Module 2 (1) – Expectation and variance, variance estimation, simple simulations

Exercise 1: Estimating the total number and its variance of Dungeness crab within three bays (A, B, and C) in SE Alaska during the personal use pot fishery.

Assume that we know the following from previous sampling:

- The probability of catching any crab in a pot after soaking it for a week has been estimated at 30% in Bay A, 50% in Bay B, and 80% in Bay C.
  [These are binomial probabilities. That is, a pot either catches crab (1) or it doesn't (0) and the variance of a single binomial event, as we will see later, is $p*(1-p)$. Assume that individual pot lifts are independent and there were a total of n pot lifts, X of which were successful and actually caught crab. Then the expected value of the number of successful pot lifts that catch crab is $E(X) = n*p$ and its variance is: $Var(X) = n*p*(1-p)$]
- The average catch per pot when crab are actually caught (averaged over only those pots with positive catches) and its standard deviation has previously been estimated as:
  - μ_A = 6 fish in Bay A (standard deviation s_A = 1.8)
  - μ_B = 8 fish in Bay B (standard deviation s_B = 2.2)
  - μ_C = 8 fish in Bay C (standard deviation s_C = 3.0)

A survey determined that the total number of pot lifts (assuming pots were checked once a week) was N_A = 86 in Bay A, N_B = 123 in Bay B and N_C = 58 in Bay C (these are assumed to be known without error).

What is the expected total number of crabs (C) caught in the personal use fishery and its variance?

Hints:

- Compute expected numbers (N_i) and variances (σ_i^2) for a single bay and then add across bays
- Let $X_A = N_A * f_A$ be the number of successful pot lifts that catch crab in Bay A (X_b, X_c for the other bays), where f_A is the fraction of pots that caught crab [$E(f_A) = p_A$].
- Let Y_A = the number of crab caught in a successful pot lift in Bay A
- Hence, the expected total number of crab caught in Bay A is: $E(C_A) = E(X_A) * E(Y_A)$
- Using the formulas provided in Mod 2(1), compute the expected value of the total number of crab caught in the fishery $C = C_A + C_B + C_C$ and its variance

See also script "Mod 2(1) Exercise 1.R", which will be posted in Canvas.

Exercise 2: A. Estimate variance using the delta method

B. Simple bootstrap estimate of the variance of the median

See script "Mod 2(1) Exercise 2.R"