A6 – MORPHOLOGICAL OPERATIONS

Introduction

Morphology refers to shape or structure. In image processing, classical morphological operations are treatments done on binary images, in particular, aggregates of 1's or 0's, that form shapes. Morphological operations act on these shapes to improve them for further processing or to extract information from the. All morphological operations affect the shape of the image in some way, for example, the shapes may be expanded, thinned, internal holes could be closed, disconnected blobs can be joined

Morphological operations make use of **Set Theory.** We review a few basic definitions. Let A be a set in 2-D integer space Z^2. The elements of Z^2 in our case are the x-y location of pixels in the image. If a = (x,y) is an *element* of A then

$$a \in A \, . \tag{1}$$

See Illustration 1. If b is *not an element* of A we denote the fact as

$$b \notin A \, . \tag{2}$$

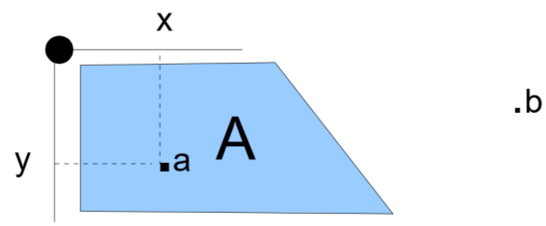

Illustration 1: The point a is an element of the set A while b is not an element of A.

If A is a *subset* of another set B then we write

$$A \subset B \tag{3}$$

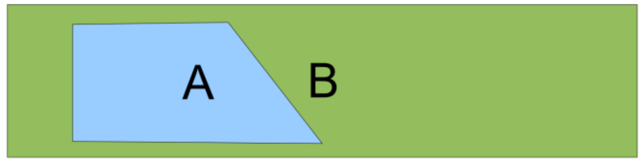

Illustration 2: Set A is a subset of Set B.

The *union* of two sets is the set of all elements that belong to either A or C and the set is written as

$$A \cup C. \tag{4}$$

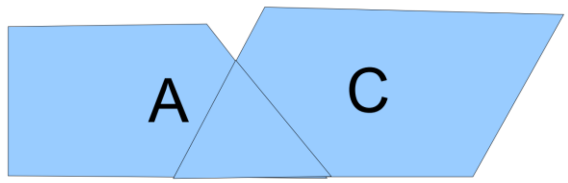

Illustration 3: The union of Sets A and C are the points in blue.

The *intersection* of two sets is the set of all elements that are both in A and C only and is denoted as

$$A \cap C\ . \tag{5}$$

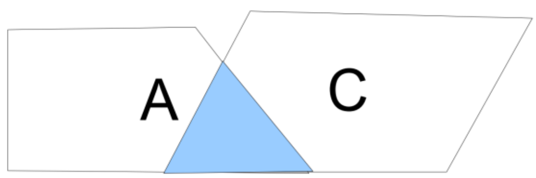

Illustration 4: The intersection of Sets A and C are the points in blue.

Two sets are said to be mutually exclusive if they have no common elements. In this case their intersection is the *empty set*, Ø. This fact is expressed as,

$$A \cap D = \emptyset\ . \tag{6}$$

Illustration 5: The complement of Set A are the points in Set A^C.

The *complement* of a set A is the set of elements not contained in A:

$$A^c = \{w : w \notin A\}. \tag{7}$$

The *difference* of two sets A and C denoted by A-C is defined as all elements of A excluding all elements of C,

$$A - C = \{w: w \in A, w \notin C\} = A \cap C^c \tag{8}$$

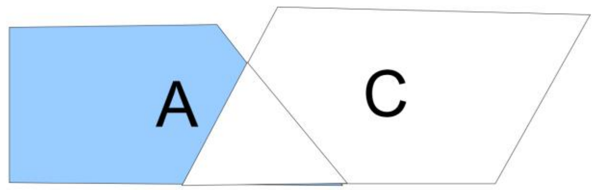

Illustration 6: The difference between A and C is the set in blue.

The *reflection* of set A is defined as

$$\hat{A} = \{w: w = -a, fora \in A\} \tag{9}$$

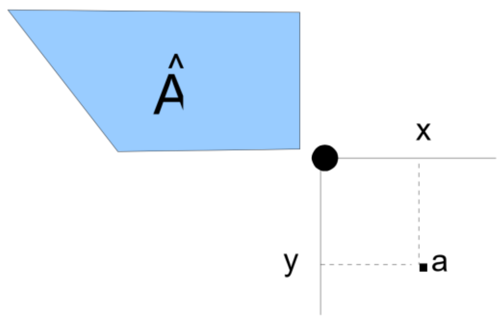

Illustration 7: The reflection of A is shown in blue.

The *translation* of set A by point z = (z_1, z_2) denoted $(A)_z$ is defined as

$$(A)_z = \{c: c = a + z, fora \in A\}\,. \tag{10}$$

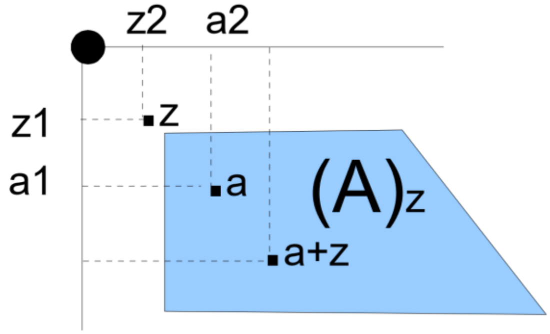

Illustration 8: The translation of A are the set of points in A translated by z.

Logical operations are complementary to morphological operations on binary images. The NOT operator is the same as complement. The AND is equal to intersection. **Question**: What is the XOR operator equal to? How about [NOT (A)] AND B?

Dilation and Erosion

The *dilation* of A by B denoted by A dilation B is defined as

$$A \oplus B = \{ z \mid (\hat{B})_z \cap A \neq \emptyset \} \tag{11}$$

This involves all z's which are translations of a reflected B that when intersected with A is not the empty set. B is known as a structuring element. The effect of a dilation is to expand or elongate A in the shape of B. Notice there is a dot in the middle of the structuring element below. It defines the origin of positions in B. It can be placed anywhere in B.

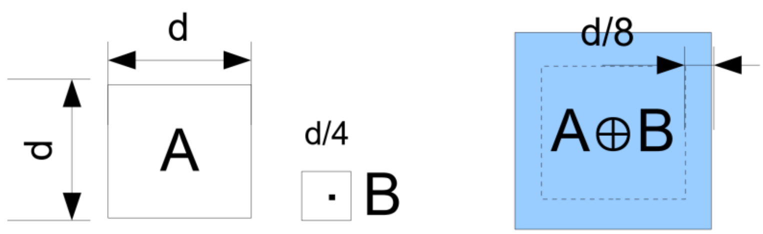

Illustration 9: The dilation of A by the structuring element B results in an expanded version of A.

The *erosion* operator is defined as

$$A \ominus B = \{z \mid (B)_z \subseteq A \}. \tag{12}$$

The erosion of A by B is the set of all points z such that B translated by z is contained in A. The effect of erosion is to reduce the image by the shape of B.

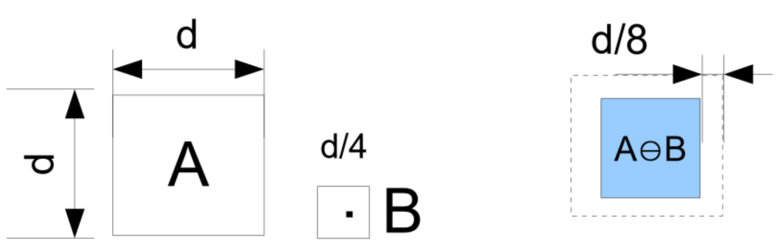

Illustration 10: The erosion of A by the structuring element B results in a reduced version of A.

Erosion and dilation are duals of each other and the following set operations are true:

$$(A \ominus B)^c = A^c \oplus B \tag{13}$$

Things to remember about Morphological operations

1. Morphological operations can be combined in any sequence. For example, morphological **closing** operator is a dilation followed by an erosion. It can "close" small holes in a binary image while preserving its size. The morphological **opening** operator on the other hand is an erosion followed by a dilation. It can separate touching regions.

2. Morphological operations can be repeated several times. For example, repeated erosions can cause the binarized blobs to reduce in size.

3. You must design your structuring element properly to achieve the results you want.

4. Finding the proper combination of morphological operations and structuring element to perform a task (e.g. remove artifacts, separate touching blobs, etc.) is sometimes an art.

Activity 6.1

The following activity will increase your understanding of set theory and morphological operations. This is the only activity where you need to hand-draw your output. You will need to Predict, Observe, and Explain.

1. On a piece of graphing paper draw the following shapes:
   a. A 5x5 square
   b. A hollow 10 x10 square, 2 boxes thick
   c. A plus sign, one box thick, 5 boxes along each line
   d. A dumbbell – two 5x5 squares connected by a 3x1 line
2. Draw the resulting image if the following structuring elements are used to (a) ERODE and (B) dilate the images above. Scan your hand-drawn predictions and paste in your slide reports.
   a. 2x2 ones. Origin is the upper left box.
   b. 2x1 ones. Origin is leftmost box.
   c. A 5x5 square. Origin is at the center of the pattern.
3. Validate your predictions in MATLAB. Generate your test images as an array. Use the **strel** function to create the structuring elements. Use the functions, **imerode** and **imdilate** to perform erosion and dilation.
4. Comment on how well you predicted the results of the morphological operation.

FOR MORE INFORMATION ON MORPHOLOGICAL OPERATIONS, VISIT:

https://www.mathworks.com/help/images/morphological-filtering.html?s_tid=CRUX_lftnav

Activity 6.2

Morphological operations are ideal for cleaning up binary images in preparation for feature extraction. After segmentation, it may happen that apart from the regions of interest, there may be several pixels that light up because they satisfy your thresholding conditions.

1. Open the image malaria(1).jpeg and convert to grayscale.

```
I = imread("malaria(1).jpeg");
Igray = rgb2gray(I);
figure(1); subplot(1,2,1); imshow(I);
subplot(1,2,2); imshow(Igray);
```

2. Display its grayscale histogram and decide on a threshold to isolate the cells.

```
figure (2); imhist (Igray);
BW= and(Igray < 164, Igray>100); %you may change these thresholds
figure (3); imshow(BW);
```

3. Try different morphological operations to remove the small "dusts". Here are several examples.

```
BW2 = bwmorph(BW,"majority");
figure(4); imshow(BW2);
SE = strel("disk",2);
figure(5);
BW3 = imopen(BW,SE);imshow(BW3);
figure(6);
BW4 = imclose(BW,SE); imshow(BW4);
```