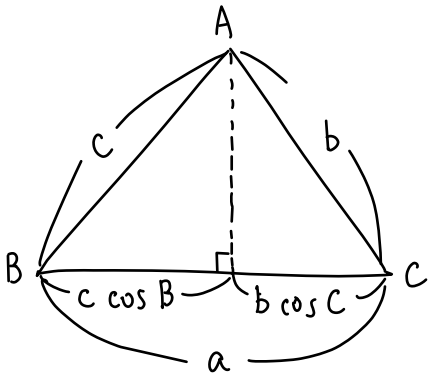
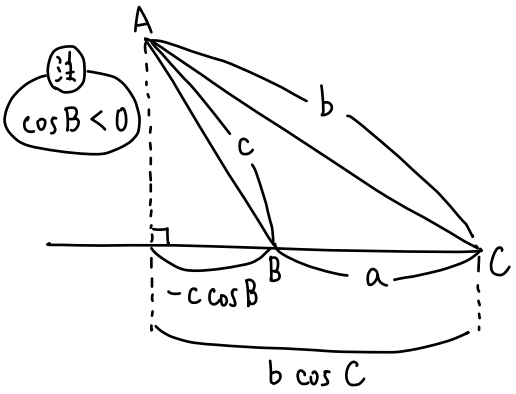
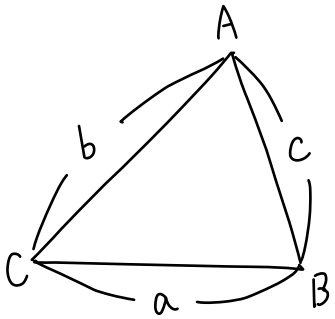
第一および第二余弦定理



$a = c\cos B + b\cos C$



$a = \underbrace{c\cos B}_{=-(-\cos B)} + b\cos C$ ← 第一余弦定理.



第一余弦定理

$a = c\cos B + b\cos C \xrightarrow{\times a} a^2 = ac\cos B + ab\cos C$

$b = a\cos C + c\cos A \xrightarrow{\times b} b^2 = ab\cos C + bc\cos A$

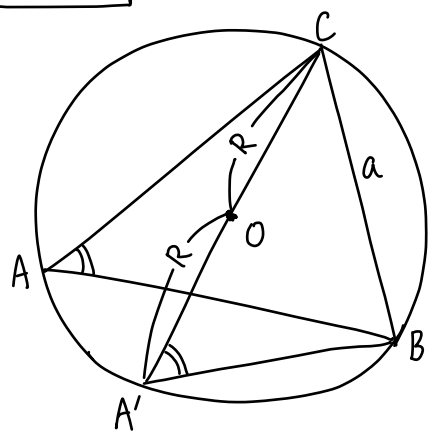
$c = b\cos A + a\cos B \xrightarrow{\times(-c)} -c^2 = -bc\cos A - ac\cos B$

$a^2 + b^2 - c^2 = 2ab\cos C.$

ゆえに, $c^2 = a^2 + b^2 - 2ab\cos C$. ← 第二余弦定理

特に $C = 90°$ のとき, $\cos C = 0$ なので $a^2 + b^2 = c^2$. ← 三平方の定理

正弦定理



$a = 2R\sin A' = 2R\sin A$

$\therefore \dfrac{a}{\sin A} = 2R$ (R は △ABC の外接円の半径).

同様にして,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R.$$