1. 由题 $\begin{cases} 2\cdot\frac{V}{(2\pi)^3}\cdot\frac{4}{3}\pi k_F^3 = N \\ V = \frac{N}{N_A} = \frac{\rho V}{m} \end{cases}$ 故 $E_F = \frac{\hbar k_F^2}{2m_{He}} = \frac{\hbar^2}{2m_{He}}\left(\frac{3\pi^2\rho N_A}{M}\right)^{\frac{2}{3}} = 4.21\times10^{-4}\,eV$

$\therefore T_F = \frac{E_F}{k_B} = 4.88K$

2. $C_e = \frac{\pi^2}{2}\left(\frac{T}{T_F}\right)Nk_B = \frac{\pi^2}{2}\left(\frac{T}{T_F}\right)$

$T_F = \frac{\pi^2}{2}\frac{N_A k_B T}{C_n} = 1.97\times10^4 K$

取晶格常数 $a = 5.333A$

则 $\begin{cases} E_F = \frac{\hbar^2 k_F^2}{2m^*} \\ 2\frac{V}{(2\pi)^3}\cdot\frac{4\pi k_F^3}{3} = N = \frac{2V}{a^3} \end{cases}$ $\therefore m^* = \frac{\hbar^2}{2E_F}\left(\frac{6\pi^2}{a^2}\right)^{\frac{2}{3}} = \frac{(6\pi^2)^{\frac{2}{3}}\cdot\hbar^2}{2E_F a^2}$

$\therefore N(E_F) = 4\pi\frac{(2m^*)^{\frac{3}{2}}}{h^3}E_F^{\frac{1}{2}} = \frac{3}{E_F a^3} = 7.26\times10^{46}\,J^{-1}m^{-3}$

3. $\vec{F}=-q\vec{v}\times\vec{B}=\frac{d\vec{p}}{dt}=-q\frac{d\vec{r}}{dt}\times\vec{B}=\hbar\frac{d\vec{k}}{dt}$

则 $|d\vec{r}|=\frac{\hbar}{qB}|d\vec{k}|$ 故 $A_n=\left(\frac{\hbar}{qB}\right)^2 S_n$

4. $E(k)=\frac{\hbar^2k^2}{2m}$ 设费米球半径 $k_F=2\times\frac{V}{(2\pi)^3}\cdot\frac{4\pi}{3}k_F^3=N$

则 $k_F=2\pi\cdot\left(\frac{3}{8\pi}\right)^{\frac{1}{3}}\left(\frac{N}{V}\right)^{\frac{1}{3}}=(3\pi^2n)^{\frac{1}{3}}$

∵钾为体心立方原胞内两个电子 ∴ $n=\frac{2}{a^3}$

∴ $k_F=(3\pi^2\frac{2}{a^3})^{\frac{1}{3}}=0.62\frac{2\pi}{a}$

$\Delta\left(\frac{1}{B}\right)=\frac{2\pi q}{\hbar S_F}$ $S_F=\pi k_F^2$ 故 $\Delta\left(\frac{1}{B}\right)=5.469\times10^{-9}T^{-1}$

当 $B=1T$ 时, $A_n=\left(\frac{\hbar}{Bq}\right)^2 S_n=7.562\times10^{-11}m^2$