ISE 5113: Assignment #1

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Dr. Charles Nicholson

Genwei Zhang

Question 1: Smullyan's Island Revisited (6 points)

Answer: (The truth table)

Gregor	Tywin	Catelyn	Gregor's statement	Tywin's statement	Catelyn's statement
T	T	T	F	F	F
T	T	F	F	F	F
T	F	T	F	F	F
T	F	F	T	F	F
F	T	T	F	F	F
F	T	F	T	F	F
F	F	T	T	F	T
F	F	F	F	T	T

From the above table, we can see that Gregor is telling the truth, while Tywin and Catelyn are lying.

Question 2: Wildlings and Humans (10 points)

Answer: (The truth table)

Arya	Sansa	Arya's statement	Sansa's statement
Sane human (T)	Sane wildling (F)	F	T
Sane human (T)	Insane wilding (T)	F	T
Insane human (F)	Sane wildling (F)	F	T
Insane human (F)	Insane wilding (T)	T	F
Sane wildling (F)	Sane human (T)	F	T
Insane wilding (T)	Sane human (T)	F	T
Sane wildling (F)	Insane human (F)	F	T
Insane wilding (T)	Insane human (F)	T	F

From the above table, we can see that Arya is the wildling.

Question 3: Working Capital Management

a) The decision variable is the amount of money chosen to invest onto a specific investment option at the beginning of each month.
There are 11 decision variables exist, namely:
Month 1: D16, D13, D11
Month 2: D23, D21
Month 3: D33, D31
Month 4: D43, D41
Month 5: D51
Month 6: D61

b) The objective is to maximizing the company's interest income at the end of month six.
Objective function: maximize month6: D16*1.035+D51*1.005+D43*1.021+13000

c) Constraints:

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s.t. start: D16+D11+D13=300000;
s.t. month1: D23+D21=D11*1.005-50000;
s.t. month2: D33+D31=D21*1.005+12000;
s.t. month3: D43+D41=D13*1.021+D31*1.005-23000;
s.t. month4: D51=D23*1.021+D41*1.005+20000;
s.t. month5: D61=D33*1.021+D51*1.005-41000;
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d) After using AMPL, the optimized investment strategy is:
At the beginning of month 1, invest $229,982 with 3-month option and 70,018 with 1-month option; at the beginning of month 2, invest $20,368 with 3-month option; at the beginning of month 3, invest $12,000 with 1-month option; at the beginning of month 4, invest $223,871 with 3-month option; and at the beginning of month 5, invest $40,796 with 1-month option. This gives the maximum income at the end of month 6, which is $282,572.72.

Question 4. Seasonal Demand

The best annual production plan solved by AMPL is:
Produce 1200 sandals in the first season and use the holding, 1600, from last year's fourth season to meet the demand. In the second season, produce 650 and hold 150, meet the demand 500. Third season, produce 1200 and hold 1250 (150 are from holding of the second season), meet the demand 100. The fourth season produce 1200 and meet the demand 850, while holding 1600 (1250 are from the third season). This way gives the minimum holding cost $450 annually.

Question 5. Golden Canning Co

(a)
Since the 'A' tomatoes averaged 9 points per pound and 'B' averaged 5 points per pound, but requirement to produce the whole tomatoes is minimum averaged 8. So we can also use some 'B' grade tomato since 'A' grade tomato is above 8. Let's assume we need x pounds 'B' tomato:
So, we have (3000000*20%*9+ x*5)/ (3000000*0.2+ x)>=8
Solve a, we get: x<=200000
Therefore, the maximum tomato we can use from 'B' equals to 200000, and in total the whole tomato production is limited to 800000 pounds.

(b)

Bollman computed the tomato cost considering the facts to meeting the demand, no exceed of the availability and also satisfying the requirements of rating.
For the whole tomato: she did the following calculation:
(600000*$0.0932+200000*$0.0518)/800000*18=$1.4913≈$1.49
For the tomato juice, she did:
(250000*$0.0932+750000*$0.0518)/1000000*20=$1.243≈$1.24
For the tomato paste, since it can be entirely produced from B, so:
$0.0518*25=$1.295 ≈1.30

Once she calculated the marginal profit, she was thinking to maximize the production of tomato paste, as it is most profitable. So she did:
80000*25=2000000
And then she used the rest of the B, 400000, and all A, 600000, to produce juice to meet the demand:
(400000+600000)/20=50000.

The holes in her reasoning are that:

1. It is incorrect the way that how she calculated the unit price for A tomato and B tomato (9.32 cents for A tomato and 5.18 cents for B tomato) because she misused the combination of the averaged 6 cents per pound and the rating points.
2. Although the unit price holds, when she actually calculated the tomato cost per case for the tomato juice, she did not use the same A/B tomato ratio for computing the overall profit, this inconsistency may also result in her incorrect marginal profit calculations.

(c)
The LP formulation of this problem:
Decision variables:
Wa: the quantity of whole tomato of grade A;
Wb: the quantity of whole tomato of grade B;
Ja: the quantity of tomato juice of grade A;
Jb: the quantity of tomato juice of grade B;
Pa: the quantity of tomato paste of grade A;
Pb: the quantity of tomato paste of grade B.

i). Based on the AMPL result, the company should produce whole tomato 700,000lb, tomato juice 300,000lb and tomato paste 2,000,000lb.

ii) Contribution=selling-cost
= 700000/18*1.48+300000/20*1.32+2000000/25*1.85-3000000*0.06
= $45,355.56

iii) No tomato left over.

iv)The average quality point count for whole tomato is (525000*9+175000*5)/700000=8; for juice is (75000*9+225000*5)/300000=6; and for tomato paste it equals to the average quality point of B grade, which is 5.

v) Function 'display limitA;' gives us 0.0903, which is the shadow price of the constraint (the availability of grade A tomato). Thus, the worth of one additional pound of A-grade tomato is 0.0903.

vi) Solve this problem in AMPL using limitA=680000, we get the new production plan: produce whole tomato 820,000lb, tomato juice 260,000lb and tomato paste still 2,000,000lb, with the total profit increased to $45,782.22. Therefore, we should purchase this extra 80,000 pounds of A-grade tomato at the offered price.

(d)
i)
If using Thomas' contribution figure, the net profit per case of producing whole tomato, tomato juice and paste are 0.12, -0.09 and 0.12, respectively.
Solving this in AMPL, we can get the product mix are: produce whole tomato 800,000lb, tomato juice 0lb and tomato paste still 2,000,000lb, but there are 200,000lb unused B-grade tomato. And the profit contribution is only $14,933.33.
If we also subtract the cost of unused B-grade tomato, about $12,000, the net profit would be only $2,933.33. So, it is less than my solution in part (c).
The difference comes from the initial calculation of the net profit per case.

ii)
If Bollman's profit figure were used, the net profit per case of producing whole tomato, tomato juice and paste would be -0.01, 0.08 and 0.55, respectively.
Solving this in AMPL, we can get the production plan: produce whole tomato 0lb, tomato juice 1,000,000lb and tomato paste still 2,000,000lb. And the profit contribution is $48,000, which is more than my solution in part (c).

iii)
From part (c) vi step, we know that the 'display limitA.up' gives us 1200000, indicating the upper limit of grade-A tomato is 1,200,000 lb. Currently we have 600,000 pounds of grade-A tomato, that means the maximum additional A-grade tomato at $0.085 per lb we should purchase is 1200000-600000=600000 pounds.
If purchase another 600,000 pounds of grade-A tomato, solve it in AMPL, we get the production plan is: 1,600,000 pounds of whole tomato, 0 pounds of tomato juice, and 2,000,000 pounds of tomato paste. And this gives us the optimal profit $48,555.56.