

CS2308

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# **Lecture 2.1**

## **Algorithmic Analysis**

# What is an Algorithm

- **Step-by-step instructions that tell a computing agent how to solve some problem using resources.**
- **Resources:**
  - Memory
  - Time (measured as CPU cycles)
- **Type of Instructions:**
  - Sequential (1 action)
  - Conditional (0 or 1 action)
  - Loops (0 or more actions)

# Pseudo-Code

- a human-language description of the sequential, conditional, and iterative operations of an algorithm.
- No rigid syntax.
- Clarity, organization, and completeness are important.
- Can be implemented in any computer language.

# Example: Find the Largest Number

1. Given a list **L** of integers of size **n**.
2. Set **Largest** to the first value in **L**, **L<sub>0</sub>**.
3. For all remaining elements in **L**:
  - a) If **L<sub>i</sub>** is larger than largest:
    - i. Set **Largest** to **L<sub>i</sub>**
4. Output **Largest**

# Measuring Time Efficiency

- For this list : **{2, 6, 3, 4, 8}**
    - Step 1 happens 0 times.
    - Step 2 happens 1 time.
    - Step 3a makes 4 comparisons.
    - Step 3ai does 2 assignments.
    - Step 4 happens 0 times.
  - For a list with length  $n=5$ , our algorithm performs 7 actions.
1. Given a list **L** of integers of size **n**.
  2. Set **Largest** to the first value in **L**, **L0**.
  3. For all remaining elements in **L**:
    - a) If **Li** is larger than largest:
      - i. Set **Largest** to **Li**
  4. Output **Largest**

# Generalizing This Measure

- For a list with length  $n$ :
  - Step 0 always happens 0 times.
  - Step 1 always happens 1 time.
  - Step 3a always does  $n-1$  comparisons.
  - Step 3ai will happen at most  $n-1$  times or as little as 0 times.
- Our algorithm will perform at least  $n$  actions and at most  $2n-1$  actions.

# Big O notation

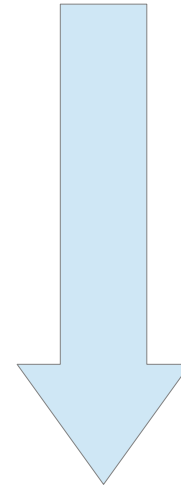
- It would be cumbersome to tell a colleague that our algorithm uses “n to  $2n-1$ ” steps.
- Big O is a simplified notation:
  - Drop all constants
  - Keep only the highest order operation
- Examples:
  - $2n-1$  is  $O(n)$
  - $n^2 + 5n$  is  $O(n^2)$
  - $1000000$  is  $O(1)$  or  $O(c)$



# Orders of Big O

- $O(1)$  constant time
- $O(\log(n))$  log time
- $O(n)$  linear time
- $O(n\log(n))$  log linear time
- $O(n^2)$  polynomial time
- $O(2^n)$  exponential time
- $O(n!)$  factorial time

Fastest



Slowest

# Comparison of Orders

Let's assume that one operation takes one second.

	2 step	8 step	32 step	128 step	512 step	2048 step	8192 step
<b>constant</b>	1 sec.	1 sec.	1 sec.	1 sec.	1 sec.	1 sec.	1 sec.
<b>log(n)</b>	1 sec.	3 sec.	5 sec.	7 sec.	9 sec.	11 sec.	13 sec.
<b>n</b>	2 sec.	8 sec.	32 sec.	2 min.	8 min.	34 min.	2 hours
<b>nlog(n)</b>	2 sec.	24 sec.	2 min.	14 min.	1 hour	6 hours	1 day
<b>n<sup>2</sup></b>	4 sec.	1 min.	17 min.	4 hours	3 days	48 days	2 years
<b>2<sup>n</sup></b>	4 sec.	1 min.	136 years	1e31 years	...	...	...
<b>n!</b>	2 sec.	11 min.	8e27 years	1e208 year	...	...	...

## 2<sup>nd</sup> Example: Find the most frequent number

- 1) Given a list of integers **L** of length **n**
- 2) Create lists **V** and **C** of length **n**
- 3) For every value in **L**:
  - a) Try to find  $L_i$  in **V**
    - i. If  $L_i$  is stored at  $V_j$ , increment  $C_j$
    - ii. Otherwise set  $V_m$  to  $L_i$  and set  $C_m$  to 1 where  $m$  is the next empty position in the two lists.
- 4) Find the index  $k$  of the maximum value in **C**
- 5) Output  $V_k$

# Space Complexity

- We are given the list **L**, it doesn't count against the space complexity.
- We create 2 lists **V** and **C** which both have size **n**.
- To process **n** values, we need to store **2n** values in memory.
- Our algorithm is **O(n)** space complexity.

- 1) Given a list of integers **L** of length **n**
- 2) Create lists **V** and **C** of length **n**
- 3) For every value in **L**:
  - a) Try to find  $L_i$  in **V**
    - i. If  $L_i$  is stored at  $V_j$ , increment  $C_j$
    - ii. Otherwise set  $V_m$  to  $L_i$  and set  $C_m$  to 1 where  $m$  is the next empty position in the two lists.
- 4) Find the index  $k$  of the maximum value in **C**
- 5) Output  $V_k$

# Big O Considerations

- Not meaningful for small values of  $n$ . The Big O measurement only needs to be true after some **threshold**.
- We have assumed so far that our computer only has one processor.
- Big O is an **upper bound** so a linear time algorithm is also in the set of polynomial time algorithms.



Questions or Comments?