MATH1241 Problem Set Solutions - Calculus

Yue Yu

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Contents

Chapter 1

Functions of Several Variables

1.13 Problem 13

a)

$$\frac{\partial F}{\partial x} + 2x\frac{\partial F}{\partial y} = f'(x^2 - y)(2x) + 2xf'(x^2 - y)(-1) = 0.$$

b)

$$f(-y) = F(0, y) = \sin y,$$

which implies that $F(x, y) = f(x^2 - y) = \sin(y - x^2)$.

1.14 Problem 14

a)

$$\begin{aligned} u_{xx} &= g''(x + \lambda t), \\ u_{tt} &= \lambda^2 g''(x + \lambda t). \end{aligned}$$

b)

Plugging the results from part **a** into the given differential equation, we obtain

$$\lambda^2 g''(x + \lambda t) - 16g''(x + \lambda t) = 0.$$

Solving for λ, we have $\lambda = \pm 4$.

1.15 Problem 15

Let the equation $f(tx, ty) = t^n f(x, y)$ be (*).

Differentiating both sides of (*) with respect to x and y respectively, we have

$$\begin{aligned} \frac{\partial f(tx, ty)}{\partial (tx)} \frac{d(tx)}{dx} &= \frac{\partial f(tx, ty)}{\partial (tx)} t = t^n \frac{\partial f}{\partial x}, \\ \frac{\partial f(tx, ty)}{\partial (ty)} \frac{d(ty)}{dy} &= \frac{\partial f(tx, ty)}{\partial (ty)} t = t^n \frac{\partial f}{\partial y}, \end{aligned}$$

which implies that

$$\frac{\partial f(tx,ty)}{\partial(tx)} = t^{n-1}\frac{\partial f}{\partial x},$$
$$\frac{\partial f(tx,ty)}{\partial(ty)} = t^{n-1}\frac{\partial f}{\partial y}.$$

Hence, if we differentiate both sides of (*) with respect to t, we obtain

$$\begin{aligned}\frac{\partial f(tx,ty)}{\partial(tx)}\frac{\partial(tx)}{\partial t} + \frac{\partial f(tx,ty)}{\partial(ty)}\frac{\partial(ty)}{\partial t} &= t^{n-1}\frac{\partial f}{\partial x}x + t^{n-1}\frac{\partial f}{\partial y}y \\ &= nt^{n-1}f.\end{aligned}$$

Since $t > 0$, cancelling t^{n-1} on both sides gives

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf.$$

1.16 Problem 16

a)

If $a\alpha + b\beta = 0$, then

$$\begin{aligned}a\frac{\partial u}{\partial x} + b\frac{\partial u}{\partial y} &= af'(\alpha x + \beta y)\alpha + bf'(\alpha x + \beta y)\beta \\ &= f'(\alpha x + \beta y)(a\alpha + b\beta) \\ &= 0.\end{aligned}$$

b)

$$u(x_1, ..., x_n) = f(\lambda_1 x_1 + \cdots \lambda x_n),$$

where $\lambda_1 x_1 + \cdots + \lambda_n x_n = 0$.

1.17 Problem 17

a)

$$\begin{aligned}\frac{\partial u}{\partial t} &= \frac{\partial f(x - tu(x,t))}{\partial(x - tu(x,t))}\frac{\partial(x - tu(x,t))}{\partial t} \\ &= f'(x - tu(x,t))\left(-u(x,t) - t\frac{\partial u}{\partial t}\right) \\ &= -u(x,t)f'(x - tu(x,t)) - tf'(x - tu(x,t))\frac{\partial u}{\partial t}.\end{aligned}$$

Rearranging the equation and solving for $\frac{\partial u}{\partial t}$, we obtain

$$\frac{\partial u}{\partial t} = \frac{-u(x,t)f'(x - tu(x,t))}{1 + tf'(x - tu(x,t))}.$$

$$\begin{aligned}\frac{\partial u}{\partial x} &= \frac{\partial f(x-tu(x,t))}{\partial(x-tu(x,t))}\frac{\partial(x-tu(x,t))}{\partial x}\\ &= f'(x-tu(x,t))\left(1-t\frac{\partial u}{\partial x}\right).\end{aligned}$$

Rearranging the equation and solving for $\frac{\partial u}{\partial x}$, we obtain

$$\frac{\partial u}{\partial x} = \frac{f'(x-tu(x,t))}{1+tf'(x-tu(x,t))}.$$

b)

Plugging in the results from part **a**, we can easily prove that the given differential equation holds.

c)

To let $\frac{\partial u}{\partial x}$ be undefined, we have

$$0 = 1+tf'(x-tu(x,t)) = 1+t(\tanh^2(x-tu(x,t))-1).$$

Noticing that $0 \leqslant \tanh^2(\cdot) \leqslant 1$. If we want $\frac{\partial u}{\partial x}$ to be undefined for precisely one value of x, we have $t \geqslant 1$. Hence, $t_m = 1$.

e)

f)

1.18 Problem 18

a)

$$x^2+y^2-z^2 = t^2+y^2-t^2 = y^2 = 1,$$

which implies that $y = \pm 1$, and hence, $\frac{\partial y}{\partial t} = 0$.

b)

$$\nabla F = \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z}\right)^T = (2x, 2y, -2z)^T.$$

c)

$$\begin{aligned}\frac{dF}{dt} &= (\nabla F)\cdot\left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right)^T\\ &= 2x(t)\cdot 1 + 2y\cdot 0 - 2z(t)\cdot 1\\ &= 0.\end{aligned}$$

d)

Skipped for now.

Chapter 2

Integration Techniques

2.3 Problem 3

a)

$$\int_0^{\pi/2} \sin^7 x \cos x \, dx = \int_0^{\pi/2} \sin^7 x \, d(\sin x)$$
$$= \left[\frac{1}{8} \sin^8 x\right]_0^{\pi/2}$$
$$= \frac{1}{8}.$$

b)

$$\int_0^{\pi} \sin^3 x \cos^2 x \, dx = \int_0^{\pi} (1 - \cos^2 x) \cos^2 x \sin x \, dx$$
$$= \int_0^{\pi} (\cos^4 x - \cos^2 x) \, d(\cos x)$$
$$= \left[\frac{1}{5} \cos^5 x\right]_0^{\pi} - \left[\frac{1}{3} \cos^3 x\right]_0^{\pi}$$
$$= \frac{4}{15}.$$

c)

$$\int \sec^3 x \tan x \, dx = \int \sec^2 x \, d(\sec x) = \frac{1}{3} \sec^3 x + C.$$

d)

$$\int \cos^2 \theta \, d\theta = \int \frac{1 + \cos 2\theta}{2} \, d\theta = \frac{1}{2}\theta + \frac{1}{4} \sin(2\theta) + C.$$

e)

$$\int \cos x \cos(10x)\, dx = \frac{1}{2}\int (\cos(9x) + \cos(11x))\, dx = \frac{1}{18}\sin(9x) + \frac{1}{22}\sin(11x) + C.$$

f)

$$\int \sin(2x)\cos(3x)\, dx = \frac{1}{2}\int (\sin(5x) - \sin x)\, dx = -\frac{1}{10}\cos(5x) + \frac{1}{2}\cos x + C.$$

2.4 Problem 4

a)

$$\begin{aligned}\int \sec x\, dx &= \int \frac{\sec x(\tan x + \sec x)}{\tan x + \sec x}\, dx \\ &= \int \frac{1}{\tan x + \sec x}\, d(\tan x + \sec x) \\ &= \ln|\tan x + \sec x| + C.\end{aligned}$$

b)

i)

$$\int \sec^4 x\, dx = \frac{\sec^2 x \tan x}{3} + \frac{2}{3}\int \sec^2 x\, dx = \frac{\sec^2 x \tan x}{3} + \frac{2\tan x}{3} + C.$$

ii)

$$\begin{aligned}\int \sec^5 x\, dx &= \frac{\sec^3 x \tan x}{4} + \frac{3}{4}\int \sec^3\, dx \\ &= \frac{\sec^3 x \tan x}{4} + \frac{3}{4}\left(\frac{\sec x \tan x}{2} + \frac{1}{2}\int \sec x\, dx\right) \\ &= \frac{1}{4}\sec^3 x \tan x + \frac{3}{8}\sec x \tan x + \frac{3}{8}\ln|\tan x + \sec x| + C.\end{aligned}$$

c)

For $n \geqslant 2$, we have

$$\begin{aligned}\int \sec^n x\, dx &= \int \sec^{n-2} x \sec^2 x\, dx \\ &= \sec^{n-2} x \tan x - (n-2)\int \sec^{n-2} x \tan^2 x\, dx && \text{(Integration by parts)} \\ &= \sec^{n-2} x \tan x - (n-2)\int (\sec^n x - \sec^{n-2} x)\, dx.\end{aligned}$$

Rearranging the equation, we have

$$(n-1)\int \sec^n x\, dx = \sec^{n-2} x \tan x + (n-2)\int \sec^{n-2} x\, dx.$$

Dividing $(n-1)$ on both sides, we obtain

$$\int \sec^n x\,dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx.$$

2.5 Problem 5

a)

$$I_{6,4} = \frac{5}{10} I_{4,4} = \frac{5}{10}\cdot\frac{3}{8} I_{4,2} = \cdots = \frac{5}{10}\cdot\frac{3}{8}\cdot\frac{3}{6}\cdot\frac{1}{4}\cdot\frac{1}{2} I_{0,0} = \frac{3\pi}{512}.$$

b)

$$I_{1,1} = \int_0^{\pi/2} \cos x \sin x\,dx = \frac{1}{2}\int_0^{\pi} \sin(2x)\,dx = \left[-\frac{1}{4}\cos(2x)\right]_0^{\pi/2} = \frac{1}{2}.$$

Hence,

$$I_{5,5} = \frac{4}{10}\cdot\frac{4}{8}\cdot\frac{2}{6}\cdot\frac{2}{4} I_{1,1} = \frac{1}{60}.$$

c)

$$I_{1,0} = \int_0^{\pi/2} \cos x\,dx = [\sin x]_0^{\pi/2} = 1.$$

Hence,

$$I_{3,4} = \frac{3}{7}\cdot\frac{2}{5}\cdot\frac{1}{3} I_{1,0} = \frac{2}{35}.$$

2.6 Problem 6

Here we only prove the formula and skip the calculation.

$$\begin{aligned} I_n &= \left[-x^n e^{-x}\right]_0^1 - \int_0^1 (-e^{-x})\cdot n x^{n-1}\,dx && \text{(Integration by parts)} \\ &= -\frac{1}{e} + n I_{n-1}. \end{aligned}$$

2.11 Problem 11

$$I_{m+n,0} = \int_0^1 x^{m+n}\,dx = \left[\frac{1}{m+n+1} x^{m+n+1}\right]_0^1 = 1.$$

Hence,

$$\begin{aligned}
I_{m,n} &= \left[\frac{1}{m+1}x^{m+1}(1-x)^n\right]_0^1 + \frac{n}{m+1}\int_0^1 x^{m+1}(1-x)^{n-1}\,dx && \text{(Integration by parts)}\\
&= \frac{n}{m+1}I_{m+1,n-1}\\
&= \frac{n}{m+1}\cdot\frac{n-1}{m+2}I_{m+2,n-2}\\
&= \cdots\\
&= \frac{n}{m+1}\cdot\frac{n-1}{m+2}\cdot\cdots\cdot\frac{1}{m+n+1}I_{m+n,0}\\
&= \frac{m!n!}{(m+n+1)!}.
\end{aligned}$$

2.12 Problem 12

a)

We first derive the reduction formula.

$$\begin{aligned}
I_n &= \int_0^{\pi/2} \cos^{n-1}\cos x\,dx\\
&= \left[\cos^{n-1}x\sin x\right]_0^{\pi/2} + (n-1)\int_0^{\pi/2}\cos^{n-2}x\sin^2 x\,dx\\
&= (n-1)\int_0^{\pi/2}\left(\cos^{n-2}x - \cos^n x\right)\,dx\\
&= (n-1)(I_{n-2}-I_n),
\end{aligned}$$

which implies that

$$I_n = \left(1-\frac{1}{n}\right)I_{n-2}.$$

Hence,

$$\begin{aligned}
I_{2m} &= \left(1-\frac{1}{2m}\right)I_{2m-2} = \cdots = I_0\prod_{k=1}^{m}\left(1-\frac{1}{2k}\right) = \frac{\pi}{2}\left(1-\frac{1}{2k}\right),\\
I_{2m+1} &= \left(1-\frac{1}{2m+1}\right)I_{2m-1} = \cdots = I_1\prod_{k=1}^{m}\left(1-\frac{1}{2k+1}\right) = \prod_{k=1}^{m}\left(1-\frac{1}{2k+1}\right).
\end{aligned}$$

b)

$$\begin{aligned}
\frac{I_{2m}}{I_{2m+1}} &= \frac{\frac{\pi}{2}\prod_{k=1}^{m}\frac{2k-1}{2k}}{\prod_{k=1}^{m}\frac{2k}{2k+1}}\\
&= \frac{\pi}{2}\prod_{k=1}^{m}\frac{(2k-1)(2k+1)}{(2k)^2}\\
&= \frac{\pi}{2}\prod_{k=1}^{m}\left(1-\frac{1}{(2k)^2}\right).
\end{aligned}$$

c)

Because $0 \leq \cos x \leq 1$, we have $I_{2m+2} \leq I_{2m+1} \leq I_{2m}$.

d)

Since $I_{2m+2} \leq I_{2m+1} \leq I_{2m}$, we have $I_{2m+1} \leq I_{2m} \leq I_{2m-1}$. Dividing I_{2m+1} gives

$$1 \leq \frac{I_{2m}}{I_{2m+1}} \leq \frac{I_{2m-1}}{I_{2m+1}}.$$

Noticing that

$$\lim_{m\to\infty} \frac{I_{2m-1}}{I_{2m+1}} = 1,$$

by the pinching theorem, we obtain

$$\lim_{m\to\infty} \frac{I_{2m}}{I_{2m+1}} = 1.$$

e)

According to part **b** and **d**,

$$\lim_{m\to\infty} \prod_{k=1}^{m} \left(1 - \frac{1}{(2k)^2}\right) = \frac{2}{\pi} \lim_{m\to\infty} \frac{I_{2m}}{I_{2m+1}} = \frac{2}{\pi} \cdot 1 = \frac{2}{\pi}.$$

f)

The first part can be easily proved by induction, and hence the assertion holds. I may come back and update this at some point.

2.13 Problem 13

a)

$$\begin{aligned} \int_m^{2m} \ln x \, dx &= [x \ln x]_m^{2m} - \int_m^{2m} dx && \text{(Integration by parts)} \\ &= 2m \ln(2m) - m \ln(m) - m \\ &= m \ln m + 2m \ln 2 - m. \end{aligned}$$

b)

$$\begin{aligned} \int_m^{2m} \ln x \, dx &\approx \frac{1}{2}(\ln m + 2\ln(m+1) + \cdots + 2\ln(2m-1) + \ln(2m)) \\ &= \sum_{k=m}^{2m} \ln k - \frac{1}{2}\ln m - \frac{1}{2}\ln(2m) \\ &= \sum_{k=m+1}^{2m} \ln k - \frac{1}{2}\ln 2 \\ &= \ln\left(\frac{(2m)!}{m!}\right) - \frac{1}{2}\ln 2. \end{aligned}$$

c)

$$\lim_{m\to\infty} \frac{(b-a)^3 M}{12m^2} = 0.$$

d)

$$\begin{aligned}
0 &= \lim_{m\to\infty} \left(\ln \left(\frac{(2m)!}{m!} \right) - \frac{1}{2}\ln 2 - (m\ln m + 2m\ln 2 - m) \right) \\
&= \lim_{m\to\infty} \left(\ln \left(\frac{(2m)!}{m!} \right) - (\ln \sqrt{2} + \ln m^m + \ln 2^{2m} + \ln e^{-m}) \right) \\
&= \lim_{m\to\infty} \left(\ln \left(\frac{(2m)!}{m!} \Big/ \left\{ \sqrt{2}\, 2^{2m} m^m e^{-m} \right\} \right) \right) \\
&= \ln \left(\lim_{m\to\infty} \left(\frac{(2m)!}{m!} \Big/ \left\{ \sqrt{2}\, 2^{2m} m^m e^{-m} \right\} \right) \right),
\end{aligned}$$

which implies that

$$\frac{(2m)!}{m!} \Big/ \left\{ \sqrt{2}\, 2^{2m} m^m e^{-m} \right\} \to 1$$

as $m \to \infty$.

2.14 Problem 14

By the results from Problem 13 and 14, we have

$$\begin{aligned}
\frac{\pi}{2} &= \lim_{m\to\infty} \frac{2^{4m}(m!)^4}{(2m+1)((2m)!)^2} \\
&= \lim_{m\to\infty} \frac{2^{4m}(m!)^2}{(2m+1)\left(\frac{(2m)!}{m!}\right)^2} \\
&= \lim_{m\to\infty} \frac{2^{4m}(m!)^2}{(2m+1)(\sqrt{2}\cdot 2^{2m} m^m e^{-m})^2} \\
&= \lim_{m\to\infty} \frac{(m!)^2}{(4m+2)m^{2m}e^{-2m}} \\
&= \lim_{m\to\infty} \frac{(m!)^2}{4m m^{2m} e^{-2m}}.
\end{aligned}$$

Taking the square roots on both sides and rearranging the equation, we obtain

$$\frac{m!}{\sqrt{2\pi}\, m^{m+\frac{1}{2}} e^{-m}} \to 1.$$

2.15 Problem 15

a)

$$\begin{aligned}
\int_0^1 \frac{x^2}{\sqrt{4-x^2}}\,dx &= \int_0^{\pi/6} \frac{(2\sin\theta)^2}{2\cos\theta}\,d(2\sin\theta) \\
&= \int_0^{\pi/6} 4\sin^2\theta\,d\theta \\
&= \Big[2\theta - \sin 2\theta\Big]_0^{\pi/6} \\
&= \frac{\pi}{3} - \frac{\sqrt{3}}{2}.
\end{aligned}$$

b)

Let $x = 2\tan\theta + 3$ where $\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$. Then,

$$\begin{aligned}
\int \frac{dx}{\sqrt{x^2-6x+13}} &= \int \frac{d(2\tan\theta+3)}{\sqrt{(2\tan\theta)^2+4}} \\
&= \int \sec\theta\,d\theta \\
&= \ln|\tan\theta + \sec\theta| + C_0 \\
&= \ln\left|\frac{x-3}{2} + \frac{1}{2}\sqrt{x^2-6x+13}\right| + C_0 \\
&= \ln\left|x - 3 + \sqrt{x^2-6x+13}\right| + C.
\end{aligned}$$

c)

$$\begin{aligned}
\int_0^3 \sqrt{9-x^2}\,dx &= \int_0^{\pi/2} \sqrt{9-(3\sin\theta)^2}\,d(3\sin\theta) \\
&= 9\int_0^{\pi/2} \cos^2\theta\,d\theta \\
&= 9\int_0^{\pi/2} \frac{1+\cos 2\theta}{2}\,d\theta \\
&= \left[\frac{9}{2}\theta + \frac{9}{4}\sin 2\theta\right]_0^{\pi/2} \\
&= \frac{9}{4}\pi.
\end{aligned}$$

d)

$$\begin{aligned}
\int \frac{dx}{x^2\sqrt{x^2+16}} &= \int \frac{d(4\sinh u)}{(4\sinh u)^2\sqrt{(4\sinh u)^2+16}} \\
&= \int \frac{4\cosh u\,du}{16\sinh^2 u\, 4\cosh u} \\
&= \frac{1}{16}\int \frac{du}{\sinh^2 u} \\
&= -\frac{\coth u}{16} + C \\
&= -\frac{\sqrt{x^2+16}}{16x} + C.
\end{aligned}$$

e)

$$\begin{aligned}
\int (1-x^2)^{-3/2}\,dx &= \int (1-\sin^2\theta)^{-3/2}\,d(\sin\theta) \\
&= \int \sec^2\theta\,d\theta \\
&= \tan\theta + C \\
&= \frac{x}{\sqrt{1-x^2}} + C.
\end{aligned}$$

f)

Let $x = \tan\theta - 1$ where $\theta \in [0, \arctan 2]$. Then,

$$\begin{aligned}
\int_{-1}^{1} \frac{dx}{x^2+2x+2} &= \int_0^{\arctan 2} \frac{d(\tan\theta - 1)}{\tan^2\theta + 1} \\
&= \int_0^{\arctan 2} d\theta \\
&= \arctan 2 + C.
\end{aligned}$$

2.17 Problem 17

c)

The answer given in the textbook is probably wrong.

Let

$$\frac{x+1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}.$$

Using the cover-up method, we obtain $B = -1, C = 2$ and $A = -2$. Hence,

$$\begin{aligned}
\int \frac{x+1}{x^2(x-1)}\,dx &= \int \frac{-2}{x}\,dx + \int \frac{-1}{x^2}\,dx + \int \frac{2}{x-1}\,dx \\
&= -2\ln|x| + \frac{1}{x} + 2\ln|x-1| + C.
\end{aligned}$$

2.18 Problem 18

d)

Let $x = u^6$, then

$$\begin{aligned}
\int_1^{64} \frac{1}{x^{1/2} + x^{1/3}}\,dx &= \int_1^2 \frac{6u^5\,du}{u^3 + u^2} \\
&= \int_1^2 \frac{6u^3}{u+1}\,du \\
&= \int_1^2 \frac{6(u^3+1) - 6}{u+1}\,du \\
&= 6\int_1^2 (u^2 - u + 1)\,du - 6\int_1^2 \frac{1}{u+1}\,du \\
&= 11 + 6\ln\frac{2}{3}.
\end{aligned}$$

2.20 Problem 20

a)

$$\begin{aligned}
I &= \int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx \\
&= \int_{\pi}^{0} \frac{(\pi - u)\sin(\pi - u)}{1 + \cos^2(\pi - u)}\,d(\pi - u) \\
&= \int_0^{\pi} \frac{(\pi - u)\sin u}{1+\cos^2 u}\,du \\
&= \pi\int_0^{\pi} \frac{\sin u}{1+\cos^2 u}\,du - I.
\end{aligned}$$

Hence,

$$\begin{aligned}
I &= \frac{\pi}{2}\int_0^{\pi} \frac{\sin u}{1+\cos^2 u}\,du \\
&= \frac{\pi}{2}\int_{-1}^{1} \frac{1}{1+t^2}\,dt && \text{(Let } t = \cos u\text{)} \\
&= \frac{\pi}{2}\Big[\arctan t\Big]_{-1}^{1} \\
&= \frac{\pi^2}{4}.
\end{aligned}$$

b)

Let $x = \dfrac{1-u}{1+u}$, then

$$\begin{aligned}
I &= \int_0^1 \frac{\ln(1+x)}{1+x^2}\,dx \\
&= \int_1^0 \frac{\ln\left(\frac{2}{1+u}\right)}{1+\left(\frac{1-u}{1+u}\right)^2}\frac{-2}{(1+u)^2}\,du \\
&= \ln 2\int_0^1 \frac{1}{1+u^2}\,du - I.
\end{aligned}$$

Hence,

$$\begin{aligned}
I &= \frac{1}{2}\ln 2\int_0^1 \frac{1}{1+u^2}\,du \\
&= \frac{1}{2}\ln 2\Big[\arctan u\Big]_0^1 \\
&= \frac{1}{2}\ln 2\cdot\frac{\pi}{4} \\
&= \frac{\pi}{8}\ln 2.
\end{aligned}$$

2.21 Problem 21

$$\begin{aligned}
\int \frac{x^2-1}{x^2+1}\frac{1}{\sqrt{1+x^4}}\,dx &= \int \frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)\frac{1}{x}}\frac{1}{\sqrt{1+x^4}}\,dx \\
&= \int \frac{1-\frac{1}{x^2}}{x+\frac{1}{x}}\frac{1}{\sqrt{x^2+\frac{1}{x^2}}}\,dx \\
&= \int \frac{du}{u\sqrt{u^2-2}} && (\text{Let } u = x+\frac{1}{x}) \\
&= \int \frac{d(\sqrt{2}\sec\theta)}{\sqrt{2}\sec\theta\sqrt{2\sec^2\theta-2}} && (\text{Let } u=\sqrt{2}\sec\theta) \\
&= \frac{\sqrt{2}}{2}\int d\theta \\
&= \frac{\sqrt{2}}{2}\arctan\sqrt{\frac{u^2}{2}-1}+C \\
&= \frac{\sqrt{2}}{2}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{x^2+\frac{1}{x^2}}\right)+C.
\end{aligned}$$