1. (1) $\phi_1 = \frac{1}{\sqrt{N}}\sum_n [e^{ikna} w_1(x-na) + e^{ik(na+b)} w_2(x-na-b)]$

(2) $E^s(k) = \varepsilon_s - J_0 - \sum_{R_s} J(R_s) e^{-ik_s R_s} = \varepsilon_s - J_0 - J_1 e^{-ikb}(1+e^{ika})$

2. 当 $T=0K$，有 $\int_{E_F(0)}^{E_1(0)} \frac{dW}{dE_1} dE_1 = \int_{E_2(k_{sw})}^{E_F^0} \frac{dW}{dE_2} dE_2$ (*)

$$\begin{cases} E_1(k) = E_1(0) - \frac{\hbar^2 k^2}{2m_1}, \quad m_1 = 0.18m \\ E_2(k) = E_2(k_0) + \frac{\hbar^2}{2m_2}(k_s - k_w)^2, \quad m_2 = 0.06m \end{cases}$$

则有 $\begin{cases}\frac{dE_1}{dk_1} = -\frac{\hbar^2 k}{m_1}, \quad k=\sqrt{\frac{2m_1}{\hbar^2}(E_1(0)-E_1(k))} \\ \frac{dE}{d(k_s-k_w)} = \frac{\hbar^2(k_s-k_w)}{m_2}, \quad k_1-k_w = \sqrt{\frac{2m_2}{\hbar^2}(E_2(k)-E_2(k_w))}\end{cases}$ 解得 $\begin{cases}\frac{dW}{dE_1} = \frac{m_1 V}{\hbar^2\pi^2}\sqrt{\frac{2m_1}{\hbar^2}[E_1(0)-E_1(k)]} \\ \frac{dW}{dE_2} = \frac{m_2 V}{\hbar^2\pi^2}\sqrt{\frac{2m_2}{\hbar^2}[E_2(k)-E_2(k_w)]}\end{cases}$ 代入(*)

$\frac{m_1^{3/2}}{\frac{3}{2}}[E_1(0)-E_F^0]^{\frac{3}{2}} = \frac{m_2^{3/2}}{\frac{3}{2}}[E_F^0 - E_1(k)]^{\frac{3}{2}}$ ∴ $E_F^0 = E_1(w) - 0.025eV$

3. (1) $E(k) = \frac{\hbar^2}{ma^2}(\frac{7}{8} - \cos ka + \frac{1}{8}\cos 2ka) = \frac{\hbar^2}{ma^2}(\frac{3}{4} - \cos ka + \frac{1}{4}\cos^2 ka)$

∵ $E_{max} = \frac{2\hbar^2}{ma^2}$ $\quad E_{min}=0$ $\quad$ ∴ $d = E_{max}-E_{min} = \frac{2\hbar^2}{ma^2}$

(2) $v(k) = \frac{1}{\hbar}\frac{dE}{dk} = \frac{\hbar}{ma}\sin ka(1-\frac{1}{2}\cos ka)$ $\quad \frac{d^2E}{dk^2} = \frac{\hbar^2}{m}(\cos ka - \frac{1}{2}\cos 2ka)$

(3) ∵ $\frac{dE}{dk}=0$ ∴ $k=\frac{n\pi}{a}$ ∴ $E(\frac{n\pi}{a}) = \frac{\hbar^2}{ma^2}[1-(-1)^n]$ ∴

∴ $m_{顶} = \frac{\hbar^2}{\frac{d^2E}{dk^2}\big|_{k=\frac{(2n+1)\pi}{a}}} = -\frac{2}{3}m$ $\quad m_{底} = \frac{\hbar^2}{\frac{d^2E}{dk^2}\big|_{k=\frac{2n\pi}{a}}} = 2m$