



$$p(a, b, c) = p(c|b)p(b|a)p(a)$$

Does $p(b|a, c) \stackrel{?}{=} p(b|c)$?

$$p(b|a, c) = \frac{p(a, b, c)}{p(a, c)} = \frac{p(c|b)p(b|a)p(a)}{\sum_b p(c|b)p(b|a)p(a)}$$

can pull out of sum

$$\Rightarrow \frac{p(c|b)p(b|a)p(a)}{p(a) \sum_b p(c|b)p(b|a)}$$

(Note: we can't just divide by num., b.c. b in num. is fixed by LHS)

$$p(b|c) = \frac{p(b, c)}{p(c)} = \frac{\sum_a p(a, b, c)}{\sum_{a, b} p(a, b, c)} = \frac{\sum_a p(c|b)p(b|c)p(a)}{\sum_{a, b} p(c|b)p(b|c)p(a)}$$

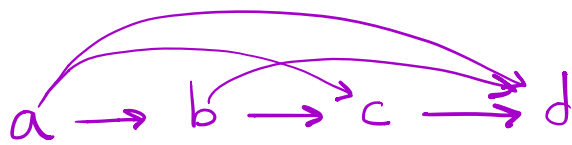
$\sum_a p(a) = 1$

$$= \frac{p(c|b)p(b|c)}{\sum_{a, b} p(c|b)p(b|c)p(a)}$$

\neq

Graphs represent independence structure / assumptions

Consider two fully connected DAGs:



("cascade graph")

$$p(a, b, c, d) = p(d|a, b, c) p(c|a, b) p(b|a)$$



$$p(a, b, c, d) = p(b|c, d, a) p(a|c, d) p(c|a)$$

Any joint distr. can be factored like this
by repeatedly applying $p(a, b) = p(a|b)p(b)$.

Contrast this with:



$$p(a, b, c, d) = p(d|c) p(c|b) p(b|a)$$

The DAG is a statement of the cond. indep. (or assumptions)

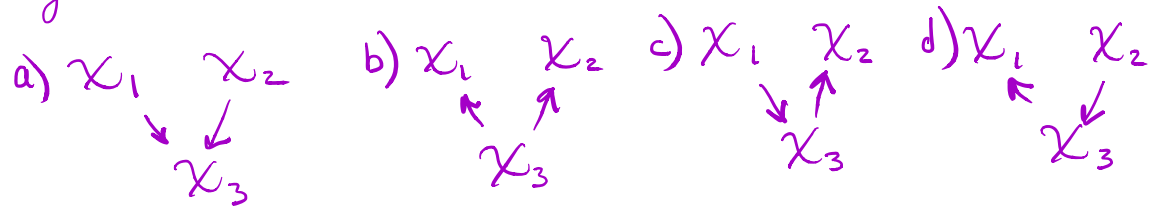
- Any joint distributed can be factored in several ways:

$$p(x_1, x_2, x_3) = p(x_{i_1} | x_{i_2}, x_{i_3}) p(x_{i_2} | x_{i_3}) p(x_{i_3})$$

Each represents $p(x_1, x_2, x_3)$ with no independ. assumptions.

(i_1, i_2, i_3) is any of 6 permutations of $(1, 2, 3)$

If we drop one connection (e.g. between X_1 & X_2), we get:



Do any of these represent the same distr.?
 That is: Are any of them equivalent?

graph c)

$$\begin{aligned}
 p(X_2|X_3) \underbrace{p(X_3|X_1)p(X_1)}_{p(X_3, X_1)} &= \frac{p(X_2, X_3)}{p(X_3)} \cdot p(X_3, X_1) = p(X_1|X_3) p(X_2, X_3) \\
 &= p(X_1|X_3) p(X_3|X_2) p(X_2) = \underbrace{p(X_1|X_3)}_{\text{graph d)}} \underbrace{p(X_2|X_3)p(X_3)}_{\text{graph b)}}
 \end{aligned}$$

Graphs b, c, and d represent same CI assumptions:


$$X_1 \perp\!\!\!\perp X_2 \mid X_3$$

Graph a) is different. $p(X_3|X_1, X_2) p(X_1) p(X_2)$
 can't be transformed into the others.

Keep in mind:

$a \rightarrow b$ says b is dep on a)
in general but we could have
 (define) $p(b|a) = p(a) \Rightarrow a \perp\!\!\!\perp b$

Barber hg 3.5

 $p(a, b, c, d) = p(d|a) p(c|a, b) p(b) p(a)$

$c \perp\!\!\!\perp d | a$?

Have to show: $p(c, d | a) = p(c|a) p(d|a)$

$$\begin{aligned} p(c, d | a) &= \frac{1}{p(a)} \sum_b p(d|a) p(c|a, b) p(b) p(a) \\ &= \frac{\overset{\textcircled{1}}{p(d|a) p(a)}}{\cancel{p(a)}} \sum_b p(c|a, b) p(b) \end{aligned}$$

$$\begin{aligned} p(c|a) &= \frac{1}{p(a)} \sum_{b, d} p(d|a) p(c|a, b) p(b) p(a) \\ &\quad \text{(rearranging)} \\ &= \frac{\cancel{p(a)}}{\cancel{p(a)}} \sum_b p(c|a, b) p(b) \overset{\textcircled{2}}{\sum_d p(d|a)} \end{aligned}$$

$$\Rightarrow p(c, d | a) = p(c|a) p(d|a)$$