow desired trajectories $p_i^d(t)$ that are given by

$$p_i^d(t) = p_{vc}^d(t) + R(\theta_{vc}^d(t)) l_i^d(t), \; R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, \tag{2}$$

By the no-side-slip constraint, this also gives desired heading angles θ_i^d.

Now, let the position tracking error e_i and angular error be defined as

$$e_i(t) = p_i^d(t) - p_i(t), \; \theta_i^e(t) = \theta_i^d(t) - \theta_i(t) \qquad (i \in \mathcal{I}). \tag{3}$$

Bearing in mind (1) and (3), the error dynamics in global coordinates are given by

$$\dot{e}_i = R(\theta_i^d - \theta_i^e) \begin{pmatrix} v_i^d \cos\theta_i^e - v_i \\ v_i^d \sin\theta_i^e \end{pmatrix}, \qquad \dot{\theta}_i^e = \omega_i^d - \omega_i. \tag{4}$$

Here the desired velocities v_i^d, ω_i^d of a robot in the formation are derived by differentiating (2) and using (1).

As mentioned in the Introduction, the objective of the control algorithm proposed in this paper is twofold. First, robots need to create a given formation geometry. Secondly, the formation as a whole needs to follow a prescribed trajectory. It follows from earlier discussions that the first objective is satisfied if there exist a time-varying vector $p_{vc}(t)$ and a function $\theta_{vc}(t)$ such that

$$p_i(t) = p_{vc}(t) + R(\theta_{vc}(t)) l_i^d(t) \quad (\forall i \in \mathcal{I}) \tag{5}$$

In addition, it follows from (2) that the second objective is satisfied if (5) holds with $p_{vc}(t) = p_{vc}^d(t)$ and $\theta_{vc}(t) = \theta_{vc}^d(t)$. It is therefore clear that both objectives are satisfied if all error variables e_i are equal to zero. In order to derive conditions in terms of the error variables e_i for the first objective to be satisfied, we consider a stricter requirement than (5) in that we require the actual orientation of the formation to coincide with the desired orientation, i.e., (5) is required to hold with $\theta_{vc}(t) = \theta_{vc}^d(t)$ and an arbitrary $p_{vc}(t)$. If we define *coordination errors* between agents i and j by

$$\sigma_{ij} = e_i - e_j, \quad \forall i, j \in \mathcal{I}, \tag{6}$$

it may be shown that robots create a desired formation shape if $\forall i, j \in \mathcal{I}$, $\sigma_{ij} = 0$. Based on this, we define a pair of agents i and j to be *coordinated* if $\sigma_{ij} = 0$ and call the whole formation coordinated if all pairs of agents are coordinated.

In terms of the error variables e_i, θ_i^e and σ_{ij}, we can then formulate the following problems that will be studied in the remainder of the paper.

Problem Statement 1 (Formation Control): Consider n unicycle robots (1), a desired formation shape defined by $l_i^d(t)$, $i \in \mathcal{I}$ and a desired trajectory $p_{vc}^d(t)$ of the virtual center. The Formation Control problem is solved if the origin of the error dynamics (4) is rendered globally asymptotically stable.

Problem Statement 2 (Coordination): Consider n unicycle robots (1), a desired formation shape defined by $l_i^d(t)$, $i \in \mathcal{I}$ and a desired trajectory $p_{vc}^d(t)$ of the virtual center. The Coordination problem is solved if the set $\{(\theta_i^e, \sigma_{ij}) = (0,0) | i, j \in \mathcal{I}\}$ is rendered a globally asymptotically stable set ([14]) for the error dynamics (4).

III. Formation Control Design

In this section, we will introduce our formation control algorithms. The first control objective of this control algorithm is to drive the tracking errors e_i and θ_i^e to zero for all $i \in \mathcal{I}$ while the error dynamics are stable (Problem Statement 1). In other words, we aim to globally asymptotically stabilize the origin of the tracking error dynamics (4) (therefore also $\sigma_{ij} \to 0$ as $t \to \infty$). To this end, we propose the following control algorithm:

$$v_i(t) = v_i^d(t) + \begin{pmatrix} 1 & 0 \end{pmatrix} R^T(\theta_i) \mu_i \tag{7}$$

$$\omega_i(t) = \omega_i^d(t) + c_i^\theta \theta_i^e + v_i^d(t) \mu_i^T R(\theta_i) \eta(\theta_i^e), \tag{8}$$

where $\mu_i = \mathbf{C}_i^e e_i + \sum_{j \neq i} \mathbf{C}_{ij} \sigma_{ij}$ and $\eta(\theta_i^e) = \mathrm{col}(\frac{\cos\theta_i^e - 1}{\theta_i^e}, \frac{\sin\theta_i^e}{\theta_i^e})$ for $\theta_i^e \neq 0$ and $\eta(\theta_i^e) = \mathrm{col}(0, 1)$ for $\theta_i^e = 0$. We note that (7, 8) constitutes a smooth control law. The control law (7, 8) is distributed with $\mathbf{C}_{ij} \neq 0$ iff robot i communicates with robot j.

Theorem 3.1: Consider n unicycle mobile robots with kinematics described by (1) and the formation control law (7, 8), with $v_i^d(t)$ bounded away from zero, uniformly continuous and bounded and $\omega_i^d(t)$ bounded. Assume that $c_i^\theta > 0$, $\mathbf{C}_i^e = \mathrm{diag}(c_i^{xe}, c_i^{ye})$, $\mathbf{C}_{ij} = \mathrm{diag}(c_{ij}^x, c_{ij}^y)$, where $c_i^{xe} > 0$, $c_i^{ye} > 0$, $c_{ij}^x \geq 0$ and $c_{ij}^y \geq 0$ subject to $\mathbf{C}_{ij} = \mathbf{C}_{ji}$ for $i, j \in \mathcal{I}$. Then, the origin of the closed–loop error dynamics (4, 7, 8) is a globally asymptotically stable equilibrium.

Proof: Consider a Lyapunov function candidate

$$V = \frac{1}{2}\sum_{i=1}^{n}\left((\theta_i^e)^2 + e_i^T\mathbf{C}_i^e e_i + \frac{1}{2}\sum_{j\neq i}\sigma_{ij}^T\mathbf{C}_{ij}\sigma_{ij}\right). \quad (9)$$

The time derivative of V in (9) along trajectories of (4) in closed loop with (7, 8) can be shown to be given by

$$\dot{V} = -\sum_{i=1}^{n}\left(c_i^\theta(\theta_i^e)^2 + \left\|\begin{pmatrix}1 & 0\end{pmatrix}R^T(\theta_i)\mu_i\right\|^2\right) \leq 0, \quad (10)$$

where $\|\cdot\|$ denotes the Euclidean norm of a vector. Therefore, the equilibrium $(e_i,\theta_i^e) = (0,0)$, for all $i \in \mathcal{I}$, is stable. Moreover, by Barbalat's lemma [6] we can show that $\lim_{t\to\infty}\dot{V} = 0$ and hence the system trajectories converge to the manifold defined by

$$\theta_i^e = 0, \qquad \begin{pmatrix}1 & 0\end{pmatrix}R^T(\theta_i)\mu_i = 0. \quad (11)$$

Note that the closed–loop error dynamics of θ_i^e are given by $\dot{\theta}_i^e = -c_i^\theta\theta_i^e - v_i^d(t)\mu_i^T R(\theta_i)\eta(\theta_i^e)$. As θ_i^e converges to zero, the first term of the right–hand side of the error dynamics converges to zero. It then follows from [5, Lemma 2] that also the second term converges to zero. Making use of the facts that $\lim_{s\to 0}\eta(s) = \mathrm{col}(0,1)$ and $v_i^d(t)$ is bounded away from zero, this then implies that $\begin{pmatrix}0 & 1\end{pmatrix}R^T(\theta_i)\mu_i \to 0$ as $t\to\infty$. Consequently, using (11) and given that $R(\theta)$ is invertible, we conclude that $\mu_i \to 0$ as $t \to \infty$. Considering the definition of $\mu_i = \mathbf{C}_i^e e_i + \sum_{j\neq i}\mathbf{C}_{ij}\sigma_{ij}$, the fact that $\mu_i \to 0$ may be written in terms of tracking errors e_i for all $i \in \mathcal{I}$ as follows:

$$A^\nu e^\nu \to 0 \qquad \text{as} \qquad t\to\infty, \quad (12)$$

where $A_{ii}^\nu = c_i^{\nu e} + \sum_{j\neq i}c_{ij}^\nu$, $A_{ij}^\nu = -c_{ij}^\nu$, $i \neq j$, $\nu \in \{x,y\}$, $e^\nu = \mathrm{col}(e_1^\nu,\ldots,e_n^\nu)$ and $e_i = \mathrm{col}(e_i^x, e_i^y)$. Using the Geršgorin disc theorem, we conclude that the matrices in (12) are non-singular which in turn implies that both e^x and e^y converge to 0 as $t\to\infty$. Thus, the origin of (4, 7, 8) is globally asymptotically stable. ∎

Note that although it is not necessary in Theorem 3.1 for the communication topology to be connected, if the connectivity condition is satisfied, the formation behavior is enhanced. This may be observed in particular when we assume that for all $i\in\mathcal{I}$ the position tracking control gains are set to be zero, i.e. $c_i^{xe} = 0$ and $c_i^{ye} = 0$, for all $i \in \mathcal{I}$. This case is considered in the next theorem, which proposes a solution to the coordination problem in Problem Statement 2.

Theorem 3.2 (Coordination): Consider a formation consisting of n unicycle mobile robots with kinematics (1). Define $\Theta_e = \mathrm{col}(\theta_i^e \,|\, i \in \mathcal{I})$, $e = \mathrm{col}(e_i \,|\, i\in\mathcal{I})$ and $\mathcal{E} = \{e \,|\, \sigma_{ij} = 0, i,j\in\mathcal{I}\}$ and let the formation control law be given in (7, 8) in which for all $i \in \mathcal{I}$, $c_i^{xe} = 0$ and $c_i^{ye} = 0$, i.e.

$$v_i(t) = v_i^d(t) + \begin{pmatrix}1 & 0\end{pmatrix}R^T(\theta_i)\sum_{j\neq i}\mathbf{C}_{ij}\sigma_{ij} \quad (13)$$

$$\omega_i(t) = \omega_i^d(t) + c_i^\theta\theta_i^e + v_i^d(t)\sum_{j\neq i}\sigma_{ij}^T\mathbf{C}_{ij}R(\theta_i)\eta(\theta_i^e) \quad (14)$$

where $c_i^\theta > 0$, $\forall i \in \mathcal{I}$, $\mathbf{C}_{ij} = \mathrm{diag}(c_{ij}^x, c_{ij}^y)$, in which $c_{ij}^x > 0$ and $c_{ij}^y > 0$ satisfy $\mathbf{C}_{ij} = \mathbf{C}_{ji}$, $\forall i,j\in\mathcal{I}$. Assume further that for $i\in\mathcal{I}$, $v_i^d(t)$ is bounded away from zero, uniformly continuous and bounded and $\omega_i^d(t)$ is bounded. If the communication topology of the formation is connected, then the set $\Omega = \{(\theta_i^e, e_i)\,|\,\Theta_e = 0, e\in\mathcal{E}\}$ is globally asymptotically stable; hence the desired formation shape is attained for all robots in the formation.

Proof: (Sketch) Consider the Lyapunov function candidate (9) with $\mathbf{C}_i^e = 0$, $i\in\mathcal{I}$. Then, following the same lines of reasoning as in the proof of Theorem 3.1, we can show that $\theta_i^e \to 0$, $i \in\mathcal{I}$, and (12) holds with $A_{ii}^\nu = \sum_{j\neq i}c_{ij}^\nu$. The matrices in (12) are Laplacian matrices [9] and if the communication topology of the formation is connected they have a single zero eigenvalue associated with the right eigenvector $\mathbf{1} = \mathrm{col}(1,\ldots,1)$. Thus, we conclude that for all i,j, $e_i(t)\to e_j(t)$ as $t\to\infty$, or equivalently $\sigma_{ij}\to 0$, which implies that the set Ω is globally asymptotically stable. Hence, all robots in the formation create the desired formation shape. ∎

Remark 3.3: When the topology of the communication network of the formation is disconnected, we can observe the formation shape being restored partially within subgroups of robots $\mathcal{I}_\ell \subset \mathcal{I}$. More specifically, for a disconnected communication network, the set $\mathcal{I}$ may be split into the union of k connected components $\mathcal{I} = \mathcal{I}_1\cup\ldots\cup\mathcal{I}_k$, where $\mathcal{I}_\ell$ is the largest set of vertices of the ℓ-th connected component, $\ell \in \{1,\ldots,k\}$. Consequently, under the conditions in Theorem 3.2 we obtain that the set $\Omega_k = \{(\theta_i^e, e_i)\,|\,\Theta_e = 0, e\in\mathcal{E}_k\}$ is a globally asymptotically stable set, where $\mathcal{E}_k = \bigcap_{\ell=1}^{k}\{e\,|\,\sigma_{ij} = 0, i,j\in\mathcal{I}_\ell\}$.

Remark 3.4: Note that in Theorem 3.2 we do not prove that $e_i(t)\to 0$ as $t\to\infty$, hence trajectory tracking is not necessarily obtained. Instead, the group of robots reaches the prescribed formation geometry and, due to the effect of the feedforward terms $v_i^d(t)$ and $\omega_i^d(t)$, ultimately follows a trajectory that is translated with respect to the desired one.

Based on Theorems 3.1 and 3.2, we can see that if the communication topology of the formation is connected and both tracking control gains $\mathbf{C}_i^e$ and coordination gains $\mathbf{C}_{ij}$ are nonzero, we may act simultaneously upon the two control objectives - coordination and trajectory tracking. This is in spite of the fact that in Theorem 3.1, the connectivity of the communication topology is not required. In this sense, $\mathbf{C}_i^e$ and $\mathbf{C}_{ij}$ may be seen as weighting parameters. Clearly, the prevalent behavior for a particular application can be determined by choosing appropriate values of the tracking gains $\mathbf{C}_i^e$ and coordination gains $\mathbf{C}_{ij}$. In particular, one may choose larger tracking gains $\mathbf{C}_i^e$ to prioritize trajectory tracking. On the other hand, if it is desired to keep the desired formation geometry rather than to track individual robot trajectories, this may be achieved by using larger coordination gains $\mathbf{C}_{ij}$ and smaller tracking gains $\mathbf{C}_i^e$. In the extreme case when coordination gains are the only nonzero parameters, we obtain coordination, see Theorem 3.2. To illustrate results of this paper, we will show the existing trade-off between tracking and coordination in simulations in Section IV.

IV. SIMULATION STUDY

In this section we present a validation of the proposed control algorithms by means of simulations of a formation of three robots. We examine two communication topologies: a disconnected one and a connected one. Control parameters used in the simulations are as follows: $c_1^{xe} = 5$, $c_{12}^{x\sigma} = 50$, $c_{13}^{x\sigma} = 0$, $c_1^{ye} = 30$, $c_{12}^{y\sigma} = 120$, $c_{13}^{y\sigma} = 0$, $c_1^{\theta} = 0.5$, $c_2^{xe} = 3$, $c_{21}^{x\sigma} = 50$, $c_{23}^{x\sigma} = 45$, $c_2^{ye} = 30$, $c_{21}^{y\sigma} = 120$, $c_{23}^{y\sigma} = 110$, $c_2^{\theta} = 0.5$, , $c_3^{xe} = 4$, $c_{31}^{x\sigma} = 0$, $c_{32}^{x\sigma} = 45$, $c_3^{ye} = 29$, $c_{31}^{y\sigma} = 0$, $c_{32}^{y\sigma} = 110$, $c_3^{\theta} = 0.5$. For the disconnected communication topology, we set all coupling gains c_{ij}^{x} and c_{ij}^{y} to zero.

A. Tracking and Coordination

In this subsection, we illustrate the behavior of robots controlled by the control algorithm given in Theorem 3.1. In all simulations in this subsection, the desired trajectory of the virtual center is given by $x_{vc}^d(t) = 3\sin\theta_{vc}^d(t) + 3.5$, $y_{vc}^d(t) = 3\cos\theta_{vc}^d(t) + 0.5$ and $\theta_{vc}^d(t) = 0.13t - \frac{\pi}{2}$. The desired formation shape is time–invariant and forms an equilateral triangle given by $l_1^d = \left(-0.3, -\frac{0.3}{\sqrt{3}}\right)^T$, $l_2^d = \left(0.3, -\frac{-0.3}{\sqrt{3}}\right)^T$ and $l_3^d = \left(0, \frac{0.6}{\sqrt{3}}\right)^T$. The initial conditions of robots are $q_1(0) = (4.65, -1.28, 0.43)^T$, $q_2(0) = (-2.24, -3.73, 0.62)^T$, $q_3(0) = (-2.43, 0.97, 0.52)^T$. Moreover, in both simulations we apply a perturbation at time $t = 25$ to observe robots' behavior after the perturbation. This perturbation is equivalent to displacing Robot 1 along $(\delta x, \delta y) = (1, -0.5)$.

The simulation results are shown in Fig. 1–3. In particular, in Fig. 1 we present robots' paths in the plane. As shown in this figure, robots initially converge to the desired formation geometry. When the topology of the communication network is disconnected, see Fig. 1(a), none of the unperturbed robots in the formation (Robots 2 and 3) reacts to the perturbation to maintain the formation shape. In contrast, when the communication topology is connected, see Figure 1(b), both unperturbed robots also diverge from their desired trajectories in favor of formation keeping.

We can also observe the advantageous influence of the communication topology being connected in Fig. 2–3 that show tracking errors. Not only are the errors smaller but also robots aim to achieve coordination as well as tracking their desired trajectories simultaneously when the communication topology is connected (observe the close matching of e_1^{ν}, e_2^{ν} and e_3^{ν}, $\nu \in \{x, y\}$, in Fig. 2(b) and 3(b)). This may be compared to the phenomena observed by Rodriguez-Angeles and Nijmeijer [12] for robotic manipulators. They noticed that by coupling the robotic manipulators, the manipulators tend to act in synchrony. Indeed, a similar behaviour may be observed in our simulations. Due to the connectivity of the communication topology, it is possible that robots maintain their desired formation geometry despite the lack of tracking of individual robots' trajectories (after the perturbation). As a matter of fact, in Fig. 2 and 3, we clearly see that tracking errors coincide with each other before they converge to zero. This means that owing to relatively strong coupling gains

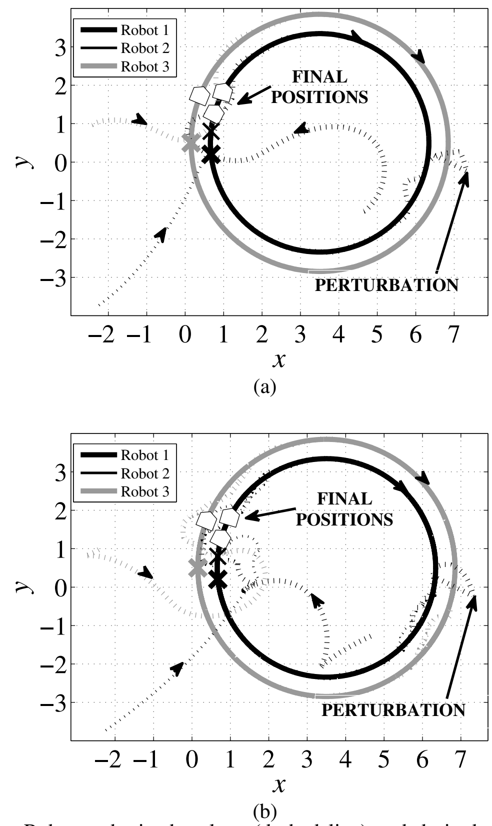


Fig. 1. Robot paths in the plane (dashed line) and desired paths in the plane (solid line): (a) disconnected communication topology (b) connected communication topology.

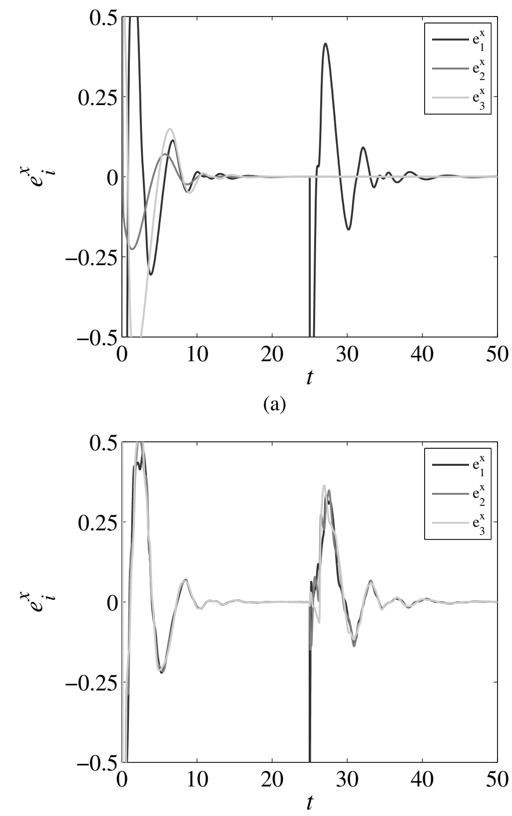


Fig. 2. Tracking errors (x – coordinate): (a) disconnected communication topology (b) connected communication topology.

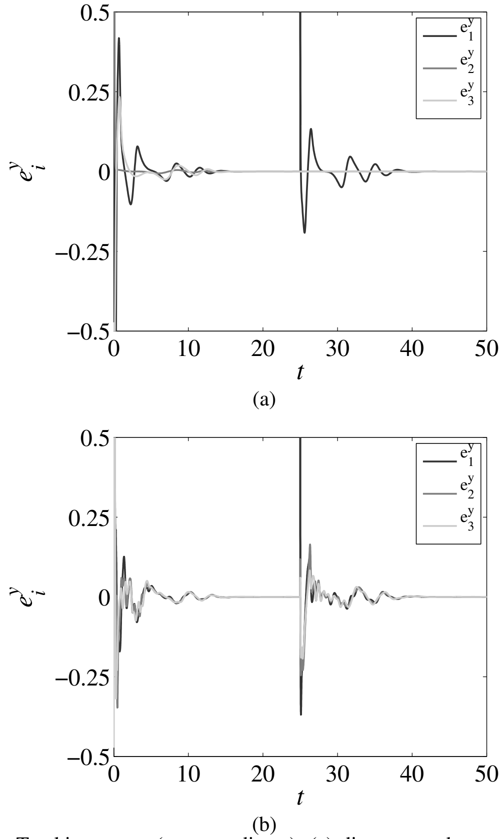


Fig. 3. Tracking errors (y – coordinate): (a) disconnected communication topology (b) connected communication topology.

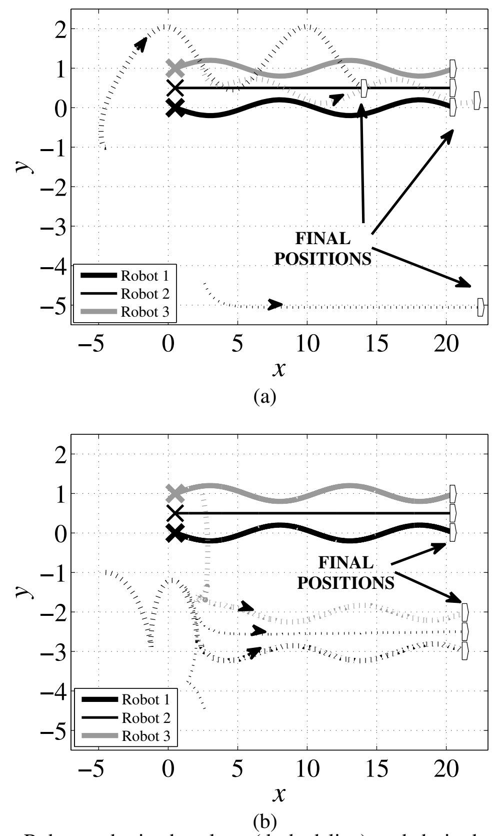


Fig. 4. Robot paths in the plane (dashed line) and desired paths in the plane (solid line): (a) disconnected communication topology (b) connected communication topology.

$\mathbf{C}_{ij}$ as compared to the tracking gains $\mathbf{C}_i^e$ robots aim to first restore the formation geometry before they return back to their desired individual trajectories.

B. Coordination

In this section, we present simulation results for a group of mobile robots under the coordination control algorithm (13, 14). The control parameters are given at the beginning of Section IV. However, all Cartesian tracking gains are zero, i.e. $\mathbf{C}_i^e = 0$, for all $i \in \mathcal{I}$, as per the requirements of the coordination algorithm.

The desired trajectory of the virtual center of the formation is taken to be a straight line given by $x_{vc}^d(t) = 0.5 + 0.4t$, $y_{vc}^d(t) = 0.5$ and $\theta_{vc}^d(t) = 0$. To illustrate further advantages of the control algorithms proposed in this paper, the desired formation geometry is now time-varying, and is created by $l_1^d = (0, -0.5 - 0.2\sin(0.25t))^T$, $l_2^d = (0,0)^T$ and $l_3^d = (0, 0.5 + 0.2\sin(0.25t))^T$. The initial robots' states are $q_1(0) = (-4.5, -1, \pi)^T$, $q_2(0) = (2.62, -4.45, -\frac{\pi}{4})^T$, $q_3(0) = (2.47, 0.98, \frac{\pi}{3})^T$.

The simulation results are given in Fig. 4–6. It can be seen that if the communication topology of the formation is connected, robots create a given desired formation shape but, as expected, they do not follow the desired trajectory of the formation, see Fig. 4(b). In fact they follow the desired trajectory in a translated sense. In contrast, when robots are decoupled, see Fig. 4(a) the robots' heading angles converge to their desired values but they neither track their individual

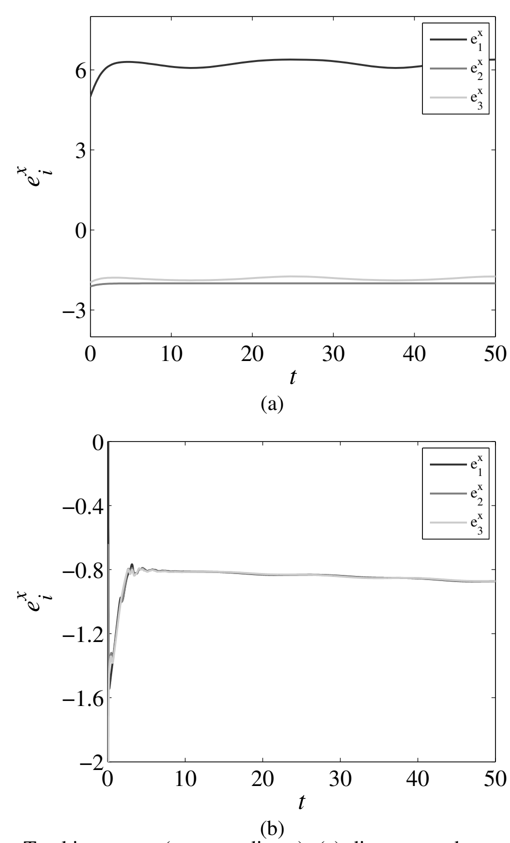
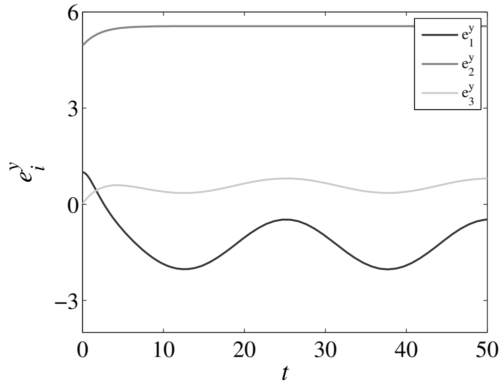
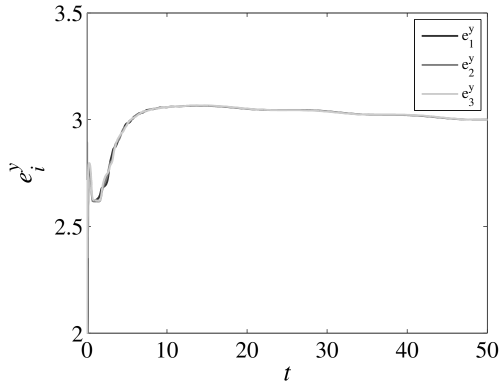


Fig. 5. Tracking errors (x – coordinate): (a) disconnected communication topology (b) connected communication topology.



(a)



(b)

Fig. 6. Tracking errors (y – coordinate): (a) disconnected communication topology (b) connected communication topology.

trajectories, nor are coordinated with each other.

Similar conclusions can be drawn with the aid of Fig. 5–6 that present the tracking errors. We can see that when the communication topology is connected, the tracking errors are in consensus with each other and so the coordination errors go to zero, see Fig. 5(b) and 6(b). However, robots do not track their individual trajectories, hence the position errors do not converge to zero. Conversely, when the communication topology is disconnected the tracking errors are different from each other, see Fig. 5(a) and 6(a). This shows that in order to achieve consensus of the tracking error variables robots need to be able to sufficiently exchange information with each other.

V. Conclusions

In this paper, we have studied a distributed formation control problem for unicycle robots with time-varying desired formation shapes. We have expressed position tracking errors in a common coordinate frame and from that we were able to show that the physical coordination of robots is associated with the consensus of the tracking errors given in a common coordinate system. By combining terms in the control law relating to both tracking of the desired trajectories of individual robots and coordination with the robots' neighbors, we have proposed a control law that ensures both tracking and coordination simultaneously. Moreover, we have observed that formation behaviour of multiple robots in the group can be enhanced if robots can communicate with each other. If this is the case, the robots in the formation not only aim to track their individual trajectories but they also explicitly act towards maintaining a desired formation shape.

In addition to the formation control problem in which robots create a desired formation shape and track, as a whole, a desired trajectory, we have also studied the case of coordination without converging of the formation to the desired trajectory. We have shown that for coordination to occur, it is crucial that robots have sufficient communication between each other.

Our future work will include dealing with some practical aspects of the formation control. More specifically, we want to change the formation controller to accommodate for the actuator limitations of robots as well as introduce inter–agent collision avoidance scheme. In addition, experimental validation of the proposed control algorithm will be performed.

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