Observational Effects

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1 Introduction

In this handout, we will learn the ways in which atmospheric and positional phenomena alter our observational measurements and how to correct for them.

2 Atmospheric Effects

While we neglect the effect of atmosphere while solving physics problems from our class, it turns out that this non-uniform, gaseous, finitely extending blanket that surrounds us can alter what we observe to a degree. The positions of stars are altered and their magnitude is dimmed by the manner in which light interacts with this medium.

2.1 Refraction

The atmosphere affects the ray of light as per the laws of refraction. Our aim in for this section is to correct for this effect. We will start with two assumptions that state:

- The atmosphere is composed of a finite set of layers.
- Each layer has uniform physical characteristics

Once we make this assumption, our problem is now reduced to calculating the refraction from n slabs, a classic exercise in optics.

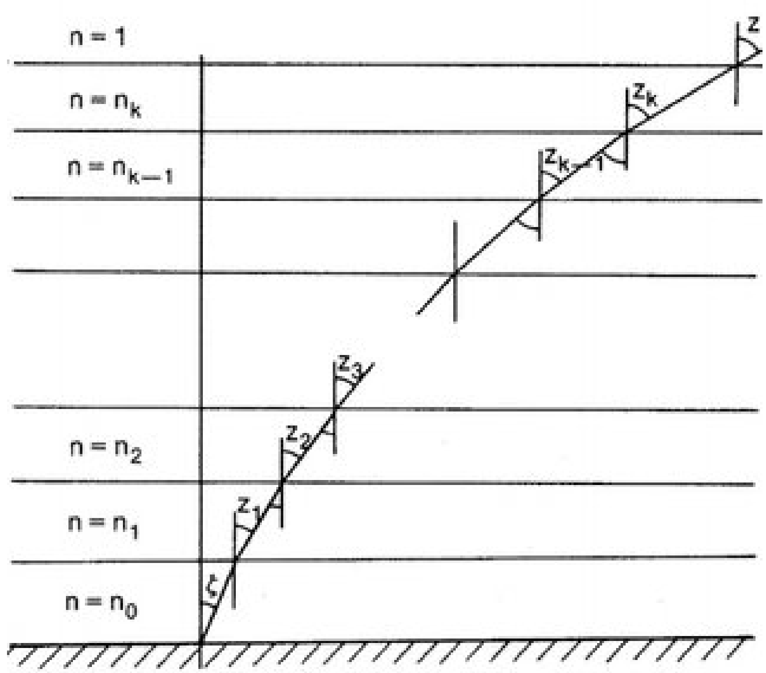

Figure 1: Refraction in the atmosphere (Source: Karttunen)

By snell's law,

$$\begin{aligned} 1 \cdot \sin z &= n_k \sin z_k \\ n_k \sin z_k &= n_{k-1} \sin z_{k-1} \\ &\vdots \\ n_1 \sin z_1 &= n_0 \sin \zeta \end{aligned}$$

We finally conclude that,

$$n_0 \sin \zeta = 1 \cdot \sin z$$

As observed, the zenith distance has reduced due to refraction, let this reduction be R

$$z = \zeta + R$$

Let's substitute this in for z and use the sum to product formula,

$$n_0 \sin \zeta = \sin(\zeta + R)$$

$$n_0 \sin \zeta = \sin \zeta \cos R + \cos \zeta \sin R$$

If we assume R to be small, then $\cos R \to 1$ and $\sin R \to R$, and we finally end up with

$$R = (n_0 - 1) \tan \zeta$$

The coefficent of $\tan \zeta$ is calculated to be around $58.2^{"}$ and is termed as the constant of mean refraction.

Unfortunately, this equation is only for small zenith distances (up to $30°$). To calculate for refraction at large angles or near the horizon, we must submit to using standard tables. A common value to remember is the value of refraction near the horizon, which is about $35'21^{"}$

2.2 Extinction

As light passes through the atmosphere, it gets scattered and absorbed by its constituents. This results in reduction of the flux that we observe from a source. This is found out to be exponential,

$$F = F_0 10^{-ax}$$

Let's calculate it for a star at zenith angle ζ. If the thickness of this layer is $\mathrm{d}h$ then the path traversed by light is,

$$\mathrm{d}x = \mathrm{d}h \sec \zeta$$

Now, we can integrate over the lengths,

$$F = F_0 10^{\sec \zeta \int_0^\infty -a(h) \mathrm{d}h}$$

Taking the log to the base of 10 on both sides,

$$\log F = \log F_0 - \sec \zeta \int_0^\infty -a(h) \mathrm{d}h$$

We can write this in terms of magnitudes,

$$m_{ob} = m_0 + \Delta m \sec\zeta$$

where Δm is a constant averaged over the atmosphere for a particular wavelength (not for extreme conditions, of course) and so we can make a substitution. An interesting point to note is that the increase is proportional to $\sec\zeta$.

The extinction that occurs due to the Interstellar Medium(ISM) is termed as **Interstellar Extinction**. For calculating the extinction due to the ISM in problems, $\Delta m \cdot a$ is a given constant (a is assumed to be uniform) so we can easily calculate the extinction given the distance to the source.

$$m_{obs} = m_0 + A \cdot d$$

2.3 Reddening

It turns out that scattering is more for shorter wavelengths, and so the value of Δm crudely increases for shorter and shorter wavelengths. What this means is that from a source, the shorter wavelengths have a higher affinity to scattering,[1] which implies that the source turns redder, because red has the longest wavelengths in the visible range and therefore scatters the least. Reddening is of two types: interstellar and atmospheric.

Interstellar Reddening: Because we want to quantify reddening in some way, we define,

$$E(B-V) = (B - B_0) - (V - V_0)$$

as the extent of reddening. We can also write the RHS as,

$$E(B-V) = (B-V) - (B-V)_0$$

We know that $(B-V)_0$ is the colour index of the source, which is a characteristic of the source. We define R, the ratio of total-to-selective absorption as

$$R = \frac{A_V}{E(B-V)}$$

A_V is the absorption in the visual. It turns out that for our galaxy, this ratio is a constant equal to 3.2:

$$A_V = 3.2 \cdot E(B-V)$$

Example 2.1: (USAAAO Round One, 2021)
An astronomer observes that a Solar type star has an apparent V magnitude of 6.73 when seen from the Earth. Assuming that the average interstellar extinction in V is 1.00 mag/kpc, determine the distance between this star and the Solar system. Given that the absolute V magnitude of the sun is 4.83

[1] The sky is blue!

Solution: We will use the distance modulus formula,

$$m - M = -5\log\left(\frac{d}{10}\right) + A \cdot d$$

Substituting for the magnitudes and the extinction (10^{-3} mag/pc)

$$6.73 - 4.83 = -5\log\left(\frac{d}{10}\right) + \frac{d}{10^3}$$

Then, we get following form for d

$$d = 10^{\frac{6.9 - \frac{d}{1000}}{5}}$$

which you can then solve by iterating on your calculator, to get $d = 23.7$ pc □

3 Positional Effects

3.1 Dip

The horizon for an observer is defined by the plane which is tangential to the earth at the observer's position. This plane changes as the height of the observer changes. As our horizon changes, the altitude of our observations also changes because they are measured from the horizon. θ termed as the angle of dip is calculated as follows.

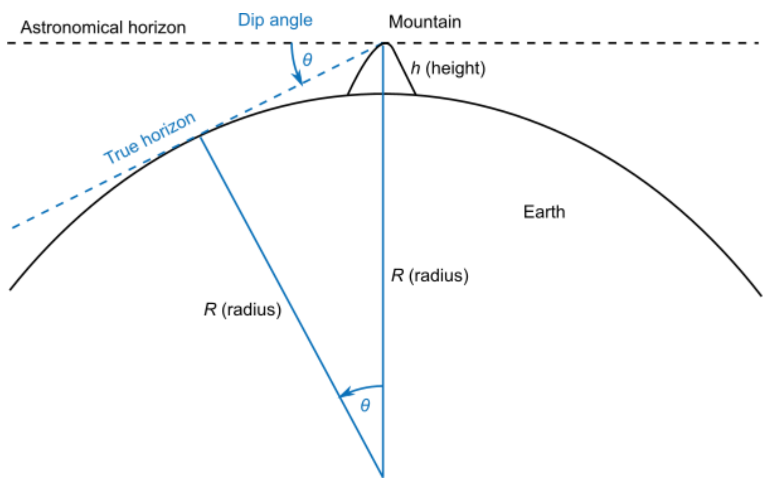

Figure 2: Angle of dip (Source: Karttunen)

From the diagram and some trigonometry, we can write

$$\cos\theta = \frac{R}{R+h}$$

because θ is small,

$$1 - \frac{\theta^2}{2} = \frac{R}{R+h}$$

$$\theta = \sqrt{\frac{2h}{R+h}}$$

$$\theta \approx \sqrt{\frac{2h}{R}}$$

$\sqrt{\frac{2}{R}}$ is a constant for which we can plug in its value to get the angle of dip as a function of height, in arc minutes

$$\theta = 1.93\sqrt{h}'$$

3.2 Parallax

You know that the position of the earth is not fixed. Forget about revolution around the milky way's centre, even in small enough timescales like the year, we're revolving around the sun. How does this affect your observations? If affects your observations because your position of making observations has changed and so every object is shifted by that vector.

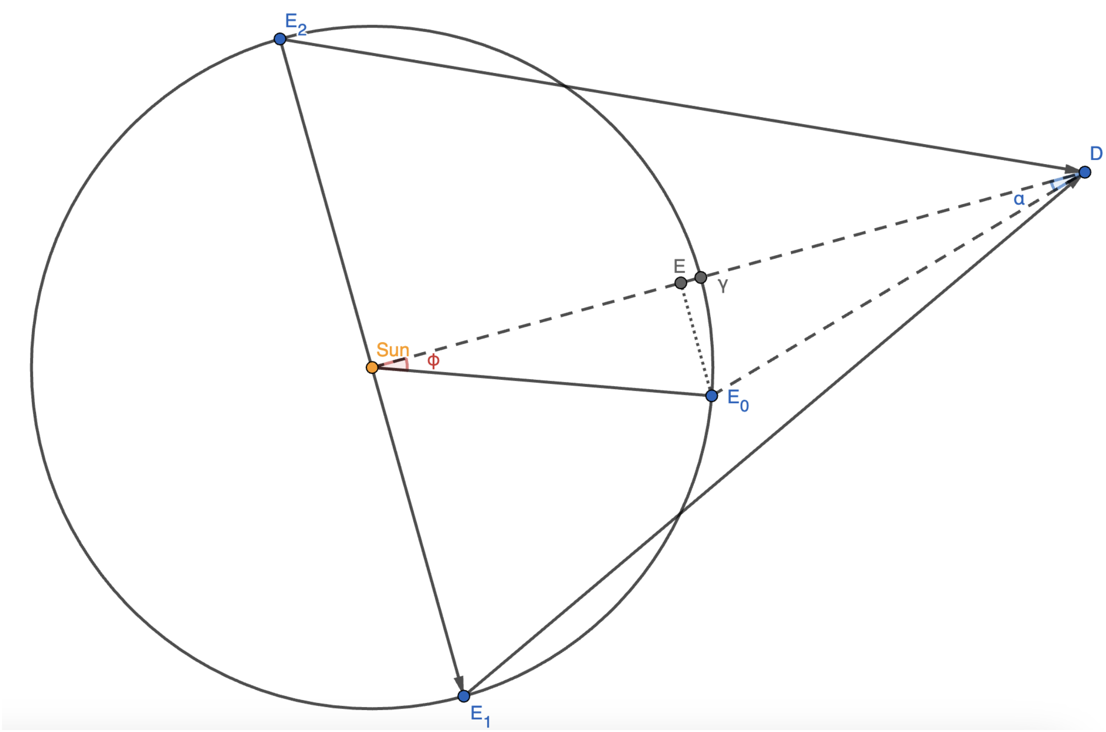

Figure 3: Parallax

Let's quantify this change in position by something measurable, which happens to be the change in angle. From the diagram, we can write the change in ecliptic longitude as,

$$\tan\alpha = \frac{R\sin\phi}{d - R\cos\phi}$$

If we assume the object to be far enough, we can simplify this as,

$$\alpha = \Delta\lambda = \frac{R\sin\phi}{d}$$

Or if you write d in parsecs,

$$\Delta\lambda = \left(\frac{\sin\phi}{d}\right)''$$

This is all fine and cool, but what if the object lies out of the ecliptic plane?(has a non-zero ecliptic latitude) How do you track the change in ecliptic longitude and latitude then? (Hint: use spherical trigonometry)

3.2.1 Geocentric Parallax

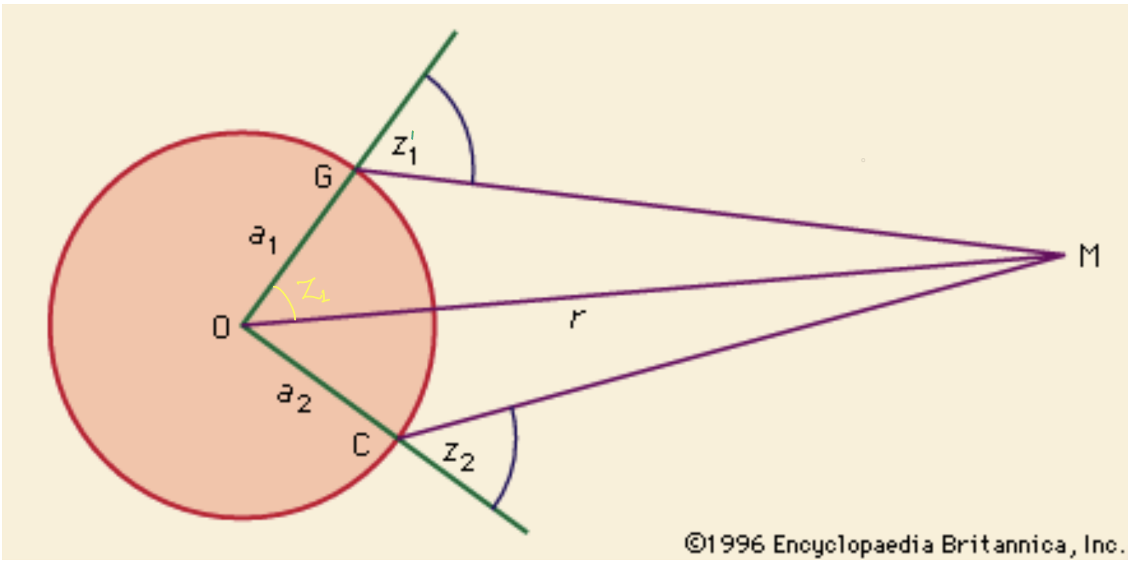

Figure 4: Geocentric Parallax

Besides not being flat, the earth is also not a point object, so the zenith distance at the celestial horizon differs from the observer's horizon.

Similar to how we derived heliocentric parallax,

$$\sin \Delta z = \frac{a_1}{r} \sin z_1$$

Assuming $z_1' - z_1$ to be small,

$$\Delta z = \frac{a_1}{r} \sin z_1$$

3.3 Aberration

Aberration is the change in position that occurs due to the motion of the observer through space. As the speed of light is finite, objects appear to be shifted from their position in the inertial frame of the observer. You might also recall the classic rain-main problem from your physics textbook. Aberration is crudely a rain-main problem in action.

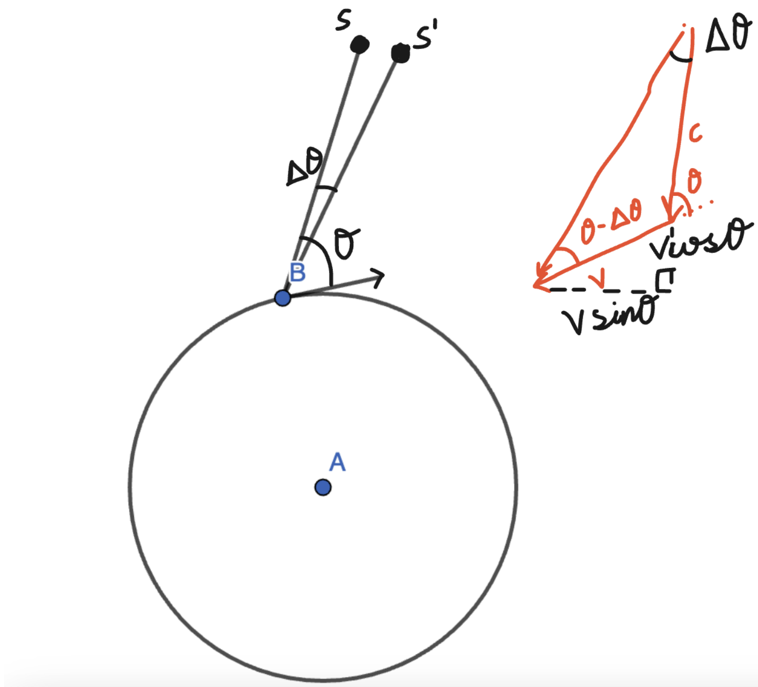

Figure 5: Aberration

Here's a classical derivation for you. From the image,

$$\tan(\Delta\theta) = \frac{v\sin\theta}{c + v\cos\theta}$$

Assuming $\Delta\theta$ to be small and $v << c$, we get,

$$\frac{v}{c}\sin\theta = \Delta\theta$$

Note that this doesn't depend on the distance between the observer and the object, just the observer's velocity.

An interesting observation is, for far enough objects,

$$\theta \approx 90 - \phi$$

where ϕ is the phase angle we saw in heliocentric parallax. This means that

$$\Delta\lambda = \frac{v}{c}\cos\phi$$

or that the effects from aberration are 90 degrees out of phase with the ones from parallax. However, for faraway objects aberration becomes larger as compared to parallax, mostly because

$$\frac{v}{c} \approx 20.49''$$

3.4 Precession

Precession of the vernal equinox occurs due to the revolution of the earth's rotation axis around the ecliptic poles. The north pole traces out a small circle around the north ecliptic pole at the ecliptic latitude of 23.5°N with a period of 26,000 years. As a result, the RA and declination also keep changing.

However, the path that precession traces out is not a smooth circle. It wobbles a little due to an additional effect known as **nutation**, which is caused by the Moon's orbit. Since the Moon's orbit is slightly angled compared to the ecliptic plane, the Moon will exert a torque on the Earth, resulting in nutation.

3.5 Proper Motion

As a star moves through space, its velocity can be split into two components: the *radial velocity*, which is motion directly towards or away from us, and the **proper motion**, which we observe as the star's change in position in the sky over many years. The star with largest known proper motion is **Barnard's star**.

4 Conclusion

The observational effects we discussed add nuance to the field of astronomy. While we usually ignore these effects when dealing with theoretical ideas, keep in mind that due to our observations occurring on Earth, which has an atmosphere and moves about, these effects present real issues that must be addressed in real applications.