Time

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1 Introduction

In this handout, we'll learn about the different ways in which units of time are measured, then we'll learn about the effects that affect our observation and how to correct for it.

2 Time

We studied that the hour angle of a star was the time since the last culmination of the star, or the time since the star was last seen on the local meridian.

Similarly, we define the **Local Sidereal Time** or LST to be the hour-angle of the vernal equinox. How do we figure that out? We need to know the hour angle corresponding to a particular RA and we're done. This can be known by just observing a star. How do we proceed from here? Because you know a particular RA (say, α) culminated at some t hours from now, it must've been $\alpha + t$ hours from now such that the vernal equinox was on the local meridian (Remember that as you go from west to east, you're moving along higher RAs). Hence, we can now write the equation for LST formally as,

$$\text{LST} = \alpha + t$$

Let's skim through some more definitions,
Greenwich Hour Angle: Hour angle of a star from the Greenwich local meridian
Greenwich Sidereal Time: Hour angle of the vernal equinox from the Greenwich local meridian. Where,

$$\text{GST} = \text{GHA} + \alpha$$

and

$$\text{GST} = \text{LST} \pm \left(\frac{\Delta\lambda^\circ}{15^\circ}\right)^h$$

2.1 The Day

A **solar day** is the time between two successful culminations of the mean sun[1].

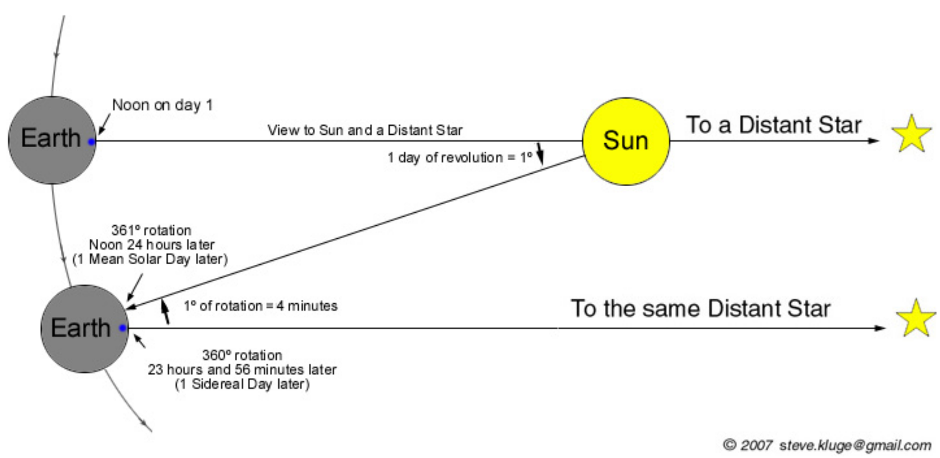

Figure 1: Diagram representing the difference between solar and sidereal day.

[1]I should write this formally, but mean sun is the hour angle assuming that the sun moves uniformly while the true sun is the time shown by a sundial.

A **sidereal day** is the time between two successful culminations of a distant star. It is equivalent to the period of rotation of the earth. The length of a sidereal day is about $23^h 56^m 4^s$ while the length of the solar day is about 24^h, because the earth orbits around the Sun and therefore must rotate about $1°$ extra to make the sun appear at the same position in the sky.

A consequence of this is that as the vernal equinox culminates on the local meridian at local noon, it moves westwards by around $3^m 56^s$ every day. Using this information we can figure out the LST at a particular date and time.

Example 2.1: (GeCAA)
A stargazer in Chiayi, Chinese Taipei (23.5°N, 120.4°E, GMT+8) saw two meteors streaking through the sky at 21:00 (Chinese Taipei time) on 25th September 2020. What is the Local Sidereal Time (LST) at the time of observation? The Greenwich Sidereal Time (GST) at 00:00 UT on 1st January 2020 is 6h 40m 30s. Sidereal day = 23.9344 hrs, Sidereal year = 365.2422 days.

Solution: Let us first calculate the Greenwich Sidereal Time on 25th September 2020 00:00 UT. As the vernal equinox moves $3^m 56^s$ westwards each day, this is simply,

$$\text{GST} = 6^h 40^m 30^s + 268 \cdot \frac{24}{365.24}$$

because 25th September and 1st January are separated by an interval of 268 days. Now, we will find the GST corresponding to 13:00 UT because from there we can simply account for the longitude difference and find the LST. So,

$$\text{GST} = 6^h 40^m 30^s + 268 \cdot \frac{24}{365.24} + 13 \cdot \frac{24}{23.93}$$

Calculating it gives us,

$$\text{GST} \approx 13^h 19^m 14^s$$

Finishing off,

$$\text{LST} = \text{GST} + \left(\frac{120.4°}{15°}\right)^h$$

$$\text{LST} \approx 21^h 20^m 50^s \quad \square$$

2.2 The Month

Lunar sidereal month: This is time it takes for the moon to revolve around the earth once with respect to the distant stars. The lunar sidereal month is about $27^d 7^h 43^m$ in length.

Lunar synodic month: This is also the time taken for the moon to complete a cycle of its phases. The lunar synodic month is about $29^d 12^h 44^m$ in length. Note that this is slightly longer than the sidereal month because the Earth orbits around the Sun.

Lunar draconic month: The lunar draconic month is the time between two successive passages of the moon through the ascending nodes. This is different from the sidereal month because the orbit of the moon precesses. The lunar draconic month is about $27^d 5^h 5^m$ in length.

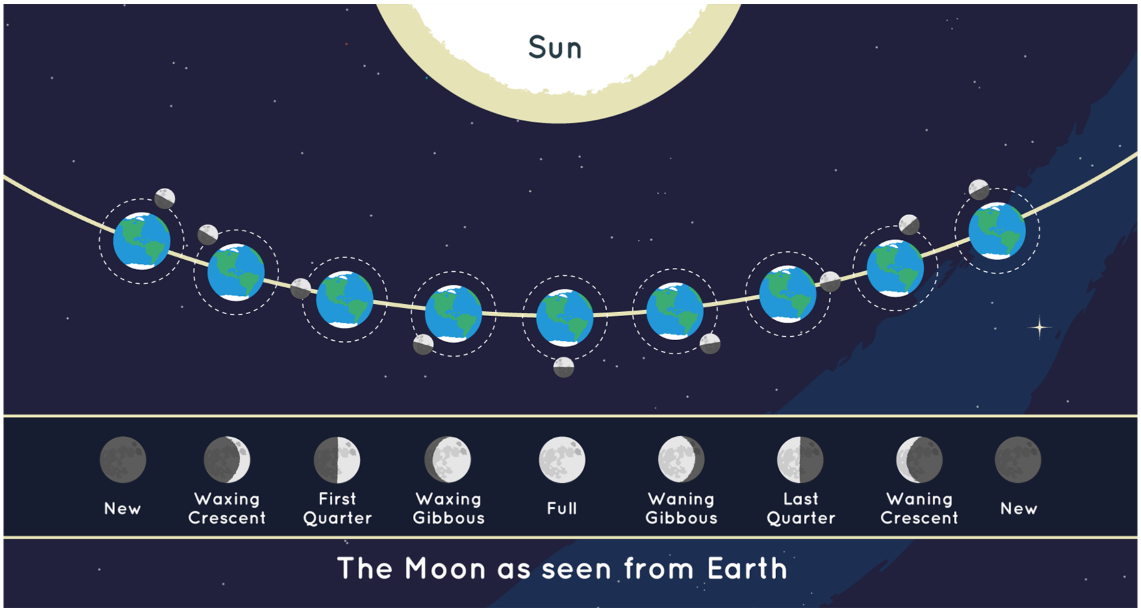

Figure 2: The Lunar phases. This also shows how the phases change as Earth orbits around the Sun, illustrating why the syndoic month is longer than the sidereal month. (Source: NASA)

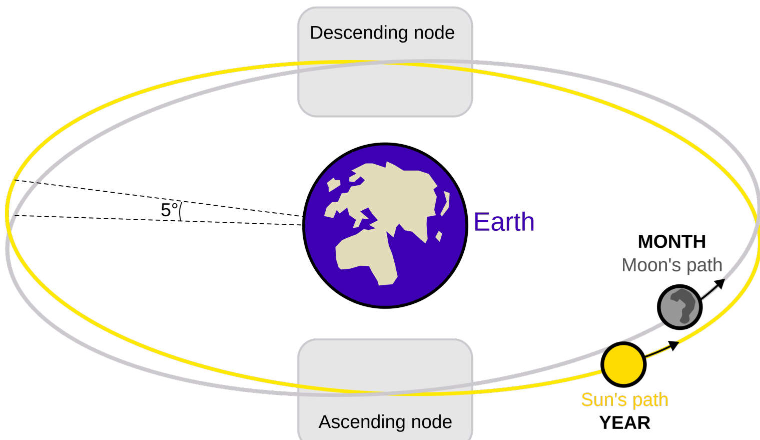

Figure 3: Lunar nodes. The plane of the Moon's path is slightly tilted with respect to the plane of the Sun's path. The points where these two paths intersect are called the lunar nodes. These nodes precess in the opposite direction of the Moon's orbit, causing the draconic month to be slightly shorter than the sidereal month. (Source: Wikipedia)

> **Example 2.2:** (Folklore)
> Calculate the time period of precession (also known as the nodal period) knowing the difference between the lunar synodic month and the lunar draconic month.

Solution: Because the angles are small,

$$\delta\theta = \omega\Delta t$$

where

$$\delta\theta = \frac{T_{side} - T_{drac}}{T_{drac}} \cdot 2\pi$$

and,

$$\omega = \frac{2\pi}{T_{nodal}} \; ; \; \Delta t = T_{side}$$

So,

$$T_{nodal} = \frac{T_{drac} \cdot T_{side}}{T_{side} - T_{drac}} \approx 18.6 \text{ years } \square$$

Some other relevant cycles are the **Metonic cycle** and the **Saros period**. Metonic cycle corresponds to the interval of time after which lunar phase occurs at the same time of the year again. The length of the Metonic cycle is about 235 lunar synodic months, or 19 years. A Saros is the interval after which the sun, the moon and the earth are in approximately the same geometry. It corresponds to about 223 lunar synodic months, or 18 years.

2.3 The Year

Tropical Year: Time between two successive passages of the Sun through the vernal equinox. It's about $365.242d$ in length

Sidereal Year: Time taken to revolve around the sun with respect to distant stars. It's about $365.256d$ in length. The tropical year is shorter than the sidereal year because of precession of the vernal equinox.

Anomalistic Year: Time between two successive passages of the earth through the perihelion or the aphelion. The length of the anomalistic year is about $365.257d$

2.4 Julian Dates

Julian Date 0 corresponds to the noon of 1/1/4713 BCE, which was a Monday. Day change in JD occurs at noon in UT.

The JD for 1 Jan 2000, 00:00 UT would be JD 2,451,544.50. The JD for 1 Jan 2023, 12:00 UT would be JD 2,457,056.00. To find the JD for a specific time you just need to subtract for the dates and account for the time difference in UT.

There's also the **Modified Julian Date**, which is analogous to how the Julian Date is calculated except that it now corresponds to the midnight of 17^{th} November, 1858. So,

$$\text{MJD} = \text{JD} - 2400000.5$$

2.5 Time zones

A time zone corresponds to the collection of land that keeps the same time. This time usually corresponds to a meridian within this area. Universal Time(UT) corresponds to the time at the prime meridian.

Obviously, the difference in longitude will give rise to a difference in the local solar time, so we might observe local noon at 12:00 from the place where time is calibrated but the easternmost and westernmost places in the time zone would've had their local noons before and after 12:00 respectively, or precisely, at

$$t = 12^h \pm \left(\frac{\Delta\lambda^\circ}{15^\circ}\right)^h$$

As we know that a difference of 15° in longitude corresponds to difference of 1 hour in time.

3 Equation of Time

The equation of time is given by,

$$E.T = T_{true} - T_{mean}$$

or to put it in words, the difference it's the difference between the true sun and the mean sun as a function of time. Why do they vary you might ask?

Two reasons: obliquity of earth's axis and eccentricity of the earth's orbit.

Obliquity of earth's axis: We know that the celestial equator is at a tilt of $\epsilon = 23.5^\circ$ with the ecliptic, hence this would mean that the daily motion of the sun is not exactly along the small-RA circle. So the 1°/day change that we assumed while deriving the solar day is not exactly 1° along the small-RA circle, it's 1° along the small-longitude circle, and it varies throughout the year, the fastest being near the equinoxes and slowest near the solstices.

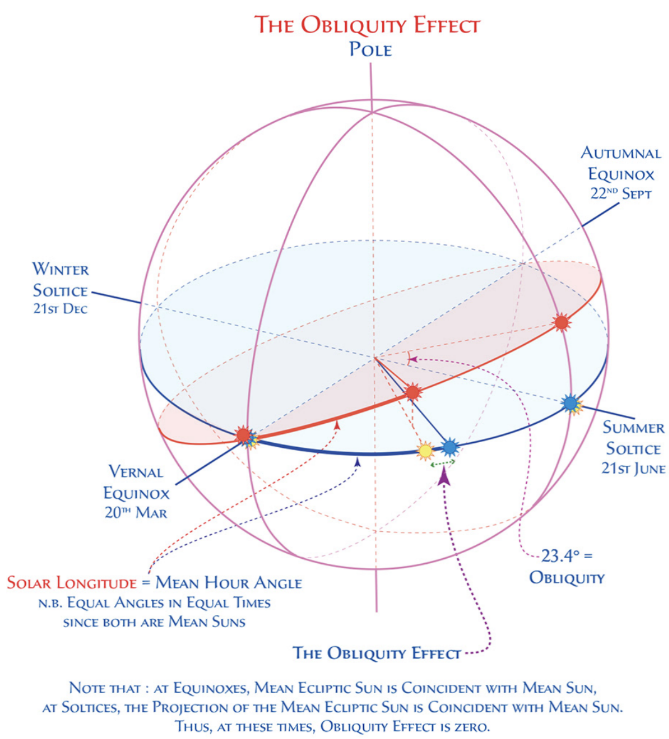

Figure 4: Obliquity Effect (Source: The Equation of Time.info)

Eccentricity of earth's orbit: This is more intuitive than the previous component. We know that planets must be at their fastest near the perihelion and slowest near the aphelion, which means the projection of the sun on the celestial sphere moves fastest near the perihelion and slowest near the aphelion, hence this also gives rise to an offset from the mean sun.

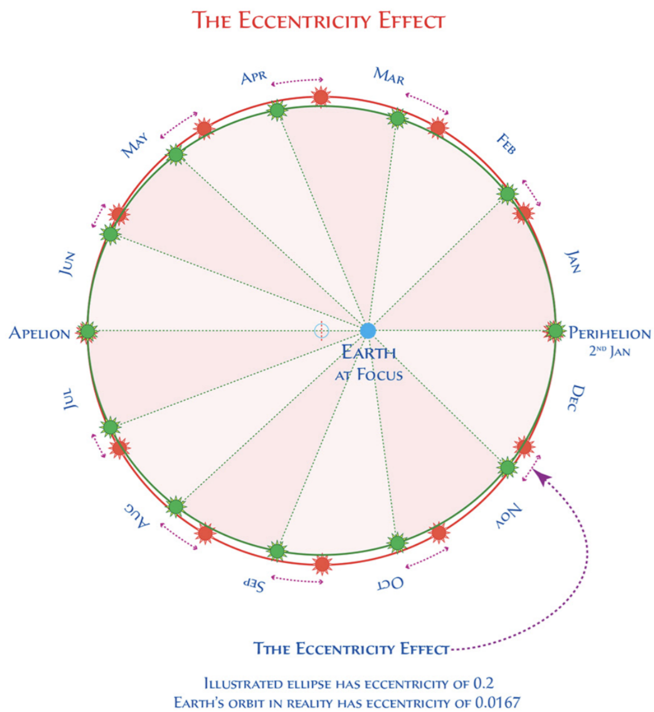

Figure 5: Eccentricity Effect (Source: The Equation of Time.info)

The combined effect from both of these effects gives us the offset of the true sun from the mean sun and effectively the equation of time, which is plotted as shown:

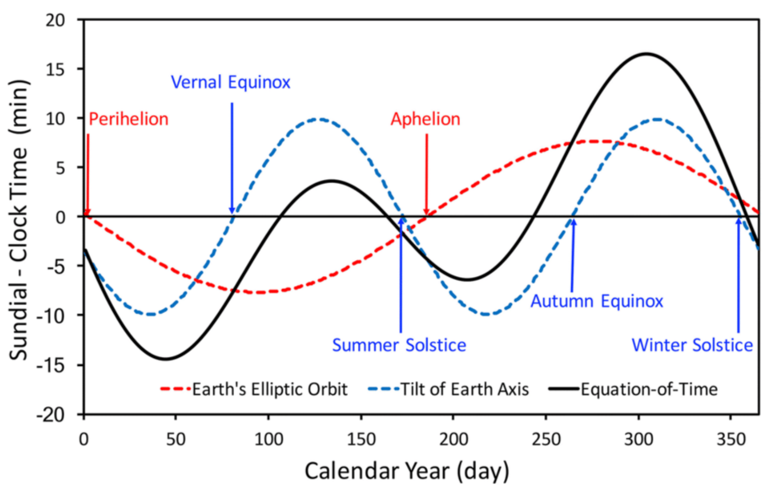

Figure 6: Equation of time (Source: The Equation of Time.info)

As observed from the graph, the difference between the True Sun and the Mean Sun ranges from the true sun being ahead by +16 minutes on 3rd November to lagging behind by about 14 minutes on 11th February. Zeroes are achieved on 15th April, 13th June, 1st September and 25th December.

3.1 Analemma

An **analemma** is the shape created by plotting the position of the sun in the sky at the same civil time throughout the year.

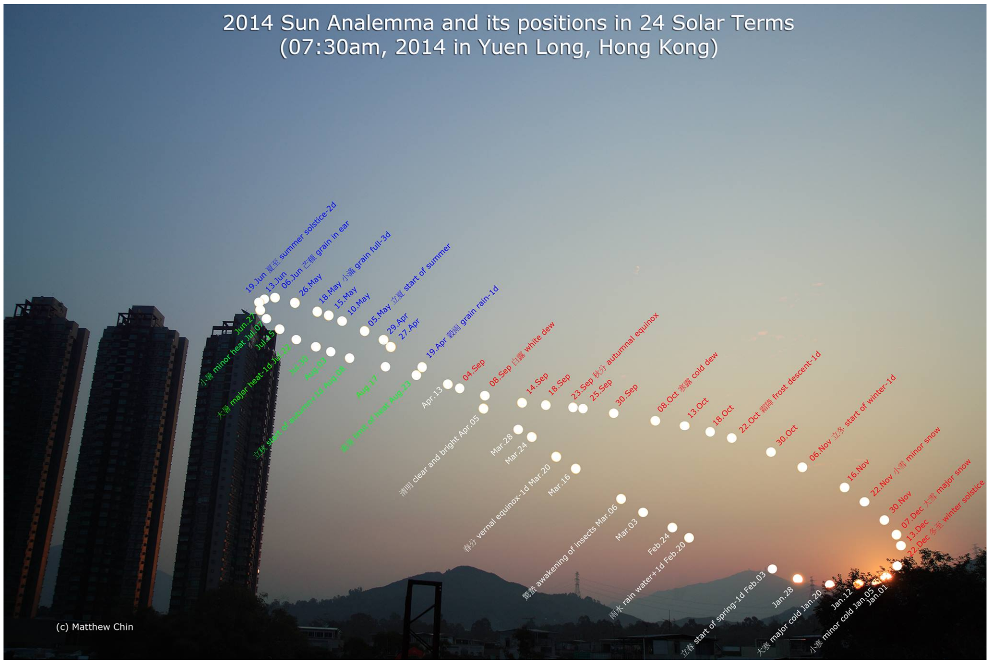

Figure 7: Analemma (Source: EarthSky)

The northernmost and southernmost ends of the Analemma are the points of solstices. Deviation from the arc joining these two points is due to the difference between true sun and mean sun.

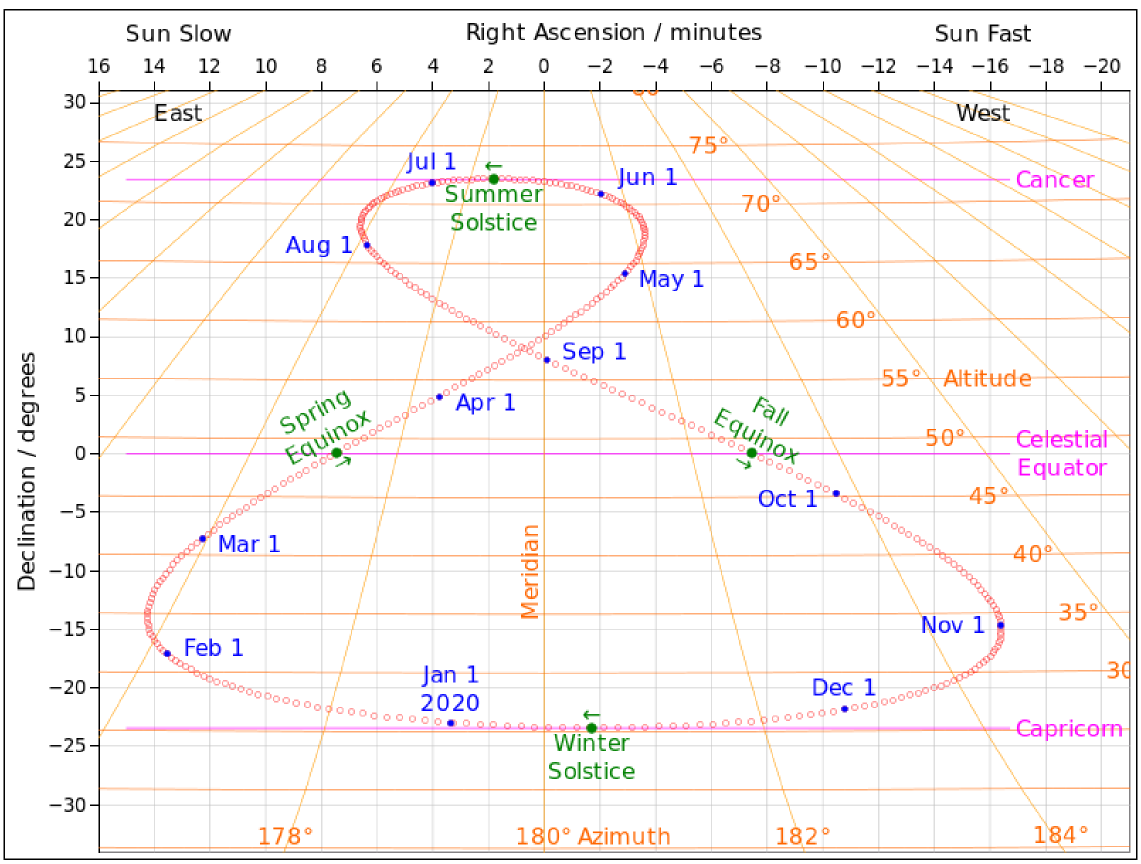

Figure 8: Equation of time and Analemma (Source: av8n.com)

Example 3.1: (USAAAO Round One 2021)
What would happen to the analemma of the Sun if the obliquity of the Earth's orbit suddenly went to zero degrees and its eccentricity remained unchanged?

(a) The analema would be perfectly symmetric in both axes and would have the shape of an "8".
(b) The analemma would look like a dot.
(c) The analemma would be the arc of a great circle.
(d) The analemma would look like a circle.
(e) The analemma would be a spherical triangle

Solution: Based on our discussion in the previous section, you know that the ends of the analemma would coalesce into a point and the deviations from that point would arise from the eccentricity effects. Hence, the correct answer would be option **(c)**.

Option (a) would be true for the case when the orbit is perfectly circular, in that case the only contribution to the equation of time would be the obliquity effect.

Option (b) would be true for the case when there's no obliquity or eccentricity or the true sun is the same as the mean sun.

4 Conclusion

We learned the various ways in which time was measured, the deviations that arise between these ways, and how to correct for them.