ACTIVITY 2 (PART 2 OF 2) – PROPERTIES AND APPLICATIONS OF THE 2D FOURIER TRANSFORM

Introduction

The Fourier Transform (FT) of a signal gives the spatial frequency distribution of that signal – a sort of histogram of spatial frequencies which make up the signal. Unlike ordinary histograms, however, FT's are reversible, meaning you can recover the original image from the FT by using inverse FT.   In this second part of Activity2, we will explore more properties and applications of the Fourier Transform.

2.2.1Rotation Property of the FT

In Unique to the 2D FT (as compared to the 1D FT) is the fact that rotation of the sinusoids result to the rotation of their FT's.

1. Create a 2D sinusoid using MATLAB in the X direction (similar to a corrugated roof).

```
nx = 100; ny = 100;
x = linspace(-1,1,nx);
y = linspace(-1,1,ny);
[X,Y] = meshgrid(x,y);
f = 4 %frequency – you may change this later.
Z = sin(2*pi*f*X);
mesh(Z);
```

2. Rotate the sinusoid and take its FFT (be sure to FFTshift!). Discuss what happens to its FT.

```
theta = 30;
Z = sin(2*pi*f*(Y*sin(theta) + X*cos(theta)));
```

3. Create a pattern which is a combination of sinusoids in X and Y and observe its FT. For example, the code below is a product of two corrugated roofs, one running in the X-direction, the other in Y.

```
Z = sin(2*pi*f*X).*sin(2*pi*f*Y);
```

4. Add several rotated sinusoids of different frequencies to the pattern in 3 and observe the FT.

Filtering in Fourier Space

Unwanted repetitive patterns in an image can be removed by masking their frequencies in the Fourier domain. Alternatively, desired frequencies in the image may also be enhanced.

We can create filter masks to block out unwanted frequencies. In creating a filter mask, careful consideration of the convolution theorem is necessary. Remember that:

1. The FT of a convolution of two functions f and g in space is the product of the two functions' FT F and G, i.e.,

$$f ** g = FG .$$

2. The convolution of a dirac delta at xo,yo and a function f(x,y) results in a replication of f(x,y) in the location of the dirac delta, i.e.,

$$\int_{-\infty}^{+\infty} \delta(x - x_o - x',\, y - y_o - y') f(x', y') dx' dy' \;=\; f(x - x_o,\, y - y_o)$$

2.2.2 Application: Canvas Weave Modeling and Removal

Suppose we want to investigate the brush strokes of a painter. However, the texture of the canvas obscures our view of the brushstrokes. Let’s use FFT to remove the canvas weave.

1. Open the image **185-8526.jpg** and convert into a grayscale image. To enhance the detection of repeating patterns we first subtract the mean grayscale from the image. This will remove the DC bias of intensity images. The image is from a painting by Dr. Vincent Daria.

2. Take the FT of the mean-subtracted image and take note of the symmetric peaks. These are the sinusoids that make up the canvas weave image, similar to the rotation property of sinusoids we did earlier.
3. Manually create a filter mask in the FT space to remove the canvas weave patterns. This is a matrix which is the same size as the image. The filter is all 1's but are zero at the locations of the sinusoidal peaks. FFT shift this filter and multiply this to the complex FT of the Red, Green and Blue channels of the original image (not mean subtracted). The filter essentially "erases" the peaks . Take the inverse FT of the filtered FT and overlay the filtered R,G,B images . Did the canvas weave vanish enough that the paint and brushstrokes remain?
4. Invert the filter mask (0's become 1's and vice versa) and take the inverse Fourier transform. Observe the generated modulus image. Is it close to the appearance of the canvas weave?

Extra Challenge 1

Look for a material that has repeating texture and on top of it is a print, for example, ecobags with supermarket logos, t-shirt with a brand, etc. Capture an image of the material with print or logo and filter out the texture. The resulting image should just be the print or logo.

Extra Challenge 2

Kaketsugi is the Japanese are of invisible mending. To repair a hole in a garment, a piece of fabric from the same garment is cut and patched onto the hole by painstakingly weaving the fabric into the hole following the weave pattern of the cloth. Watch this youtube video to appreciate the art. Practitioners study the weaving pattern of different fabrics to inform the excution of their craft. Take a close-up image of a piece of fabric and take its FT. Filter out everything except the peaks corresponding to the weave patterns. Take the inverse FT of the filtered image. Does it look like an enhanced version of the weave pattern?

2.2.3 Convolution Theorem Redux

1. Create a binary image of two dots (one pixel each) along the x-axis symmetric about center. Take the FT and display the modulus.
2. Replace the dots with circles of some radius. Discuss what you observe in the FT modulus as you vary the radius.
3. Replace the dots with squares of some width. Discuss what you observe in the FT modulus as you vary the width.
5. Create a 200×200 array of zeros. Put 10 1's in random locations in the array. These ones will approximate dirac deltas. Call this array **A**. Create an arbitrary 9×9 pattern, call it **d**. Convolve **A** and **d**. What do you observe?

6. Create another 200×200 array of zeros but this time put equally spaced 1's along the x- and y-axis in the array. Get the FT and display the modulus. Change the spacing the 1's and repeat. Explain what you observe.

2.2.4 Fingerprints : Ridge Enhancement

1. Prepare an image of your own fingerprint in grayscale. You may do this by taking a picture of your stamped-ink fingerprint on paper. If you fail to prepare your own fingerprint, download grayscale images from the web. Make sure the image is NOT YET BINARIZED like the figure below. Remember to cite your sources, like Examples of different classes of fingerprints. (a) right loop (b) whorl... | Download Scientific Diagram (researchgate.net).

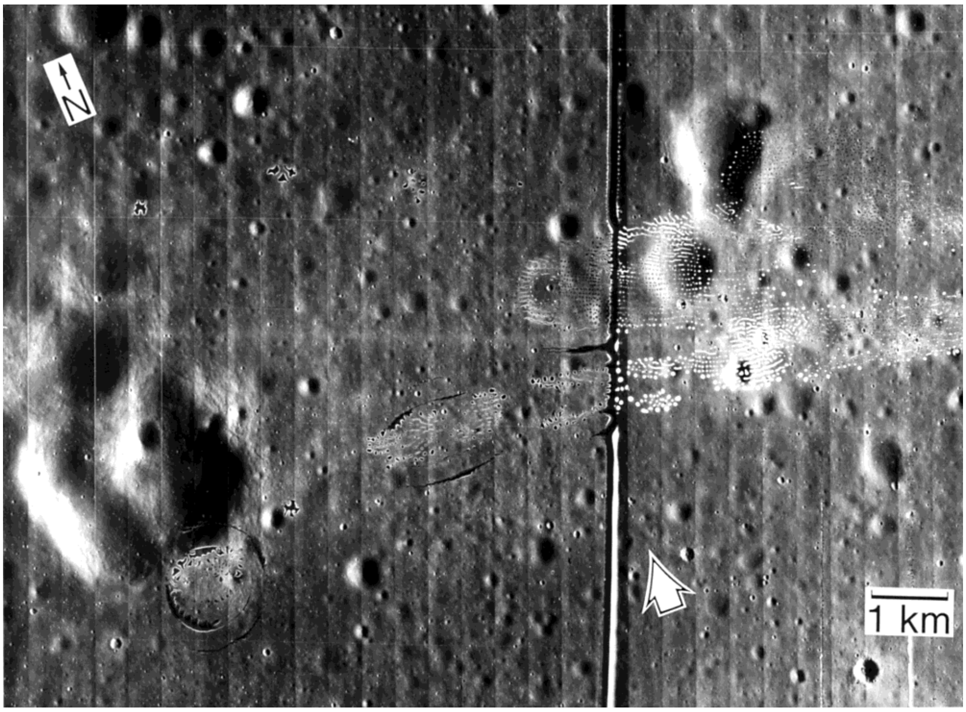
2. Open ithe image as grayscale and mean-center the grayvalues of the image.
3. Obtain the FT of the mean-centered grayscale image and investigate where the frequencies of the fingerprint ridges lie. **Tip:** The modulus of the FT image might span several orders of magnitude sp use the log scale to display the FT image. Example :

```
imagesc(log(abs(fftshift(FT_A))));
```

4. Explain why the FT of fingerprints look the way they do.

2.2.5 Lunar Landing Scanned Pictures : Line removal

1. Download the image below from the website: 5. Apollo 11 site: High resolution vertical view (usra.edu)



" The two groups of irregularly shaped craters north and west of the landing site are secondaries from Sabine Crater. This view was obtained by the unmanned Lunar Orbiter V spacecraft in 1967 prior to the Apollo missions to the Moon. The black and white film was automatically developed onboard the spacecraft and subsequently digitized for transmission to Earth. The regularly spaced vertical lines are the result of combining individually digitized 'framelets' to make a composite photograph and the irregularly-shaped bright and dark spots are due to nonuniform film development. [NASA Lunar Orbiter photograph]"

2. Remove the vertical lines in the image by filtering in the Fourier Domain.

