Gentry Atkinson
CS5318: Spring 2019
Homework #3

1) Correctly parenthesize the following lambda expression and find its set of free variables.
λ x . x λ y . y λ z . z λ w . w z y x

(λ x . x (λ y . y(λ z . z (λ w . w)))) z y x
Free variables: z, y, x

2) Perform the following substitutions.

1. **{ g / x } (f (λ x . x y) λ z . x y z)**

   ((g/x)(λ x . x y) λ z . x y z)

   (g (λ x . x y) λ z . x y z) /x

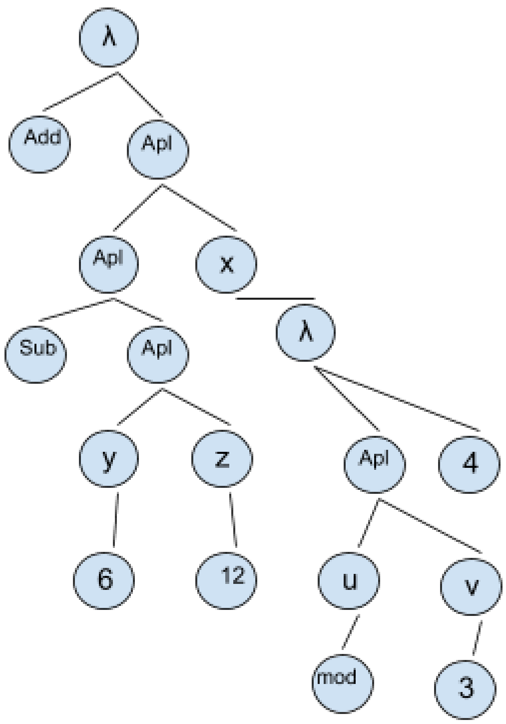
   (g (λ x . 1 y) λ z . 1 y z)

2. **{ g x / f } ((λ x . f x) λ f . f x)**

   ((λ x . (gx/f) x) λ (gx/f) . (gx/f) x)

3)Draw the abstract syntax tree of the following lambda expression and identify its innermost and outermost beta-redices.

(λ x y z . (add x (sub y z))) 6 12 ((λ u v . u 4 v) mod 3)



Innermost: 4 mod 3

Outermost: add 6 11

4) Use both normal order reduction and applicative order reduction to reduce the following lambda expressions. Reach a normal form representation if possible.

1. (λ x . x x) (λ x . x x x)

   => (λ x . x x x) (λ x . x x x)

   => (λ x . x x x) (λ x . x x x) (λ x . x x x)

   ...

2. (λ c . c (λ a . λ b . b)) ((λ a . λ b . λ f . f a b) p q)

   Normal:

   => (λ a . λ b . b) ((λ a . λ b . λ f . f a b) p q)

   =>(λ b . b) ((λ a . λ b . λ f . f a b) p q)

   => ((λ a . λ b . λ f . f a b) p q)

   =>paq

   Applicative:

   => (λ c . c (λ a . λ b . b)) ((λ a . λ b . λ f . f a b) p q)

   => (λ c . c (λ a . a)) ((λ a . λ b . p a b) q)

   =>(a) ((λ a . p a q))

   =>paq

5) Constants in the pure lambda calculus can be defined as functions. For example, T for the "true" constant and F for the "false" constant can be defined as follows:

T = λ x . λ y . x
F = λ x . λ y . y

Prove that the following function:

AND = λ x . λ y . x y F is the logical "and" function.

X	**Y**	**X AND Y**	**λ x . λ y . x y F**
T	T	T	(λ x . λ y .xyF) T T TTF (λ x . λ y . x)TF T
T	F	F	(λ x . λ y .xyF) TF TFF (λ x . λ y . x)TF F
F	T	F	(λ x . λ y .xyF) FT FTF (λ x . λ y . y)FT F
F	F	F	(λ x . λ y .xyF) FT FFF (λ x . λ y . y)FF F