Chapter 6: Relational Algebra and Relational Calculus

Last Lecture:

- SQL nested subqueries
- Correlated subqueries
- Updates in SQL
- Introduction to Relational Algebra

Today:

- More Relational Algebra
- Division Operator
- Null Values and Bags in Relational Algebra
- Relational Calculus

Formal Definition

- A basic expression in the relational algebra consists of either one of the following:
  - A relation in the database
  - A constant relation
- Let E_1 and E_2 be relational-algebra expressions; the following are all relational-algebra expressions:
  - $E_1 \cup E_2$
  - $E_1 - E_2$
  - $E_1 \times E_2$
  - $\sigma_p (E_1)$, P is a predicate on attributes in E_1
  - $\prod_s (E_1)$, S is a list consisting of some of the attributes in E_1
  - $\rho_x (E_1)$, x is the new name for the result of E_1

Additional Operations

We define additional operations that do not add any power to the

relational algebra, but that simplify common queries.

- Set intersection
- Natural join
- Assignment
- Outer join

Set-Intersection Operation

- Notation: $r \cap s$
- Defined as:
- $r \cap s = \{ t \mid t \in r \text{ and } t \in s \}$
- Assume:
  - r, s have the *same arity* (i.e., number of attributes)
  - attributes of r and s are compatible
- **Note: $r \cap s = r - (r - s)$**

Set-Intersection Operation – Example

- Relation *r*, *s*:

A	*B*
α	1
α	2
β	1

r

A	*B*
α	2
β	3

s

A	*B*
α	2

- $r \cap s$

Natural-Join Operation

- Notation: $r \bowtie s$
- Let r and s be relations on schemas R and S respectively. Then, $r \bowtie s$ is a relation on schema $R \cup S$ obtained as follows:
  - Consider each pair of tuples t_r from r and t_s from s.
  - If t_r and t_s have the same value on each of the attributes in $R \cap S$, add a tuple t to the result, where
    - t has the same value as t_r on r
    - t has the same value as t_s on s
- Example:

  $R = (A, B, C, D)$

  $S = (E, B, D)$

  - Result schema = (A, B, C, D, E)

Natural Join Example

Relations r, s:

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	ε

s

r ⋈ s

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

Natural Join and Theta Join

Find the names of all instructors in the Comp. Sci. department together with the course titles of all the courses that the instructors teach

$$\Pi_{name,\ title}(\sigma_{dept\_name=\text{“Comp. Sci.”}}(instructor \bowtie teaches \bowtie course))$$

Natural join is associative

$$(instructor \bowtie teaches) \bowtie course \equiv instructor \bowtie (teaches \bowtie course)$$

Natural join is commutative

$$instructor \bowtie teaches \equiv teaches \bowtie instructor$$

- The **theta join** operation $r \bowtie_{\theta} s$ is defined as

Natural join of *instructor* relation with *teaches* relation

ID	*name*	*dept_name*	*salary*	*course_id*	*sec_id*	*semester*	*year*
10101	Srinivasan	Comp. Sci.	65000	CS-101	1	Fall	2009
10101	Srinivasan	Comp. Sci.	65000	CS-315	1	Spring	2010
10101	Srinivasan	Comp. Sci.	65000	CS-347	1	Fall	2009
12121	Wu	Finance	90000	FIN-201	1	Spring	2010
15151	Mozart	Music	40000	MU-199	1	Spring	2010
22222	Einstein	Physics	95000	PHY-101	1	Fall	2009
32343	El Said	History	60000	HIS-351	1	Spring	2010
45565	Katz	Comp. Sci.	75000	CS-101	1	Spring	2010
45565	Katz	Comp. Sci.	75000	CS-319	1	Spring	2010
76766	Crick	Biology	72000	BIO-101	1	Summer	2009
76766	Crick	Biology	72000	BIO-301	1	Summer	2010
83821	Brandt	Comp. Sci.	92000	CS-190	1	Spring	2009
83821	Brandt	Comp. Sci.	92000	CS-190	2	Spring	2009
83821	Brandt	Comp. Sci.	92000	CS-319	2	Spring	2010
98345	Kim	Elec. Eng.	80000	EE-181	1	Spring	2009

name	*course_id*
Srinivasan	CS-101
Srinivasan	CS-315
Srinivasan	CS-347
Wu	FIN-201
Mozart	MU-199
Einstein	PHY-101
El Said	HIS-351
Katz	CS-101
Katz	CS-319
Crick	BIO-101
Crick	BIO-301
Brandt	CS-190
Brandt	CS-319
Kim	EE-181

Result of

$\Pi_{name,\ course\_id}$ (*instructor* ⋈ *teaches*)

name	*title*
Brandt	Game Design
Brandt	Image Processing
Katz	Image Processing
Katz	Intro. to Computer Science
Srinivasan	Intro. to Computer Science
Srinivasan	Robotics
Srinivasan	Database System Concepts

Result of

$\Pi_{name,\ title}\ (\sigma_{dept\_name=\text{“Comp. Sci”}}\ ((instructor \bowtie teaches) \bowtie course))$

Assignment Operation

- The assignment operation (←) provides a convenient way to express complex queries.
  - Write query as a sequential program consisting of
    - a series of assignments
    - followed by an expression whose value is displayed as a result of the query.
  - Assignment must always be made to a temporary relation variable.

$$\text{temp1} \leftarrow R \times S$$

$$\text{temp2} \leftarrow \sigma_{r.A1 = s.A1 \wedge r.A2 = s.A2 \wedge \ldots \wedge r.An = s.An}(\text{temp1})$$

$$\text{result} = \prod_{R \cup S}(\text{temp2})$$

Outer Join

- An extension of the join operation that avoids loss of information.
- Computes the join and then adds tuples form one relation that does not match tuples in the other relation to the result of the join.
- Uses *null* values:
  - *null* signifies that the value is unknown or does not exist
  - All comparisons involving *null* are (roughly speaking) **false** by definition.

Outer Join – Example

Relation *instructor1*

ID	*name*	*dept_name*
10101	Srinivasan	Comp. Sci.
12121	Wu	Finance
15151	Mozart	Music

Relation *teaches1*

ID	*course_id*
10101	CS-101
12121	FIN-201
76766	BIO-101

Outer Join – Example

Natural Join

instructor ⋈ *teaches*

ID	*name*	*dept_name*	*course_id*
10101	Srinivasan	Comp. Sci.	CS-101
12121	Wu	Finance	FIN-201

Left Outer Join

instructor ⟕ *teaches*

ID	*name*	*dept_name*	*course_id*
10101	Srinivasan	Comp. Sci.	CS-101
12121	Wu	Finance	FIN-201
15151	Mozart	Music	*null*

Outer Join – Example

Right Outer Join *instructor* ⟖ *teaches*

ID	*name*	*dept_name*	*course_id*
10101	Srinivasan	Comp. Sci.	CS-101
12121	Wu	Finance	FIN-201
76766	null	null	BIO-101

Full Outer Join *instructor* ⟗ *teaches*

ID	*name*	*dept_name*	*course_id*
10101	Srinivasan	Comp. Sci.	CS-101
12121	Wu	Finance	FIN-201
15151	Mozart	Music	*null*
76766	null	null	BIO-101

Instructor ⟕ teaches (left outer join)

ID	*name*	*dept_name*	*salary*	*course_id*	*sec_id*	*semester*	*year*
10101	Srinivasan	Comp. Sci.	65000	CS-101	1	Fall	2009
10101	Srinivasan	Comp. Sci.	65000	CS-315	1	Spring	2010
10101	Srinivasan	Comp. Sci.	65000	CS-347	1	Fall	2009
12121	Wu	Finance	90000	FIN-201	1	Spring	2010
15151	Mozart	Music	40000	MU-199	1	Spring	2010
22222	Einstein	Physics	95000	PHY-101	1	Fall	2009
32343	El Said	History	60000	HIS-351	1	Spring	2010
33456	Gold	Physics	87000	*null*	*null*	*null*	*null*
45565	Katz	Comp. Sci.	75000	CS-101	1	Spring	2010
45565	Katz	Comp. Sci.	75000	CS-319	1	Spring	2010
58583	Califieri	History	62000	*null*	*null*	*null*	*null*
76543	Singh	Finance	80000	*null*	*null*	*null*	*null*
76766	Crick	Biology	72000	BIO-101	1	Summer	2009
76766	Crick	Biology	72000	BIO-301	1	Summer	2010
83821	Brandt	Comp. Sci.	92000	CS-190	1	Spring	2009
83821	Brandt	Comp. Sci.	92000	CS-190	2	Spring	2009
83821	Brandt	Comp. Sci.	92000	CS-319	2	Spring	2010
98345	Kim	Elec. Eng.	80000	EE-181	1	Spring	2009

instructor ⟖ teaches (right outer join)

ID	*course_id*	*sec_id*	*semester*	*year*	*name*	*dept_name*	*salary*
10101	CS-101	1	Fall	2009	Srinivasan	Comp. Sci.	65000
10101	CS-315	1	Spring	2010	Srinivasan	Comp. Sci.	65000
10101	CS-347	1	Fall	2009	Srinivasan	Comp. Sci.	65000
12121	FIN-201	1	Spring	2010	Wu	Finance	90000
15151	MU-199	1	Spring	2010	Mozart	Music	40000
22222	PHY-101	1	Fall	2009	Einstein	Physics	95000
32343	HIS-351	1	Spring	2010	El Said	History	60000
33456	*null*	*null*	*null*	*null*	Gold	Physics	87000
45565	CS-101	1	Spring	2010	Katz	Comp. Sci.	75000
45565	CS-319	1	Spring	2010	Katz	Comp. Sci.	75000
58583	*null*	*null*	*null*	*null*	Califieri	History	62000
76543	*null*	*null*	*null*	*null*	Singh	Finance	80000
76766	BIO-101	1	Summer	2009	Crick	Biology	72000
76766	BIO-301	1	Summer	2010	Crick	Biology	72000
83821	CS-190	1	Spring	2009	Brandt	Comp. Sci.	92000
83821	CS-190	2	Spring	2009	Brandt	Comp. Sci.	92000
83821	CS-319	2	Spring	2010	Brandt	Comp. Sci.	92000
98345	EE-181	1	Spring	2009	Kim	Elec. Eng.	80000

Outer Join using Joins

Outer join can be expressed using basic operations

e.g. $r \supset\!\bowtie s$ can be written as

$$(r \bowtie s) \cup (r - \Pi_R(r \bowtie s) \times \{(null, \ldots, null)\}$$

Find out tuples in r but not in $r \bowtie s$

Division Operator

Example: let $r(ID, course\_id) = \prod_{ID,\ course\_id}(takes)$ and
$s(course\_id) = \prod_{course\_id}(\sigma_{dept\_name=\text{“Biology”}}(course))$

then r / s gives us students who have taken all courses in the Biology department

- Can write r / s as

$$temp1 \leftarrow \prod_{R\text{-}S}(r)$$

$$temp2 \leftarrow \prod_{R\text{-}S}((temp1 \times s) - \prod_{R\text{-}S,S}(r))$$

$$result = temp1 - temp2$$

All the tuples over R-S attributes

All possible combinations with r and s

All the tuples in r that have all the course ids will be removed

Why not R

Reorganize attribute

Division

Useful for expressing universal quantification:

*Find professors who have taught **all** courses.*

- Let *R* have 2 fields, *x* and *y*; *S* have only field *y*:
  - *R/S* contains all *x* tuples (professors) such that for *every* *y* tuple (course) in *S*, there is an *xy* tuple in *R*.

Examples of Division R/S

sno	pno
s1	p1
s1	p2
s1	p3
s1	p4
s2	p1
s2	p2
s3	p2
s4	p2
s4	p4

R

pno
p2

S1

sno
s1
s2
s3
s4

R/S1

pno
p2
p4

S2

sno
s1
s4

R/S2

pno
p1
p2
p4

S3

sno
s1

R/S3

Extended Relational-Algebra-Operations

- Generalized Projection
- Aggregate Functions

Generalized Projection

- Extends the projection operation by allowing arithmetic functions to be used in the projection list.

$$\Pi_{F_1, F_2, \ldots, F_n}(E)$$

- E is any relational-algebra expression
- Each of F_1, F_2, …, F_n are arithmetic expressions involving constants and attributes in the schema of E.
- Given relation *instructor(ID, name, dept_name,* salary) where salary is annual salary, get the same information but with monthly salary

$\Pi_{ID, name, dept\_name, salary/12}$ *(instructor)*

Aggregate Functions and Operations

- **Aggregation function** takes a collection of values and returns a single value as a result.

  avg: average value
  min: minimum value
  max: maximum value
  sum: sum of values
  count: number of values

- **Aggregate operation** in relational algebra

$$_{G_1,G_2,\ldots,G_n}\mathcal{G}_{F_1(A_1),F_2(A_2),\ldots,F_n(A_n)}(E)$$

  E is any relational-algebra expression
  - G_1, G_2 …, G_n is a list of attributes on which to group (can be empty)
  - Each F_i is an aggregate function
  - Each A_i is an attribute name

- Note: Some books/articles use γ instead of $\mathcal{G}$ (Calligraphic G)

Aggregate Operation – Example

- Relation *r*:

A	*B*	*C*
α	α	7
α	β	7
β	β	3
β	β	10

$\mathcal{G}_{\text{sum(c)}}(r)$

sum(*c*)
27

ID	*name*	*dept_name*	*salary*
76766	Crick	Biology	72000
45565	Katz	Comp. Sci.	75000
10101	Srinivasan	Comp. Sci.	65000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000
12121	Wu	Finance	90000
76543	Singh	Finance	80000
32343	El Said	History	60000
58583	Califieri	History	62000
15151	Mozart	Music	40000
33456	Gold	Physics	87000
22222	Einstein	Physics	95000

Tuples of the instructor relation grouped by the *dept_name* attr

Aggregate Operation – Example

- Find the average salary in each department

$$_{dept\_name}\mathcal{G}_{\textbf{avg}(salary)}\ (instructor)$$

ID	*name*	*dept_name*	*salary*
76766	Crick	Biology	72000
45565	Katz	Comp. Sci.	75000
10101	Srinivasan	Comp. Sci.	65000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000
12121	Wu	Finance	90000
76543	Singh	Finance	80000
32343	El Said	History	60000
58583	Califieri	History	62000
15151	Mozart	Music	40000
33456	Gold	Physics	87000
22222	Einstein	Physics	95000

dept_name	*avg_salary*
Biology	72000
Comp. Sci.	77333
Elec. Eng.	80000
Finance	85000
History	61000
Music	40000
Physics	91000

Aggregate Functions (Cont.)

Result of aggregation does not have a name

- Can use rename operation to give it a name
- For convenience, we permit renaming as part of aggregate operation

$$dept\_name \quad \mathcal{G}_{\textbf{avg}(salary)\ \textbf{as}\ avg\_sal}(instructor)$$

Multiset Relational Algebra

- Pure relational algebra removes all duplicates
  - e.g. after projection
- Multiset relational algebra retains duplicates, to match SQL semantics
  - SQL duplicate retention was initially for efficiency, but is now a feature
- Multiset relational algebra defined as follows
  - selection: has as many duplicates of a tuple as in the input, if the tuple satisfies the selection
  - projection: one tuple per input tuple, even if it is a duplicate
  - cross product: If there are *m* copies of *t1* in *r*, and *n* copies of *t2* in *s*, there are *m* x *n* copies of *t1.t2* in *r* x *s*
  - Other operators similarly defined
    - E.g. union: *m* + *n copies,* intersection: min(*m, n*) copies
      difference: min(0, *m* – *n*) copies

SQL and Relational Algebra

- **select** *A1, A2, .. An*
  from *r1, r2, …, rm*
  where P

  is equivalent to the following expression in multiset relational algebra

  $$\Pi_{A1, .., An} (\sigma_{P} (r1 \times r2 \times .. \times rm))$$

- **select** *A1, A2,* **sum***(A3)*
  from *r1, r2, …, rm*
  where P
  group by *A1, A2*

  is equivalent to the following expression in multiset relational algebra

  $$_{A1, A2}\mathcal{G}_{\textbf{sum}(A3)} (\sigma_{P} (r1 \times r2 \times .. \times rm)))$$

Next: Tuple Relational Calculus

Domain Relational Calculus

Safety of Expressions