POL213 TA Session

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```r
## Clear Workspace
rm(list = ls())

## Set Working Directory to the File location
## (If using RStudio, can be set automatically)
setwd(dirname(rstudioapi::getActiveDocumentContext()$path))
getwd()
```

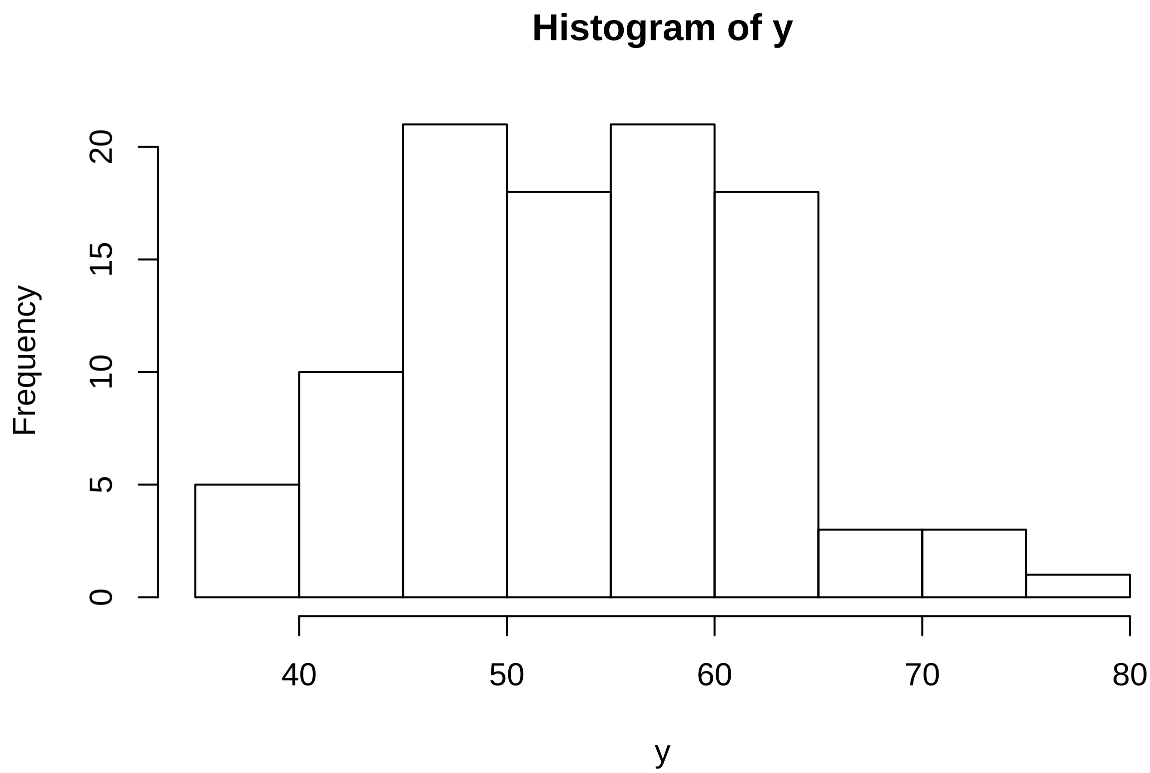
```
## [1] "C:/GoogleDrive/Lectures/2019_04to06_UCD/POL213_TA/POL213_TA_resource"
```

```r
## Required packages
library(readr) # Reading csv file
library(ggplot2) # Plotting
library(faraway) # for ilogit function
```

Coarse Grid Search

Think about the voter turnout of counties within a state, follows a normal distribution with mean 53.2 and standard deviation 8

```r
set.seed(780)
y <- rnorm(100, mean = 53.2, sd = 8)
hist(y)
```



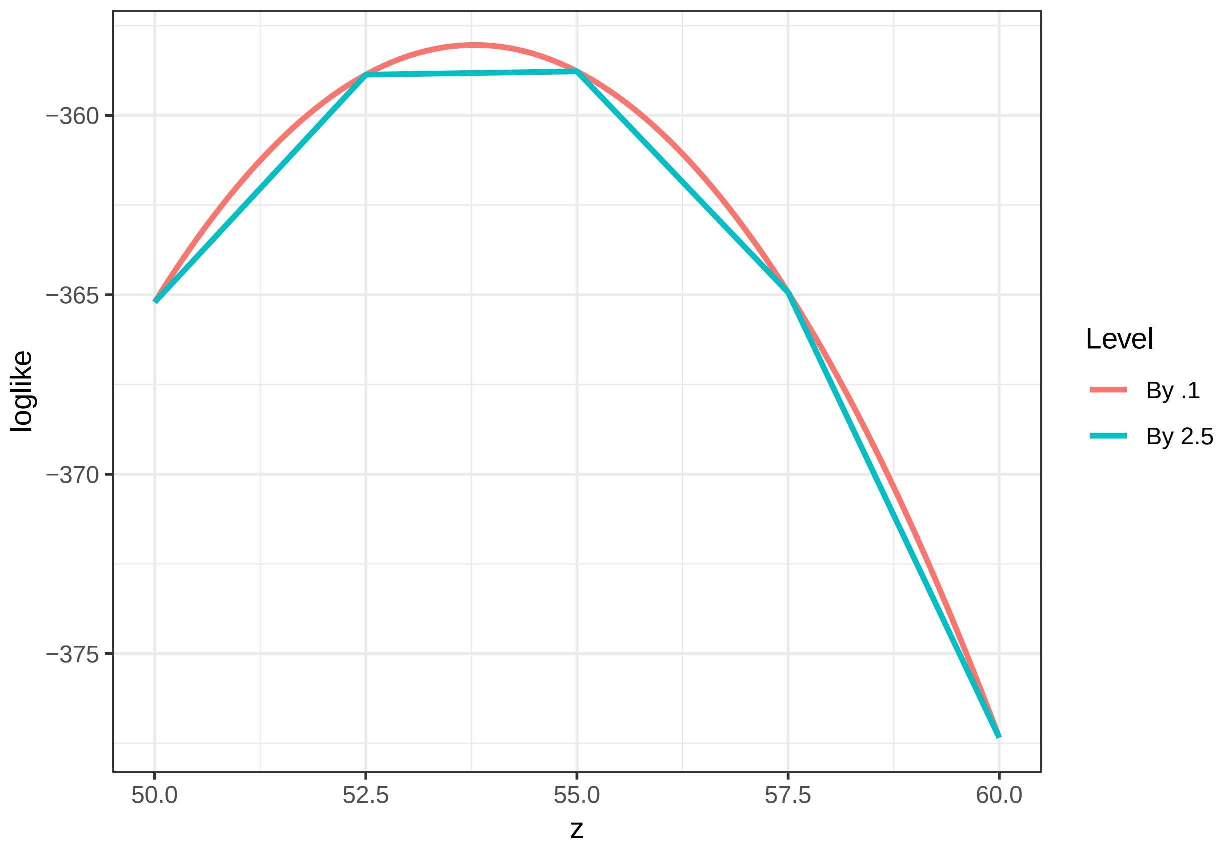
Assuming that standard deviation is 10, conduct a coarse grid search of theta parameter.

```
z1 <- seq(50,60, by = 2.5)
loglike1 <- sapply(z1, function(z) log(prod((1/sqrt(2*100*pi))*exp(-((y-z)^2/(2*100))))))

z2 <- seq(50,60, by = .1)
loglike2 <- sapply(z2, function(z) log(prod((1/sqrt(2*100*pi))*exp(-((y-z)^2/(2*100))))))

# Make it a data.frame
searchdata <- data.frame(z = c(z1,z2),
                         loglike = c(loglike1, loglike2),
                         Level = c(rep("By 2.5",length(z1)),
                                 rep("By .1", length(z2))))

# Export plot
ggplot(searchdata, aes(x=z, y=loglike, color=Level)) +
  geom_line(size=1) + theme_bw()
```




```
# Find Max
z1[which.max(loglike1)]
```

```
## [1] 55
```

```
z2[which.max(loglike2)] # More fine grained
```

```
## [1] 53.8
```

Fitting Logit

The following data contains county level presidential election results 2000-2016. (Check codebook at https://github.com/gentok/POL213_TA_Resource/blob/master/data/County%2BPresidential%2BReturns%2B2000-2016.md)

```
d <- read_csv("https://raw.githubusercontent.com/gentok/POL213_TA_Resource/master/data/countypres_2000-
```

```
## Parsed with column specification:
## cols(
##   year = col_integer(),
##   state = col_character(),
##   state_po = col_character(),
##   county = col_character(),
##   FIPS = col_integer(),
##   office = col_character(),
##   candidate = col_character(),
##   party = col_character(),
```

```
##   candidatevotes = col_integer(),
##   totalvotes = col_integer(),
##   version = col_integer()
## )
```

```
d <- na.omit(d)
```

Let's subset the data and extract county-level votes for Gore (2000), Bush (2000), Obama (2008), and McCain (2008) in Texas.

```
# Gore Vote Share in TX
TX_gore <- d[d$year==2000 & d$party == "democrat" & d$state_po == "TX",]
# Bush Vote Share in TX
TX_bush <- d[d$year==2000 & d$party == "republican" & d$state_po == "TX",]
# Obama Vote Share in TX
TX_obama <- d[d$year==2008 & d$party == "democrat" & d$state_po == "TX",]
# McCain Vote Share in TX
TX_mccain <- d[d$year==2008 & d$party == "republican" & d$state_po == "TX",]
# Check if county rows match
all(TX_obama$FIPS == TX_mccain$FIPS)
```

```
## [1] TRUE
```

```
all(TX_obama$FIPS == TX_bush$FIPS)
```

```
## [1] TRUE
```

```
all(TX_obama$FIPS == TX_gore$FIPS)
```

```
## [1] TRUE
```

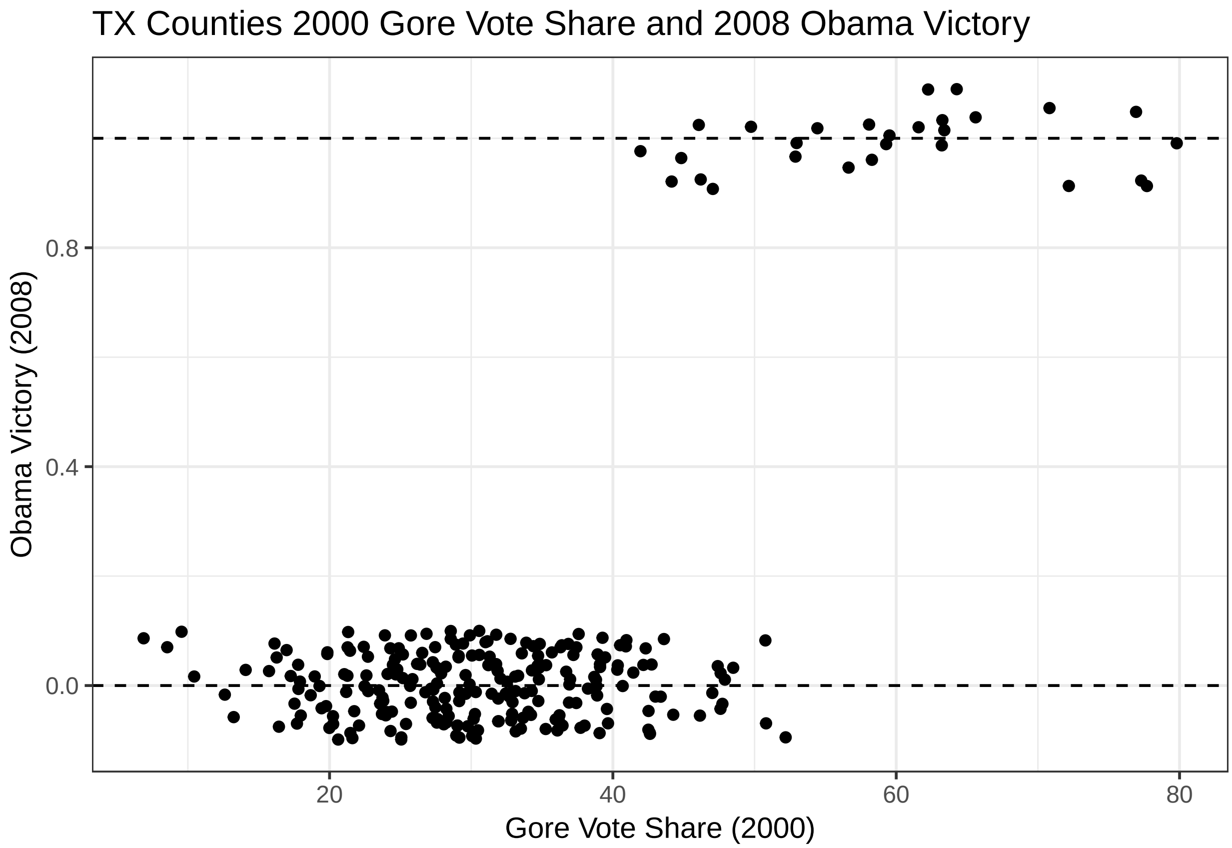
Calculate 2008 Obama win-lose and 2000 Gore Vote Share

```
# Create Data
TX_data <- data.frame(FIPS = TX_bush$FIPS)

# Gore Vote Share
TX_data$goreshare <- TX_gore$candidatevotes/(TX_gore$candidatevotes +
                                        TX_bush$candidatevotes) * 100
# Obama Win-Lose in County
TX_data$obamawin <- (TX_obama$candidatevotes >= TX_mccain$candidatevotes) * 1
```

Estimate Logistic Regression predicting Obama win-lose by Gore vote share.

```
# Plot Obama win-lose by Gore Vote Share
p <- ggplot(TX_data, aes(x=goreshare,y=obamawin)) +
  geom_jitter(height=0.1) + # Jittered points
  geom_hline(aes(yintercept=1), linetype=2) + # Horizontal dashed line @ 1
  geom_hline(aes(yintercept=0), linetype=2) + # Horizontal dashed line @ 0
  xlab("Gore Vote Share (2000)") +
  ylab("Obama Victory (2008)") +
  ggtitle("TX Counties 2000 Gore Vote Share and 2008 Obama Victory") +
  theme_bw()
p
```




```
# Estimate Logistic regression
logit.TX_obamawin <- glm(obamawin ~ goreshare, TX_data, family = binomial)
summary(logit.TX_obamawin)
```

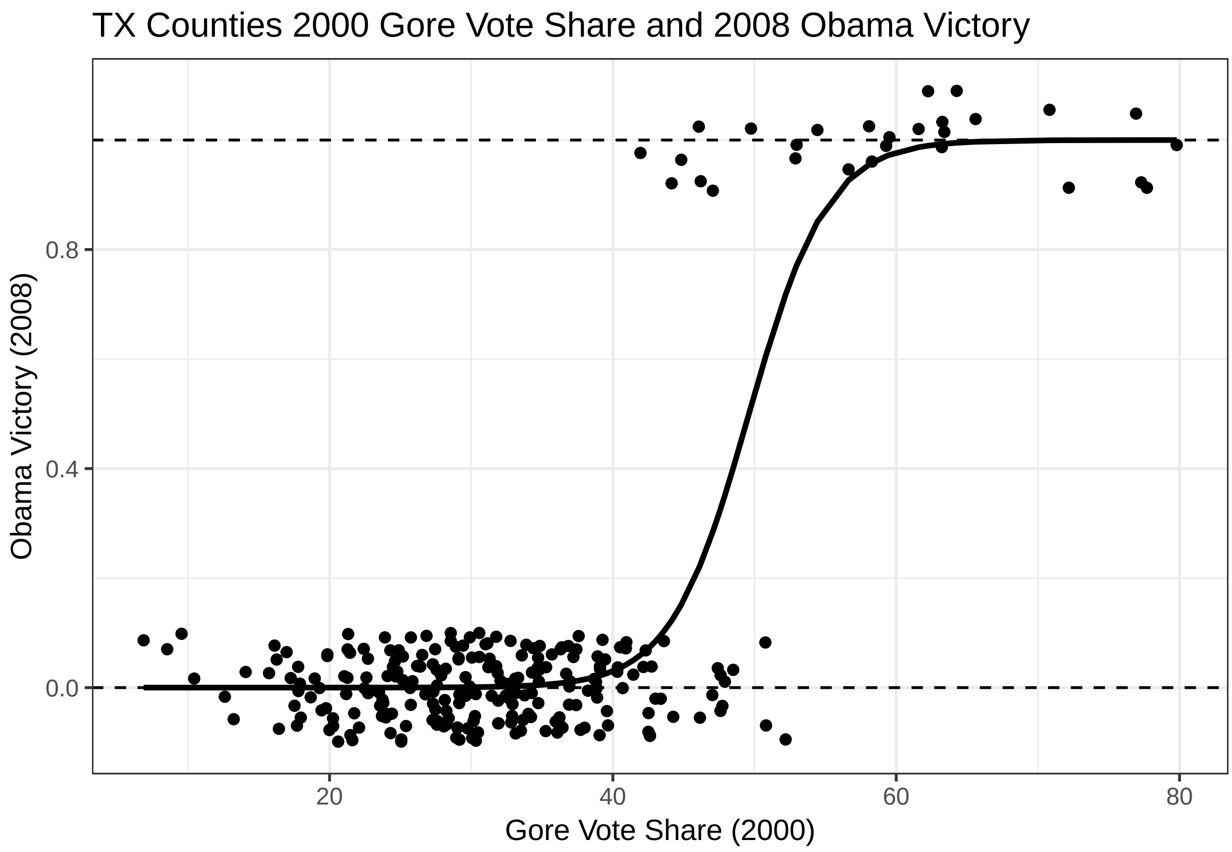
```
##
## Call:
## glm(formula = obamawin ~ goreshare, family = binomial, data = TX_data)
##
## Deviance Residuals:
##      Min        1Q    Median        3Q       Max
## -1.59085  -0.09826  -0.03393  -0.00881   2.37681
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept) -17.89819    3.59583  -4.977 6.44e-07 ***
## goreshare     0.36081    0.07651   4.716 2.41e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 176.281  on 253  degrees of freedom
## Residual deviance:  41.618  on 252  degrees of freedom
## AIC: 45.618
##
## Number of Fisher Scoring iterations: 8
```

```r
# Log-likelihood of the estimates
logLik(logit.TX_obamawin)
```

```
## 'log Lik.' -20.80918 (df=2)
```

```r
# Calculate Logit prediction
prediction <- ilogit(-17.8919 + 0.36081*TX_data$goreshare)
# OR
prediction <- predict(logit.TX_obamawin, type="response")

# Add prediction to the plot
p + geom_line(aes(y=prediction), size=1)
```



Manually Fitting Logit

Prepare Variables & functions

```r
# DV
y <- cbind(TX_data$obamawin)
# IV
x <- cbind(TX_data$goreshare)
# Constant
cons <- rep(1, length(x[,1]))
# Matrix of Constant and IV(s)
xmat<-cbind(cons, x)
```

```r
# Function to calculate Log Likelihood
llk.logit <- function(param,y,x) {
  # prepare constant
  cons <- rep(1, length(x[,1]))
  # matrix of constant and IV(s)
  x <- cbind(cons, x)
  # assigned beta parameters
  b <- param[1 : ncol(x)]
  # calculate fitted values
  xb<-x%*%b
  # calculate log-likelihood
  sum(y*log(1 + exp(-xb)) + (1-y)*log(1 + exp(xb)))
}

# Set starting values taken from OLS.
ols.result <- lm(y~x); ols.result
```

```
##
## Call:
## lm(formula = y ~ x)
##
## Coefficients:
## (Intercept)            x
##    -0.48227      0.01768
```

```r
stval <- ols.result$coeff
stval
```

```
## (Intercept)           x
## -0.48226639  0.01767522
```

First iteration

```r
# Optimize by log-likelihood
logit.result <- optim(stval, llk.logit, method="BFGS",
                      control=list(maxit=0, trace=1), hessian=TRUE, y=y, x=x)

# Printing Optimization results #
# beta paramter estimates
parm_est <- logit.result$par; parm_est
```

```
## (Intercept)           x
## -0.48226639  0.01767522
```

```r
# variance covariance matrix
var_cov <- solve(logit.result$hessian); var_cov
```

```
##              (Intercept)             x
## (Intercept)  0.130804583 -0.0034583735
## x           -0.003458373  0.0001041678
```

```r
# Standard error of beta estimates
std_err <- sqrt(diag(var_cov)); std_err
```

```
## (Intercept)           x
##  0.36166916  0.01020626
```

```
# Log-likelihood
log_like <- -logit.result$value; log_like
```

```
## [1] -176.0126
```

```
# Deviance
dev <- -2*(log_like - 0); dev
```

```
## [1] 352.0252
```

```
# Find new starting value
beta <- cbind(parm_est) # Store paramete estiamtes
plgtb <- 1/(1 + exp(-xmat%*%beta))
# score vector
score.vector <- t(xmat)%*%(y - plgtb); score.vector
```

```
##       parm_est
## cons  -105.843
##      -2993.912
```

```
# direction vector
direction.vector <- var_cov%*%score.vector; direction.vector
```

```
##                parm_est
## (Intercept) -3.49069077
## x            0.05417553
```

```
# updated starting value
update <- cbind(stval) + direction.vector; update
```

```
##                   stval
## (Intercept) -3.97295716
## x            0.07185075
```

Second iteration

```
# Optimize by log-likelihood
logit.result <- optim(update, llk.logit, method="BFGS",
                      control=list(maxit=0, trace=1), hessian=TRUE, y=y, x=x)

# Printing Optimization results #
# beta paramter estimates
parm_est <- logit.result$par; parm_est
```

```
##                   stval
## (Intercept) -3.97295716
## x            0.07185075
```

```
# variance covariance matrix
var_cov <- solve(logit.result$hessian); var_cov
```

```
##              [,1]          [,2]
## [1,]  0.286700820 -0.0067955146
## [2,] -0.006795515  0.0001791048
```

```
# Standard error of beta estimates
std_err <- sqrt(diag(var_cov)); std_err
```

```
## [1] 0.5354445 0.0133830
```

```
# Log-likelihood
log_like <- -logit.result$value; log_like
```

```
## [1] -57.34766
```

```
# Deviance
dev <- -2*(log_like - 0); dev
```

```
## [1] 114.6953
```

```
# Find new starting value
beta <- cbind(parm_est) # Store paramete estiamtes
plgtb <- 1/(1 + exp(-xmat%*%beta))
# score vector
score.vector <- t(xmat)%*%(y - plgtb); score.vector
```

```
##           stval
## cons  -23.60246
##      -562.53390
```

```
# direction vector
direction.vector <- var_cov%*%score.vector; direction.vector
```

```
##            stval
## [1,] -2.94413702
## [2,]  0.05963834
```

```
# updated starting value
update <- cbind(update) + direction.vector; update
```

```
##                  stval
## (Intercept) -6.9170942
## x            0.1314891
```

Third iteration

```
# Optimize by log-likelihood
logit.result <- optim(update, llk.logit, method="BFGS",
                      control=list(maxit=0, trace=1), hessian=TRUE, y=y, x=x)

# Printing Optimization results #
# beta paramter estimates
parm_est <- logit.result$par; parm_est
```

```
##                  stval
## (Intercept) -6.9170942
## x            0.1314891
```

```
# variance covariance matrix
var_cov <- solve(logit.result$hessian); var_cov
```

```
##             [,1]          [,2]
## [1,]  0.76343913 -0.0171157210
## [2,] -0.01711572  0.0004112217
```

```
# Standard error of beta estimates
std_err <- sqrt(diag(var_cov)); std_err
```

```
## [1] 0.8737500 0.0202786
```

```r
# Log-likelihood
log_like <- -logit.result$value; log_like
```

```
## [1] -34.51776
```

```r
# Deviance
dev <- -2*(log_like - 0); dev
```

```
## [1] 69.03552
```

```r
# Find new starting value
beta <- cbind(parm_est) # Store paramete estiamtes
plgtb <- 1/(1 + exp(-xmat%*%beta))
# score vector
score.vector <- t(xmat)%*%(y - plgtb); score.vector
```

```
##            stval
## cons    -8.44227
##       -190.09565
```

```r
# direction vector
direction.vector <- var_cov%*%score.vector; direction.vector
```

```
##             stval
## [1,] -3.19153532
## [2,]  0.06632409
```

```r
# updated starting value
update <- cbind(update) + direction.vector; update
```

```
##                   stval
## (Intercept) -10.1086295
## x             0.1978132
```

Fourth iteration

```r
# Optimize by log-likelihood
logit.result <- optim(update, llk.logit, method="BFGS",
                      control=list(maxit=0, trace=1), hessian=TRUE, y=y, x=x)

# Printing Optimization results #
# beta paramter estimates
parm_est <- logit.result$par; parm_est
```

```
##                   stval
## (Intercept) -10.1086295
## x             0.1978132
```

```r
# variance covariance matrix
var_cov <- solve(logit.result$hessian); var_cov
```

```
##             [,1]         [,2]
## [1,]  2.03563428 -0.044260128
## [2,] -0.04426013  0.001002926
```

```r
# Standard error of beta estimates
std_err <- sqrt(diag(var_cov)); std_err
```

```
## [1] 1.426757 0.031669
```

```r
# Log-likelihood
log_like <- -logit.result$value; log_like
```

```
## [1] -25.3888
```

```r
# Deviance
dev <- -2*(log_like - 0); dev
```

```
## [1] 50.77759
```

```r
# Find new starting value
beta <- cbind(parm_est) # Store paramete estiamtes
plgtb <- 1/(1 + exp(-xmat%*%beta))
# score vector
score.vector <- t(xmat)%*%(y - plgtb); score.vector
```

```
##            stval
## cons   -3.085691
##       -67.003799
```

```r
# direction vector
direction.vector <- var_cov%*%score.vector; direction.vector
```

```
##             stval
## [1,] -3.31574124
## [2,]  0.06937323
```

```r
# updated starting value
update <- cbind(update) + direction.vector; update
```

```
##                   stval
## (Intercept) -13.4243707
## x             0.2671864
```

Fifth iteration

```r
# Optimize by log-likelihood
logit.result <- optim(update, llk.logit, method="BFGS",
                      control=list(maxit=0, trace=1), hessian=TRUE, y=y, x=x)

# Printing Optimization results #
# beta paramter estimates
parm_est <- logit.result$par; parm_est
```

```
##                   stval
## (Intercept) -13.4243707
## x             0.2671864
```

```r
# variance covariance matrix
var_cov <- solve(logit.result$hessian); var_cov
```

```
##            [,1]        [,2]
## [1,]  4.8897988 -0.10460130
## [2,] -0.1046013  0.00229326
```

```r
# Standard error of beta estimates
std_err <- sqrt(diag(var_cov)); std_err
```

```
## [1] 2.211289 0.047888
```

```
# Log-likelihood
log_like <- -logit.result$value; log_like
```

```
## [1] -21.90023
```

```
# Deviance
dev <- -2*(log_like - 0); dev
```

```
## [1] 43.80046
```

```
# Find new starting value
beta <- cbind(parm_est) # Store paramete estiamtes
plgtb <- 1/(1 + exp(-xmat%*%beta))
# score vector
score.vector <- t(xmat)%*%(y - plgtb); score.vector
```

```
##            stval
## cons   -1.056987
##       -22.679856
```

```
# direction vector
direction.vector <- var_cov%*%score.vector; direction.vector
```

```
##            stval
## [1,] -2.79610904
## [2,]  0.05855136
```

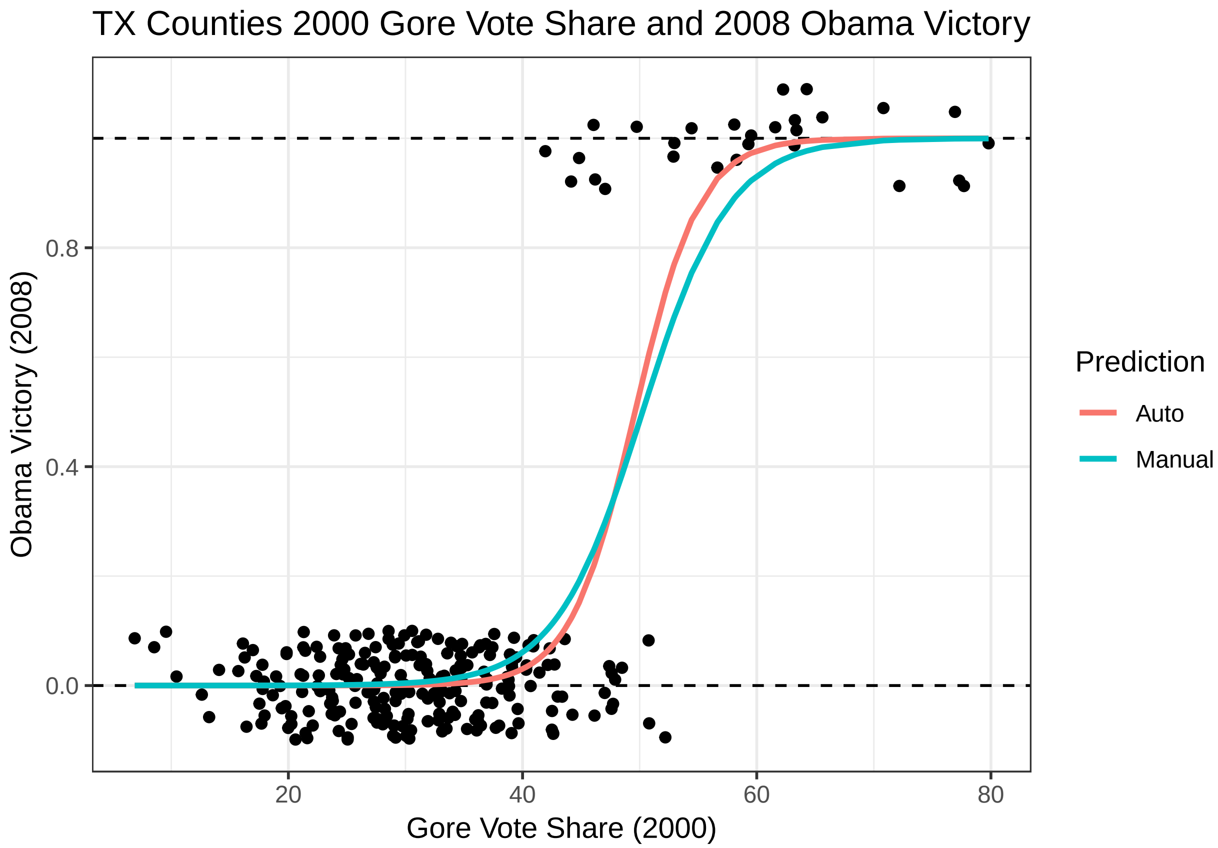
```
# updated starting value
update <- cbind(update) + direction.vector; update
```

```
##                   stval
## (Intercept) -16.2204798
## x             0.3257378
```

Compare manual and automatic results

```
# Fit Prediction
prediction_manual <- ilogit(parm_est[1] + parm_est[2]*x)

# Compare predictions
p + geom_line(aes(y=prediction, color="Auto"), size=1) +
  geom_line(aes(y=prediction_manual, color="Manual"), size=1) +
  scale_color_discrete(name="Prediction")
```



Workshop question

Fit logistic regression that predicts Trump victory by 2008 McCain vote share in California. Optimize by both automatic and manual methods and compare results.