EECS491-A1-yxs626

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0.1 EECS 491 Assignment 1

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1 Q1. Basic Probability

1.1 1.1

Prove

$$p(x, y|z) = p(x|z)p(y|x, z) \tag{1}$$

Proof:

$$p(x, y|z) = \frac{p(x, y, z)}{p(z)}$$

$$p(x, y, z) = p(y|x, z)p(x, z)$$

$$p(x, y|z) = \frac{p(y|x, z)p(x, z)}{p(z)}$$

$$\frac{p(x, z)}{p(z)} = p(x|z)$$

$$p(x, y|z) = p(y|x, z)p(x|z)$$

Therefore,

$$p(x, y|z) = p(x|z)p(y|x, z) \tag{2}$$

1.2 1.2

Prove

$$p(x|y, z) = \frac{p(y|x, z)p(x|z)}{p(y|z)} \tag{3}$$

Proof:

$$p(x|y, z) = \frac{p(x, y, z)}{p(y, z)}$$

$$p(x, y, z) = p(y|x, z)p(x, z)$$

$$p(x|y,z) = \frac{p(y|x,z)p(x,z)}{p(y,z)}$$
$$p(x,z) = p(x|z)p(z)$$
$$p(y,z) = p(y|z)p(z)$$

Therefore,

$$p(x|y,z) = \frac{p(y|x,z)p(x|z)p(z)}{p(y|z)p(z)} = \frac{p(y|x,z)p(x|z)}{p(y|z)}$$

2 Q2. Independence

2.1 2.1

Show that independence is not transitive, i.e. $abbcac$. Define a joint probability distribution $p(a,b,c)$ for which the previous expression holds and provide an interpretation.

Proof:

Let's define a probability distribution set as follow: $p(a) = 0.4$, $p(b) = 0.8$, $p(c) = 0.3$, while $p(ab) = 0.32$, $p(bc) = 0.24$, $p(ac) = 0.1$, $p(abc) = 0.12$.

According to the definition of proposition independence, x and y are independent if and only if $p(xy) = p(x) * p(y)$.

Therefore, by the defined joint probability distribution, $abbc$ since $p(ab) = 0.32 = p(a) * p(b)$ and $p(bc) = 0.24 = p(b) * p(c)$, whereas $a \not\perp c$ since $p(ac) = 0.1 \neq p(a) * p(c) = 0.12$.

Finally, we can also see that $p(abc) = 0.12 \neq p(a) * p(b) * p(c) = 0.96$, which further proves that this is not an independent set.

Therefore, we may conclude that independence is not transitive, i.e. $abbcac$.

2.2 2.2

Show that conditional independence does not imply marginal independence, i.e. $ab|cab$. Again provide an example.

Proof:

Let's define a conditional probability distribution set as follow: $p(a) = 0.4$, $p(b) = 0.8$, $p(ab) = 0.28$.

Also, $p(a|c) = 0.5$, $p(b|c) = 0.6$, $p(ab|c) = 0.3$.

By definition of conditional independence, $ab|c(a|c)(b|c)p(a|c) * p(b|c) = p(ab|c)$.

Therefore, by the defined conditional probability distribution, $ab|c$ since $p(a|c) * p(b|c) = 0.3 = p(ab|c)$.

However, $a \not\perp b$ since $p(ab) = 0.28 \neq p(a) * p(b) = 0.32$.

Therefore, we shall conclude that conditional independence does not imply marginal independence, i.e. $ab|cab$.

3 Q3. Inspector Clouseau re-revisited

3.1 3.1

Write a program to evaluate $p(B|K)$ in Example 1.3 in Barber. Write your code and choose your data representations so that it is easy to use it to solve the remaining questions. Show that it correctly computes the value in the example.