# Towards the Identifiability in Noisy Label Learning: A Multinomial Mixture Approach

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#### **Abstract**

Learning from noisy labels plays an important role in the deep learning era. Despite numerous studies with promising results, identifying clean labels from a noisily-annotated dataset is still challenging since the conventional noisy label learning problem with single noisy label per instance is not identifiable, i.e., it does not theoretically have a unique solution unless one has access to clean labels or introduces additional assumptions. This paper aims to formally investigate such identifiability issue by formulating the noisy label learning problem as a multinomial mixture model, enabling the formulation of the identifiability constraint. In particular, we prove that the noisy label learning problem is identifiable if there are at least 2C-1 noisy labels per instance provided, with C being the number of classes. In light of such requirement, we propose a method that automatically generates additional noisy labels per training sample by estimating the noisy label distribution based on nearest neighbours. Such additional noisy labels allow us to apply the Expectation - Maximisation algorithm to estimate the posterior of clean labels. We empirically demonstrate that the proposed method is not only capable of estimating clean labels without any heuristics in several challenging label noise benchmarks, including synthetic, web-controlled and real-world label noises, but also of performing competitively with many state-of-the-art methods.

# 1 Introduction

The great advances in machine learning, and especially deep learning, in the last decade has created many applications that help to solve increasingly-complex problems in computer vision [8, 22], natural language processing (NLP) [3, 38] and reinforcement learning [20, 33]. To achieve such performance, those solutions often rely on high capacity models which are trained on a massive amount of annotated data. Such large amount of data is often annotated via crowd-sourcing services, such as Amazon Mechanical Turk, or via automated approaches based on NLP or search engines, which might, generally, produce poor-quality annotated labels, particularly when data is ambiguous. This poor annotation, combined with the fact that deep neural networks can easily overfit to randomly-labelled data [50], might lead to catastrophic failures, especially in some critical applications such as autonomous vehicles or medical diagnostics.

Noisy label learning has, therefore, attracted research interest in supervised learning. Some papers have provided a more theoretical investigation of certain types of label noise, such as random label noise [2] or Massart label noise [4], from a statistical machine learning point of view to determine the sufficient number of samples to achieve certain level of performance. Other papers have focused on more practical aspects of deep-learning methods, leading to two main research directions: (i) the design of loss or regularisation functions that are robust to label noise [11, 43, 50], and (ii) the proposal of heuristics to detect and re-label samples with noisy labels [14, 23]. Despite being effective under certain simulated types of label noise (e.g., symmetric and asymmetric), these approaches tend to be challenged by more natural types of label noise, particularly the ones present in real-world datasets. Currently, methods relying on semi-supervised learning and heuristics to detect noisy samples have achieved state-of-the-art results in several label noise settings and even close to the ones trained on "pure" clean data [23]. Nevertheless, those methods still lack theoretical explanation, especially the heuristic criteria, e.g., small loss hypothesis, to detect noisy samples. Indeed, without such heuristics or any further assumptions, the label noise problem does not have a unique clean label solution and hence, becomes un-identifiable.

It is crucial to know when the label noise problem is identifiable, so we can address it properly. Despite its importance, studies about the identifiability of noisy label learning are still limited by only a few papers [25, 32, 49, 51]. In this paper, we carry out a new study on the challenging identifiability issue, where our aim is to find the condition that makes the label noise problem identifiable, and hence, address it in a principled way. Our contributions can be summarised as follows:

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- We provide a theoretically rigorous formulation of the identifiability condition of the label noise problem, which concludes that we need multiple additional labels per training sample; in particular, approximately 2C-1 labels per training sample are required to make the problem sufficiently identifiable, where C denotes the number of classes.
- We propose a method relying on nearest neighbours to generate several additional noisy labels per training sample and use the Expectation Maximisation (EM) algorithm to gradually clean noisy labels and train the model of interest simultaneously. Our empirical results show competitive performance to several state-of-the-art methods in many instant-dependent and real-world label noise benchmarks.

Our proposed method is tested on many noisy-label learning benchmarks (including synthetic, web-controlled and real-world label noise problems) and shown to successfully estimate clean labels without any heuristics. Furthermore, even though the main goal of this paper is the theoretical investigation of the identifiability condition, our method shows competitive results with several state-of-the-art techniques.

### 2 Related work

Noisy label learning has been studied since the 1980s with some early works focusing on the statistical point of view [2, 4], such as determining the number of samples to achieve certain prediction accuracy under certain types of label noise. The field has then attracted more research interest, especially in the era of deep learning where an increasing number of annotated data is required to train large deep learning models. Noisy label learning has received even more attention when Zhang et al. [47] empirically pointed out that any convolutional neural network can easily fit a randomly labelled dataset, showing a severe overfitting capability by deep learning models. There have been numerous studies aiming to propose practical methods to address the noisy label learning problem. One research direction is to design robust loss functions in which training on noisily-labelled data results in the same classifier as if training on the unobserved cleanlylabelled data [11, 43, 50]. Some other methods model the data generation process where the clean label is considered as a latent random variable, allowing to correct the training loss [16, 30] or integrating additional modules to model the label noise [12]. Another popular research direction is to employ the *small loss hypothesis* in which training samples with small loss values are assumed to be clean. Training is then carried out either on only those low-risk samples [14] or cast as a semi-supervised learning approach with those clean samples representing labelled data while the others denoting un-labelled data [23]. Although this line of research achieves state-of-the-art results in several benchmarks, they still lack theoretical foundation to understand why the *small loss hypothesis* is effective. There is one recent attempt that tries to explain the theory behind the small loss hypothesis, but it is applicable only to the class-dependent (a.k.a. instance-independent) label noise setting [13].

Despite a large number of already-published and on-going research papers, the identifiability issue in noisy label learning has not been formally studied, but only discussed and partially addressed under certain assumptions [51] or heuristic constraints [6, 25, 49]. To the best of our knowledge, the most relevant study about the identifiability issue in noisy label learning is the on-going (but still unpublished) work that investigates the identifiability of transition matrix [27]. Liu et al. [27] use the results in the mixture proportion estimation [32] to derive the identifiability condition for noisy label learning, in which at least 3 "informative" noisy labels per instance are required. This is a more optimistic than our result in Claim 1, where we theoretically show that at least almost twice the number of additional noisy labels per sample is sufficient to make the label noise problem identifiable. Our result agrees with [27] for binary classification: C=2, but deviates from [27] for multi-class classification. Please refer to Appendix A for further detailed discussion about the difference.

Our work is also connected to the identifiability in mixture models [37, Sec. 3.1] that investigates sufficient (or useful sufficient) conditions to recover the unique parameter of mixture models. Certain mixture models, such as Gaussian, Poisson or negative binomial, are identifiable up to the permutation of labels. However, some others, especially mixtures with discrete distributions, are only identifiable under some conditions. For example, mixtures of binomial or multinomial distributions are identifiable when there are sufficient number of samples generated from such mixtures [10, 21, 36]. Our work does not focus on mixture models, but cast the noisy label learning to a mixture model and impose the sufficient identifiability condition to the model.

# 3 Background

### 3.1 Noisy label learning

Let  $X \in \mathcal{X} \subseteq \mathbb{R}^d$  be a random variable that represents input data and Y be another random variable denoting the corresponding annotated label. In C-class classification problems, the label Y can be represented as a scalar, represented by  $\mathcal{Y} \subseteq \mathbb{Z}_+$ , or a one-hot vector, corresponding to  $\mathcal{Y} \subseteq \Delta_{C-1}$  (in this paper, we use the label Y either as a scalar or a probability vector interchangeably), where  $\Delta_{C-1}$  is a probability simplex defined as  $\Delta_{C-1} = \{\mathbf{y} \in \mathbb{R}^C : \mathbf{1}^\top \mathbf{y} = 1 \land \mathbf{y}_c \in [0,1], \forall c \in \{1,\dots,C\}\}$ .

Our aim is to learn a model that maps from  $\hat{X}$  to  $\hat{Y}$ , by maximising some utility function, e.g., maximum likelihood. Instead of observing the "clean" label  $\hat{Y}$  of an instance X, in practice, we are often given a noisy label  $\hat{Y}$  that might or

might not be the same as Y. The aim is to learn a good model to predict the clean label of an unseen instance correctly, even though the training dataset, denoted as  $\{(\mathbf{x}_i, \hat{\mathbf{y}}_i)\}_{i=1}^M$ , contains noisy labels. This is often known as noisy label learning or label-noise problem.

One way to model the label noise problem is to consider the clean label Y as a latent variable and then applying the sum rule of probability to obtain the following:

$$p(\hat{Y}|X) = \sum_{c=1}^{C} p(\hat{Y}|X, Y = c) \, p(Y = c|X), \tag{1}$$

where C is the number of classes. In the literature of noisy label learning, the matrix  $T(X) = [p(\hat{Y} = j | X, Y = c)]_{j,c=1}^C$  is also known as the transition matrix representing the probability to flip the label from one class to another class. As the noisy label data is observed, the left-hand side term in Eq. (1) can be estimated. Thus, the clean label probability p(Y|X) can be easily calculated if T(X) is known.

### 3.2 Mixture models

A mixture model of C distributions can be written as:

$$p(X) = \sum_{c=1}^{C} \pi_c \mathbb{P}_c(X), \tag{2}$$

where  $X \in \mathcal{X}$  is a random variable,  $\pi \in \Delta_{C-1}$  is the mixture coefficient vector in the C-1 probability simplex and  $\{\mathbb{P}_c\}_{c=1}^C$  is a set of C distributions (a.k.a. mixture components).

Compared to a single distribution, mixture models are more flexible with higher modelling capacity, and hence, widely used to provide computationally convenient representation of complex data distributions. Some of the most common ones include Gaussian-, Bernoulli- and multinomial-mixture models. And since mixture models are an instance of latent variable models, we can use the Expectation Maximisation (EM) algorithm [7] via maximum likelihood or maximum a posterior to infer their parameters.

### 3.3 Identifiability issues in mixture models

The study of identifiability is to investigate whether one may, in principle, recover the exact parameters of the distribution of interest from observed variables. In particular, we are interested in distributions defined in a family  $p(X;\theta)$  over random variable X parameterised by  $\theta \in \Theta$  where  $\Theta$  denotes a parametric space. The identifiability can be defined as follows:

**Definition 1**  $\forall \theta, \theta' \in \Theta$ : if  $\theta \neq \theta'$  then  $p(X; \theta) \neq p(X; \theta')$ .

In statistical inference for mixture models, we often encounter the identifiability issue of  $\theta$ . For example, if all the C component distributions in (2) belong to the same parametric family, p(X) is invariant under C! permutations by simply swapping the indices of the component distributions, a phenomenon known as label-switching. In practice, the identifiability issue due to label-switching (we will refer to this identifiability issue as label-switching from now on) is of no concern since one can impose an appropriate constraint on  $\theta$  to obtain a unique solution. Nevertheless, parameter identifiability up to the permutation of class labels (we will refer this as identifiability in the remaining of this paper) is still a practical problem, at least in maximum likelihood for mixture models where the distribution components of such mixtures belong to certain distribution families. According to [37, Section 3.1], most mixture models supported on continuous space, e.g., Gaussian mixture models (excluding the mixture of uniform distributions), are identifiable. However, when the support space is discrete, the identifiability of such mixtures might not always hold. For example, a mixture of Poisson distribution [36] or a mixture of negative binomial distribution [45] is identifiable, while a mixture of binomial distributions is only identifiable under certain conditions [36, Proposition 4]. Another example is multinomial mixture models which is, according to Theorem 1, identifiable only when the number of samples is at least almost twice the number of class labels.

**Theorem 1 (Lemma 2.2 in [21] and Theorem 4.2 in [10])** *If*  $Mult(\mathbf{x}; N, \mathbf{p}_c)$  *is a multinomial distribution with*  $N \in \mathbb{N}$  *being the number of trials and*  $\mathbf{p}_c \in \Delta_{d-1}$  *being the success probability vector of* d *categories, then the class of multinomial mixture models:* 

$$\mathcal{M}_{N,C} = \left\{ M(\mathbf{x}) : M(\mathbf{x}) = \sum_{c=1}^{C} \boldsymbol{\pi}_{c} \operatorname{Mult}(\mathbf{x}; N_{c}, \mathbf{p}_{c}) \right\}$$

is identifiable (up to label permutation) if  $\min_{c \in \{1,...,C\}} N_c \ge 2C - 1$ .

# 4 Methodology

Given that only the instance X and its single noisy label  $\hat{Y}$  are available, the noisy label learning problem shown in Eq. (1) is ill-defined, potentially resulting in infinite values of the clean label Y. Also, if there is a unique solution  $(p(\hat{Y}|X,Y),p(Y|X))$ , one can freely swap the columns of the transition matrix  $p(\hat{Y}|X,Y)$  and the corresponding rows in the clean label vector p(Y|X) without changing the distribution of the noisy label  $p(\hat{Y}|X)$ . This type of identifiability results into C! solution and is referred to as *label-switching*, mentioned in Sec. 3.3.

In general, there are two types of identifiability issues regarding to the noisy label learning: (i) the uniqueness of the solution  $(p(\hat{Y}|X,Y),p(Y|X))$  up to the label permutation and (ii) the label-switching problem of such unique solution. In the following subsections, we address the identifiability issue by formulating the noisy label learning to multinomial mixture models and impose the identifiable condition. We then present a mitigation solution of the label-switching problem by imposing certain constraints on the initialisation when inferring the posterior of the clean label.

### 4.1 Noisy label learning as a multinomial mixture

The likelihood of the noisy label  $p(\hat{Y}|X)$  presented in Eq. (1) can be considered to be a multinomial mixture model where  $p(\hat{Y}|X,Y=c)=\operatorname{Mult}(\hat{Y};N_c,\rho_c)$  is a multinomial component and p(Y=c|X) is the corresponding mixture coefficient with  $N_c\in\mathbb{Z}_+,\rho_c\in\Delta_{C-1}$  and  $c\in\{1,\ldots,C\}$ . We can, therefore, rewrite the likelihood of the noisy label in the form of multinomial mixture models as following:

$$p\left(\hat{Y}|X;\rho\right) = \sum_{c=1}^{C} p(Y=c|X) \operatorname{Mult}(\hat{Y};N,\rho_c),$$
(3)

where we assume that  $N_c = N, \forall c \in \{1, \dots, C\}$  to simplify the analysis. The case varying  $N_c$  can be straight-forwardly extended by considering  $N = \min_{c \in \{1, \dots, C\}} N_c$ .

As the noisy label learning can be formulated as a multinomial mixture model shown in Eq. (3), we can employ Theorem 1 to obtain the following claim about the identifiability in noisy label learning:

**Claim 1** Any noisy label learning problem where the noisy label distribution is modelled as a multinomial mixture model shown in Eq. (3) is identifiable if there are at least 2C-1 samples of noisy label  $\hat{Y}$  for an instance X with C being the number of classes.

Given Claim 1, one can derive some interesting results in noisy label learning. For example, the conventional noisy label learning has only one annotated noisy label per sample: N=1. Therefore, according to Claim 1, it is unidentifiable for  $C \geq 2$ . Another example is that binary classification on noisy labels, corresponding to C=2, is identifiable if there are at least 3 noisy labels per sample, which agrees with studies in the literature of identifiability for noisy label learning [27, 51].

Hence, to address the identifiability issue in noisy label learning, we need at least additional 2C-2 noisy labels per instance. One naive way is to annotate more labels, e.g., via crowd-sourcing, to obtain the number of noisy labels per instance that satisfies the identifiable condition in Claim 1. Such approach is, however, costly, time-consuming and poorly scalable, especially when the number of classes C is very large. For example, WebVision dataset [24] with C=1,000 will require at least an addition of 1,998 noisy labels per sample, resulting in an intractable solution.

To obtain additional noisy labels per sample without the need of additional resources, we propose to approximate the complex noisy label distribution  $p(\hat{Y}|X)$  following a data-driven approach that takes the similarity between instance features into account. Our hypothesis is that instances with similar features tend to be annotated similarly, or in other words, similar instances have similar noisy labels. Thus, we can employ the single noisy label per instance available in the training dataset to approximate the noisy label distribution  $p(\hat{Y}|X)$ . The approximated distribution is then used to generate many noisy labels that satisfy the identifiability condition specified in Claim 1. Subsequently, Expectation-Maximisation algorithm is employed to infer the parameter of the multinomial mixture model of noisy label learning in Eq. (3) (refer to Appendix C for the details of EM on multinomial mixtures).

### 4.2 Approximate noisy label distribution

To generate additional noisy labels that satisfies the identifiable condition in Claim 1, we approximate the noisy label distribution of each training sample by exploiting the information of nearest neighbours. The complex noisy label distribution of an instance  $\mathbf{x}_i$ , denoted as  $p(\hat{Y}|X=\mathbf{x}_i)$ , is silmutaneously derived not only from the one-hot noisy label vector  $\hat{\mathbf{y}}_i$ , but also the noisy labels of other instances whose features are similar to the instance. Specifically, such approximated distribution can be written as:

$$p(\hat{Y}|\mathbf{X} = \mathbf{x}_i) \approx \mu \hat{\mathbf{y}}_i + (1 - \mu) \sum_{j \neq i, j=1}^K \mathbf{A}_{ij} \hat{\mathbf{y}}_j,$$
(4)

where  $\mu \in [0,1]$  is a hyper-parameter reflecting the tradeoff between the noisy label of the instance and the noisy labels of other instance,  $K \in \{1, ..., M\}$  is the number of instances considered in the approximation, and  $\mathbf{A}_{ij} \in [0,1]$ 

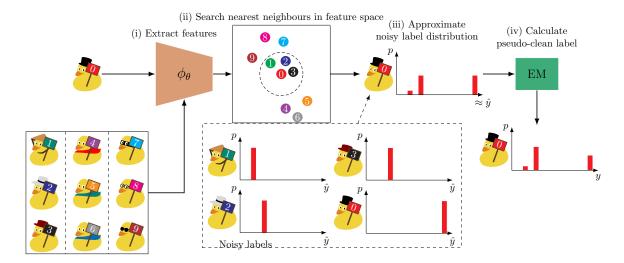


Figure 1: An illustration of the proposed method that consists of 4 steps: (i) extract features, (ii) search for nearest neighbours in the feature space, (ii) approximate the noisy label distribution, and (iv) use EM to obtain pseudo-clean label.

is a coefficient representing the similarity between  $\mathbf{x}_i$  and  $\mathbf{x}_j$ . Since  $p(\hat{Y}|\mathbf{X}=\mathbf{x}_i)$  is a probability distribution, one constraint for  $\mathbf{A}_{ij}$  is that  $\sum_{j \neq i, j=1}^K \mathbf{A}_{ij} = 1$ . There are several ways to find the similarity matrix  $[\mathbf{A}_{ij}], \mathbf{A}_{ii} = 0, i \in \{1, \dots, M\}, j \in \{1, \dots, K\}$ . For example,

There are several ways to find the similarity matrix  $[\mathbf{A}_{ij}]$ ,  $\mathbf{A}_{ii} = 0, i \in \{1, \dots, M\}, j \in \{1, \dots, K\}$ . For example, [15] employed sparse subspace clustering method [9] to approximate the label distribution when learning human age from images. In this paper, we use a slightly similar but more efficient method that utilises the nearest neighbour information: locality-constrained linear coding (LLC) [39]. In particular, the coefficient  $\mathbf{A}_{ij}$  can be determined via the following optimisation:

$$\min_{\mathbf{A}_i} \|\mathbf{x}_i - \mathbf{B}_i \mathbf{A}_i\|_2^2 + \lambda \|\mathbf{d}_i \odot \mathbf{A}_i\|_2^2$$
s.t.:  $\mathbf{1}^{\top} \mathbf{A}_i = 1, \mathbf{A}_{ij} \ge 0, \forall j \in \{1, \dots, K\},$ 
(5)

where  $\mathbf{B}_i \in \mathbb{R}^{d \times K}$  is the matrix containing K nearest neighbours of instance  $\mathbf{x}_i$  (each column is a nearest-neighbour instance),  $\mathbf{A}_i = \begin{bmatrix} \mathbf{A}_{i1} & \mathbf{A}_{i2} & \dots & \mathbf{A}_{iK} \end{bmatrix}^{\top}$  is the K-dimensional vector representing the coding coefficients,  $\mathbf{d}_i = \exp(\operatorname{dist}(\mathbf{x}_i, \mathbf{B}_i)/\sigma)$  is the locality adaptor with  $\operatorname{dist}(\mathbf{x}_i, \mathbf{B}_i)$  being a vector of Euclidean distances from  $\mathbf{x}_i$  to each of its nearest neighbours, and  $\sigma$  being used for adjusting the weight decay speed for the locality adaptor. Nevertheless, since our interest is locality, not sparsity, in our implementation, we ignore the second term in Eq. (5) by setting  $\lambda = 0$ .

Note that the optimisation in (5) is slightly different from the original LLC due to the additional constraint of non-negativity of  $A_{ij}$ . Nevertheless, the optimisation resembles a quadratic program, and therefore, can be efficiently solved by off-the-shelf solvers, such as OSQP [35] which is available in the deep learning framework JAX (refer to OSQP solver in JAXopt).

Another problem when solving the optimisation in (5) for the coding vector  $\mathbf{A}_i$  of each instance  $\mathbf{x}_i$  is to find the nearest neighbours for an instance  $\mathbf{x}_i$ . In light of efficiently obtaining nearest neighbours, we employ FAISS [19] – a library for efficient similarity search and clustering of dense vectors written in C++ with Python wrappers and able to run on GPU. In addition, for datasets that contain millions of samples, we randomly sample a subset (about 10,000 samples) and run the nearest neighbour search on that subset.

### 4.3 Infer clean label posterior with EM

Once the noisy label distribution  $p(\hat{Y}|X)$  is approximated, we can generate L sets of N noisy labels for each instance where  $N \geq 2C-1$  and perform maximum a posterior to infer the parameter of the multinomial mixture model for noisy label learning shown in Eq. (3). In particular, the objective function can be written as:

$$\max_{\rho} \ln p(\rho|X, \hat{Y}) = \max_{\rho} \ln p(\hat{Y}|X, \rho) + \ln p(\rho|\beta), \tag{6}$$

where  $\rho \in \{\rho \in \mathbb{R}^{C \times C} : \rho_c \in \Delta_{C-1}\}$  is the matrix of instance X that contains C probability vectors as its columns, and  $\beta$  is the parameter of the prior of  $\rho$ .

Maximising the objective function in (6) is difficult since the log-likelihood  $\ln p(\hat{Y}|X,\rho)$  cannot be evaluated unless the hidden variable Y is available. To optimise such latent variable model, we employ the Expectation - Maximisation algorithm – an iterative optimisation method alternating between two steps. In the E-step, we calculate the expectation

### Algorithm 1 A progressive approach to address label noise

```
1: procedure Gradually relabelling(\mathbf{X}, \hat{\mathbf{Y}}, \mu, \eta)
                       \mathbf{X} \in \mathbb{R}^{d \times M}: a matrix of instances
                       \hat{\mathbf{Y}} \in \mathbb{R}^{C \times M}: a matrix of one-hot noisy labels
  3:
                       K: number of nearest neighbours
  4:
                                                                                                                                                                                                                                                 ◁
                       L: number of N-trial multinomial samples
  5:
                       μ: trade-off coefficient
  6:
                       \eta: number of EM iterations
  7:
              initialise parameter of feature extractor: \theta
  8:
              initialise parameter of a classifier: w
  9:
              \theta, \mathbf{w} \leftarrow \text{Warm-up}(\mathbf{X}, \hat{\mathbf{Y}}, (\theta, \mathbf{w}))
10:
              while (\theta, \mathbf{w}) not converged do
11:
                      \Upsilon \leftarrow \varnothing
                                                                                                                                                                        > an empty set to store updated labels
12:
                     for each (\mathbf{x}_i, \hat{\mathbf{y}}_i) \in (\mathbf{X}, \hat{\mathbf{Y}}) do
13:
                             extract features: \phi(\mathbf{x}_i; \theta)
14:
                            \mathbf{B}_{i}, \{\hat{\mathbf{y}}_{ij}\}_{j=1}^{K} \leftarrow \text{KNN}(\phi(\mathbf{x}_{i}; \theta), K)
15:
                             \mathbf{A}_i \leftarrow \text{LLC}(\mathbf{x}_i, \mathbf{B}_i)
16:
                                                                                                                                                                                                                                 \triangleright Eq. (5)
                            \begin{aligned} \mathbf{p}_i \leftarrow \mu \hat{\mathbf{y}}_i + (1 - \mu) \sum_j \mathbf{A}_{ij} \hat{\mathbf{y}}_{ij} \\ \text{sample } N \times L \text{ samples: } \tilde{\mathbf{y}}_{iln} \sim \mathrm{Cat}(Y; \mathbf{p}_i) \end{aligned}
                                                                                                                                                                                                                                 \triangleright Eq. (4)
17:
18:
                            \tilde{\mathbf{Y}}_i = \{(\{\tilde{\mathbf{y}}_{iln}\}_{n=1}^N)\}_{l=1}^L
19:
                             \rho, \pi \leftarrow \text{EM}(\tilde{\mathbf{Y}}_i, \eta)
                                                                                                                                                                                                         \triangleright \pi = p(Y|X, \hat{Y}, \rho^t)
20:
                                                                                                                                                                                                  > store the updated label
                             \Upsilon \leftarrow \text{Append}(\pi)
21:
                     update noisy labels: \hat{\mathbf{Y}} \leftarrow \Upsilon
22:
                     \theta, \mathbf{w} \leftarrow \text{Train}(\mathbf{X}, \hat{\mathbf{Y}}, (\theta, \mathbf{w}))
23:
              return (\theta, \mathbf{w})
```

of  $\ln p(\rho|X, \hat{Y}, Y)$  w.r.t.  $p(Y|X, \hat{Y}, \rho^t)$  as follows:

$$\begin{split} Q(\rho, \rho^t) &= \mathbb{E}_{p(Y|X, \hat{Y}, \rho^t)} \left[ \ln p(\rho|X, \hat{Y}, Y) \right] \\ &= \mathbb{E}_{p(Y|X, \hat{Y}, \rho^t)} \left[ \ln p(\hat{Y}|X, Y, \rho) + \ln p(\rho|\beta) \right] + \text{const.} \\ &\propto \mathbb{E}_{p(Y|X, \hat{Y}, \rho^t)} \left[ \ln \text{Mult}(\hat{Y}; N, \rho) + \ln p(\rho|\beta) \right], \end{split} \tag{7}$$

where  $\rho^t$  denotes the parameter at t-th iteration.

In the M-step, we maximise Q to obtain the parameter  $\rho^{t+1}$  used in the next iteration:

$$\rho^{t+1} = \arg\max_{\rho} Q(\rho, \rho^t). \tag{8}$$

The algorithm iterates until  $\|\rho^{t+1} - \rho^t\|$  is sufficiently small. For the prior term, we assume that each probability column vector in  $\rho$  follows a Dirichlet distribution:

$$\ln p(\rho|\beta) = \sum_{c=1}^{C} \ln \operatorname{Dir}(\rho_c; \beta_c). \tag{9}$$

Such conjugate prior allows us to calculate both the E-step and M-step exactly for each training sample X (refer to Appendix C for the detailed derivation of EM used for multinomial mixtures). The clean label posterior  $p(Y|X,\hat{Y},\rho^t)$  obtained in the E-step at the final iteration can then be used as the soft label for training. This is the main idea of our proposed method which is visually illustrated in Fig. 1. Despite the apparent simplicity, the procedure is, however, associated with an important drawback related to the approximation of  $p(\hat{Y}|X)$ . Initially, it is modelled as a multinomial mixture, but then approximated as a categorical distribution. Thus, the performance would heavily depend on how accurate that approximation is. To overcome such weakness, we propose to slightly modify the procedure by using the clean label posterior  $p(Y|X,\hat{Y},\rho^t)$  obtained from EM as a pseudo-clean label to substitute  $\hat{Y}$ . The "cleaner"  $\hat{Y}$  is then used to approximate  $p(\hat{Y}|X)$  shown in Eq. (4), which in turn, generates many noisy labels to estimate  $p(Y|X,\hat{Y},\rho^t)$  via EM. This process is then repeated until the clean label posterior  $p(Y|X,\hat{Y},\rho^t)$  converges. The intuition of this iterative procedure is to progressively correct the noisy labels in the training set until they become clean. Further details can be referred to Algorithm 1.

As shown in Algorithm 1, the proposed method relies on the extracted features to perform nearest neighbour search. Thus, if the features extracted are biased, it will worsen the quality of the nearest neighbours, reducing the effectiveness of the proposed method. To avoid such bias, we follow a co-teaching approach [14] that trains two models simultaneously where the noisy labels being cleaned by one model are used to train the other model and vice versa.

Table 1: Prediction accuracy on instance-dependent noise on CIFAR-10 and CIFAR-100 where results are as reported in [51].

	CIFA	R-10	CIFAR-100		
Noise rate	0.2	0.4	0.2	0.4	
Cross-entropy	85.66	76.89	57.26	41.33	
Peer loss [28]	89.52	83.44	61.13	48.01	
L <sub>DMI</sub> [43]	88.67	83.65	57.36	43.06	
$L_q$ [50]	85.66	75.24	56.92	40.17	
Co-teaching [14]	88.84	72.61	43.47	23.20	
Co-teaching+ [46]	89.82	73.44	41.62	24.74	
JocoR [40]	88.82	71.13	44.55	23.92	
Forward [30]	87.87	79.81	57.69	42.62	
T-Revision [42]	90.31	84.99	58.00	40.01	
Ours	89.79	85.42	63.27	56.32	

Table 2: Comparison of prediction accuracy on various instance-dependent label noise rates for CIFAR-10 and CIFAR-100 with different network architecture including pre-trained on ImageNet and self-supervised ones trained on the corresponding un-labelled datasets.

Method	CIFAR-10				CIFAR-100			
	0.2	0.3	0.4	0.5	0.2	0.3	0.4	0.5
PTD-R-V [41] <sup>1</sup>	76.58	72.77	59.50	56.32	65.33	64.56	59.73	56.80
kMEIDTM [6] <sup>1</sup>	92.26	90.73	85.94	73.77	69.16	66.76	63.46	59.18
HOC global [51] <sup>2</sup>	89.71	-	84.62	-	68.82	-	62.29	-
HOC local [51] <sup>2</sup>	90.03	-	85.49	-	67.47	-	61.20	-
Ours (with DINO)	91.16	89.67	86.85	76.03	75.45	73.69	70.32	58.02

<sup>&</sup>lt;sup>1</sup> Resnet-34 <sup>2</sup> Resnet-50 pre-trained on ImageNet

### 4.4 Overcome label switching

Although generating more noisy labels per sample by LLC resolves the identifiability issue to obtain a "unique" solution, we still end up with C! permutations due to label-switching. This issue is, however, intrinsic to mixture models in general, unless we impose further constraints. In the context of noisy label learning, the noise is often assumed to be non-dominant [25, 49, 51]. In other words,  $\forall c, j \in \{1, \dots, C\}$ :

$$p(\hat{Y} = c|X, Y = c) \ge p(\hat{Y} = j|X, Y = c),$$
 (10)

which means that the transition matrix is diagonally dominant. Otherwise, the concept of a class label might completely be switched to another class, causing a class mismatch with the ones defined in evaluation sets.

One way to integrate the constraint in (10) is to design the prior parameter  $\beta$  to enforce the matrix  $\rho$  to be close to some diagonally dominant matrix, such as the identity matrix. Imposing such prior, however, increases the complexity of the objective function in Eq. (6), resulting in a complicated fine-tuning for  $\beta$ . In the implementation, we observe that simply initialising the parameter matrix  $\rho$  of  $p(\hat{Y}|Y,X)$  as a diagonally dominant matrix gives us quite accurate results. We, therefore, follow this approach as a simple way to mitigate the label-switching issue.

# 5 Experiments

We empirically evaluate our method using several noisy label learning benchmarks designed to test the robustness of learning methods to the most realistic type of label noise, namely: the instance-dependent noise. In particular, our experiments are carried out on both synthetic and real-world instance-dependent label noise benchmarks. In addition, since the focus of our paper is on the theory side of the identifiability in noisy label learning, we show that the proposed method is effective and competitive to other state-of-the-art methods in the literature, but we do not aim to achieve state-of-the-art results by further fine-tuning or employing highly-complex neural network architectures. All the implementation is done in PyTorch and JAX and will be released upon the acceptance of this paper.

Datasets We evaluate on two types of instance-dependent label noises: synthetic and real-world. For the synthetic noise setting, we use CIFAR-10 and CIFAR-100 as our evaluation datasets, and follow [41] to generate synthetic instance-dependent noisy labels. For real-world label noise, we use three common benchmarks: Controlled Noisy Web Labels (CNWL) [18], mini-WebVision [24] with additional evaluation on the validation of ImageNet ILSVRC 2012 [31], and Animal-10N [34]. For CNWL, we use the web label noise (or red noise) setting where the labels of internet-queried images are annotated manually. For mini-WebVision, we follow previous works that take a subset containing the first 50 classes in the WebVision 1.0 dataset for training and evaluate on the clean validation set. The model trained on mini-

Table 3: Accuracy evaluated on real-world datasets: (left) Red CNWL, (middle) mini-WebVision and ImageNet, and (right) Animal-10N.

Method Noise rate		e	Method	WebVision	ImageNet	Method	Animal-10N	
	0.2	0.4	0.6	Mixup [48]	74.96	-	Cross entropy	79.40
Cross-entropy	47.36	42.70	37.30	Co-teaching [14]	63.58	61.48	Nested-Dropout	81.30
MixUp [48]	49.10	46.40	40.58	DivideMix [23]	77.32	75.20	CE + Dropout	81.30
DivideMix [23]	50.96	46.72	43.14	ELR [26]	76.26	68.71	SELFIE	81.80
				ELR+ [26]	77.78	70.29	PLC	83.40
MentorMix [18]	51.02	47.14	43.80	MOIT [29]	78.36	-	Nested-CE	84.10
FaMUS [44]	51.42	48.06	45.10	NCR [17]	77.10	_	Ours	85.96
Ours	52.78	49.18	46.00	Ours	80.48	74.63	Ours	00.70

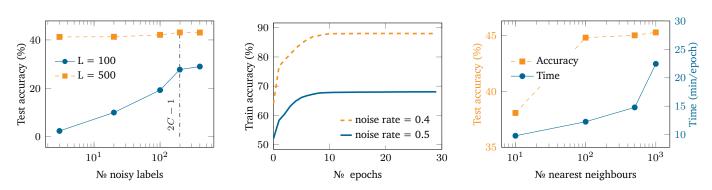


Figure 2: Ablation studies on: (*left*) the effect of number of noisy labels per sample on Red CNWL at 0.6 noise rate, (*middle*) and (*right*) the accuracy of the relabelling and the influence of nearest neighbours on CIFAR-100.

WebVision is also evaluated on the clean validation set of ImageNet ILSVRC 2012. Finally, we evaluate the proposed method on Animal-10N dataset that contains 5 pairs of confusing animals.

**Models** We use PreAct Resnet-18 as the backbone to evaluate the proposed method on CIFAR-10, CIFAR-100 and Red CNWL datasets. Note that for CNWL, we preprocess the images by resizing from 84-by-84 pixel<sup>2</sup> to 32-by-32 pixel<sup>2</sup>. For mini-WebVision, we download the small image version and further resize all images to 224-by-224 pixel<sup>2</sup> before passing the images into a Resnet-50. For Animal-10N, we keep the original image size of 64-by-64 pixel<sup>2</sup> and use VGG-19 as the backbone to obtain a fair comparison with existing baselines.

**Hyper-parameters:** refer to Appendix B for hyper-parameters and further details of the experiments.

#### 5.1 Results

Tab. 1 shows a comparison in terms of prediction accuracy for synthetic label noises between some common baselines and our proposed method. The results show that our proposed method is on par with those baselines on CIFAR-10 dataset, while out-performing competing approaches on CIFAR-100 dataset. We further evaluate our method by pretraining a PreAct Resnet-18 using the unlabelled data of each dataset with DINO [5], which is a self-supervised training method developed to initialise models with the goal of improving their performance in downstream tasks. Tab. 2 shows the results of several state-of-the-art methods with different network architectures pre-trained on ImageNet. In both datasets, our method demonstrates a competitive performance compared to the state-of-the-art methods at small noise rates and slightly better at larger noise rates.

For Red CNWL, we follow the experiment setup in [44] and show the results in Tab. 3 (*left*). The results in Tab. 3 (*left*) includes baselines evaluated on small size images (32-by-32 pixel<sup>2</sup>) for a fair comparison. In this benchmark, the proposed method consistently outperforms the state-of-the-art methods. We further evaluate the proposed method on other real-world label noise datasets: mini-WebVision and Animal-10 and show the results in Tab. 3 (*middle*) and (*right*), respectively. For mini-WebVision, we use a Resnet-50 that is pre-trained on ImageNet for 100 epochs by the self-supervised learning method DINO [5] as an initialisation. For Animal-10N, we use DINO to pre-train a VGG-19 for 800 epochs and use the pre-trained parameters as an initialisation to train our model. In general, the results show competitive performance compared to common baselines.

# 5.2 Ablation studies

We carry out additional studies to investigate the effect of the number of noisy labels per sample, the number of nearest neighbours and the effectiveness of the relabelling. Note that no self-supervised learning is used for pre-training the model to avoid potential confounding factors.

We run experiments with the same setting on Red CNWL at 0.6 noise rate with various number of noisy labels per sample  $N \in \{3, 20, 100, 199, 400\}$  and plotted the results in Fig. 2 (*left*), where L is the number of N multinomial noisy labels defined in Algorithm 1. When L is small, the more noisy labels per sample, the more effective, and the

effectiveness diminishes after the threshold of 2C-1, which in this case is 199. This empirically confirms the validity of Claim 1 about the identifiability in noisy label learning. However, when L is large, the performance difference when varying N is not as noticeable. In this regime (of large L), Claim 1 might result in a conservative requirement in terms of number of noisy labels per sample. The current setting of noisy label learning might contain some common latent structure between samples, which we have not exploited yet to bring down the number of required noisy labels per sample. Future work will need to address such issue to make the problem more practical.

We investigate the effectiveness of the proposed method by measuring the accuracy on the training set between the pseudo labels "cleaned" by EM and the ground truth labels. The results in Fig. 2 (middle) show that the proposed method improve by about 16 to 24 percent the accuracy on the CIFAR-100 training set compared to the original noisy one. This is equivalent to cleaning 33 to 67 percent of noisy labels.

We also investigate the effect of the number of nearest neighbours K used to estimate noisy label distribution of each sample and show the results evaluated on CIFAR-100 at 0.5 noise rate in Fig. 2 (right). In light of testing accuracy, the larger K, the more accurate. However, the trade-off is the running time as shown in Fig. 2 (right). Since K = 100 gives a good balance between the performance and running time, we use this value in all of our experiments in Sec. 5.1.

# 6 Conclusion

This work has formally investigated the identifiability of noisy label learning in the lens of finite mixture models by formulating the label noise problem as a multinomial mixture model, where the mixture coefficient is represented by the clean label probability and the multinomial components are denoted by the columns in the transition matrix. Under this modelling approach, we show that the conventional noisy label learning with a single noisy label per instance is un-identifiable, even up to the permutation of labels. To make it identifiable, we impose a constraint from multinomial mixture by requiring at least 2C-1 noisy labels per instance, where C is the number of classes. We then propose to employ locality-constrained linear coding approach that relies on nearest neighbours to generate additional noisy labels per sample. Such approach allows to estimate the clean labels via the EM algorithm. Experimental results show that our proposed method is competitive with many existing state-of-the-art methods in several challenging benchmarks, especially the instance-dependent and real-world label noises.

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### A Extended related work

As mentioned in Sec. 2, the closest study to our work is [27] where the authors employ the results in the mixture proportion estimation literature to derive the identifiable condition for noisy label learning, in which at least 3 "informative" noisy labels per instance are required. This is different and much more optimistic from our result in Claim 1, where we theoretically show that at least almost twice the number of additional noisy labels per sample is sufficient to make the label noise problem identifiable. Although both studies derive results from mixture models defined in Eq. (2), the distinction is at the "number of features" (or loosely speaking, the number of variables in multivariate mixture models) in [27] versus the number of samples (or noisy labels per instance) generated from the mixture model of interest in our our work. According to [1, Eq. (1)] – one of the primary references in [27], each component  $\mathbb{P}_k$  consists of p independent features, resulting in K-class p-feature models (if we follow the notation in [1], then it is equivalent to r-class p-feature (see [1, 2nd paragraph after Eq. (1)])). The minimal number of features required to ensure that such mixture model identifiable leads to the main result in [27, Theorem 4]. This is, however, arguable since the derived three noisy labels corresponding to three features are "identical" in terms of variable modelling, which in turn, conflicts with the initial assumption. In contrast, our study investigates the minimum number of samples that are generated from the mixture model of interest to make the inverse problem (inferring model parameters from samples) identifiable.

# **B** Experiment details

We use a mini-batch size of 128 training samples, perform a warm-up for 10 epochs and train for 200 epochs. The optimiser used is SGD with a momentum of 0.9 and an initial learning rate 0.01 when training without self-supervised learning and 0.001 when self-supervised learning models are used. The learning is decayed with cosine annealing with T=1,000. For the priors defined in (6), we assume both priors on the mixture coefficient (or clean label posterior) and the probability vector of the multinomial components in the transition matrix as symmetric Dirichlet distributions with  $\alpha=20$  and  $\beta=5$ . For the nearest neighbours, we first randomly sample a subset of 10,000 samples then perform nearest neighbour search and select the 100 nearest samples for LLC.

In addition, we select different  $\mu$  for different label noise rates on different datasets as shown in Tab. 4.

Dataset	Noise rate	$\mu$
	0.2	0.6
CIFAR	0.3	0.5
CIFAR	0.4	0.4
	0.5	0.2
	0.2	0.6
Red CNWL	0.4	0.6
	0.6	0.3
mini-WebVision		0.5
Animal-10N		0.5

Table 4: The value of hyper-parameter  $\mu$  used in our experiments.

# C EM for multinomial mixture models

We consider a more general case – maximum a posterior (MAP) – for a mixture of C multinomial distributions. Note that the number of trials, m, is made fixed. Note that the notations here are different from the ones presented in the main paper.

A mixture of multinomial distributions can be written as:

$$p(\mathbf{x}) = \sum_{z} p(z) p(\mathbf{x}|z) = \sum_{k=1}^{K} \pi_k \text{Mult}(\mathbf{x}; m, \rho_k).$$
(11)

### C.1 Maximum likelihood

#### C.1.1 E-step

This step is to calculate the posterior of the latent variable  $z_n$  given the data  $x_n$ :

$$\gamma_{nk} = p(\mathbf{z}_{nk} = 1 | \mathbf{x}_{n}, \pi^{(t)}, \rho^{(t)}) 
= \frac{p(\mathbf{x}_{n} | \mathbf{z}_{nk} = 1, \rho^{(t)}) p(\mathbf{z}_{nk} = 1 | \pi^{(t)})}{\sum_{k=1}^{K} p(\mathbf{x}_{n} | \mathbf{z}_{nk} = 1, \rho^{(t)}) p(\mathbf{z}_{nk} = 1 | \pi^{(t)})} 
= \frac{\pi_{k}^{(t)} \operatorname{Mult}(\mathbf{x}_{n}; N, \rho_{k}^{(t)})}{\sum_{k=1}^{K} \pi_{k}^{(t)} \operatorname{Mult}(\mathbf{x}_{n}; N, \rho_{k}^{(t)})}.$$
(12)

#### C.1.2 M-step

In the M-step, we maximise the following expected completed log-likelihood w.r.t.  $\pi$  and  $\rho$ :

$$L = \sum_{n=1}^{N} \mathbb{E}_{p(\mathbf{z}_{n}|\mathbf{x}_{n},\pi^{(t)},\rho^{(t)})} \left[ \ln p(\mathbf{x}_{n},\mathbf{z}_{n}|\pi,\rho) \right]$$

$$= \sum_{n=1}^{N} \mathbb{E}_{p(\mathbf{z}_{n}|\mathbf{x}_{n},\pi^{(t)},\rho^{(t)})} \left[ \ln p(\mathbf{z}_{n}|\pi) + \ln p(\mathbf{x}_{n}|\mathbf{z}_{n},\rho) \right]$$

$$= \sum_{n=1}^{N} \mathbb{E}_{p(\mathbf{z}_{n}|\mathbf{x}_{n},\pi^{(t)},\rho^{(t)})} \left[ \sum_{k=1}^{K} \mathbf{z}_{nk} \ln \pi_{k} + \mathbf{z}_{nk} \ln \text{Mult}(\mathbf{x}_{n};m,\rho_{k}) \right]$$

$$= \sum_{n=1}^{N} \sum_{k=1}^{K} \gamma_{nk} \left[ \ln \pi_{k} + \sum_{d=1}^{D} \mathbf{x}_{nd} \ln \rho_{kd} + \text{const.} \right]$$
(13)

The Lagrangian for  $\pi$  can be written as:

$$\mathsf{L}_{\pi} = \mathsf{L} - \lambda \left( \sum_{k=1}^{K} \pi_k - 1 \right),\tag{14}$$

where  $\lambda$  is the Lagrange multiplier. Taking derivative of the Lagrangian w.r.t.  $\pi_k$  gives:

$$\frac{\partial \mathsf{L}_{\pi}}{\partial \pi_k} = \frac{1}{\pi_k} \sum_{n=1}^{N} \gamma_{nk} - \lambda. \tag{15}$$

Setting the derivative to zero and solving for  $\pi_k$  gives:

$$\pi_k = \frac{1}{\lambda} \sum_{n=1}^{N} \gamma_{nk}. \tag{16}$$

And since  $\sum_{k=1}^K \pi_k = 1$ , one can substitute and find that  $\lambda = N$ . Thus:

$$\pi_k^{(t+1)} = \frac{1}{N} \sum_{n=1}^{N} \gamma_{nk}.$$
 (17)

Similarly, the Lagrangian of  $\rho$  can be expressed as:

$$L_{\rho} = L - \sum_{k=1}^{K} \eta_k \left( \sum_{d=1}^{D} \rho_{kd} - 1 \right), \tag{18}$$

where  $\eta_k$  is the Lagrange multiplier. Taking derivative w.r.t.  $\rho_{kd}$  gives:

$$\frac{\partial \mathsf{L}_{\rho}}{\partial \rho_{kd}} = \frac{1}{\rho_{kd}} \sum_{n=1}^{N} \gamma_{nk} \mathbf{x}_{nd} - \eta_{k}. \tag{19}$$

Setting the derivative to zero and solving for  $\rho_{kd}$  gives:

$$\rho_{kd} = \frac{1}{\eta_k} \sum_{n=1}^{N} \gamma_{nk} \mathbf{x}_{nd}.$$
 (20)

The constraint on  $\rho_k$  as a probability vector leads to  $\eta_k = m \sum_{n=1}^N \gamma_{nk}$ . Thus:

$$\rho_{kd}^{(t+1)} = \frac{\sum_{n=1}^{N} \gamma_{nk} \mathbf{x}_{nd}}{m \sum_{n=1}^{N} \gamma_{nk}}.$$
(21)

# C.2 Maximum a posterior (MAP)

The E-step in this case remains unchanged from (12).

The expected complete data log-likelihood plus the log prior can be written as:

$$\mathsf{L}^{\mathrm{MAP}} = \mathsf{L} + \ln p(\pi) + \sum_{k=1}^{K} \ln p(\rho_k),$$
 (22)

where the two priors are:

$$p(\pi) = Dir(\pi; \alpha) \tag{23}$$

$$p(\rho_k) = \text{Dir}(\rho_k; \beta_k). \tag{24}$$

The derivative of the Lagrangian for  $\pi$  can be written as:

$$\frac{\partial \mathsf{L}_{\pi}^{\mathrm{MAP}}}{\partial \pi_{k}} = \frac{1}{\pi_{k}} \left( \sum_{n=1}^{N} \gamma_{nk} + \alpha_{k} - 1 \right) - \lambda \tag{25}$$

Thus:

$$\pi_k^{(t+1)} = \frac{\sum_{n=1}^N \gamma_{nk} + \alpha_k - 1}{N + \sum_{k=1}^K \alpha_k - K}.$$
 (26)

Similarly for  $\rho_{kd}$ :

$$\frac{\partial \mathsf{L}_{\rho}^{\text{MAP}}}{\partial \rho_{kd}} = \frac{1}{\rho_{kd}} \left( \sum_{n=1}^{N} \gamma_{nk} \mathbf{x}_{nd} + \beta_{kd} - 1 \right) - \eta_{k}. \tag{27}$$

Thus:

$$\rho_{kd}^{(t+1)} = \frac{\sum_{n=1}^{N} \gamma_{nk} \mathbf{x}_{nd} + \beta_{kd} - 1}{m \sum_{n=1}^{N} \gamma_{nk} + \sum_{d=1}^{D} \beta_{kd} - K}.$$
(28)