Etude 13 Two Tales

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A League Table

We will start off by looking at B because we can tell that B can't have won a game (won = 0) because they haven't scored any goals and the game that they lost would have had to have a score of 5 - 0.

Team	Played	Won	Lost	Drawn	For	Against
A			1	0	1	2
B		0	1		0	5
C	2		1		7	4
D		1		0	2	2
E		2		0	5	2

(Red = new results, Blue = previous worked out results, Black = given results)

Next we can see that there are only 2 teams to have scored at least 5 goals, team C and team E. If team E had played B and won 5-0 they wouldn't have any more for goals to be able to win the other game that we know they won so E can't have played B. Therefore the only option for B is that they played against C. We can also tell that neither B nor C drew a game because each team can only play each other once and for a team to draw, the team that they play must draw as well.

Here is a table to keep track of the games and scores

Games Played	
B - 0	C - 5

Team	Played	Won	Lost	Drawn	For	Against
A			1	0	1	2
B		0	1	0	0	5
C	2		1	0	7	4
D		1		0	2	2
E		2		0	5	2

(Red = new results, Blue = previous worked out results, Black = given results)

With B's won, lost and drawn columns filled out, it means that they can't have played any more games so the total that they have played is 1. Knowing that C has only played 2 games we can also fill out their remaining cell with the 1 win against B.

Team	Played	Won	Lost	Drawn	For	Against
A			1	0	1	2
B	1	0	1	0	0	5
C	2	1	1	0	7	4
D		1		0	2	2
E		2		0	5	2

(Red = new results, Blue = previous worked out results, Black = given results)

The other game that C played we now know that they lost and that the score would have been 4 -2 against them and the only other team to have scored at least 4 goals is team E. Therefore we know that C definitely played E.

Updated games played

Games Played	
B - 0	C - 5
C - 2	E - 4

This leaves E with only 1 for goal and no against goals. This also means that we know E can't lose any more games and we can fill out their row fully with that information(played = 2, lost = 0).

Team	Played	Won	Lost	Drawn	For	Against
A			1	0	1	2
B	1	0	1	0	0	5
C	2	1	1	0	7	4
D		1		0	2	2
E	2	2	0	0	5	2

(Red = new results, Blue = previous worked out results, Black = given results)

We know B had no more against goals scored and E has just played C and so won't verse them again. This leaves the options for E's 1 - 0 game to be A or D. If E played A and won 1-0, that would leave A's goals being 1 for and 1 against but that would mean that they would

either have to lose another game (which they can't) or draw a game (which they also can't). This leaves the 1-0 game to be played by E against D.

Updated games played

Games Played	
B - 0	C - 5
C - 2	E - 4
D - 0	E - 1

This increases D's losses by 1 (lost += 1) and leaves their for and against goals being 2 for and 1 against. The only other team that they could play is team A, who happen to have 2 against and 1 for which matches perfectly with D's. This means that D had to have played A and won 2 - 1.

Updated games played

Games Played	
B - 0	C - 5
C - 2	E - 4
D - 0	E - 1
A - 1	D - 2

Knowing that there are no draws and specifically 0 - 0 draws which would be the only possible result given the goals scored in all the previous matches, it allows us to finish the table with D playing 2 and losing 1 (played = 2, losses = 1) and A playing 1 and winning none (played = 1, won = 0)

Team	Played	Won	Lost	Drawn	For	Against
A	1	0	1	0	1	2
B	1	0	1	0	0	5
C	2	1	1	0	7	4
D	2	1	1	0	2	2
E	2	2	0	0	5	2

(Red = new results, Blue = previous worked out results, Black = given results)

With the table now full, we can confidently say that the League Commissioner is correct.

Hats, Hats, Hats

Our constants:

- 6 hats: BBB WWW
- 5 people: A B C D E
- 5 Positions: (back) E D C B A (front), however to avoid confusion i will refer to the seats by number, (back) 5 4 3 2 1 (front)

Example positions with hats: (back) 5B 4B 3B 2W 1W (front)

First we will look at position 1. Now if you were in position 1 you would never know what colour hat you were wearing because the only clues that you could get are verbal ones of people knowing what hat they're wearing. This means that whoever was there would be quiet meaning that A or E would have had to have been there.

Here's a table so that we can keep track of who is where:

Position	Person
1	A or E
2	
3	
4	
5	

Knowing that there was a pause and no one immediately knew their hat colour, it means we can assume that there was an even amount of hats of each colour in front of position 5. This is because if the person in position 5 saw 3 hats of the same colour, they would immediately know that they were the opposite. Because of this, we can tell that the person in position 5 would never know and would stay quiet. This also means that it would have to be A or E sitting in position 5.

Position	Person
1	A or E
2	
3	
4	
5	A or E

Now that we know A and E are separately at either end, we can start thinking about the other positions. Taking into account the brief pause, being in position 4 we know now that 5 could see an even mix of hats in front of them and couldn't pick, if we can see at least 2 of the same colour in front of us then we know that we have to be the opposite and we would

know what hat we're wearing. But 2 people said they know at the same time after the pause, this means that in position 3 they could also see 2 people with the same colour and would know that 5 paused and so would know that their hat would be the opposite of the 2 in front of them e.g. (back) 5B 4B 3B 2W 1W (front). The people in positions 4 and 3 would have had to be C and D as they were both the ones to speak at the same time after the pause.

Position	Person
1	A or E
2	
3	C or D
4	C or D
5	A or E

Now the final position that we haven't looked at would be position 2. We know that B said they knew after the two that talked at the same time. And we can work out that this meant the B was in position 2 because for C and D both to say something after the pause then it would mean that the first 2 would have to be the same colour. With B taking this into account and being in position 2 they could see that they would have to be the same colour as position 1.

Position	Person
1	A or E
2	B
3	C or D
4	C or D
5	A or E

Now our second to last piece of information is that A's Hat was the same colour as the one that wasn't used. If A had been in position 5 then there would have been 3 of the same colour hats in front of them and so A would have immediately said something. This means that A must have been in position 1 and therefore E must have been in position 5.

Position	Person
1	A
2	B
3	C or D

4	C or D
5	E

Finally we know that not everyone was seated in their own labelled seats. Now if we look back at what seats I had assigned numbers you can see that the seating order is:

Position	Person
A	A
B	B
C	C or D
D	C or D
E	E

With A, B, and E all being in their own labelled seats that means that C and D would have to have been in each other's seats giving us the final ordering of:

Position	Person
A	A
B	B
C	D
D	C
E	E