Chapter 8 Graphical Models

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8.1 Bayesian Networks

(8.15)

$$
\begin{aligned}
\mathbb{E}[x_i] &= \mathbb{E}\left[\sum_{j \in \text{pa}_i} w_{ij}\mathbb{E}[x_j] + b_i\right] \\
&= \sum_{j \in \text{pa}_i} w_{ij}\mathbb{E}[\mathbb{E}[x_j]] + \mathbb{E}[b_i] \\
&= \sum_{j \in \text{pa}_i} w_{ij}\mathbb{E}[x_j] + b_i.
\end{aligned}
$$

(8.16)

$$
\begin{aligned}
\text{var}[x_i, x_j] &= \mathbb{E}[(x_i - \mathbb{E}[x_i])(x_j - \mathbb{E}[x_j])] \\
&= \mathbb{E}\left[(x_i - \mathbb{E}[x_i])\left\{\sum_{k \in \text{pa}_j} w_{jk}x_k + b_j + \sqrt{v_j}\epsilon_j - \left(\sum_{k \in \text{pa}_i} w_{jk}\mathbb{E}[x_k] + b_j\right)\right\}\right] \\
&= \mathbb{E}\left[(x_i - \mathbb{E}[x_i])\left\{\sum_{k \in \text{pa}_j} w_{jk}(x_k - \mathbb{E}[x_k]) + \sqrt{v_j}\epsilon_j\right\}\right] \\
&= \sum_{k \in \text{pa}_j} w_{jk}\mathbb{E}[(x_i - \mathbb{E}[x_i])(x_k - \mathbb{E}[x_k])] + \mathbb{E}[(x_i - \mathbb{E}[x_i])\sqrt{v_j}\epsilon_j] \\
&= \sum_{k \in \text{pa}_j} w_{jk}\mathbb{E}[(x_i - \mathbb{E}[x_i])(x_k - \mathbb{E}[x_k])] + \mathbb{E}\left[\left\{\sum_{l \in \text{pa}_i} w_{il}(x_l - \mathbb{E}[x_l]) + \sqrt{v_i}\epsilon_i\right\}\sqrt{v_j}\epsilon_j\right] \\
&= \sum_{k \in \text{pa}_j} w_{jk}\text{cov}[x_i, x_k] + \sqrt{v_j}\,\mathbb{E}\left[\sum_{l \in \text{pa}_i} w_{il}(x_l - \mathbb{E}[x_l])\right]\mathbb{E}[\epsilon_j] + \sqrt{v_i v_j}\,\mathbb{E}[\epsilon_i\epsilon_j] \\
&= \sum_{k \in \text{pa}_j} w_{jk}\text{cov}[x_i, x_k] + I_{ij}v_j
\end{aligned}
$$

where we assumed that ϵ is independent of x.