# Interpreting InSAR lines of sight and directions of motion

The aim of this exercise is to guide participants through the interpretation of multiple Interferometric Synthetic Aperture Radar (InSAR) data sets, and to estimate the directions and amplitudes of motions in three dimensions.

#### Introduction

A key concept in InSAR is that measurements of displacement (or velocity) are made in the satellite line of sight (LOS). This is defined as the vector direction between the observing SAR antenna and the ground target. InSAR detects the three-dimensional displacement of the ground surface that is resolved into the LOS direction; depending on the direction of the ground displacement, this could be a large LOS displacement, a small LOS displacement, and, in some cases, no LOS displacement at all! In this exercise, we will look at an area of landslide deformation from more than one line of sight, and see how the directions of motion can be inferred.

You are provided with a paper copy of maps of LOS velocities from two different satellites from the Berkeley Hills in California.

#### The line-of-sight vector in InSAR

Formally, the unit InSAR line-of-sight vector,  $\hat{p}$ , is defined in terms of the direction of motion (also referred to as 'heading') of the SAR satellite,  $\phi$ , and the angle of incidence that the radar makes with the ground,  $\theta$ :

$$\widehat{\boldsymbol{p}} = \begin{bmatrix} p_x & p_y & p_z \end{bmatrix} = [\cos \phi \sin \theta & -\sin \phi \sin \theta & -\cos \theta ] \tag{1}$$

The displacement (or velocity) vector of the ground,  $\mathbf{u} = [u_x \ u_y \ u_z]$ , resolves into the LOS direction to give the range change, r, by the scalar product:

$$r = \widehat{\boldsymbol{p}} \cdot \boldsymbol{u} \qquad (2)$$

In this case, positive values of *r* indicate an increase in range, i.e. an increase in the distance between the satellite and the ground target, and vice-versa.

The ERS satellites at the latitude of California had headings of 191° and 349° in descending (flying from north to south) and ascending (south to north) directions, respectively. The radar incidence in the center of the swath was 23°.

•	Calculate the three components of the line-of-sight vector for data acquired in
	a descending viewing geometry with ERS-1.

$$p_x = p_y = p_z =$$

• Which two line of sight components are correlated in terms of the sign(s) of their contributions to range change between the two viewing geometries?

• Which direction(s) of displacement would result in zero detectable range change in an ERS descending interferogram, and why?

### Multiple lines-of-sight

We can take these ideas further and consider the case where you have two interferograms of the same area, one from each viewing geometry (ascending and descending). In this case, we can start to determine the direction(s) of deformation with greater certainty.

With two observations of range change,  $r_a$  and  $r_d$ , corresponding to the ascending and descending interferograms respectively, we can write two equations with the form of (2):

$$r_a = \widehat{p_a} \cdot \mathbf{u}$$
$$r_d = \widehat{p_d} \cdot \mathbf{u}$$

If we evaluate the scalar products explicitly, we get

$$r_{a} = p_{ax}u_{x} + p_{ay}u_{y} + p_{az}u_{z}$$

$$r_{d} = p_{dx}u_{x} + p_{dy}u_{y} + p_{dz}u_{z}$$
(3)

where  $p_{ax}$ ,  $p_{ay}$ ,  $p_{az}$  are the unit pointing vector components for the ascending interferogram, and  $p_{dx}$ ,  $p_{dy}$ ,  $p_{dz}$  are the equivalents for the descending interferogram.

At this stage, we still have three unknown displacement components, and only two observations, so we need to make a simplifying assumption to make the problem well posed. A common assumption, given the lack of sensitivity of InSAR to motion in that direction, is that there is zero north-south displacement (i.e.  $u_y = 0$ ). In that case, the problem simplifies to:

$$r_a = p_{ax}u_x + p_{az}u_z$$
  

$$r_d = p_{dx}u_x + p_{dz}u_z$$
 (4)

Since  $r_a$ ,  $r_d$ ,  $p_{ax}$ ,  $p_{az}$   $p_{dx}$ ,  $p_{dz}$  are all known (e.g. from equation 1), this is now a solvable set of simultaneous equations. [Other simplifying assumptions can be made to reduce (3) to a solvable problem with two unknowns, e.g. assuming a direction that horizontal motion must occur in – for further details on this approach, see Jin and Funning (2017).]

The simultaneous equations can be solved by traditional means (i.e. by eliminating one variable), or be posed as a matrix problem:

$$\mathbf{r} = \mathbf{P} \mathbf{u'}$$
 (5)

where  $\mathbf{r} = [r_a \ r_d]^T$ ,  $\mathbf{u'} = [u_x \ u_z]^T$ , and

$$\mathbf{P} = \begin{pmatrix} p_{ax} & p_{az} \\ p_{dx} & p_{dz} \end{pmatrix} \tag{6}$$

The solution to (5) can be found by simple least squares, e.g.

$$\mathbf{u'} = [\mathbf{P^T} \ \mathbf{P}]^{-1} \mathbf{P^T} \ \mathbf{r} \tag{7}$$

Example: the Berkeley Hills, California

You are provided with a handout showing two permanent scatterer InSAR data sets from the Berkeley Hills in California, an area with several slow landslides. One data set is from the ERS mission, and is in a descending viewing geometry. The other is from data collected by the Canadian Radarsat-1 mission in an ascending viewing geometry. Acquisition parameters (incidence and heading) for both data sets are provided on the handout; **note in particular that the parameters for Radarsat-1 differ from those for ERS**. Both data sets show LOS velocity, estimated with respect to 'stable Berkeley' (the SW corner). Although the data sets cover different (but overlapping) time periods, we are going to assume that the velocities are constant over the whole time period, and thus can be directly compared (and combined).

Look at the two permanent scatterer data sets. You will see that five sub-areas have been delimited by pink outlines on each – these are the perimeters of known, mapped landslides. The velocities within these perimeters differ from the velocities from the surrounding areas.		
Given the senses of motion in the two data sets, can you determine whether the motion of the landslides is mostly horizontal or vertical? Explain your reasoning.		
• Using the provided average velocities for each landslide, estimate the eastwest and vertical velocities for each, e.g. by solving (4) or (7) above.		

Landslide 1			
E–W velocity:	vertical velocity:		
Landslide 2			
E–W velocity:	vertical velocity:		
Landslide 3			
E–W velocity:	vertical velocity:		
Landslide 4			
E–W velocity:	vertical velocity:		
Landslide 5			
E–W velocity:	vertical velocity:		
You may have noticed that the descending track data show an approximately linear change in LOS velocity in the eastern half of the data set. This is the signature of creep (slow, aseismic movement) on the Hayward fault, a strike-slip fault responsible for a M~7 earthquake in 1868 The fault trace is located at the change in velocity.			
Determine the sense of motion (i.e. left- or right-lateral) of the Hayward fault.			
Why do you think the deformation from fault creep is clear in the descending data set, but much less clear in the ascending data set?			

## Readings and acknowledgements

This exercise was developed by Gareth Funning (gareth@ucr.edu) from data processed by Alessandro Ferretti and Fabrizio Novali at TRE Altimira in collaboration with Roland Bürgmann at UC Berkeley.

For further information on the Berkeley Hills landslides (and the reference for the ERS data):

Hilley, G. E., R. Bürgmann, A. Ferretti, F. Novali, F. Rocca, 2004, Dynamics of Slow-Moving Landslides from Permanent Scatterer Analysis, Science, 304, 1952-1955, doi:10.1126/science.1098821

The Radarsat-1 data were originally presented at AGU in 2006:

Novali, F., G. J. Funning, R. Bürgmann, A. Ferretti, C. Giannico, 2006, ASF RADARSAT data reveal rates and mechanisms of contemporary surface deformation in the San Francisco Bay Area, American Geophysical Union, Fall Meeting 2006, abstract id. H24C-04

Further information on solving for components of deformation from multiple lines of sight can be found here:

- Wright, T. J., B. E. Parsons, Z. Lu, 2004, Toward mapping surface deformation in three dimensions using InSAR, Geophys. Res. Lett., 31, L01607, doi:10.1029/2003GL018827.
- Jin, L. and G. J. Funning, 2017, Testing the inference of creep on the northern Rodgers Creek fault, California, using ascending and descending persistent scatterer InSAR data, J. Geophys. Res. Solid Earth, 122, doi:10.1002/2016JB013535