Paired t-test

Task 2

Gent Rexha

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Introduction

A paired t-test is used to compare two population means where you have two samples in which observations in one sample can be paired with observations in the other sample. Examples of where this might occur are:

- Before-and-after observations on the same subjects (e.g. students' diagnostic test results before and after a particular module or course).
- A comparison of two different methods of measurement or two different treatments where the measurements and/or treatments are applied to the same subjects (e.g. blood pressure measurements using a stethoscope and a dynamap). [1]

Assumptions

- Normal distribution - there should roughly be a normal distribution in both the population and in the sample
- Approximately Similar variance - in each of the samples.
- Data points - there should be the same number of data points on either sample. And then finally this works good with low numbers, but you generally want to be in the 20 to 30 number range.

Procedure for carrying out a paired t-test

Suppose a sample of n students were given a diagnostic test before studying a particular module and then again after completing the module. We want to find out if, in general, our teaching leads to improvements in students' knowledge/skills (i.e. test scores). We can use the results from our sample of students to draw conclusions about the impact of this module in general.

Let $x =$ test score before the module, $y =$ test score after the module.

To test the null hypothesis that the true mean difference is zero, the procedure is as follows:

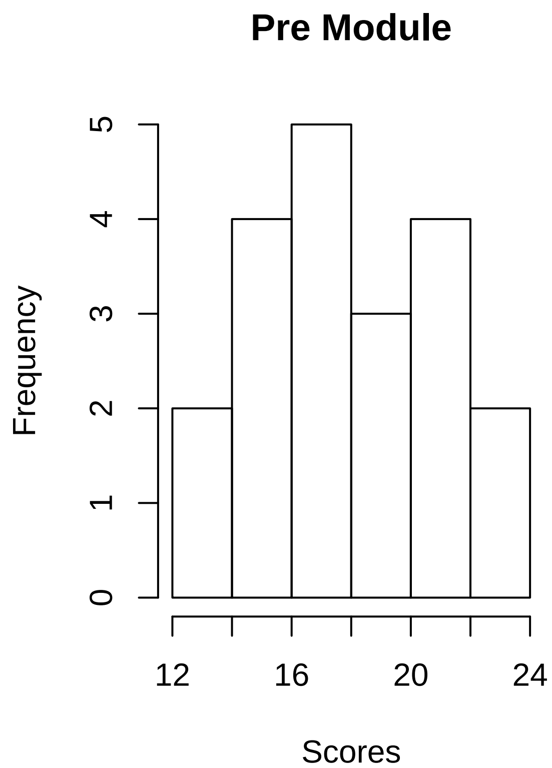
1. Calculate the difference ($d_i = y_i - x_i$) between the two observations on each pair, making sure you distinguish between positive and negative differences.
2. Calculate the mean difference, $\bar{d}$
3. Calculate the standard deviation of the differences, sd, and use this to calculate the standard error of the mean difference $SE(\bar{d}) = \frac{n_d}{\sqrt{n}}$
4. Calculate the t-statistic, which is given by $T = \frac{\bar{d}}{SE(\bar{d})}$. Under the null hypothesis, this statistic follows a t-distribution with $n-1$ degrees of freedom.
5. Use tables of the t-distribution to compare your value for T to the t_{n-1} distribution. This will give the p-value for the paired t-test.

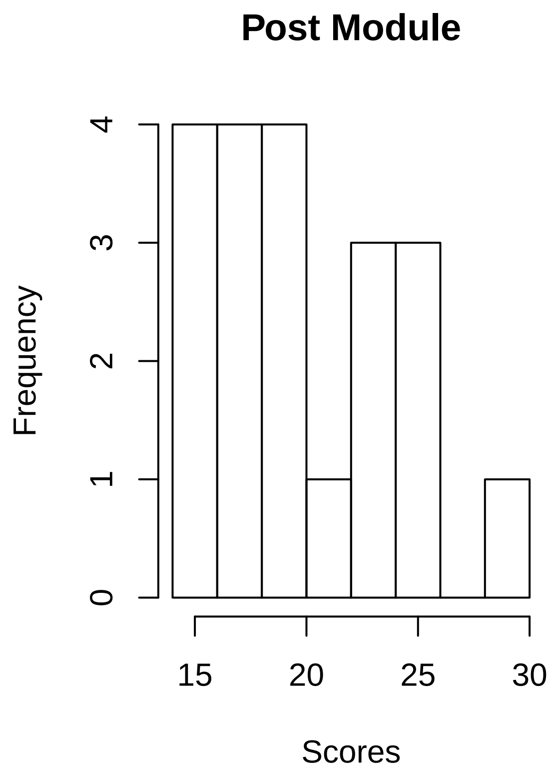
Example

Using the above example with $n = 20$ students, the following results were obtained:

```r
pre_module <- c(18, 21, 16, 22, 19, 24, 17, 21, 23, 18,
                14, 16, 16, 19, 18, 20, 12, 22, 15, 17)
post_module <- c(22, 25, 17, 24, 16, 29, 20, 23, 19,20,
                 15, 15, 18, 26, 18, 24, 18, 25, 19, 16)
df <- data.frame(Student = c(1:20), Pre_Module_Score = pre_module,
                 Post_Module_Score = post_module, Difference = post_module - pre_module)
kable(df)
```

Student	Pre_Module_Score	Post_Module_Score	Difference
1	18	22	4
2	21	25	4
3	16	17	1
4	22	24	2
5	19	16	-3
6	24	29	5
7	17	20	3
8	21	23	2
9	23	19	-4
10	18	20	2
11	14	15	1
12	16	15	-1
13	16	18	2
14	19	26	7
15	18	18	0
16	20	24	4
17	12	18	6
18	22	25	3
19	15	19	4
20	17	16	-1





Instead of calculating everything manually, R offers an already implemented function for this:

```
t.test(df$Pre_Module_Score, df$Post_Module_Score, paired = TRUE)
```

```
##
##  Paired t-test
##
## data:  df$Pre_Module_Score and df$Post_Module_Score
## t = -3.2313, df = 19, p-value = 0.004395
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -3.3778749 -0.7221251
## sample estimates:
## mean of the differences
##                   -2.05
```

Looking this up in tables gives $p = 0.004$. Therefore, there is strong evidence that, on average, the module does lead to improvements.

References

[1] "[No title]." [Online]. Available: http://www.statstutor.ac.uk/resources/uploaded/paired-t-test.pdf. [Accessed: 19-Mar-2019]