Cosmic Distance Ladder

Faraz Ahmed

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Table of Contents

1 Introduction

In this handout, we will learn how to measure distances to any object in the universe. Different methods have been developed that work best for different distance scales, so we often group all these methods together into a handy toolkit known as the **distance ladder**.

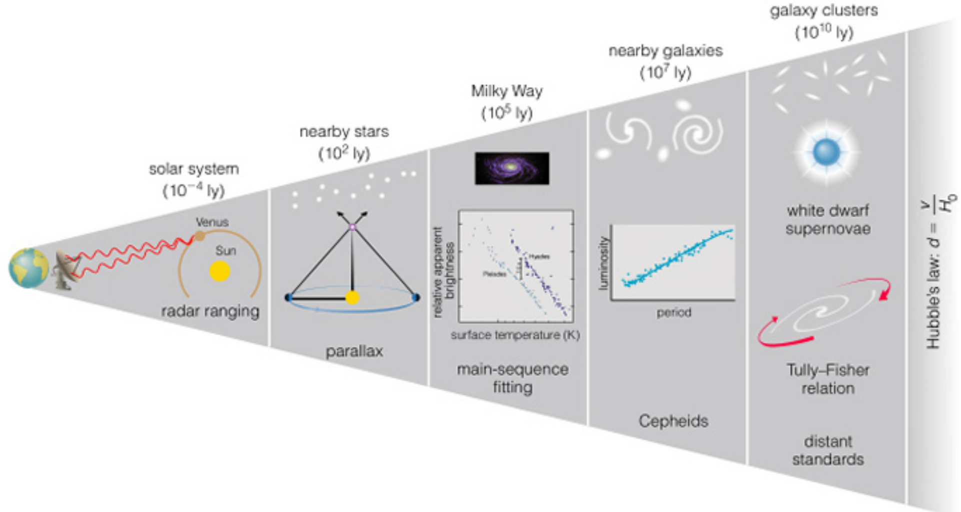

Figure 1: Distance Ladder (Source: David Darling)

2 Parallax

Hold your index finger in front of your eyes at a reasonable distance. Now, close your left eye and observe the finger and its background through your right eye. Do the same, closing your right eye and seeing with your left eye. What do you observe? The optical background behind your finger appears to have shifted. This is because the position of the observer (your eye) has changed. If you repeat this experiment, focusing on objects farther away from you, you would notice that this change becomes more and more negligible as the distance from the object being viewed becomes larger.

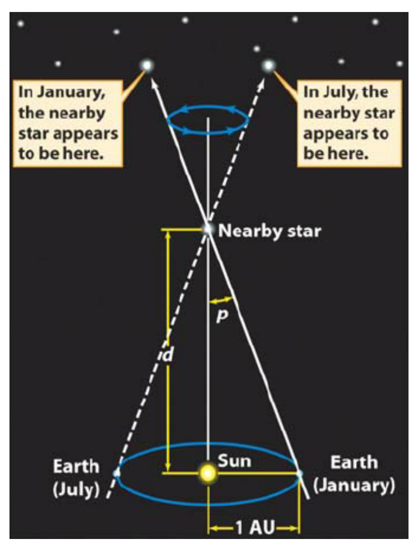

Figure 2: Parallax of a nearby star (Source: John McGraw)

Now, let's extend this to an observer on earth viewing a star at two different times in the year that are six months apart. This situation is exactly analogous to the finger example we gave in the previous paragraph: here we have the diameter of earth's orbit as the distance between your eyes, and your finger is the star. It's important that our stellar background is much more distant to us than the star we're observing (that way the background doesn't also shift).

Let's get into some terminologies now. The angle p in the diagram is termed as the parallax angle, it's usually measured in arcseconds and the distance to the star, d, is measured in units of parsecs[1]. From the diagram, we can write,

$$1 \text{ AU} = d \tan p$$

By definition,

$$\frac{1 \text{ AU}}{\tan(1")} = 1 \text{ pc}$$

Since the angles are small,[2]

$$206265 \text{ AU} = 1 \text{ pc}$$

Let's continue to write down the equation for d in terms of p, which is just,

$$d = \frac{1}{p} \text{ pc} = \frac{206265}{p} \text{ AU}$$

Example 2:1 (USAAAO Round One 2023)
Friedrich Bessel was the first person to quantitatively measure the annual change in stellar positions due to the motion of the Earth around the Sun. This change is known as stellar parallax. Bessel determined the stellar parallax for 61 Cygni to be about 1/3 arc-seconds. What is the distance to 61 Cygni from Earth?

(a) 1/3 light years
(b) 3 light years
(c) 1/3 parsec
(d) 3 parsecs
(e) 3 kiloparsecs

Solution: GG, this is what we just discussed.

$$d = \frac{1}{p} \text{ pc} = 3 \text{ pc}$$

So the answer is just **(d) 3 parsecs**.

Unforunately, it's pretty hard to make parallax measurements for most of our neighbourhood because measuring small angles is hard. For example, ESA's Gaia can measure parallaxes to the order of 10-100 nano-arcseconds accurately (What's that in parsecs assuming Gaia to be at L_1?) or just about 1% of our neighbourhood.

[1] 1 pc = 3.26 ly = $3.086 \cdot 10^{16}$ m
[2] 206265 arc seconds = 1 radian

3 HR-Diagrams

Consider two clusters of stars that have around the same age. This would mean that the HR-Diagrams of these clusters should be about identical (see Stellar Evolution handout for an explanation of the HR diagram). If for one cluster, we use m_V on y because we don't know its M_V, the difference that arises would be the uniform vertical shift due the variation in distance.[3] Which is then calculated by using (see Stellar Flux handout for an explanation of the distance modulus),

$$m_V - M_V = -5 \log\left(\frac{d}{10}\right)$$

A better known version of this technique is called Main-sequence fitting, where we fit the Main-sequence branch observed in the cluster to the actual known Main-sequence branch and get the distance as we described previously. However, this only works if the cluster has a significant diversity of stars.

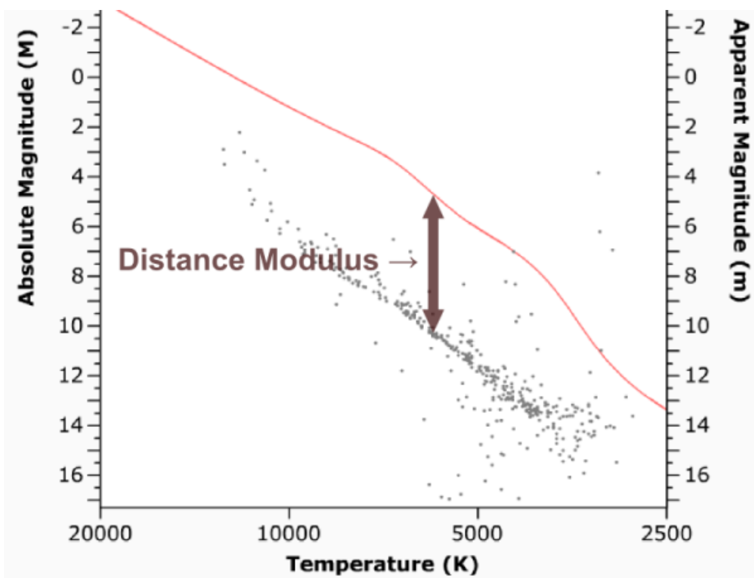

Figure 3: Main-Sequence fitting (Source: GSU.edu)

4 Standard Candles

Standard Candles are classes of objects that have a calibrated absolute magnitude M_V. That is, we know their absolute magnitude by some physical relation. Since the apparent magnitude is simply what we observe, then all we need to do is just find the distance modulus and ta-da!

4.1 Type 1a Supernovae

A Type Ia Supernova is the result of the explosion of a white dwarf after it exceeds the *Chandrasekhar limit*. Since all white dwarfs share the same limit to their mass, when they explode the

[3]We're drawing the observer's version of an HR-Diagram so the y and x-axes represent M_V and M_{B-V}

conditions are always fairly similar. As a result, the peak absolute magnitude of *every* Type 1a supernovae is the same:

$$M_V = -19.3$$

And then we use distance-modulus and determine the distance to the supernova, yay.

4.2 Cepheid Variables

Cepheid variables display a unique property known as the **period-luminosity relationship**, discovered by *Henrietta Leavitt*. By observing many cepheids and plotting their properties on a diagram, astronomers found out that *all* cepheids exhibit a linear relationship between their luminosity and their period. In other words, cepheids with longer periods have a higher absolute magnitude. As a result, just by observing the periodic variation in brightness of a Cepheid, you can figure out its absolute magnitude!

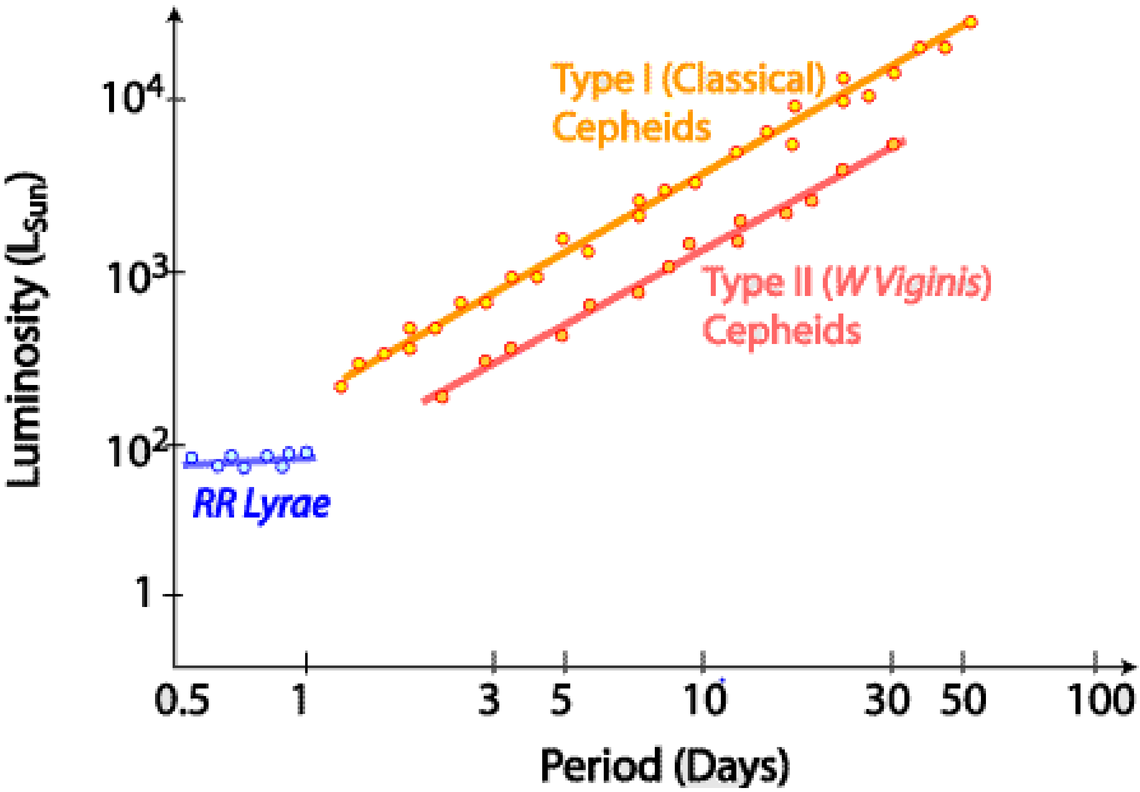

Figure 4: Period-Luminosity relation for several types of variable stars. Type I cepheids are often used. (Source: Australia Telescope)

Note that the graph above has multiple lines. This is because cepheids come in two types: Type I (Classical) and Type II (W Virginis). Recall from Stellar Evolution that Population I stars are brighter than Population II stars because they are, on average, younger and bluer. This is an important distinction that we will come back to in the next section.

5 Hubble's Law

The universe is expanding, but how did we figure that out? Redshift. When an object has a *recessional velocity* it will cause a *redshift* due to the *Doppler effect*. Mathematically, we can express this shift as follows:

$$\frac{\Delta\lambda}{\lambda_e} = z$$

where z is the **redshift** (It's named that because as objects move away from us, the wavelength is stretched, which means that it gets longer, or more *redder* for the visible spectrum). To relate it with the velocity, we will simply use the doppler-shift formula,

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

Note that this only holds true when $v \ll c$. When v is comparable to c, we use,

$$\frac{\Delta\lambda}{\lambda} = \sqrt{\frac{1+\beta}{1-\beta}} - 1$$

where $\beta = \frac{v}{c}$

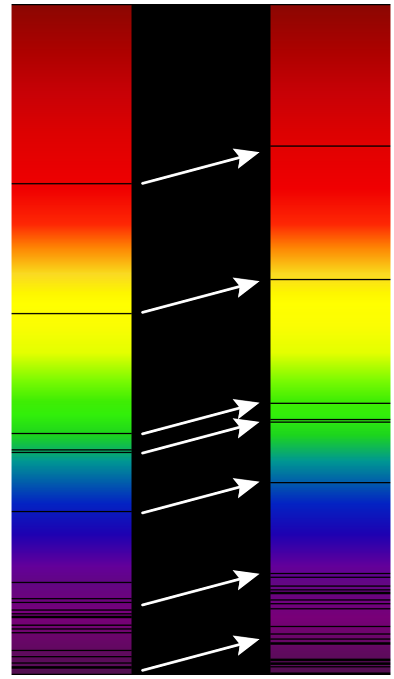

Figure 5: Redshifting of absorption lines (Source: Wikipedia)

Edwin Hubble used the Period-Luminosity relations to estimate distances to Cepheids and noticed something interesting. When he plotted the velocity-distance curve, he arrived at this:

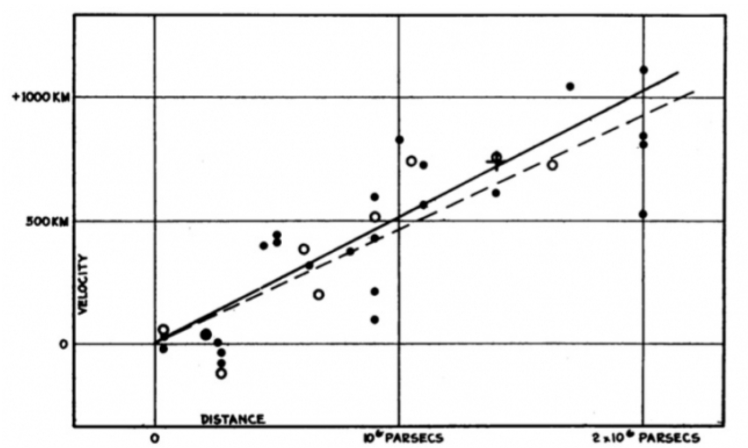

Figure 6: Hubble's graph (Source: NASA)

Objects farther from us, move away *faster* from us. Hubble explained this phenomenon with the following idea: **the universe is expanding**. Since the universe expands, the objects within it will appear to be pulled away from us in all directions since the physical distance to them is increasing. In addition, if an object is farther from us, that means there is more space in-between to expand, causing it to move away from us even faster and resulting in a higher redshift!

This relationship between distance and recessional velocity is:

$$v = H_0 \cdot d$$

Where H_0 is **Hubble's constant**. It is generally said to be about 70km/s/Mc (Let's analyze the units. km/s are the units for recessional velocity and Megaparsecs is a measure of distance. Therefore, the units are simply velocty/distance), but more recent estimates show that its value comes out to be around $H_0 = 73 \pm 1$ km/s/Mpc.

Example 5.1: (USAAAO Round One 2023)
How far from the Solar System would a galaxy with a redshift of 0.035 be?

(a) 150 Mpc
(b) 200 Mpc
(c) 250 Mpc
(d) 300 Mpc
(e) 350 Mpc

Solution: We know,

$$z = \frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

So,

$$v = cz \approx 0.105 \cdot 10^5 \text{ km/s}$$

Now, we'll use Hubble's law,

$$v = H_0 \cdot d$$

Where, $H_0 = 72$ km/s/Mpc . So,

$$d = \frac{v}{H_0} \approx 150Mpc$$

or **option (a)**.

6 Conclusion

With the techniques we learned in this handout, we can now find the distance to any object in the universe using nothing but our observations! Of course, that is not entirely true. Some stars are too dim to be seen, especially for those far away subject to lots of interstellar extinction. However, the cosmic distance ladder is still an incredibly useful tool that you will see frequently used.