Basic Physics Mechanics

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Table of Contents

1 Introduction **2**

2 Kinematics **2**

2.1 Important Quantities 2

2.2 The Kinematics Equations 3

3 Dynamics and Energy **3**

3.1 Newton's Laws 3

3.2 Energy and Work 3

3.3 Circular Motion 4

4 Momentum and Collisions **5**

4.1 Linear Momentum 5

4.2 Impulse 5

4.3 Center of Mass and Conservation of Momentum 6

5 Conclusion **6**

6 Practice Problems **6**

1 Introduction

In order to do well in astronomy and astrophysics olympiads, one must have a solid understanding of physics. In this handout, we will focus on basic mechanics, though you will likely want to study other areas of physics (such as electromagnetism and thermodynamics) in the future if you want to perform at the highest level.

2 Kinematics

We will start by studying *how* objects move. This study is called **kinematics**. When we do kinematics problems, we're not concerned with how or why the object started moving; we simply want to study its future motion.

2.1 Important Quantities

Let's begin by exploring the basic quantities in kinematics.

- **Position** is, quite simply, the position of an object relative to some origin. It is usually measured in meters and is denoted by $\overrightarrow{x}$ (the arrow is above the x because position is a vector quantity).
- **Displacement** is the change in an object's position. It is also measured in meters and is denoted by $\Delta\overrightarrow{x}$.
- **Velocity** is the rate at which position changes and is denoted by $\overrightarrow{v}$. In other words, $\overrightarrow{v} = \frac{\mathrm{d}\overrightarrow{x}}{\mathrm{d}t}$. It is important to note that velocity is not just a number; velocity also has a direction because it's a vector (e.g. 5 m/s north). **Speed** is the magnitude of velocity.
- **Acceleration** is the rate at which velocity changes: $\overrightarrow{a} = \frac{\mathrm{d}\overrightarrow{v}}{\mathrm{d}t}$.

Let's do an example to understand all these quantities.

> **Example 1**: An object's position is given by the function $x(t) = t^2 - 5t + 4$.
> a) What are the object's position, velocity, and acceleration at $t = 3$? b) At what times is the object moving to the right? c) At what times is the object speeding up?

Solution: The position is simply $x(3) = 3^2 - 5*3+4 =$ **-2**. The velocity function is $v(t) = x'(t) = 2t - 5$, so $v(3) = 2*3-5 =$ **1**. The acceleration function is $a(t) = v'(t) = 2$. Therefore, $a(3) =$ **2**. Now, for part b), we need to determine when the object is moving to the right. For an object to move to the right, it's velocity must be positive. So $v(t) > 0, 2t - 5 > 0, t > 2.5$. The object is thus moving to the right at **all times after** $t = 2.5$. Now, take a second and think about what it means for an object to be speeding up. It means that the *magnitude* of the velocity must be increasing, not necessarily that the velocity is becoming more positive. What this means is that for an object to be speeding up, it's velocity and acceleration must have the same sign. Otherwise, it will be slowing down. For example, if the velocity is initially negative and the acceleration is initially positive, the velocity will become more positive but its *magnitude* will decrease (e.g. -4 m/s to -3 m/s), meaning the object will be slowing down. In this problem, the acceleration is always positive, so for the object to be speeding up, the velocity must be positive. But we just determined that that happens for $t > 2.5$! Therefore, we can say that the object is moving to the right *and* speeding up for **all times after** $t = 2.5$.

2.2 The Kinematics Equations

There are four essential equations you must know to solve kinematics problems. Note that you can only use these equations if the acceleration is constant.

1. $v_f = v_0 + at$. This is fairly self-explanatory. The object's velocity at a later time (v_f) will be equal to its initial velocity (v_0) plus its change in velocity (at). Remember that $a = \frac{\mathrm{d}v}{\mathrm{d}t}$, so at essentially represents the change in velocity.

2. $\Delta x = v_0 t + \frac{1}{2}at^2$. This equation can be obtained by integrating the first equation with respect to time (recall that $v = \frac{\mathrm{d}x}{\mathrm{d}t}$, so $\int \mathrm{d}x = \int v \, \mathrm{d}t$.

3. $\Delta x = \frac{v_0 + v_f}{2} * t$. This equation essentially says that the displacement (Δx) is equal to the average velocity ($\frac{v_0 + v_f}{2}$) times time. This intuitively makes sense.

4. $v_f^2 = v_0^2 + 2a\Delta x$. This is the least intuitive out of all these equations. Try deriving it on your own using the other equations. Hint: start with the definition of acceleration and then use the third equation.

3 Dynamics and Energy

Dynamics is the study of *why* things move. This can be explained using forces.

3.1 Newton's Laws

Newton's first law states that an object at rest will remain at rest unless acted on by a net external force. You've likely heard of this being associated with **inertia**. Inertia is the tendency of an object to remain unchanged; if it is moving, it will want to keep moving, and if it is at rest, it will want to stay at rest.

Newton's second law states that $F_{net} = ma$. Note that this equation represents the *net* force on an object, not just any single force. The net force must be determined by treating each individual force as a vector and then adding those vectors together.

Newton's third law states that every action has an equal and opposite reaction. As an example, if you push against a wall with 10N, the wall also pushes against you with 10N.

3.2 Energy and Work

NOTE: Sections 3.2 through 4.3 of this handout are heavily based on Everaise's Astronomy Book.

Kinetic energy is the energy of an object due to its motion while **potential energy** is the energy of an object due to its position. Energy is a scalar and given in units of joules (J). For a point particle of mass m moving at velocity v, the kinetic energy K is defined as

$$K = \frac{1}{2}mv^2$$

The kinetic energy of an object in orbit of radius r, of mass m around a mass M is $K = \frac{GMm}{2r}$.

For a constant force F exerted on an object as it is displaced a distance d, the **work** done can be calculated with the equation $W = Fd\cos(\theta)$, where θ is the angle between the force and displacement vectors.

The quantity work is useful because it is directly related to kinetic energy according to the **Work-Energy Theorem**: The work done on an object is equal to its change in kinetic energy. That is $W = \Delta K$).

The **gravitational potential energy** is the energy some object has relative to the field formed by the gravitational pull of another. The formula for the gravitational potential energy of a mass m relative to a mass M a distance r away is: $U_g = -\frac{GMm}{r}$, where G is the universal gravitational constant equal to $6.67 * 10^{-11}\frac{N}{kg^2m^2}$.

The **Virial Theorem** states that for a body in a **circular** (ONLY CIRCULAR, NOT ELLIPTICAL) orbit, the kinetic and potential energies of the body are related as follows: $2K + U = 0$, and if we let E represent the *total* energy of the body, then $E = K + U = -K$. Thus, the total energy of a body in a circular orbit is always negative. This theorem is *very* useful in solving problems (see practice problem 4 for a simple application).

The reason energy is useful is because of the **law of conservation of energy**. We will explore this more in the orbital mechanics handout.

3.3 Circular Motion

So far, we have dealt mainly with *translational motion*, with accelerations constant in both magnitude and direction. In this section, however, we will deal with a very different kind of motion: one in which the magnitude of the velocity and acceleration is constant, but the direction of both the acceleration and velocity constantly changes. This situation is called **circular motion** and includes many examples such as the earth rotating around the sun and a ball being whirled around on a string.

Since the velocity (and hence kinetic energy) of the particle is constant, we can then conclude that the net force and velocity vectors for the particle must be perpendicular. This leaves us with two possible directions for the acceleration vector, towards the center of the circle, or away from the center of the circle. Intuitively, since acceleration is the rate of change of velocity, we should conclude the acceleration vector is directed towards the center of the circle. We will call this inward direction the centripetal direction.

For an object traveling in a circle of radius r at a constant speed v, the acceleration experienced by the object has magnitude

$$a = \frac{v^2}{r}$$

The **angular frequency** ω is defined as $\omega = \frac{d\theta}{dt}$, or (because $s = r\theta$) $\omega = \frac{v}{r}$. For an object undergoing uniform circular motion, we also have that the period

$$T = \frac{2\pi}{\omega}$$

From Newton's second law, the **centripetal force** F_c is equal to

$$ma_c = \frac{mv^2}{r}$$

It is very important to understand that the centripetal force is *not* an actual force; it is simply the net force in the centripetal direction for an object undergoing uniform circular motion. What this equation says is that if you add all the centripetal components of the forces on an object undergoing uniform circular motion, that number will be equal to $\frac{mv^2}{r}$.

4 Momentum and Collisions

When studying collisions, it is often difficult to analyze the motion of objects using only forces. We thus introduce the concept of **momentum**.

4.1 Linear Momentum

The **linear momentum** of an object is defined as the product of its mass and velocity; $\overrightarrow{p} = m\overrightarrow{v}$. Using this definition, we can also express Newton's second law as $F_{net} = \frac{\mathrm{d}p}{\mathrm{d}t}$.

4.2 Impulse

We now define a quantity known as **impulse** for a force acting on an object. We like to think of one object applying an impulse on another object when it applies a force on it.

The impulse $\overrightarrow{J}$ of a force $\overrightarrow{F}$ acting on a particle over a specific time interval is defined as

$$\overrightarrow{J} = \int_{t_i}^{t_f} \overrightarrow{F}\mathrm{d}t = F_{avg}\Delta t$$

For forces of constant magnitude, $J = Ft$. The impulse of the net force acting on a particle during a given time interval is equal to the change in momentum of the particle during that interval. $J_{net} = \Delta p$. This can easily be proven using $F = \frac{\mathrm{d}p}{\mathrm{d}t}$ and integrating.

4.3 Center of Mass and Conservation of Momentum

The center of mass is a position defined relative to an object or system of objects. It is the average position of all the parts of the system, weighted according to their masses.

When two objects collide, they exert equal and opposite impulses on each other. This is because they exert equal and opposite *forces* on each other by Newton's third law, and these forces are exerted over the same time interval since both objects are experiencing the collision. Therefore, the total momentum of the system will not change, nor will the velocity of the center of mass. This principle is known as the **law of conservation of momentum**.

In an **elastic** collision, the kinetic energy of the system is conserved in addition to the linear momentum. Furthermore, in an elastic collision, the velocities of the objects in the center of mass reference frame are reversed. The reason that this trick works is that kinetic energy is conserved in an elastic collision. In an **inelastic** collision, however, kinetic energy is *not* conserved, but linear momentum is still conserved.

> **Example Problem 4.1:** Ball A of mass 2 kg is moving at 3 m/s to the right and collides with Ball B of mass 1 kg, which is at rest. If the collision is elastic, what are the final velocities of the balls? If the collision in inelastic (the balls stick together), what is the final velocity of the balls?

Solution: By conservation of momentum, the initial center of mass velocity in both cases is $v_{com} = \frac{m_A v_A + m_B v_B}{m_A + m_B} = \frac{6}{3} = 2$ m/s. If the collision is inelastic, the two balls will be moving together after the collision. Therefore, both balls will be travelling at the same speed as the center of mass. Since the velocity of the center of mass doesn't change, this means that both balls will have a final velocity of **2 m/s**.

If the collision is elastic, we want to go to the center of mass frame to simplify things. To do this, we subtract 2 m/s from the velocities of the balls. This means that in the center of mass frame, the initial velocity of ball A is 1 m/s and the initial velocity of ball B is -2 m/s. Now, we just flip the signs to get the final velocities of the balls *in the center of mass frame*. To get back to our initial frame, we must add back the center of mass velocity. Thus, the final velocity of ball A is $-1 + 2 = 1$ m/s and the final velocity of ball B is $2 + 2 =$ **4 m/s**.

5 Conclusion

In this handout, we learned many essential physics concepts. Of course, this handout was a very quick overview of basic mechanics and was meant to be more a review than a lesson. If you want to learn more physics in-depth, either take a physics class at your school or learn by yourself! There are many, many resources (textbooks, videos, courses) online to help you get started.

6 Practice Problems

Practice Problem 1: Two trains initially 500 meters apart are moving towards each other, both at a constant speed of 5 m/s. How long does it take the trains to collide?

Practice Problem 2: You swing a ball attached to a string around in a horizontal circle. Suppose the tension in the string is 50 N, the mass of the ball is 5 kg, and the radius of the circle is 2 m. What is the speed of the ball? What is the ball's period of revolution?

Practice Problem 3: A ball of mass 5 kg moving at 10 m/s enters a rough surface (friction is present) and comes to rest. How much work did friction do on the ball?

Practice Problem 4: An asteroid in a circular orbit around a planet has a kinetic energy of 50 J. If the radius of the asteroid's orbit is doubled, what is the new kinetic energy of the rocket? By how much does the gravitational potential energy of the change? (hint: use the Virial Theorem for the second part)

Practice Problem 5 (2020 USAAAO First Round): Calculate the speed of the sun around the center of mass due to the presence of Jupiter. You may look up the mass of Jupiter, the mass of the Sun, and the distance from Jupiter to the Sun.

Practice Problem 6: Search up gravitational slingshots and try to understand the concept. Here is a good place to get started.