Integral Averages

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1 Introduction

How do you take the average of $0.5x^2 + 1$ from 3 to 8? Well, we can use something called integral averages!

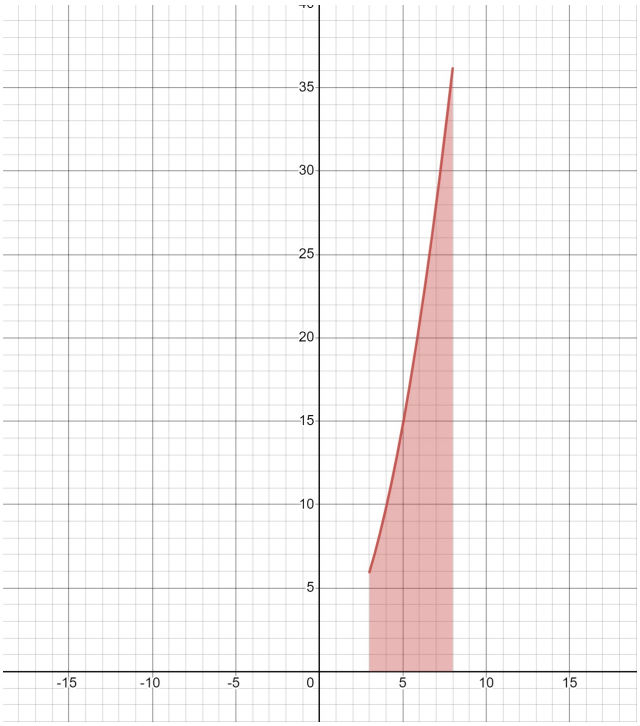

Figure 1: A depiction of the actual meaning of an integral as the area under a curve (the author, using Desmos)

An integral average is a method of taking the average of a function across a range by using integrals. It relies on the following formula:

$$average = \frac{\int_{lowerbound}^{upperbound} f(x)\,\mathrm{d}x}{upperbound - lowerbound}$$

This simply finds the area under the relevant portion of the curve and divides it by the length - to find the height it would be if the shape were a rectangle, which is the average height and thus

the average value. Applying the formula to this problem, we get:

$$average = \frac{\int_3^8 0.5x^2 + 1 \, \mathrm{d}x}{8 - 3} = \frac{126.25}{5} = 25.25$$

And there's our answer! Here are a few practice examples: Take the average of the following from -4 to 4:

$$8x^3 + 4x + 1$$

Ans: 1

$$x^2 + 2x + 3$$

Ans: 25/3

$$9x^3 + x + 7$$

Ans: 7

$$4x + 1024$$

Ans: 1024

$$8x^4$$

Ans: 409.6

$$1834567892345679x + 45678183$$

Ans: 45678183

$$1$$

Ans: 1