1. 解: 最近邻 $A_{12}: 12\times\frac{1}{1^{12}}=12$ $\quad A_6: 12\times\frac{1}{1^{6}}=6$

最近邻、次近邻 $A_{12}: 12\times\frac{1}{1^{12}}+6\times\frac{1}{(\sqrt{2})^{12}}=\frac{387}{32}$ $\quad A_6: 12\times\frac{1}{1^{6}}+6\times\frac{1}{(\sqrt{2})^{6}}=\frac{51}{4}$

最近邻、次近邻、第三近邻 $A_{12}: 12\times\frac{1}{1^{12}}+6\times\frac{1}{(\sqrt{2})^{12}}+24\times\frac{1}{(\sqrt{3})^{12}}=\frac{94296}{7776}$

$A_6=12\times\frac{1}{1^{6}}+6\times\frac{1}{(\sqrt{2})^{6}}+24\times\frac{1}{(\sqrt{3})^{6}}=\frac{491}{36}$

2. $V=-\frac{N}{2}\left(\frac{\alpha e^2}{4\pi\varepsilon_0 r}-\frac{\beta}{r^n}\right)$

$$\begin{cases} r_1=\left(\frac{4\pi\varepsilon_0 n\beta}{\alpha e^2}\right)^{\frac{1}{n-1}} \\ V_1=\frac{\beta}{r_1^n} \end{cases}$$

由 $-\frac{N}{2}\left(\frac{\alpha e^2}{4\pi\varepsilon_0 r}-Ce^{-\frac{r}{\rho}}\right)$ 得

$$\begin{cases} \frac{\alpha e^2}{4\pi\varepsilon_0 r_2^2}=\frac{C}{\rho}e^{-\frac{r_2}{\rho}} \\ V_2=Ce^{-\frac{r_2}{\rho}} \end{cases}$$

势能贡献相同
平衡位置一致

$$\begin{cases} r_1=r_2 \\ V_1=V_2 \\ r_1=\left(\frac{4\pi\varepsilon_0 n\beta}{\alpha e^2}\right)^{\frac{1}{n-1}} \\ \frac{\alpha e^2}{4\pi\varepsilon_0 r_2^2}=\frac{C}{\rho}e^{-\frac{r_2}{\rho}} \end{cases}$$

即

$$\begin{cases} r_2=\sqrt{\frac{\alpha e^2\rho}{4\pi\varepsilon_0\beta}r_1^n} \\ r_1=\left(\frac{4\pi\varepsilon_0 n\beta}{\alpha e^2}\right)^{\frac{1}{n-1}} \end{cases}$$

$$\therefore n^{n-2}\rho^{n-1}=\frac{4\pi\varepsilon_0\beta}{\alpha e^2}$$

3. $$\begin{cases} \rho \dfrac{\partial^2 u}{\partial t^2} = C_{11}\dfrac{\partial^2 u}{\partial x^2} + C_{44}\left(\dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2}\right) + (C_{11}+C_{44})\left(\dfrac{\partial^2 u}{\partial x \partial y} + \dfrac{\partial^2 w}{\partial x \partial z}\right) \\ \rho \dfrac{\partial^2 v}{\partial t^2} = C_{11}\dfrac{\partial^2 v}{\partial y^2} + C_{44}\left(\dfrac{\partial^2 v}{\partial x^2} + \dfrac{\partial^2 v}{\partial z^2}\right) + (C_{11}+C_{44})\left(\dfrac{\partial^2 u}{\partial x \partial y} + \dfrac{\partial^2 w}{\partial y \partial z}\right) \\ \rho \dfrac{\partial^2 w}{\partial t^2} = C_{11}\dfrac{\partial^2 w}{\partial z^2} + C_{44}\left(\dfrac{\partial^2 w}{\partial x^2} + \dfrac{\partial^2 w}{\partial y^2}\right) + (C_{12}+C_{44})\left(\dfrac{\partial^2 u}{\partial x \partial z} + \dfrac{\partial^2 v}{\partial y \partial z}\right) \end{cases}$$

代入：$$\begin{cases} u = v = w \\ u = e^{\frac{ik}{\sqrt{3}}(x+y+z)} e^{-i\omega t} \cdot u_0 \end{cases}$$

则 $$\rho \frac{\partial^2 u}{\partial t^2} = C_{11}\frac{\partial^2 u}{\partial x^2} + C_{44}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + (C_{12}+C_{44})\left(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 w}{\partial x \partial z}\right)$$

可得 $\dfrac{\omega^2}{k^2} = V_3^2 = \dfrac{C_{11}+2C_{12}+4C_{44}}{3\rho}$ $\quad\therefore V_3 = \sqrt{\dfrac{C_{11}+2C_{12}+4C_{44}}{3\rho}}$