1. Let's formulate this problem as an optimal control problem.
Define the state equations :

$\dot{x} = V_x$
$\dot{y} = V_x$
$\dot{V_x} = \frac{1}{m} \cdot (f_{F_x} \cdot cos(\delta) - f_{F_y} \cdot sin(\delta) + f_{R_x}) - V \cdot r$
$\dot{V_y} = \frac{1}{m} \cdot (f_{F_x} \cdot sin(\delta) + f_{F_y} \cdot cos(\delta) + f_{R_y}) - V_x \cdot r$
$\dot{r} = \frac{1}{I_z} \cdot (f_{F_y} \cdot cos(\delta) + f_{F_x} \cdot sin(\delta)) \cdot l_F - f_{R_y} \cdot l_R$
$\dot{\psi} = r$

Define the inputs :
f_{F_x}, f_{R_x}, δ

Define the performance measure :
$J = \alpha \cdot \ y(t_f)^2 + t_f = \alpha \cdot \ y(t_f)^2 + \int_0^{t_f} 1dt$

The boundary conditions are :
$\beta(0) = 0$
$V_x(0) = 55kmperhour$
$V_y(0) = 0$
$\beta(t_f) \simeq -90degrees$
$\dot{\psi}(t_f) = 0$

2. Assuming the rear-wheel driven (RWD) vehicle, let us derive the necessary conditions for this optimization problem.
For a rear-drive vehicle, $f_{F_x} = 0$ when driving and $f_{F_x} < 0$ when braking, while f_{R_x} can take both positive (when driving) and negative (when braking) values. Ideally, $f_{F_x} < 0$ if and only if $f_{R_x} < 0$.

Define the Hamiltonian for the optimal-control problem :
$H = 1 + \lambda_x \cdot V_x + \lambda_y \cdot V_y + \lambda_{V_x} \cdot \frac{1}{m} \cdot [(f_{F_x} \cdot cos(\delta) - f_{F_y} \cdot sin(\delta) + f_{R_x}) - V \cdot r] + \lambda_{V_y} \cdot [\frac{1}{m} \cdot (f_{F_x} \cdot sin(\delta) + f_{F_y} \cdot cos(\delta) + f_{R_y}) - V_x \cdot r] + \lambda_r \cdot [\frac{1}{I_z} \cdot (f_{F_y} \cdot cos(\delta) + f_{F_x} \cdot sin(\delta)) \cdot l_F - f_{R_y} \cdot l_R] + \lambda_\psi \cdot r$
The co-state equations are the following :
$\dot{\lambda_x} = -\frac{\partial H}{\partial x} = 0$
$\dot{\lambda_y} = -\frac{\partial H}{\partial y} = 0$
$\dot{\lambda_{V_x}} = -\frac{\partial H}{\partial V_x} = \lambda_{V_y} \cdot r$
$\dot{\lambda_{V_y}} = -\frac{\partial H}{\partial V_y} = -\lambda_{V_x} \cdot r$
$\dot{\lambda_r} = -\frac{\partial H}{\partial r} = -\lambda_{V_x} \cdot V_y + \lambda_{V_y} \cdot V_x$
$\dot{\lambda_\psi} = -\frac{\partial H}{\partial \psi} = 0$

Looking for a singular control :
$\frac{\partial H}{\partial u} = 0$ which means
$\frac{\partial H}{\partial f_{F_x}} = 0$ that is

$\frac{\lambda_{V_x}}{m} \cdot cos(\delta) + \frac{\lambda_{V_y}}{m} \cdot sin(\delta) + \frac{\lambda_r}{I_z} \cdot sin(\delta) = 0$

$\frac{\partial H}{\partial f_{R_x}} = \frac{\lambda_{V_x}}{m} = 0$

$\frac{\partial H}{\partial \delta} = 0$ that is
$-\frac{\lambda_{V_x}}{m} \cdot f_{F_x} \cdot sin(\delta) - \frac{\lambda_{V_x}}{m} \cdot f_{F_y} \cdot cos(\delta) + \frac{\lambda_{V_y} m}{\cdot} f_{F_x} \cdot cos(\delta) - \frac{\lambda_{V_y} m}{\cdot} f_{F_y} \cdot sin(\delta) -$
$\frac{\lambda_r}{I_z} \cdot f_{R_y} \cdot sin(\delta) + \frac{\lambda r}{I_z} \cdot f_{F_x} \cdot cos(\delta) = 0$
i 3. Let us gve the boundary conditions for the states $r(t_f) = 0$

The transversality condition for the Hamiltonian is :
$\frac{\partial \phi}{\partial t}(t_f, x_f) + H(t_f, x_f) = 0$
that is $H(t_f, x_f) = -1$

The transversality conditions for the co-states are :
$\frac{\partial \phi}{\partial X}(t_f, x_f) - \lambda(t_f) = 0$
that is :
$\frac{\partial \phi}{\partial x}(t_f, x_f) = \lambda_x(t_f) = 0$
$\frac{\partial \phi}{\partial y}(t_f, x_f) = \lambda_y(t_f) = \alpha \cdot 2 \cdot y(t_f)$
$\frac{\partial \phi}{\partial V_x}(t_f, x_f) = \lambda_{V_x}(t_f) = 0$
$\frac{\partial \phi}{\partial V_y}(t_f, x_f) = \lambda_V(t_f) = 0$
$\frac{\partial \phi}{\partial r}(t_f, x_f) = \lambda_r(t_f) = 0$
$\frac{\partial \phi}{\partial \psi}(t_f, x_f) = \lambda_\psi(t_f) = 0$

4. Let us determine the optimal control strategy :