1. $w < w_0$时，q为实数 $\frac{dq}{dw} = -\frac{1}{2[A(w_0-w)]^{\frac{1}{2}}}$

代入模式密度普遍表达式 $f(w) = \left(\frac{L}{2\pi}\right)^3 \int_{w=C} \frac{dS}{|\nabla_q W|} = \frac{V}{4\pi^2 A^{\frac{3}{2}}}\sqrt{W_0 - W}$

当$w > w_0$ q为虚数 波在晶体中被阻尼，无此振动，$f(w) = 0$

2. 在面积为S的二维晶格波矢空间中，q到$q+dq$内的格波数

$dn = \frac{S}{(2\pi)^2} \cdot 2\pi q\, dq$ 将德拜模型 $w = vq$代入，$dn = \frac{S w\, dw}{2\pi v^2}$

格波总的模式密度 $g(w) = \frac{dn}{dw} = \frac{Sw}{\pi \bar{c}^2}$ 其中 $\frac{2}{\bar{c}^2} = \frac{1}{V_L^2} + \frac{1}{V_T^2}$

德拜截止频率 $w_m = \int_0^{w_m} g(w)\, dw = \frac{S w_m^2}{2\pi \bar{c}^2} = 2N$ $\therefore w_m = 2\bar{c}\sqrt{\frac{\pi N}{S}}$

格波振动能 $E = \int_0^{w_m} E \cdot g(E) f(E)\, dE = \int_0^{w_m} g(w) \frac{\hbar w}{e^{\frac{\hbar w}{k_B T}} - 1} dw = \int_0^{w_m} \frac{S\hbar w^2\, dw}{\pi \bar{c}^2 (e^{\frac{\hbar w}{k_B T}} - 1)}$

晶体热容 $C_V = \left(\frac{\partial E}{\partial T}\right)_V = \frac{S}{\pi \bar{c}^2} \int_0^{w_m} \frac{\hbar^2 w^2}{k_B T^2} \frac{e^{\frac{\hbar w}{k_B T}}}{(e^{\frac{\hbar w}{k_B T}} - 1)^2} w\, dw$

温度高 $e^{\frac{\hbar w}{k_B T}} \to 1 + \frac{\hbar w}{k_B T}$ $C_V = 2N k_B$

温度低 $\frac{\hbar w}{k_B T} \to \infty$ $C_V = \frac{S k_B^3}{\pi \bar{c}^2 \hbar^2} T^2 \int_0^{\infty} \frac{\left(\frac{\hbar w}{k_B T}\right)^3 e^{\frac{\hbar w}{k_B T}}}{(e^{\frac{\hbar w}{k_B T}} - 1)^2} d\left(\frac{\hbar w}{k_B T}\right) \propto T^2$