

3-1_DimensionalityReduction_Clustering

2018 年 11 月 20 日

1 次元削減とクラスタリング

2 0. 事前準備

```
In [1]: %matplotlib inline
```

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns

sns.set(style='whitegrid')
pd.options.display.float_format = '{:,.2f}'.format
pd.set_option('display.width', 800)

import matplotlib
import sklearn

print('pandas',pd.__version__)           # VM+Ubuntu: 0.23.4
print('numpy',np.__version__)             # VM+Ubuntu: 1.15.1
print('matplotlib',matplotlib.__version__) # VM+Ubuntu: 2.2.3
print('seaborn',sns.__version__)           # VM+Ubuntu: 0.9.0
print('scikit-learn',sklearn.__version__) # VM+Ubuntu: 0.19.2
```

```
pandas 0.23.4
numpy 1.15.4
matplotlib 3.0.1
seaborn 0.9.0
scikit-learn 0.20.0
```

```
In [2]: # データ読み込み
```

```
df = pd.read_table('./input/count_tpm.tsv', index_col=0)
print(df.head())
print(len(df))

    batch_1  batch_2  batch_3  chemostat_1  chemostat_2  chemostat_3
```

```

gene_id
gene_0001    0.00    0.73    3.13    0.00    0.00    0.50
gene_0002    0.00    0.00    0.00    0.00    0.00    0.00
gene_0003    0.00    0.00    0.00    0.00    0.00    0.00
gene_0004    0.00    0.00    0.00    0.00    0.00    0.00
gene_0005    0.95    2.80    4.97    4.69    4.37    8.66
5983

```

```

In [3]: # 全サンプルで TPM がゼロの遺伝子のレコードを削除
        all_zero_index = df.index[df.sum(axis=1) == 0]
        df = df.drop(all_zero_index)
        print(df.head())
        print(len(df))

```

```

        batch_1  batch_2  batch_3  chemostat_1  chemostat_2  chemostat_3
gene_id
gene_0001    0.00    0.73    3.13    0.00    0.00    0.50
gene_0005    0.95    2.80    4.97    4.69    4.37    8.66
gene_0009    1.46    1.24    1.57    3.23    3.78    3.88
gene_0010    8.55    7.96    8.13   159.95   159.68   147.90
gene_0011   17.15   12.23   13.44   147.00   166.23   154.75
5892

```

```

In [4]: # サンプルをプロットするときの色を設定
        sample_colors = {'batch_1': 'b', # b : blue
                          'batch_2': 'b',
                          'batch_3': 'b',
                          'chemostat_1': 'g', # g: green
                          'chemostat_2': 'g',
                          'chemostat_3': 'g'}
        colors = df.columns.map(sample_colors)

```

3 1. 次元削減

3.1 1. 1 行列分解に基づく次元削減

3.1.1 1.1.1 PCA (Principal Component Analysis; 主成分分析)

```

In [5]: import sklearn.decomposition

```

```

In [6]: # PCA 実行
        pca = sklearn.decomposition.PCA()
        coords = pca.fit_transform(df.transpose().values)

```

```

In [7]: def scatter_plot(coords, sample_labels, colors, xlabel=None, ylabel=None, title=

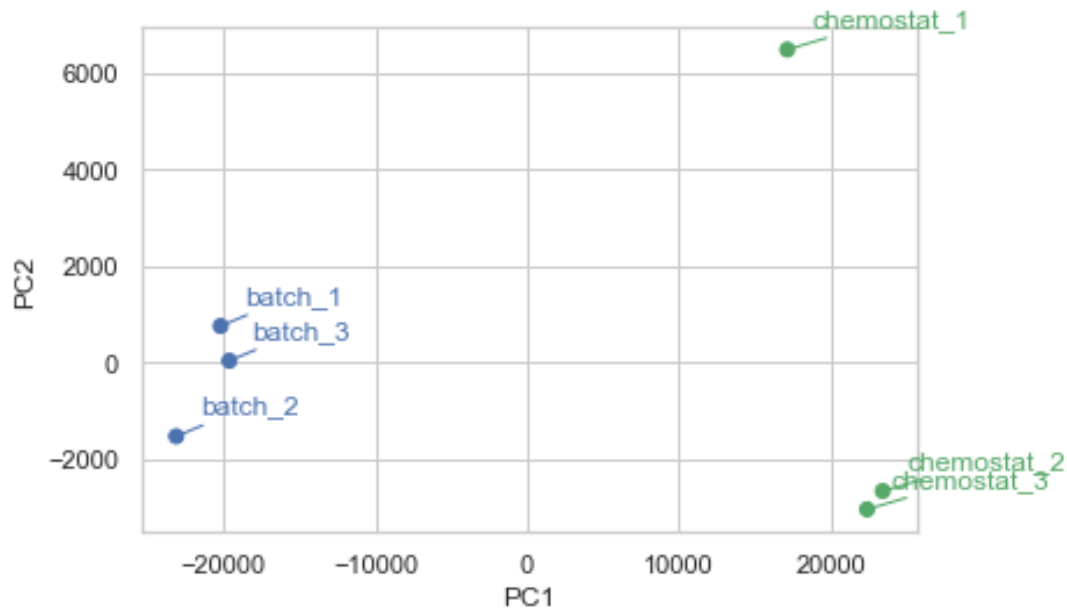
```

```

fig = plt.figure()
ax = fig.add_subplot(111)
ax.scatter(coords[:, 0], coords[:, 1], color=colors)
for i, sample_label in enumerate(sample_labels):
    ax.annotate(sample_label, xy=(coords[i, :2]), xytext=(10,10),
                textcoords='offset points', color=colors[i],
                arrowprops={'arrowstyle':'-', 'edgecolor':colors[i]})
ax.set_xlabel(xlabel)
ax.set_ylabel(ylabel)
ax.set_title(title)
plt.show()

```

In [8]: scatter_plot(coords, df.columns, colors, xlabel='PC1', ylabel='PC2')



In [9]: # z-score 標準化

```

import sklearn.preprocessing
values = df.transpose().values
scaler = sklearn.preprocessing.StandardScaler(with_mean=True, with_std=True)
std_values = scaler.fit_transform(values)
std_df = pd.DataFrame(std_values.T, index=df.index, columns=df.columns)

print('\nRaw values')
print('TPM Average :')
print(df.mean(axis=1)[:10])
print('\nTPM Standard deviation :')
print(df.std(ddof=0, axis=1)[:10])

```

```

print('\n\nStandardized values')
print('TPM Average :')
print(std_df.mean(axis=1)[:10])
print('\n\nTPM Standard deviation :')
print(std_df.std(ddof=0, axis=1)[:10])

#おんなじことを自分で計算する場合
#values = df.transpose().values
#std_values = (values - values.mean(axis=0)) / values.std(axis=0)
#std_df2 = pd.DataFrame(std_values.T, index=df.index, columns=df.columns)
#print('\n\nStandardized values 2')
#print('TPM Average :')
#print(std_df2.mean(axis=1)[:10])
#print('\n\nTPM Standard deviation:')
#print(std_df2.std(ddof=0, axis=1)[:10])

```

Raw values

TPM Average :

gene_id

gene_0001	0.73
gene_0005	4.41
gene_0009	2.53
gene_0010	82.03
gene_0011	85.13
gene_0012	214.09
gene_0013	80.77
gene_0014	32.10
gene_0015	33.83
gene_0016	27.58

dtype: float64

TPM Standard deviation :

gene_id

gene_0001	1.11
gene_0005	2.35
gene_0009	1.12
gene_0010	73.92
gene_0011	71.10
gene_0012	22.93
gene_0013	13.01
gene_0014	4.40
gene_0015	12.19

```
gene_0016    12.55
dtype: float64
```

Standardized values

TPM Average :

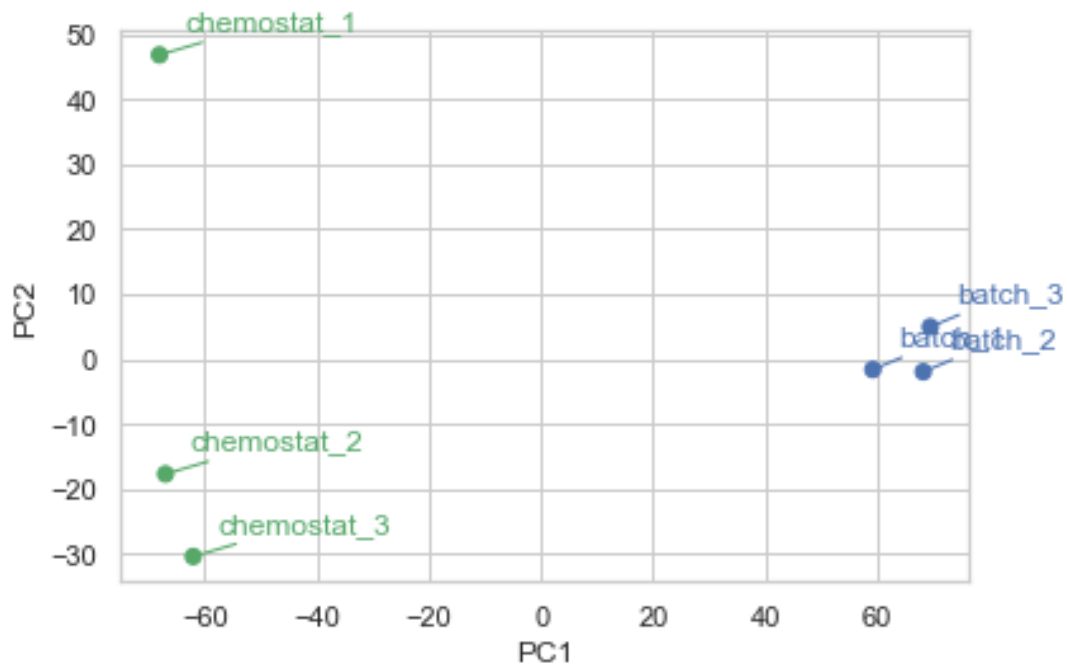
```
gene_id
gene_0001    0.00
gene_0005   -0.00
gene_0009   -0.00
gene_0010   -0.00
gene_0011   -0.00
gene_0012    0.00
gene_0013    0.00
gene_0014   -0.00
gene_0015    0.00
gene_0016   -0.00
dtype: float64
```

TPM Standard deviation :

```
gene_id
gene_0001    1.00
gene_0005    1.00
gene_0009    1.00
gene_0010    1.00
gene_0011    1.00
gene_0012    1.00
gene_0013    1.00
gene_0014    1.00
gene_0015    1.00
gene_0016    1.00
dtype: float64
```

In [10]: # 標準化されたデータで PCA 実行

```
pca = sklearn.decomposition.PCA()
coords = pca.fit_transform(std_df.transpose().values)
scatter_plot(coords, std_df.columns, colors, xlabel='PC1', ylabel='PC2')
```



```
In [11]: # 各主成分の「寄与率」
          print([' {:.2f}%'.format(x*100) for x in pca.explained_variance_ratio_])

['73.55%', '9.78%', '7.09%', '5.54%', '4.04%', '0.00%']
```

3.1.2 1.1.2 NMF (Non-negative Matrix Factorization; 非負値行列因子分解)

$X \sim W * H$

X: non-negative matrix. (n, m)

W: non-negative matrix. (n, k)

H: non-negative matrix. (k, m)

```
In [12]: # 擬似データで NMF のデモ
```

```
def plot_W_H_X(W, H, X):
    classes = ['Class%d'%x for x in range(W.shape[1])]
    genes = ['Gene%d'%x for x in range(W.shape[0])]
    samples = ['Sample%d'%x for x in range(H.shape[1])]
    fig = plt.figure()
    ax1 = plt.axes([0.0, 0.1, 0.1, 0.8])
    sns.heatmap(pd.DataFrame(W, index=genes, columns=classes), cmap='Greys', cbar_kws={'label': 'W'})
    ax2 = plt.axes([0.2, 0.3, 0.4, 0.5])
    pd.DataFrame(H, index=classes, columns=samples).transpose().plot.bar(stacked=True, ax=ax2)
    plt.legend(loc=(0., 1.04), ncol=3, fontsize=8)
    ax3 = plt.axes([0.8, 0.1, 0.4, 0.8])
    X.plot(ax=ax3, legend=False)
```

```

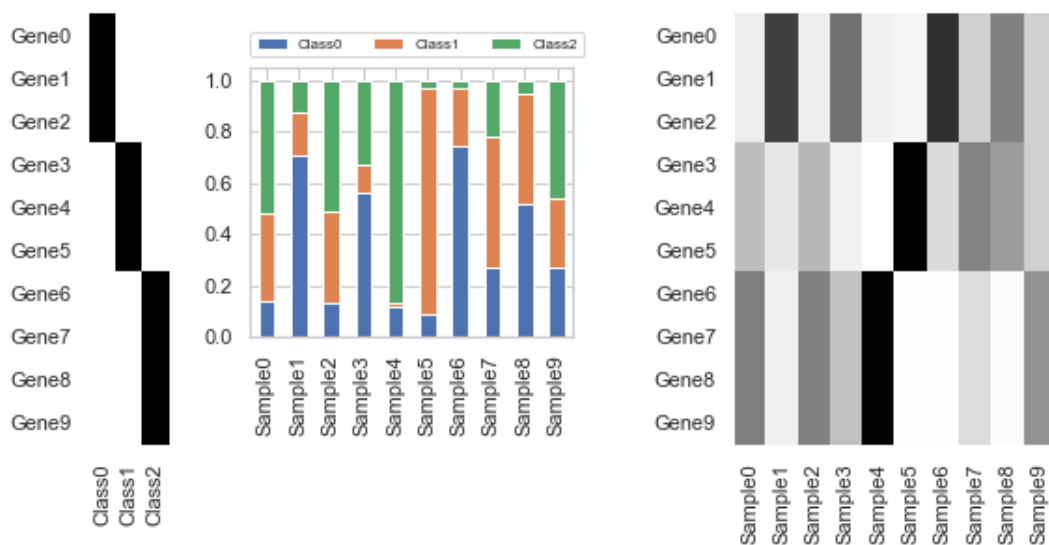
sns.heatmap(pd.DataFrame(X, index=genes, columns=samples), cmap='Greys', ch
plt.show()

print('Original')
# class-0 は最初の 3 個の遺伝子を発現
genes = np.array([[1.0, 1.0, 1.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0],
# class-1 はまんなか 3 個の遺伝子を発現
[0.0, 0.0, 0.0, 1.0, 1.0, 1.0, 0.0, 0.0, 0.0, 0.0],
# class-2 はうしろ 4 個の遺伝子を発現
[0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 1.0, 1.0, 1.0, 1.0]]).T
# 各サンプルが class-0, class-1, class2 をどういう割合で持っているか
samples = np.random.dirichlet(alpha=[1.0]*3, size=10).T
# 遺伝子発現テーブルはその掛け算で決まっている (と仮定する)
original_expression_data = np.dot(genes, samples)
plot_W_H_X(genes, samples, original_expression_data)

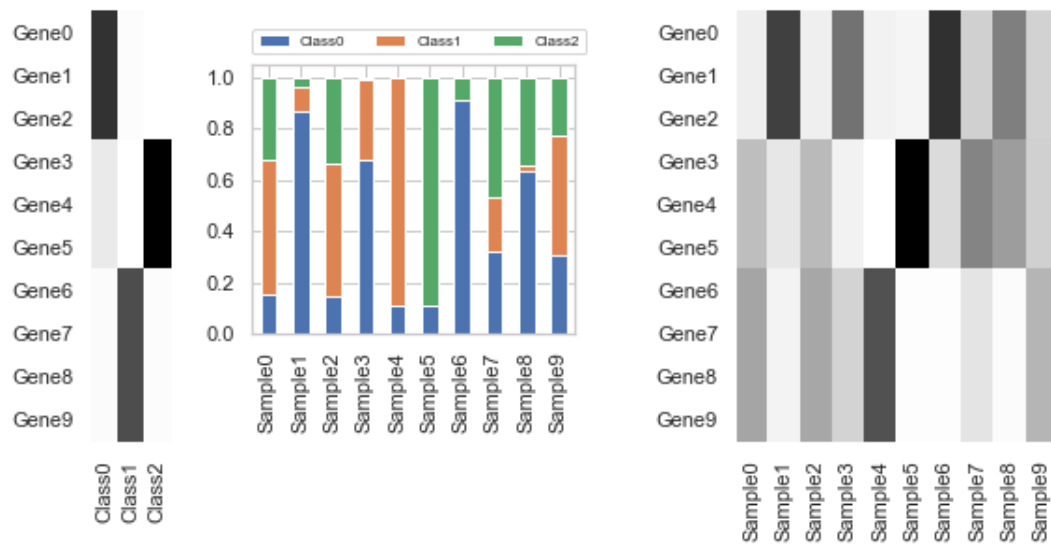
print('Reconstructed')
# 遺伝子発現テーブルだけを使って、クラスごとの発現パターンベクトル、サンプルごとのクラス割合を復元する
model = sklearn.decomposition.NMF(n_components=3)
W = model.fit_transform(original_expression_data)
W /= W.sum(axis=0)
H = model.components_
H /= H.sum(axis=0)
X = np.dot(W, H)
plot_W_H_X(W, H, X)

```

Original



Reconstructed



```
In [13]: model = sklearn.decomposition.NMF(n_components=2)
```

```
W = model.fit_transform(df.values)
```

```
H = model.components_
```

```
print('Original shape=', df.values.shape)
```

```
print('W shape =', W.shape)
```

```
print('H shape =', H.shape)
```

```
Original shape= (5892, 6)
```

```
W shape = (5892, 2)
```

```
H shape = (2, 6)
```

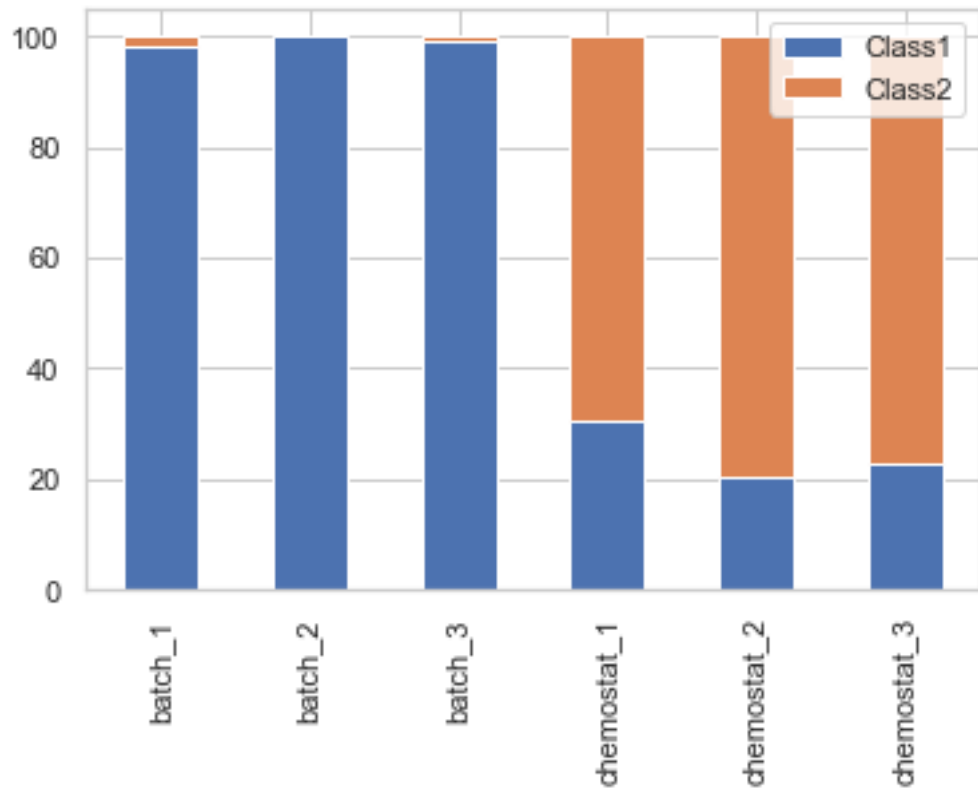
```
In [14]: # 次元削減としての利用
```

```
H_percentage = 100.0 * H / H.sum(axis=0)
```

```
pd.DataFrame(H_percentage.T, index=df.columns, columns=['Class1', 'Class2']).plot
```

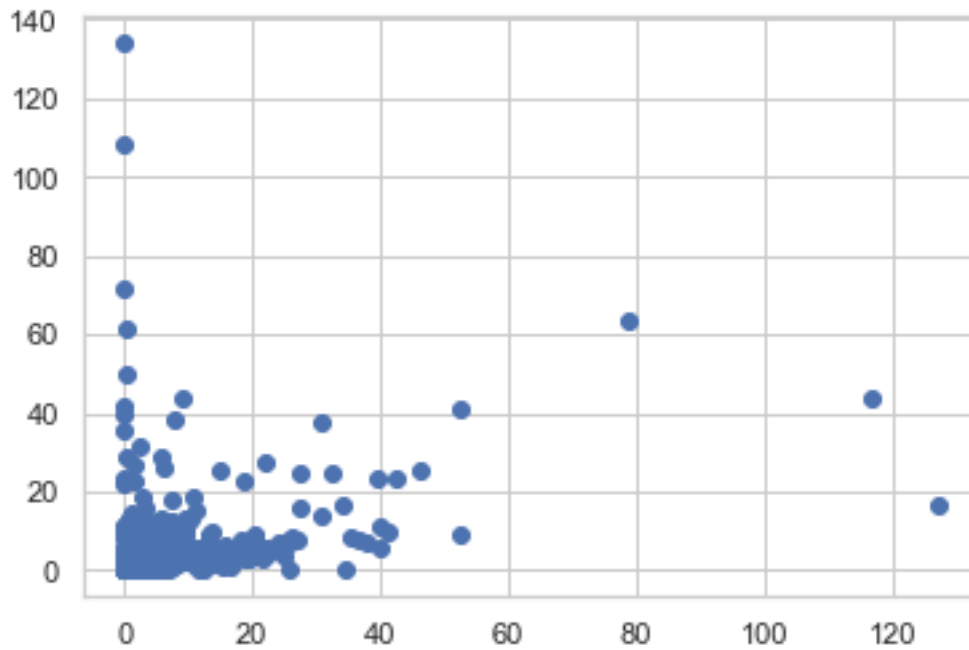
```
#各サンプル、もっとも値の高い要素に割り当てることでクラスタリングの代わりとしてしまうこともある。
```

```
Out[14]: <matplotlib.axes._subplots.AxesSubplot at 0x1a235d96d8>
```

In [15]: # それぞれの因子に強く寄与している遺伝子はなにか？

```
fig = plt.figure()
ax = fig.add_subplot(111)
ax.scatter(W[:,0], W[:,1])
plt.show()
```



In [16]: # 遺伝子の *product* 情報をロード

```
gene_products = pd.read_table('./input/gene_id_product.tsv', index_col=0, names
```

In [17]: # 因子1 に強く寄与する遺伝子

```
topN = 50
```

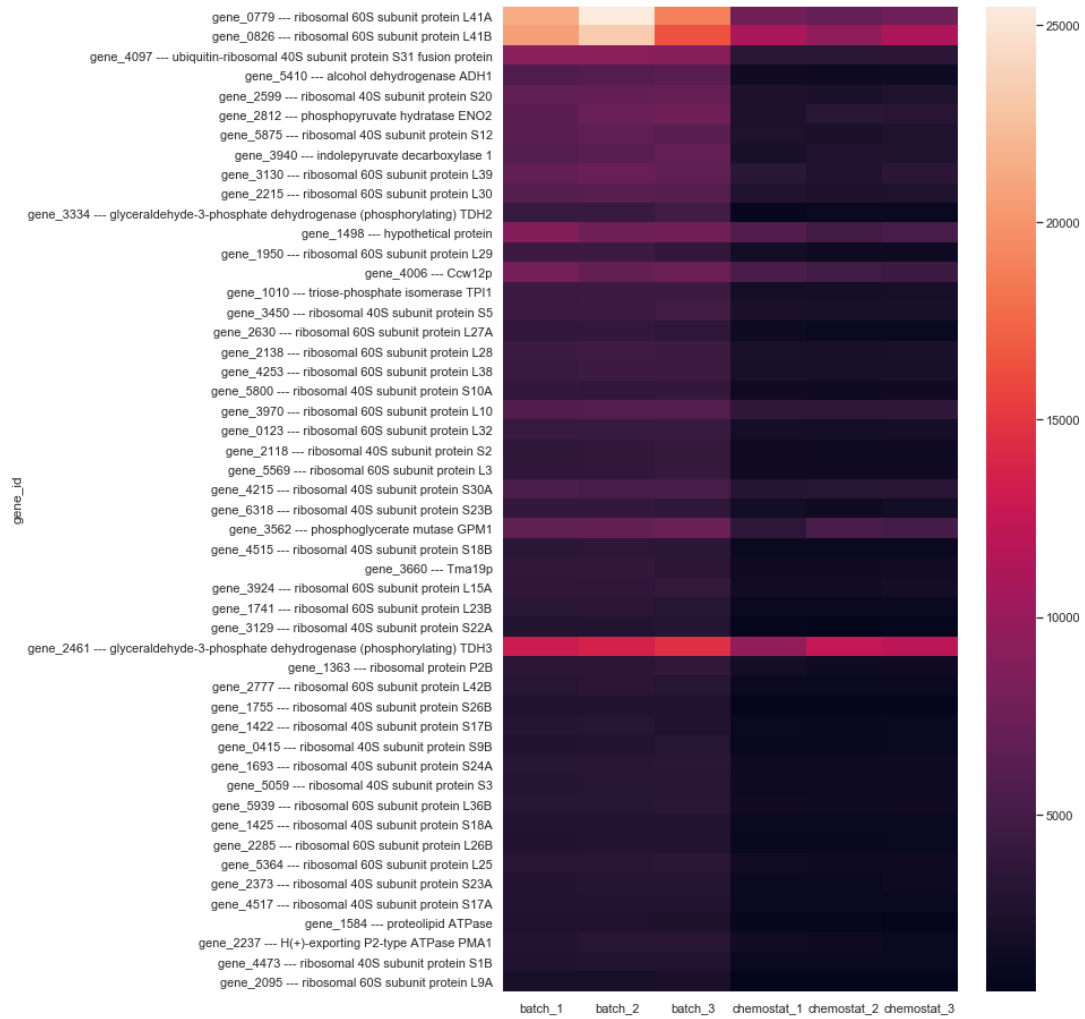
```
top_factor1 = df.index[ np.argsort(W[:,0] - W[:,1])[:, -1][:topN] ]
```

```
gene_labels = top_factor1 + ' --- ' + gene_products.loc[top_factor1, 'product']
```

```
fig = plt.figure(figsize=(9,16))
```

```
sns.heatmap(df.loc[top_factor1, :], yticklabels=gene_labels)
```

Out [17]: <matplotlib.axes._subplots.AxesSubplot at 0x1a2379edd8>

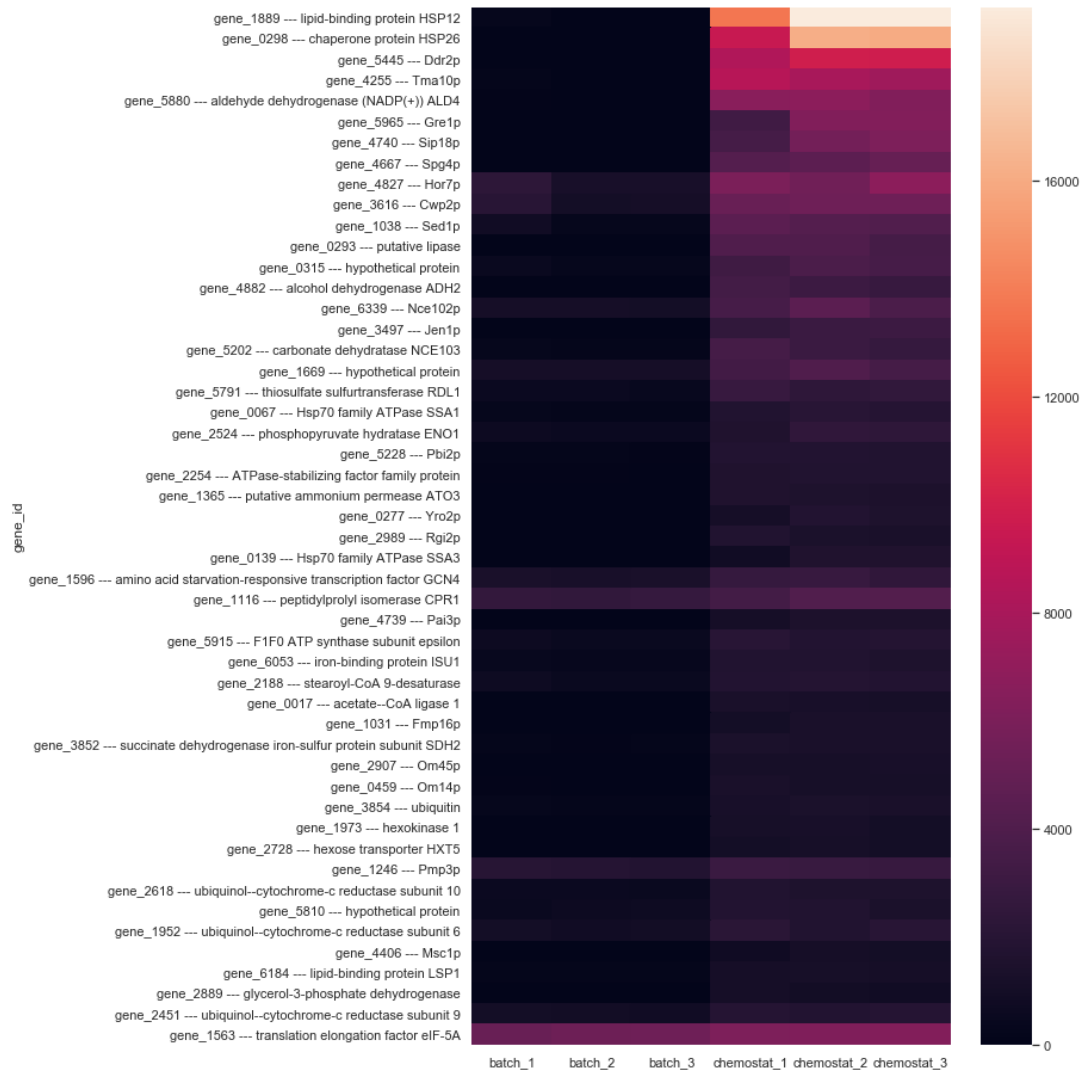


In [18]: # 因子 2 に強く寄与する遺伝子

```
top_factor2 = df.index[ np.argsort(W[:,0] - W[:,1])[:topN] ]
gene_labels = top_factor2 + ' --- ' + gene_products.loc[top_factor2, 'product']

fig = plt.figure(figsize=(9,16))
sns.heatmap(df.loc[top_factor2, :], yticklabels=gene_labels)
```

Out[18]: <matplotlib.axes._subplots.AxesSubplot at 0x1a239b3400>



3.1.3 1.1.3 LSI (Latent semantic indexing; 潜在意味解析)

In [19]: # *TF-IDF* 変換

```
# TF ... Term Frequency   いくつかの流儀がある。ここではサンプルごとの max に対する割合
TF = df.values / df.values.max(axis=0)

# IDF ... Inverse Document Frequency   これもいくつかの流儀あり。
n_samples = len(df.columns)
IDF = np.log2(1.0 + (float(n_samples) / df.values.astype(bool).sum(axis=1)))

# TF-IDF
TFIDF = TF * IDF[:, np.newaxis]
df_tfidf = pd.DataFrame(TFIDF, index=df.index, columns=df.columns)
```

In [20]: # *SVD* (Singular Value Decomposition; 特異値分解) を実行

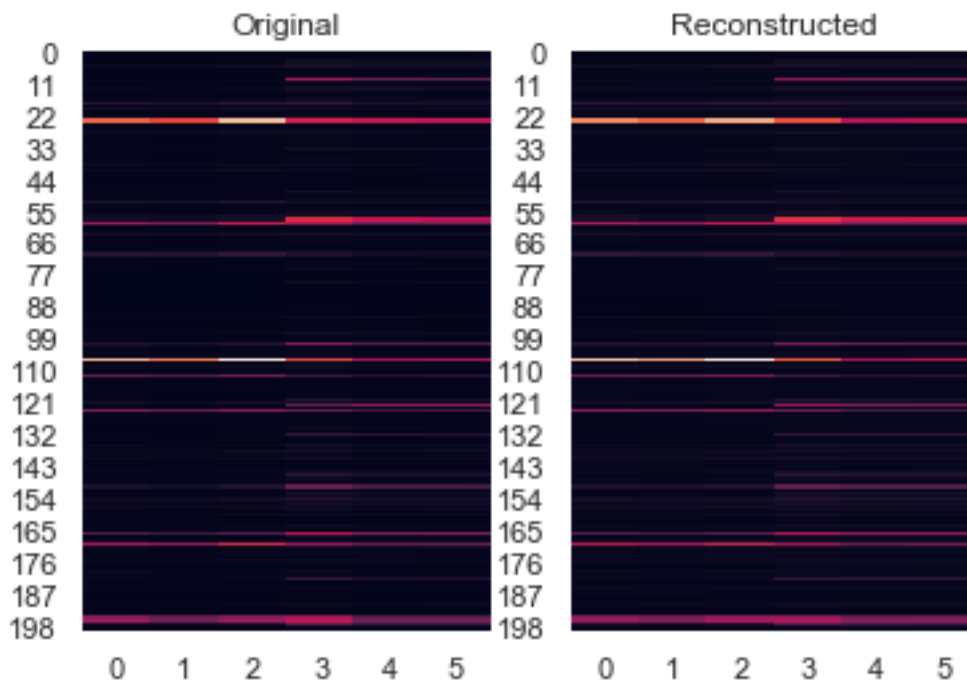
```
import numpy.linalg
U, Sig, V = np.linalg.svd(df_tfidf.values, full_matrices=False)
Weights = np.dot(np.diag(Sig), V)
```

```
# U, Weights は元の行列を分解したもののなので、U と Weights の掛け算は元の行列を近似
np.allclose(df_tfidf.values, np.dot(U, Weights))
```

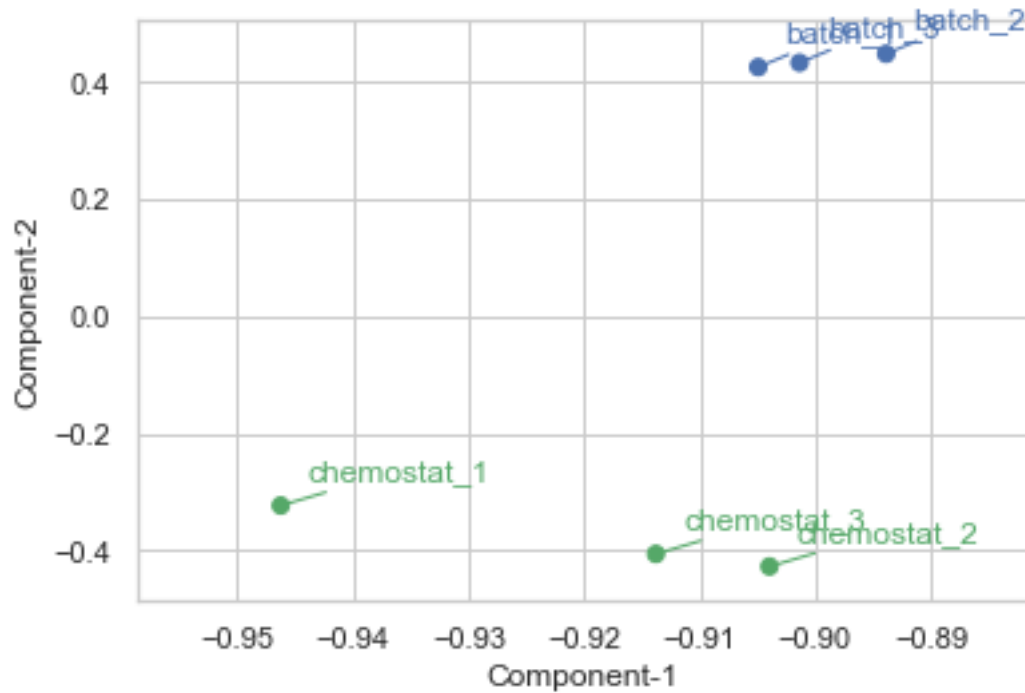
Out [20]: True

```
In [21]: U_reduced = U[:, :2]
         W_reduced = np.dot(np.diag(Sig[:2]), V[:2, :])
```

```
In [22]: fig = plt.figure()
         ax1 = fig.add_subplot(121)
         sns.heatmap(df_tfidf.values[:200,:], cbar=False, ax=ax1)
         ax1.set_title('Original')
         ax2 = fig.add_subplot(122)
         sns.heatmap(np.dot(U_reduced, W_reduced)[:200,:], cbar=False, ax=ax2)
         ax2.set_title('Reconstructed')
         plt.show()
```

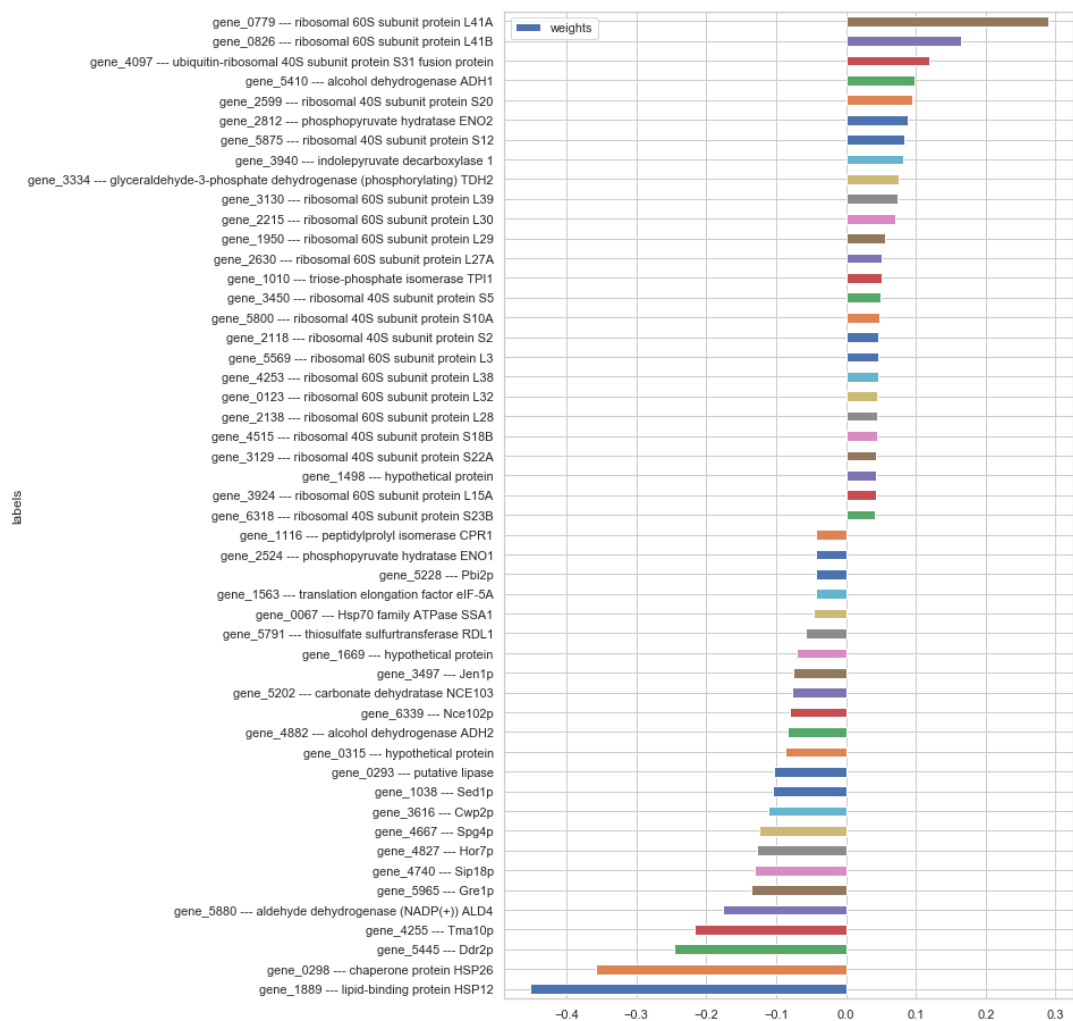


```
In [23]: # 長さ1のベクトルにノーマライズする
         W_norm = W_reduced / np.sqrt((W_reduced**2).sum(axis=0))
         # 2つの component の「重み」空間でサンプルをプロット
         scatter_plot(W_norm.T, df_tfidf.columns, colors, xlabel='Component-1', ylabel='Component-2')
```



```
In [24]: # batch と chemostat の違いは、 Component-2 がプラスに寄与するかマイナスに寄与するか。
# Components-2 にどういう遺伝子の合成ベクトルなのか、係数が大きい遺伝子をいくつか見てみる
C2_genes = pd.DataFrame(U_reduced[:, 1], index=df_tfidf.index, columns=['weights'])
# 係数の絶対値が大きいトップ 50 の遺伝子を取得
topN = 50
C2_genes['abs_weights'] = np.abs(C2_genes['weights'].values)
C2_genes = C2_genes.sort_values(by=['abs_weights'], ascending=False)
C2_genes = C2_genes.head(topN)
C2_genes = C2_genes.sort_values(by=['weights'])
C2_genes['labels'] = C2_genes.index + ' --- ' + gene_products.loc[C2_genes.index].name
C2_genes.plot.barh(y='weights', x='labels', figsize=(9, 16))

Out[24]: <matplotlib.axes._subplots.AxesSubplot at 0x1a24413898>
```



3.2 1. 2 距離行列の最適化に基づく次元削減

In [25]: #非線形次元圧縮、各手法の比較

#他の手法は以下の URL を参照

#https://scikit-learn.org/stable/auto_examples/manifold/plot_compare_methods.html

```
from time import time
from mpl_toolkits.mplot3d import Axes3D
from matplotlib.ticker import NullFormatter
from sklearn import manifold, datasets

Axes3D
```

```
n_points = 1000
```

```
X, color = datasets.samples_generator.make_s_curve(n_points, random_state=0)
```

```
n_neighbors = 10
```

```
n_components = 2
```

```

fig = plt.figure(figsize=(18, 6))
plt.suptitle("Manifold Learning with %i points, %i neighbors" % (1000, n_neighb

ax = fig.add_subplot(141, projection='3d')
ax.scatter(X[:, 0], X[:, 1], X[:, 2], c=color, cmap=plt.cm.Spectral)
ax.view_init(4, -72)

t0 = time()
pca = sklearn.decomposition.PCA()
Y = pca.fit_transform(X)
t1 = time()
print("PCA: %.2g sec" % (t1 - t0))
ax = fig.add_subplot(142)
plt.scatter(Y[:, 0], Y[:, 1], c=color, cmap=plt.cm.Spectral)
plt.title("PCA (%.2g sec)" % (t1 - t0))
ax.xaxis.set_major_formatter(NullFormatter())
ax.yaxis.set_major_formatter(NullFormatter())
plt.axis('tight')

t0 = time()
mds = manifold.MDS(n_components, max_iter=100, n_init=1)
Y = mds.fit_transform(X)
t1 = time()
print("MDS: %.2g sec" % (t1 - t0))
ax = fig.add_subplot(143)
plt.scatter(Y[:, 0], Y[:, 1], c=color, cmap=plt.cm.Spectral)
plt.title("MDS (%.2g sec)" % (t1 - t0))
ax.xaxis.set_major_formatter(NullFormatter())
ax.yaxis.set_major_formatter(NullFormatter())
plt.axis('tight')

t0 = time()
tsne = manifold.TSNE(n_components=n_components, init='pca', random_state=0)
Y = tsne.fit_transform(X)
t1 = time()
print("t-SNE: %.2g sec" % (t1 - t0))
ax = fig.add_subplot(144)
plt.scatter(Y[:, 0], Y[:, 1], c=color, cmap=plt.cm.Spectral)
plt.title("t-SNE (%.2g sec)" % (t1 - t0))
ax.xaxis.set_major_formatter(NullFormatter())
ax.yaxis.set_major_formatter(NullFormatter())
plt.axis('tight')

```

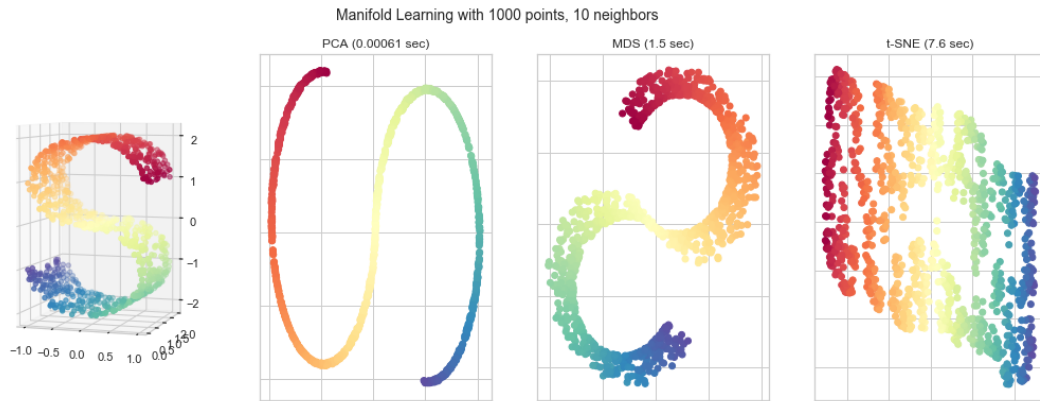


```
plt.show()
```

PCA: 0.00061 sec

MDS: 1.5 sec

t-SNE: 7.6 sec



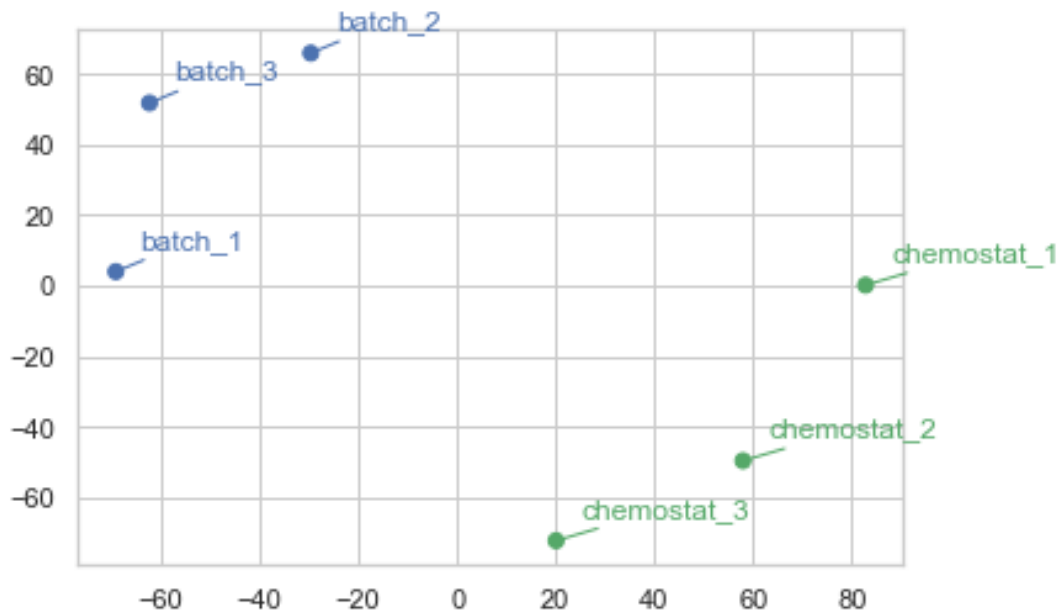
3.2.1 1.2.1 MDS (Multidimensional scaling; 多次元尺度構成法)

■metric MDS = PCoA (Principal Coordinate Analysis; 主座標分析) 構成する「距離行列」が、距離関数の要件を満たしている場合に適用可能な手法。

Bray-Curtis dissimilarity など、三角不等式の要件を満たさない非類似性指標があるので注意。こういう場合は PCoA ではなく nMDS を使う。

```
In [26]: import sklearn.manifold
```

```
In [27]: # デフォルトではユークリッド距離でサンプル間距離行列を計算。この場合、数学的には主成分分析と等価。
# ただし scikit-learn の MDS 実装は iterative に最適化する metric MDS であるため (classical
# ランダムな初期値の影響で実行のたびに結果が若干変わる
mds = sklearn.manifold.MDS(n_components=2, dissimilarity='euclidean')
coords = mds.fit_transform(std_df.transpose().values)
scatter_plot(coords, std_df.columns, colors)
```



In [28]: # 距離行列の計算

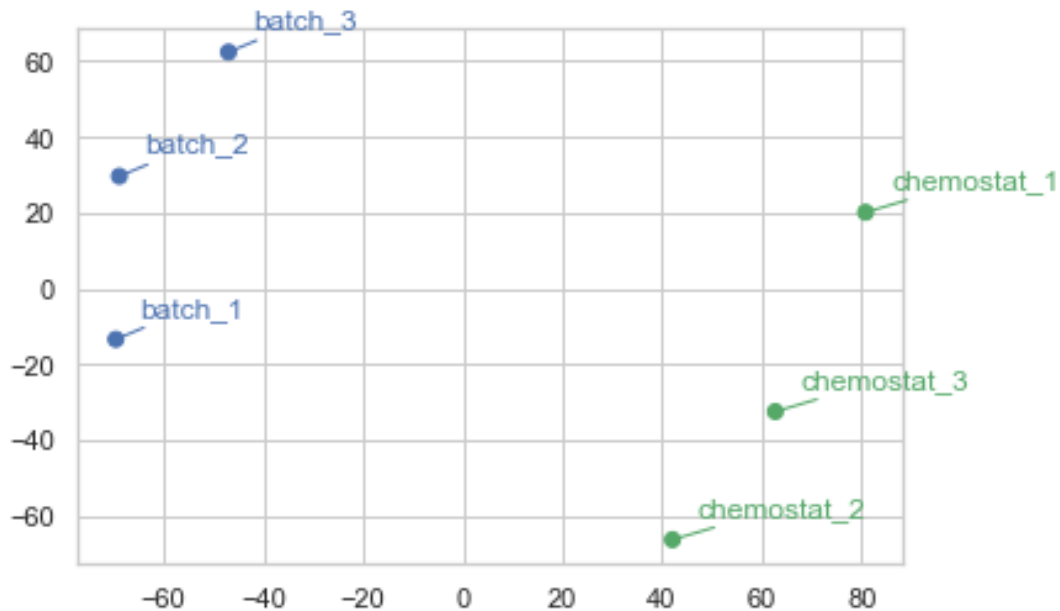
```
from scipy.spatial.distance import pdist, squareform
distance_matrix = squareform(pdist(std_df.transpose().values))
print(distance_matrix)
```

```
#自分で距離行列を作る場合（上の一行と同じ計算結果）
#from scipy.spatial.distance import euclidean
#values = std_df.transpose().values
#distance_matrix_2 = []
#for i in range(values.shape[0]):
#    vec = []
#    for j in range(values.shape[0]):
#        vec.append(euclidean(values[i, :], values[j, :]))
#    distance_matrix_2.append(vec)
#print(np.array(distance_matrix_2))
```

```
[[ 0.          61.3886715  69.82784729 141.72006779 139.27185618
 133.98870325]
 [ 61.3886715   0.          56.11255212 148.032513   143.22201069
 140.71825784]
 [ 69.82784729  56.11255212   0.          149.09664857 146.08961444
 143.53272672]
 [141.72006779 148.032513   149.09664857   0.          76.11398372
 80.8830421 ]
 [139.27185618 143.22201069 146.08961444  76.11398372   0.
 64.40382016]
```

```
[133.98870325 140.71825784 143.53272672 80.8830421 64.40382016
 0.
 ]]
```

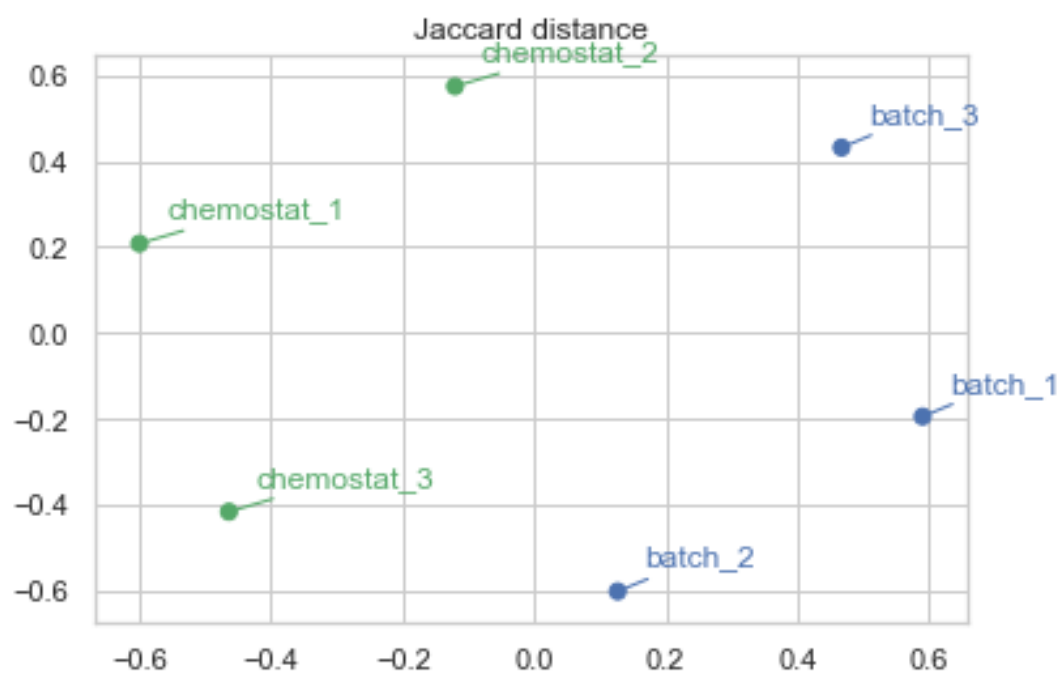
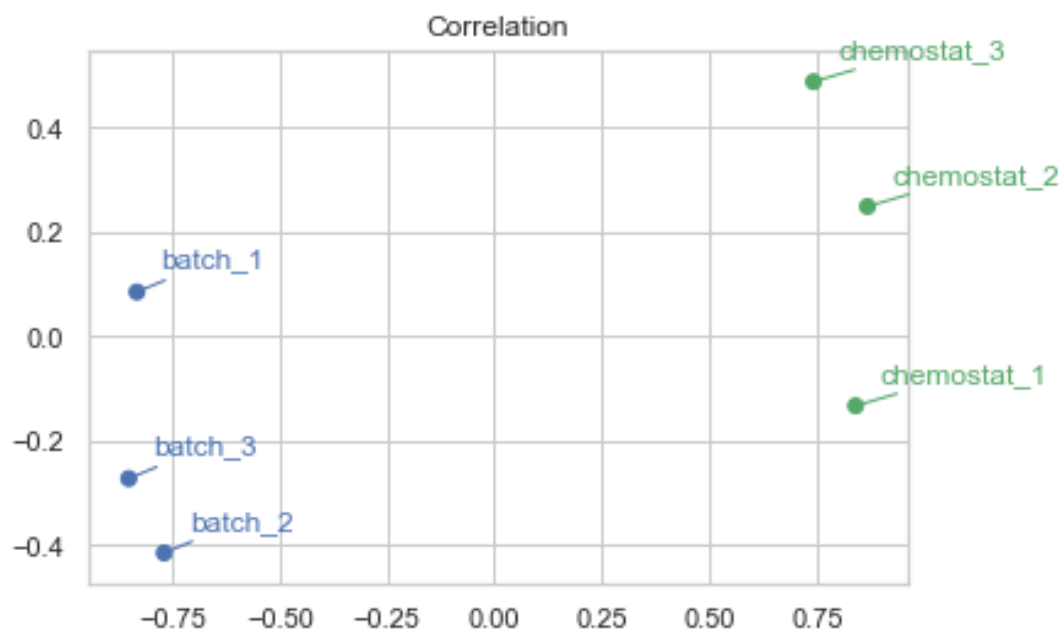
```
In [29]: mds = sklearn.manifold.MDS(n_components=2, dissimilarity='precomputed')
        coords = mds.fit_transform(distance_matrix)
        scatter_plot(coords, std_df.columns, colors)
```

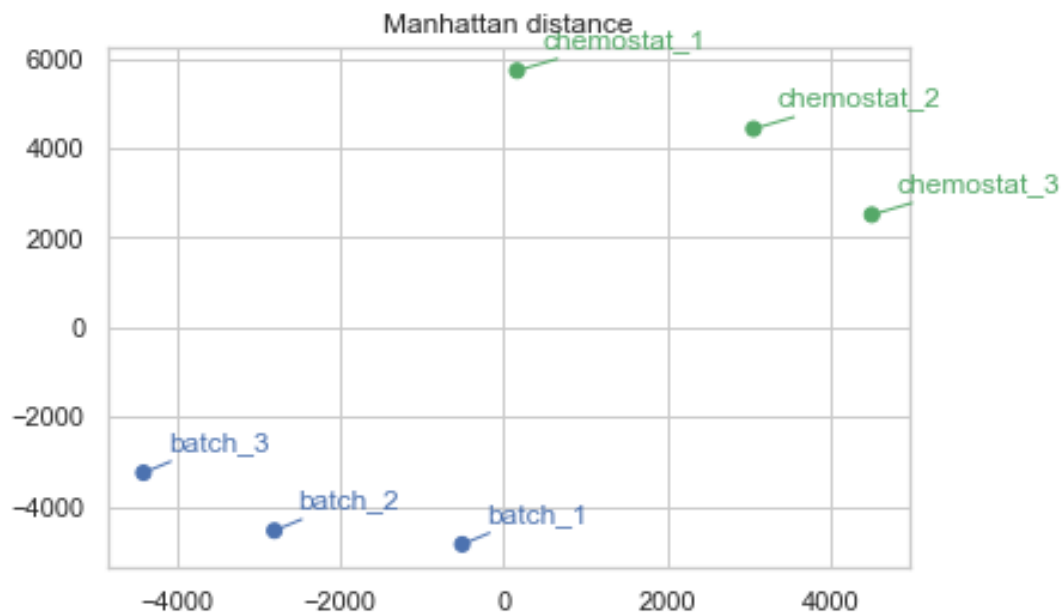


```
In [30]: # 相関係数で計算した距離 (1 - 相関係数)
        distance_matrix = squareform(pdist(std_df.transpose().values, 'correlation'))
        mds = sklearn.manifold.MDS(n_components=2, dissimilarity='precomputed')
        coords = mds.fit_transform(distance_matrix)
        scatter_plot(coords, std_df.columns, colors, title='Correlation')

        # Jaccard 距離
        distance_matrix = squareform(pdist(std_df.transpose().values, 'jaccard'))
        mds = sklearn.manifold.MDS(n_components=2, dissimilarity='precomputed')
        coords = mds.fit_transform(distance_matrix)
        scatter_plot(coords, std_df.columns, colors, title='Jaccard distance')

        # マンハッタン距離
        distance_matrix = squareform(pdist(std_df.transpose().values, 'cityblock'))
        mds = sklearn.manifold.MDS(n_components=2, dissimilarity='precomputed')
        coords = mds.fit_transform(distance_matrix)
        scatter_plot(coords, std_df.columns, colors, title='Manhattan distance')
```

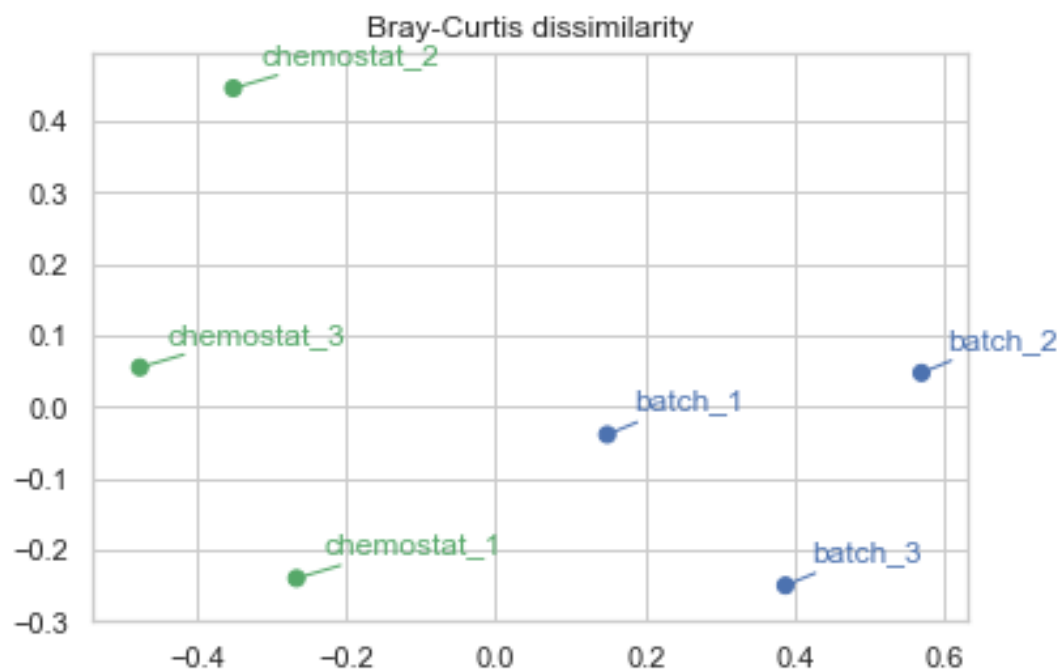




■non-metric MDS (非計量多次元尺度構成法) 構成する「距離行列」が、距離関数の要件を満たさない場合に適用する。

In [31]: # Bray-Curtis 非類似度指標による計算

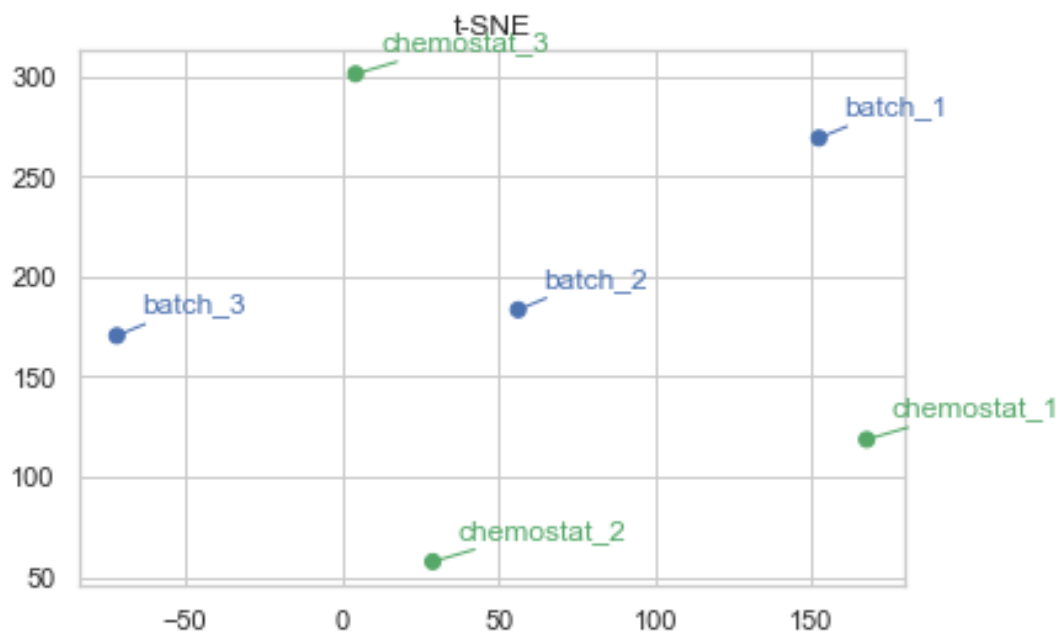
```
distance_matrix = squareform(pdist(std_df.transpose().values, 'braycurtis'))
nmfs = sklearn.manifold.MDS(n_components=2, metric=False, dissimilarity='precomputed')
coords = nmfs.fit_transform(distance_matrix)
scatter_plot(coords, std_df.columns, colors, title='Bray-Curtis dissimilarity')
```



3.2.2 1.2.2 多様体学習

■t-SNE (t-distributed Stochastic Neighbor Embedding)

```
In [32]: tsne = sklearn.manifold.TSNE(n_components=2)
        coords = tsne.fit_transform(std_df.transpose().values)
        scatter_plot(coords, std_df.columns, colors, title='t-SNE')
```

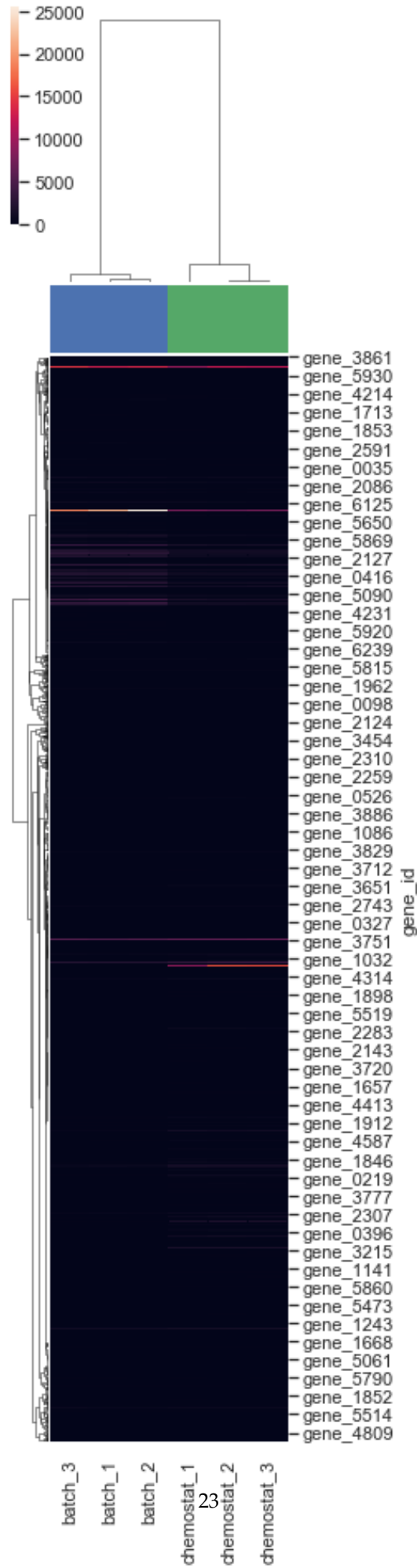


4 2. クラスタリング

4.1 2.1 階層的クラスタリング

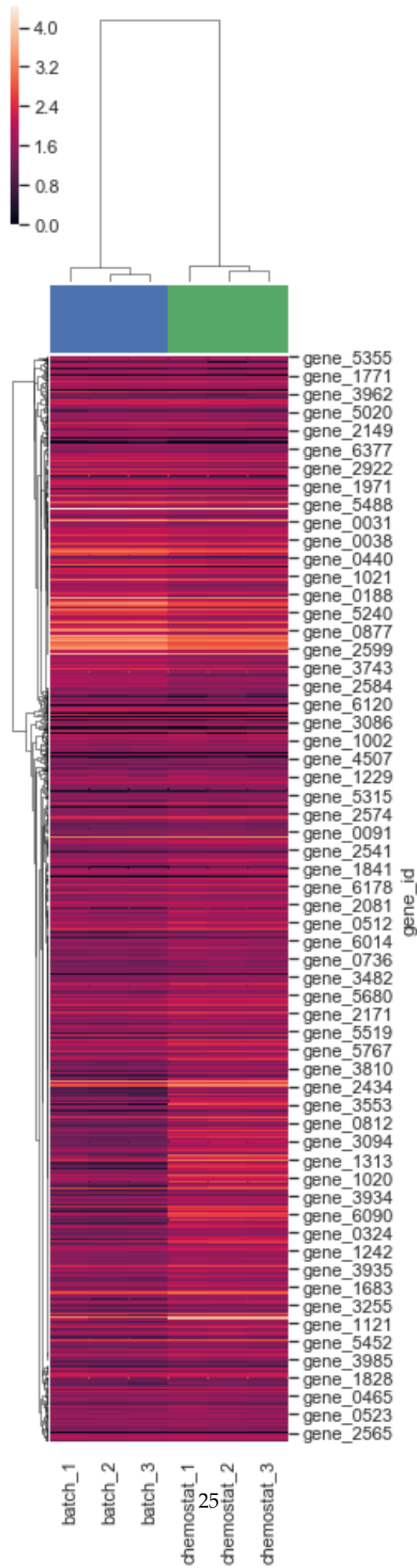
```
In [33]: sns.clustermap(df, method='average', metric='correlation', col_colors=colors, f
```

```
Out[33]: <seaborn.matrix.ClusterGrid at 0x1a23c45940>
```



```
In [34]: log_transform = lambda x: np.log10(x + 1.0)
sns.clustermap(df.apply(log_transform), method='average', metric='correlation',
```

```
Out[34]: <seaborn.matrix.ClusterGrid at 0x1a221a4198>
```

```
In [35]: # 平均 TPM のサンプル間分散が Top-20 の遺伝子だけ抜き出し
```

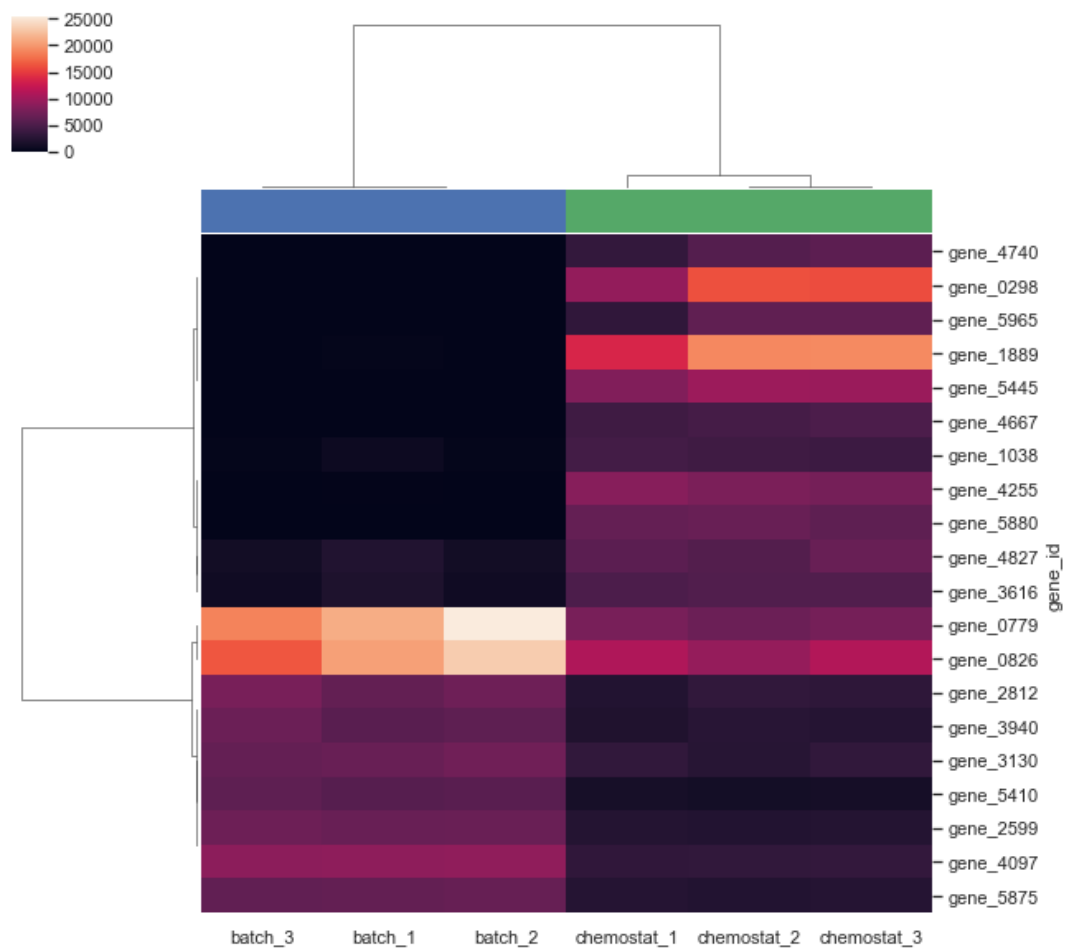
```
top20_df = df.loc[df.var(axis=1).sort_values(ascending=False).index[:20], :]
```

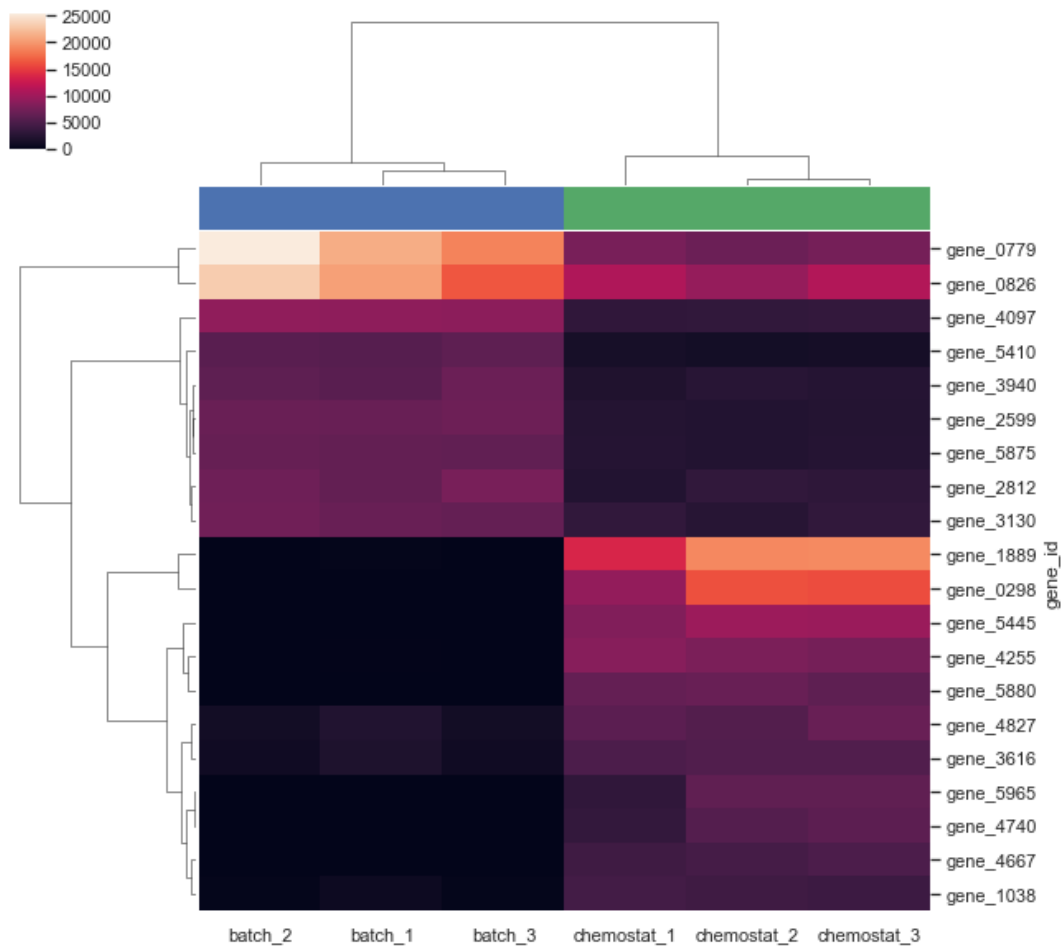
```
In [36]: # サンプル間の距離計算手法、クラスター間の距離計算手法による違い
```

```
sns.clustermap(top20_df, method='average', metric='correlation', col_colors=col
```

```
sns.clustermap(top20_df, method='ward', metric='euclidean', col_colors=colors)
```

```
Out[36]: <seaborn.matrix.ClusterGrid at 0x1a26977358>
```





4.2 2.2 K-means & K-medoids (クラスタ「中心」との距離に基づくクラスタリング)

4.2.1 K-means clustering

```
In [37]: import sklearn.cluster
         model = sklearn.cluster.KMeans(n_clusters=2)
         y = model.fit_predict(df.transpose().values)
         print(y)
```

```
[0 0 0 1 1 1]
```

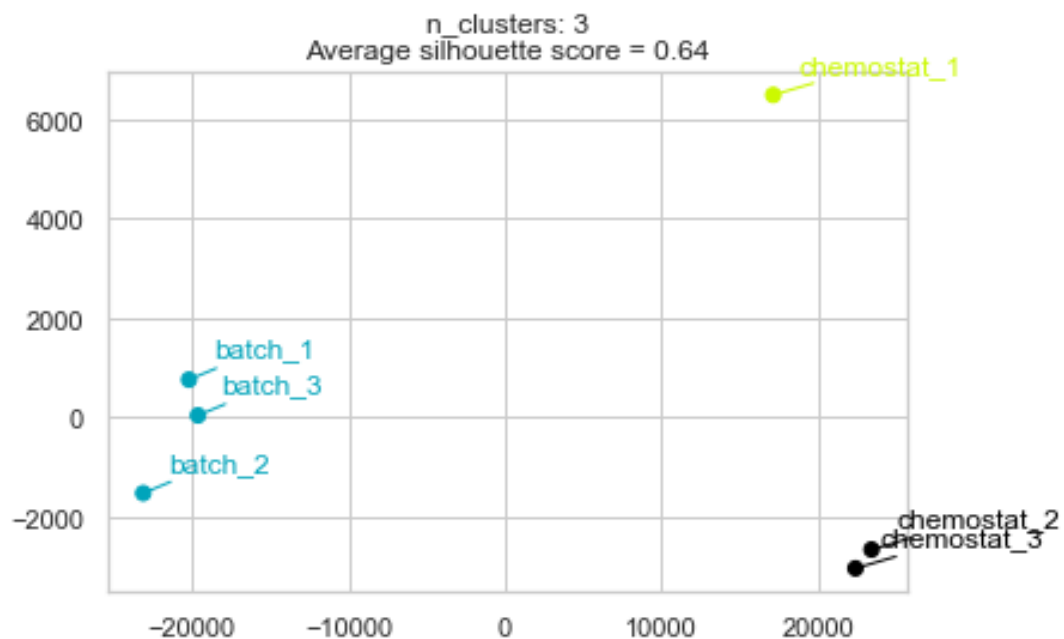
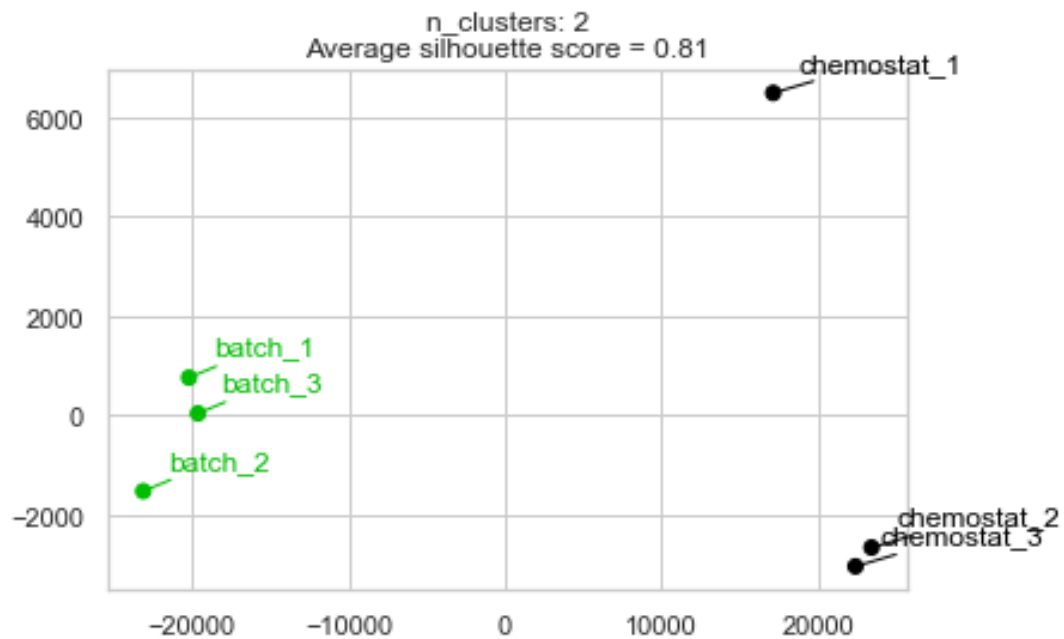
```
In [38]: # silhouette 解析による適切なクラスター数の推定
         from sklearn.metrics import silhouette_score
         import matplotlib.cm as cm

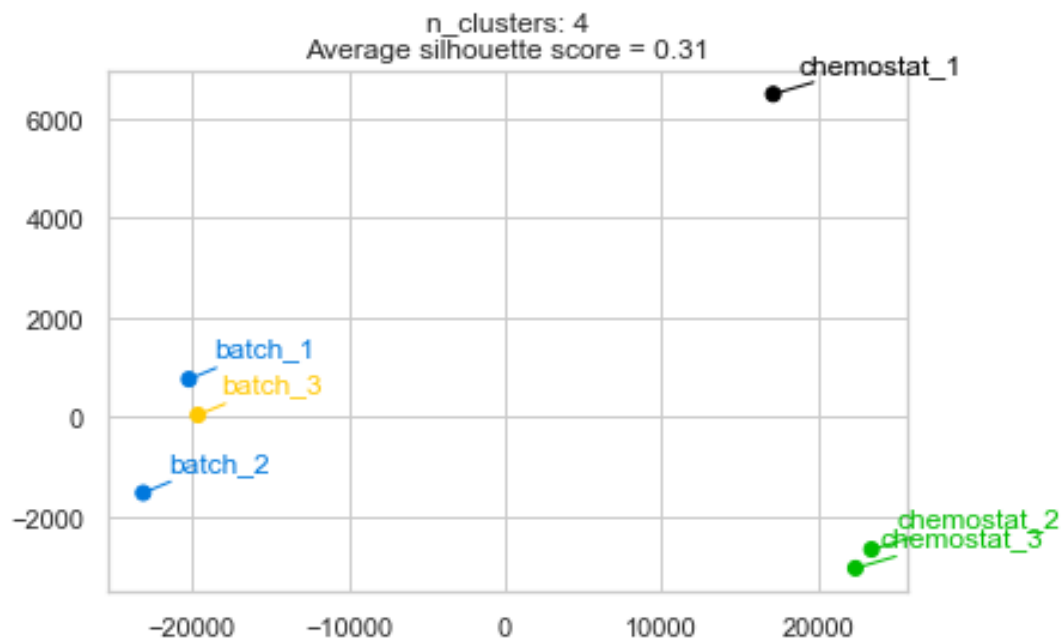
         X = df.transpose().values
         pca = sklearn.decomposition.PCA()
         pca_coords = pca.fit_transform(X)
```

```

for n_clusters in [2, 3, 4]:
    model = sklearn.cluster.KMeans(n_clusters=n_clusters)
    y = model.fit_predict(X)
    silhouette_avg = silhouette_score(X, y)
    title = 'n_clusters: {} \n Average silhouette score = {:.2f}'.format(n_clusters, silhouette_avg)
    cluster_colors = cm.nipy_spectral(y.astype(float) / n_clusters)
    scatter_plot(pca_coors, std_df.columns, cluster_colors, title=title)

```





```
In [39]: # もっとサンプル数がたくさんある場合は、サンプルごとのシルエットスコアの分布を比較できる
# https://scikit-learn.org/stable/auto_examples/cluster/plot_kmeans_silhouette.html
from sklearn.datasets import make_blobs
from sklearn.metrics import silhouette_samples, silhouette_score
import matplotlib.cm as cm
```

```
X, y = make_blobs(n_samples=500, n_features=2, centers=4, cluster_std=1, center_box=(-10000, 10000))
```

```
def plot_silhouette(n_clusters):
    fig = plt.figure(figsize=(12,4))
    ax1 = fig.add_subplot(121)
    ax2 = fig.add_subplot(122)
    model = sklearn.cluster.KMeans(n_clusters=n_clusters)
    cluster_labels = model.fit_predict(X)
    silhouette_avg = silhouette_score(X, cluster_labels)
    print('n_clusters:',n_clusters, ' Average silhouette score =',silhouette_avg)
    sample_silhouette_values = silhouette_samples(X, cluster_labels)
    y_lower = 10
    for i in range(n_clusters):
        ith_cluster_silhouette_values = np.sort(sample_silhouette_values[cluster_labels==i])
        size_cluster_i = ith_cluster_silhouette_values.shape[0]
        y_upper = y_lower + size_cluster_i
        cluster_colors = cm.nipy_spectral(float(i) / n_clusters)
```

```

ax1.fill_betweenx(np.arange(y_lower, y_upper), 0, ith_cluster_silhouette)
ax1.text(-0.05, y_lower + 0.5 * size_cluster_i, str(i))
y_lower = y_upper + 10 # 10 for the 0 samples
ax1.set_xlabel("The silhouette coefficient values")
ax1.set_ylabel("Cluster label")
ax1.axvline(x=silhouette_avg, color="red", linestyle="--")
ax1.set_yticks([])
ax1.set_xticks([-0.1, 0, 0.2, 0.4, 0.6, 0.8, 1])
cluster_colors = cm.nipy_spectral(cluster_labels.astype(float) / n_clusters)
ax2.scatter(X[:, 0], X[:, 1], c=cluster_colors)
plt.show()

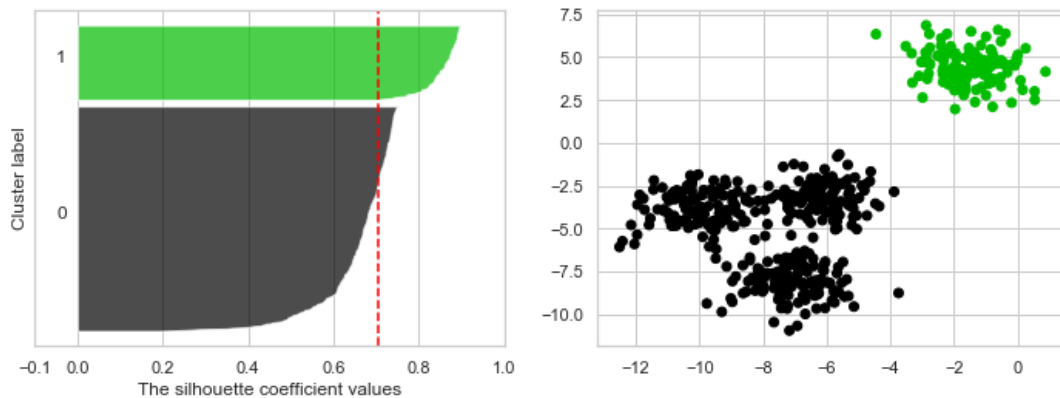
```

```

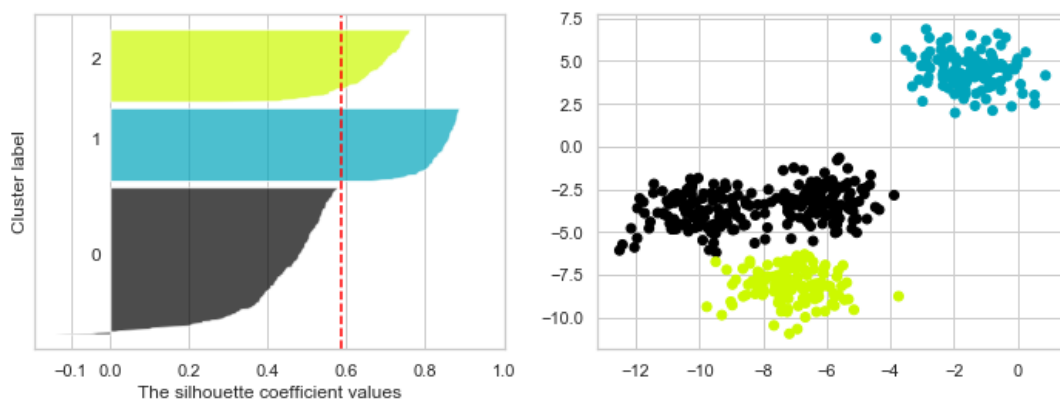
for n_clusters in [2, 3, 4]:
    plot_silhouette(n_clusters)

```

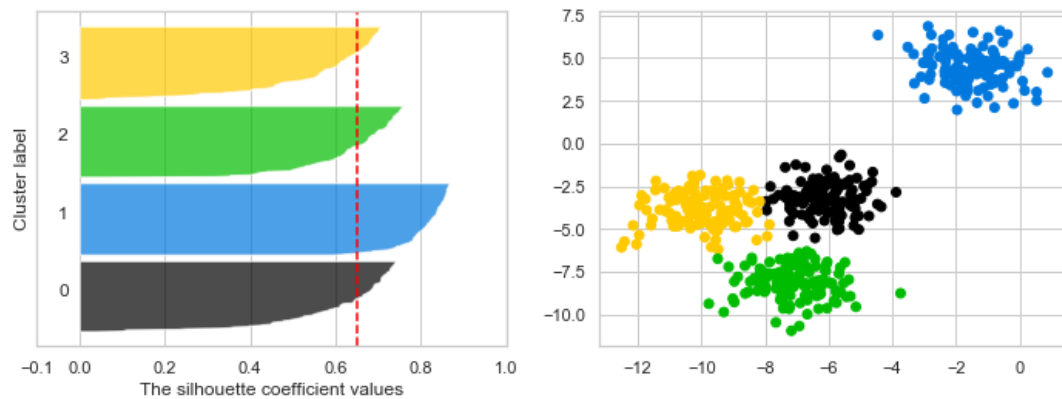
n_clusters: 2 Average silhouette score = 0.7049787496083262



n_clusters: 3 Average silhouette score = 0.5882004012129721



```
n_clusters: 4 Average silhouette score = 0.6505186632729437
```



4.2.2 K-medoids clustering

平均 (mean) ではなく medoid (すべてのデータ点との距離の和が最小になるデータ点) を使う点だけが K-means と異なる。

実データの平均を計算する必要がないため、実データを入力する必要がなく、距離行列があればいい
→ ユークリッド距離である必要はなく、任意の距離関数を使える。

```
In [40]: # K-medoids clustering を scikit-learn の使用感っぽく使えるように実装した。  
# エラーハンドリングとか全然してないので注意。また、初期値依存性も高い。sklearn の KMeans メソッドの  
# 参考: https://www.researchgate.net/publication/272351873\_NumPy\_SciPy\_Recipes\_for\_Data\_Scientists
```

```
import numpy as np  
from scipy.spatial.distance import pdist, squareform  
  
class KMedoids():  
    """K-Medoids clustering  
  
    Parameters  
    -----  
    n_clusters : int, optional, default: 2  
        Number of clusters to form.  
    max_iter : int, optional, default: 100  
        Maximum number of iterations of the k-medoids algorithm.  
    verbose : boolean, optional, default: False  
        Verbosity mode.  
    random_state : int, RandomState instance or None, optional, default: None  
    dissimilarity : 'euclidean' | 'precomputed', optional, default: 'euclidean'  
        Dissimilarity measure to use:
```

```

- 'euclidean':
    Pairwise Euclidean distances between points in the dataset.
- 'precomputed':
    Pre-computed dissimilarities are passed.

Attributes
-----
cluster_medoids_ : array, [n_clusters]
    Indices of cluster medoids
labels_ :
    Labels of each point
"""
def __init__(self, n_clusters=2, max_iter=100, verbose=False,
              random_state=None, dissimilarity='euclidean'):
    self.n_clusters = n_clusters
    self.max_iter = max_iter
    self.verbose = verbose
    self.random_state = random_state
    self.dissimilarity = dissimilarity

def _init_medoids(self, X, n_clusters, random_state):
    n_samples = X.shape[0]
    init_indices = random_state.choice(n_samples, n_clusters, replace=False)
    return np.sort(init_indices)

def _k_medoids(self, X, n_clusters, max_iter, random_state, verbose):
    # 初期 medoids をランダムに n_clusters 個選択。
    medoids = self._init_medoids(X, n_clusters, random_state)
    if self.verbose:
        print('initial medoids =', medoids)
    new_medoids = np.copy(medoids)
    # ループ開始。medoids が収束するまで繰り返す。
    for i in range(max_iter):
        # 各サンプルをもっとも近い medoids に割り当てる。
        assigned_cluster_labels = np.argmin(X[:, medoids], axis=1)
        for c in range(n_clusters): # クラスごとに新たな medoids を選択。
            # クラス c に割り当てられたサンプルのインデックスを取得。
            sample_indices_in_c = np.where(assigned_cluster_labels == c)[0]
            # クラス c に割り当てられたサンプル内の距離行列を抽出。
            distance_matrix_of_c = X[np.ix_(sample_indices_in_c, sample_indices_in_c)]
            # クラス c 内の各サンプルについて、同一クラス内サンプルとの平均距離を計算
            average_distance_in_c = np.mean(distance_matrix_of_c, axis=1)
            # 新たな medoid は平均距離がもっとも小さいサンプル。

```



```

        new_medoid_index_in_c = np.argmin(average_distance_in_c)
        new_medoid_index = sample_indices_in_c[new_medoid_index_in_c]
        new_medoids[c] = new_medoid_index
    np.sort(new_medoids)
    if self.verbose:
        print('\titeration:', i, '\n\t\tnew medoids=', new_medoids)
    if np.array_equal(new_medoids, medoids):
        # medoids のインデックスが更新されなかったらループ終了。
        break
    medoids = np.copy(new_medoids)
    assigned_cluster_labels = np.argmin(X[:, medoids], axis=1)
    return medoids, assigned_cluster_labels

def fit(self, X):
    """Compute k-medoids clustering.
    Parameters
    -----
    X : array, shape (n_samples, n_features) or (n_samples, n_samples)
        Input data. If ``dissimilarity == 'precomputed'``, the input should
        be the dissimilarity matrix.
    """
    if self.dissimilarity == 'precomputed':
        self.dissimilarity_matrix_ = X
    elif self.dissimilarity == 'euclidean':
        self.dissimilarity_matrix_ = squareform(pdist(X))
    else:
        raise ValueError("dissimilarity must be 'precomputed' or 'euclidean'")
    if self.random_state is None or self.random_state is np.random:
        self.random_state = np.random.mtrand._rand
    elif isinstance(self.random_state, int):
        self.random_state = np.random.RandomState(self.random_state)
    elif isinstance(self.random_state, np.random.RandomState):
        self.random_state = np.random.RandomState
    else:
        raise ValueError("%r cannot be used to seed a numpy.random.RandomState")
    self.cluster_medoids_, self.labels_ = \
        self._k_medoids(self.dissimilarity_matrix_, self.n_clusters, self.random_state)
    return self

def fit_predict(self, X):
    """Compute k-medoids clustering and predict cluster index for each sample.
    """
    return self.fit(X).labels_

```

```

In [41]: # K-medoids クラスタリングの実行
distance_matrix = squareform(pdist(df.transpose().values, metric='correlation'))
n_clusters = 2

model = KMedoids(n_clusters=n_clusters, dissimilarity='precomputed', verbose=True)
model.fit(distance_matrix)
medoids = model.cluster_medoids_
labels = model.labels_
print('\nMedoids :', df.columns[medoids].values)
print('Clusters :')
for cluster in range(n_clusters):
    print('\tcluster-', cluster, ': ', df.columns[labels == cluster].values)

initial medoids = [1 3]
iteration: 0
    new medoids= [0 5]
iteration: 1
    new medoids= [0 5]

Medoids : ['batch_1' 'chemostat_3']
Clusters :
cluster- 0 : ['batch_1' 'batch_2' 'batch_3']
cluster- 1 : ['chemostat_1' 'chemostat_2' 'chemostat_3']

```

4.3 2.3 混合正規分布（確率分布に基づくクラスタリング）

実データが、なんからの確率分布がいくつか混ざった混合確率分布から生成されたものであると仮定する手法。

微生物群集構造データではここ数年、Dirichlet Multinomial Mixture がよく使われている。

```

In [42]: import sklearn.mixture
import matplotlib as mpl

n_samples = 500
C = np.array([[0., -0.1], [1.7, .4]])
X = np.r_[np.dot(np.random.randn(n_samples, 2), C),
          .7 * np.random.randn(n_samples, 2) + np.array([-6, 3])]

gmm = sklearn.mixture.GaussianMixture(n_components=2, covariance_type='full')
gmm.fit(X)

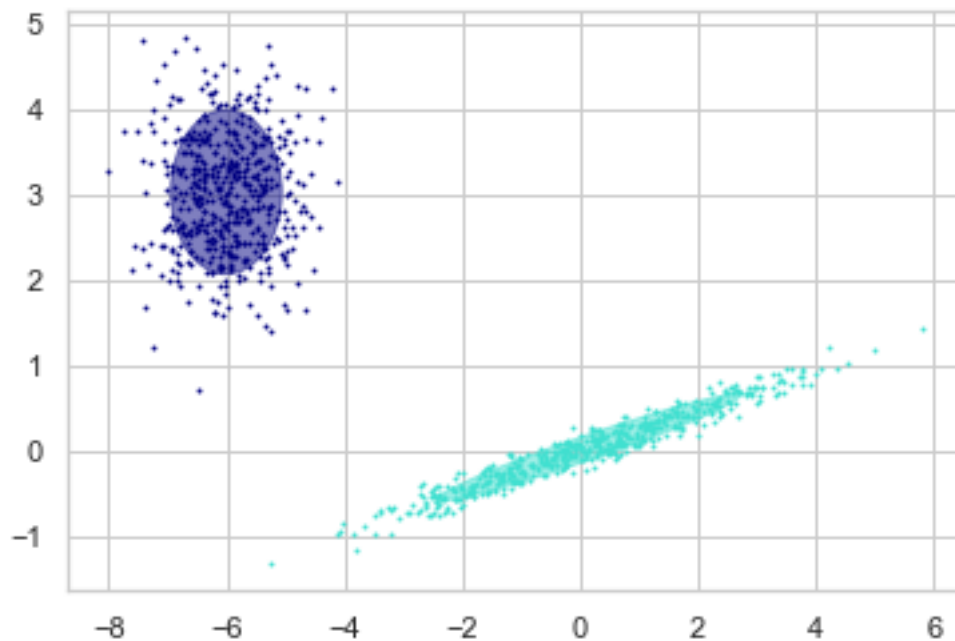
plt.figure()
ax = plt.subplot(111)

```

```

Y_ = gmm.predict(X)
cluster_colors = ['navy', 'turquoise']
for i, (mean, cov, color) in enumerate(zip(gmm.means_, gmm.covariances_, cluster_colors)):
    v, w = np.linalg.eigh(cov)
    if not np.any(Y_ == i):
        continue
    ax.scatter(X[Y_ == i, 0], X[Y_ == i, 1], .8, color=color)
    angle = np.arctan2(w[0][1], w[0][0])
    angle = 180. * angle / np.pi
    v = 2. * np.sqrt(2.) * np.sqrt(v)
    ell = mpl.patches.Ellipse(mean, v[0], v[1], 180. + angle, color=color)
    ell.set_clip_box(ax.bbox)
    ell.set_alpha(.5)
    ax.add_artist(ell)

```



```

In [43]: import itertools

cluster_colors = itertools.cycle(['navy', 'turquoise', 'cornflowerblue', 'darkorange'])

# n_components=10 とすることで、実際の構造よりかなり大きな 10 個ぶんのクラスターを仮定して推論をば
# 最終的には、2 つ以外のクラスターが推論の過程で勝手につぶれてくれる
vbgmm = sklearn.mixture.BayesianGaussianMixture(n_components=10, max_iter=300,
                                                  covariance_type='full', weight_concentration_prior_type='dirichlet')
vbgmm.fit(X)

for i, cluster_weight in enumerate(vbgmm.weights_):

```

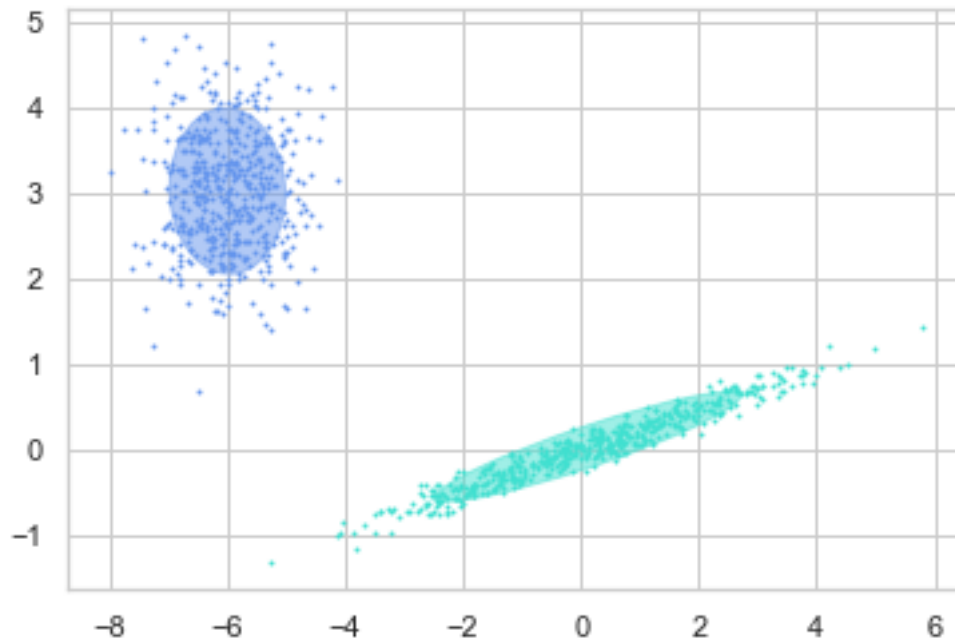
```

print('Cluster-', i, ' weight =', cluster_weight)

plt.figure()
ax = plt.subplot(111)
Y_ = vbgmm.predict(X)
for i, (mean, cov, color) in enumerate(zip(vbgmm.means_, vbgmm.covariances_, cl
    v, w = np.linalg.eigh(cov)
    if not np.any(Y_ == i):
        continue
    ax.scatter(X[Y_ == i, 0], X[Y_ == i, 1], .8, color=color)
    angle = np.arctan2(w[0][1], w[0][0])
    angle = 180. * angle / np.pi
    v = 2. * np.sqrt(2.) * np.sqrt(v)
    ell = mplt.patches.Ellipse(mean, v[0], v[1], 180. + angle, color=color)
    ell.set_clip_box(ax.bbox)
    ell.set_alpha(.5)
    ax.add_artist(ell)

Cluster- 0 weight = 9.990101490188931e-05
Cluster- 1 weight = 9.990101490188931e-05
Cluster- 2 weight = 9.990101490188931e-05
Cluster- 3 weight = 9.990101490188931e-05
Cluster- 4 weight = 9.990101490188931e-05
Cluster- 5 weight = 9.990101490188931e-05
Cluster- 6 weight = 0.4996003981229847
Cluster- 7 weight = 0.4996003937563627
Cluster- 8 weight = 9.990101633951395e-05
Cluster- 9 weight = 9.990101490188931e-05

```



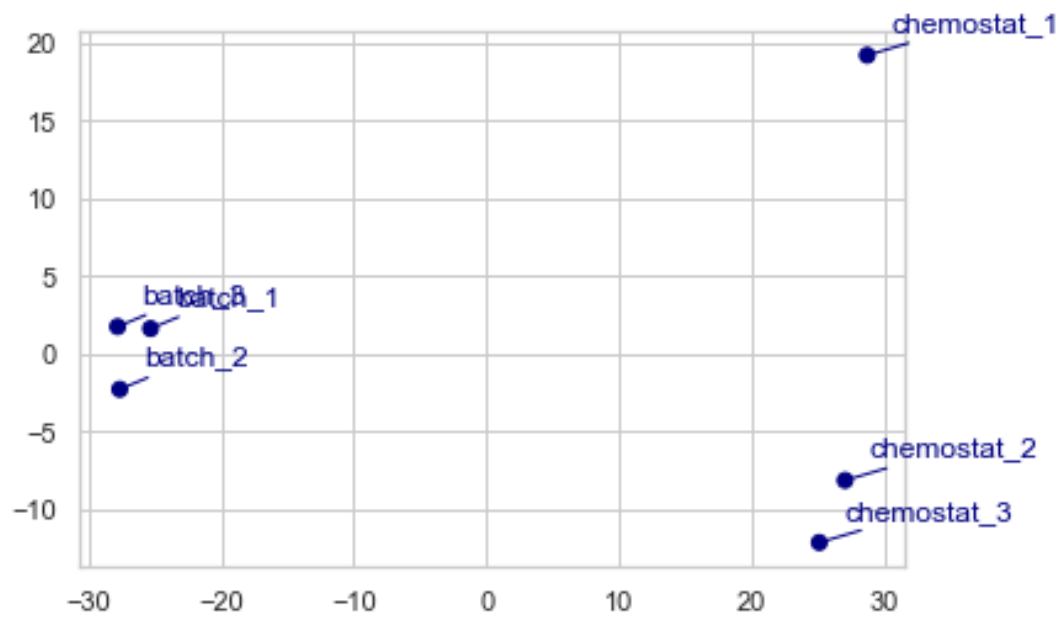
```
In [44]: # サンプル間で分散の大きいトップ 1000 の遺伝子のみを使って推定
var1000_genes = std_df.var(axis=1).sort_values(ascending=False).index[:1000]
X = std_df.loc[var1000_genes, :].transpose().values

pca = sklearn.decomposition.PCA()
pca_coords = pca.fit_transform(X)
cluster_colors = np.array(['navy', 'turquoise', 'cornflowerblue', 'darkorange'])

for n_clusters in [1, 2, 3, 4]:
    vbghmm = sklearn.mixture.BayesianGaussianMixture(n_components=n_clusters,
                                                    weight_concentration_prior_

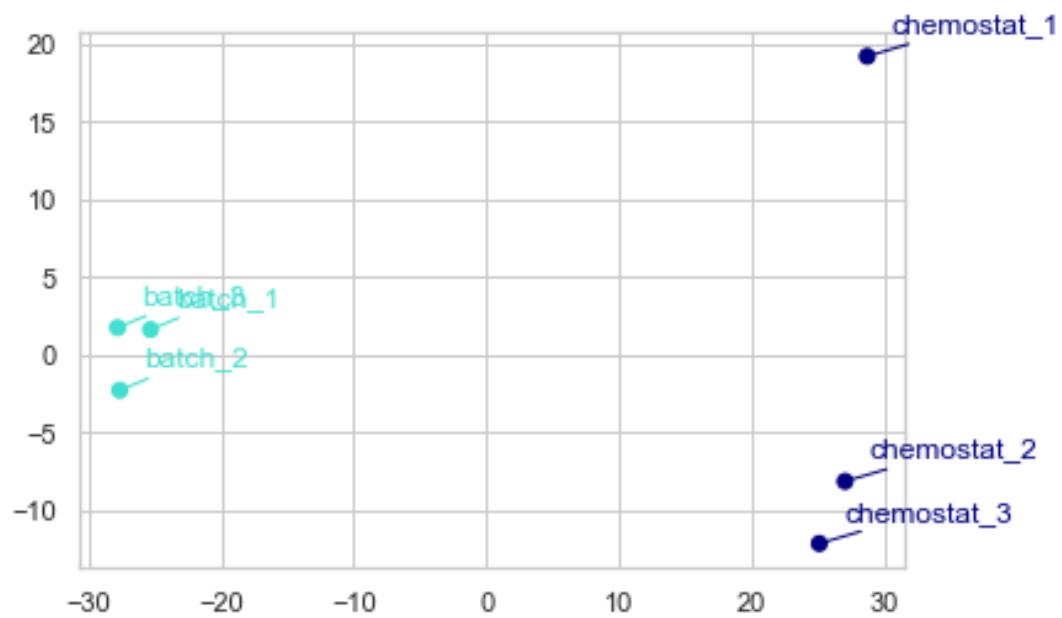
    vbghmm.fit(X)
    print('Number of clusters estimated:', n_clusters)
    for c in range(n_clusters):
        print('\tcluster-', c, ' weights =', vbghmm.weights_[c])
    y = vbghmm.predict(X)
    scatter_plot(pca_coords, std_df.columns, cluster_colors[y])
```

```
Number of clusters estimated: 1
cluster- 0 weights = 1.0
```



```

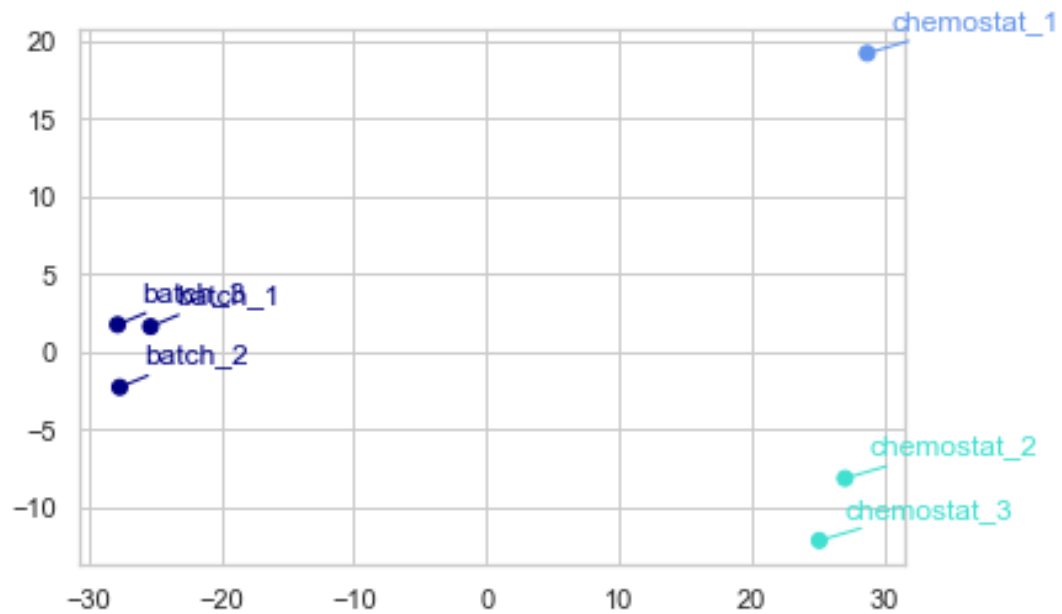
Number of clusters estimated: 2
  cluster- 0  weights = 0.5
  cluster- 1  weights = 0.5
  
```



```

Number of clusters estimated: 3
  cluster- 0  weights = 0.47619047619047605
  cluster- 1  weights = 0.3333333333333333
  
```

cluster- 2 weights = 0.19047619047619058



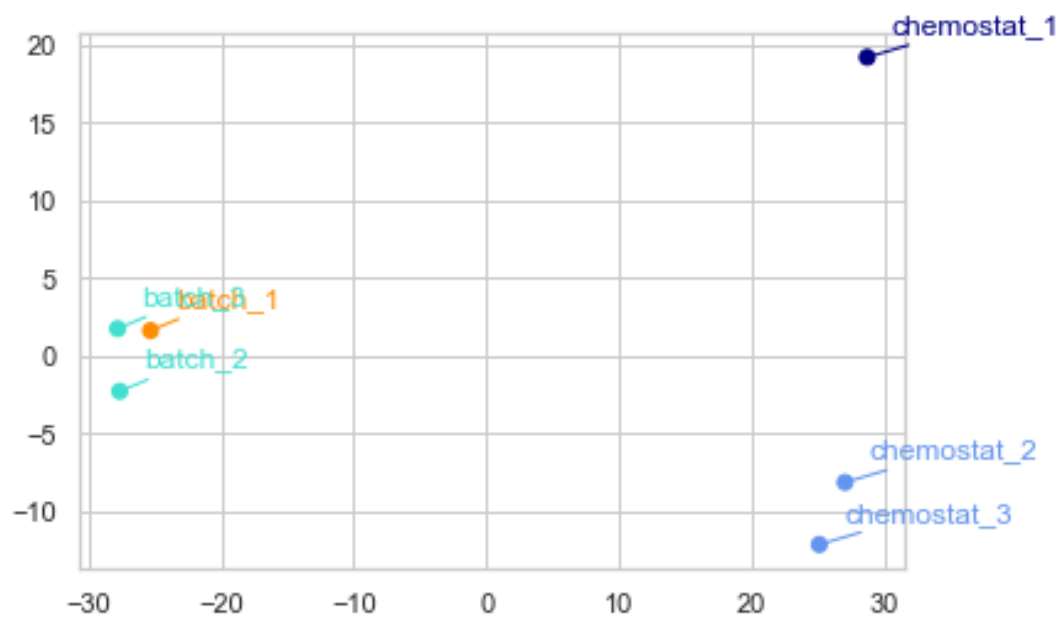
Number of clusters estimated: 4

cluster- 0 weights = 0.17857142857142866

cluster- 1 weights = 0.32142857142857134

cluster- 2 weights = 0.32142857142857134

cluster- 3 weights = 0.17857142857142866



```
In [ ]:
```