INSTITUTO POLITÉCNICO NACIONAL

Escuela Superior de Cómputo (ESCOM)

TEORÍA DE COMUNICACIONES Y SEÑALES.

GUÍA DEL 2$^{\underline{do}}$ PARCIAL

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3CV6

02 – JULIO – 2020

Sección I. Convolución Continua.

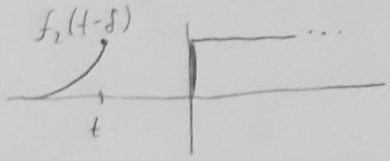
Problema 1. Calcular las siguientes integrales de convolución:

a) $u(t) * e^{-t} u(t)$; f_1

<u>Solución</u>

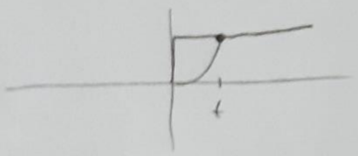
$$z(t) = \int_{-\infty}^{\infty} f_1(\delta) \cdot f_2(t-\delta)\, d\delta$$

Caso Ø



$$\int_{-\infty}^{\infty} f_1(\delta) \cdot f_2(t-\delta)\, d\delta = \emptyset$$

caso 1



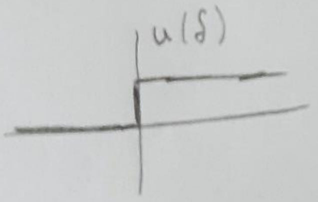
$$\int_0^t e^{t \cdot \delta}\, d\delta = \frac{1}{2t} e^{2t} - \frac{1}{t} \cdot e^{t} = \frac{e^{t}}{2t}\left[e^{t} - t\right]$$

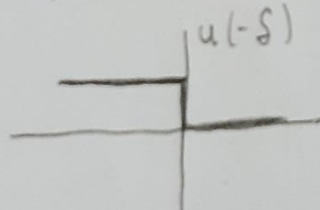
$t \geq 0$

b) $u(t) * u(t)$

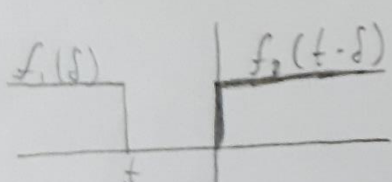
<u>solución</u>

$f_1(\delta) = u(\delta) \Rightarrow f_2(-\delta) = u(-\delta)$



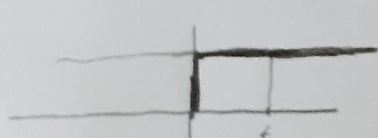


Caso 0: $t < 0$



$$z(t) = \int_{-\infty}^{\infty} f_1(\delta) \cdot f_2(t-\delta) \cdot d\delta = 0$$

Caso 1: $t \geq 0$



$$z(t) = \int_0^t f_1(\delta) \cdot f_2(t-\delta)\, d\delta$$

$$z(t) = \int_0^t 1\, d\delta = t$$

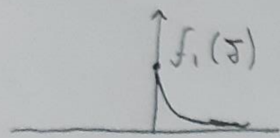
$\therefore t$ en $t \geq 0$

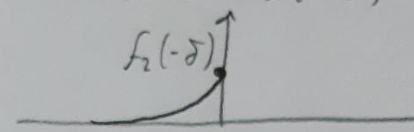
c) $e^{-t}u(t) * e^{-3t}u(t)$

Solución

$$f_1(t) = e^{-t}\cdot u(t)$$
$$f_2(t) = e^{-3t}u(t) \Rightarrow \int_{-\infty}^{\infty} f_1(\tau)\cdot f_2(t-\tau)\,d\tau$$

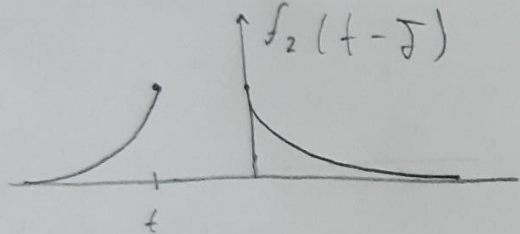
$$f_1(\tau) = e^{-\tau}\cdot u(\tau) \qquad f_2(-\tau) = e^{3\tau}\cdot u(-\tau)$$





$$f_2(t-\tau) = e^{3(t-\tau)}\cdot u(t-\tau)$$

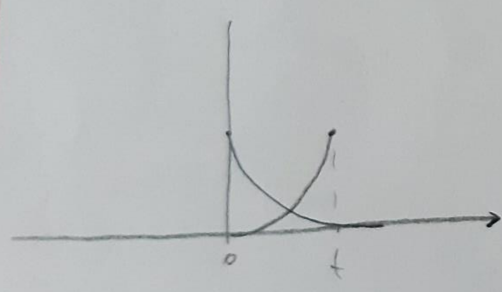
Caso 0:



$t<0$

$$\int_{-\infty}^{\infty} f_1(t)\cdot f_2(t-\tau)\,d\tau = 0$$

Caso 1:



$$\int_0^t f_1(t)\cdot f_2(t-\tau)\,d\tau = \int_0^t e^{3(t-\tau)}\cdot e^{-\tau}\,d\tau$$

$$-\int_0^t e^{3t-4\tau}\,d\tau = \frac{1}{3t-4t}\cdot e^{3t-4t} - \frac{1}{3t}e^{3t} =$$

$$= \frac{1}{-t}e^{-t} - \frac{1}{3t}\cdot e^{3t} \Rightarrow -\frac{1}{3t}\left[3e^{-t}+e^{3t}\right]$$

$t \geq 0$

j) $u(t) * t\,u(t)$

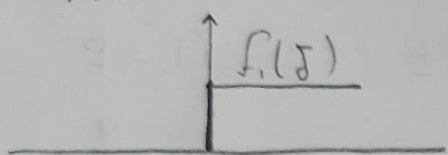
Solución

$f_1(t) = u(t)$

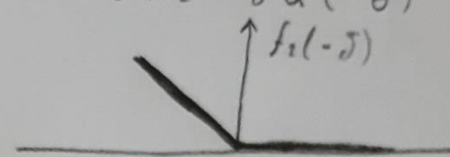
$f_2(t) = t\,u(t)$

$$z(t) = \int_{-\infty}^{\infty} f_1(\tau) \cdot f_2(t-\tau)\, d\tau$$

$f_1(\tau) = u(\tau)$

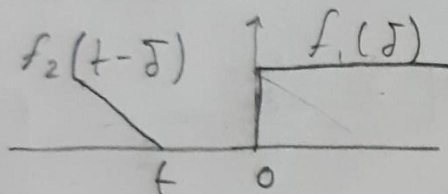


$f_2(-\tau) = -\tau\, u(-\tau)$



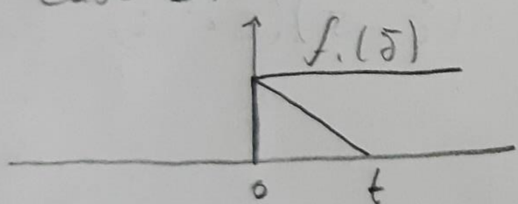
$f_2(t-\tau) = (t-\tau)\,u(t-\tau)$

Caso 0:



$$\int_{-\infty}^{\infty} f_1(\tau) \cdot f_2(t-\tau)\, d\tau = 0$$

Caso 1:



$$\int_{0}^{t} f_1(\tau) \cdot f_2(t-\tau)\, d\tau$$

$$= \int_0^t u(\tau) \cdot (t-\tau) \cdot u(t-\tau)\, d\tau = \int_0^t (t-\tau)\, d\tau$$

$$= \left(t\tau - \frac{\tau^2}{2}\right)\Big|_0^t = t^2 - \frac{t^2}{2} \Rightarrow \underline{\frac{t^2}{2}}, \quad t \geq 0$$

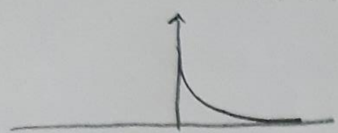
e) $e^{-t}u(t) * t\,u(t)$

$f_1(t) = e^{-t}u(t)$

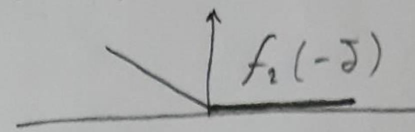
$f_2(t) = t \cdot u(t)$

$$z = \int_{-\infty}^{\infty} f_1(\tau)\cdot f_2(t-\tau)\,d\tau$$

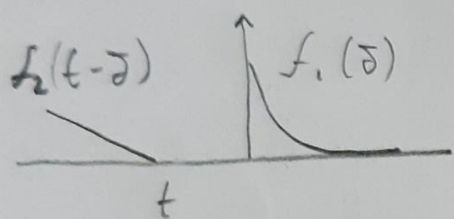
$f_1(\tau) = e^{-\tau}u(\tau)$



$f_2(-\tau) = -\tau\, u(-\tau)$

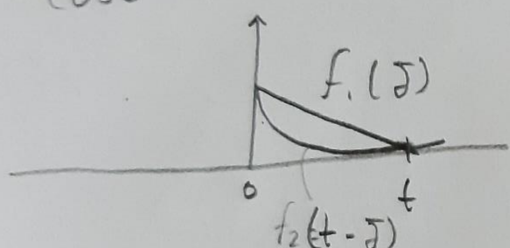


Caso 0:



$$\int_{-\infty}^{\infty} f_1(\tau)\cdot f_2(t-\tau)\,d\tau = 0 \qquad t<0$$

Caso 1:



$$= \int_0^t f_1(\tau)\cdot f_2(t-\tau)\,d\tau$$

$$= \int_0^t e^{-\tau}u(\tau)\cdot(t-\tau)\cdot u(t-\tau)\,d\tau$$

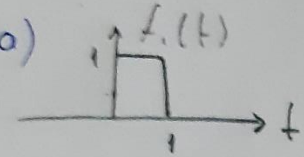
$$= -\int_0^t e^{-\tau}(\tau-t)\,d\tau = \left[\frac{1}{-1}(\tau-t)\cdot e^{-\tau} - \frac{1}{(-1)^2}e^{-\tau}\right]_0^t$$

$$= \left[-(\tau-t)\cdot e^{-\tau} - e^{-\tau}\right]_0^t = -e^{-t} - (t-1)$$

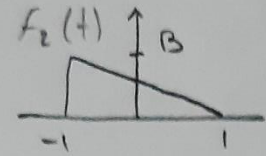
$$= -e^{-t} - t + 1 \qquad t \geq 0$$

Problema 2: Calcular $f_1(t) * f_2(t)$ para cada par de señales de la figura 1.

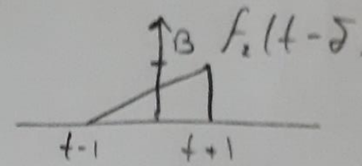
a)



;

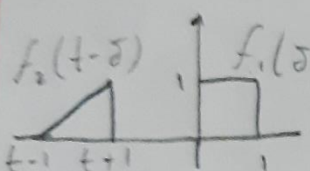


$\Rightarrow$



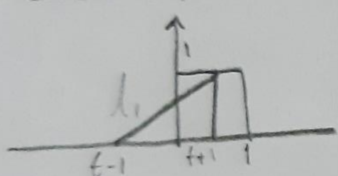
$$f_2(\tau) = \begin{cases} -\frac{B}{2}(\tau-1) \\ -1 \le \tau \le 1 \end{cases} \qquad f_2(-\tau) \begin{cases} -\frac{B}{2}(-\tau-1) \\ -1 \le \tau \le 1 \end{cases} \Rightarrow f_2(t-\tau) = \begin{cases} \frac{B}{2}[\tau+1-t] \\ t+1 \le \tau \le -1 \end{cases}$$

Caso 0:



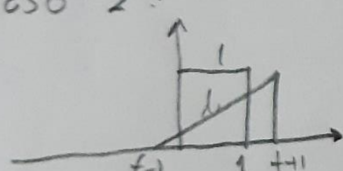
$$\int_{-\infty}^{\infty} f_1(\tau) \cdot f_2(t-\tau)\, d\tau = 0$$

caso 1:



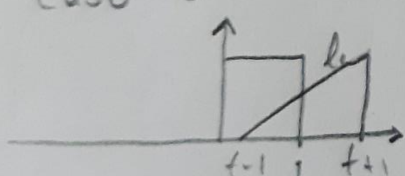
$$\int_0^{t+1} 1\, l_1\, d\tau \quad \begin{matrix} t+1>0 \\ t+1 \le 1 \end{matrix} \Rightarrow \underline{-1 < t \le 0}$$

Caso 2:



$$\int_0^{1} 1 l_1\, d\tau \quad \begin{matrix} t+1>1 \\ t-1 \le 0 \end{matrix} \Rightarrow 0 < t \le 1$$

Caso 3

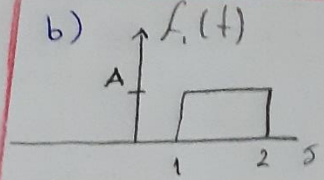


$$\int_{t-1}^{1} 1 \cdot l_1\, d\tau \Rightarrow \begin{matrix} t+1>1 \\ t-1 \le 1 \end{matrix} \quad \underline{0 < t \le 2}$$

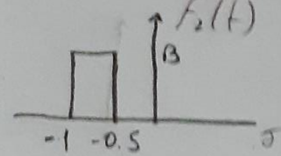
Entonces:

$$\underbrace{-\int_0^{t+1} \frac{B}{2}(\tau-1)\,d\tau}_{-1 < t \le 0} + \underbrace{\int_0^{1} -\frac{B}{2}(\tau-1)\,d\tau}_{0 < t \le 1} + \underbrace{\int -\frac{B}{2}(\tau-1)\,d\tau}_{0 < t \le 2}$$

b)

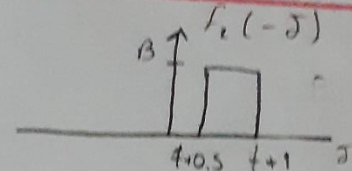


$A ; 1 < t \leq 2$



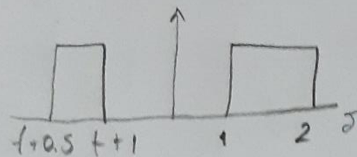
$B ; -1 < \tau < 0.5$

$\Rightarrow$



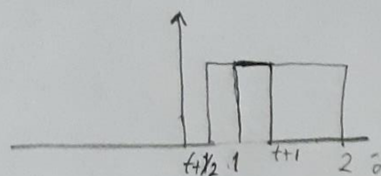
$B ; t+1 < \tau < 0.5$

Caso 0:



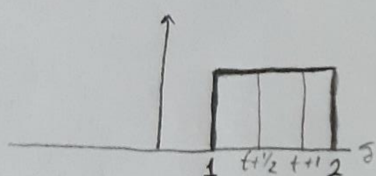
$$\int_{-\infty}^{\infty} f_1(\tau) \cdot f_2(t-\tau)\, d\tau = 0$$

Caso 1:



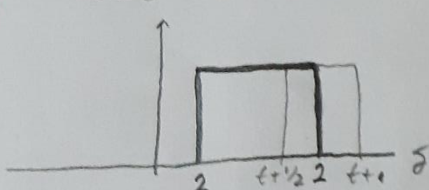
$$\int_{1}^{t+1} A \cdot B\, d\tau \Rightarrow \begin{array}{l} t+1 > 1 \\ t+\tfrac{1}{2} \leq 1 \end{array} \Rightarrow \underline{0 < t \leq 1}$$

Caso 2:



$$\int_{t+1/2}^{t+1} A \cdot B\, d\tau \quad \begin{array}{l} t+0.5 > 1 \\ t+1 \leq 2 \end{array} \Rightarrow \underline{1/2 < t \leq 1}$$

Caso 3:

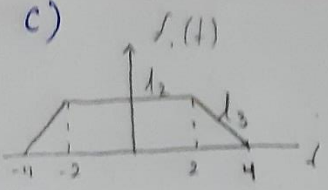


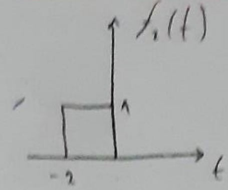
$$\int_{t+1/2}^{2} A \cdot B\, d\tau \quad \begin{array}{l} t+1 > 2 \\ t+1/2 \leq 2 \end{array} \Rightarrow 1 < t \leq 1.5$$

Entonces

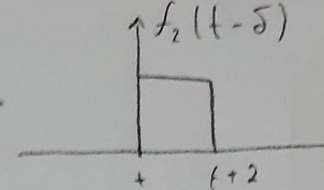
$$f_1(t) \cdot f_2(t) = \underbrace{\int_{1}^{t+1} A \cdot B\, d\tau}_{0 < t < 1/2} + \underbrace{\int_{1}^{2} A \cdot B\, d\tau}_{1/2 < t \leq 1} + \underbrace{\int_{t+1/2}^{2} A \cdot B\, d\tau}_{1 < t \leq 1.5}$$

c)

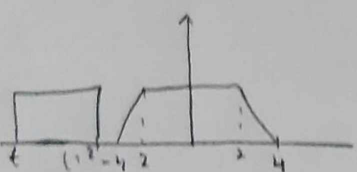




$\Rightarrow$

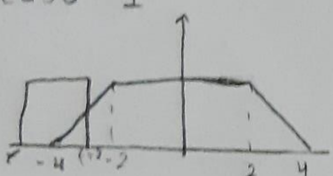


Caso $\emptyset$:



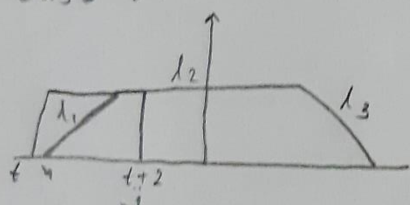
$$\int_{-\infty}^{\infty} f_1(t)\, f_2(t-\delta) = \emptyset$$

Caso 1



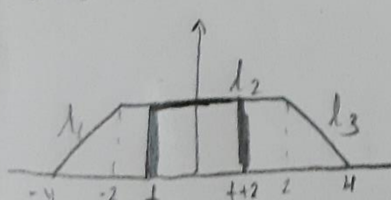
$$\int_{-4}^{t+2} 1\, l_1\, d\delta \quad \text{en} \quad \begin{array}{l} t+2 > -4 \\ t+2 \le -2 \end{array} \Rightarrow -6 < t \le -4$$

Caso 2:



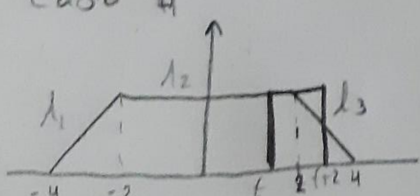
$$\int_{4}^{-1} 1\, l_1\, d\delta + \int_{-1}^{t+2} 1\, d\delta \quad \text{en} \quad \begin{array}{l} t+2 > -2 \\ t \le -2 \end{array} \Rightarrow -4 < t \le -2$$

Caso 3:



$$\int_{t}^{t+2} 1 \cdot l_2\, d\delta \quad \begin{array}{l} t > -2 \\ t+2 \le 2 \end{array} \Rightarrow -2 < t \le 0$$

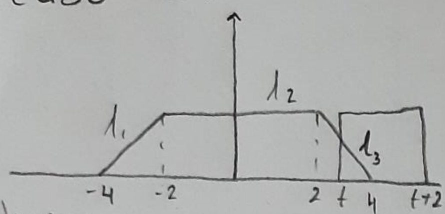
Caso 4



$$\int_{4}^{2} 1\, l_2\, d\delta + \int_{2}^{t+2} 1\, l_3\, d\delta \quad \begin{array}{l} t+2 > 2 \\ t \le 2 \end{array}$$

$0 < t \le 2$

Caso 5:



$$\int_{t}^{4} 1\, l_3\, d\tau \qquad \begin{matrix} t+2>4 \\ t\le 4 \end{matrix} \Rightarrow \underline{2<t\le 4}$$

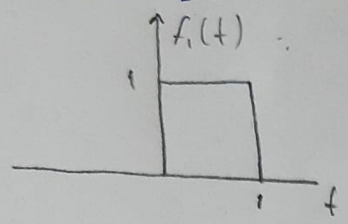
Entonces:

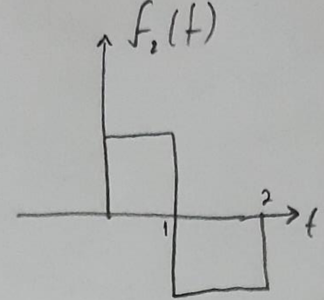
$$\underbrace{\int_{-4}^{t+2} 1\, \lambda_1\, d\tau}_{-6<t\le -4} + \underbrace{\int_{t}^{-1} (1)\lambda_1\, d\tau + \int_{-1}^{t+2} 1\, \lambda_2\, d\tau}_{-4<t\le 2} + \underbrace{\int_{t}^{t+2} (1)\, \lambda_2\, d\tau}_{-2<t\le 0} +$$

$$+ \underbrace{\int_{t}^{2} 1\, \lambda_2\, d\tau + \int_{2}^{t+2} 1\, l_3\, d\tau}_{0<t\le 2} + \underbrace{\int_{t}^{4} 1\, l_3\, d\tau}_{2<t\le 4}$$

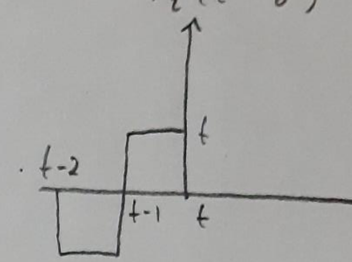
Problema 3: Evalúe las funciones de convolución para las señales mostradas

a) $f_1(t) * f_2(t)$

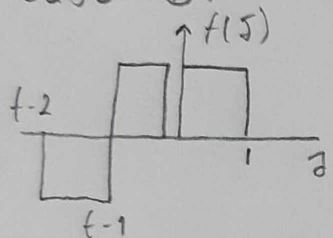




$\Rightarrow$

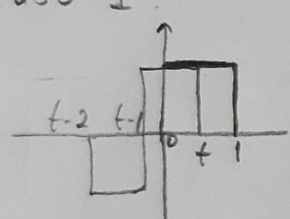


Caso 0:



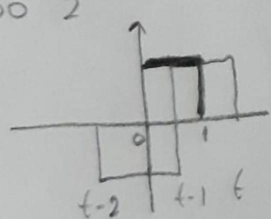
$$\int_{-\infty}^{\infty} f_1(\tau)\cdot f_2(t-\tau)\,d\tau = 0$$

Caso 1:



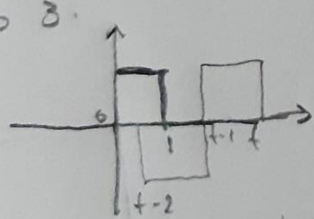
$$\int_{0}^{t} (-1)(1)\,d\tau \qquad \begin{matrix} t>0 \\ t\leq 1 \end{matrix} \Rightarrow \underline{0<t\leq 1}$$

Caso 2



$$\int_{0}^{t-1} (1)(-1)\,d\tau + \int_{t-1}^{1} (1)(1)\,d\tau \qquad \begin{matrix} t>1 \\ t-1\leq 1 \end{matrix} \Rightarrow \underline{1<t\leq 2}$$

Caso 3.



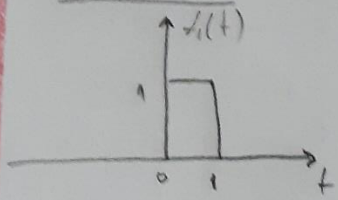
$$\int_{t-2}^{1} (1)(-1)\,d\tau \qquad \begin{matrix} t-1>1 \\ t-2\leq 1 \end{matrix} \Rightarrow \underline{2<t\leq 3}$$

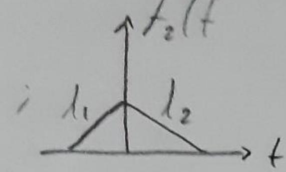
Entonces:

$$f_1(t) * f_2(t) = \underbrace{\int_{0}^{1} (-1)(1)\,d\tau}_{0<t\leq 1} + \underbrace{\int_{0}^{t-1} 1(-1)\,d\tau + \int_{t-1}^{1} (1)(1)\,d\tau}_{1<t\leq 2} + \underbrace{\int_{t-2}^{1} (1)(-1)\,d\tau}_{2<t\leq 3}$$

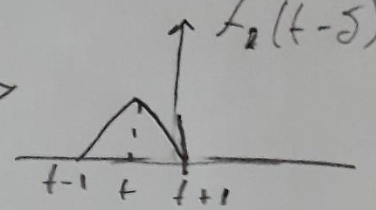
b) $f_1(t) * f_3(t)$

Solución

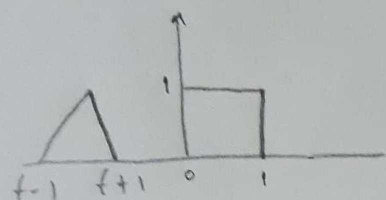




$\Rightarrow$

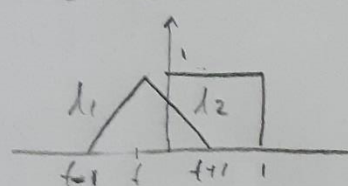


Caso 0:



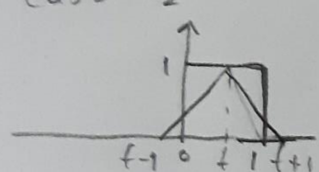
$$\int_{-\infty}^{\infty} f_1(\tau)\cdot f_2(t-\tau)\,d\tau = 0$$

Caso 1



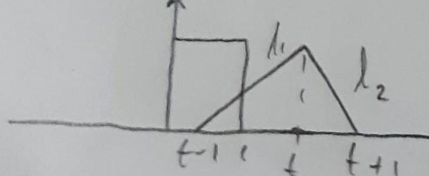
$$\int_0^{t+1} (1)\, l_2\, d\tau \qquad \begin{matrix} t+1>0 \\ t \le 0 \end{matrix} \Rightarrow -1<t\le 0$$

Caso 2



$$\int_0^{t} (1)\, l_1\, d\tau + \int_t^{1} (1)\, l_2\, d\tau \qquad \begin{matrix} t>0 \\ t \le 1 \end{matrix} \Rightarrow 0<t\le 1$$

Caso 3

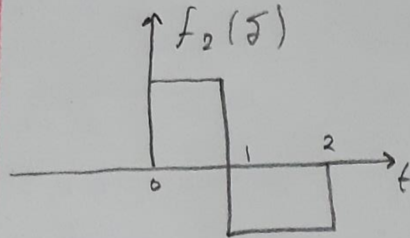


$$\int_{t-1}^{1} l_1\, d\tau \qquad \begin{matrix} t>1 \\ t \le 1 \end{matrix} \Rightarrow 1<t\le 2$$

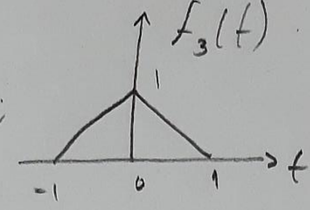
Entonces:

$$f_1(t) * f_3(t) = \underbrace{\int_0^{t+1} l_2\, d\tau}_{-1<t\le 0} + \underbrace{\int_0^{t} l_1\, d\tau + \int_t^{1} l_2\, d\tau}_{0<t\le 1} + \underbrace{\int_{t-1}^{1} l_1\, d\tau}_{1<t\le 2}$$

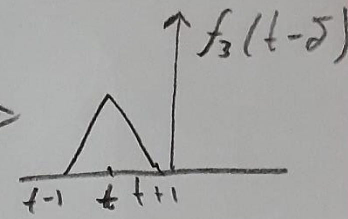
c) $f_2(t) * f_3(t)$



;

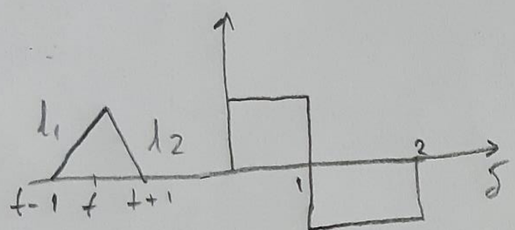


$\Rightarrow$



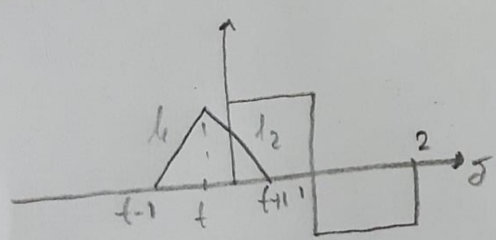
solución

Caso $\emptyset$:



$$\int_{-\infty}^{\infty} f_1(\delta)\cdot f_2(t-\delta)\,d\delta = \emptyset$$

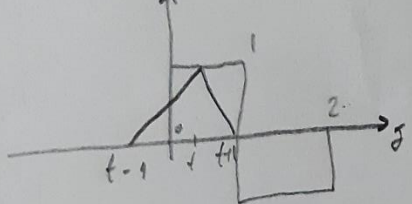
Caso 1:



$$\int_{0}^{t+1} l_2(1)\,d\delta$$

$t+1>0$
$t\leq 0$ $\Rightarrow$ $t<1\leq 0$

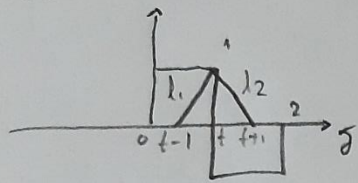
Caso 2:



$$\int_{0}^{t} l_1(1)\,d\delta + \int_{t}^{1} l_2(1)\,d\delta$$

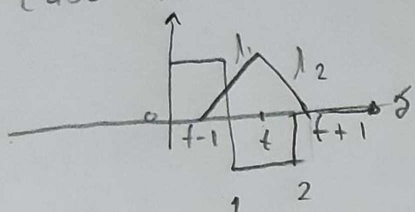
$t>0$
$t+1\leq 1$ $\Rightarrow$ $0<t\leq 0$

Caso 3:



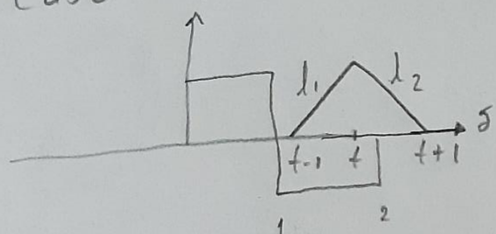
$$\int_{t-1}^{1} \lambda_1(1)\,d\sigma + \int_{1}^{t+1} \lambda_2(-1)\,d\sigma \;;\; \begin{matrix} t+1>1 \\ t\le 1 \end{matrix} \Rightarrow 0<t\le 1$$

Caso 4:



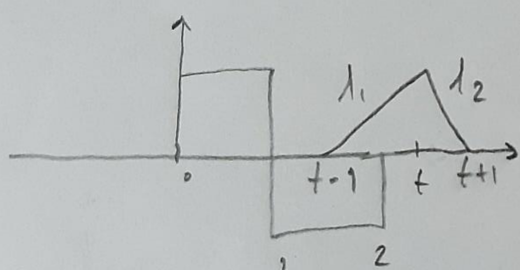
$$\int_{t-1}^{1} \lambda_1(1)\,d\sigma + \int_{1}^{t} \lambda_2(-1)\,d\sigma \;;\; \begin{matrix} t>1 \\ t-1\le 1 \end{matrix} \Rightarrow 1<t\le 2$$

Caso 5:



$$\int_{1}^{t} \lambda_1(-1)\,d\sigma + \int_{t}^{2} \lambda_2(-1)\,d\sigma \;;\; \begin{matrix} t-1>1 \\ t\le 2 \end{matrix} \Rightarrow 2<t\le 2$$

Caso 6:

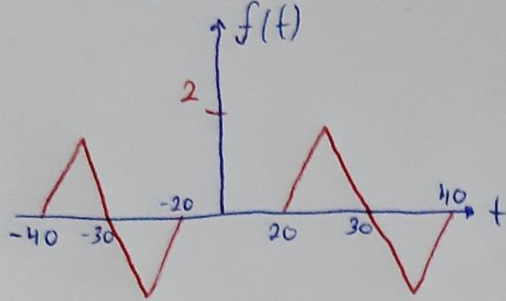


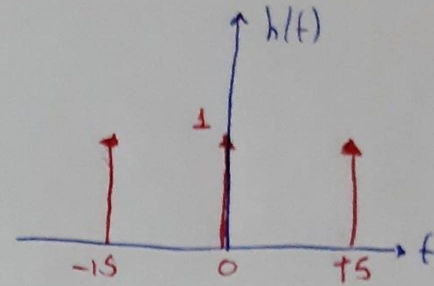
$$\int_{t-1}^{2} \lambda_1(-1)\,d\sigma \quad \begin{matrix} t>2 \\ t-1\le 2 \end{matrix} \Rightarrow 2<t\le 3$$

Entonces:

$$f_2(t)*f_3(t) = \underbrace{\int_{0}^{t+1} \lambda_2(1)\,d\sigma}_{-1<t<0} + \underbrace{\int_{t-1}^{1} \lambda_1(1)\,d\sigma + \int_{1}^{t+1} \lambda_2(-1)\,d\sigma}_{0<t\le 1} + \underbrace{\int_{t-1}^{1} \lambda_1(1)\,d\sigma + \int_{1}^{2} \lambda_2(1)\,d\sigma}_{1<t\le 2} + \underbrace{\int_{t+1}^{2} \lambda_1(-1)\,d\sigma}_{2<t\le 3}$$

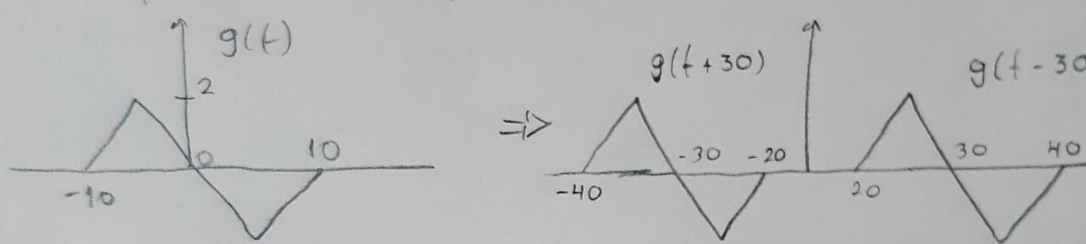
Problema 4: Obtener y dibujar $f_1(t) * h(t)$ para la siguiente figura.





Solución

Se representa a $f(t)$



$$f(t) * g(t) = f(t) * [\delta(t+15) + \delta(t-15)] =$$

$$= [g(t+30) + g(t-30)] * [\delta(t+15) + \delta(t-15)]$$

$$= g(t+30) * \delta(t+15) + g(t+30) * \delta(t-15) + g(t-30) * \delta(t+15) + g(t-30) * \delta(t-15)$$

$$= g(t+30+15) + g(t+30-15) + g(t-30+15) + g(t-30-15)$$

Entonces:

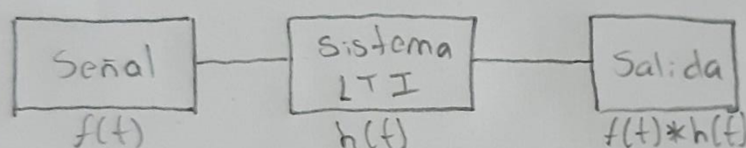
$\therefore$

$$f_1(t) * f_2(t) = g(t+45) + g(t+15) + g(t-15) + g(t-45)$$

Sección 2.

1. Defina Procesamiento Digital de señales, procesador digital de señales y dibuje el diagrama a bloques de un sistema de procesamiento digital de señales (adquisición de datos)

- Procesamiento digital es una manipulación matemática de una señal de información para modificarla y mejorarla. Caracterizado por su representación en tiempo discreto o frecuencia discreta.
- Procesador digital: Es un sistema basado en un procesador que posee un conjunto de instrucciones para optimizaciones que requieran operaciones a muy alta velocidad.



2.- Enuncie el teorema del muestreo.

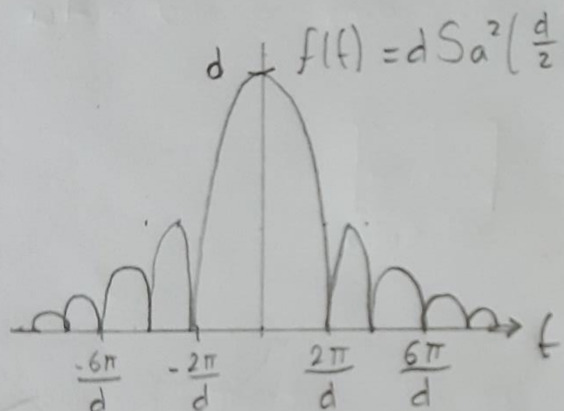
Toda señal limitada en banda que no contiene frecuenciales mayores a Hz y $\frac{rad}{s}$ esta completamente representada por el conjunto de sus muestras formadas a intervalos uniformes no mayores de $1/2\,fm$ (s)

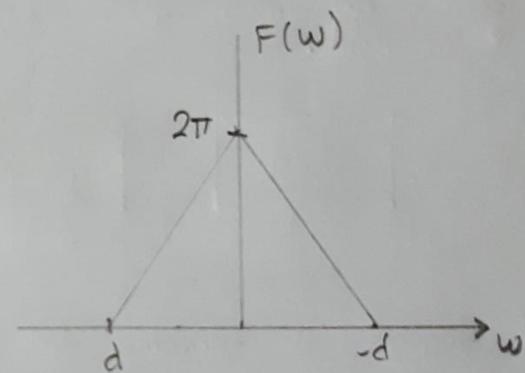
Es decir $T \leq 1/2fm$ ó $f \geq 2fm$

3: ¿A que se refiere el efecto Alias?

Surge cuando no se cumple el teorema de muestreo y provoca un deformamiento en la señal, partiendo que se quiera volver a la señal original, es decir, se traslapan las señales y nos da una señal distinta a la original.

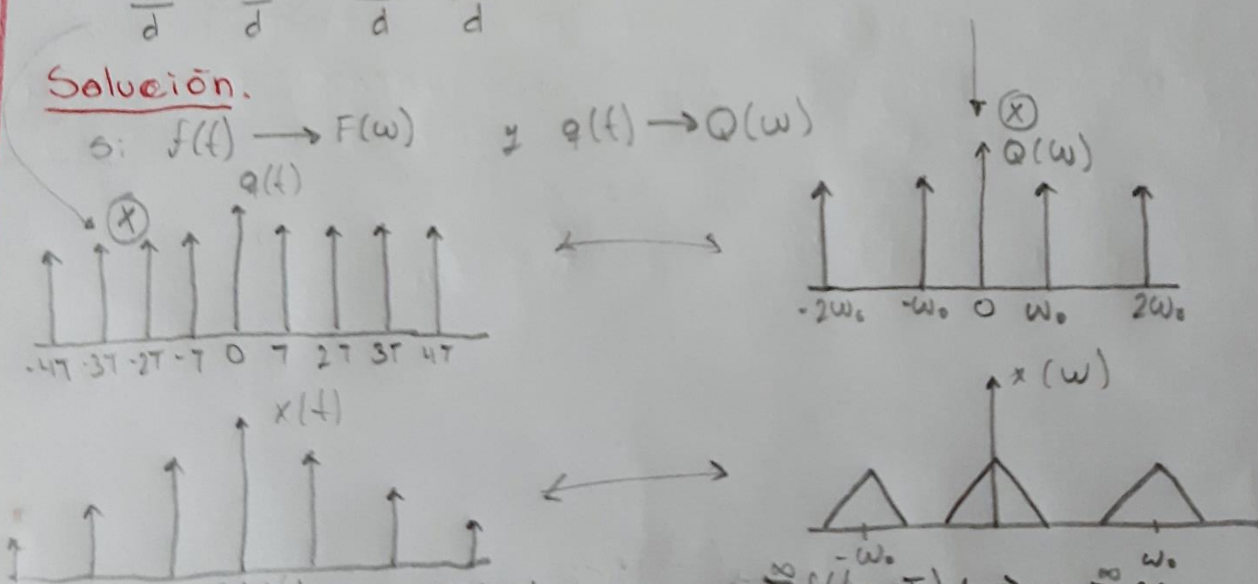
4. Considere la siguiente función en el tiempo $f(t)$ y su transformada $F(\omega)$. Desarrolle gráfica y matemáticamente el muestreo ideal de $f(t)$.





Solución.

Si $f(t) \longrightarrow F(\omega)$ y $q(t) \longrightarrow Q(\omega)$



$dSa^2\left(\frac{dt}{2}\right) \longleftrightarrow 2\pi\Lambda\left(\frac{\omega}{d}\right)$

$\sum_{n=-\infty}^{\infty} \delta(t-nT) \longleftrightarrow \omega_0 \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$

$$\sum_{n=-\infty}^{\infty} f(nT)\delta(t-nT) \longleftrightarrow \frac{1}{T}\sum_{n=-\infty}^{\infty} F(\omega - n\omega_0)$$

5.- Determine la rapidez y el intervalo de Nyquist de las siguientes señales.

a) $Sa(100t)$

Solución

De tablas. $A C_d(t) \longleftrightarrow A d Sa\left(\frac{\omega d}{2}\right)$

$$C_d(t) \longleftrightarrow d\,Sa\left(\frac{\omega d}{2}\right)$$

$$d\,Sa\left(\frac{td}{2}\right) \longleftrightarrow 2\pi C_d(\omega)$$

Si $d = 200$

$$200\,Sa(100t) \longleftrightarrow 2\pi C_{200}(\omega)$$

La componente frecuencial $\omega_m = 200 \frac{rad}{s}$

Así $f_m = \frac{200}{2\pi} = 100/\pi$ Hz

$$2f_m = \frac{200}{\pi}\ Hz \qquad T \leq \frac{1}{2f_m} = \frac{\pi}{200}\ s$$

$$T_N = \frac{1}{2f_m} = \frac{\pi}{200}\ s$$

b) $Sa^2(100t)$

solución

$$\Lambda\left(\frac{t}{d}\right) \longleftrightarrow d\,Sa^2\left(\frac{\omega d}{2}\right)$$

$$d\,Sa^2\left(\frac{td}{2}\right) \longleftrightarrow 2\pi \Lambda\left(\frac{\omega}{d}\right)$$

Si $d = 200$

$$200\,Sa^2(100t) \longleftrightarrow 2\pi \Lambda\left(\frac{\omega}{200}\right)$$

$$Sa^2(100t) \longleftrightarrow \frac{\pi}{100} \Lambda\left(\frac{\omega}{200}\right)$$

Componente frecuencial.

$$\omega_m = 200 \frac{rad}{s}$$

$$f_m = \frac{200}{2\pi} = \frac{100}{\pi}\ Hz$$

$$2f_m = \frac{200}{\pi}\ Hz$$

$$T \leq \frac{1}{2f_m} = \frac{\pi}{200}\ seg$$

$$T_N = \frac{1}{2f_m} = \frac{\pi}{200}\ s$$

c) $Sa(100t) + Sa(50t)$

solución

Para $Sa(100t) \longleftrightarrow ?$

$$C_d(t) \longleftrightarrow d\,Sa\left(\frac{\omega d}{2}\right)$$

$$d\,Sa\left(\frac{td}{2}\right) \longleftrightarrow 2\pi C_d(\omega)$$

Si $d = 200$

$$200\,Sa(100t) \longleftrightarrow 2\pi C_{200}(\omega)$$

$$Sa(100t) \longleftrightarrow \frac{\pi}{100} C_{200}(\omega)$$

Para: $Sa(50t)$

$$C_d(t) \longleftrightarrow d\,Sa\left(\frac{\omega d}{2}\right)$$

$$d\,Sa(t) \longleftrightarrow 2\pi C_d(\omega)$$

Si $\omega = 100$

$$100\,Sa(50t) \longleftrightarrow 2\pi C_{100}(\omega)$$

$$Sa(50t) \longleftrightarrow \frac{\pi}{50} C_{100}(\omega)$$

Entonces: $Sa(100t) + Sa(50t) \longleftrightarrow \frac{\pi}{100} C_{200}(\omega) + \frac{\pi}{50} C_{100}(\omega)$

La componente frecuencial más grande es:

$W_m = 200 \frac{rad}{s}$

Así:

$f_m = \frac{200}{2\pi} = \frac{100}{\pi} Hz$

$2 f_m = \frac{200}{\pi} Hz$

$T = \frac{1}{2f_m} = \frac{\pi}{200} s$

$T_N = \frac{1}{2f_m} = \frac{\pi}{200}$ seg

6.- Se sabe que una señal de valor real $x(t)$ ha sido determinada solo por sus muestras cuando la frecuencia de muestreo es $W_s = 10{,}000\pi t$, ¿Para que valores de W se garantiza que $F(W)$ sea cero.

Solución

Sea: $W_s = 10000\pi t$; $\Rightarrow$ Dividiendo entre 2π: $W_s = \frac{10000}{2\pi}\pi t =$

$W_s = 5000t$ $\therefore$ para valores de $5000t$

7.- Aquella frecuencia que de acuerdo con el teorema de muestreo, debe ser excedida por la frecuencia de muestreo se llama razón de Nyquist. Determine la razón de Nyquist correspondiente a cada una de las siguientes señales:

a) $x(t) = 10 \text{ sen } wt + 5 \text{ sen } 2wt$

Solución

$10 \text{ sen } wt \longleftrightarrow ?$

Por tablas

$10 \text{ sen}(wt) \longleftrightarrow 10\pi i[\delta(\omega - w) - \delta(\omega - w)]$

$5 \text{ sen }(wt) \longleftrightarrow ?$

Por tablas:

$5 \text{ sen }(wt) \longleftrightarrow 5\pi[\delta(\omega - w) - \delta(w - w)]$

La componente frecuencial es $W_m = W$

Así $f_m = \frac{W}{2\pi}$ $\quad 2f_m = \frac{W}{\pi} Hz$

$T_N = \frac{1}{2f_m} = \frac{\pi}{W}$ seg $\quad f_N = 2f_m = \frac{W}{\pi} Hz$

b) $x(t) = 1 + \cos(2000\pi t) + \text{sen}(4000\pi t)$

Solución

$1 \longleftrightarrow ?$

Por tablas

$\delta(t) \longleftrightarrow 1$

$1 \longleftrightarrow 2\pi\delta(\omega)$

$\cos(2000\pi t) \longleftrightarrow ?$

Por tablas

$\cos 2000\pi t \longleftrightarrow \pi[\delta(\omega + 2000\pi) + \delta(\omega - 2000\pi)]$

$\text{sen}(4000\pi t) \longleftrightarrow ?$

Por tablas

$\text{sen}(4000\pi t) \longleftrightarrow \pi i[\delta(\omega + 4000\pi) - \delta(\omega - 4000\pi)]$

$x(t) \longleftrightarrow 2\pi\delta(\omega) + \pi i[\delta(\omega + 4000\pi) - \delta(\omega - 4000\pi)] + \pi[\delta(\omega + 2000\pi + \delta(\omega - 2000\pi)]$

La componente frecuencial más grande es $\omega_m = 4000\pi$

$f_m = \frac{4000\pi}{2\pi} = 2000$ $\qquad$ $2f_m = 4000 \text{ Hz}$

$T_N = \frac{1}{2f_m} = \frac{1}{4000} \text{ Hz} = 4 \times 10^{3} \text{ Hz} = 250 \mu s$

$f_N = 2f_m = 4000 \text{ Hz}$

8) Una señal continua $x(t)$ se obtiene a la salida de un filtro pasa bajo ideal con frecuencia de corte $W_c = 1000\pi$. Si el muestreo con tren impulsos se realiza sobre $x(t)$: ¿Cuál de los siguientes periodos de muestreo garantiza que $x(t)$ se puede recuperar a partir de sus versiones muestreadas usando un filtro paso bajas adecuado?

a) $T = 0.5 \times 10^{-3}$

b) $T = 2 \times 10^{-3}$

c) $T = 10^{-4}$

Solución

Si $W_c = 1000\pi \Rightarrow f_m = \frac{1000\pi}{2\pi} = 500 \text{ Hz}$

$2f_m = 1000 \text{ Hz}$

$T \leq \frac{1}{1000 \text{ Hz}} = 1 ms$

a) $0.5 \text{ mseg} \leq 1 ms \Rightarrow$ Sí cumple.

b) $4 \text{ mseg} \geq 1 ms \Rightarrow$ No cumple.

c) $0.1 ms \leq 1 ms \Rightarrow$ Sí cumple.

9. Dibuje la función de transferencia $H(\omega)$ de un filtro pasa bajas con ganancia 5 y frecuencia de corte $f = 100\,Hz$

Solución

$f_m = 100\,Hz$

$\omega_m = 200\pi \frac{rad}{s}$

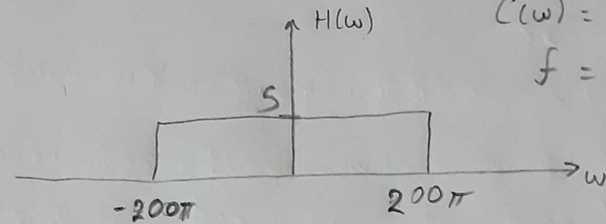
$H(\omega) = T\,C_{2\omega_m}(\omega) = 5\,C_{400\pi}$

$H(\omega) = 5$

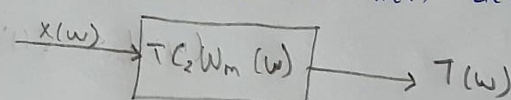
$C(\omega) = 5_{400}\,C(\omega)$

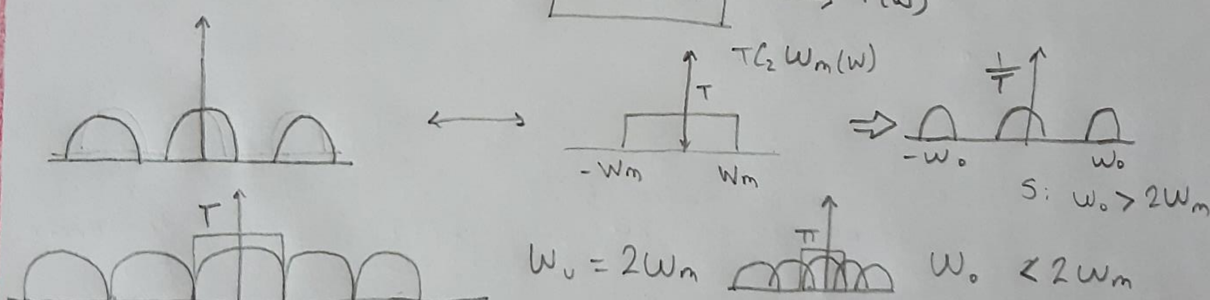
$f = 100\,Hz$

$\Rightarrow \omega = 2\pi(100\,Hz)$



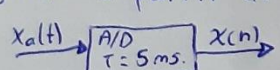
10) Dibuje la función característica $h(t)$ del filtro anterior.





11) Considere el sistema mostrado en la figura siguiente. La señal de entrada al sistema es: $x_a(t) = 3\cos 100\pi t + 2\,sen\,250\pi t$

Determine la versión discreta de $x_a(t)$ ¿Es posible recuperar la señal original a partir de $x(n)$ usando un filtro pasa bajas adecuado?



Solución

$Si\; f_m = \frac{100\pi}{2\pi} = \cancel{50} \Rightarrow 2f_m = 100\,Hz$

$T_N = \frac{1}{2f_m} = \frac{1}{\cancel{200}}\,\cancel{seg} \Rightarrow f_N = 2f_m = 100\,Hz$

$\therefore$ Si se puede recuperar usando un filtro pasa bajas

12. Analice las siguientes secuencias (esto es, su secuencia digital); e indique si son o no periódicas. En caso de ser periódicas halle su periodo.

a) $10\,sen\left(\frac{3}{2}\pi n\right)$

Solución

$$10\,sen\left(\frac{3}{2}\pi n\right) = 10\cos\left(\frac{3}{2}\pi n + \frac{3}{2}\pi\right)$$

$$2\pi F N = \frac{3}{2}\pi N$$

$$F = \frac{3}{4} \Rightarrow \underline{N = 4}$$

b) $5\cos\left(\frac{4}{9}\pi n\right)$

Solución

$$5\cos\left(\frac{4}{9}\pi n\right) \Rightarrow A\cos(2\pi f_N + d)$$

$$2\pi F_n = \frac{4}{9}\pi n$$

$$F = \frac{4}{18} \Rightarrow \underline{N = 18}$$

c) $2\cos\left(\frac{4}{9} n\right)$

Solución

$$2\cos\ \frac{4}{9} n \Rightarrow A\cos(2\pi F_n + d)$$

$$2\pi F = \frac{4}{9} n$$

$$F = \frac{4}{18\pi} \ \therefore \ \text{No es periódica}$$

d) $x(n) = \cos \frac{2\pi n}{3} + e^{\pi n}$

Solución

$x_a(n) : \cos \frac{2\pi n}{3}$; es periodica $T=3$

$x_b(n) = e^{\pi n}$; no es periodico

$\therefore$ $x(n)$ no es periodica

e) $y(n) = 2 + Re[e^{i\frac{n\pi}{3}}] + \cos \frac{3\pi n}{2}$

Solución

$y_1(n) = 2 \; : \{\ldots, 2, 2, 2, 2, 2, \bar{2}, 2, 2, 2, 2, 2, \ldots\}$

$y_2(n) = Re\{e^{i\frac{n\pi}{3}}\} = \cos\frac{n\pi}{3} + i sen \frac{n\pi}{3}$

$\cos\frac{n\pi}{3} = \{\ldots, 1, \frac{1}{2}, \frac{1}{2}, -1, \frac{1}{2}, \frac{1}{2}, \underbrace{\bar{1}, \frac{1}{2}, \frac{1}{2}, -1, \frac{1}{2}, \frac{1}{2}}_{N=6}, \ldots\}$

$y_3(n) = \cos\left(\frac{3n\pi}{2}\right) = \{\ldots, -1, 0, 1, 0, -1, 0, 1, 0, -1, 0, 1, 0, -1, 0, \ldots\}$

$N = 4$

$\therefore$ $N = 12$

13.- Gráfique lo siguiente señal $y(n)$ que es una suma sinusoides, indique su periodo. ¿Cuál es el periodo de la suma de 2 sinusoides de periodo N_1 y N_2?

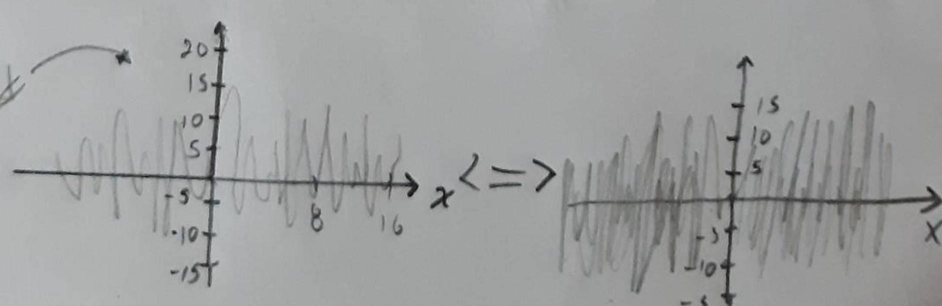
$$y(n) = \underbrace{10 sen\left(\tfrac{3}{2}\pi n\right)}_{N_1} + \underbrace{5\cos\left(\tfrac{4}{a}\pi n\right)}_{N_2}$$

Solución

$N_1 = 4 \; ; \; N_2 = 18$

4	18	2
2	9	2
1	9	3
1	3	3
1	1	

$2 \cdot 2 = 4$, $3 \cdot 3 = 9$ > 36



Sección 3: Operaciones Básicas entre secuencias.

Problema 1. Considere las secuencias siguientes y realice con ellas las operaciones indicadas.

$x[n] = \{\bar{0}, 1, 2, 3, 4, 5, 6, 7, 8\}$

$y[n] = \{2, 4, 8, 16, \overline{32}\}$

$z[n] = \sum_{k=-3}^{3} \delta(n-k) =$

$g[n] = \{\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \overline{\frac{1}{12}}, \frac{1}{15}, \frac{1}{18}, \ldots\}$

$k[n] = 2u(n-2)$

Encuentre:

a) $g[-n]$

Solución

$g[-n] = \{\ldots, \frac{1}{18}, \frac{1}{15}, \overline{\frac{1}{12}}, \frac{1}{9}, \frac{1}{6}, \frac{1}{3}\}$

b) $z[n] + y[n]$

Solución

$z[n] + y[n] = \{1, 1, 1, 1, 2, 3, 4, 4, 5, 6, 7, 8\}$

c) $3\,g[n] - 6\,z[n]$

Solución

$3\,g[n] = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}\}$

$-6\,z[n] = \{-6, -6, -6, -6, -6, -6\}$

$\therefore = \{-5, -\frac{11}{2}, -\frac{17}{3}, -\frac{23}{4}, -\frac{29}{5}, -\frac{35}{6}, \ldots\}$

d) $y[n-6]$

Solución

$y[n-6] = \{\bar{0}, 0, 2, 4, 8, 16, 32\}$

e) $\frac{1}{2} y[n+3]$

solución

$y[n+3] = \{2, 4, 8, 16, 32, 0, 0, \bar{0}\}$

$\frac{1}{2} y[n+3] = \{1, 2, 4, 8, 16, 0, 0, \bar{0}\}$ //

f) $x[n-2]$

solución

$x[n-2] = \{\bar{0}, 0, 0, 1, 2, 3, 4, 5, 6, 7, 8\}$

g) $x[3n-3]$

solución

$x[n-3] = \{\bar{0}, 0, 0, 0, 1, 2, 3, 4, 5, 6, 7, 8\}$

$x[3n-3] = \{\bar{0}, 0, 3, 6\}$ ///

h) $x[\frac{n}{2}+5]$

solución

$x[n+5] = \{0, 1, 2, 3, 4, \bar{5}, 6, 7, 8\}$

$x[\frac{n}{2}+5] = \{0, 0, 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 6, 6, 7, 7, 8, 8\}$

i) $y[\frac{n-3}{3}]$

solución

$y[n-3] = \{2, \bar{4}, 8, 16, 32\}$

$y[\frac{n-3}{3}] = \{2, 2, 2, 4, 4, 4, 8, 8, 8, 16, 16, 16, 32, 32, 32\}$ ///

j) $k[\frac{4n-3}{10}]$

solución

$k[n] = \{\bar{0}, 0, 2, 2, 2, \ldots\}$

$k[n-3] = \{\bar{0}, 0, 0, 0, 0, 2, 2, 2\}$

$k[\frac{n-3}{10}] = \{\bar{0}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,$
$0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 2, 2, 2, 2,$
$2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2\}$

$k[\frac{4n-3}{10}] = \{\bar{0}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 2, 2, 2, 2, 2\}$ ///

$k)\ k\left[-\frac{n}{4}+10\right]$

Solución

$$k[n]=\{\bar{0},0,2,2,2,2,\ldots\}$$

$$k[-n]=\{\bar{0},0,0,2,2,2,2,\ldots\}$$

$$k\left[\frac{-n}{4}\right]=\{\bar{0},2\}$$

$$k\left[-\frac{n}{4}+10\right]=\{0,2,0,2,0,2,0,2,0,2,0,\bar{2},\ldots\}$$

$l)\ x\left[\frac{3n-3}{3}\right]-g\left[-\frac{8n-7}{3}\right]$

Solución

$$x[3n]=\{\bar{0},1,2,3,4,5,6,7,8\}=\{\bar{0},3,6\}$$

$$x[3n-3]=\{\bar{0},0,0,0,3,6\}$$

$$x\left[\frac{3n-3}{3}\right]=\{\bar{0},0,0,0,0,0,0,0,0,0,0,0,3,3,3,3,6,6,6,6\}$$

$$g[8n]=\{\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{6},\tfrac{1}{6},\tfrac{1}{6},\tfrac{1}{6},\tfrac{1}{6},\tfrac{1}{6},\tfrac{1}{6},\tfrac{1}{6},\tfrac{1}{9},\tfrac{1}{9},\tfrac{1}{9},$$
$$\tfrac{1}{9},\tfrac{1}{9},\tfrac{1}{9},\tfrac{1}{9},\tfrac{1}{9},\tfrac{1}{9},\overline{\tfrac{1}{12}},\tfrac{1}{12},\tfrac{1}{12},\tfrac{1}{12},\tfrac{1}{12},\tfrac{1}{12},\tfrac{1}{12},\tfrac{1}{12},\tfrac{1}{12},\tfrac{1}{15},$$
$$\tfrac{1}{15},\tfrac{1}{15},\tfrac{1}{15},\tfrac{1}{15},\tfrac{1}{15},\tfrac{1}{15},\tfrac{1}{15},\tfrac{1}{15}\}$$

$$g[-8n-7]=\{\tfrac{1}{15},\tfrac{1}{15},\tfrac{1}{15},\tfrac{1}{15},\tfrac{1}{15},\tfrac{1}{15},\tfrac{1}{15},\tfrac{1}{13},\tfrac{1}{15},\tfrac{1}{12},\overline{\tfrac{1}{12}},\tfrac{1}{12},\tfrac{1}{12},\tfrac{1}{12},$$
$$\tfrac{1}{12},\tfrac{1}{12},\tfrac{1}{12},\tfrac{1}{12},\tfrac{1}{9},\tfrac{1}{9},\tfrac{1}{9},\tfrac{1}{9},\tfrac{1}{9},\tfrac{1}{9},\tfrac{1}{9},\tfrac{1}{9},\tfrac{1}{9},\tfrac{1}{9},$$
$$\tfrac{1}{6},\tfrac{1}{6},\tfrac{1}{6},\tfrac{1}{6},\tfrac{1}{6},\tfrac{1}{6},\tfrac{1}{6},\tfrac{1}{6},\tfrac{1}{6},\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3}\}$$

$$g\left[-\frac{8n-7}{3}\right]=\{\tfrac{1}{15},\tfrac{1}{15},\tfrac{1}{15},\tfrac{1}{12},\tfrac{1}{12},\tfrac{1}{12},\tfrac{1}{9},\tfrac{1}{9},\tfrac{1}{9},\tfrac{1}{6},\tfrac{1}{6},\tfrac{1}{6},\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3}\}$$

$$x\left[\frac{3n-3}{3}\right]-g\left[-\frac{8n-7}{3}\right]=\{-,\ldots,-\overline{\tfrac{1}{18}},-\tfrac{1}{9},\tfrac{2}{3},2,3,4,5,6,7,8\}$$

m) $u[n] \cdot g[n] + z[n]$

Solución

$$u[n] = \{\bar{1}, 1, 1, 1, 1, 1, \ldots\}$$
$$g[n] = \{\tfrac{1}{3}, \tfrac{1}{6}, \tfrac{1}{9}, \overline{\tfrac{1}{12}}, \tfrac{1}{15}, \tfrac{1}{18}, \ldots\}$$
$$u(n) \cdot g(n) = \{\overline{\tfrac{1}{12}}, \tfrac{1}{15}, \tfrac{1}{18}, \tfrac{1}{21}, \tfrac{1}{24}, \ldots\}$$
$$+ \quad z(n) = \{1, 1, 1, \bar{1}, 1, 1, 1\}$$
$$u(n) \cdot g(n) + z(n) = \{1, 1, 1, \overline{\tfrac{13}{12}}, \tfrac{16}{15}, \tfrac{19}{18}, \tfrac{22}{21}, \tfrac{25}{24}, \ldots\}$$

n) $x\left[\frac{n}{2}\right] * y\left[\frac{n}{3}\right]$

Solución

$$x(n) = \{\bar{0}, 1, 2, 3, 4, 5, 6, 7, 8\} \qquad y(n) = \{2, 4, 8, 16, \overline{32}\}$$
$$x\left(\tfrac{n}{2}\right) = \{\bar{0}, 0, 1, 1, 2, 2, 3, 3, 4, 4, 5, 5, 6, 6, 7, 7, 8, 8\}$$
$$y\left(\tfrac{n}{3}\right) = \{2, 2, 2, 4, 4, 4, 8, 8, 8, 16, 16, 16, \overline{32}, 32, 32\}$$

$$x\left(\tfrac{n}{2}\right) * y\left(\tfrac{n}{3}\right) = \{0, 0, 2, 4, 8, 14, 22, 32, 48, 66, 90, \overline{124}, 164, 214, 286, 308, 442, 559, 640, 709, 788, 832, 896, 936, 960, 960, 970, 896, 832, 766, 256\}$$

ñ) $g\left(\frac{3n-1}{2}\right) * y\left(\frac{2n}{2} + 2\right)$

Solución

$$g(3n) = \{\tfrac{1}{3}, \tfrac{1}{6}, \tfrac{1}{9}, \overline{\tfrac{1}{12}}, \tfrac{1}{15}, \tfrac{1}{18}, \ldots\} = \{\tfrac{1}{3}, \overline{\tfrac{1}{12}}\}$$
$$g(3n-1) = \{\overline{\tfrac{1}{3}}, \tfrac{1}{12}\}$$
$$g\left(\tfrac{3n-1}{2}\right) = \{\overline{\tfrac{1}{3}}, \tfrac{1}{3}, \tfrac{1}{12}, \tfrac{1}{12}\}$$
$$y(2n) = \{2, 4, 8, 16, \overline{32}\} = \{2, 8, \overline{32}\}$$
$$y\left(\tfrac{2n}{2}\right) = \{2, 2, 8, 8, \overline{32}, 32\}$$
$$y\left(\tfrac{2n}{2} + 2\right) = \{2, 2, 8, 8, 32, 32, \bar{0}\}$$

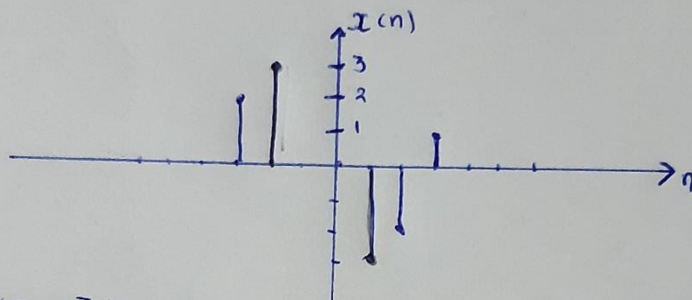
$$g\left(\tfrac{3n-1}{2}\right) * y\left(\tfrac{2n}{2} + 2\right) = \{\tfrac{1}{3}, \tfrac{8}{9}, \tfrac{86}{45}, \tfrac{59}{15}, \tfrac{118}{45}, \tfrac{76}{15}, \tfrac{136}{45}, \overline{\tfrac{16}{9}}, 0, 0\}

Problema 2. Encuentre la gráfica de la secuencia de convolución de dos secuencias definidas como:
$x[n] = \{1, \bar{2}, 3, 4, 3, 2, 1\}$ y $y(n) = \{-2, -1, \bar{0}, 1, 2\}$. Use cualquier método.

Solución

$$x(n) * y(n) = \{-2, -5, -8, -10, -6, 0, 6, \overline{10}, 8, 5, 2\}$$

Problema 3. A partir de la secuencia mostrada en la figura.



Encuentre: $x\left(\frac{3n+3}{-4}\right)$ y $g(n) = x(n) * \{x[\frac{n}{2}+1] \cdot (u[n+1] - u[n-3])\}$

Solución

$$x(n) = \sum_{n=-\infty}^{\infty} x(n) e^{i\Omega n} \Rightarrow x(n) = \sum_{n=-\infty}^{\infty} x(n) e^{-i\Omega n} = x(-2) e^{i2\Omega} + x(-1) e^{-i\Omega}$$

$$+ x(0) e^{0} + x(1) e^{-i\Omega} + x(2) e^{-2i\Omega}$$

$$= 2(e^{i2\Omega} + e^{-2i\Omega}) + e^{i\Omega} + e^{-i\Omega}$$

$$= 4\cos 2\Omega + 2\cos\Omega$$

Problema 4. Las extensiones periódicas de dos secuencias, $x_1(n)$ y $x_2(n)$, se definen como:

$x_1(n) = \{1, 2, 0, -1, 1\}$, $x_2(n) = \{1, 3, -1, -2\}$. Halle la secuencia de convolución.

Solución

```
      1 2 0 -1 1
      1 3 -1 -2
      -----------------
      1 2 0 -1 1
        3 6 0 -3 3
          -1 -2 0 1 -1
             -2 -4 0 2 -2
      -----------------------
      1 5 5 -5 -6 4 1 -2
```

```
   1 5 5 6 -6
   4 1 -2 0 0
   ----------
   5 6 3 6 -6
```

$$\therefore x_1(n) * x_2(n) = \{5, 6, \bar{3}, 6, -6\}$$

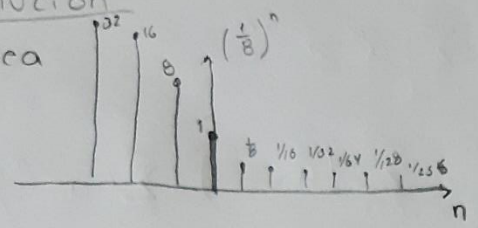
Sección 4: Transformado de Fourier y Transformada Z.

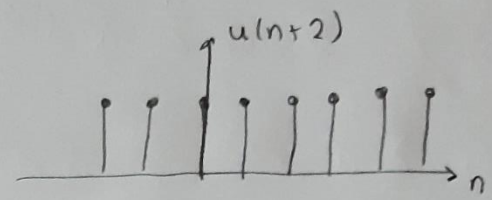
Problema 1: Halle la transformada de Fourier de:

$$y(n) = \left(\tfrac{1}{8}\right)^n u(n+2) + 2^n u(-n)$$

Solución

Sea

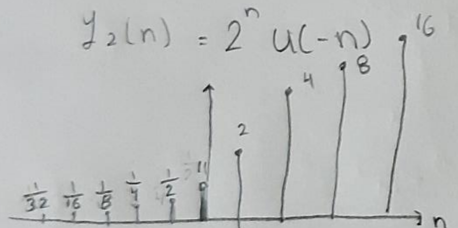


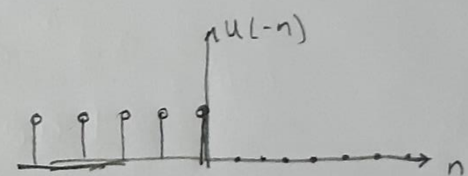


$$y_1(n) = \begin{cases} 0 & n<-2 \\ \left(\frac{1}{8}\right)^n & n \geq -2 \end{cases} \qquad Y_1(\Omega) = \sum_{n=-2}^{\infty} \left(\frac{1}{8}\right)^n e^{-i\Omega n} = \sum_{n=-2}^{\infty}\left(\frac{1}{8} e^{-i\Omega}\right)^n \quad n+2<0$$

$$\left|\frac{1}{8} e^{-i\Omega}\right| = \frac{1}{8} < 1 \implies Y_1(\Omega) = \frac{\left(\frac{1}{8} e^{-i\Omega}\right)^{-2}}{1-\frac{1}{8}e^{-i\Omega}} = \frac{8^2 e^{-i2\Omega}}{\frac{8-e^{-i\Omega}}{8}} = \frac{8^3 e^{2i\Omega}}{8-e^{-i\Omega}}$$

$$y_2(n) = 2^n u(-n)$$





$$y_2(n) = \begin{cases} 0 & n>0 \\ 2^n & n \leq 0 \end{cases}$$

$$Y_2(\Omega) = \sum_{n=-\infty}^{0} 2^n e^{-i\Omega n} = \sum_{n=-\infty}^{0} (2e^{-i\Omega}) = \sum_{n=0}^{n=\infty} (2e^{-i\Omega})^{-n} = \sum_{n=0}^{\infty}\left(\frac{1}{2}e^{i\Omega}\right)^n$$

$$\left|\frac{1}{2}e^{i\Omega}\right| = \frac{1}{2} < 1 \implies Y_2(\Omega) = \frac{1}{1-\frac{1}{2}e^{i\Omega}} = \frac{2}{2-e^{i\Omega}}$$

$$\therefore\ Y(n) = Y_1(n) + Y_2(n) = \frac{8^3 e^{2i\Omega}}{8-e^{-i\Omega}} + \frac{2}{2-e^{i\Omega}} = \frac{(8^3 e^{2i\Omega})(2-e^{i\Omega}) + 2(8-e^{-i\Omega})}{(8-e^{-i\Omega})(2-e^{i\Omega})}$$

$$= \frac{(2\cdot 8^3)e^{2i\Omega} - 8e^{3i\Omega} + 16 - 2e^{-i\Omega}}{17 - 8e^{i\Omega} - 2e^{-i\Omega}}$$

Problema 2: Halle la transformada de Fourier de:

$$y(n) = \begin{cases} a^{n} & n \geq 0 \\ a^{-n} & n < 0 \end{cases}$$

Solución

$$y(n) = \sum_{n=-\infty}^{\infty} \Rightarrow y(n) = \sum_{n=-\infty}^{-1} a^{-n} e^{-i\Omega n} + \sum_{n=0}^{\infty} a^{n} e^{-i\Omega n}$$

$$y(n) = \sum_{n=1}^{\infty} a^{n} e^{in\Omega} + \sum_{n=0}^{\infty} (a e^{-i\Omega})^{n} = \sum_{n=1}^{\infty} (a e^{i\Omega})^{n} + \sum_{n=0}^{\infty} (a e^{-i\Omega})^{n}$$

Por series geométricas...

$$y(n) = \frac{a e^{i\Omega}}{1 - a e^{i\Omega}} + \frac{1}{1 - a e^{-i\Omega}} \quad \text{para } |a| < 1$$

$$y(n) = \frac{a e^{i\Omega}(1 - a e^{i\Omega}) + 1 - a e^{i\Omega n}}{(1 - a e^{i\Omega})(1 - a e^{-i\Omega})}$$

$$y(n) = \frac{a e^{i\Omega} - a \cancel{e}^{0=1} + 1 - a e^{i\Omega}}{1 - a e^{i\Omega} - a e^{i\Omega} + a^{2} e^{0}} =$$

$$y(n) = \frac{1 - a^{2}}{1 + a^{2} - 2a\cos\Omega}$$

Calcule la transfomada discreta de Fourier (DFT) de $x(n)$. $x(n) = \{0, \frac{1}{2}, 1, 2, -3, -4\}$

Solución

$$x(K) = \sum_{n=0}^{N-1} x(n) e^{-i\frac{2\pi}{N}Kn}$$

$N=6$ $$x(K) = \sum_{n=0}^{6} x(n) e^{i\frac{2\pi}{N}Kn}$$

con $K=0$

$$x(0) = \sum_{n=0} x(n) e^{0}$$

$$x(0) = x(0) + x(1) + x(2) + x(3) + x(4) + x(5)$$

$$x(0) = 0 + \frac{1}{2} + 1 + 2 - 3 - 4 = -\frac{7}{2}$$

$K=1$

$$x(1) = \sum_{n=0}^{5} x(n) e^{-i\frac{2\pi}{5}n}$$

$$x(1) = x(6) e^{0} + x(1) e^{-i\frac{2\pi}{5}} + x(2) e^{-i\frac{4\pi}{5}} + \cancel{x(3) e^{-i\frac{2\pi}{6}}}^{0} + \cancel{x(4) e^{-i\frac{8\pi}{6}}}^{1} + \cancel{x(5) e^{-i3\pi}}^{1}$$

$$x(1) = 0 - \frac{1}{2}i - 1 - i - 3i - 4 = -5 - \frac{9}{2}i$$

$K=2$

$$x(2) = \sum_{n=0}^{5} x(n) e^{-i\frac{4\pi}{5}n}$$

$$x(2) = x(0) e^{0} + x(1) e^{-i\frac{4\pi}{5}} + x(2) e^{-i\frac{8\pi}{5}} + x(3) e^{-i\frac{12\pi}{5}} + x(4) e^{-i\frac{16\pi}{5}} + x(5) e^{-i3\pi}$$

$$= -\frac{3}{2} + \frac{3}{2}i$$

$$X(3) = -1 - 4i$$

$$x(4) = x(0)e^{0} + x(1)e^{-i\frac{8}{5}\pi} + x(2)e^{-i\frac{16\pi}{5}} + x(3)e^{-i\frac{24\pi}{5}} + x(4)e^{-i\frac{32\pi}{5}} + x(5)e^{-18\pi}$$

$$x(4) = \frac{1}{2} - 1 - i - 2 - 2i + 3i - 4 = \frac{15}{2}$$

$$x(5) = \sum_{n=0}^{5} x(n)\, e^{-i2\pi n}$$

$$x(5) = x(0)e^{0} + x(1)e^{-i2\pi} + x(2)e^{-i4\pi} + x(3)e^{-i8\pi} + x(4)e^{-i\pi} + x(5)e^{-20\pi} =$$

$$x(5) = \frac{1}{2} + 1 + 2 - 3 - 4 = -\frac{7}{2}$$

$$x(k) = \left\{ \dots, -\frac{7}{2}, -5, -\frac{9}{2}i, -\frac{3}{2} + \frac{3}{2}i, -1 - 4i, \frac{15}{2}, -\frac{7}{2}, \dots \right\}$$

Problema 5: Use la forma matricial de la DFT y calcule:

$$g(n) = \{0.5, 0, 2, 5\}$$

Solución

$$x(k) = \sum_{n=0}^{N-1} x(n)\, e^{-i\frac{2\pi}{N}kn} \quad ; \quad x(k) = \sum_{n=0}^{3} x(n)\, e^{-i\frac{2\pi}{3}kn}$$

$$x(k) = \sum_{n=0}^{N-1} x(n)\, W^{kn} \qquad X_k = W_N X_n$$

$$W_N = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{vmatrix} \begin{vmatrix} 5 \\ 0 \\ 2 \\ 5 \end{vmatrix} = \begin{vmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0 \\ 2 & -2 & 2 & -2 \\ 5 & 5i & -5 & -5i \end{vmatrix} \begin{vmatrix} 2 \\ 0 \\ 0 \\ 0 \end{vmatrix}$$

$$g(k) = \{2, 0, 0, 0\}$$

Halle la FFT para la siguiente secuencia:

$$x[n] = [0.5, 1, 0, 12]$$

Solución

$N = 4$; $\log_2 N = \log_2 4 = \underline{2}$; $W^2 = \left(e^{-i\frac{2\pi}{4}}\right) = \left(e^{-i\frac{\pi}{2}}\right) = \underline{-i}$

$\frac{N}{2} = 2$;

$x(0)$ $x'(0)$ W^0

$x(1)$ $x'(1)$ W^0

$x(2)$ $x'(2)$ W^0

$x(3)$ $x'(3)$ $-W^0$

$x'(0) = 0.5 + W^0(0) = 0.5$

$x'(1) = 0.5 + W^0(0) = 0.5$

$x'(2) = 1 + W^0(12) = 13$

$x'(3) = 1 + W^0(12) = -1$

$X_0 = 0.5 + 13 = 13.5$

$X_1 = 0.5 + W^1(-11) = 0.5 + (-i)(11) = 0.5 + 11i$

$X_2 = 0.5 + 13(-1) = -8$

$X_3 = 0.5 - (W^1)(-11) = 0.5 - (-i)(-11) = 0.5 - 11i$

Encuentre la tranformada z y grafique la región de convergencia

$$h(n) = \begin{cases} \left(\frac{1}{6}\right)^n & n \geq 0 \\ 0 & n \leq 0 \end{cases}$$

Solución

$$z\{h(n)\} = h(z) = \sum_{n=-\infty}^{\infty} h(n)\, z^{-n}$$

$$z\{h(n)\} = h(z) = \sum_{n=-\infty}^{0} 0 + \sum_{n=0}^{\infty} \left(\frac{1}{6}\right)^n z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{6} z^{-1}\right)^n$$

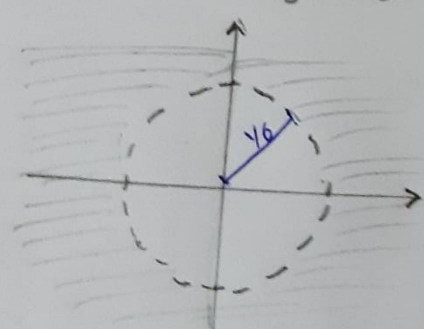
Por series geometricas

$$\sum_{k=m}^{\infty} \alpha^k = \frac{\alpha^m}{1-\alpha} \qquad |\alpha| < 1$$

$$h(z) = \frac{\frac{1}{6z}}{1 - \frac{1}{6z}} \; ; \; \left|\frac{1}{6z}\right| < 1 \Rightarrow h(z) = \frac{\cancel{6z}}{(6z-1)\cancel{(6z)}} = \frac{1}{6z-1}$$

$$\hookrightarrow \frac{6z-1}{6z}$$

región de convergencia



Polos de $h(z)$

$z = \frac{1}{6}$

Encuentre la transformada z y grafique la región de convergencia de h(n)

$$h(n) = \begin{cases} (10)^n & n<0 \quad \text{anticausal} \\ \left(\frac{1}{10}\right)^n & n \text{ par} \quad \text{causal} \\ \left(\frac{1}{5}\right)^n & n \text{ impar} \quad \text{causal} \end{cases}$$

Solución

$$z\{h(n)\} = h(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n}$$

$$h(z) = \sum_{n=-\infty}^{n=-1} 10^n z^{-n} + \underbrace{\sum_{n=0}^{\infty} \left(\frac{1}{10}\right)^n z^{-n}}_{\text{"par"}} + \underbrace{\sum_{n=0}^{\infty} \left(\frac{1}{5}\right)^n z^{-n}}_{\text{"impar"}}$$

$$h(z) = \sum_{n=-\infty}^{n=-1} (10z^{-1})^n + \sum_{n=0}^{\infty} \left(\frac{1}{10}\right)^{2n} z^{-2n} + \sum_{n=0}^{\infty} \left(\frac{1}{5}\right)^{2n+1} z^{-(2n+1)}$$

$$h(z) = \sum_{n=-\infty}^{n=-1} \left(\frac{10}{z}\right)^n + \sum_{n=0}^{\infty} \left[\left(\frac{1}{10}\right)^2 \frac{1}{z^2}\right]^n + \sum_{n=0}^{\infty} \left(\frac{1}{5}\right)^{2n} \left(\frac{1}{5}\right) \cdot z^{-2n} z^{-1}$$

$$h(z) = \sum_{n=1}^{\infty} \left(\frac{z}{10}\right)^n + \sum_{n=0}^{\infty} \left[\frac{1}{100z^2}\right]^n + \frac{1}{5z} \sum_{n=0}^{\infty} \left(\frac{1}{25z^2}\right)$$

De series geométricas

$$\sum_{k=0}^{\infty} \alpha^k = \frac{1}{1-\alpha} \quad |\alpha| < 1 \quad ; \quad \sum_{k=m}^{\infty} \alpha^k = \frac{\alpha^m}{1-\alpha} \quad |\alpha| < 1$$

Entonces:

$$h(z) = \underbrace{\frac{\frac{z}{10}}{1-\frac{z}{10}}}_{\left|\frac{z}{10}\right|<1} + \underbrace{\frac{1}{1-\frac{1}{100z^2}}}_{\left|\frac{1}{100z^2}\right|<1} + \frac{1}{5z} \cdot \underbrace{\frac{1}{1-\frac{1}{25z^2}}}_{\left|\frac{1}{25z^2}\right|<1}$$

$$h(z) = \frac{\frac{z}{10}}{\frac{10-z}{10}} + \frac{1}{\frac{100z^2-1}{100z^2}} + \frac{1}{5z} \cdot \frac{1}{\frac{25z^2-1}{25z^2}}$$

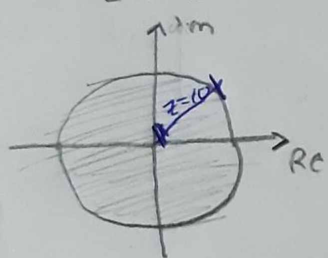
$$h(z) = \frac{z}{10-z} + \frac{100z^2}{100z^2-1} + \frac{1}{5z} \cdot \frac{25z^2}{25z^2-1}$$

$$h(z) = \underbrace{\frac{z}{10-z}}_{|z|<10} + \underbrace{\frac{z^2}{z^2-\frac{1}{100}}}_{\frac{1}{100}<|z^2|} + \underbrace{\frac{1}{5z} \cdot \frac{z^2}{z^2-\frac{1}{25}}}_{\frac{1}{25}<|z^2|}$$

Convergencias:

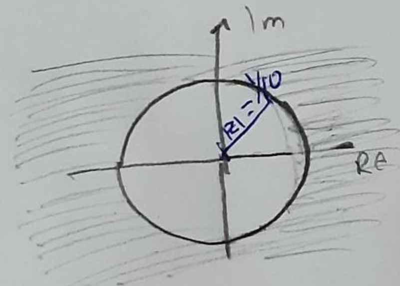
Polos

$z = 10$



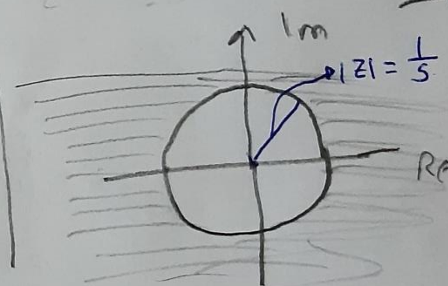
Polo

$z^2 = \frac{1}{100} \Rightarrow z = \pm\frac{1}{10}$



Tiene 2 polos

$z_1 = 0$; $|z_2^2| = \frac{1}{25} \Rightarrow z_2 = \pm\frac{1}{5}$



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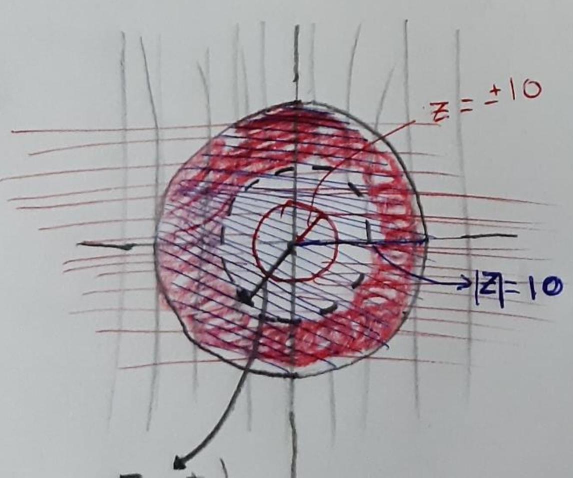

$\frac{1}{5} < |z| < 10$