

Formal Methods for Secure Systems

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0 Introduction

0.1 Outline of the course

1. Dependability

- Building high reliable computer-based systems
- Quantitative evaluation of dependability
- Threat modeling and risk assessment
- Malware analysis
- Cybersecurity engineering

2. Formal methods for security

- Formal methods applied to security
- Case studies: Data confidentiality, Security protocols, Cyber-physical systems security

0.2 Computer-based systems

Computer-based systems are everywhere, and the services they offer are very diverse. From that, we can easily understand why the **dependability**, which is the ability of the system to deliver the expected service, is a critical aspect of these systems, in particular in a security point of view.

A system should (or must) be able to deliver the expected service, even in the presence of faults, errors, and **attacks**. This is the main goal of dependability, which is as important as the functionality of the system, perhaps even more. To achieve that, we have **Formal Methods** that provide to us a set of techniques and tools to design, verify, and validate computer-based systems, in a rigorous and systematic way, even in presence of faults and attacks.

1 Basic concepts and terminology

All the concept and the terminology that will be presented in this section derives directly from the paper “Basic Concepts and Taxonomy of Dependable and Secure Computing” by Avizienis et al. (2004). This paper is a fundamental reference in the field of dependability and security and it is the basis for the definition of the concepts and terminology that will be used in the entire course, as suggested by the professor.

1.1 Dependability

We can give to **dependability** a simple definition: given a system, which is designed to provide a certain service, the dependability is the ability of that system to deliver the specified service also in presence of faults and malfunctions. In other words *dependability is that property of a computer-based system such that reliance can justifiably be placed on the service it delivers*. Note that the latter definition stresses the need for a justified reliance on the service, which is a key aspect of dependability.

1.1.1 Computer-based systems

A computer-based system is a system that includes a certain number of components: each of them can be interconnected and have its own functionality. The components can be hardware, software, humans and the environment in which the system operates.

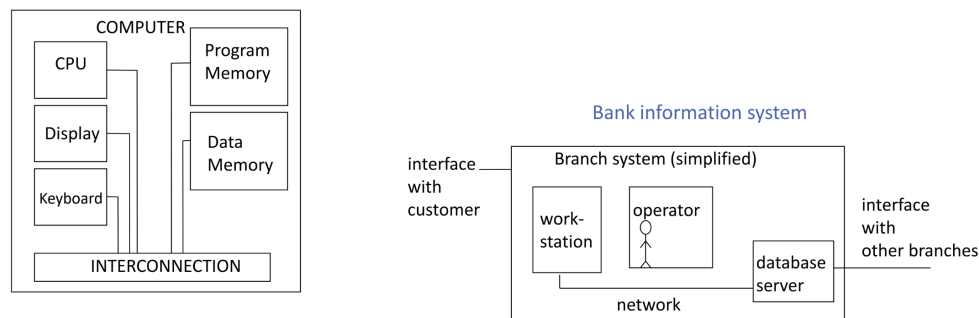


Figura 1.1: Computer-based system - C. Bernardeschi

1.1.2 Faults and Failures

We call a **failure** the inability of the system to deliver the expected service, and a **fault** the cause of that failure.

Example: if a cash machine delivers the wrong amount of money, we can say that the system has failed.

A fault causes an **error** in the state of the system, which lead to a **failure**. A failure can have different nature, such as physical, logical, human error or even as consequence of an attack.

Example: Logic Bomb. It's a piece of code that is inserted into a software system that will execute a malicious function when specified conditions are met.

```

1 legitimate_code();
2 if (date == "01/01/2020") {
3     crash_system();
4 }
5 legitimate_code();

```

Computer Faults vs. Other Equipment Faults Computer faults differ from those of other equipment in several ways:

- **Subtler Failures:** computer failures are more subtle than outright crashes or sudden stops.
- **Information Storage:** computers store information in various ways, leading to a multitude of possible errors, both internally and externally.
- **Hidden Small Defects, Big Effects:** even small hidden defects can have significant impacts, especially in digital systems.
- **Complex Hierarchies:** computer systems are intricate hierarchies built upon hidden components.

1.1.3 Achieving Dependability

The dependability of a system can be achieved going through a rigorous and engineered steps. Two main figures are involved, system and software engineers:

- **System engineers** are responsible for the design of the system, and they have to use analysis to model the dependability of their design. From these, the software specifications are derived, and the possible changes to the system are evaluated, in order to accommodate software limitations;
- **Software engineers** are responsible for the implementation of the software, and they have to use the specifications to develop the software, and to test it in order to verify that it meets the requirements.

In general, it's crucial to understand that **dependability is not something that can be added to a system as an afterthought**. It must be considered from the very beginning of the design process, and it must be an integral part of the system, using a scientific and engineering approach.

1.2 The system entity

A simple but effective definition of a system is the following: a system is an entity that interacts with the environment and other systems; its boundaries are the common frontier between the system and the environment.

1.2.1 System's properties

A system has a **function**, which is the service that it provides to the environment, and it's described by its own functional specification. It also has a **behavior**, visualized as the sequence of states that the system goes through during its operation, and it's how the system implements its function. Then there is the **structure**, which is the way the system is organized, and it's described by its own structural specification.

From the user point of view, the system has a **delivered service**, which is the result of the interaction between the user and the system, that is the behavior of the system as perceived by the user. Obviously, the user can be seen as another system that interacts with the system under consideration.

1.2.2 System's requirements

First of all, we need to define the problem that the system has to solve, and then we have to define the requirements that the system has to meet, and then we have to define both functional and dependability requirements. Pay attention to the difference between the system's function and the system's specification: the former is the service that the system provides, while the latter is the solution implemented to provide that service. In the end, we define the **correctness** of the system, which is the ability of the system to deliver the specified service.

1.3 Dependability tree

Take in consideration the following dependability tree:

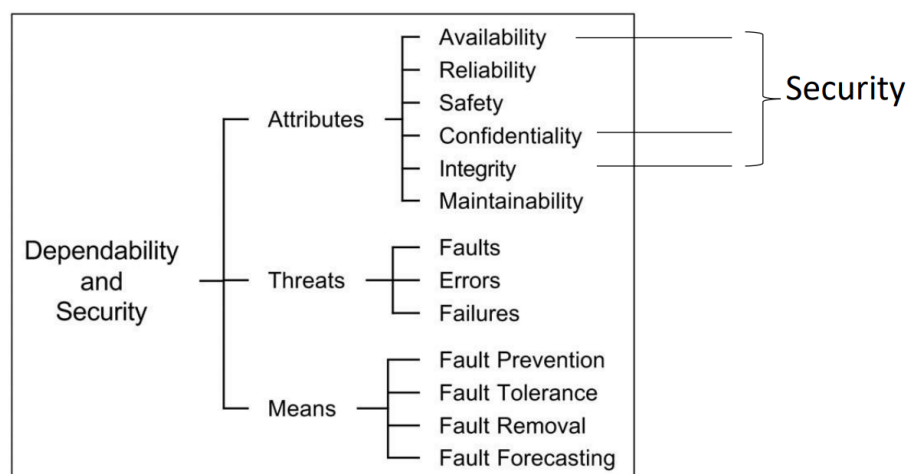


Figura 1.2: Dependability tree - Avizienis et al., 2004

1.3.1 Threats to dependability

As we said before, a **correct service** is delivered if the service is delivered in accordance with the system's specification. When this doesn't happen, we have a **service failure**, which is one the possible states of the system:

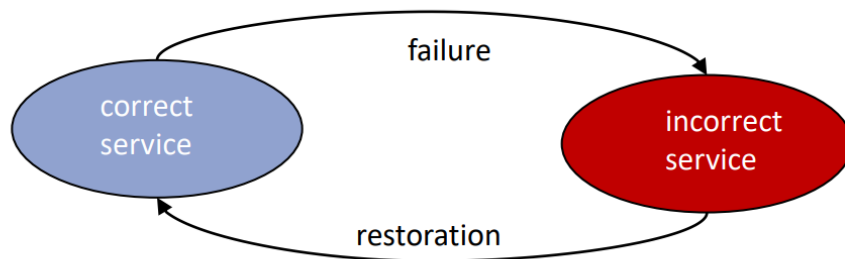


Figura 1.3: Service failure - C. Bernardeschi

We call **service outage** the period during which the system is not able to deliver the service, a **service degradation** the period during which the system delivers a service that is not in accordance with the specification, such as a subset of the services. We also recall the **chain of threats** to dependability: **faults** causes **errors**, which lead to **failures**: note that many errors don't cause failures because they don't reach the external state of the system.

Faults can be **dormant**, that is they are present in the system but they don't cause errors, and **active**, that is they cause errors, and they can be **external** or **internal**: from the latter we can extract the definition of **vulnerability**, which is the property of the system that allows an external agent to cause a fault.

Example: Trapdoor. It's a hidden entry within a system that allows an attacker to bypass security measures.

```
1 username = read_username();
2 password = read_password();
3 if (username == 'dummy_user'){
4     //note that the password is not checked
5     grant_access();
6 }
7 if (username.isValid() && password.isValid()){
8     grant_access();
9 }
```

From the past example we can learn that, having in mind the dependability tree, to achieve security only the authorized actions have to be allowed, and the confidentiality and the integrity of the data have appears in case of improper or unauthorized actions.

1.3.2 Dependability attributes

The dependability of a system can be described by a set of attributes, measurable and quantifiable in terms of probabilities:

- **Availability:** the readiness for correct service;
- **Reliability:** the continuity of correct service;
- **Safety:** the absence of catastrophic consequences on the user and the environment;
- **Confidentiality:** the absence of unauthorized disclosure of information;
- **Integrity:** the absence of improper system state alterations;
- **Maintainability:** the ability to undergo modifications and repairs.

We also briefly present the concept of **trust** between systems, that express the dependance of dependability of the system A from the dependability of the system B.

From them, we can gave a new definition of dependability, based on the frequency of the service failures:

Dependability is the ability of the system to deliver the service in accordance with the specification, avoiding that service failures occur with a frequency that is greater than a certain threshold.

The threshold should be derived from the system requirements, and should consider frequency, duration and severity of the service failures.

1.4 Taxonomy of faults

1.4.1 The system life cycle

We define the **system life cycle** as the period of time that starts from the conception of the system and continues until the system is decommissioned. We're going to consider only two phases of the system life cycle: the **development phase**, during which the system is designed and implemented, and the **use phase**, during which the system is used to deliver the service.

1.4.1.1 Development phase

In this phase the system only interact with the **development environment**, such as physical world, human developers, their tools and possible facilities: in this phase development faults can be introduced in the system.

1.4.1.2 Use phase

In this phase the system interacts with the **use environment**:

- **Physical environment**: the system is subject to physical stress, such as temperature, humidity, vibrations, etc;
- **administrator**: who manages the system;
- **users**: who interact with the system;
- **providers**: who delivers the system to the users;
- **infrastructure**: everything that is needed to support the system;
- **intruders**: who try to attack the system.

The use phase alternates period of **correct delivery** of the service, **service outage** and **service shutdown**:

- **service outage**: when the system has a service failure, and it's not able to deliver the service correctly;
- **service shutdown**: when the system is stopped for maintenance or for other reasons. Note that maintenance may take place during every period of time, and includes both repair and modification.

In fact, there is a taxonomy for the maintenance: if the system is stopped for repair an active fault, then we have a **corrective maintenance**; if the system is stopped for repair a dormant fault, then we have a **preventive maintenance**; if the system is stopped for modification, then we have an **adaptive maintenance**, or **augmentative maintenance** if the system is stopped for improvement.

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1.4.2 Faults classification

Given the fact that faults can't be enumerated, it's useful to classify them in order to understand their nature and their effects, because we can also identify the mechanisms that can be used to prevent that specific class of faults.

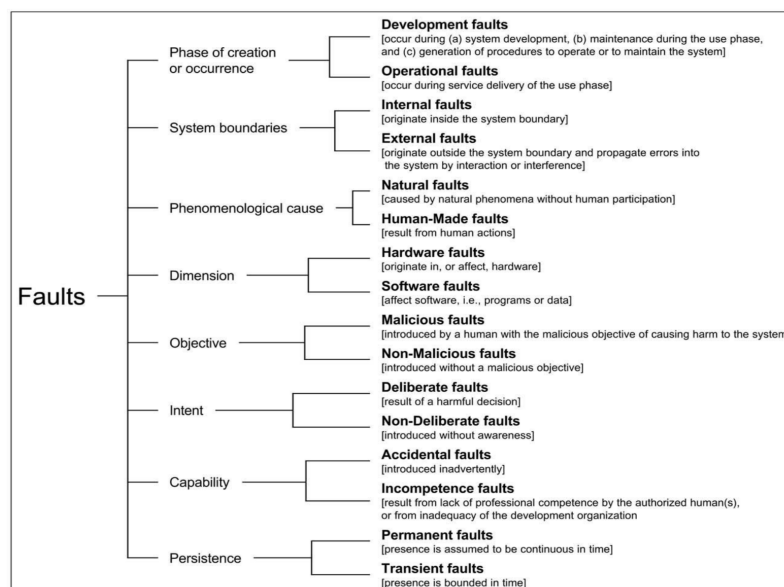


Figura 1.4: Faults classification - Avizienis et al., 2004

We identifies three main classes of faults:

- **development faults:** faults that are introduced during the development phase;
- **physical faults:** faults that affect the physical components of the system;
- **interaction faults:** faults that includes all the external faults.

Overlapping classes are possible, so we're able to identify 31 possible combinations of faults.

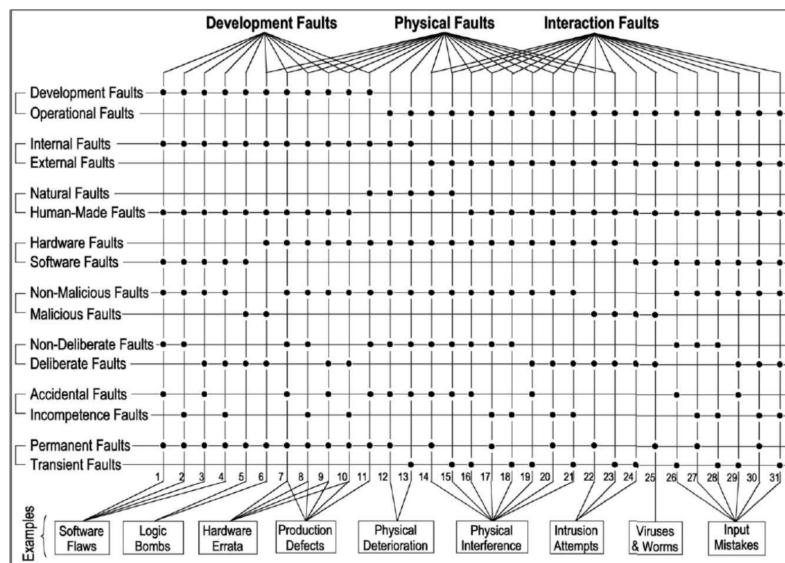


Figura 1.5: Faults classification - Avizienis et al., 2004

We're going to analyze the most important classes of faults, having in mind the previous figure.

1.4.2.1 Natural faults

They're the faults numbered from 11 to 15 in the previous figure, and they're are mainly hardware faults caused by natural phenomena, without the intervention of humans.

- **Production defect**, 11: a fault that is introduced during the development;
- **internal defect**, 12 and 13: a fault that is introduced during the use phase, and it's caused by the physical degradation of the components;
- **external defect**, 14 and 15: a fault that is introduced during the use phase, and it's caused by the physical stress of the environment, outside the system boundaries.

1.4.2.2 Human-made faults

We can distinguish between **malicious faults**, introduced with the intent to harm the system, and **non-malicious faults**, introduced without any malicious intent.

The formers have as a goal to harm the system, the latter are divided in two more classes:

- **non-deliberate faults**: faults that are introduced as human errors (1, 2, 7, 8, 16-18, 26-28);
- **deliberate faults**: faults that are introduced as a consequence of a deliberate action, such as a bad decision (3, 4, 9, 10, 19-21, 29-31).

The latter can be introduced during **development** or by **interaction**:

- **deliberate development faults**: are generally the result of a trade-off, both in terms of performance and economy (3, 4, 9, 10);
- **deliberate interaction faults**: when operational procedures are deliberately violated (19-21, 29-31).

In general, **deliberate faults** shows up only after an unacceptable behavior of the system, and it can be difficult to realize the actual faults because, when it happened, who introduced the fault can be not conscious of the consequences of his action.

However, not all mistakes and bad decisions by non-malicious humans are accidental: we can distinguish between **accidental faults** and **incompetence faults**.

1.4.2.3 Interaction faults

They can also be named as **operational faults**, given the fact that they occur during the use phase, and they're all external, because they're caused by the interaction between the system and the environment. Classes from 16 to 31 are human made, and only 14 and 15 are natural.

A common feature of these faults is the fact that they usually need the presence of a vulnerability in the system, that is a property of the system that allows an external agent to cause a fault, both intentional and unintentional.

Lastly, we recognize the **permanent faults**, that are continuous and stable, and the **transient faults**, that are temporary, even for very short periods of time.

1.5 Failures

In order to characterize the failures, we use four different dimensions, such that each of them can describe a different aspect of the failure.

1.5.1 Failure domain

This point of view leads us to distinguish between **content failures**, that are the result of the system's inability to deliver the correct service, and **timing failures**, that are the result of the system's inability to deliver the correct service at the correct time (early, late, or never).

1.5.2 Consistency domain

When a system has more than one user, it's important to understand if a failure shows up identically for each user, and we call this situation **consistent failure**, or if the failure shows up differently for each user, and we call this situation **inconsistent failure**.

1.5.3 Detectability domain

It's the property of the system to check the correctness of the service, and it's the ability of the system to detect the failure. These mechanisms have two failure modes:

- **false alarm**: the system detects a failure when there isn't;
- **missed detection**: the system doesn't detect a failure when there is.

1.5.4 Consequences domain

Consequences of a failure are divided in two classes, based on their severity and impact:

- **minor failure**: the failure has a similar cost to the benefit of the service;
- **catastrophic failure**: the failure has a cost that is much greater than the benefit of the service.

1.5.5 Criteria to evaluate the severity of a failure

We can use the following criteria to evaluate the severity of a failure:

- **availability**: the duration of the service outage;
- **safety**: possible loss of life or injury;
- **confidentiality**: possible unauthorized disclosure of information;
- **integrity**: data corruption and/or inability to recover;

1.5.6 System failures

When a system has a failure, it is usually caused by different coexisting faults; we talk about **single fails** if the failure is caused by a single fault, and **multiple fails** if the failure is caused by multiple faults. In the latter we can also divide the faults in **independent** and **dependent**, respectively if the faults are not related to each other, and if the faults have a common cause.

1.5.7 Dependability and security failures

These failures occurs if the system suffers service failures more frequently than an acceptable threshold; even the specifications of the system can contain faults:

- **omission**: a requirement is not included in the specification;
- **unjustified requirements**: choice of requirements that are not justified by the system's function, that raises the cost of the system.

Systems can have different type of failures:

- **fail-controlled**: the system is designed to fail in a controlled way, described by the system's specification;
- **fail-stopped**: the system in which possible failures are only *haltings*;
- **fail-silent**: the system in which possible failures are only *silents*;
- **fail-safe**: the system in which there are only minor failures;

1.6 Errors

As we saw, an error is a part of the system state that is able to lead to a failure. An error is detected if a signal to indicate its presence in raised, otherwise the error is undetected and it's called **latent error**. It's not true that every error leads to a failure, and this depends on:

- the **structure of the system**, specially the presence of redundancy;
- the **behavior of the system**, for example the part of the state that contains the error is never reached during the delivery of the service, then the error is not able to lead to a failure.

The classification of the error is done according to the damage pattern (single, double or triple bit, burst, etc.), and how many components are affected by the error (single, multiple, etc.).

1.7 Chain of threats - Relationship between faults, errors and failures

When systems have to interact with each other, an error propagation can occur:

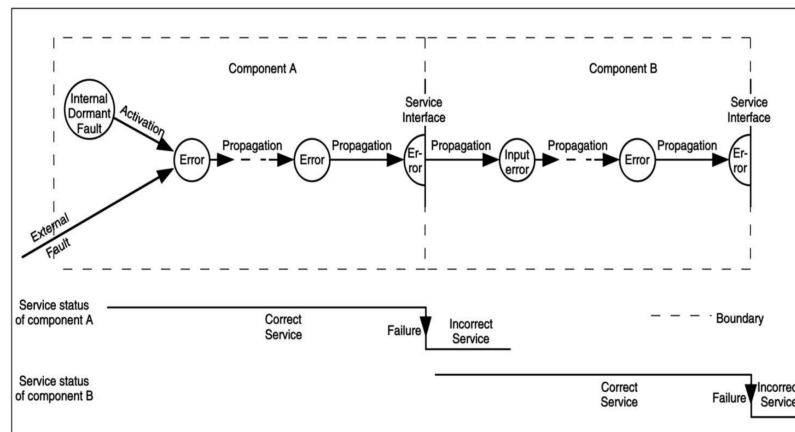


Figura 1.6: Chain of threats - Avizienis et al., 2004

An **active fault** is when it leads to an error, both if it's internal or external, and it can lead to a failure, and a **fault activation** occurs when a particular input activates a dormant fault in a specific component. This can also be a **propagation** between the components, where an error of a component can lead to an error of another component.

Example: error propagation

Take a sensor which reports the spinning speed of a turbine. If the sensor fails and starts to report that the turbine is no longer spinning, it inject incorrect data (fault) into the control system, that will send to the turbine the wrong commands, and the turbine could be damaged (failure).

1.8 Dependability means

When we talk about dependability means, we refer to the mechanisms that are used to prevent, detect and tolerate, or in general deal, with faults.

1.8.1 Fault prevention

These techniques are usually related to general system design, and they're used to avoid the introduction of faults in the system. They can be divided in two classes:

- **development fault prevention:** techniques that are used to avoid the introduction of faults during the development phase;
- **improved development process:** techniques that are used to improve the development process, such as the use of formal methods, the use of a rigorous testing and so on.

1.8.2 Fault tolerance

It's the ability to deal with faults at run-time, and ensure that the system is able to deliver the service even in presence of faults. There are a lot of techniques that can be used to achieve fault tolerance, as we can see in the following figure:

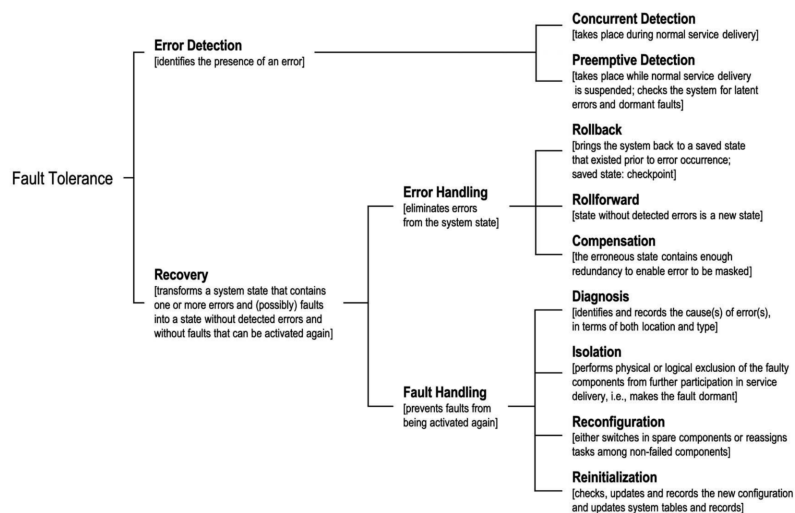


Figura 1.7: Fault tolerance techniques - Avizienis et al., 2004

1.8.2.1 Example of error detection

For simplicity, let's state that *when the error reaches the boundaries of the systems, then we have a failure*. In this context, the most challenging aspect are:

- the **identification of the error**, that is the ability to detect the presence of the error;
- **ensure that status containing the error is never reached**, that is the ability to avoid that the error leads to a failure;
- the **prevention of the error propagation**, that is the ability to avoid that the error of a component leads to the error of another component.

Example: error detection with two systems

Take two systems, A and B, that should provide the same service. If they get the same input, then they should provide the same output. If the outputs are different, then we have a detection of the error, within the hypothesis that the systems are independent and it's very unlikely that they have the same error at the same time.

1.8.3 Fault handling

When we talk about fault handling, we refer to the mechanisms that **prevents faults from being activated again**. It's composed by different phases:

1. **diagnosis**: the phase in which the system detects the presence of the fault. Usually, a component is made in order to test another components, and the aim is to identify and records the cause of the error, in terms of location and type;
2. **isolation**: to obtain the physical and/or logical exclusion of the faulty component from the rest of the system;
3. **reconfiguration**, such as thw switch to a redundant component, or the use of a different path to reach the same component;
4. **reinitialization**: to restore the system to update the system to the new configuration.

In conclusion we state that the **system recover is composed by the error handling phase and the fault handling phase**.

1.8.4 Fault removal

The fault removal is the process that is used to remove the faults from the system, and it's usually done during the development phase. The main goal is to remove the faults that are present in the system, and to prevent the introduction of new faults. It's composed by different phases, that we'll see in the next sections:

1.8.4.1 Verification phase

The verification phase is the phase in which the system is tested to verify that it meets the **verification conditions**. To do that, there are two main methods:

- verification **without execution**: the system is tested without executing it, and it's usually done via inspection or theory-proving. A state-transition diagram can be used to verify the correctness of the system, and it's applicable to various type of the system, and applicable to fault tolerance mechanisms. Worth to mention the fact that, in this type of verification errors and faults are artificially injected as part of the test pattern;
- verification **by execution**: the system is tested by executing it, and it's usually done via **dynamic verification** (e.g. symbolic execution, testing both for hardware and software), **deterministic testing** and **statistical testing**.

Another two steps are crucial:

- **verification of the mechanism:** the verification of the fault tolerance mechanism, and it's usually done via **fault injection**;
- **verification of the system:** ensure that the system cannot do more than what is supposed to do, and it's usually done via **penetration testing**.

To remove a fault during the exercise, both **corrective maintenance** and **preventive maintenance**.

1.8.5 Fault forecasting

The fault forecasting is done by performing an evaluation of the system behavior, with respect to fault occurrence and activation, and it's usually done via **qualitative evaluation** and **quantitative evaluation**.

1.8.6 Error recovery

1.8.6.1 Error compensation

When we talk about fault tolerance, we're in fact talking about **fault masking**: a general method to achieve this goal is **performing multiple computations** through replicas, and then apply a vote mechanism to the results. It's worth to remember that hardware faults **fails independently** and, on the contrary, software faults **fails dependently**: to achieve a sort of independence, we can use **design diversity**, that is the use of different design techniques to implement the same function.

Example: error compensation with TMR

Take a system that uses a **Triple Modular Redundancy** (TMR) to achieve fault tolerance. The system has three replicas, and the output is the result of a majority vote. If one of the replicas fails, then the output is still correct, because the other two replicas are still able to provide the correct output.

1.8.6.2 Organization of fault tolerance

We can summarize the organization of fault tolerance in the following figure:

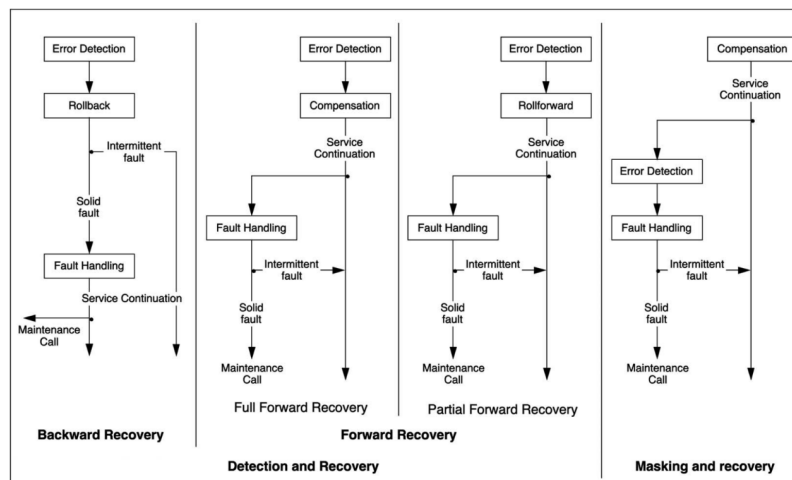


Figura 1.8: Organization of fault tolerance - Avizienis et al., 2004

WE just have to add some definitions:

- **solid faults** are those that are permanent and their activation is repeatable;
- **elusive faults** are those that are permanent and their activation is not systematically reproducible;
- **intermittent faults** are those with transient physical or interaction faults, and their activation is not systematically reproducible.

Remember that the classes of faults that can be actually tolerated depend on the fault assumption that is being considered in the development process, and on the independence of the redundant components that are used to achieve fault tolerance.

1.9 Error detection

The error detection is the ability of the system to detect the presence of an error, and different strategies can be used to achieve this goal, such as:

- **replication checks:** the use of multiple replicas to perform the same computation, and then compare the results, under the assumption that the replicas fail independently;
- **reasonability checks:** the use of a model of the system to check the reasonability of the output, and then compare the output with the model;
- **run-time checks:** mechanisms provided via hardware or software, like division by zero, array bounds, etc.

- **specification-based checks:** the use of the problem specification to check the correctness of the output (e.g. to find a solution to an equation, we can substitute the result in the equation and check if the result is correct);
- **reversal checks:** the use of the inverse function to check the correctness of the output.
- **structural checks:** the use of known properties of the system to check the correctness of the output.
- **timing checks:** the use of watchdogs to check the timing of the system.
- **codes:** the use of codes to check the correctness of the output (e.g. parity, checksum, etc.).

1.9.1 Structural approach to error detection

The main goal is to prevent the propagation of the error, and to achieve that some structural properties should be set to help the system.

1.9.1.1 Principle of least privilege

the concept of **minimum privilege** is crucial, and it's the idea that a component should have the minimum privilege to perform its function, and nothing more. Following this idea, we should consider the fact that **no action is permissible unless it is explicitly allowed**, also known as the concept of **mutual suspicion**.

1.9.1.2 System modularization and partitioning

Remembering the fact that a system should be modularized, we can use the **modularization** to prevent the propagation of the error, adding to each module an error detection (and possibly recovery) mechanism, in order to confine the error to the module in which it occurred and don't let it spread to the other modules. The last reasoning is also valid for the **partitioning** of the system, when modules act independently and the error can't spread to the other modules.

1.9.1.3 Temporal structuring

Another thing to take in consideration is the **temporal structuring** of the activities between the modules, for those operations only between two specific modules that don't communicate with the rest of the system. We also introduce the concept of **atomic action**, that is an action that is performed in a single step, and it's not possible to interrupt it: if a failure occurs, only the participating actions are affected.

1.9.2 Measurement of effectiveness of error detection

Different metrics can be used to measure the effectiveness of the error detection, such as:

- **coverage**: the probability that an error is detected, given that it actually occurs;
- **latency**: the time that elapses between the occurrence of the error and its detection;
- **damage confinement**: the probability that the error is confined to the component in which it occurred;
- **forward recovery**: the probability that the system is able to recover from the error, transforming the erroneous state into a **new** correct state;
- **backward recovery**: the probability that the system is able to recover from the error, transforming the erroneous state into the **previous** correct state.

It's worth to spent some words for the last two metrics, that will be discussed in the next section.

1.9.2.1 Forward recovery

This technique requires to **asses the damage cause** by the detected error **propagates before detection**, and it's usually implemented ad-hoc for the specific system. An effective **example** is the following:

In a real time control system, a situation when input a sensor input is occasionally missed is tolerable, and the system should implement a forward recovery by skipping its response of the missed input.

1.9.2.2 Backward recovery

This technique is a little bit more complex, because **requires a previous correct state** to be restored, also called **checkpoint**, and can be tedious, especially in case when multiple modules are involved, because we need to restore a **consistent checkpoint** for each of them, as we can see in the following figure:

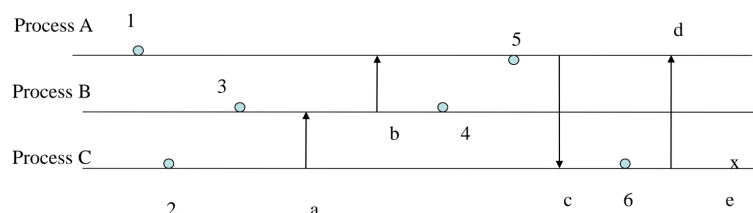


Figura 1.9: Backward recovery - C. Bernardeschi

In this image we see the checkpoint, as circle, the passed messages between the modules, and the error that occurs with a X : to avoid a **domino effect**, we need to restore a consistent checkpoint for each module, also considering their communications, remembering the concept of atomic action.

The basic issues of backward recovery are:

- loss of computation time between the checkpoint and the rollback;
- loss of data between the checkpoint and the rollback;
- the need of a specific mechanism that implements the rollback;
- the increase of the overhead of the system, in order to restore the correct state.

The class of faults that gain benefits from the backward recovery are the **transient faults**, because they usually disappear after a short period of time, in **parallel computing**, to avoid a complete restart of the system, and in **real-time systems**, to avoid the loss of the real-time constraints.

On the other hand, the class of faults that are not suitable for the backward recovery are the **hardware and design faults**, because the system will always do the same action, resulting in the same error.

1.9.3 The exception handling

The exception handling is a mechanism that is used to deal with the errors, and it's usually implemented via software, and it's used to deal with the errors that are detected at run-time. The main goal of the exception handling is to avoid the propagation of the error, and to restore the system to a consistent state. Three are the main classes of exceptions:

- **interface exceptions**: exceptions that are raised when the system receives an input that is not in accordance with the specification, handled by the module that requests the service;
- **internal local exceptions**: exceptions that are raised when the system detects an error in its own state, handled by the module itself;
- **failure exceptions**: exceptions that are not handled by the mechanism, communicated to the user.

2 Redundancy in Fault Tolerant Computing

In this chapter we'll see how to use redundancy to improve the reliability of a system. There are various types of redundancy, and we'll go through them one by one.

2.1 Hardware Redundancy

Briefly, hardware redundancy is the physical replication of the hardware components of a system. This is done to ensure that if one component fails, the system can continue to operate using the redundant component. In this category we can find **passive**, **active** and **hybrid** redundancy.

2.1.1 Passive fault tolerance techniques

In passive tolerance, the **fault masking** is the keystone of the technique: a **voting mechanism** is implemented upon the redundant components, and the output of the system is the result of the majority of the components. This way, if one component fails, the system can continue to operate using the redundant component, and **without any external intervention**.

2.1.1.1 Triple Modular Redundancy (TMR)

This technique is schematized in the following figure:

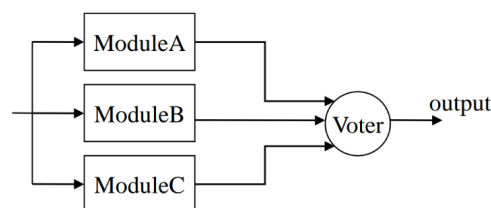


Figura 2.1: Scheme of a TMR implementation - C.Bernardeschi

The main idea is to **triplicate** the specific component, and perform a **majority voting** on the outputs. This technique obviously needs the assumption that the **probability of failure of the components is independent**, but ensure a fault neutralization without the need of any external intervention, and this for each of the components.

TMR is very effective when we're dealing with **transient faults**, but on the contrary isn't very helpful when dealing with **permanent faults**, and this because the fault tolerance decreases since the faulty component remains in the system.

2.1.1.2 Cascaded TMR with triplicated voters

This technique is a further development of the TMR, and is schematized in the following figure:

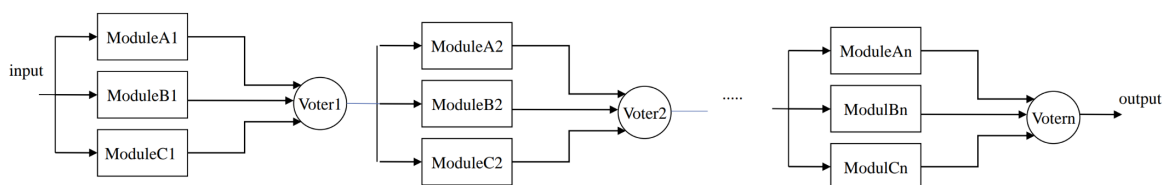


Figura 2.2: Scheme of a cascaded TMR with triplicated voters - C.Bernardeschi

The effect of partitioning the modules is that the design can now withstands more failures than the TMR, thanks to the multiple voters. Unfortunately, we can let this technique arbitrarily reliable, because the reliability of the system is limited by the **reliability of the voters**, which also are a **single point of failure**.

2.1.1.3 The Voter

The voter is a crucial component in the previous techniques: it's usually implemented as an hardware digital circuiting, and it's responsible for the majority voting. This leads to difficulties on its implementation:

- **delay on signal propagation:** the voter must wait for the inputs to be stable before performing the voting, waiting for the synchronization of the inputs;
- trade-off between **achieved fault tolerance** and **cost** in hardware complexity.

There is also another main problem, that come out when we're dealing with **analog signals**: in this case, the majority voting is not so straightforward, and the voter must be implemented as a **digital-to-analog converter**. Given that these converter could produce inconsistent results, another techniques are implemented to ensure the correct voting, such as:

- **average of the signals;**
- choose the **mean of similar signals;**
- choose the **median of the signals.**

2.1.1.4 N-Modular Redundancy (NMR)

This technique is a generalization of the TMR, when the number of redundant components, which **must be odd**, is arbitrary. Using this method, we can cover up to m faulty modules, such that $N = 2m + 1$. The main problem of this technique is that the number of voters grows with the number of redundant components, and this leads to a **high cost**.

2.1.2 Active fault tolerance techniques

In active tolerance, redundancy is used in a **dynamic way**, through fault detection, location and recovery. Briefly, the existence of faults is detected, and some actions are taken in order to remove the faulty component from the system, and replace through a reconfiguration. These techniques are used in context where temporary faults while the system is being reconfigured are acceptable.

2.1.2.1 Duplication with comparison scheme

Two identical pieces of hardware are used, performing the same computation in parallel, with a comparator that takes as input their results. If the results differs, a failure is detected and an error signal is generated. The technique has a good coverage ratio, because it's able to detect **every faults**, except for those that affect the comparator: worth to mention is the simplicity and the low cost, with a minimal impact on the performance. On the other hand, we have to deal with the possibility that the comparator itself could fails, leading to both false positives and false negatives.

2.1.2.2 Reconfigurable Duplication

This technique is a further development of the previous one, and is schematized in the following figure:

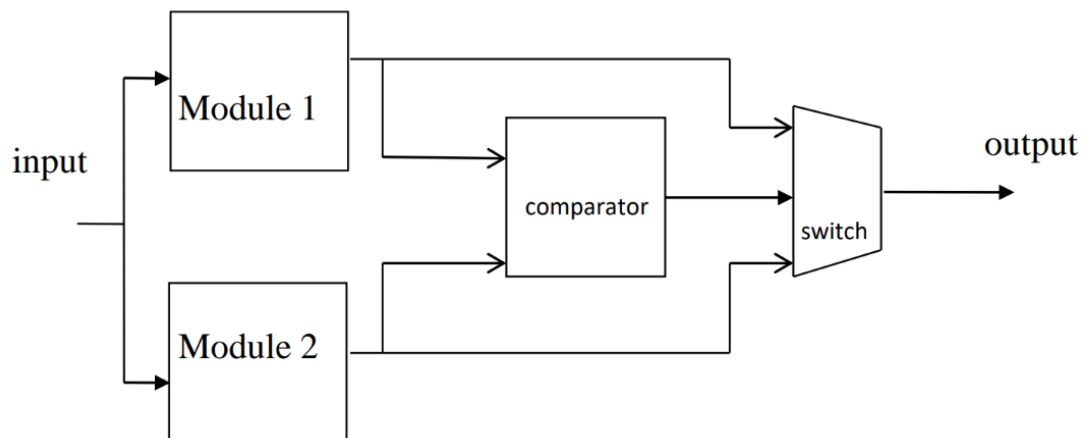


Figura 2.3: Scheme of a reconfigurable duplication - C.Bernardeschi

The circuit works like the previous one, with the difference that the output of the comparator is used as input of the switch, in order to select the module that will be used as output, with the implicit hypothesis that the comparator is able to select the correct value in case of disagreement. To do that, checks such as coding, reversal or reasonable checks are performed, and the switch is used to select the correct output disconnecting the faulty module. This configuration is also called **duplex system**.

2.1.2.3 Stand-by sparing

As we can see in the schema, modules can be **operational** or **spares**:

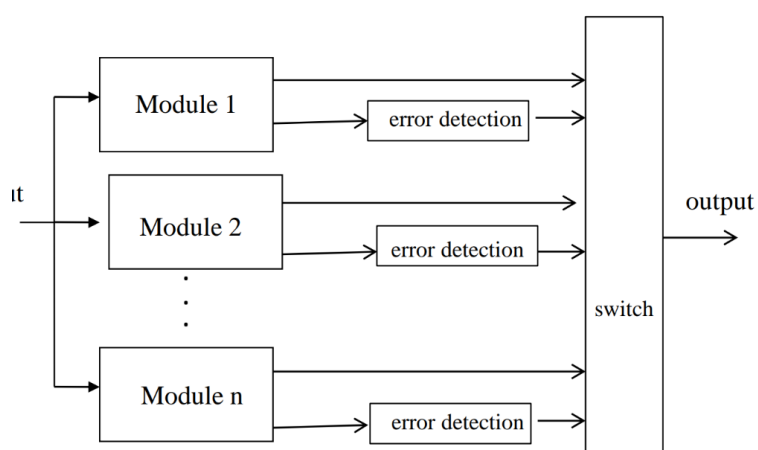


Figura 2.4: Scheme of a stand-by sparing - C.Bernardeschi

The switch implements the **fault detection and localization**, and can decide to no longer use a module if it's faulty, and to replace it with a spare. Spares are divided in three categories:

- **hot spares:** they are always active, and ready to replace a faulty module in every moment;
- **warm spares:** they are running, but receives inputs from the system only when they are activated;
- **cold spares:** they are off, and must be activated before being used.

A **pair-and-spare** approach can also be used, when every module is a **duplex system**, connected to the switch with a comparator: as long as the outputs agree, the spares are not used, but in case of disagreement, the switch operates the replacement.

2.1.3 Hybrid fault tolerance techniques

In this category, we can find the **combination** of the previous techniques, in order to obtain a better fault tolerance. The main idea is to combine the advantages of the previous techniques, and to mitigate their disadvantages. Intuitively, we can say that costs increase dramatically, such as the complexity of the implementations, so this techniques are used only in **critical systems**.

2.1.3.1 Reconfigurable NMR

This technique is a combination of the NMR and the reconfigurable duplication, and is schematized in the following figure:

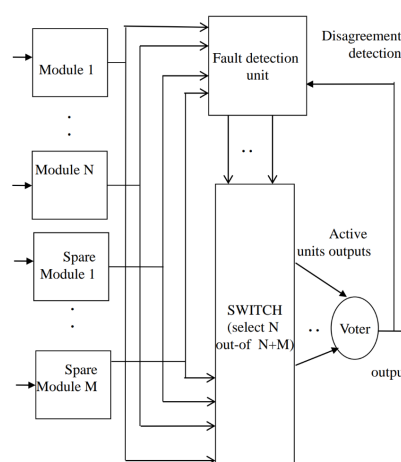


Figura 2.5: Scheme of a reconfigurable NMR - C.Bernardeschi

There are N redundant modules, and the voter; the **fault detection units** compare the outputs of the voters with the output of the active modules, and replace the faulty module with a spare, isolating it from the system. The reliability of the circuit holds as long as the number of spare modules isn't zero, and the coverage depends on the number of spare: a TMR with one spare can tolerate up to two faults, by masking the first module, replacing it and possibly mask the second one.

2.1.4 Summary of the hardware redundancy techniques

The key differences between the techniques are summarized:

- **passive** rely on the **fault masking**, and requires high investments in the hardware;
- **active** rely on the **error detection**, **fault localization** and **recovery**, but has the disadvantage of needing additional hardware to detect and recover from the faults, and can produce **transient errors**;
- **hybrid** are a combination of the previous techniques. The reliability of the system is increased, but the costs are the highest.

2.2 Information Redundancy

When we talk about information redundancy, we're referring to **coding**, that is the application of the redundancy within the information itself. This is done using more bits than strictly necessary to represent the information: if n is the number of bits needed to represent the information, we use $m = n + c$ bits, such that among all the possible 2^m combinations, only 2^n represents acceptable information, and these combinations are called **codewords**; if a non-codeword is received, an error is detected.

Coding needs a phase of **encoding**, where the c bits are calculated and added to the information, and a phase of **decoding**, where the information is extracted from the codeword. We refer to **separable code** when the information and the redundancy are clearly separated, such as a concatenation of the information and the redundancy, and to **non-separable code** when the information and the redundancy are mixed together.

2.2.1 Parity codes

The simplest form of coding is the **parity code**, where a single bit is added to the information, such that the number of 1s in the codeword is even or odd. This technique is very simple, but is able to detect only **single bit errors**, and can't correct them.

Example:

with $n = 2$ and $m = 3$, 8 are the possible combinations, and 4 are the codewords. We can easily get that the codewords are 001, 010, 100 and 111.

2.2.2 Complemented duplication

In this technique, the information is duplicated, having n bits of information and n bits of redundancy, such that the redundancy is the complement of the information. In this case, having $n = 4$, we can have 16 possible combinations, and 4 are the codewords.

2.2.3 Hamming distance

We define the **Hamming distance** as it follows:

Given two codewords x and y , the Hamming distance between them is the number of bits in which they differ.

The **minimum Hamming distance** between two codewords is the number of independent single bit errors that the code can detect; a code such that the Hamming distance is $\geq k$ will detect up to k erroneous bits.

The main concept is that **the corrupted data is closer to the correct data than to any other codeword**. In the following figure, we can see a generic example of the Hamming distance, with the differences between predictable and unpredictable errors.

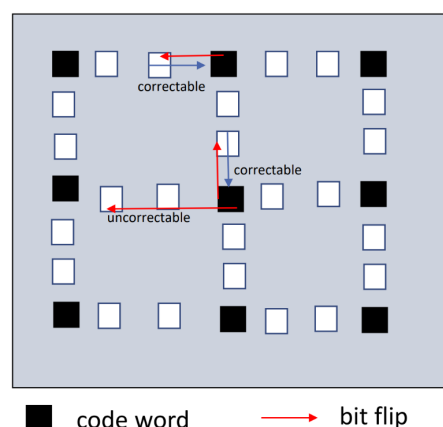


Figura 2.6: Hamming distance - C.Bernardeschi

2.2.4 Checksums

This method is usually applied to large blocks of data, and it's able to cover a single fault. The checksum for a block of n words is formed by adding together all the words modulo k , with k arbitrary. The checksum is then stored in the data block, and when the block is transmitted, the checksum is recalculated and compared with the stored one; the codeword in this case is composed by the entire block of data, and the actual checksum.

The main disadvantages of the technique are:

- the need to recalculate the checksum every time the data is modified;
- the error detection is limited is allowed, but the error localization is not possible.
- it represents a single point of failure.

2.2.5 ECC

This technique enables the **error location** for single-bit error, as we can see in the following figure:

Odd parity		n-bit words	row parity	
k words		1 0 1 ... 0	1	
		0 0 1 ... 1	1	← parity error
		1 1 1 ... 0	0	
		1 0 0 ... 0	0	
	column parity			
		↑		parity error

Figura 2.7: ECC - C.Bernardeschi

2.2.6 Hamming codes

This technique is usually used in databases, with specific disks that implements that mechanism, and it's based on **spreading parity bits across the data**. The main idea is to use the **parity bits**, that are all bit in positions that are a power of 2, to check the data, and to use the **data bits** to store the information. In this way, the data bits are included in a set of two or more parity bits, and the parity bits are included in a set of two or more data bits, implementing a **detection and correction** mechanism. In the figure we can see an example of the Hamming code:

Bit position	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
Encoded data bits	p1	p2	d1	p4	d2	d3	d4	p8	d5	d6	d7	d8	d9	d10	d11	p16	d12	d13	d14	d15	
Parity bit coverage	p1	✓		✓		✓		✓		✓		✓		✓		✓		✓		✓	
	p2		✓	✓			✓	✓			✓	✓			✓	✓			✓	✓	
	p4				✓	✓	✓	✓					✓	✓	✓	✓					✓
	p8								✓	✓	✓	✓	✓	✓	✓	✓					
	p16																✓	✓	✓	✓	✓

Figura 2.8: Hamming code - Wikipedia

The parity bit p_j covers all the bit whose position has the j -th bit set to 1, so every data bit is covered by a set of parity bits.

2.2.7 Self-checking circuitry

In order to implements all the previous methods, we have the necessity of reliability on the modules that performs checks and comparisons, and this is done through the **self-checking circuitry**.: given a set of faults, the circuit has the ability to **automatically detect** the fault, during the normal course of its operations; this is usually achieved by implementing coding techniques in the circuitry, such that input and outputs are encoded. . Three are the main techniques:

- **self-testing** circuitry: if, for every fault from the set, does exists a non-code output;
- **fault-secure** circuitry: if, for every fault from the set, the circuit never produces an incorrect output for any input;
- **totally self-checking** circuitry: if the circuit is both self-testing and fault-secure.

2.3 Time Redundancy

To avoid the need (and the expense) of additional hardware, we can use another type of redundancy, called **time redundancy**: the main idea is to exchange the **expense of the hardware** with the **expense of the time**.

2.3.1 Repetition of the computation

If we implement the time redundancy in this way, we have to **perform multiple time** the same computation: it appears obvious that **permanent faults cannot be detected**, given the fact that

the latter lead the system to always have the same error, but instead it's effective against **transient faults**. We have also the problem to guaranteeing that the computation is performed in the same way and with the same data, and the non-negligible overhead.

2.4 Software Redundancy

As we briefly said in the previous chapter, the software is subject to both **operational faults** and **design flaws**. In particular, the latter is due to **ambiguities** in the specifications, or in mistakes made during the implementation. They're hard to visualize, and they're also closely related to a human factor: if, for example, only specific inputs trigger a fault, the number of failures depends on the number of times the inputs are used; it's also true that the **apparent reliability** of a software is more due to the number of the exercised design faults, rather than the actual number of design faults present in the software. Given these premises, and the fact that the software has a large cost of development, the main focuses of software reliability are made on the **fault prevention** and **testing strategies**, usually with a **multi-version** approach.

2.4.1 Software diversity

Implementing two (or more) identical version of the software, these versions will always fail not in an independent way, leading to an inability to detect possible software faults. For this reason, the versions must be **diverse**: they obviously must be functionally equivalent, but their development must be carried out by different teams, using different tools and different algorithms. Developing N different versions, the need of a decision mechanism is necessary, and this is usually implemented through a **voting mechanism**.

2.4.1.1 Disadvantages and practical considerations

When using software diversity, we have to deal with the higher cost for development and concurrent execution of the software, and the possibility to have **correlated errors**, due to specification mistakes that shouldn't be tolerated. A practical concern over the voting mechanism is that every version will have different compilers and formats for data types, that must be taken into account when implementing the voting mechanism.

2.4.2 N-version programming

2.4.2.1 The voter

Some considerations we made in the previous sections, when we talked about hardware redundancy, are also valid for software redundancy: the voter is still a **single point of failure**, and it's not replicated, in order to remain simple and verifiable.

New tasks for the voter are that must **verify the consistency of input data** within the different versions, and must be able to receive data in a identical format from every version, both implementing a communication protocol and/or a efficient data conversion.

2.4.2.2 N-version self-check programming

This circuit is based on the **acceptance tests** rather than the comparison between equivalent version: the voter implements some sort of selection logic that takes as input only the results from the versions whose output passed the specific-version acceptance tests, tolerating up to $N - 1$ faults.

2.4.3 Design diversity

This technique is based on the fact that the **same specification can be implemented in different ways**, and the different implementations can be used to check each other. The main idea is to use different algorithms, data structures and programming languages, and to use the same inputs to check the outputs. Unfortunately, as we said before, it's not possible to assume that these versions will fail independently, and this is due to the fact that the same specification can be implemented in different ways, but the same design flaws can be present in the different implementations: it's in fact true that, from empirical studies, that common faults are pretty common, but at the same time, implementing the diversity delivers actual improvements in the reliability.

3 Basic building blocks in Fault tolerant distributed systems

3.1 Atomic actions

We intend as atomic an action that is indivisible, that is, it is executed in its entirety or not at all. In a distributed system, an atomic action is generally executed at more than one node, so these nodes have to cooperate in order to guarantee the atomicity property: if one of the nodes fails, the action shouldn't let its effects be visible to the entire system.

3.1.1 Atomic actions in a database

In a database, an atomic action is a transaction, that can be composed by more than one operation such as read, write, update or deleted a record in the database.

3.1.1.1 Commit

When a transaction is committed, all its effects are made visible to the entire system. This means that the transaction is finished and its effects are permanent. An example of a transaction is a bank transfer, where the money is removed from one account and added to another account, as we can see in the following example:

```
1  t: begin transaction
2
3      UPDATE account
4      SET balance = balance + 500
5      WHERE account_number = 45;
6
7      UPDATE account
8      SET balance = balance - 500
9      WHERE account_number = 35;
10
11 t: commit
12 end transaction
```


3.1.1.2 Abort or Rollback

When a transaction is aborted, all its effects are discarded and the system is restored to the state it was before the transaction started. An example of a transaction that is aborted is a bank transfer where the money is added to one account but not removed from the other account because of balance insufficiency, as we can see in the following example:

```
1  t: begin transaction
2
3      UPDATE account
4      SET balance = balance + 500
5      WHERE account_number = 45;
6
7      UPDATE account
8      SET balance = balance - 500
9      WHERE account_number = 35;
10
11     SELECT balance
12     INTO V FROM account
13     WHERE account_number = 35;
14
15     IF V >= 0 THEN
16         commit;
17     ELSE
18         abort;
19     END IF;
20 ;
21 end transaction
```

3.1.2 Atomic actions in a distributed system

Imagine now that the bank database is distributed, and the two accounts are stored in two different nodes. In this case, the transaction has to use data stored in different nodes: a possible rollback of the transaction must leave both nodes in a consistent state, that is, the state they were in before the transaction started. Using the previous codes as an example, the DBMS has to guarantee that if the transaction is aborted, the money is not added to the first account and the money is not removed from the second account, considering the fact that these information are stored in different nodes.

3.1.3 Two-phase commit

The two-phase commit is a protocol that guarantees that a transaction is either committed or aborted in a distributed system. We have some crucial elements to consider:

- the **transaction manager TM**, that is the entity that coordinates the transaction;

- some **resource managers RM**, that are the entities that manage the resources involved in the transaction.
- a **log file**, stored in persistent memory, that is used to recover the state of the transaction in case of failure;
- a **timeout**, that is the maximum time that the TM waits for a response from the RMs.

A scheme of the two-phase commit is the following:

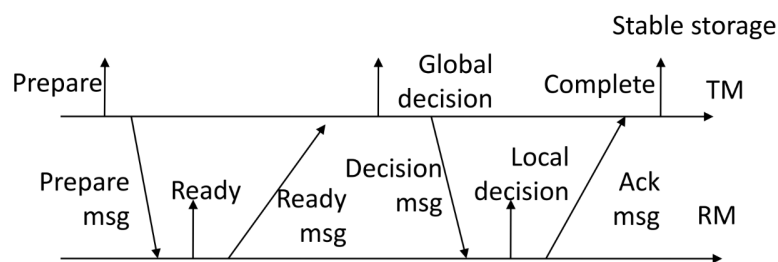


Figura 3.1: Two-phase commit

In case of fail of the TM, we have an **uncertain period**, during which a RM with a ready state cannot terminate the transaction. Note that this mechanism is able to tolerate both loss of messages and crash of nodes, simply rolling back the transaction.

3.1.4 Three-phase commit

This protocol is an evolution of the previous one, where a **pre commit** phase is added. Assuming a permanent crash of the TM, a participant is able to substitute the coordinate, in order to terminate the transaction, with a **global commit** or a **global abort**.

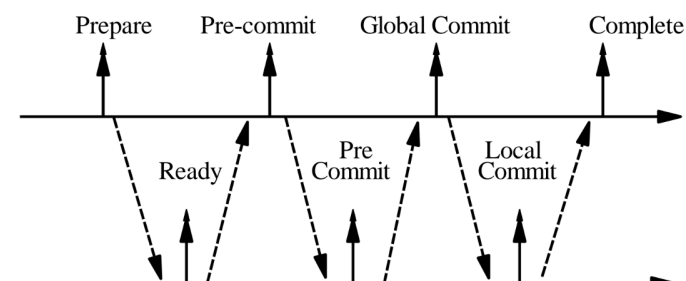


Figura 3.2: Three-phase commit

3.1.5 Recovery and atomicity

Given a block B , that can be a **physical block** (a disk block) or a **buffer block** (temporary storage in memory), the operations to move a block from a disk to a buffer are **input(B)** and **output(B)**. A transaction T_i has its own private work-area, in which local copies of all accessed items are stored, performing the operations **read_item(X)** and **write_item(X)**. The system can perform the **output** of the operation, through the **output(B_X)** operation, that need not immediately follows the **write_item(X)** operation.

In general, several outputs are required by a transaction, and the latter can be aborted after one of these operations have been made permanent; it's also true that a transaction can be committed, and a failure can occur before all the outputs are made permanent. In order to guarantee the atomicity property, the system has to perform the **output** operation in a way that the transaction is either committed or aborted, and the **output** operation is either made permanent or not, using a **log file** to recover the state of the transaction in case of failure.

3.1.6 Log file

The log file is a file stored in persistent memory, that is used to recover the state of the transaction in case of failure. Before the commit of a transaction, system has to save into the log file all the records of the operations that have been made permanent, and after the commit of a transaction, system has to save into the log file a record that indicates the commit of the transaction, executing **UNDO** and **REDO** operations in case of failure, to restore the state of the transaction.

3.2 Consensus problem

The consensus problem describes **how a set of distributed systems can agree on a value**, despite failures; to help its description, the *Byzantines Generals* metaphor is used. In this scenario, generals must agree on an attack plan or a retreat, but not all of them are loyal: we have the presence of traitors among them, who may lie in order to influence the plan to take an advantage to the enemies. Each general observes the enemy and communicates their observations to the others: traitors may lie in order to support their own plan, or even misrepresent the observations of the loyal generals, making it difficult to reach an agreement.

3.2.1 The Byzantine Generals problem

As we just said, generals can be loyal or traitors, and they reach the consensus when:

1. all loyal generals agree on the same plan;
2. a small number of traitors cannot cause the loyal generals to adopt a bad plan.

3.2.1.1 Assumptions

The general assumptions of the Byzantine Generals problem are:

- let n be the number of generals;
- the opinion of a general is either **attack** or **retreat**, described by the function $v(i)$;
- each general i share their value $v(i)$ to each other general;
- each general final decision is obtained by a majority vote among the opinions shared by the generals.

3.2.1.2 Role of the traitors

We have **absence of traitors** when the generals have the same value $v(1), v(2), \dots, v(n)$, a situation that results in taking the same decision. On the contrary we have **presence of traitors** when the generals have different values, and the traitors can influence the decision of the loyal generals. In particular, in presence of traitors:

- to satisfy the first condition, the loyal generals have to agree on the same value. In other words, every general must apply the majority function to the same set of values $v(1), v(2), \dots, v(n)$;
- to satisfy the second condition, for each general i , if they are loyal, the the value they $v(i)$ they sent must be used by every loyal general as the value of general i .

3.2.1.3 Interactive consistency

Interactive consistency is a concept where a commanding general C and $n - 1$ lieutenants L_1, \dots, L_{n-1} must agree of a action plan. Then, we have two situations:

- IC_1 : all loyal lieutenants obey the order of the commanding general;
- IC_2 : the decision of loyal lieutenants must be the same as the decision of the commanding general, if they're loyal.

In simple terms, the Interactive consistency ensures that all loyal lieutenants follow the same command issued by the general, maintaining consistency in the decision.

3.2.1.4 Case study: 3 generals and 1 lieutenant traitor

First of all, note that **there are no solutions** for this case. Let's now observe the scheme of the generals:

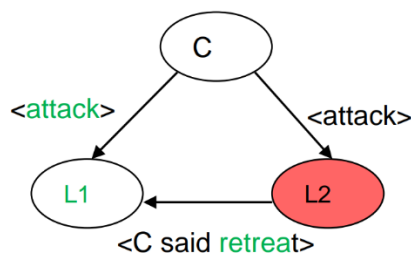


Figura 3.3: Byzantine Generals

In this situation, with two different commands, we assume that L_1 must obey the command of C :

- if L_1 decides to attack, both IC_1 and IC_2 are satisfied;
- if L_1 decides to retreat, IC_1 is satisfied, but IC_2 is not.

From the latter, we can get the following rule: **if L_i receives different commands, they will always takes the decision received by C .**

3.2.1.5 Case study: 3 generals and 1 general traitor

The situation is the same as the previous one, and the same inferred rule is applied:

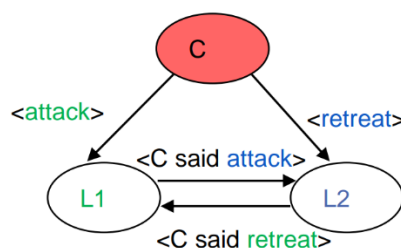


Figura 3.4: Byzantine Generals

- L_1 must obey to G , and decides to attack;

- L_2 must obey to G , and decides to retreat.

In this case, the IC_1 is violated, because the lieutenants have different decisions, but the IC_2 is satisfied. In general, **four generals are required to cope with one traitor**.

3.2.1.6 Generalization of the Byzantine Generals problem

A general scheme for the Byzantine Problem is the following:

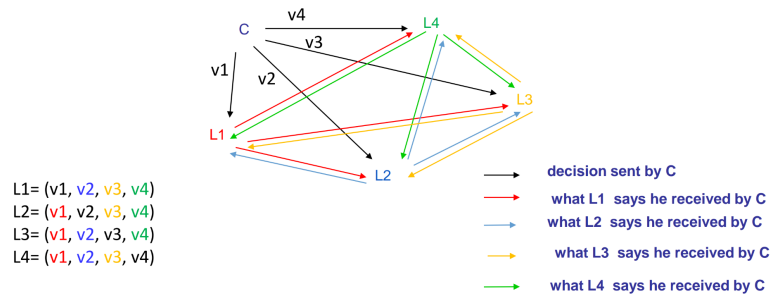


Figura 3.5: Byzantine Generals

In case there isn't any traitor, each L_i will receive three copies of the same order they've already received from C .

3.2.2 Oral message algorithm

In this section, we'll go through the oral message algorithm, that is a solution to the consensus problem. The algorithm is based on the following assumptions:

- the system is **synchronous**;
- any two processes can communicate with each other, and the **communication is reliable and instantaneous**;
- the **receiver** can identify the **sender** of the message;
- every message sent by a non-faulty process is **eventually delivered**;
- the **absence of messages** can be detected.

This algorithm is able to solve the Byzantine problem for $n \geq (3m + 1)$ general, and includes a majority vote on the values received from the lieutenants: note that, if a traitor doesn't send any message, we assume that the message is *retreat*; consider also the fact that the majority vote returns *retreat* if a majority is not reached.

3.2.2.1 The algorithm for $OM(0)$

1. The commander C sends their value to every lieutenant L_i , with $i \in \{1, \dots, n-1\}$;
2. Each lieutenant uses the received value.

3.2.2.2 The algorithm for $OM(m)$, with $m > 0$

1. C sends their value to every lieutenant L_i , with $i \in \{1, \dots, n-1\}$;
2. Let v_i the value received by L_i from C : L_i acts as C in $OM(m-1)$ to send V_i to each of the $n-2$ lieutenants;
3. For each i , with $j \neq i$, let v_j be the value that goes from L_j to L_i in the previous step;
4. Perform a majority vote among values $\{v_1, \dots, v_{n-1}\}$

The algorithm is **recursive**, and it invokes $n-1$ calls for $OM(m-1)$, $n-2$ calls for $OM(m-2)$ and so on.

3.2.3 Consideration about the algorithm

Solution for consensus problem are highly costly, requiring a minimum of $3m+1$ nodes and $m+1$ rounds, with message sizes $O(n^{m+1})$ growing at each round. Different metrics are used to evaluate algorithms, including the number of faulty processors, the number of rounds, and the message size; some of these algorithms have been proven optimal for specific aspects.

3.2.4 Signed messages

The problem is made challenging due to the ability of the traitors to lie: in order to address this aspect, a solution involving signed messages has been proposed, enabling generals to send unforgeable and authenticated messages. This is a huge simplification of the problem, that relies on the following assumptions:

- the **signature cannot be forged**, and any alteration of a signed message can be detected;
- the authenticity verification can be made by everyone.

However, no assumptions on the signature of traitor generals have been made.

3.2.4.1 The choice algorithm

Let V be a set of orders, with a function `choice(V)` that obtain a single order from a set. The function will returns:

- *retreat* if V is empty;
- v if V is composed only by a single element v ;
- *retreat* if V consists of more than one element.

General 0 is the commander, and for each i , V_i is the set of properly signed orders that Lieutenants L_i received so far.

3.2.4.2 Signed algorithm $SM(m)$

The algorithm ensures that at most m lieutenants can send an order to their subordinates. The algorithm is based on the following steps:

1. at the start, all lieutenants have an empty set V_i ;
2. C signs and sends their value to every lieutenant L_i , except for the last one;
3. each L_i collects and verifies the received signed messages, appending their signatures if necessary, and broadcast the updated message to other lieutenants;
4. when no more messages are received, each lieutenant applies the **choice** function to the set of received messages.

Some key observations about the algorithm:

- lieutenants will discard any messages that have been already received;
- if L_I is the m -th lieutenant that adds their signature to the message, the message is not broadcasted anymore;
- time-outs techniques can be used to detect the absence of messages.

There is an algorithm, that has been formally proved, that states: **for any m , the algorithm $SM(m)$ solves the Byzantine Generals problem for $n \geq 2m + 1$ generals and m traitors.**

3.2.5 Impossibility results

In **asynchronous distributed systems**, where no timing assumptions are made, making them easier to port application and handle variable workloads, the consensus problem **cannot be deterministically solved**, and this because it's difficult to distinguish between a slow process and a failed one, and deciding to stop a single process in an inopportune moment can lead to a failure of the consensus algorithm. In order to circumvent this issue, different strategies can be used, such as:

- **loosely synchronized systems**, where different processors allocated to a task are executing the same iteration, so they don't need tight synchronization;
- **median clock algorithm**; each clock observes every other clocks, and sets its time to the median value of the observed clocks.

3.2.6 Clock synchronization

Let's observe the following figure:

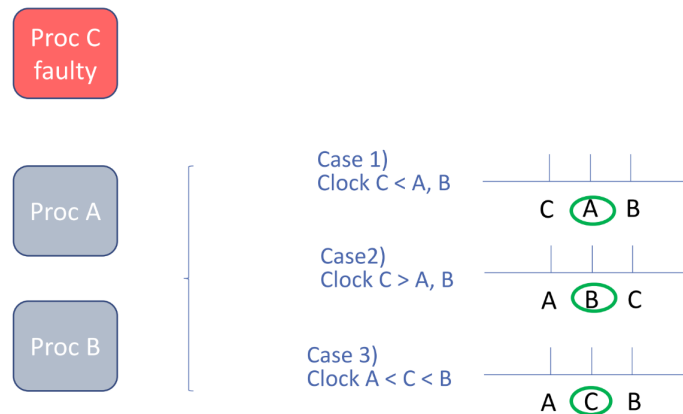


Figura 3.6: Clock synchronization

Without loss of generality, we can state that $\text{clock}(A) \leq \text{clock}(B)$, and assume C as faulty. Assume that A and B observe the same value of C : we have a **non Byzantine failure**, with processes that may obtain different values from the faulty process.

Another case can be shown with the following hypothesis:

- $\text{clock}(A) = 10 < \text{clock}(B) = 20$;
- assume a Byzantine failure of C .

If A sees a value for clock C that is slightly lower than its own value, and B sees a value for clock C that is slightly higher than its own value, both A and B will see their own value as the median value, and the algorithm will fail, and this because **the median computed by two different processes will be the same if the set of clock values they obtain is the same**.