

# Formation Control of Unicycle Mobile Robots: a Virtual Structure Approach

Thijs H.A. van den Broek, Nathan van de Wouw, Henk Nijmeijer

**Abstract**—In this paper, the formation control problem for unicycle mobile robots is studied. A virtual structure control strategy with mutual coupling between the robots is proposed. The rationale behind the introduction of the coupling terms is the fact that these introduce additional robustness with respect to perturbations as compared to typical leader-follower approaches. The applicability of the proposed approach is shown in experiments with a group of mobile robots controlled over a wireless communication network.

## I. INTRODUCTION

In this paper, the formation control problem (i.e. cooperative control problem) for unicycle mobile robots is considered. Formation control problems arise when groups of mobile robots are employed to jointly perform certain tasks. The benefits of exploiting groups of robots, as opposed to a single robot or a human, become apparent when considering spatially distributed tasks, dangerous tasks, tasks which require redundancy, tasks that scale up or down in time or tasks that require flexibility. Various areas of application of cooperative control of mobile robots are e.g. simultaneous localization and mapping [1], automated highway systems [2], payload transportation [3] and enclosing an invader [4]. [5] presents a recent overview on cooperative control of robotic systems.

Before a wide application of cooperative mobile robotics will become feasible, many technical and scientific challenges must be faced such as the development of cooperative and formation control strategies, control schemes robust to communication constraints, the localization of the robot position, sensing and environment mapping, etc. In the current paper, the focus is on the aspect of cooperative control. In the recent literature, see e.g. [6], [7], three different approaches towards the cooperative control of mobile robots are described: the behaviour-based approach, the leader-follower approach and the virtual structure approach. In the behaviour-based approach, a so-called behaviour (e.g. obstacle avoidance, target seeking) is assigned to each individual robot [8]. This approach can naturally be used to design control strategies for robots with multiple competing objectives. Moreover, it is suitable for large groups of robots, since it is typically a decentralized strategy. A disadvantage is that the complexity of the dynamics of the group of

robots does not lend itself for simple mathematical stability analysis. To simplify the analysis, the dynamics of individual robots are commonly simplified as being described by a single integrator. Clearly, even kinematic models of mobile robots is more complex, limiting the applicability of this approach in practice.

In the leader-follower approach some robots will take the role of leader and aim to track predefined trajectories, while the follower robots will follow the leader according to a relative posture [9]–[11]. An advantage of this approach is the fact that it is relatively easy to understand and implement. A disadvantage, however, is the fact that there is no feedback from the followers to the leaders. Consequently, if a follower is being perturbed, the formation cannot be maintained and such a formation control strategy lacks robustness in the face of such perturbations.

A third approach in cooperative control is the virtual structure approach, in which the robots' formation no longer consists of leaders nor followers, i.e. no hierarchy exists in the formation. In [12], a general controller strategy is developed for the virtual structure approach. Using this strategy, however, it is not possible to consider formations which are time-varying. Moreover, the priority of the mobile robots, either to follow their individual trajectories or to maintain the groups formation, can not be changed. In [7], a virtual structure controller is designed for a group of unicycle mobile robots using models involving the dynamics of the robots. Consequently, the controller design tends to be rather complex, which is unfavorable from an implementation perspective, especially when kinematic models suffice. An advantage of the virtual structure approach is, as we will show in this paper, that it allows to attain a certain robustness of the formation to perturbations on the robots.

The first contribution of this paper is the design of a virtual structure controller for nonholonomic unicycle mobile robots, including a stability proof for the formation error dynamics of a group of two unicycle mobile robots. To limit the complexity of the virtual structure controller, the controller design is based on the kinematics of unicycle mobile robots. Moreover, so-called mutual coupling terms will be introduced between the robots to ensure robustness of the formation with respect to perturbations. The second contribution of this paper is the validation of the controller design in an experimental setting.

This paper is organized as follows. In Section II, the virtual structure control design, which uses the tracking controller of [13] as a stepping stone, is presented and a stability result for the formation error dynamics is provided. In Section III,

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experiments are presented validating the proposed approach in practice. Section IV present concluding remarks.

## II. VIRTUAL STRUCTURE FORMATION CONTROL WITH MUTUAL COUPLING

In Section II-A, the kinematic model of a unicycle mobile robot is presented. In Section II-B, a generic virtual structure formation controller, with mutual coupling between the robots, will be presented, which is based on the kinematic model of a unicycle. Moreover, in Section II-C, a stability theorem for the formation error dynamics for the case of two robots is given.

### A. Kinematics of a Unicycle Mobile Robot

The kinematics of the  $i$ th nonholonomic unicycle mobile robot, in a group of  $N$  robots, is described by the following differential equation:

$$\begin{aligned}\dot{x}_i &= v_i \cos(\phi_i), \\ \dot{y}_i &= v_i \sin(\phi_i), \\ \dot{\phi}_i &= \omega_i,\end{aligned}\quad (1)$$

with  $i = 1, \dots, N$  and where the coordinates  $x_i$  and  $y_i$  describe the position of the center of the  $i$ th mobile robot with respect to the fixed coordinate frame  $\bar{e}^0 := [\bar{e}_1^0 \ \bar{e}_2^0]^T$  and the orientation  $\phi_i$  is the angle between the heading of the  $i$ th robot and the  $x$ -axis of the fixed coordinate frame  $\bar{e}^0$ , see Figure 1. For the sake of generality we consider a group of  $N$  mobile robots. The forward velocity and rotational velocity are given by  $v_i$  and  $\omega_i$ , respectively, which are the control inputs of the  $i$ th mobile robot. The reference trajectory is

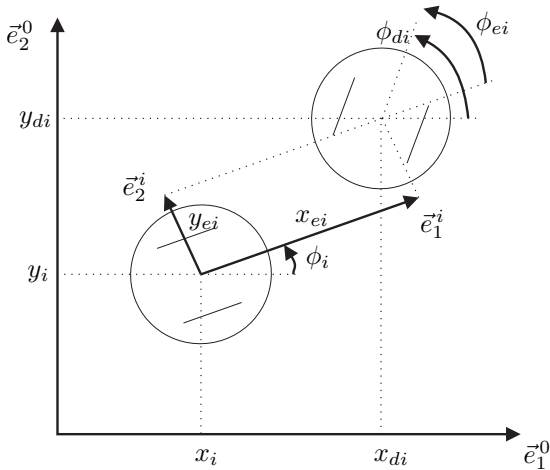


Fig. 1: The unicycle coordinates, desired coordinates and error coordinates of the  $i$ th mobile robot.

given by  $(x_{di}(t), y_{di}(t), \phi_{di}(t))$ . Due to the nonholonomic constraint of unicycle robots, the desired orientation  $\phi_{di}(t)$  satisfies  $-\dot{x}_{di} \sin(\phi_{di}) + \dot{y}_{di} \cos(\phi_{di}) = 0$ , see Figure 1. The desired forward and rotational velocity  $(v_{di}(t), \omega_{di}(t))$  are defined as

$$\begin{aligned}v_{di} &= \sqrt{\dot{x}_{di}^2 + \dot{y}_{di}^2}, \\ \omega_{di} &= \frac{\dot{x}_{di} \dot{y}_{di} - \dot{y}_{di} \dot{x}_{di}}{\dot{x}_{di}^2 + \dot{y}_{di}^2},\end{aligned}\quad (2)$$

for  $\dot{x}_{di}, \dot{y}_{di} \neq 0$ . Define the tracking error coordinates  $(x_{ei}, y_{ei}, \phi_{ei})$  as follows:

$$\begin{aligned}x_{ei} &= (x_{di} - x_i) \cos(\phi_i) + (y_{di} - y_i) \sin(\phi_i) \\ y_{ei} &= -(x_{di} - x_i) \sin(\phi_i) + (y_{di} - y_i) \cos(\phi_i), \\ \phi_{ei} &= \phi_{di} - \phi_i\end{aligned}\quad (3)$$

see also Figure 1. We will exploit these error coordinate definitions in the stability result of the formation error dynamics in Section II-C.

### B. Virtual Structure Control Design

We design a virtual structure controller, with mutual coupling between  $N$  individual robots, such that a desired formation is achieved. The main goals of the virtual structure controller are twofold. Firstly, the formation as a whole should follow a predefined trajectory; i.e. a so-called virtual center should follow a predefined trajectory and the  $i$ -th unicycle robot,  $i \in \{1, \dots, N\}$ , should follow at a certain predefined, and possibly time-varying, location  $(l_{xi}, l_{yi}) \bar{e}^i$  relative to the virtual center. Secondly, if the individual robots suffer from perturbations, the controller should mediate between keeping formation and ensuring the tracking of the individual robots' desired trajectories, which is facilitated by introducing mutual coupling between the robots. In Figure 2, for the purpose of illustration two mobile robots and the virtual center  $VC$  of the formation are shown. The reference

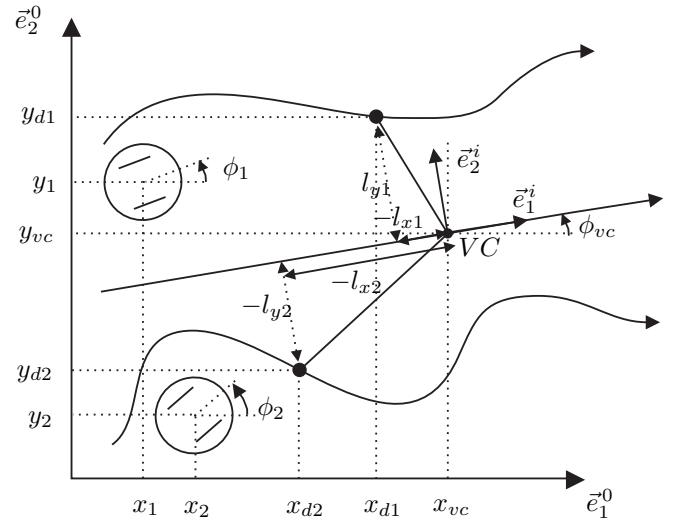


Fig. 2: The virtual structure approach with two unicycle mobile robots.

trajectory of the virtual center is described by the coordinates  $(x_{vc}(t), y_{vc}(t))$  defining the position of the virtual center with respect to the fixed coordinate frame  $\bar{e}^0$ . The desired trajectories of the individual robots  $(x_{di}(t), y_{di}(t))$ ,  $i = 1, \dots, N$ , are described as

$$\begin{aligned}x_{di}(t) &= x_{vc}(t) + l_{xi}(t) \cos(\phi_{vc}(t)) \\ &\quad - l_{yi}(t) \sin(\phi_{vc}(t)) \\ y_{di}(t) &= y_{vc}(t) + l_{xi}(t) \sin(\phi_{vc}(t)) \\ &\quad + l_{yi}(t) \cos(\phi_{vc}(t))\end{aligned}\quad (4)$$

where  $\phi_{vc}(t)$  is the orientation of the virtual center along its trajectory and  $(l_{xi}(t), l_{yi}(t))\bar{e}^i$  is possibly time-varying to allow for time-varying formation shapes. Now, the tracking controller of [13] is expanded with so-called mutual coupling terms. In [14], such terms were introduced at the level of the desired trajectories to achieve mutual synchronization between industrial robots. This type of mutual coupling, which is located at the desired trajectory level, is not possible for unicycle mobile robots due to the nonholonomic constraints. Here, we propose to introduce the coupling directly in the feedback control strategy arriving at the following control law:

$$\begin{aligned}\omega_i &= \omega_{di} + \alpha_i \sin(\phi_{ei}) \\ &\quad + \sum_{j=1, i \neq j}^N \tilde{\alpha}_{i,j} \sin(\phi_{ei} - \phi_{ej}) \\ v_i &= v_{di} + \beta_i x_{ei} - \gamma_i \omega_{di} y_{ei} \\ &\quad + \sum_{j=1, i \neq j}^N \tilde{\beta}_{i,j} (x_{ei} - x_{ej}) \\ &\quad - \sum_{j=1, i \neq j}^N \tilde{\gamma}_{i,j} \omega_{di} (y_{ei} - y_{ej}),\end{aligned}\quad (5)$$

$i = 1, \dots, N$ , where the feedforward velocities  $(v_{di}, \omega_{di})$  and the error coordinates  $(x_{ei}, y_{ei}, \phi_{ei})$  are defined in (2) and (3), respectively, and with  $\alpha_i > 0$ ,  $\beta_i > 0$  and  $\gamma_i > -1$ ,  $i = 1, \dots, N$ . Moreover,  $\tilde{\alpha}_{i,j} > 0$ ,  $\tilde{\beta}_{i,j} > 0$  and  $\tilde{\gamma}_{i,j} > 0$ , which represent mutual coupling parameters, and the subscript  $i = 1, \dots, N$  denotes the  $i$ th mobile robot, which is mutually coupled to the  $j$ th mobile robot,  $j = 1, \dots, N$ .

Before a stability result for the formation error dynamics of a group of unicycles under application of the virtual structure controller of (5) is presented in the next section, let us explain the working principle of the controller in (5). For the sake of simplicity, we limit ourselves to the case of two mobile robots. Assume that robot 2 resides on its desired trajectory, i.e.  $(x_{e2}, y_{e2}, \phi_{e2}) = 0$ . According to (5) this results in the following individual control inputs of robots 1 and 2:

$$\begin{aligned}\omega_1 &= \omega_{d1} + \alpha_1 \sin(\phi_{e1}) + \tilde{\alpha}_{1,2} \sin(\phi_{e1}), \\ v_1 &= v_{d1} + \beta_1 x_{e1} - \gamma_1 \omega_{d1} y_{e1} + \tilde{\beta}_{1,2} x_{e1} \\ &\quad - \tilde{\gamma}_{1,2} \omega_{d1} y_{e1},\end{aligned}\quad (6)$$

and

$$\begin{aligned}\omega_2 &= \omega_{d2} - \tilde{\alpha}_{2,1} \sin(\phi_{e1}), \\ v_2 &= v_{d2} - \tilde{\beta}_{2,1} x_{e1} + \tilde{\gamma}_{2,1} \omega_{d2} y_{e1},\end{aligned}\quad (7)$$

respectively. Moreover, it is assumed that robot 1 is not on its desired trajectory, e.g.  $x_{e1}, y_{e1}, \phi_{e1} > 0$ . Note that the terms  $-\tilde{\alpha}_{2,1} \sin(\phi_{e1})$ ,  $-\tilde{\beta}_{2,1} x_{e1}$  and  $\tilde{\gamma}_{2,1} \omega_{d2} y_{e1}$  in (7), with  $x_{e1}, y_{e1}, \phi_{e1} > 0$ , have a similar effect as terms  $\alpha_2 \sin(\phi_{e2})$ ,  $\beta_2 x_{e2}$  and  $-\gamma_2 \omega_{d2} y_{e2}$  would, with  $x_{e2}, y_{e2}, \phi_{e2} < 0$ . In other words, the mutual coupling terms are acting as if robot 2 is behind its desired trajectory (i.e. as if  $x_{e2} < 0$ ), below its desired trajectory (i.e. as if  $y_{e2} < 0$ ) and orientated in clockwise direction relative to the desired trajectory (i.e. as if  $\phi_{e2} < 0$ ). Consequently, the controller for robot 2 will try

to compensate for these errors, which results in the fact that the formation will remain (partly) intact. The second effect of the mutual coupling term is that robot 1 in this case is subject to effective gains  $\alpha_1 + \tilde{\alpha}_{1,2}$ ,  $\beta_1 + \tilde{\beta}_{1,2}$  and  $\gamma_1 + \tilde{\gamma}_{1,2}$  in (6).

### C. Stability Analysis of the Formation Error Dynamics

In this section, the stability of the resulting formation error dynamics under application of the controller (5) is analyzed for the specific case of a formation of two mobile robots. The formation error dynamics of two mobile robots, described by (1) and the controller (5), can be written in the following cascaded form:

$$\begin{aligned}\begin{bmatrix} \dot{x}_{e1} \\ \dot{y}_{e1} \\ \dot{x}_{e2} \\ \dot{y}_{e2} \end{bmatrix} &= f_1(t, x_{e1}, y_{e1}, x_{e2}, y_{e2}) \\ &\quad + g(t, x_{e1}, y_{e1}, x_{e2}, y_{e2}, \phi_{e1}, \phi_{e2}) \begin{bmatrix} \phi_{e1} \\ \phi_{e2} \end{bmatrix} \\ \begin{bmatrix} \dot{\phi}_{e1} \\ \dot{\phi}_{e2} \end{bmatrix} &= f_2(t, \phi_{e1}, \phi_{e2}),\end{aligned}\quad (8)$$

where

$$f_1(t, x_{e1}, y_{e1}, x_{e2}, y_{e2}) = \begin{bmatrix} y_{e1} \omega_{d1} - \beta_1 x_{e1} + \gamma_1 \omega_{d1} y_{e1} \\ -\tilde{\beta}_{1,2}(x_{e1} - x_{e2}) + \tilde{\gamma}_{1,2} \omega_{d1}(y_{e1} - y_{e2}) \\ -\omega_{d1} x_{e1} \\ y_{e2} \omega_{d2} - \beta_2 x_{e2} + \gamma_2 \omega_{d2} y_{e2} \\ -\tilde{\beta}_{2,1}(x_{e2} - x_{e1}) + \tilde{\gamma}_{2,1} \omega_{d2}(y_{e2} - y_{e1}) \\ -\omega_{d2} x_{e2} \end{bmatrix}, \quad (9)$$

$$f_2(t, \phi_{e1}, \phi_{e2}) = \begin{bmatrix} -\alpha_1 \sin(\phi_{e1}) - \tilde{\alpha}_{1,2} \sin(\phi_{e1} - \phi_{e2}) \\ -\alpha_2 \sin(\phi_{e2}) - \tilde{\alpha}_{2,1} \sin(\phi_{e2} - \phi_{e1}) \end{bmatrix}, \quad (10)$$

and

$$g(t, x_{e1}, y_{e1}, x_{e2}, y_{e2}, \phi_{e1}, \phi_{e2}) \begin{bmatrix} \phi_{e1} \\ \phi_{e2} \end{bmatrix} = \begin{bmatrix} \alpha_1 y_{e1} \sin(\phi_{e1}) + y_{e1} \tilde{\alpha}_{1,2} \sin(\phi_{e1} - \phi_{e2}) \\ -v_{d1} + v_{d1} \cos(\phi_{e1}) \\ -\alpha_1 x_{e1} \sin(\phi_{e1}) - x_{e1} \tilde{\alpha}_{1,2} \sin(\phi_{e1} - \phi_{e2}) \\ +v_{d1} \sin(\phi_{e1}) \\ \alpha_2 y_{e2} \sin(\phi_{e2}) + y_{e2} \tilde{\alpha}_{2,1} \sin(\phi_{e2} - \phi_{e1}) \\ -v_{d2} + v_{d2} \cos(\phi_{e2}) \\ -\alpha_2 x_{e2} \sin(\phi_{e2}) - x_{e2} \tilde{\alpha}_{2,1} \sin(\phi_{e2} - \phi_{e1}) \\ +v_{d2} \sin(\phi_{e2}) \end{bmatrix}, \quad (11)$$

with  $\alpha_i > 0$ ,  $\beta_i > 0$ ,  $\gamma_i > -1$ ,  $i = 1, 2$ , and  $\tilde{\alpha}_{i,j} > 0$ ,  $\tilde{\beta}_{i,j}$  and  $\tilde{\gamma}_{i,j}$ ,  $i = 1, 2$ ,  $j = 1, 2$ ,  $i \neq j$ , the coupling parameters. The following theorem gives sufficient conditions under which the equilibrium point  $(x_{ei}, y_{ei}, \phi_{ei}) = 0$ ,  $i = 1, 2$ , of the formation error dynamics (8)-(11) is locally exponentially stable. In other words, the formation control problem is solved locally.

### Theorem 1

Consider two non-holonomic unicycle mobile robots whose kinematics are described by (1). Suppose that the desired tracking state trajectories of the individual robots

$(x_{di}(t), y_{di}(t))$ ,  $i = 1, 2$ , are given by (4) for a given trajectory  $(x_{vc}(t), y_{vc}(t))$  for the virtual center. Moreover, the desired orientations  $\phi_{di}(t)$ ,  $i = 1, 2$ , are imposed by the non-holonomic constraint. Consider controller (5) for  $N = 2$ , with the feedforwards  $v_{di}$ ,  $\omega_{di}$ ,  $i = 1, 2$ , satisfying (2). If

- the desired rotational velocities of both mobile robots are persistently exciting and identical, i.e.  $\omega_{d1} = \omega_{d2}$ ;
- $\beta_1 = \beta_2$ ,  $\gamma_1 = \gamma_2$ ,  $\tilde{\beta}_{1,2} = \tilde{\beta}_{2,1}$  and  $\tilde{\gamma}_{1,2} = \tilde{\gamma}_{2,1}$ ;
- the control parameters  $\alpha_i > 0$ ,  $\beta_i > 0$  and  $\gamma_i > -1$ ,  $i = 1, 2$ ;
- the coupling parameters  $\tilde{\alpha}_{i,j} > 0$ , for  $i = 1, 2$ ,  $j = 1, 2$ ,  $i \neq j$ ,  $\tilde{\beta}_{1,2} = \tilde{\beta}_{2,1} > -\frac{\beta_1}{2}$ ,  $\tilde{\gamma}_{1,2} = \tilde{\gamma}_{2,1} > \frac{-1-\gamma_1}{2}$  and  $(1 + \gamma_1)\tilde{\beta}_{1,2}^2 - \tilde{\gamma}_{1,2}\tilde{\beta}_{1,2}\beta_1 > 0$ ,

then the equilibrium point  $(x_{e1}, y_{e1}, \phi_{e1}, x_{e2}, y_{e2}, \phi_{e2}) = 0$  of the formation error dynamics (8)-(11) is locally exponentially stable.

*Proof:* For the sake of brevity the proof is omitted here, but the interested reader is referred to [15]. ■

#### Remark 1

In practice we typically choose the coupling parameters such that  $\tilde{\alpha}_{i,j} > 0$ ,  $\tilde{\beta}_{i,j} > 0$ ,  $\tilde{\gamma}_{i,j} > 0$  (which reflect more strict conditions than those in the theorem), because if we would opt for  $-\frac{\beta_1}{2} < \tilde{\beta}_{i,j} \leq 0$  and  $(\tilde{\gamma}_{i,j} \leq 0) \wedge (\tilde{\gamma}_{i,j} > \frac{-1-\gamma_1}{2})$ , then (although stability is not endangered) undesirable transient behaviour of the formation may be induced.

#### Remark 2

Simulations, with more than two robots, moving with different desired rotational velocities  $\omega_{di}$ , different control parameters  $(\beta_i, \gamma_i)$  and different coupling parameters  $(\tilde{\beta}_{i,j}, \tilde{\gamma}_{i,j})$ , show that the error dynamics of the virtual structure controller is stable in a more general setting. In the current paper, we refrain from such technical extensions, but rather focus on the experimental validation of the proposed approach, which is shown in the next section.

### III. EXPERIMENTS

In this section experiments are performed to validate in practice the controller design, proposed in the previous section. In Section III-A, the experimental setup is presented and experimental results are discussed in Section III-B.

#### A. Experimental Setup

The experimental setup is shown in Figure 3. The experiments are performed with two E-Puck mobile robots [16]. The E-Puck robot has two driven wheels, which are individually actuated by means of stepper motors. Velocity control commands are sent to both stepper motors over a wireless Bluetooth connection. The absolute position measurement of the mobile robots is performed using a Firewire camera AVT Guppy F-080b b/w [17], in combination with reacTIVision software [18]. We note that the achieved position and orientation accuracy of these position measurements are 0.0019 m in  $x$ - and  $y$ -direction and 0.0524 rad in  $\phi$ -direction, and the driving area of the mobile robots is 1.75 by 1.28 m. The sample rate is given as 25 Hz. Both signal processing and controller implementation is executed in Python [19].

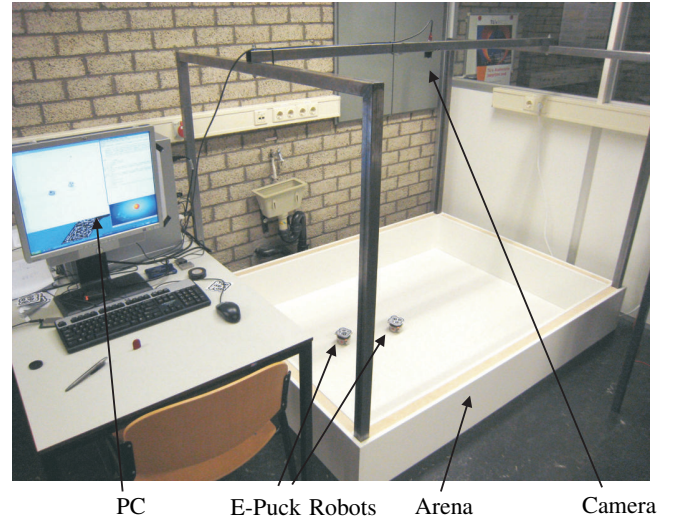


Fig. 3: The experimental setup.

#### B. Experimental Results

In this section, the results of an experiment with two mobile robots driving in formation are discussed. The trajectory of the virtual center is given by  $x_{vc}(t) = 0.9 + 0.3 \cos(2\pi \cdot 0.02t)$  [m] and  $y_{vc}(t) = 0.6 + 0.3 \sin(2\pi \cdot 0.02t)$  [m]. The desired trajectories of the mobile robots are defined according to (4), with  $l_{x1} = 0.1$  m,  $l_{y1} = 0.1$  m,  $l_{x2} = -0.1$  m and  $l_{y2} = 0$  m, respectively. In other words, the virtual center moves in a circular motion, robot 1 is positioned ahead and left of the virtual center and robot 2 is positioned behind the virtual center. The controllers of both robots are of the form (5), where the control and coupling parameters satisfy the conditions of Theorem 1 and given by  $\alpha_i = 0.3$ ,  $\beta_i = 0.275$ ,  $\gamma_i = 1.3$ ,  $i = 1, 2$ ,  $\tilde{\alpha}_{i,j} = 3$ ,  $\tilde{\beta}_{i,j} = 2.75$  and  $\tilde{\gamma}_{i,j} = 13$ ,  $i = 1, 2$ ,  $j = 1, 2$ ,  $i \neq j$ . The control and coupling parameters are tuned to demonstrate that the two mobile robots attain formation asymptotically and prefer to maintain formation, as opposed to following their individual desired trajectories. This type of behaviour is due to the relatively strong coupling parameters  $(\tilde{\alpha}_{i,j}, \tilde{\beta}_{i,j}, \tilde{\gamma}_{i,j})$ . Two types of perturbations are applied to illustrate the behaviour of the mobile robots in the face of perturbations. The first perturbation involves both the forward velocity  $v_1$  and rotational velocity  $\omega_1$  of robot 1 as follows:

$$\begin{aligned} \omega_1 &= \omega_{d1} + \alpha_1 \sin(\phi_{e1}) + \tilde{\alpha}_{1,2} \sin(\phi_{e1}) + 0.5, \\ v_1 &= v_{d1} + \beta_1 x_{e1} - \gamma_1 \omega_{d1} y_{e1} + \tilde{\beta}_{1,2} x_{e1} - \tilde{\gamma}_{1,2} \omega_{d1} y_{e1} + 0.3, \end{aligned} \quad (12)$$

for  $t \in [35, 36]$  s. The second perturbation takes place at  $t = 56$  s; here, robot 1 is repositioned manually. In Figure 4, the desired trajectories and actual trajectories of robots 1 and 2 are shown. Robot 1 initially moves backwards and away from its desired trajectory, thereby aiming to achieve the desired formation with robot 2 as fast as possible. A closer inspection of the trajectory of robot 2 reveals that the effects of the disturbances on robot 1 are clearly noticeable in the behaviour of robot 2. In Figure 5, the error coordinates of the

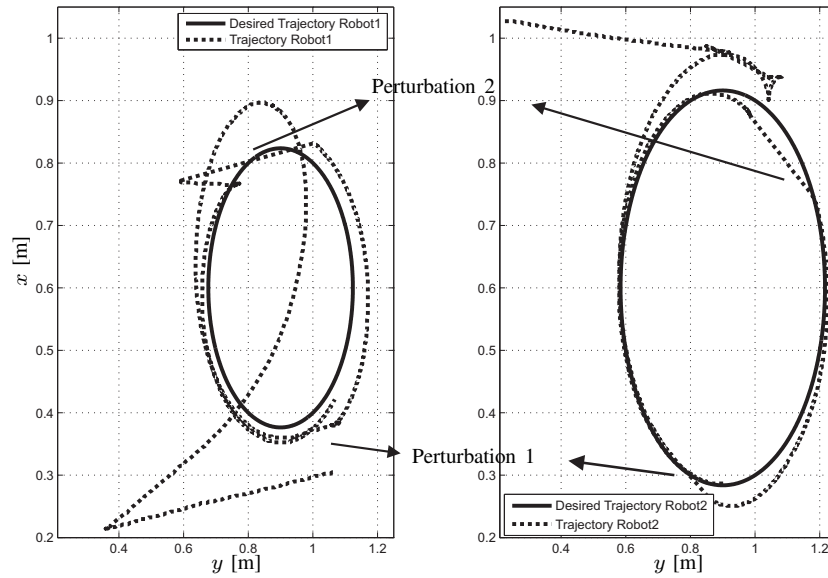


Fig. 4: The measured trajectories and desired trajectories of robots 1 and 2.

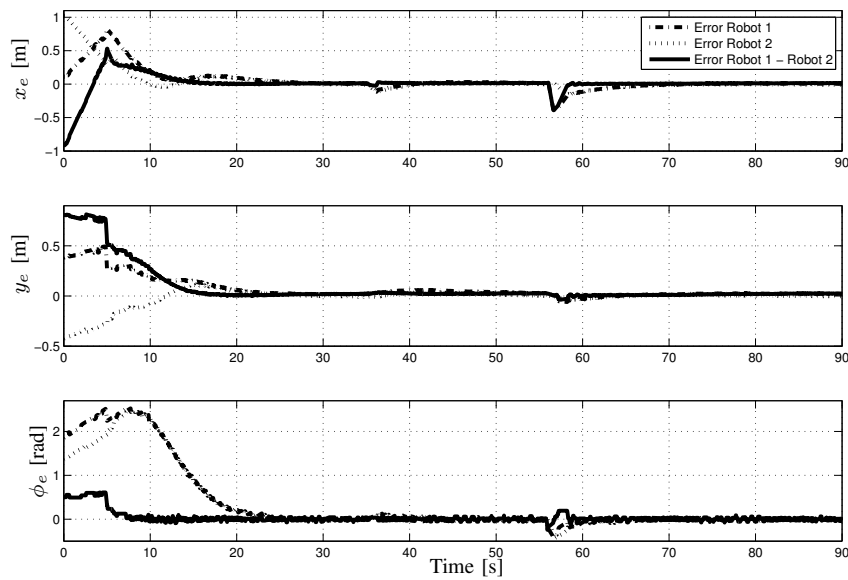


Fig. 5: Experimental evolution of the error coordinates of robots 1 and 2.

individual robots  $(x_{e1}, y_{e1}, \phi_{e1}, x_{e2}, y_{e2}, \phi_{e2})$  and the error coordinates of the formation  $(x_{e1} - x_{e2}, y_{e1} - y_{e2}, \phi_{e1} - \phi_{e2})$  are shown. This figure clearly shows that the robots converge to the desired formation within 15 s. Within 25 s, the robots have also converged to their desired trajectories. Clearly, both in transients and after perturbations the robots first converge to their desired formation, and then converge to their desired trajectories. This behaviour is due to the choice for strong coupling parameters, i.e. the robots priority is to maintain

the formation. In Figure 6 (a zoomed version of Figure 5), the error coordinates of robots 1 and 2 are displayed for the time interval  $t \in [30, 90]$  s. During the perturbations, robot 2 is reacting to the error of robot 1, thereby trying to remain in formation. Clearly, this experiment shows that the mutual coupling terms in the proposed controlled strategy provides robustness to the formation in the face of perturbations. Moreover, the tuning of the coupling control gains provides a means to mediate between the individual tracking of

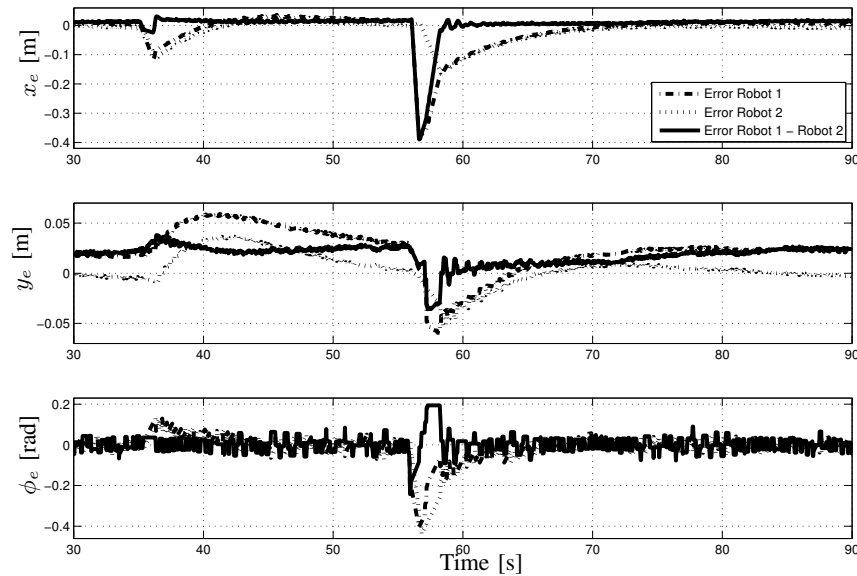


Fig. 6: Experimental evolution of the error coordinates of robots 1 and 2 for the time  $\in [30, 90]$  s.

the robots' desired trajectories and the goal of achieving formation.

#### IV. CONCLUSIONS

In this paper a virtual structure controller is designed for the formation control of unicycle mobile robots. We have proposed a controller, which introduces mutual coupling between the individual robots, thereby providing more robustness to the formation in the face of perturbations as compared to leader-follower (i.e. master-slave) type approaches. Moreover, sufficient conditions for the stability of the formation error dynamics are given for the case of two cooperating mobile robots. Experiments performed with an experimental setup for multi-robot systems demonstrate the practical applicability of the approach. Moreover, these experiments also show that the tuning of the mutual coupling parameters provides a means to weigh the importance of maintaining formation versus the importance of the individual robots tracking their individual desired trajectories.

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