Interstellar Medium

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1 The Interstellar medium

The interstellar medium (**ISM**) is is located in the region between stars and comprised of dust, gas and cosmic rays. Its existence was discovered in 1930 by Robert Trumpler when he saw a discrepancy in his research. For finding the distances to the stars, he used the distance modulus:

$$m - M = 5\log(\frac{r}{10pc})$$

Furthermore, using the equation

$$D = dr$$

where d is the apparent angular diameter of the star. He found that the diameter of the stars far from us are relatively bigger than closer stars. So, he concluded that there is a medium between the stars and us that is obscuring the background stars and scattering some of their light. This in turn leads to the magnitude of a star to be more positive (and therefore dimmer) than the real magnitude of a star (in an ideal case where there is no medium). After this he found the equation:

$$m - M = 5\log(\frac{r}{10pc}) + A \tag{1}$$

where A is called **extinction** and is a positive value because the medium absorbs and scatters the light coming from a star, which shows that the light intensity coming from a star is decreasing. The interstellar medium is crucial to astronomy because it helps us understand how the medium behaves and affects the flux density of a star. Learning the behaviour of the medium, we can find A and find the real magnitudes of stars, the distance to them and so on.

1.1 Physics behind extinction

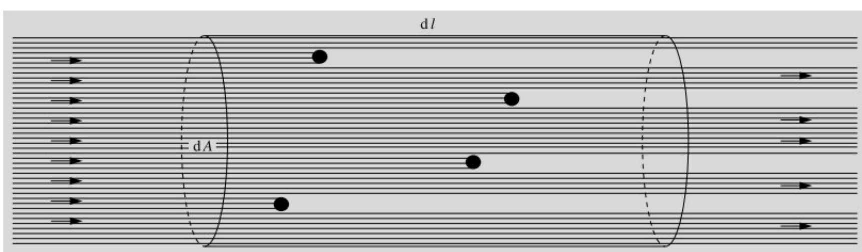

Figure 1: Extinction by distribution of particles (Karttunen)

As shown in Figure 1, particles in a medium decrease the intensity of light by absorbing a fraction of it. We will assume that no particle is overshadowing another and that the particles are small spheres with geometrical cross section πa^2 where a is the radius of the particle. However, remember that because of the effect of diffraction, we have to add a Q_{ext} factor into our equation:

$$s = Q_{ext}\pi a^2$$

After this, we say that:

$$\mathrm{d}L = N\mathrm{d}s\frac{L}{\mathrm{d}A}$$

where dL is the intensity absorbed by the medium. Moreover, as we know the concentration of the particles n, we can multiply it with the volume and equate $N = n\mathrm{d}A\mathrm{d}l$:

$$\mathrm{d}L = nLs\mathrm{d}l$$

Equating $\frac{\mathrm{d}L}{L}$ with $\mathrm{d}\tau$, called optical depth which is the fraction of intensity absorbed, we find:

$$\mathrm{d}\tau = ns\mathrm{d}l \tag{2}$$

After integrating, we find :

$$\tau = \bar{n}sr$$

where $\bar{n}$ is the average concentration and r is the distance between the star and us. $\bar{n}s$ is called the *opacity* of the medium. Note that after finding how τ depends on r, we can now find how extinction A depends on distance r:

$$\int_{L_0}^{L} \frac{\mathrm{d}L}{L} = -\int_{0}^{\tau} \mathrm{d}\tau$$

Integrating,

$$L = \frac{L_0}{e^{\tau}}.$$

Relating flux with luminosity,

$$F_m = \frac{L_0}{4\pi r^2 e^{\tau}}$$

Substituting r with $10pc$ and removing the effect of extinction,

$$F_M = \frac{L_0}{4\pi(10pc)^2}$$

We find the flux corresponding to absolute magnitude. We eliminated e^{τ} because we need to find the flux of the star in the distance of $10pc$ if there was no medium between the star and the point of observation. Which gives us the flux corresponding to absolute magnitude and helps us find extinction A:

$$m - M = -2.5lg\frac{F_m}{F_M} = 5lg\frac{r}{10pc} + 2.5\log(e^{\tau}).$$

Looking at our equation Eq. 1, we can see that $A = 2.5\tau\log(e)$. When opacity, the term ns is constant in equation Eq. 2, as τ will be proportional to r, we can substitute A with ar where a will be equal to $2.5ns\log(e)$:

$$m - M = 5\log(\frac{r}{10pc}) + ar$$

By knowing the constant a, we can find the distance to the star from this equation. Usually, $2mag/kpc$ is used as the value of a which means that magnitude of the star increases by 2 per thousand pc. But this value depends on the direction greatly. For example, in our galactic plane we can observe extinction of 30 magnitudes per 8-9 kpc. The reason for this is that more dust is concentrated in the galactic plane than in the galactic halo.

2 Extinction for different wavelengths

2.1 Magnitude Systems

Besides the distance, apparent magnitude also depends on the instrument we are using. Some instruments are sensitive to a specific range of wavelengths. That is why the flux detected by the instrument is only a fraction of the total flux. This is why different magnitude systems are created where F_0 in the equation $-2.5\log(\frac{F}{F_0})$ is different for different systems. These values of F_0 are defined by a few selected reference stars.

One magnitude system is called the **visual magnitude** - m_v, and is the magnitude corresponding to the sensitivity of the eye. In daylight, our eyes are most sensitive to green light, and we detect light in the region 350 - 700 nm wavelength. So, the magnitude corresponding to this region is called visual magnitude.

Another magnitude system is multicolour magnitude system named **UBV** which involves 3 filters: Ultraviolet, Blue, Visual. This system is used in photoelectric photometry, which is one of the most accurate ways to measure magnitudes (the other best way is CCDs). It is evident from their names that U, B, V magnitudes have wavelength bands centered in wavelengths corresponding to ultraviolet, blue and green light. More information about these bandgaps are shown in Figure 2 and 3. Later, UBV system was expanded by adding R-red and I-infrared magnitudes.

There are other multicolour systems too, but this system is used more than the others. In every multicolour system, we can define colour indices (colour index). In UBV system when we subtract V magnitude from B magnitude we get **B-V color index** and the same idea applies to others. If we know U-B and B-V color indexes we can find U and B when we know V.

The F_0 value in U, B, V magnitudes is such that for spectral A0 type stars, B-V and U-B are zero. For example, A0V star **Vega** (α Lyr) has $V = 0.04$ and $B - V = U - B = 0.00$. But for other stars, for example Sun, $V = -26.8$ and $B - V = 0.64$ and $U - B = 0.12$

The magnitudes U, B, V that we have talked about are the apparent magnitudes of the stars. When expressing corresponding absolute magnitudes, we will use M_V. M_B, M_U, respectively.

There is also another magnitude called **bolometric magnitude** m_{bol}. Bolometric magnitude is the magnitude corresponding to all the flux coming from the star. To measure the m_{bol} of a star, we need to measure the flux in all wavelengths. In practice, this is very difficult as some part of the light is absorbed by the atmosphere and different instruments measure different wavelength regions. We can derive the bolometric magnitude if we know bolometric correction:

$$m_{bol} = m_V - BC.$$

By definition, bolometric correction is zero for F5 spectral type stars. Note that, when $m_{bol} = m_V$, the flux density corresponding to them will not be equal. Bolometric flux density will always be bigger. The reason for this is the difference in the F_0 value of those systems. In F5 type stars, for example, even though bolometric and visual magnitudes are equal, the flux density corresponding to bolometric magnitude is bigger. Also, note that BC should always be greater than or equal to zero (as it should be because it captures more region in the electromagnetic spectrum).

So, you may think that due to these last two sentences, the discrepancy in the F_0 value can lead to BC being lower than zero. But this will not happen because ratio of flux densities of visual and bolometric magnitude is the closest to one for F5 type stars as their spectrum is peaked at the center of visual bandgaps. In the spectrum of these stars, flux density in the regions out of the visual band is the smallest. That is why, considering this small flux density to be equal to 0, we will not encounter any case where spectrum of another star will have less flux density outside of the visual band, which would cause BC to be smaller than zero as we would say that the flux density outside of the visual band is smaller than zero.

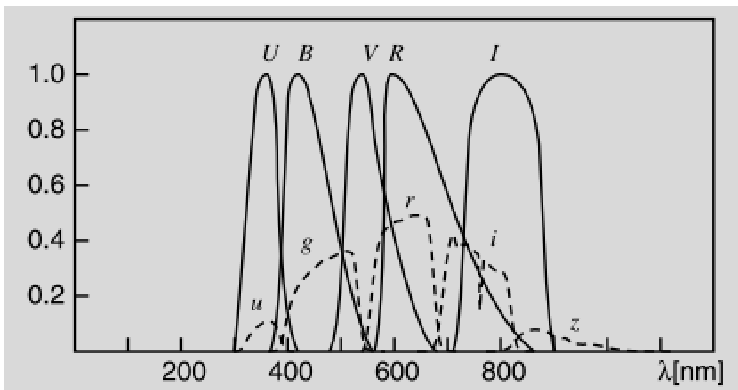

Figure 2: Relative transmission profiles of filters used in the UBVRI magnitude system. The maxima of the bands are normalised to unity. The R and I bands are based on the system of Johnson, Cousins and Glass, which also includes infrared bands J, H, K, L and M. Previously used R and I bands differ considerably from these. The curves of the ugriz magnitudes (dashed lines) give quantum efficiencies. (Source: Karttunen)

Table 4.1 Wavelength bands of the UBVRI and uvby filters and their effective (≈ average) wavelengths

	Magnitude	Band width [nm]	Effective wavelength [nm]
U	ultraviolet	66	367
B	blue	94	436
V	visual	88	545
R	red	138	638
I	infrared	149	797
u	ultraviolet	30	349
v	violet	19	411
b	blue	18	467
y	yellow	23	547

Figure 3: Wavelength bands of the UBVRI and uvby filters and their effective (average) wavelengths (Source: Karttunen)

2.2 Color Excess and reddening of light

Now that we have covered how U,B,V magnitudes are measured, we can talk how extinction by ISM affects these values. Extinctions for different magnitudes are different as lights of different wavelength are scattered differently by the medium. For example, blue light is scattered more than red light and this makes the sky blue. Going from blue to red, scattering is inversely proportional to wavelength. Extinctions for each of the magnitude A_U,A_B,A_V are different and A_U is the biggest. The dependence of extinction on wavelength in detail will be covered in next subsection. So, using the equation Eq. 1, we can write :

$$V - M_v = 5\log(\frac{r}{10pc}) + A_V \tag{3}$$

We can write same equation for B magnitude:

$$B - M_B = 5\log(\frac{r}{10pc}) + A_B \tag{4}$$

Subtracting Eq. 3 from Eq. 4 :

$$B - V = (M_B - M_V) + (A_B - A_V).$$

Here, $(M_B - M_V)$ is usually written as $(B_0 - V_0)$ which is called the **intrinsic color** of the star. $(A_B - A_V)$ in turn, is called **colour excess** and is written as E_{B-V}. Studies show that $\frac{A_v}{E_{B-V}}$ is approximately equal to 3. This is a widely used equation in competitive astronomy because knowing the value of E_{B-V} we can find colour excess A_V and find the distance to the star from equation Eq. 3 if we know the value of V and M_V.

2.3 Dependence of the extinction on wavelength

Dependence of the extinction on wavelength can be found by studying the magnitudes with different filter (U,B,V). The measurements have shown that extinction approaches zero as wavelength becomes too large. $A(\lambda)$ is shown in the figure 4. This figure is normalised to make $E_{B-V} = 1$ and here we can see also find value of $\frac{A_V}{E_{B-V}}$ which is approximately equal to 3. From the second graph we can see that scattering is biggest in short wavelengths.

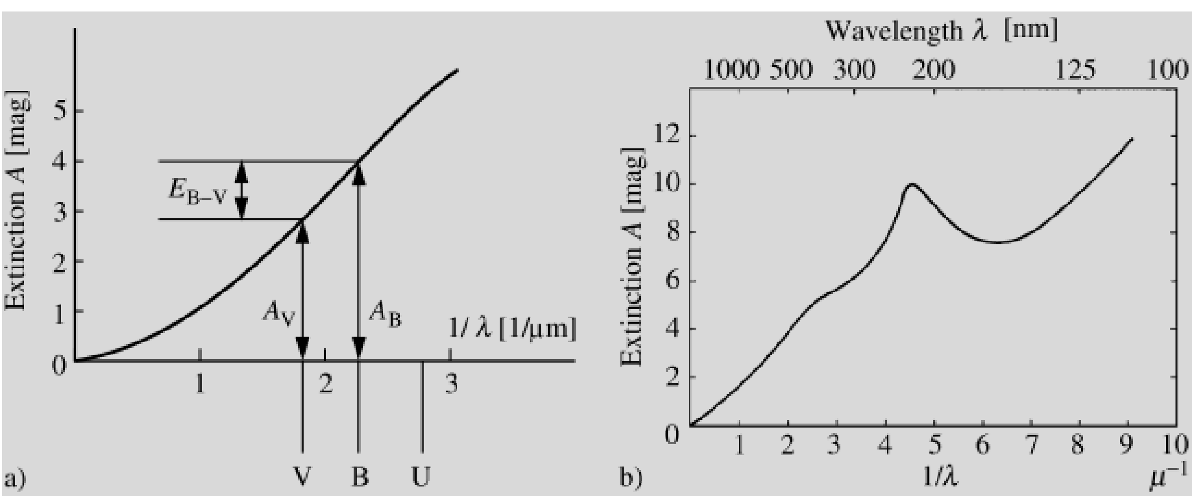

Figure 4: Schematic representation of the interstellar extinction. As the wavelength increases, the extinction approaches zero. (b) Measured extinction curve, normalised to make $E_{B-V} = 1$. (Source: Karttunen)

3 Atmospheric Extinction

One of the causes of extinction is the Earth's atmosphere. Here, zenith distance z of the star and the location of the observer affects the value of the observed magnitude as this determines the distance light has to travel. Now to find the magnitude of the star we should remove the atmospheric effects, which seem difficult at first since changing the zenith distance can lower the distance traveled by light no further than H, which is the height of the atmosphere on Earth. By doing some linearization, we can find this value. As shown in Figure 5, when zenith distance is equal to z, the distance light travels is equal to $\frac{H}{cosz}$. Assuming that ns in the equation is constant, we can say that:

$$m_0 = m + \frac{k}{cosz},$$

where $k = H\alpha$. α is just another constant showing that extinction is proportional to distance traveled. Now, plotting the graph of m_0 versus $\frac{1}{cosz}$, we can find the intercept and find the value of m which is the magnitude unaffected by the extinction of atmosphere.

Note that, in practice, zenith distances higher than 70° should not be used in the plot since at low altitudes, the atmosphere starts to complicate matters. Also, the value of k depends on the observation site and wavelength as shorter wavelengths scatter light more.

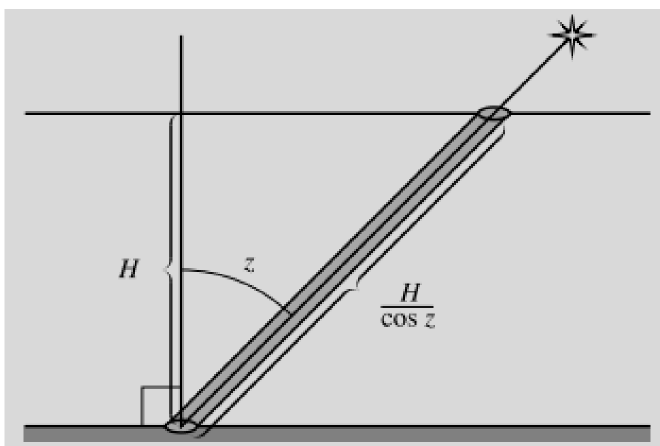

Figure 5: Diagram showing how the distance light travels through the atmosphere varies with angle z. (Source: Karttunen)

4 Problems and Solutions

Problem 1: (Azerbaijan IOAA team selection test question) The radius of the star is 0.897 R_{sun}. During the transition of the orbiting exoplanet, it is detected that the luminosity of the star decreases by 0.47 percent. Find the radius of the exoplanet in terms of solar mass R_{sun}. Assume that luminosity is distributed uniformly across the disc of the star.
Hint: you can use the idea in section 1.1, or read the Exoplanets handout for similar examples.

Solution: When the exoplanet transits the star, it eliminates a part of the intensity flux coming from the star. As this part is equal to $I_0\frac{A_1}{A_2}$ where A_1 is the area of the exoplanet and A_2 is the area of the star, we can write:

$$\frac{I}{I_0} = \frac{\pi\sigma T^4(R_*^2 - R^2)}{\pi\sigma T^4 R_*^2} = \frac{R_*^2 - R^2}{R_*^2} = 0.9953.$$

Here, I is the intensity when the exoplanet is in transit and I_0 is the previous intensity. R_* and R are the star's and exoplanet's radius respectively. Finding the relation between R and R_*:

$$R^2 = 0.0047R_*^2$$

$$R = 0.062R_{sun} \quad \square$$

Problem 2: (Azerbaijan IOAA team selection test question) The star's V magnitude is 15.1^m and absolute magnitude is equal to 1.3^m. Extinction of the interstellar medium per kpc is equal to 1^m ($a_v = \frac{1^m}{kpc}$)

(a) Find the approximate distance from the star to the Earth.

(b) If $(B - V) = 1.6^m$, find the intrinsic colour of the star $(B_0 - V_0)$.

(c) If $BC = 0.6$, find the luminosity of the star. Sun's bolometric absolute magnitude is equal to 4.75^m and Sun has the luminosity of $3.846 * 10^{26} Vt$.

Solution: a)

$$V = V_0 + 5\log(\frac{r}{10 \times 10^{-3}}) + a_v r$$

$$V - V_0 = 5\log(100r) + r$$

$$V - V_0 = 5(\log r + 2) + r$$

$$V - V_0 - 10 = \log r + r$$

$$15.1 - 1.3 - 10 = \log r + r$$

$$r \approx 2.15 \quad \square$$

b)

$$B = B_0 + 5\log(\frac{r}{1010^{-3}}) + A_v$$

$$B - v = B_0 - V_0 + A - B - A_V$$

$$R = \frac{A_v}{A_B - A_v}$$

$$B_0 - V_0 = (B - V)_0 = (B - V) - (A\_A_v) = 1.6 - \frac{A_v}{R} \approx 0.88 \quad \square$$

c)

$$M_{Bol} = V_0 + BC$$

$$M_{Bol} - M_{Bol_\odot} = -2.5\log\frac{L}{L_\odot}$$

$$V_0 + BC - M_{Bol_\odot} = -2.5 \log \frac{L}{L_\odot}$$

$$\frac{L}{L_\odot} = 10^{\frac{V_0 + BC - M_{Bol_\odot}}{-2.5}}$$

$$L = L_\odot 10^{\frac{V_0 + BC - M_{Bol_\odot}}{-2.5}} = 41.68 L_\odot \ \square$$

Problem 3: (IOAA 2008 question) A UBV photometric (UBV Johnson's) observation of a star gives U = 8.15, B = 8.50, and V = 8.14. Based on the spectral class, one gets the intrinsic colour $U_0 - B_0 = -0.45$. If the star is known to have radius of $2.3 R_{sun}$, absolute bolometric magnitude of -0.25, and bolometric correction (BC) of -0.15 ,determine:

(a) the intrinsic magnitudes U, B, and V of the star,

(b) the effective temperature of the star,

(c) the distance to the star.

Note: Take, for typical interstellar matter, the ratio of total to selective extinction $R_V = 3.2$ and the colour excess in (B - V) to be about 72 percent of the colour excess in (U - B)

Solution: $U = 8.15, B = 8.50$, and $V = 8.14$. $(U - B)_0 = -0.45$. $R = 2.3, R_{sun} = 1.60 * 10^{11}$cm, $M_{bol} = -0.25, BC = -0.15$

a) U - B = 8.15 - 8.50 = - 0.35

The color excess for U-B: $E(U - B) = (U - B) - (U - B)_o = -0.35 - (-0.45) = 0.10$

The relation between color excess of U - B and of B - V:

$$E(U - B) = 0.72 E(B - V)$$

$$E(B - V) = \frac{0.10}{0.72} = 0.14$$

Let Av be the interstellar extinction and $R = 3.2$, then

$$Av = 3.2 E(B - V) = 3.2(0.14) = 0.45$$

$$V - V_0 = A_v$$

$$V_o = V - A_v = 8.14 - 0.45 = 7.69$$

$$E(B - V) = (B - V) - (B - V)_0$$

$$(B - V)_0 = (B - V) - E(B - V) = (8.50 - 8.14) - 0.14 = 0.22$$

$$(B - V)_0 = B_0 - V_0 = 0.22$$

$$B_0 = 0.22 + V_0 = 0.22 + 7.69 = 7.91$$

$$(U - B)_0 = U_0 - B_0 = -0.45$$

$$U_0 = B_0 - 0.45 = 7.91 - 0.45 = 7.46 \ \square$$

b) Luminosity - absolute magnitude relation:

$$M_{bol} - M_{bolsun} = -2.5 log L/L_{sun}$$

$$L/L_{sun} = 10^{-(M_{bol}-M_{bolsun})/2.5 = 10^-(0.25-4.75)/2.5 = 100}$$

$$L = 100$$

$$L_{sun} = 3.90 * 10^{35} erg/det$$

$$L = 4\pi\sigma R^2 T_{ef}^4$$

$$T_{ef} = (\frac{L}{4\pi\sigma R^2})^{\frac{1}{4}} = 12092K \quad \square$$

c)

$$M_v - M_{bol} = BC$$

$$M_v = M_{bol} + BC = -0.25 - 0.15 = -0.40$$

$$m_v - M_v = -5 + 5 log d + A_v$$

$$5 log d = m_v - M_v + 5 - A_v = 8.14 + 0.40 + 5 - 0.45 = 13.09$$

$$d = 10^{13.09/5} = 10^{2.62} = 414.9 pc \quad \square$$

Problem 4: (IOAA 2012 question) An old planetary nebula, with a white dwarf (WD) in its center, is located 50 pc away from Earth. Exactly in the same direction, but behind the nebula, lies another WD, identical to the first, but located at 150 pc from the Earth.

Consider that the two WDs have absolute bolometric magnitude +14.2 and intrinsic color indexes B - V = 0.300 and U - V = 0.330. Extinction occurs in the interstellar medium and in the planetary nebula. When we measure the color indices for the closer WD (the one who lies at the center of the nebula), we find the values B - V = 0.327 and U - B = 0.038. In this part of the Galaxy, the interstellar extinction rates are 1.50, 1.23 and 1.00 magnitudes per kiloparsec for the filters U, B and V, respectively.

Calculate the color indices as they would be measured for the second star.

Solution: The intrinsic color indexes are U- V = 0.300 and U- V= 0.330. Therefore, $U - B = (U - V) - (B - V) = 0.030$.

The difference between the real and the measured color indexes are due to the absorption in the interstellar medium, as well as the stronger absorption of the planetary nebula (in fact, during the calculations, one can easily see that the interstellar absorption is not enough to account for the measured values). So, for the first (nearest) star:

$$(B - V)_1 = (B - V) + (E_B - E_v) * D + (E'_B - E'_v) * R = 0.327$$

$$(U - V)_1 = (U - V) + (E_U - E_v) * D + (U'_B - E'_v) * R = 0.328$$

$$(U - B)_1 = (U - B) + (E_U - E_B) * D + (E'_U - E'_B) * R = 0.365$$

where 2nd term represents interstellar absorption and 3rd term represents nebular absorption.

Calculating the interstellar absorption, we can find the nebula values:

$$(E'_B - E'_v)R = 0.0155$$

$$(E'_u - E'_v)R = 0.0100$$

$$(E'_u - E'_B)R = -0.0055$$

So, for the second (farthest) star,

$$(B - V)_2 = (B - V) + (E_B - E_V) * 3D + (E'_B - E'_V) * 2R = 0.3655$$

$$(U - V)_2 = (U - V) + (E_U - E_V) * 3D + (E'_U - E'_V) * 2R = 0.425$$

$$(U - B)_2 = (U - B) + (E_U - E_B) * 3D + (E'_U - E'_B) * 2R = 0.0595$$

5 Conclusion

As we have seen in this handout, the cosmos can play tricks on our eyes. Due to the ISM, extinction and scattering can cause stars to appear dimmer and redder than they really are. However, we can correct for this using the equations we learned!