2111033艾明旭原子物理第一次作业

1.1 $\frac{1}{2}M_\alpha V^2 = \frac{1}{2}M_\alpha V'^2 + \frac{1}{2}m_e v^2$ ①

$M_\alpha V = M_\alpha V'\cos\theta + m_e v\cos\varphi$ ②

$M_\alpha V'\sin\theta = m_e v\sin\varphi$ ③

② × $\sin\theta$ + ③ × $\cos\theta$　　$m_e v = M_\alpha V\frac{\sin\theta}{\sin(\theta+\varphi)}$ ④

② × $\sin\theta$ − ③ × $\cos\theta$　　$M_\alpha V' = M_\alpha V\frac{\sin\varphi}{\sin(\theta+\varphi)}$ ⑤

联立 ④⑤① 消除 得角关系　$\sin^2(\theta+\varphi) = \sin^2\varphi + \frac{M_\alpha}{m_e}\sin^2\theta$

若记 $\mu = \frac{m_e}{M_\alpha}$　可得 $\mu\sin^2(\theta+\varphi) = \mu\sin^2\varphi + \sin^2\theta$

$\frac{d\theta}{d\varphi}[\sin 2\theta - \mu\sin(\theta+\varphi)] = \mu[-\sin 2\varphi + \sin 2(\theta+\varphi)]$ ⑥

令 $\frac{d\theta}{d\varphi} = 0$　$\sin 2(\theta+\varphi) = \sin 2\varphi$　即 $2\cos(\theta+2\varphi)\sin\theta = 0$

若 $\sin\theta = 0$，$\theta = 0$

若 $\cos(\theta+2\varphi) = 0$　$\theta = \frac{\pi}{2} - 2\varphi$，代入⑥

$\mu\sin^2(\frac{\pi}{2} - \varphi) = \mu\sin^2\varphi + \sin^2\theta$

$\sin\theta = \mu = \frac{m_e}{M_\alpha} = \frac{1}{4\times 1836} \approx 10^{-4}\ \text{rad}$

1.5 $d\Omega = \frac{S}{r^2} = \frac{1.5}{10^2} = 1.5\times 10^{-2}$

$n = \frac{m}{V}\cdot\frac{1}{A}N_A = \frac{\rho}{A}N_A$

$\int_{\theta_1}^{\theta_2}\frac{2\pi\sin\theta\, d\theta}{\sin^4\frac{\theta}{2}} = 2\pi\int_{\theta_1}^{\theta_2}\frac{\sin\theta\, d\theta}{\sin^4\frac{\theta}{2}} = 2\pi\int_{\theta_1}^{\theta_2}\frac{\cos\frac{\theta}{2}\, d\theta}{\sin^3\frac{\theta}{2}} = -4\pi\,\frac{1}{\sin^2\frac{\theta}{2}}\Big|_{\theta_1}^{\theta_2}$

$\frac{dN'}{N} = nt\frac{a^2}{16}\frac{d\Omega}{\sin^4\frac{\theta}{2}} = \frac{1.5}{197}\times 6.02\times 10^{23}\times\frac{(79\times 1.44\times 10^{-15})^2}{16}\times\frac{1.5\times 10^{-2}}{(\frac{1}{2})^4} = 8.9\times 10^{-6}$

1.8 由 1.1. $2\cos(\theta+2\varphi)\sin\theta=0$

$$\mu\sin^2(\frac{\pi}{2}-\varphi)=\mu\sin^2\varphi+\sin^2\theta$$

$\sin\theta=\mu=\frac{m_2}{m_1}$ 若 $m_2=m_1$ $\sin\theta=1,\theta=\frac{\pi}{2}$

1.9 先求金箔厚度 $\rho t=(0.7\rho_{Au}+\rho_{Ag}\cdot 0.3)t=1.5mg/cm^2$

$$t=\frac{1.5\times10^{-2}}{0.7\times1.888\times10^4+0.3\times1.05\times10^4}m=0.916\mu m$$

金原子数 $\frac{\rho_{Au}t}{A_{Au}}N_A$ 银原子数 $\frac{\rho_{Ag}t}{A_{Ag}}N_A$

$$\eta_{Au}=\int\frac{dN'_{Au}}{N}=\int_{\frac{\pi}{6}}^{\pi}nt\frac{\alpha^2}{16}\frac{2\pi\sin\theta d\theta}{\sin^4\frac{\theta}{2}}=\frac{\rho_{Au}t}{A_{Au}}\left(N_A\frac{\pi z_1^2z_2^2 1.44^2}{2}-\frac{1}{2}\frac{1}{\sin\frac{\theta}{2}}\Big|_{\frac{\pi}{6}}^{\pi}\right)\cdot\frac{\rho_{Au}}{\frac{\rho_{Au}}{A_{Au}}+\frac{\rho_{Ag}}{A_{Ag}}}$$

$\approx 1.028\times10^{-5}$

2.2 $N=\frac{\rho}{V}=\frac{0.13g/cm^3}{4.003g/mol}\approx0.0324mol/cm^3$

$0.034\times N_A\ \cancel{\times74\%}\cdot\frac{4}{3}\pi r^3=1\times74\%$

~~1.52 1.5×10⁻¹⁰~~ ~~1.76×10⁻¹⁰ m~~

$r\approx2.08\times10^{-8}cm\approx2.08\times10^{-10}m$

2.4. $0.0234L/mol$

$\frac{4}{3}\pi r^3\cdot0.0234\times N_A=1$

$$r=\left(\frac{3}{4\pi\times0.0234\times N_A}\right)^{\frac{1}{3}}\approx2.57\times10^{-9}dm=2.57\times10^{-10}m$$

$\frac{4}{3}\pi r^3\cdot0.01696\cdot N_A=1$

$$r=\left(\frac{3}{4\pi\times0.01696\cdot N_A}\right)^{\frac{1}{3}}\approx2.86\times10^{-9}dm=2.86\times10^{-10}m$$

4.5 $\frac{1}{2}mv^2 = \frac{keZ}{r}$ $r = \frac{keZ}{2mv^2}$

(1) $v = \sqrt{\frac{2 \cdot 10MeV}{m}}$ $\therefore r \approx 3.4 \times 10^{-14} m$

(2) $v = \sqrt{\frac{2 \cdot 80MeV}{m}}$ $\therefore r \approx 2.1 \times 10^{-14} m$

由于金原子核半径约为 $7\times10^{-15} m$ 较小

质子接触原子核时动能转化为势能，质子得以进入

金原子核的静电势垒

4.6 由 Rutherford 散射公式 $\frac{d\sigma}{d\Omega} = \left(\frac{zZe^2}{4E}\right)^2 \frac{1}{\sin^4\frac{\theta}{2}}$

$r(\theta) = R + R\sin\frac{\theta}{2}$

$\cancel{\frac{1}{2}mv^2 = 12.75MeV}$ 由 $\frac{dN'}{N} = nt \frac{z^2Z^2e^4}{16E^2} \cancel{\frac{1}{\sin^4\frac{\theta}{2}}} \int_{54^\circ}^{180^\circ} \frac{d\Omega}{\sin^4\frac{\theta}{2}}$

$\therefore \phi_0 = (180-\theta)/2 = 63^\circ$

$\frac{d\sigma}{d\Omega} = \left(\frac{zZe^2}{4E}\right)^2 \cdot \frac{1}{\sin^4\frac{\theta}{2}} = \left(\frac{2\times13\times(1.6\times10^{-19})^2}{4\times12.75\times10^6}\right)^2 \cdot \frac{1}{\sin^4 27^\circ} \approx 1.57\times10^{-21} m^2/sr$

$r(\phi_0) = 2\times10^{-15} + 2\times10^{-15}\sin\frac{63^\circ}{2} \approx 2.235\times10^{-15} m$

$\therefore r(\phi_0) = R_{Al}$ $R_{Al} = 2.235\times10^{-15} m$