https://colab.research.google.com/drive/1oX1Uwq00S473M4Jcy_nCRZMFAN3F1UyN?usp=sharing

[5]
```
n = 5
s = [sympy.Symbol("s$i"; commutative=false) for i
```

$$\begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix}$$

[6]
```
A = [j ≤ i+1 ? s[i] : oftype(s[i], 0) for i in 1:
```

$$\begin{bmatrix} s_1 & s_1 & 0 & 0 \\ s_2 & s_2 & s_2 & 0 \\ s_3 & s_3 & s_3 & s_3 \\ s_4 & s_4 & s_4 & s_4 \end{bmatrix}$$

[7]
```
x1 = s
```

$$\begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix}$$

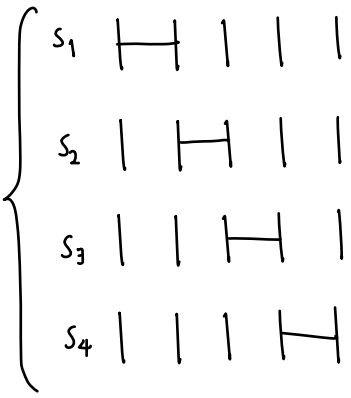
```
x2 = A*x1
```

$$\begin{bmatrix} s_1 s_2 + s_1^2 \\ s_2 s_1 + s_2 s_3 + s_2^2 \\ s_3 s_1 + s_3 s_2 + s_3 s_4 + s_3^2 \\ s_4 s_1 + s_4 s_2 + s_4 s_3 + s_4^2 \end{bmatrix}$$

[9]
```
x3 = A*x2 .|> expand
```

$$\begin{bmatrix} s_1 s_2 s_1 + s_1 s_2 s_3 + s_1 s_2^2 + s_1^2 s_2 + s_1^3 \\ s_2 s_1 s_2 + s_2 s_1^2 + s_2 s_3 s_1 + s_2 s_3 s_2 + s_2 s_3 s_4 + s_2 s_3^2 + s_2^2 s_1 + s_2^2 s_3 + s_2^3 \\ s_3 s_1 s_2 + s_3 s_1^2 + s_3 s_2 s_1 + s_3 s_2 s_3 + s_3 s_2^2 + s_3 s_4 s_1 + s_3 s_4 s_2 + s_3 s_4 s_3 + s_3 s_4^2 + s_3^2 s_1 + s_3^2 s_2 + s_3^2 s_4 + s_3^3 \\ s_4 s_1 s_2 + s_4 s_1^2 + s_4 s_2 s_1 + s_4 s_2 s_3 + s_4 s_2^2 + s_4 s_3 s_1 + s_4 s_3 s_2 + s_4 s_3 s_4 + s_4 s_3^2 + s_4^2 s_1 + s_4^2 s_2 + s_4^2 s_3 + s_4^3 \end{bmatrix}$$

$S = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix}$

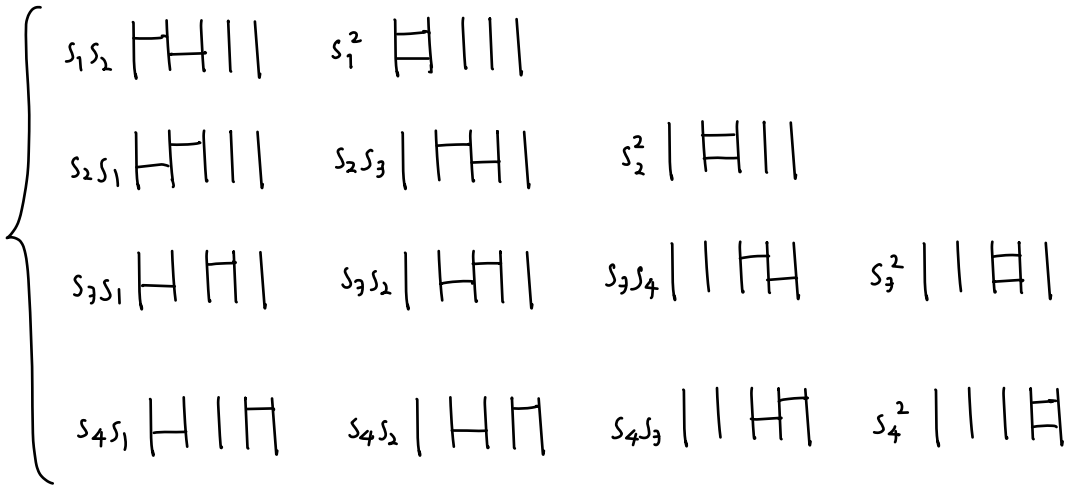


$S = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix}$, $A = \begin{bmatrix} s_1 & s_1 & 0 & 0 \\ s_2 & s_2 & s_2 & 0 \\ s_3 & s_3 & s_3 & s_3 \\ s_4 & s_4 & s_4 & s_4 \end{bmatrix}$

s_i's are non-commutative variables.

AS

$$\begin{bmatrix} s_1 s_2 + s_1^2 \\ s_2 s_1 + s_2 s_3 + s_2^2 \\ s_3 s_1 + s_3 s_2 + s_3 s_4 + s_3^2 \\ s_4 s_1 + s_4 s_2 + s_4 s_3 + s_4^2 \end{bmatrix}$$



$A^2 S$

$$\begin{bmatrix} s_1 s_2 s_1 + s_1 s_2 s_3 + s_1 s_2^2 + s_1^2 s_2 + s_1^3 \\ s_2 s_1 s_2 + s_2 s_1^2 + s_2 s_3 s_1 + s_2 s_3 s_2 + s_2 s_3 s_4 + s_2 s_3^2 + s_2^2 s_1 + s_2^2 s_3 + s_2^3 \\ s_3 s_1 s_2 + s_3 s_1^2 + s_3 s_2 s_1 + s_3 s_2 s_3 + s_3 s_2^2 + s_3 s_4 s_1 + s_3 s_4 s_2 + s_3 s_4 s_3 + s_3 s_4^2 + s_3^2 s_1 + s_3^2 s_2 + s_3^2 s_4 + s_3^3 \\ s_4 s_1 s_2 + s_4 s_1^2 + s_4 s_2 s_1 + s_4 s_2 s_3 + s_4 s_2^2 + s_4 s_3 s_1 + s_4 s_3 s_2 + s_4 s_3 s_4 + s_4 s_3^2 + s_4^2 s_1 + s_4^2 s_2 + s_4^2 s_3 + s_4^3 \end{bmatrix}$$