Z. Phys. B 95, 407–414 (1994)

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Competition and cooperation on a toy Autobahn model

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Received: 15 April 1994

Abstract. Traffic on an one-lane freeway is simulated using a continuous space-discrete time probabilistic cellular automata model. The effect of different individual driving patterns is estimated by monitoring the traffic flow, the velocity and acceleration distributions, the average number of accidents, and the distribution of density-waves (traffic jams) as a function of traffic density. The number of accidents, traffic jams, and the fuel consumption are drastically reduced by driving strategies adapting to local traffic conditions. At high traffic densities this leads, however, to a decrease in the global traffic throughout.

1. Introduction

Numerical simulations, hydrodynamical models, and queuing theory are but a few of the basic theoretical tools used to describe vehicle traffic on freeway networks – see [1–3]. Traffic control has been intensively studied also from the point of view of operational research [4]. As expected, the theory of vehicle traffic has many intersection points with telecommunications and computer network dynamics. Perhaps more surprising is that many basic concepts of the traffic flow theory have their origins in physics – and have been actually developed by physicists [5, 7–10].

In fact, vehicles moving on the Autobahn (a freeway without speed limit) can be considered as a system of particles with stochastic dynamics and exclusion interactions (as long as accidents do not occur). Many real properties of traffic on a long freeway streak can be qualitatively described using macroscopic, hydrodynamical arguments [5, 1]. Microscopic models are based on a more detailed description of the vehicle-vehicle or jam-jam interactions. Recently, simple cellular automata models have been shown to reproduce quantitatively measurements regarding the basic traffic flow vs. traffic density diagram [11–15]. These models have strong similarities to the dynamics of driven fluids, lattice-gas models in external fields [16], and indirectly to models of surface growth [17, 18]. Many of these models are described in the continuum limit by the (noisy) Burgers Eq. [19], which is exactly soluble in one-dimension. Simple lattice-gas models with exclusion interactions, drift, and special boundary conditions are currently actively investigated [20–24]. Traffic-jam waves are also very similar in cause and nature to density-waves observed in granular media transport [26].

Control of flow in urban traffic is a more complex endeavor. However, by using analogies to biological neurons described by networks of interacting oscillators, a 'firefly' approach has been recently proposed for the optimal control of signal lights in urban traffic [25]. Hence, not only has traffic flow a lot of important physical analogies but – as a typical example of 'everyday physics' – it continues to receive a lot of attention from the physics community.

In this paper we consider a slightly expanded version of the cellular automata model developed in Köln [11], which describes traffic on a single lane freeway. The model is a good approximation for the traffic on a two – or more lane highway at small and large traffic densities, when lane changes are either not necessary or impossible. The numerical load is only marginally increased when allowing a continuum simulation of the Autobahn, updated as before in small synchronous discrete time steps [1]. We were interested in understanding how global cooperation and competition patterns emerge from individual driving strategies. Hence, we consider here different mixtures of fixed strategies, characteristic for archetypical vehicle-driver combinations. Together with stochastic noise added to the velocity field, this mixture of strategies leads to density- (shock- or Stau-) waves, an all too familiar signature of the european weekends.

[1] We did not consider parallel and/or multibit implementations of the algorithm – see [15]

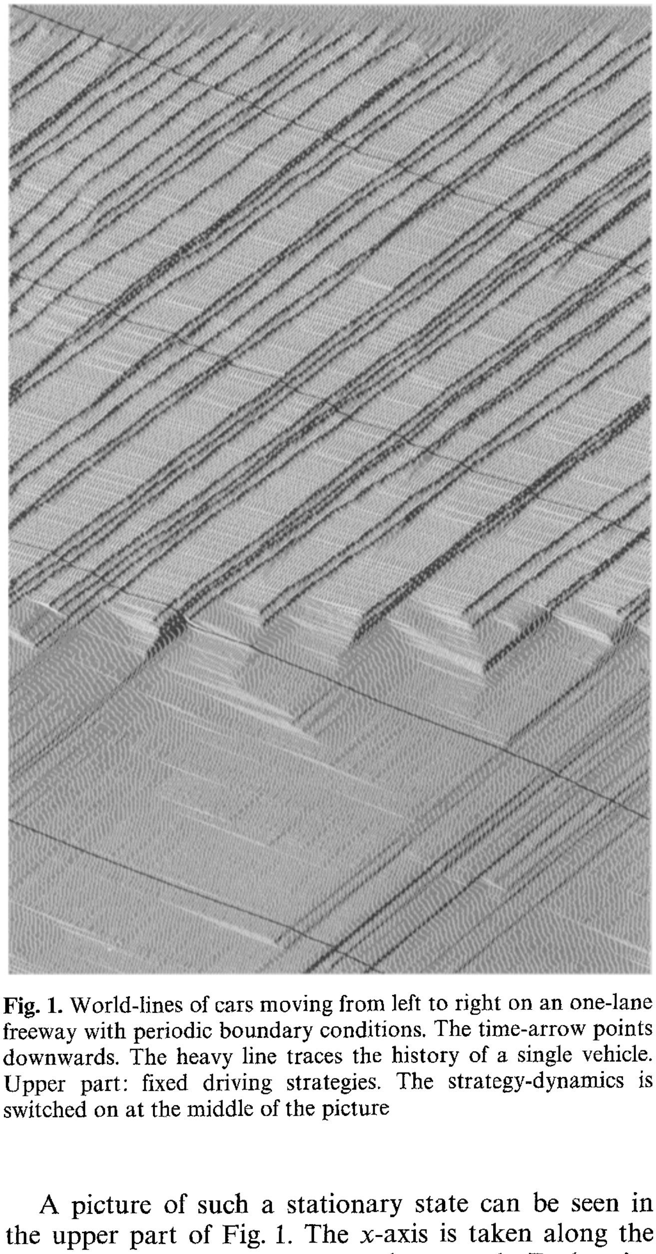

Fig. 1. World-lines of cars moving from left to right on an one-lane freeway with periodic boundary conditions. The time-arrow points downwards. The heavy line traces the history of a single vehicle. Upper part: fixed driving strategies. The strategy-dynamics is switched on at the middle of the picture

A picture of such a stationary state can be seen in the upper part of Fig. 1. The x-axis is taken along the freeway flow. The time increases downwards. Each point represents the position of a car at a given time. Hence, the shown picture consists of a series of snapshots of the whole freeway as a function of time. Traffic jam (density) waves are clearly visible as thick lines and flow backwards, against the traffic.

Although individual driving patterns seem particularly stable against rational self-control, we will nevertheless assume that our drivers are able to switch strategies when the traffic conditions so demand. At the middle of the Fig. 1, each driver start to adapt her/his strategy to the local traffic conditions. This process will be defined on the next section. The lower part of Fig. 1 shows what happens if such a 'miracle' occurs, all other parameters of the simulation remaining unchanged. Clearly, the traffic jams disappear and the traffic flow becomes more homogeneous. Note that the white stripes correspond to fronts of free highway segments (rarefaction waves) and are running ahead the traffic flow. As expected, the number of accidents and the total fuel consumption are drastically reduced. However, compared to the case when the strategies remain fixed, the total traffic throughput *decreases* at high flow densities.

The paper is organized as follows: Section 2 defines the model and explains the way the model is simulated. Section 3 contains a detailed analysis of the simulation results. Section 4 contains a brief summary and discussion.

2. Model and simulation

With the event of fast and cheaply available computing power, the accurate and realistic numerical simulations of complex, real-life phenomena is becoming a reality. At least for the problem considered here, one can argue that for practical purposes numerical simulations provide a more powerful tool than analytic theories.

Note that in hydrodynamical simulations [27] the cellular automata dynamics itself is *virtual*, since it does not describe the microscopic interactions of fluid particles. Only in the continuum limit does it lead to the correct hydrodynamics. The cellular automata model suggested in [11] and expanded below is, however, based on a correct 'atomistic' model of reality, at least for the traffic on a one-lane Autobahn.

2.1. The basic algorithm

The motion of each i^{th} car is evaluated at every time interval $\Delta t = 1$, the dynamics is synchronously updated. Although such parallel updating can generate spurious effects, the presence of stochastic forces prevent the system from running into artificial states. The dynamical rules are local: they take into account the position x_i and velocity v_i of car i and the position x_{i+1} and velocity v_{i+1} of the followed car. The next position and velocity of car i depends on the acceleration $\ddot{x}_i$. The following constraints are enforced:

1. uniform boundaries for the acceleration: $-b_{\max} < \ddot{x}_i < \ddot{x}_{\max}$
2. Security distance D_s:

$$D_s = l - x'_i + x_{i+1} - \frac{v'^2_i}{2b_{\max}} + \frac{v^2_{i+1}}{2b_{\max}} \tag{1}$$

where x'_i, v'_i are the position and the velocity of the car i *after* some given ($\tau_r = 1$ *sec*) reaction time. The minimal stopping distance is estimated by assuming breaking with maximal deacceleration $b_{\max}$ and l is the length of the vehicle;

3. phase space boundaries for the velocity: $0 \leqq v_i \leqq v_{\max}$

The acceleration is estimated first from term 2 and is truncated as required by the conditions 1 and 3. The resulting speed is the maximal allowed velocity not violating the security condition 2. Note that the parameters $v_{\max}$, $a_{\max}$, and l can be made specific to each car i.

According to these rules, every car keeps a distance larger than the security distance D_s from the followed car. Small perturbations induced by spontaneous changes on the road conditions and on the attention state of drivers are modeled by a random variable forcing randomly and independently chosen cars to slow down. On each time step, car i will break with probability 0.25 and deacceleration $0.2 b_{\max}$, independently from other cars. This stochastic force 'thermalizes' the system and at high densities acts as a source of discontinuity interfaces (shock- or kinetic-waves).

In order to measure the number of accidents, we define a crash as occuring when one car violates the *minimal* security zone of the car in front of it. The minimal security zone is three times the length of a car, one length backwards and one length to the front. This definition works rather well, except at very dense traffic, where it overestimates the number of real accidents. After a (simulated) accident takes place, the velocity and the acceleration of the car causing the crash are set to zero and its position is set back to one car length behind the front car. Further measurements are made only after the highway traffic relaxes back to its stationary state.

Table 1 summarizes the global parameters used throughout our simulations:

Table 1. Global model parameters

Parameter	Value
Length L	10000 m
Number of cars N	50–900
$b_{\max}$	12 m/s^2
Lowest $v_{\max}$	27.5 $\frac{\mathrm{m}}{\mathrm{s}}$
Highest $v_{\max}$	33.5 $\frac{\mathrm{m}}{\mathrm{s}}$
Lowest security factor	96%

The initial conditions were in all cases generated as following: at $t=0$ the cars are equidistantly placed on a highway with periodic boundary conditions. Their parameters (car type, driver type), are generated at random and independently, using different random number sets for different experiments. The initial velocities are all set to zero. The Autobahn is then 'thermalized' (brought into the steady state) with this fixed set of strategies. The results shown in this paper are averages obtained over four independent experiments.

2.2. Strategies

The model described above can be easily extended to simulate different cars and different driving stragies. Car types are defined by their maximal velocity and maximal acceleration (the maximal breaking deacceleration is assumed to be the same for all cars). Two types of experiments were made: in one we use different sets of driving strategies but the same car type. In the second type of experiments one has three different car classes and a single set of five driving strategies. Since the results follow the expected trends, we show below only the results for three cars – five strategies type experiments.

Table 2. Parametrization of car classes

Car type	Probability	$v_{\max}$[m/s]	$a_{\max}$[m/s^2]
fast	0.25	33.5	8
average	0.5	30.0	6
slow	0.25	27.5	5

As seen in Table 2 the cars are classified according to their maximal velocity and maximal acceleration.
A driving strategy is defined by the following three factors:
1. the percentage of the car's maximal velocity the driver is actually using, $V_{\max}=0.8$–$1.0\ v_{\max}$;
2. the percentage of the car's maximal acceleration (power) the driver is actually using $A_{\max}=9.9$–$1.0\ a_{\max}$;
3. the percentage of the security distance the driver is actually keeping, the security factor: $SF=0.9$–1.0 min D_s.
The following set of strategies was used (Table 3):

Table 3. Strategy parameters

	Occurrence probability	$A_{\max}/a_{\max}$	$V_{\max}/v_{\max}$	SF/D_s
Strategy 1	0.2	1.0	1.0	0.96
Strategy 2	0.2	0.95	0.97	0.97
Strategy 3	0.2	0.90	0.94	0.98
Strategy 4	0.2	0.85	0.91	0.99
Strategy 5	0.2	0.8	0.88	1.0

It is somewhat difficult to separate the effect of car diversity from that of driving strategy diversity. However, the two sets of experiments (one allowing different strategy classes but one car type, the other one more car types but a single set of strategies) reveal that the general trends can be easily estimated even for mixed situations.

2.3. Strategy-dynamics

One way of introducing dynamical changes of strategies is by using payoff matrices, as in game theory [28, 29]. Instead of going through the process of deriving perceived payoffs associated with a given strategy, we use a simple but realistic model. On each time step, the drivers evaluate the local traffic conditions and change to a different strategy according to the following protocol. Assume that the driver of a given car type aims at driving with some maximal velocity $V_{\max}(i)$, which is a given fraction of its car maximal velocity $v_{\max}(i)$. Note that while $V_{\max}(i)$ and $A_{\max}(i)$ are the limits set by a given strategy, they can never exceed the physical boundaries $v_{\max}(i)$, $a_{\max}(i)$. After the usual time-step evaluation, the driver 'looks' forward, to see if there is a traffic-jam within the horizon. The significant events signaling a possible traffic jam are a standing car, a car breaking with b_{max}, or an accident site. The horizon is proportional to the car velocity and the traffic density. A typical number is 200 m for a 5 km Autobahn and a density of 0.4, at average velocity.

1. *Changing to a faster strategy.* Assume that car i is already driving with the maximal velocity $V_{max}(i)$ allowed by the current strategy and that the distance between car i and car $i+1$ is larger than $2V_{max}(i)\Delta t$. Then the driver is allowed to change its $V_{max}(i)$ to $V_{max}(i)+1$ but not the other parameters of its strategy. If, however, the actual $V_{max}(i)$ is larger than the maximal velocity allowed by the next (faster driving) strategy, then the whole parameter space is changed to that of the faster strategy, with the consequences of higher A_{max} and lower safety factor.

2. *Changing to a slower strategy.* In this case the driver notices a traffic jam within the horizon. Then $V_{max}(i)$ is reduced by 1. If the new $V_{max}(i)$ is less than V_{max} of the next 'slower' strategy, the driver adopts it with the consequences of lower A_{max} and a higher safety factor (SF).

3. Simulation results

In order to obtain a detailed description of the behavior of our model we measured the following quantities:
1. Traffic flow (number of vehicles per unit length and unit time) *vs* traffic density (number of vehicles per unit length);
2. Total number of accidents *vs* density;
3. Total force (acceleration) *vs* density;
4. Velocity distribution;
5. Evolution of strategies *vs* density;

In all simulation experiments we run in total 1000 time steps from which the first 200 were discarded. Figure 2 displays the typical flow-density diagrams for the fixed set of strategies and car-types given in Tables 2 and 3. The curves show the traffic flow when the strategies are fixed (diamonds) or dynamically adapting (crosses).

At low densities the structure of stationary traffic consists of car columns. A column consists of a slow vehicle followed closely by faster cars. However, these car columns do not interact with each other because their average distance is larger than D_s. Hence, the effects of the stochastic breaking force cannot propagate. The traffic flow is stable against small and large perturbations. Also, the traffic flow increases linearly with increasing density. In hydrodynamical language, the flow is supersonic, the compression waves appear as overdamped modes. As the flow density increases, the average distance between columns of cars decreases to the same order of magnitude as the typical D_s for slow cars. The columns start to interact, and the resulting decrease in the average velocity overweights the increase in density. As the density further increases, kinematic density waves [5] appear, which represent shock (Stau-) waves separating supersonic flow from subsonic one [30].

The introduction of dynamic strategies has two main effects. First, the strategies for slow cars evolve towards the maximal velocity strategy (strategy 1), as clearly seen in Fig. 11, while the faster cars must adopt slower strategies Fig. 13. This increases the overall stability and maximizes the flow on the stable traffic phase. On the high density phase the adaptation happens always towards more defensive strategies. At densities around 0.35 almost all cars have switched to strategy 5 – see Figs. 11–13. As a result, the global flow decreases compared to the fixed strategy case.

The second observation is that the sharp cusp seen for quenched strategies at maximal flow is now rounded. This is also indicated by the large fluctuations in the choice of driving strategy characteristic for this region (Figs. 11–13). This type of fluctuations are further enhanced by the possibility of changing lanes and is clearly seen in measurements of real highway traffic, shown in Figs. 3–4, [31]–[32], respectively.

As mentioned above, at densities near the maximal flow the cars start to interact because the average dis-

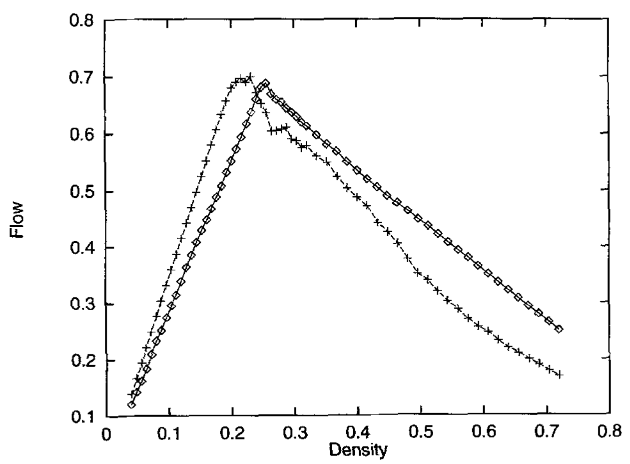

Fig. 2. The basic diagram for fixed strategies: traffic flow vs traffic density. Diamonds: fixed strategies. Crosses: dynamic strategies

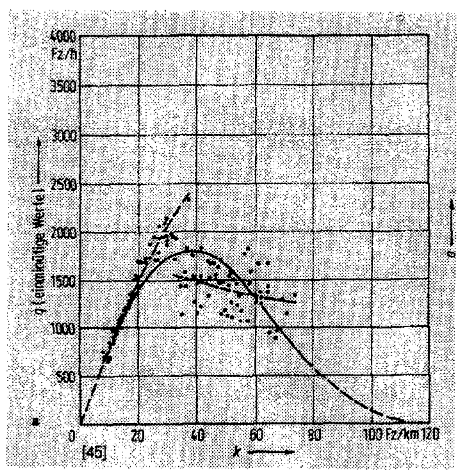

Fig. 3. Experimental data on the basic traffic-flow diagram: [31]

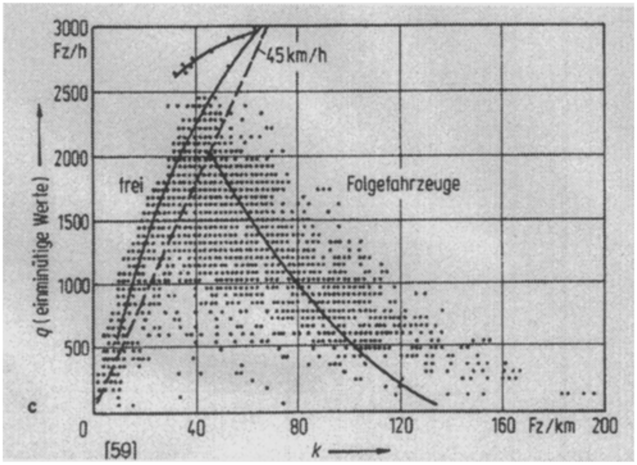

Fig. 4. Experimental data on the basic traffic-flow diagram: [32]

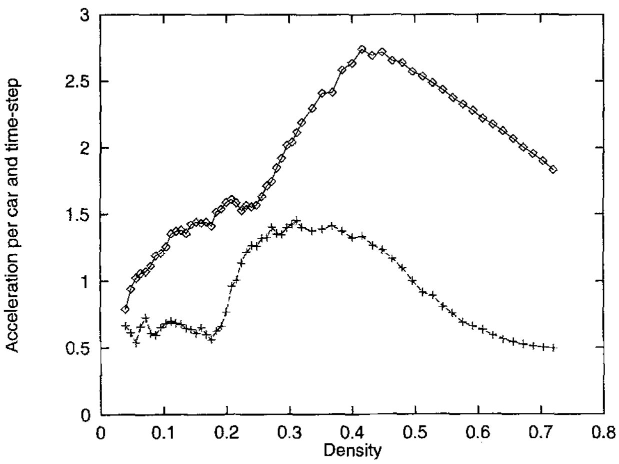

Fig. 5. Total acceleration per car and time unit for fixed (diamonds) and adaptive (crosses) strategies

tance between cars is reduced to the necessary security distance. Hence, accidents can occur at this point as can be seen in Fig. 6 for fixed strategies. The same plot, Fig. 7, for adaptive strategies shows clearly a forty fold reduction in the number of accidents. It is also interesting to observe that the number of accidents is maximal close to the maximal flow: this is due to the persistence of fast driving strategies in a situation when this is only locally optimal. Note that our definition of 'crash' overestimates the number of accidents at high density.

The total fuel consumption can be evaluated by measuring the total acceleration (force) per time unit along the highway (work done in accelerating vehicles) and the velocity distribution (energy dissipated by wind, road, and engine resistance). Figure 5 compares the distribution of the total acceleration for both fixed strategies (diamonds) and adaptive strategies (crosses). Note that close to the maximal flow density the dynamics of strategies is not stable, requiring a continuous reevaluation of the driving strategy. This leads, in turn, to a high acceleration rate.

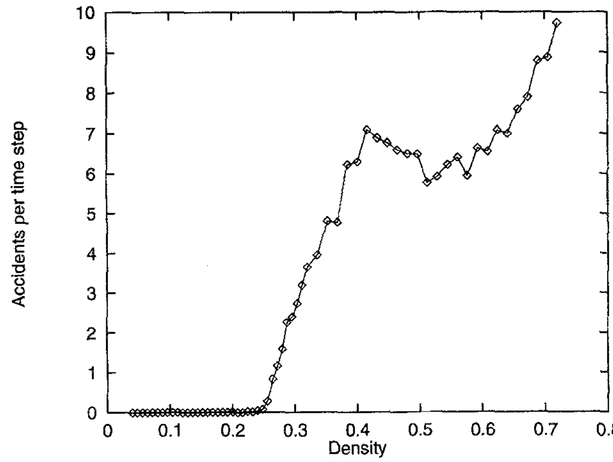

Fig. 6. Total number of accidents per unit time step: fixed strategies

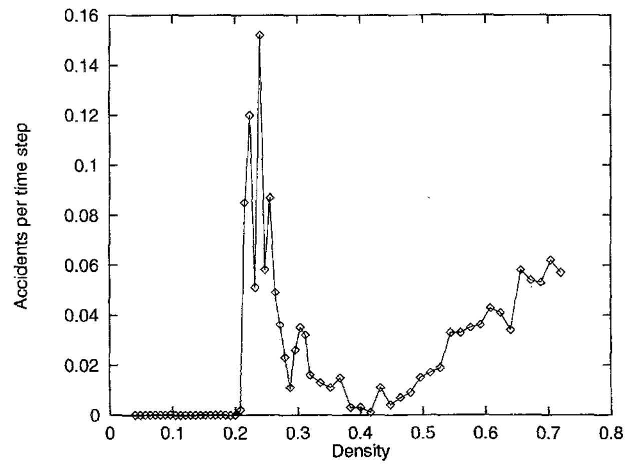

Fig. 7. Total number of accidents per unit time step: adaptive strategies

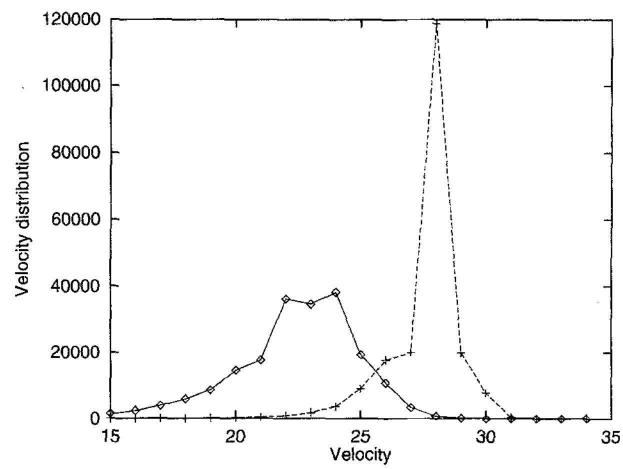

Fig. 8. Velocity distribution (in km/h) for fixed (diamonds) and adaptive (crosses) strategies. Density = 0.15

Figure 8 compares the velocity distribution for fixed strategies (diamonds) and adaptive strategies (crosses) at density 0.15 and Fig. 9 at density 0.45. In both cases the adaptive strategies lead to a more uniform flow, signaled by a stronger localization of the distribution

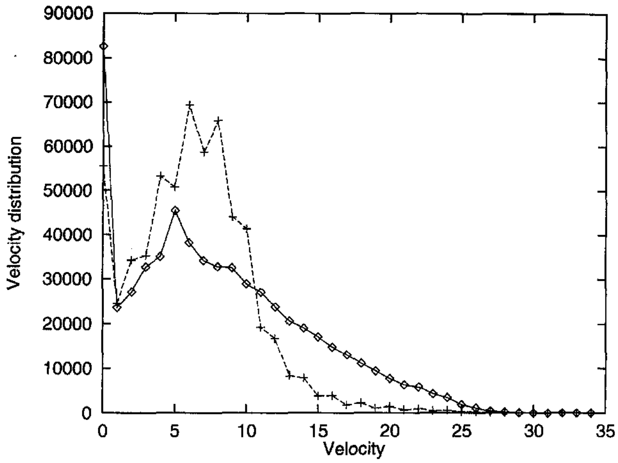

Fig. 9. Velocity distribution (in km/h) for fixed (diamonds) and adaptive (crosses) strategies. Density = 0.45

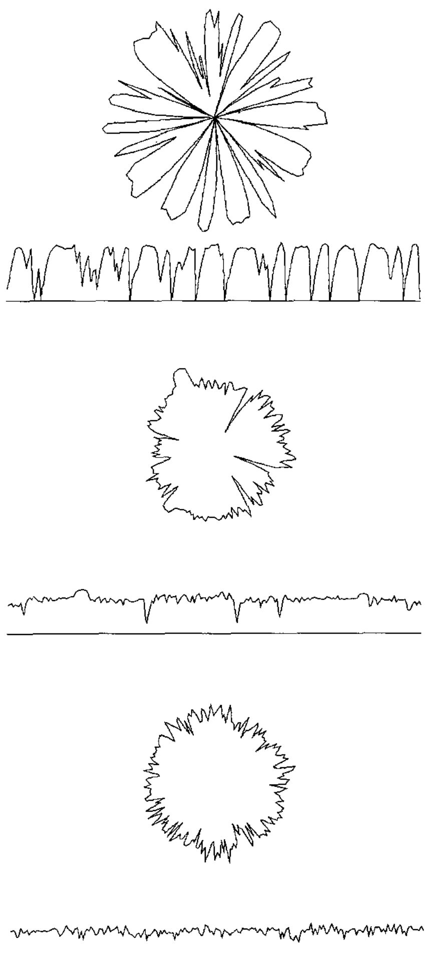

Fig. 10. Three snapshots of the velocity distribution along the freeway (lower curves) and in polar coordinates (upper closed curves). The top picture corresponds to fixed strategies. The middle picture is taken shortly after the strategies are allowed to change. The lower picture corresponds to stationary adaptive strategies

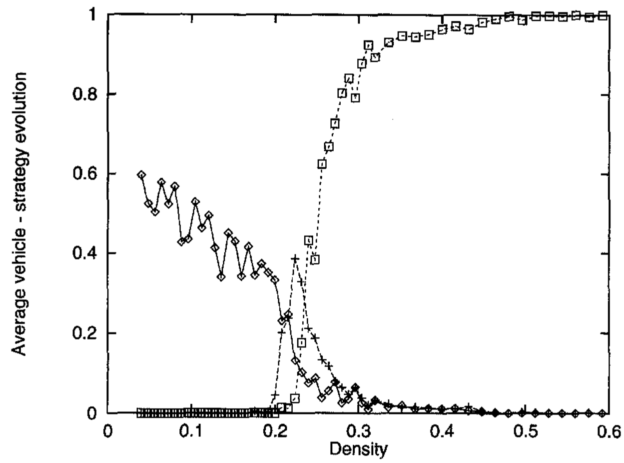

Fig. 11. Distribution of strategies for slow cars. Diamonds, crosses, and squares represent strategies 1, 3, and 5, respectively

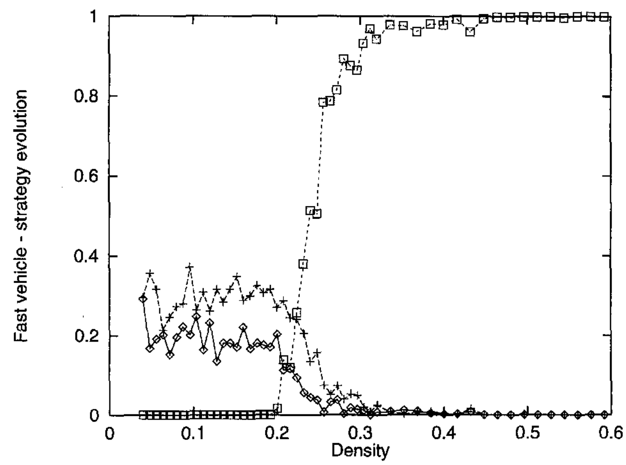

Fig. 12. Distribution of strategies for average cars. Diamonds, crosses, and squares represent strategies 1, 3, and 5, respectively

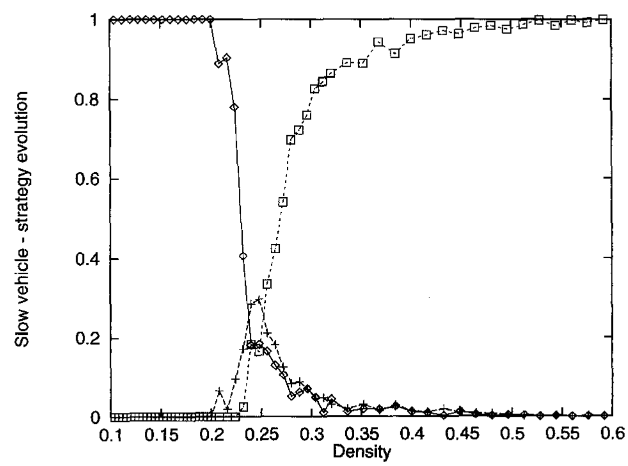

Fig. 13. Distribution of strategies for fast cars. Diamonds, crosses, and squares represent strategies 1, 3, and 5, respectively

around the average velocity. The flow is becoming more constant, as it can be also visually seen in Fig. 1.

Another way of looking at the velocity distribution is by taking a snapshot of the cars' velocity along the highway. This is shown in Fig. 10 both along the x-direction (lower graph) and in polar coordinates (traffic-jam flowers). The three snapshots represent the same process as seen in Fig. 1: the upper picture shows a fixed strategy stationary highway. In the middle we see a snapshot taken shortly after strategy dynamics has been switched on. The lower part is a snapshot of the stationary flow with adaptive strategies, respectively.

Figures 11–13 show the distribution of the strategies for the three different type of cars. From these figures it is clear that on the stable traffic phase slow cars adopt to fast driving strategies, while fast cars are forced to drive more defensively. On the high density phase, all cars adopt a defensive driving strategy. The large fluctuations seen close to the critical density indicate that here the strategy-dynamics is unstable.

4. Discussion and conclusions

There are many theoretical aspects which make the problem of traffic on a single lane rather interesting. Consider first the case of small densities. In this case, the flow is determined basically by the slowest cars. Behind each slow car a column of faster cars builds up, following more or less closely (within or a bit above D_s) the leading vehicle. Hence, at low densities the flow is determined by independent car columns, whose average velocity and length can be calculated without difficulty from the initial conditions. The main point is that the mean distance between the different columns is large enough so that the effect of stochastic breaking cannot propagate from one column to another. This explains the linear form of the basic flow-density diagram seen in Fig. 2. The flow is stable against perturbations. As the density increases, the car columns come close enough one to another so that the effect of stochastic breaking will influence the next column. When such an effect propagates across the whole freeway, density waves may appear. In hydrodynamical terms, the density modes are fully damped on the stable phase but start to oscillate in the 'unstable' traffic phase. The stationary distribution of vehicles (seen for example in Fig. 1) is rather similar to the walls seen in the context of commensurate-incommensurate phase transitions or surface steps. Hence, similar methods can be borrowed from equilibrium statistical mechanics to describe this stationary state far from equilibrium.

At very high densities the vehicles are basically standing: although in prinicple it is possible that the cars move in parallel with some velocity, the effect of random breaking will force at least one car to stop. Hence, a standing highway is an absorbing state of the dynamics. Motion happens through the drift of 'holes' (small free vehicle-vehicle distances). A description of the traffic in this regime is hence possible by considering the drift of independent 'holes'.

The above arguments can be made more precise: this is, however, beyond the scope of this article. Another interesting line of thought is the use of macroscopic hydrodynamic balance equations for mass, momentum, and energy transport. The rich class of phenomena encountered on the rather similar problem of one-dimensional compressible flow through a pipe [30] might be partially realized also on the freeways. Another task is to extend standard theoretical physical methods to include 'clever particles', which can exert themselves 'judgement' and act according to some appropriate strategies. Recent attempts at quantifying competition and cooperation processes between intelligent 'agents' [35, 36] did produce some highly interesting results.

Although our simulation reproduces *quantitatively* many aspects of Autobahn traffic, we are fully aware of its limitation and must warn from undue generalizations. There are some aspects of our simulation, however, which might be valid beyond the frame of a toy model. It seems obvious that adapting its own driving style to the local traffic conditions will improve the safety. Less obvious is the existence of a 'dangerous' density zone close to the maximal traffic flow, where a surge in the number of accidents can be observed. We suspect that this phenomenon is intrinsically related to a phase transition in the strategy dynamics. Hence, the region of maximal traffic flow remains a dangerous zone, no matter what (local) strategy we follow. Another general observation is that the crisp cusp predicted by fixed-strategy simulations and cellular automata theories [14, 23, 22] is rounded in the presence of adaptable strategies. The additional freedom of changing lanes on a many-lane highway enhances further this rounding effect, as seen in experimental measurements [31, 32] (Figs. 3, 4). As expected, slightly more defensive driving will help homogenize the traffic flow and many traffic jams will disappear. This involves less fuel consumption, *much* less accidents but NOT a shorter average traveling time.

Finally, let us point out that the purpose of this article was certainly not to teach the traffic engineers how to simulate traffic. We are fully aware that more realistic and complex models are simulated in Karlsruhe, Los Alamos, and elsewhere. However, considering its simplicity and flexibility, this toy model performs surprisingly well when compared to real experimental data (note that all simulations were run on 486 PC's). Hence, simulation methods originating from statistical physics seem useful in understanding and controlling real-life problems. Last but not least, such simple models provide a fruitful didactic background to physics students, who are early confronted with collective effects emerging from the interactions of many degrees of freedom.

This work started in 1993 as a third semester Theorie Praktikum Project at the University of Oldenburg. We thank J. Hellmers and M. Eisermann for their contribution to the original project. We are grateful to B.A. Huberman, K. Nagel, and M. Schreckenberg for sending their work prior publication and for discussions. A copy of this article (amok.ps.Z) and of the simulation program executables (amok.tar.Z) can be retrieved anonymously via ftp from ftp.uni-oldenburg.de, directory/pub/billboard/amok.

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