1. 对于一维双原子链 $\omega^2=\beta\frac{M+m}{Mm}\{1\pm[1-\frac{4mM}{(m+M)^2}\sin^2 aq]^{\frac{1}{2}}\}$

光学波 $\omega_{max}^2=2\beta\frac{M+m}{Mm}=6.70\times10^{13}s^{-1}$ $\omega_{min}^2=\frac{2\beta}{m}=5.99\times10^{13}s^{-1}$

声学波：$\omega_{max}^2=\frac{2\beta}{M}=3.00\times10^{13}s^{-1}$

(b) $\varepsilon_{max}=\hbar\omega_{光max}=4.41\times10^{-2}eV$ $\varepsilon_{min}=\hbar\omega_{光min}=3.94\times10^{-2}eV$ 声：$\varepsilon_{max}=\hbar\omega_{声max}=1.97\times10^{-2}eV$

(c) 光学波：$\bar{n}_{max}=\frac{1}{e^{\frac{\hbar\omega}{k_BT}}-1}=0.222$ 同理 $\bar{n}_{min}=0.279$ 声学波：$\bar{n}_{max}=\frac{1}{e^{\frac{\hbar\omega}{k_BT}}-1}=0.876$

(d) $\lambda=\frac{2\pi c}{\omega_{max}}=28.1\mu m$

2. 由题 $\begin{cases} g(\omega)d\omega=\frac{V}{2\pi^2}(\frac{1}{v_1^3}+\frac{1}{v_2^3})\omega^2d\omega=\frac{2V}{2\pi^2\bar{c}^3}\omega^2d\omega \\ \int_0^{\omega_m}g(\omega)d\omega=3N \end{cases}$

假设 v_1 v_2 波数相等

$\omega_m=[\frac{18N\pi^2}{V}(\frac{1}{v_1^3}+\frac{2}{v_2^3})^{-1}]^{\frac{1}{3}}=16\pi^2\frac{N}{V}^{\frac{1}{3}}\bar{c}$

$U_0=\int_0^{\omega_m}\frac{1}{2}\hbar\omega g(\omega)d\omega=\frac{9}{8}N\hbar\omega_m$