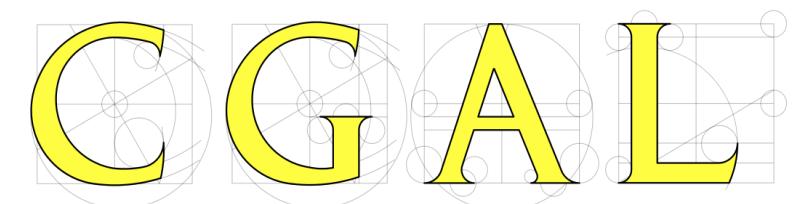
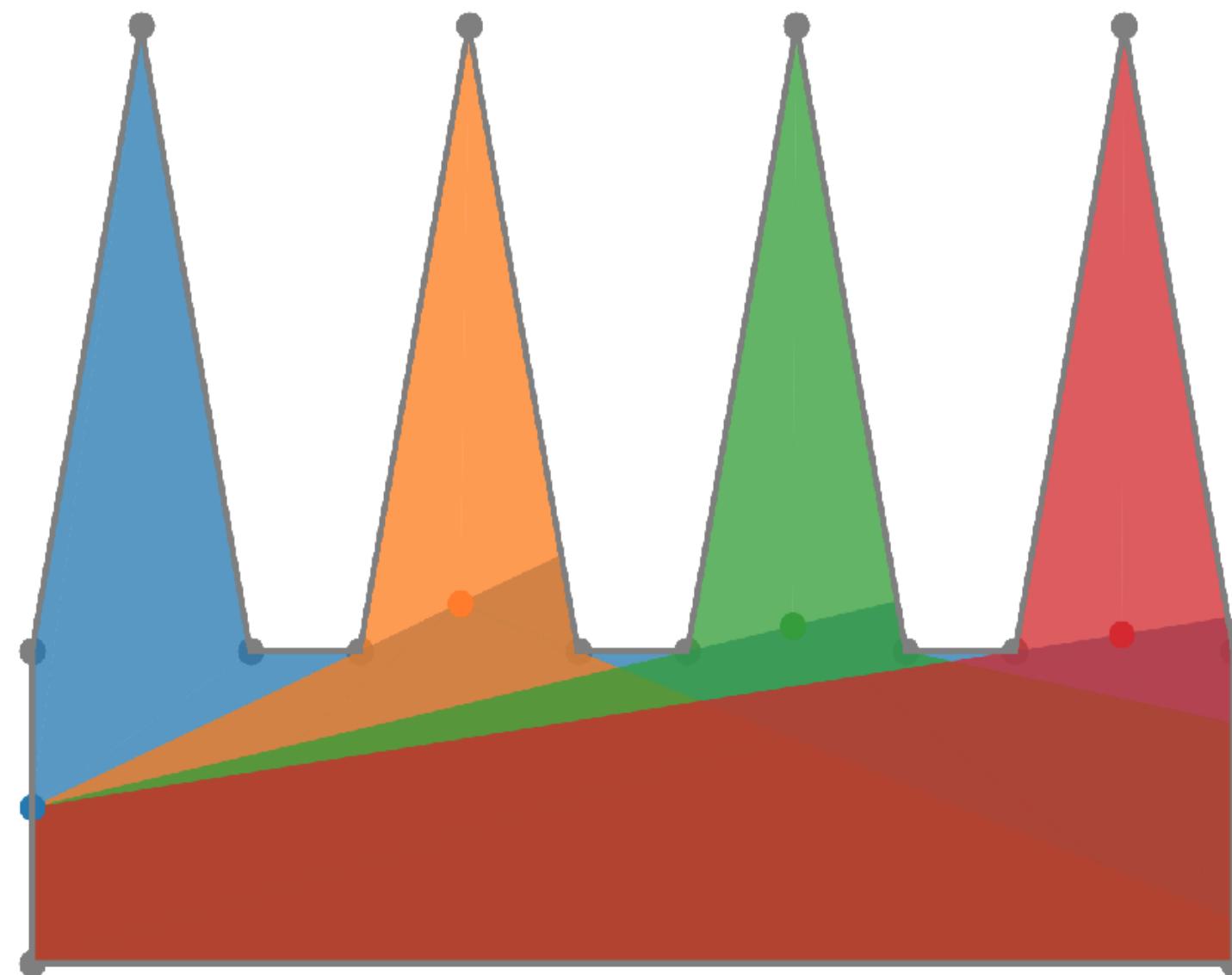


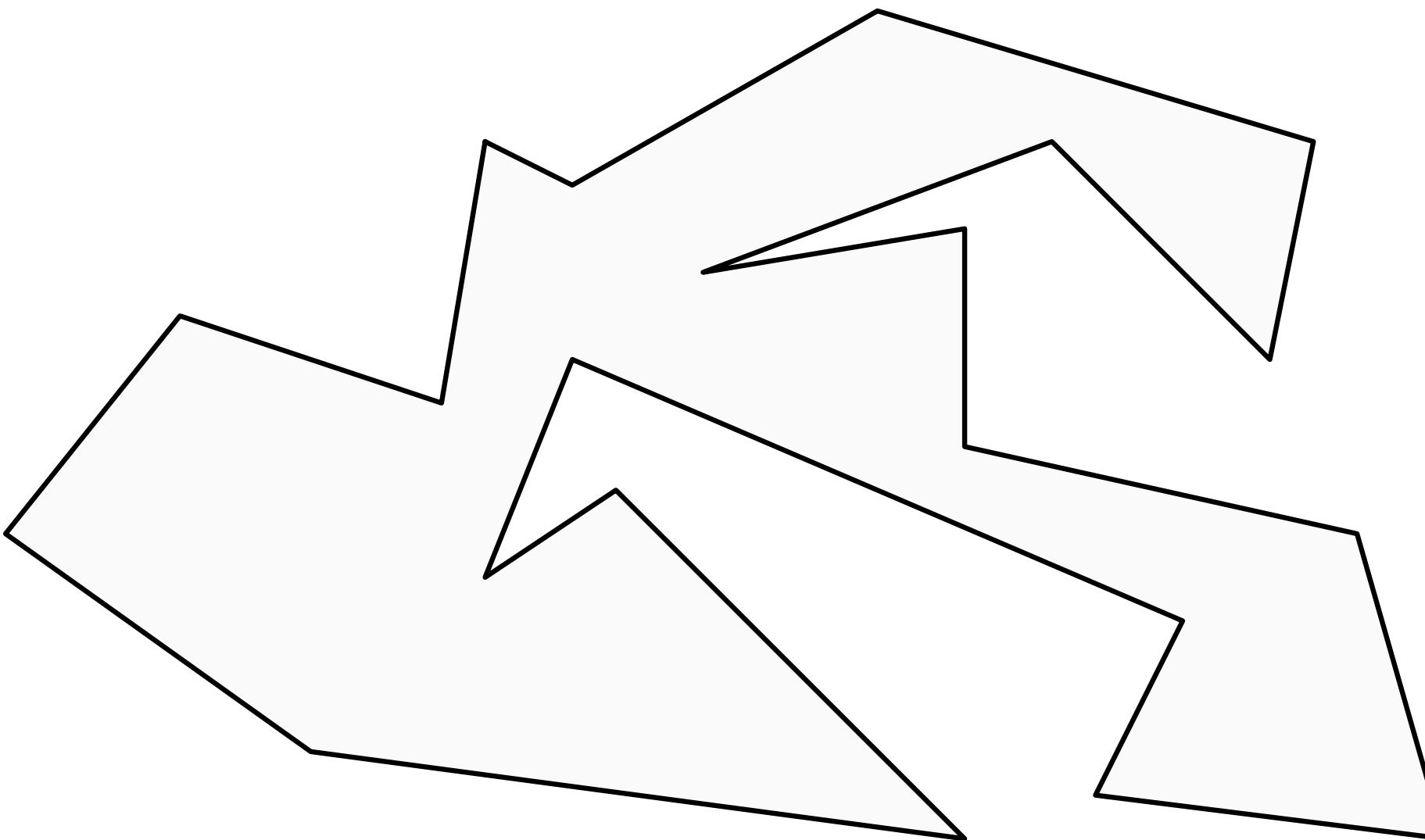
Solving the Art Gallery Problem Using Gradient Descent

Master's Thesis Defense

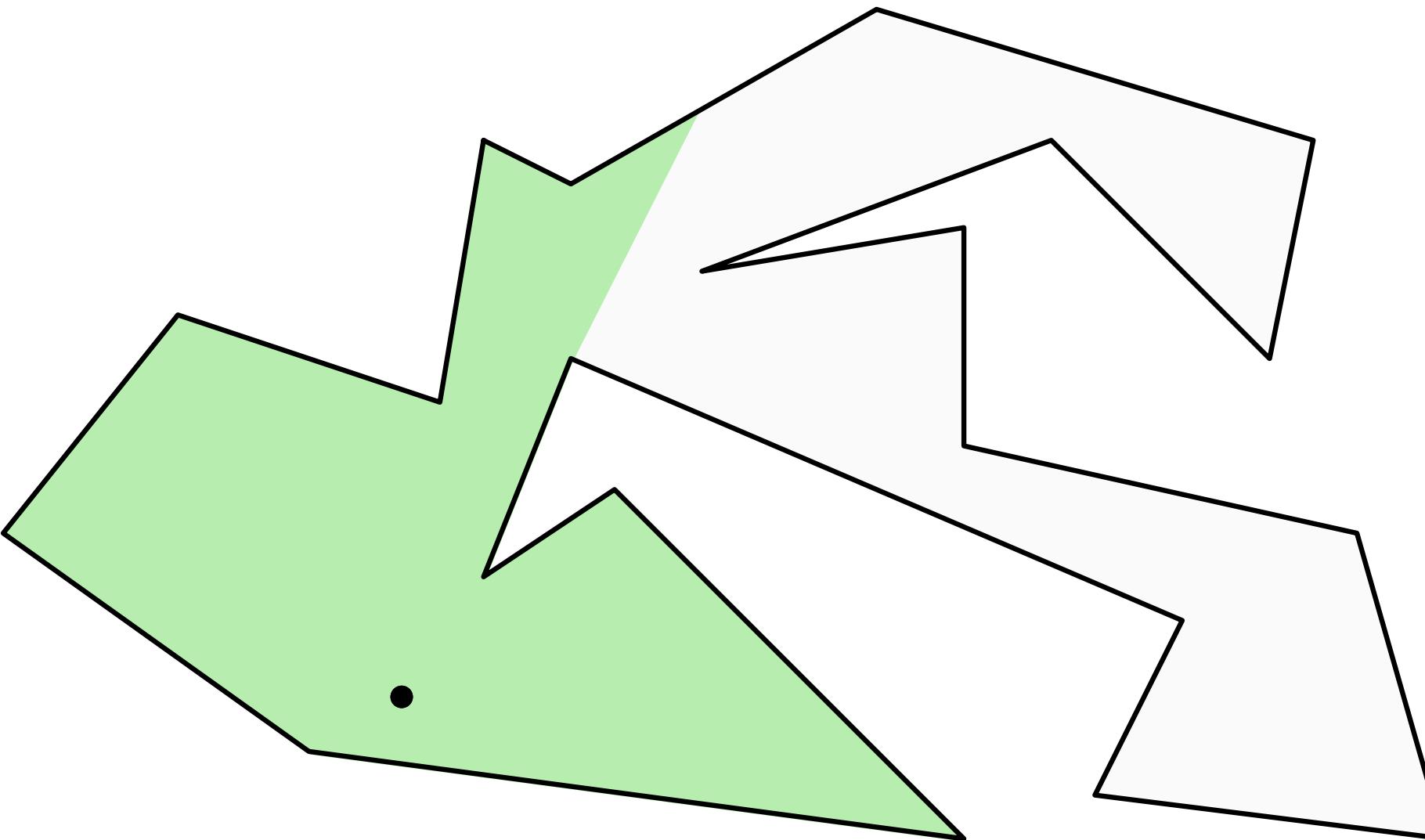
Geo Juglan



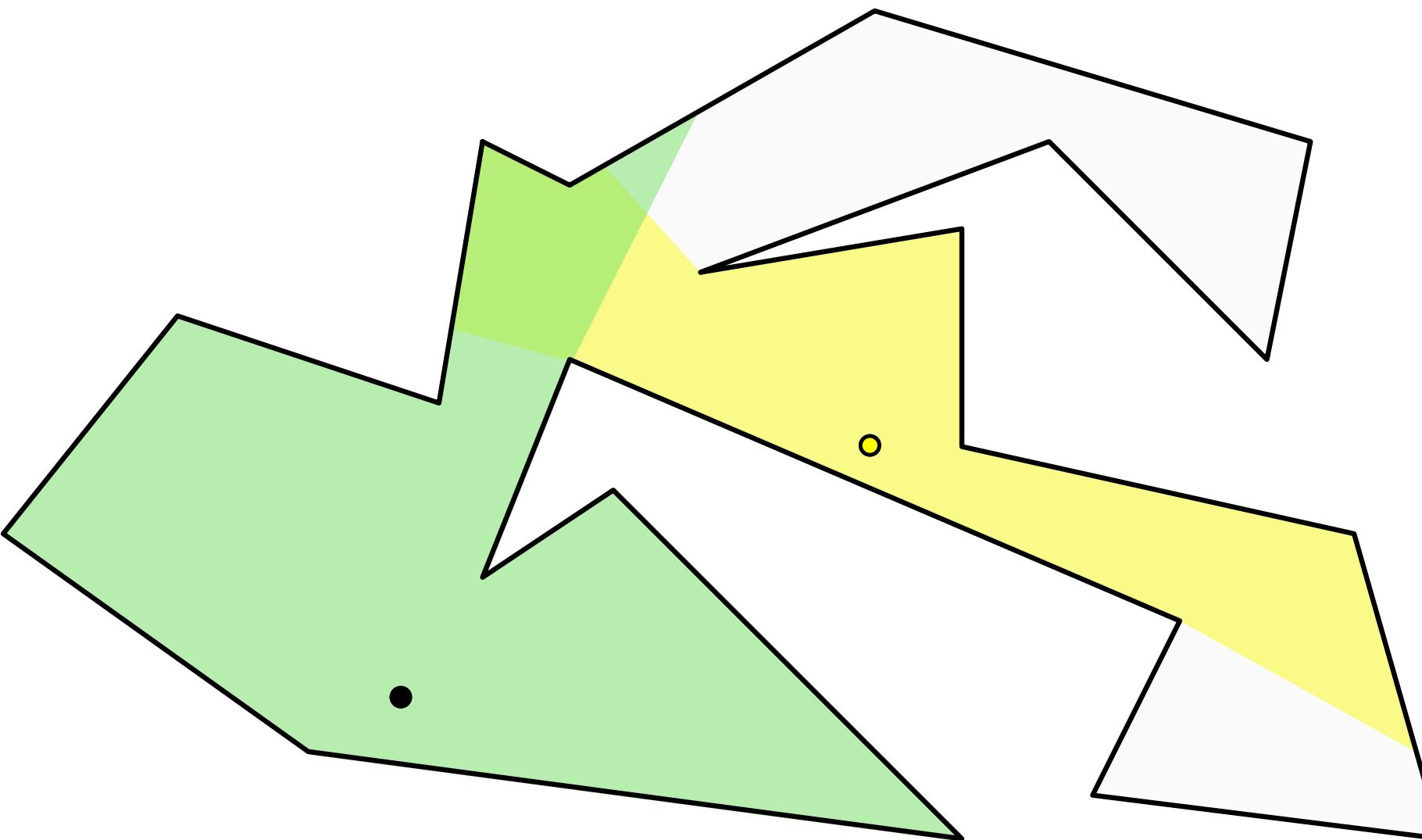
The Art Gallery Problem



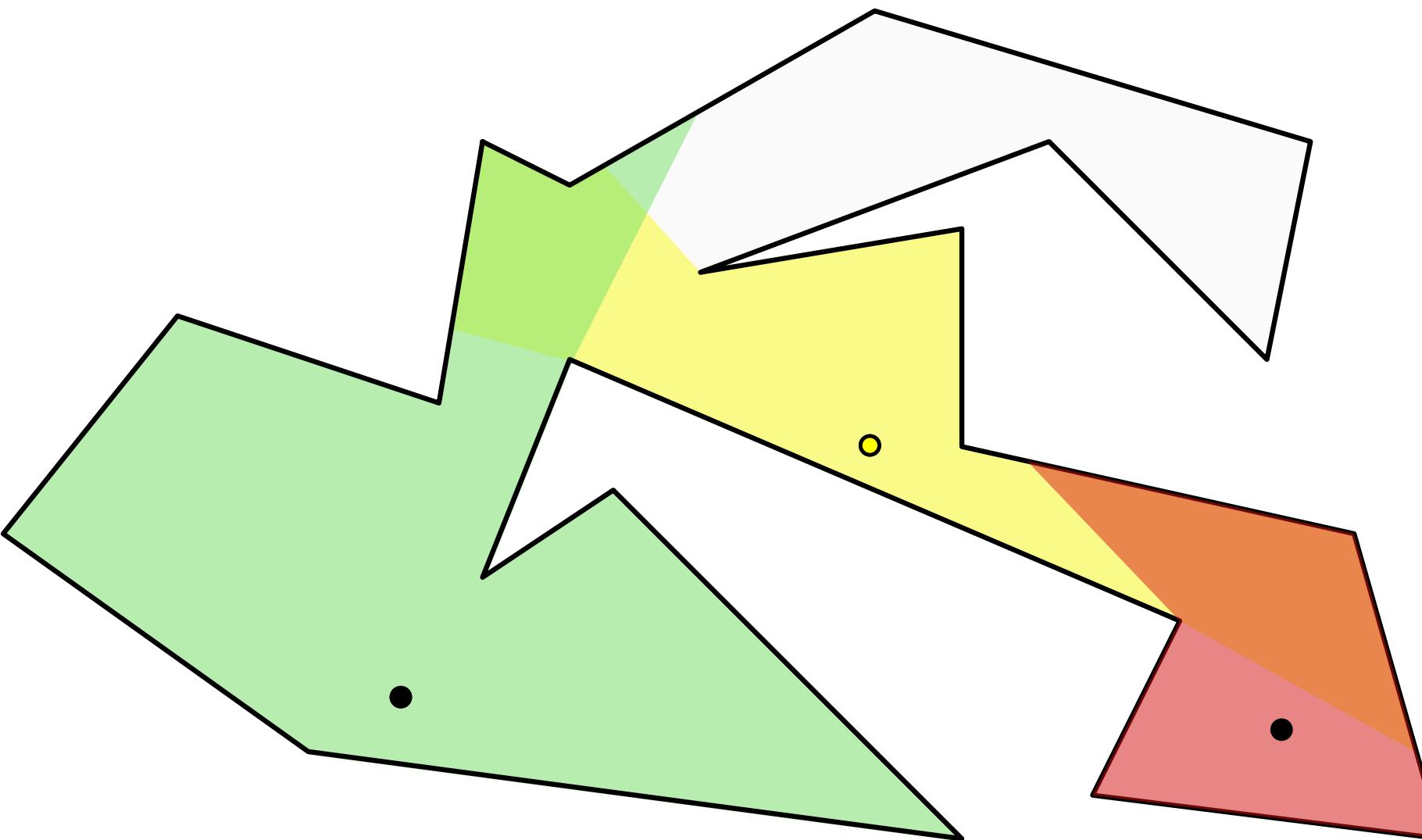
The Art Gallery Problem



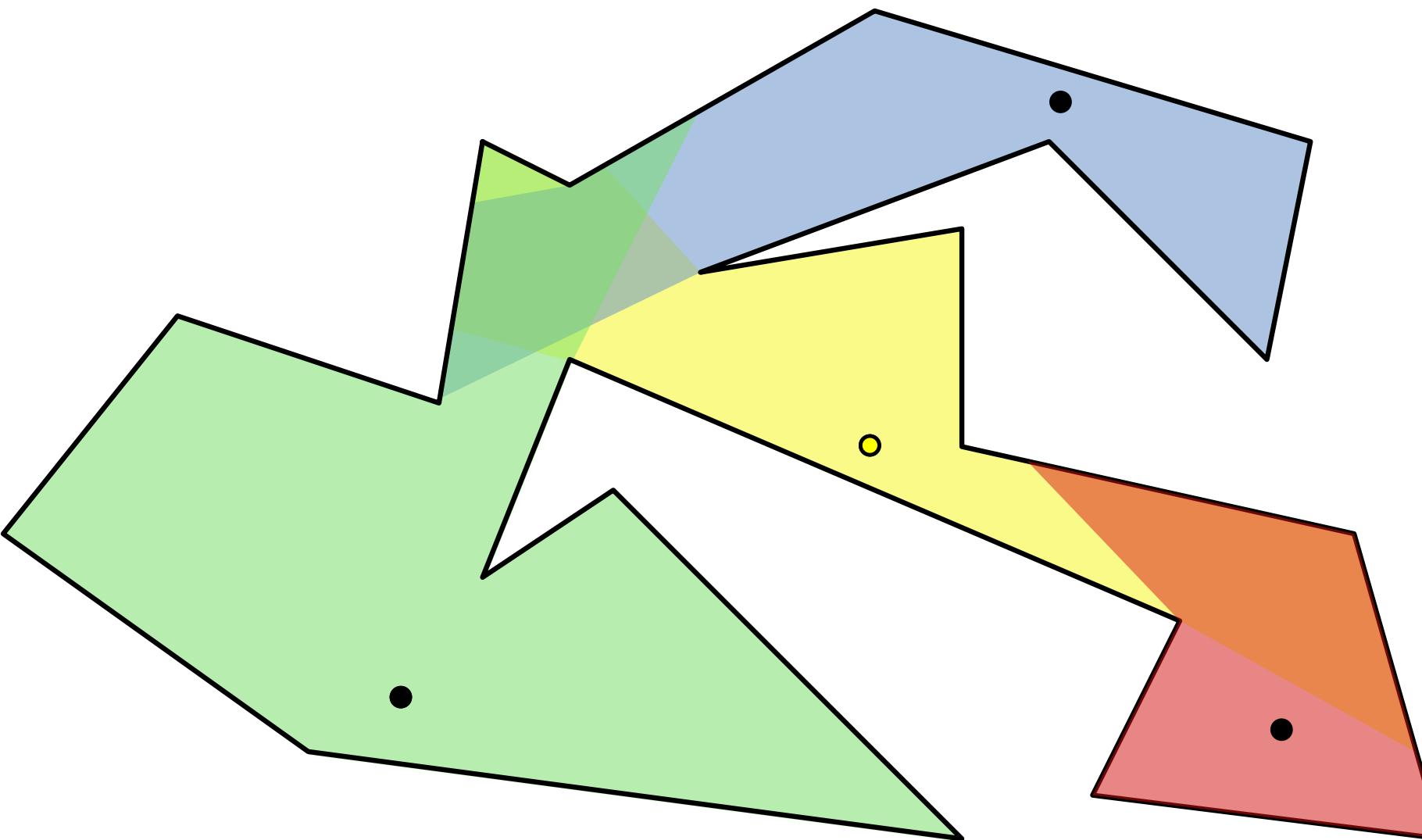
The Art Gallery Problem



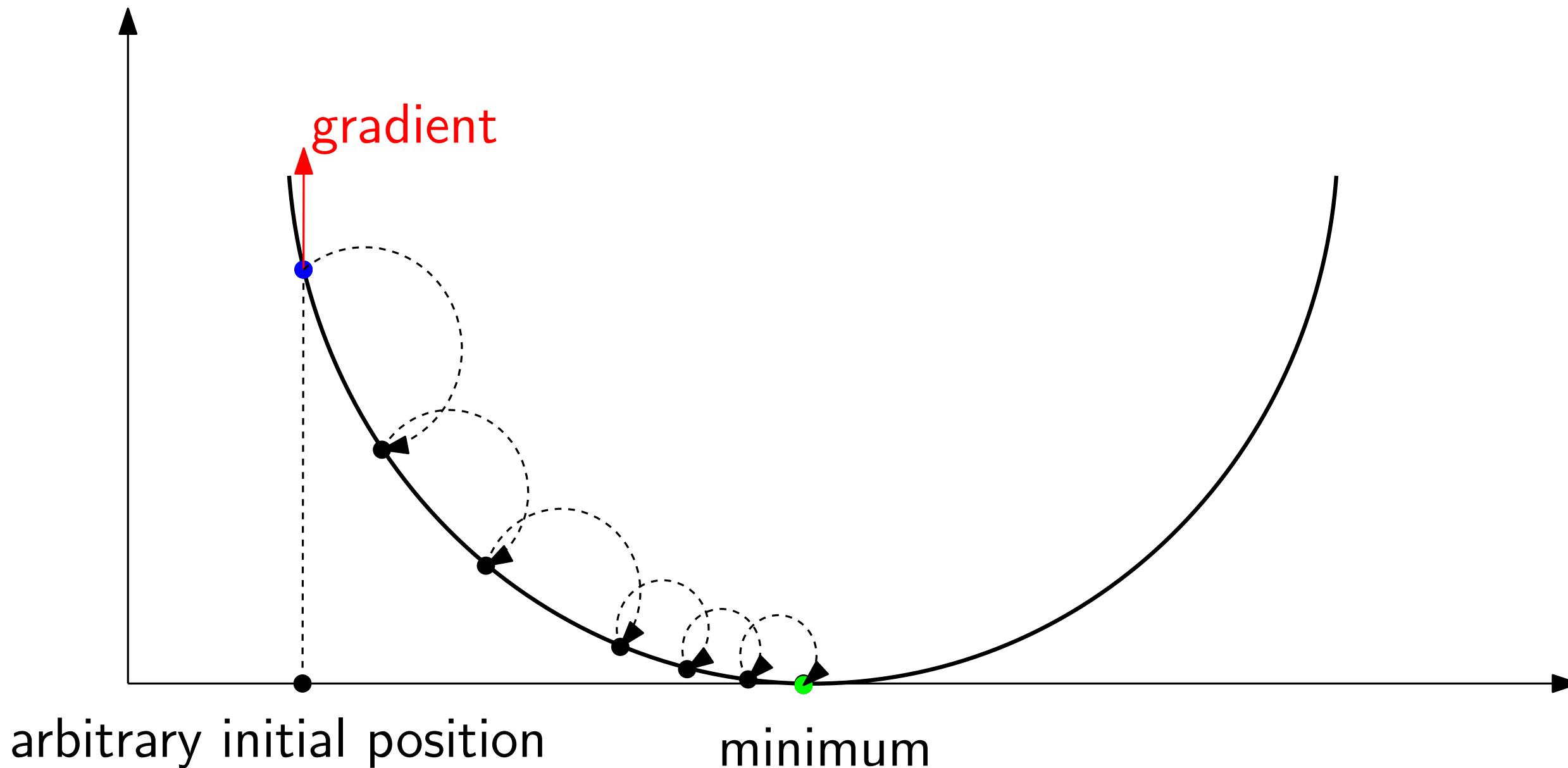
The Art Gallery Problem



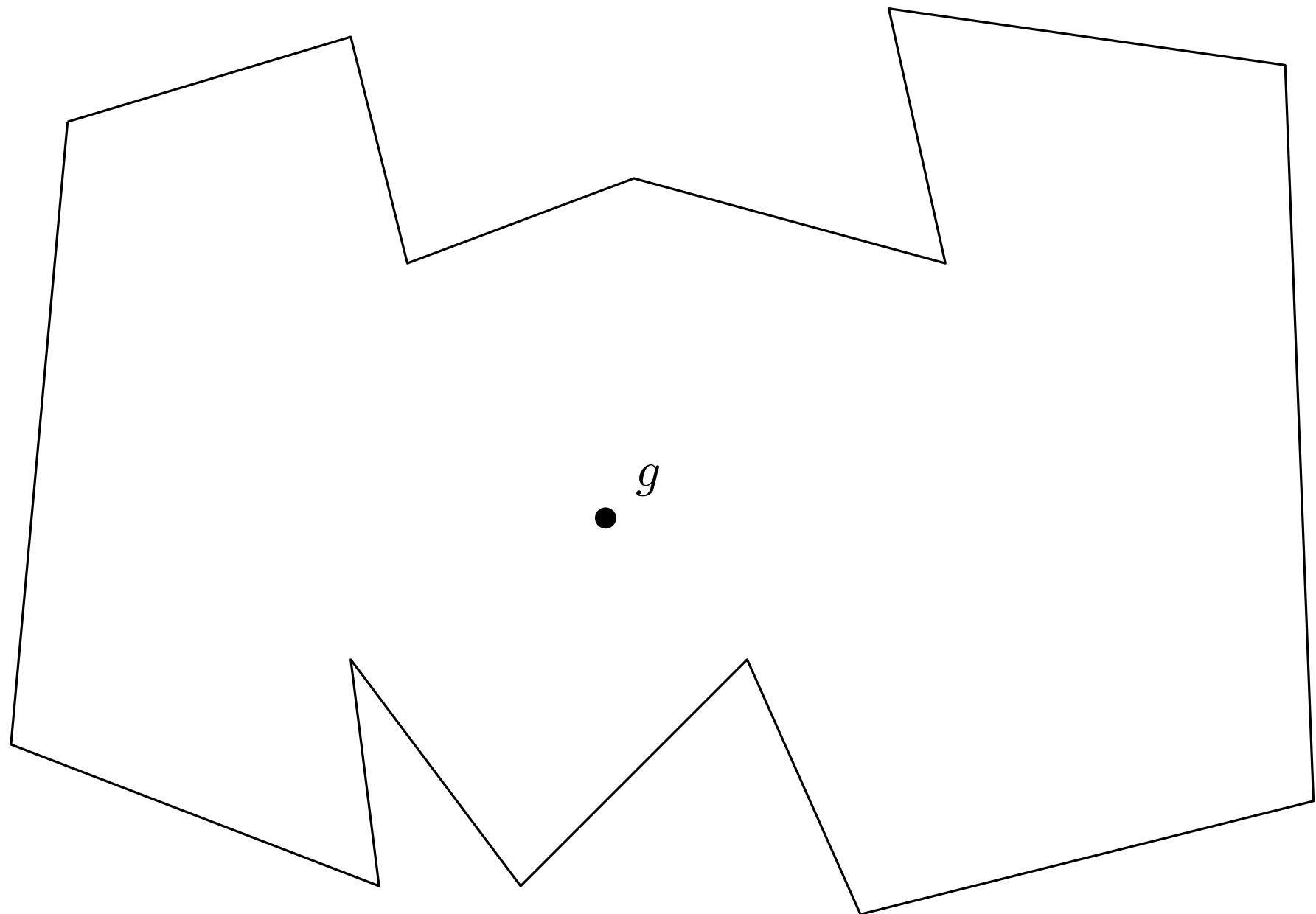
The Art Gallery Problem



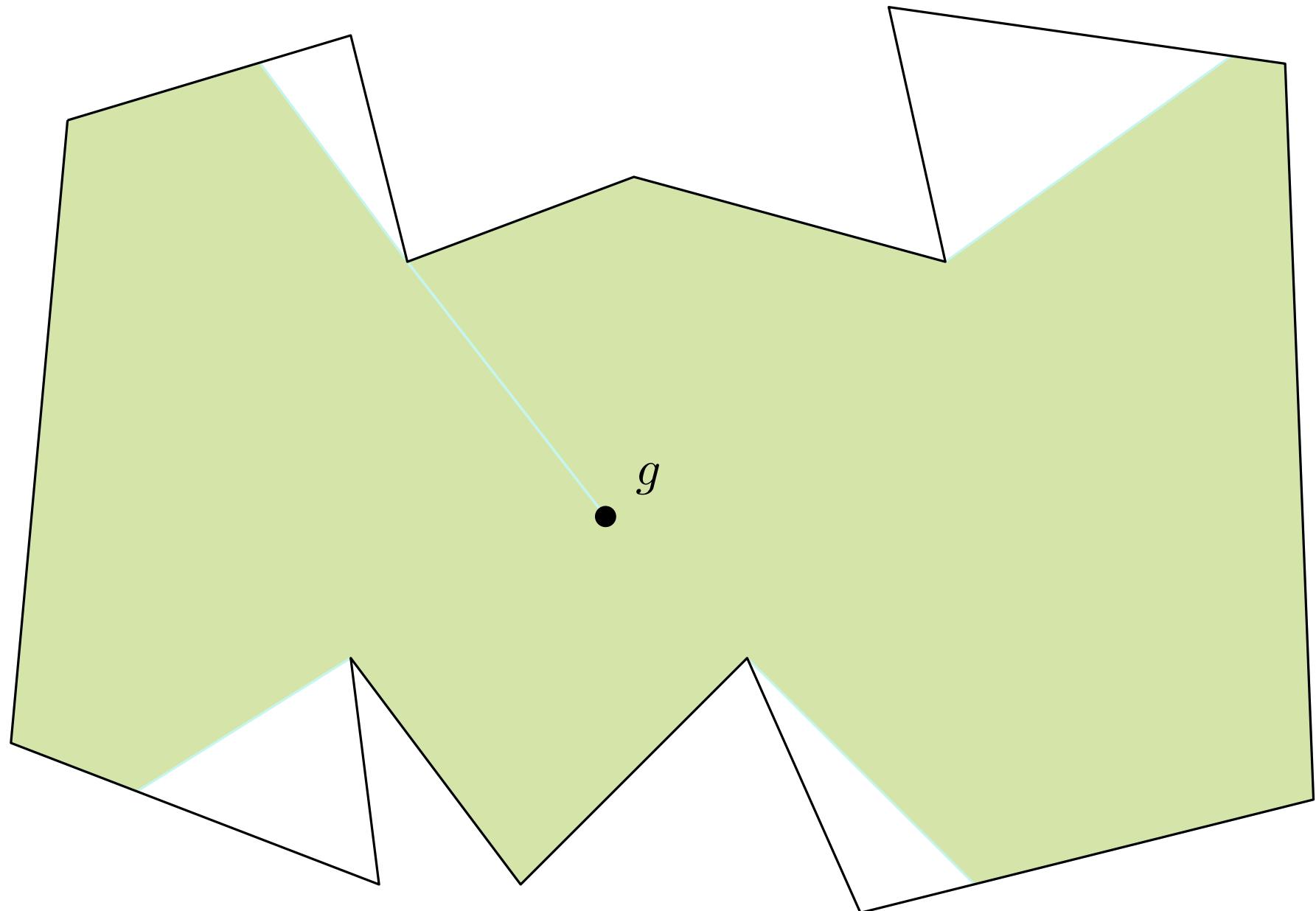
Gradient Descent



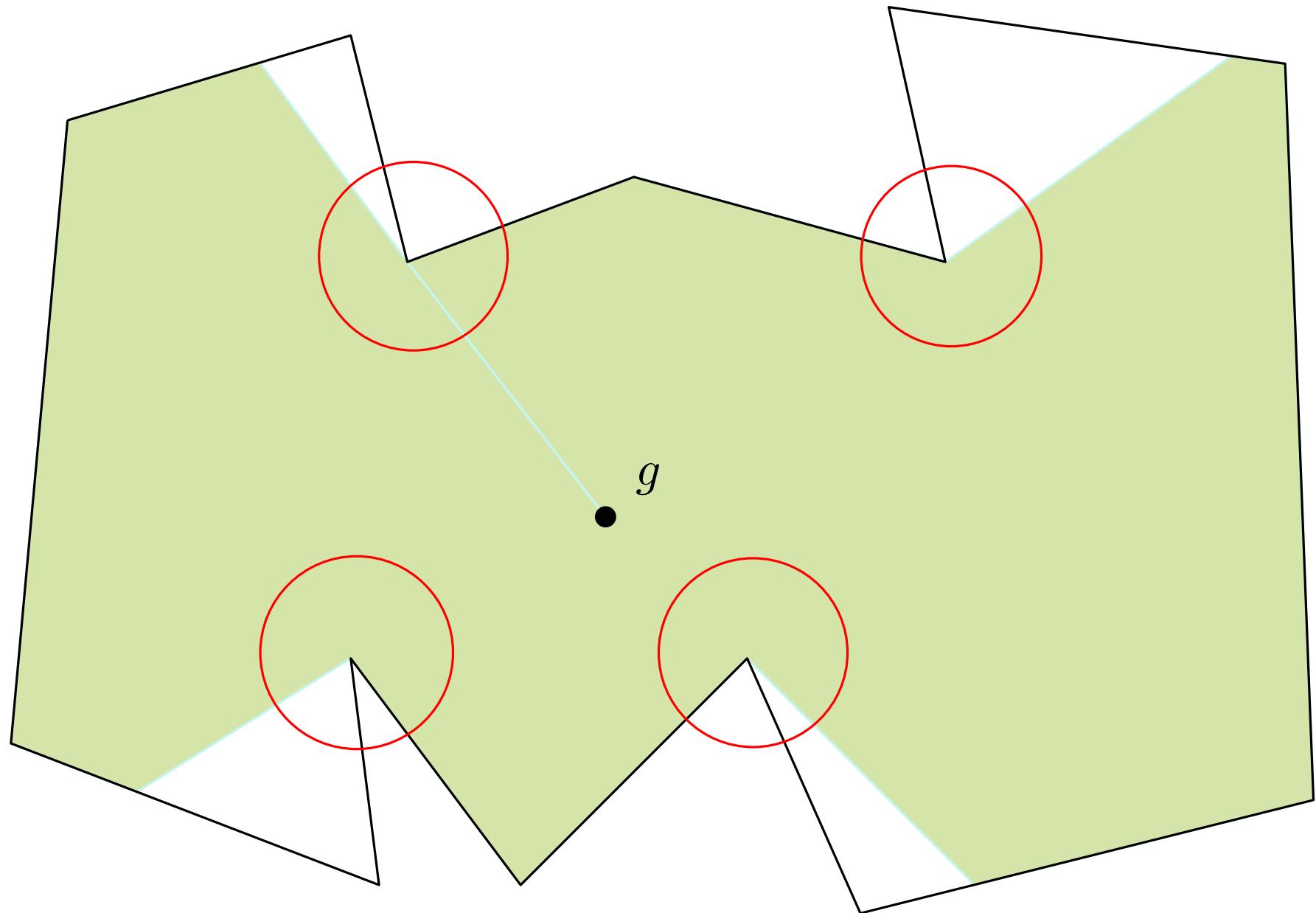
Computing the gradient



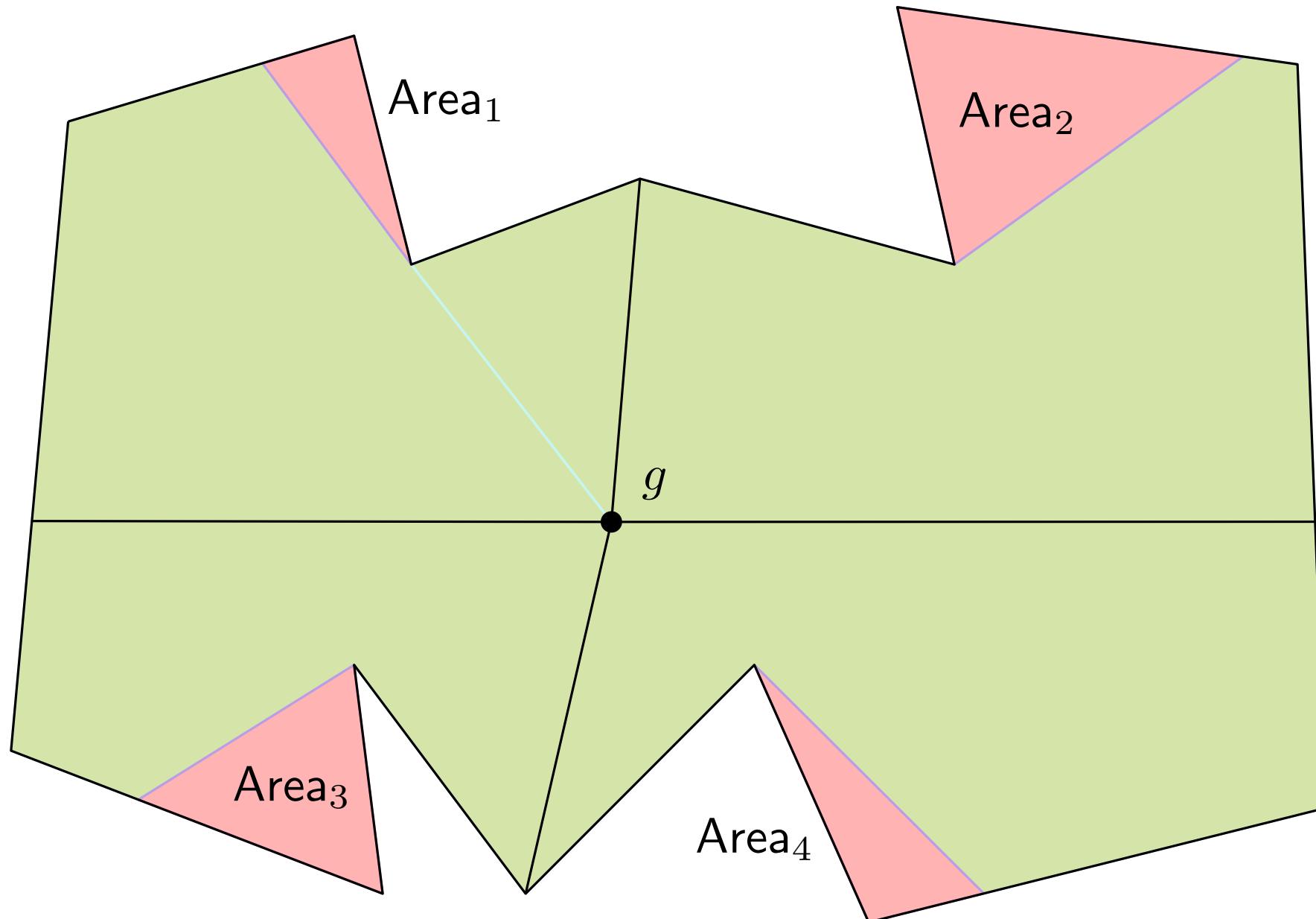
Computing the gradient



Computing the gradient

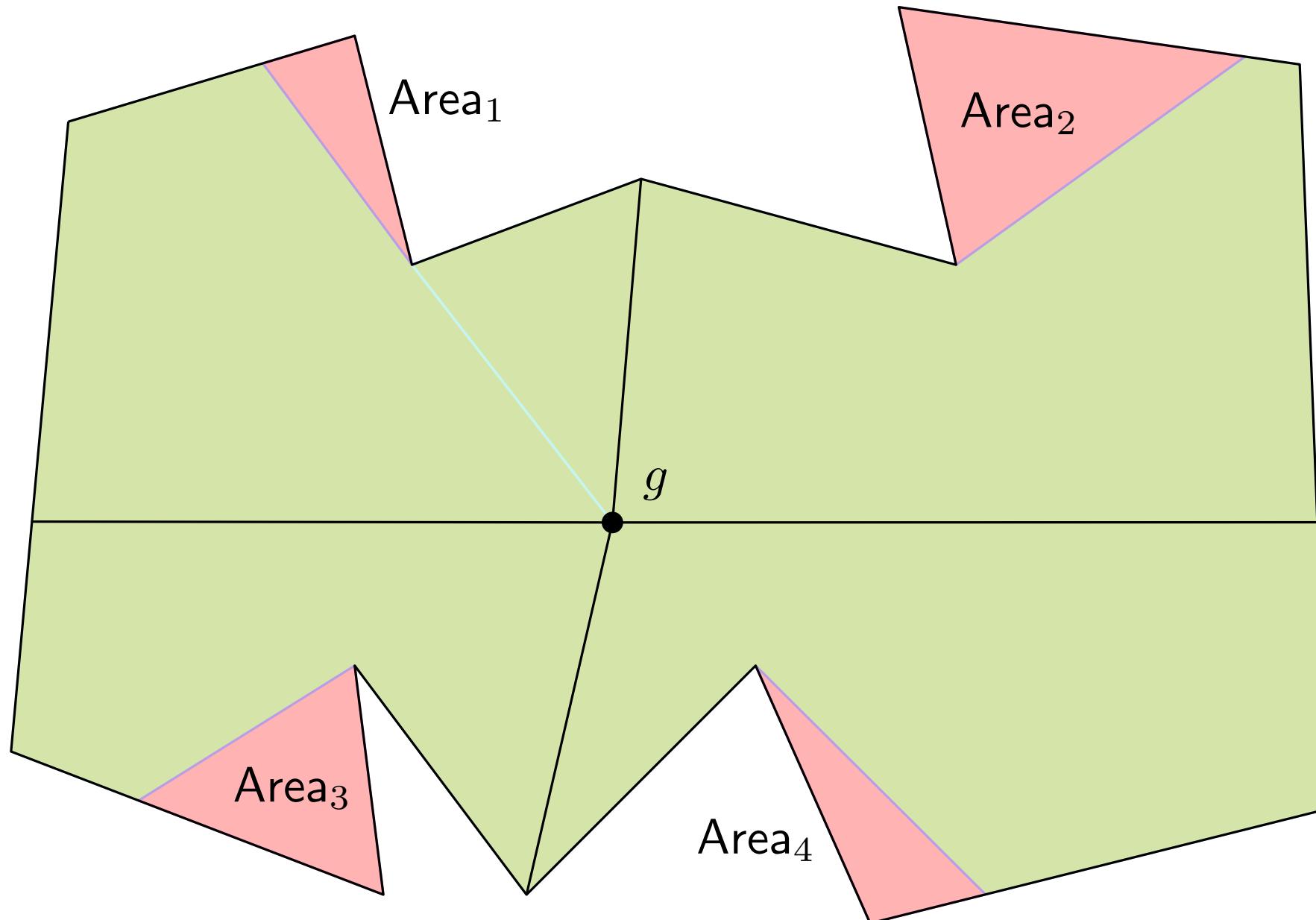


Computing the gradient



$$f(g) = \text{Area}_1 + \text{Area}_2 + \text{Area}_3 + \text{Area}_4$$

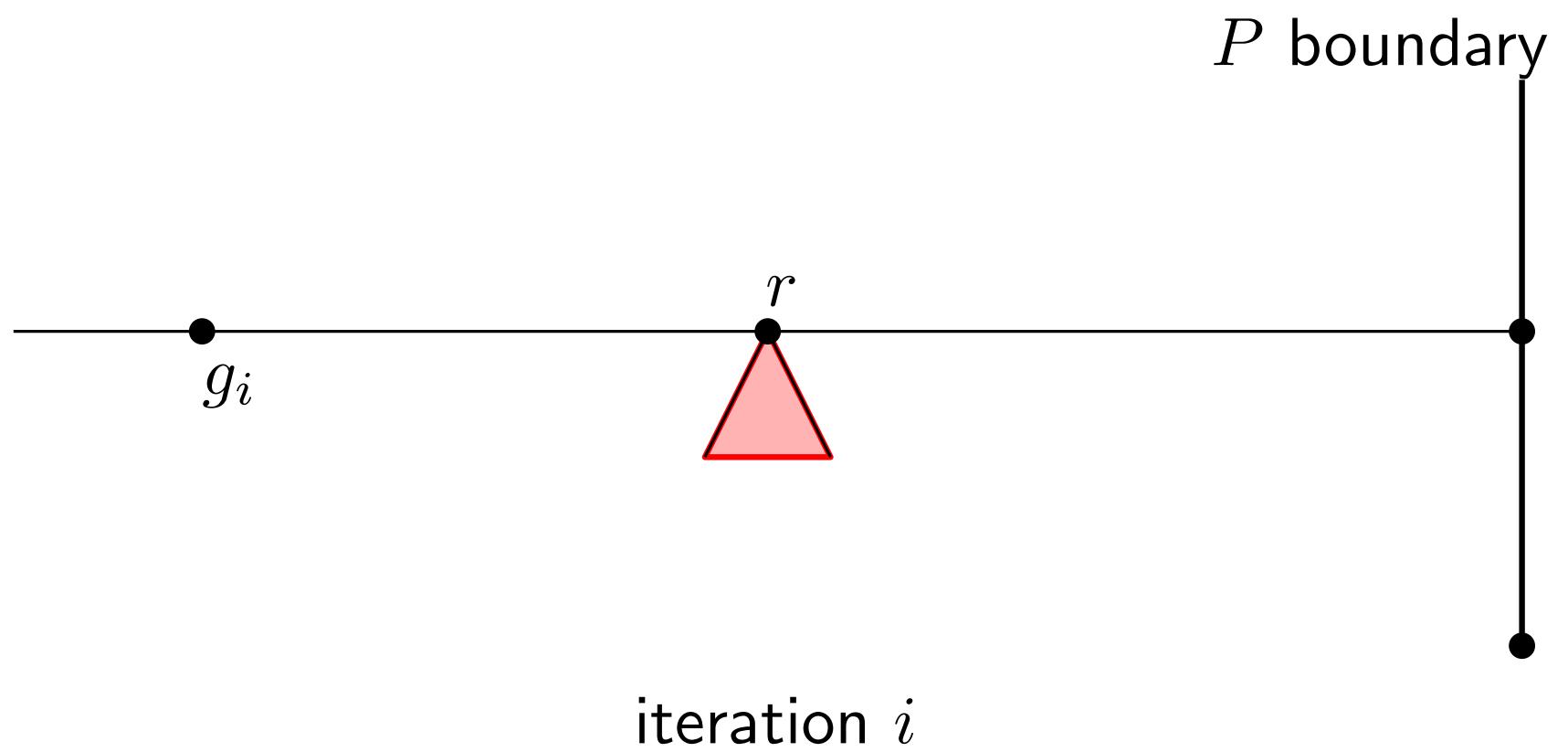
Computing the gradient



$$f(g) = \text{Area}_1 + \text{Area}_2 + \text{Area}_3 + \text{Area}_4$$

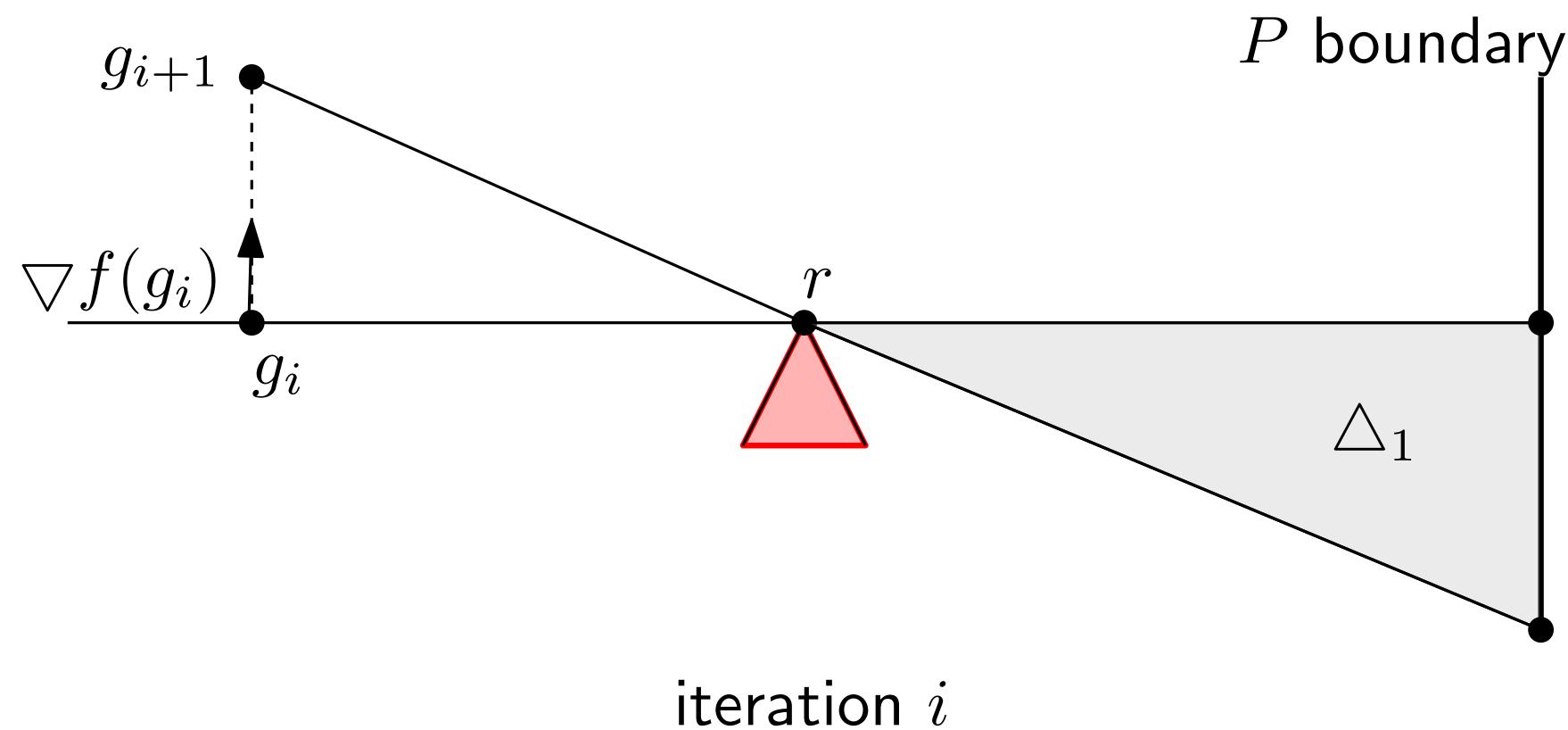
$$\nabla f(g) = \nabla \sum_i \text{Area}_i = \sum_i \nabla \text{Area}_i$$

Computing the gradient



Computing the gradient

$$\nabla f(g_i) = \nabla \text{Area}_{\triangle_1}(g_i)$$

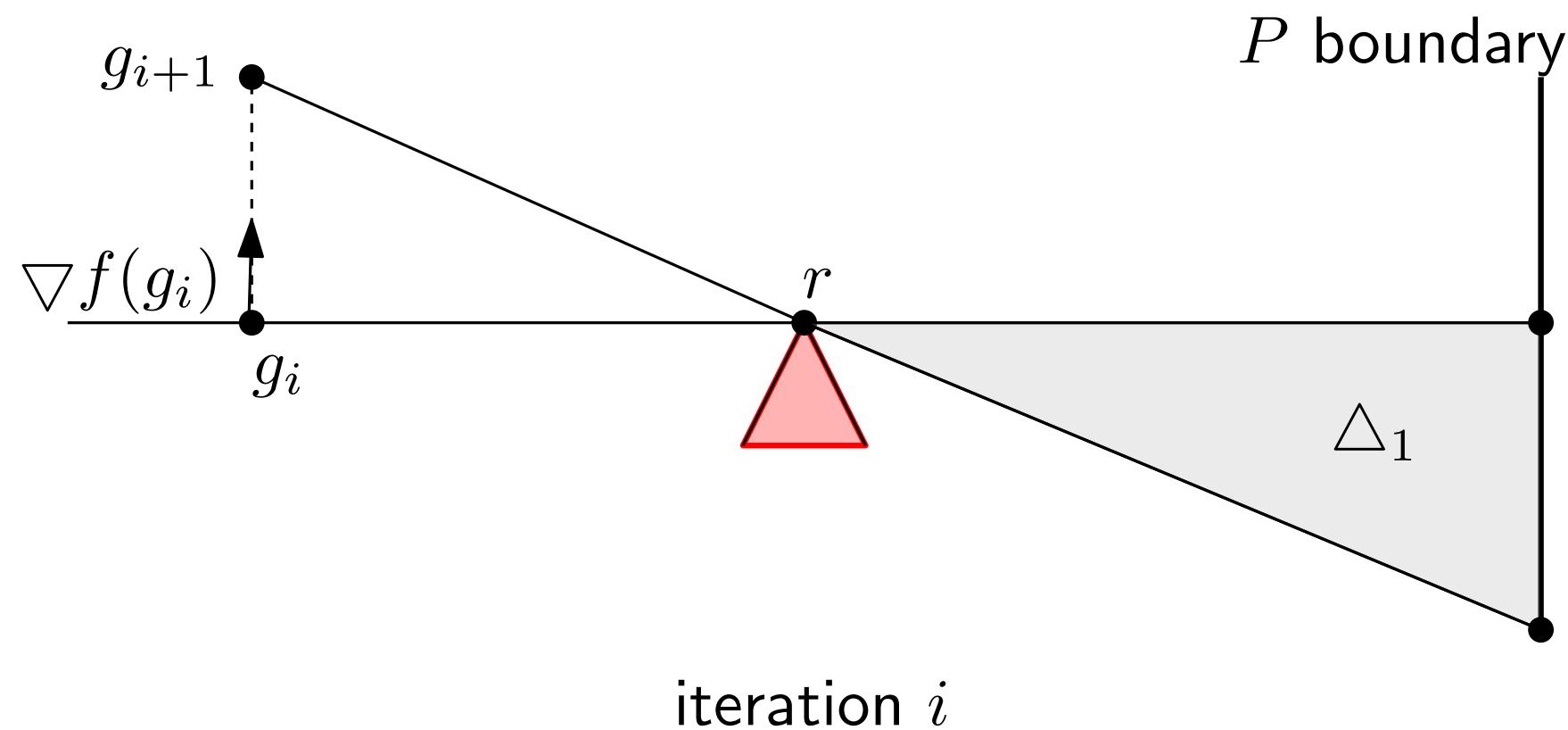


Computing the gradient

$$\nabla f(g_i) = \nabla \text{Area}_{\triangle_1}(g_i)$$

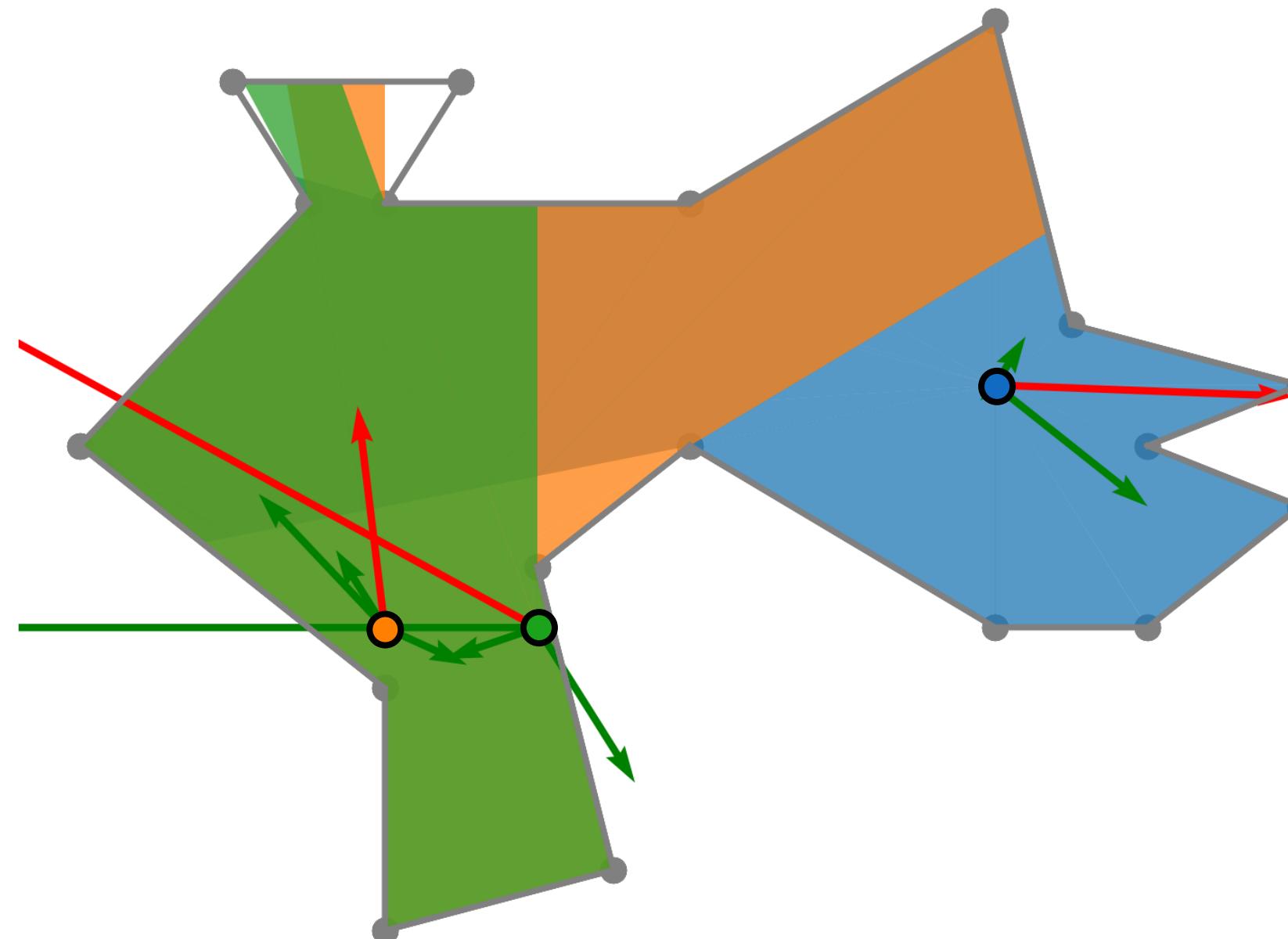
$$g_{i+1} = g_i + \alpha \nabla f(g_i)$$

α - learning rate



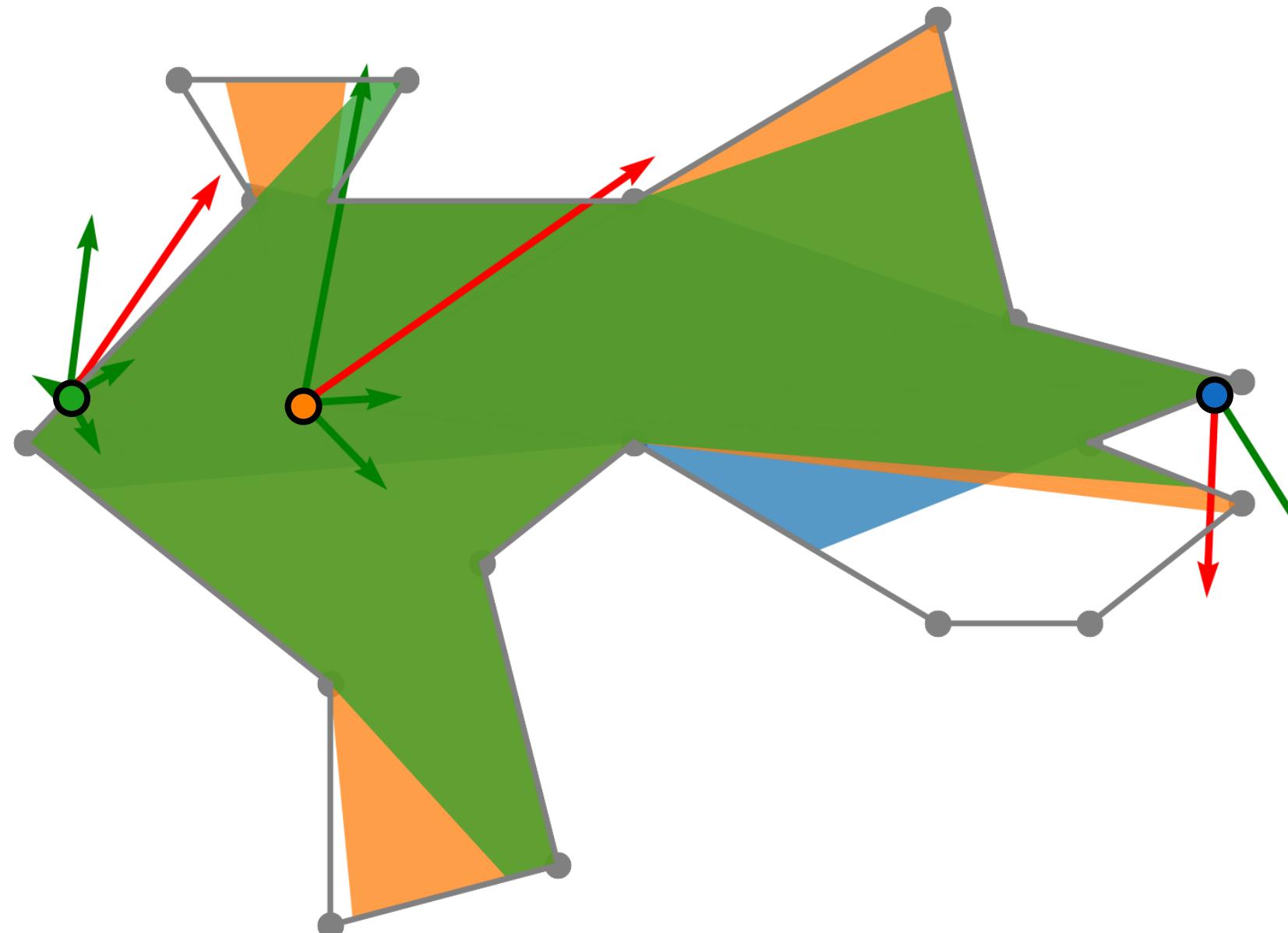
Solving the art gallery problem using the gradient

Gradient Computation for Iteration #0



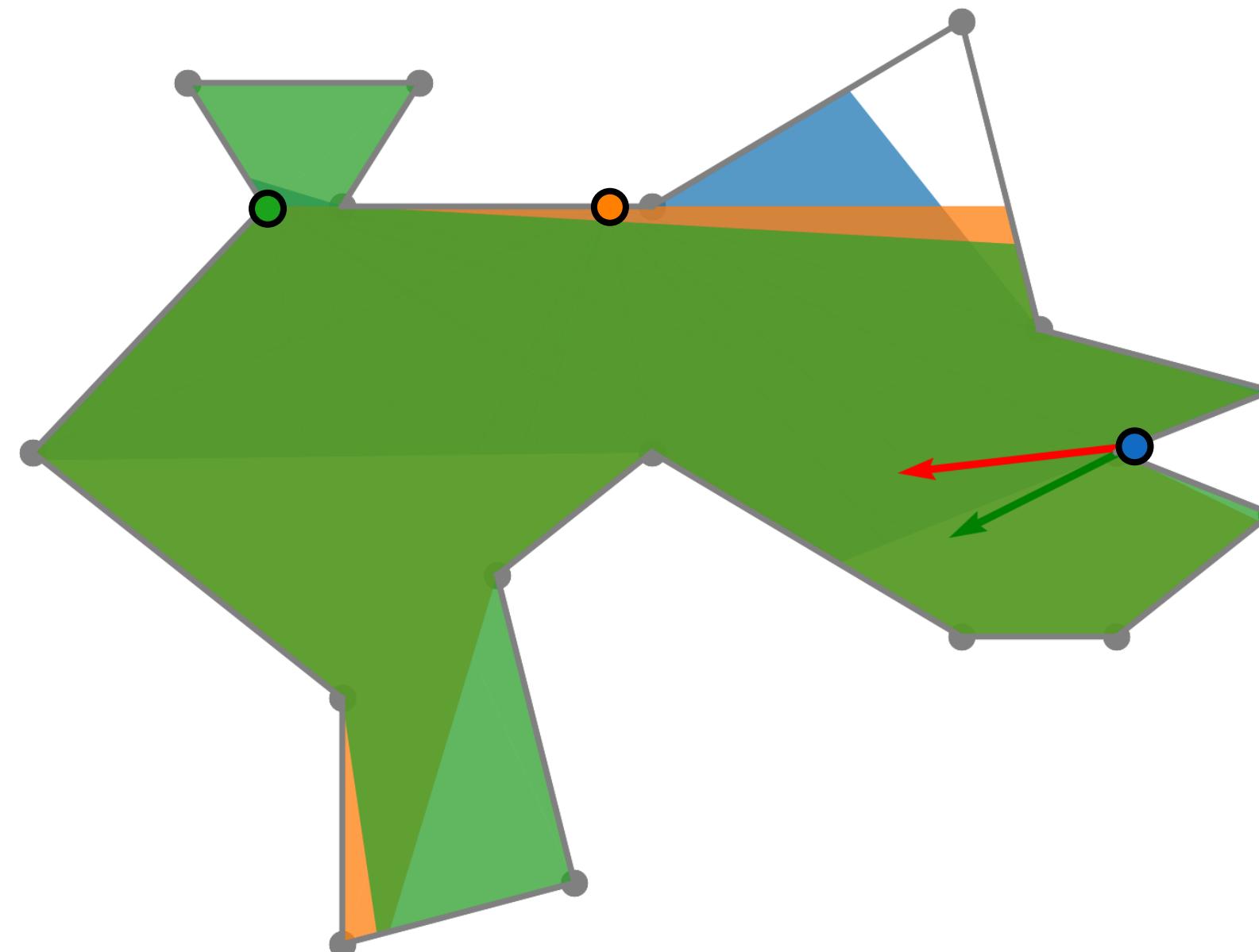
Solving the art gallery problem using the gradient

Gradient Computation for Iteration #1



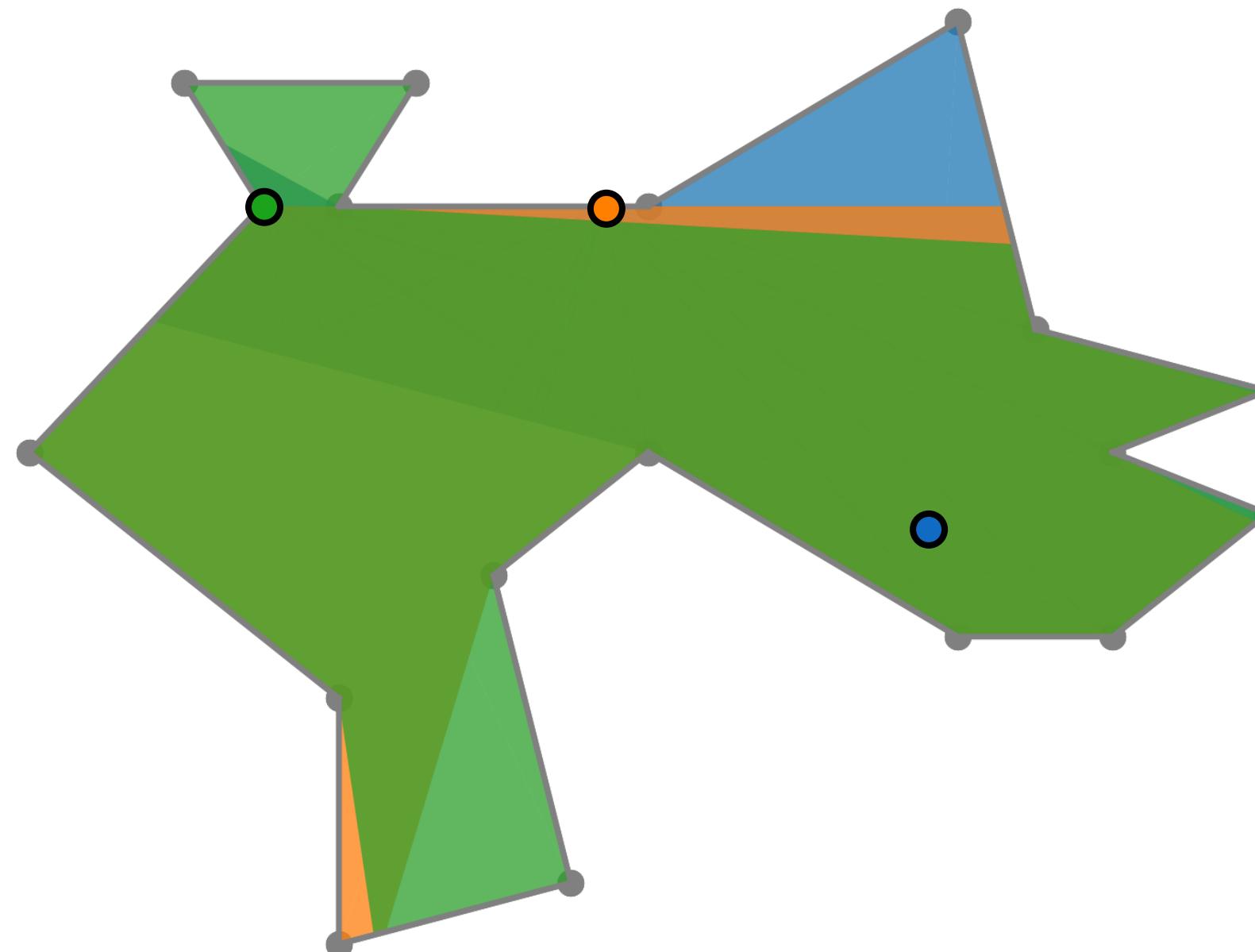
Solving the art gallery problem using the gradient

Gradient Computation for Iteration #2



Solving the art gallery problem using the gradient

Gradient Computation for Iteration #3



Heuristics

momentum

pull towards reflex vertex

pull onto the reflex vertex

pull capping

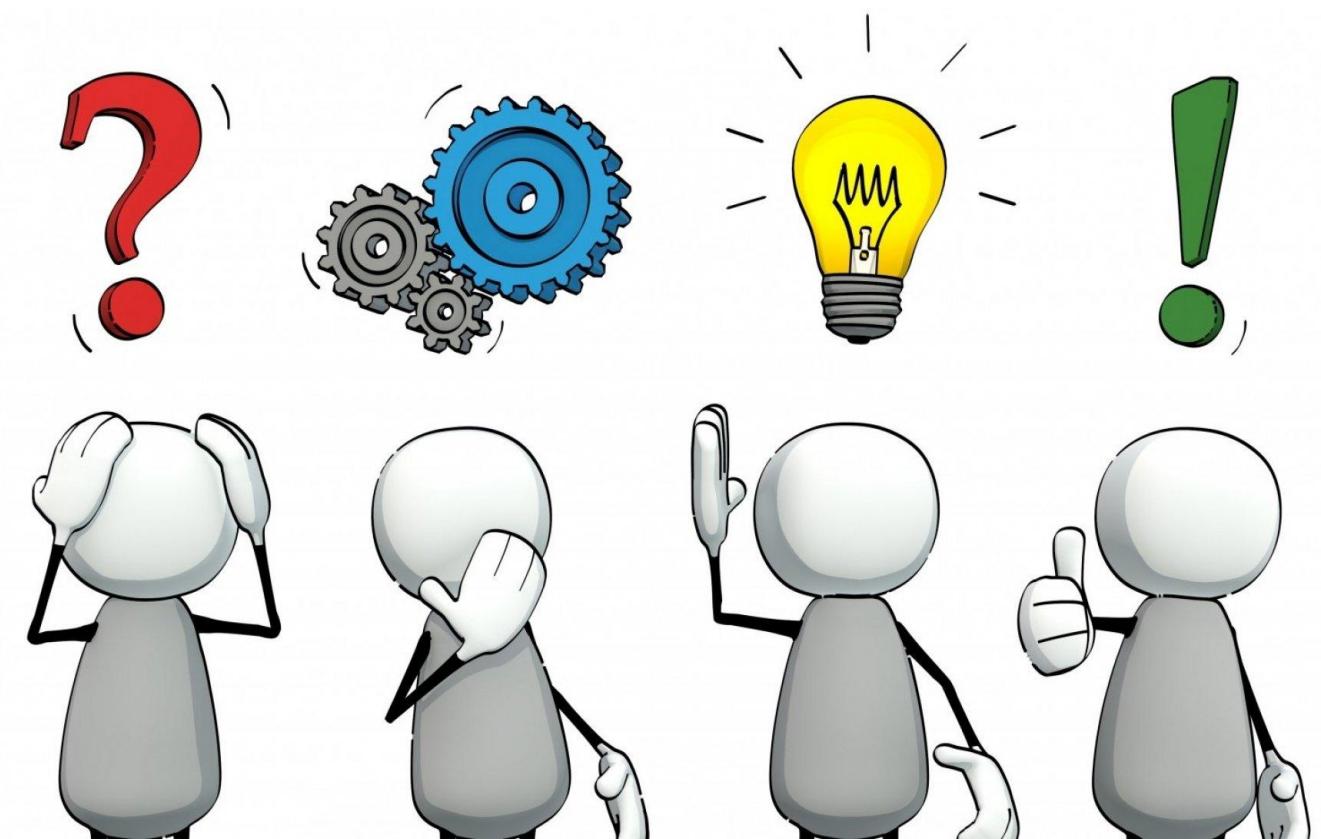
reflex area

line search

angle behind reflex vertex

hidden movement

greedy initialisation



Heuristics

momentum

pull towards reflex vertex

pull onto the reflex vertex

pull capping

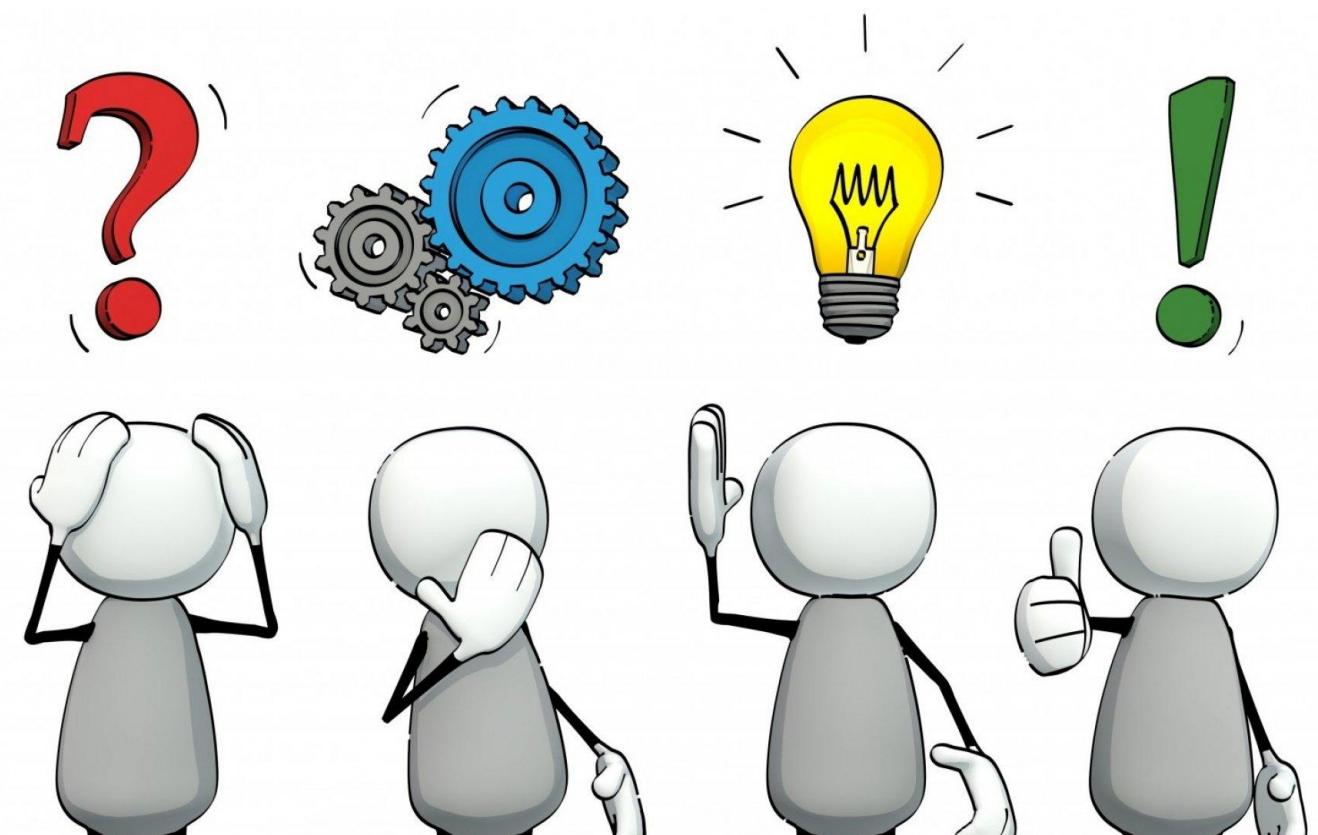
reflex area

line search

angle behind reflex vertex

hidden movement

greedy initialisation



Heuristics

momentum

pull towards reflex vertex

pull onto the reflex vertex

pull capping

reflex area

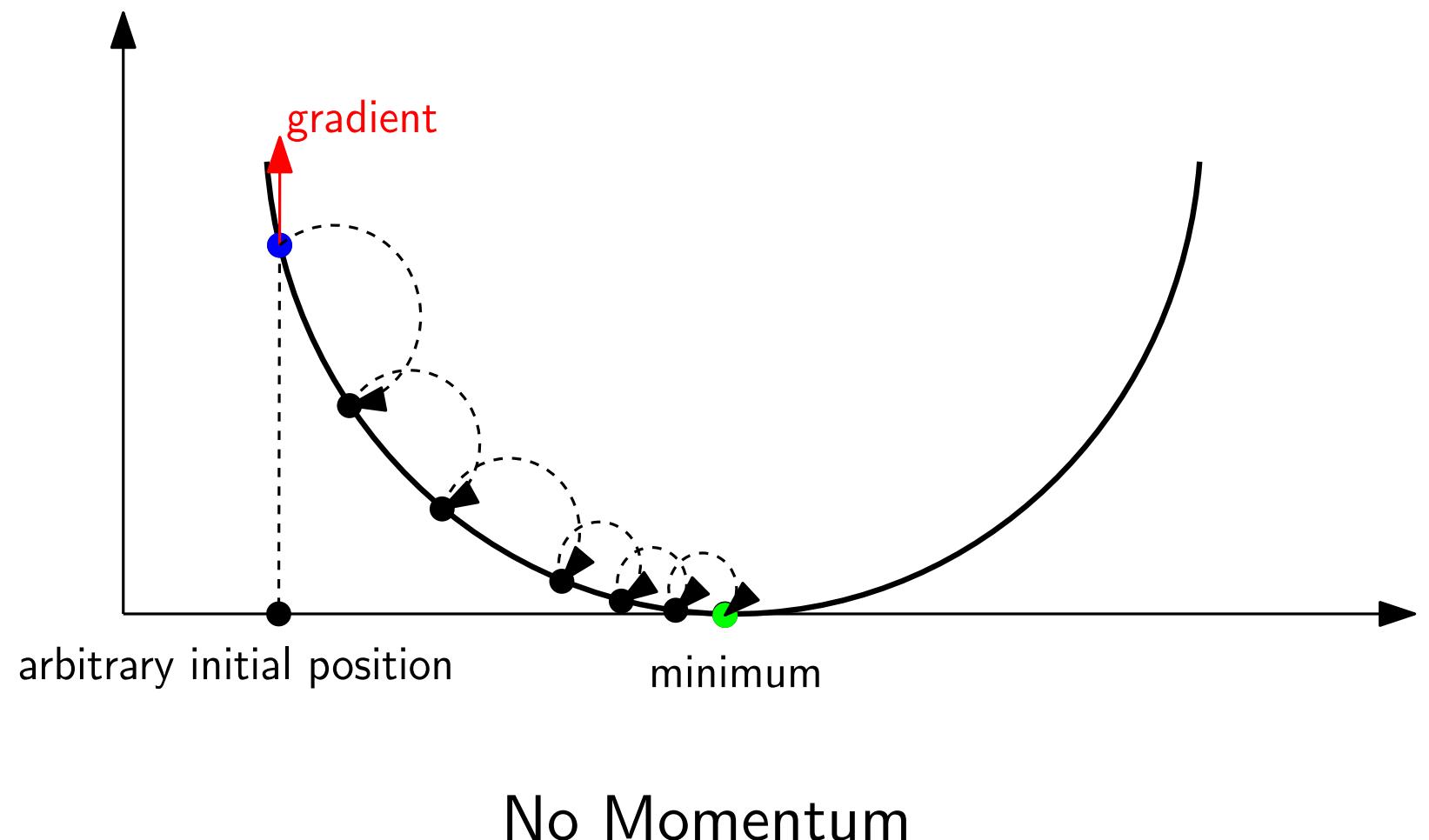
line search

angle behind reflex vertex

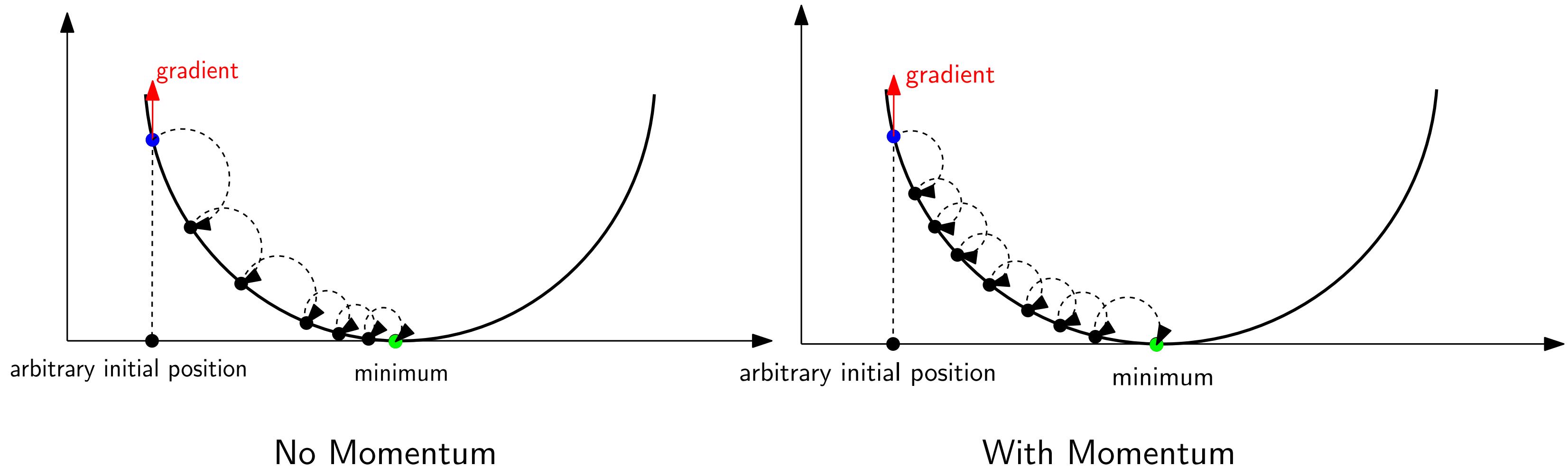
hidden movement

greedy initialisation

Heuristics: Momentum



Heuristics: Momentum

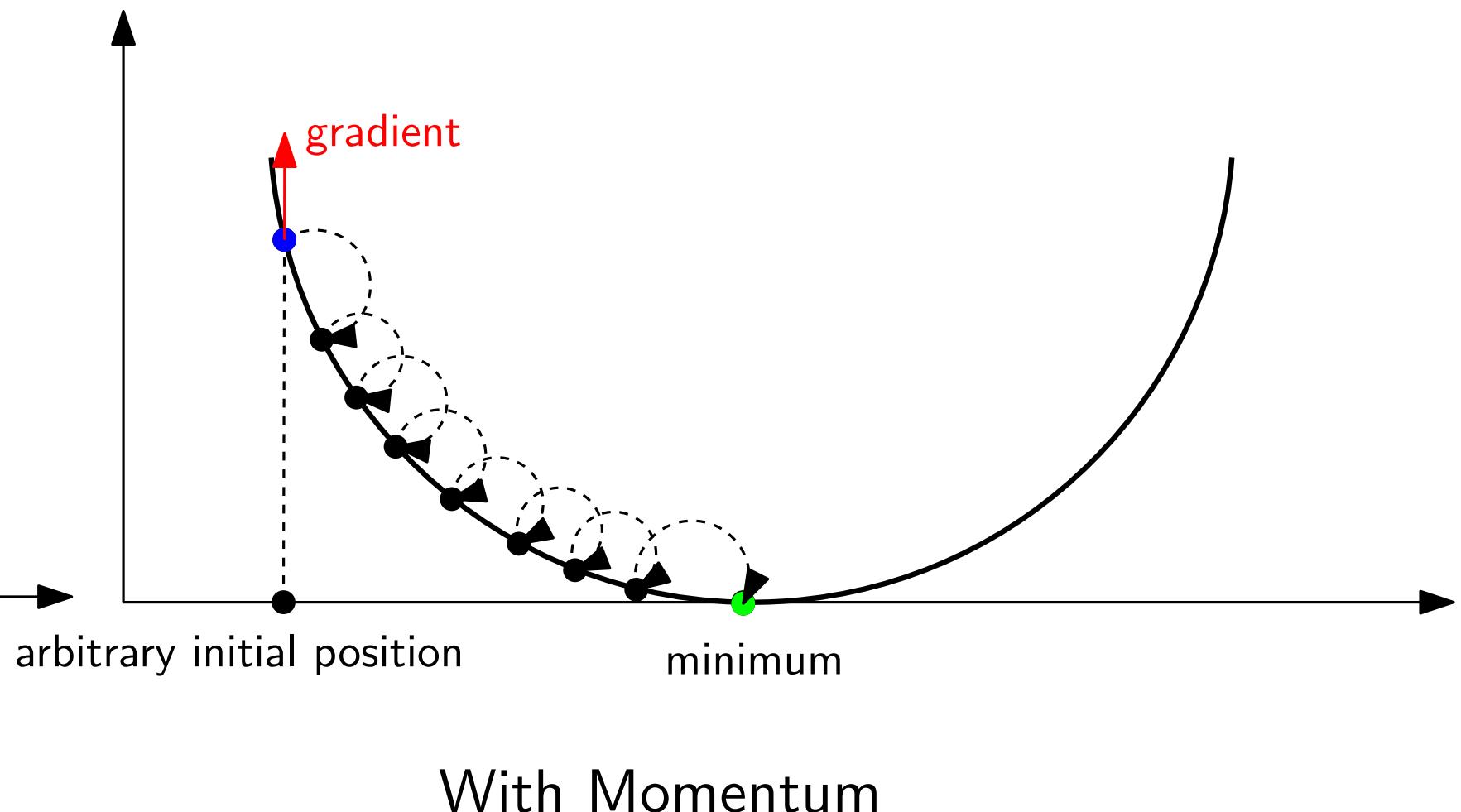
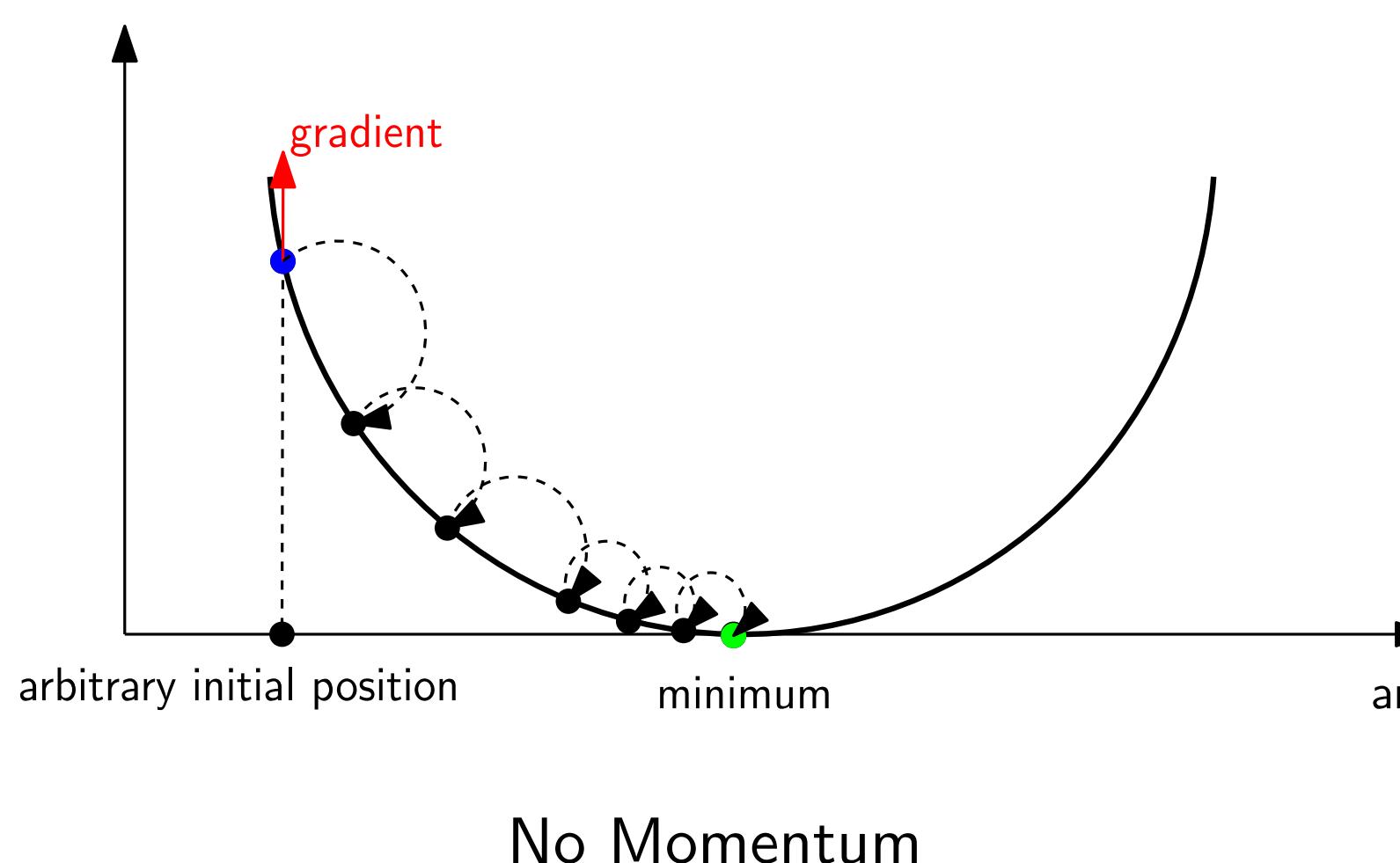


Heuristics: Momentum

iteration i :

γ - past state weight ("inertia")

$$M(g_{i+1}) = \gamma M(g_i) + (1 - \gamma) \nabla f(g_i)$$



Heuristics: Momentum

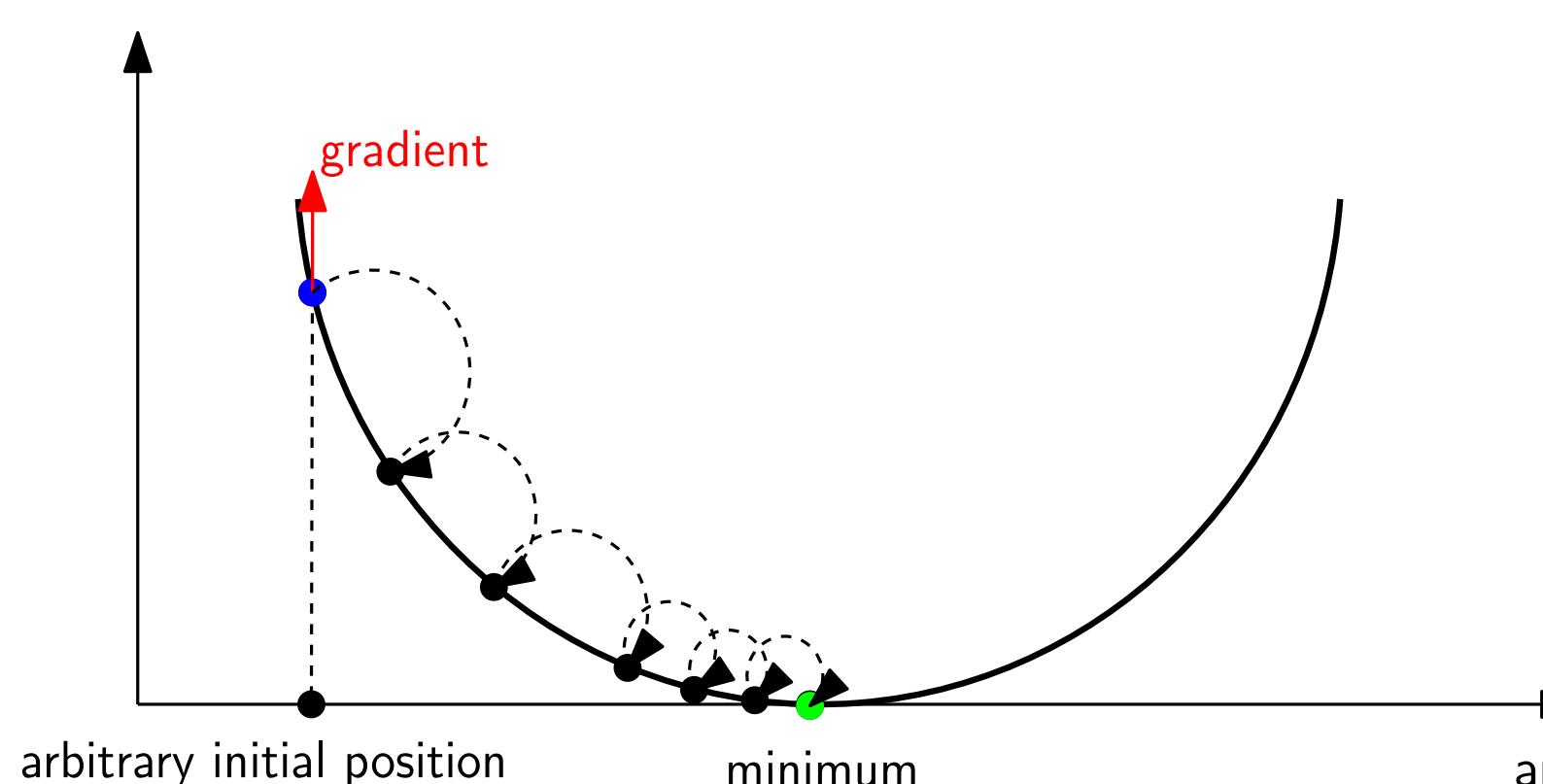
$$g_{i+1} = g_i + \alpha \nabla f(g_i)$$

iteration i :

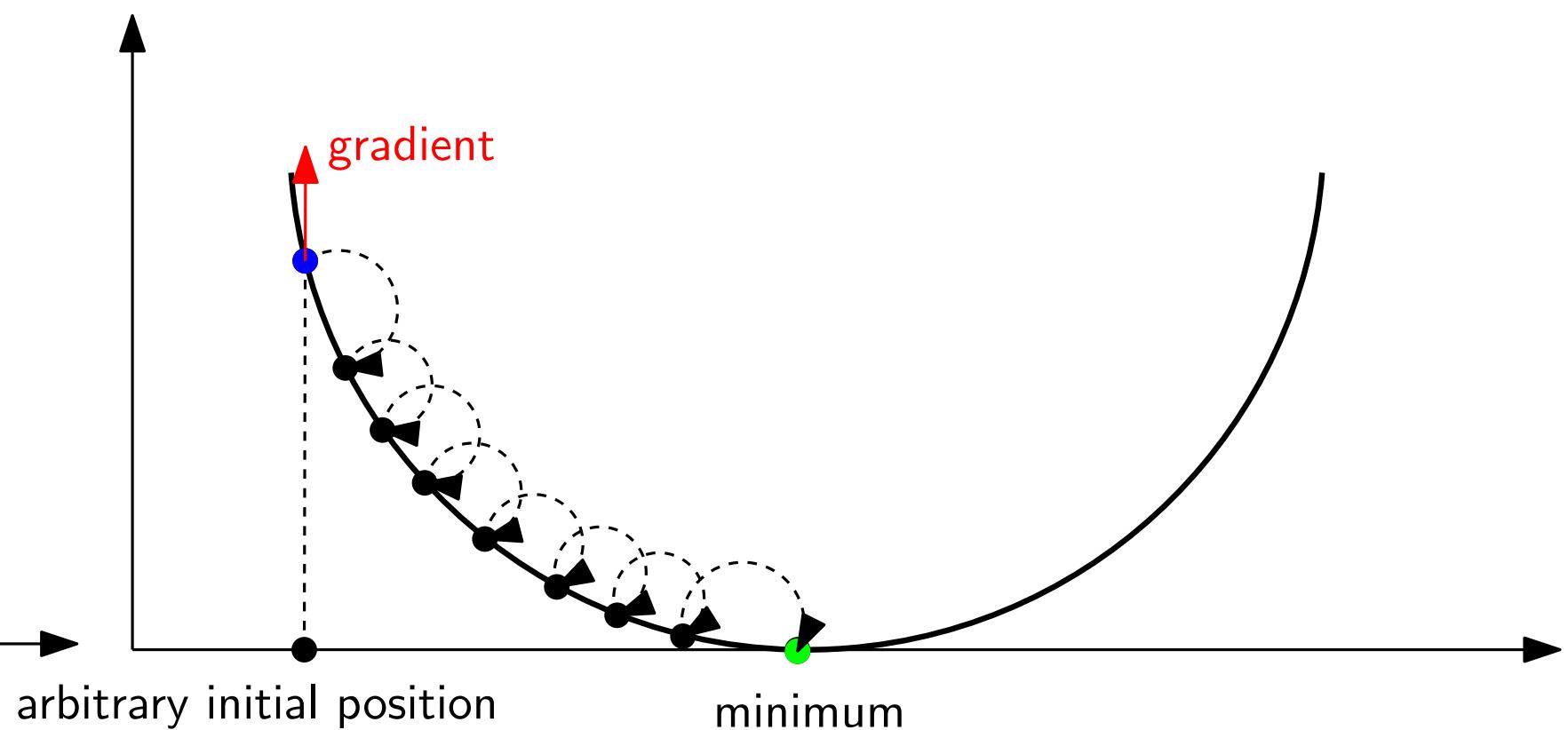
γ - past state weight ("inertia")

$$M(g_{i+1}) = \gamma M(g_i) + (1 - \gamma) \nabla f(g_i)$$

$$g_{i+1} = g_i + \alpha M(g_i)$$



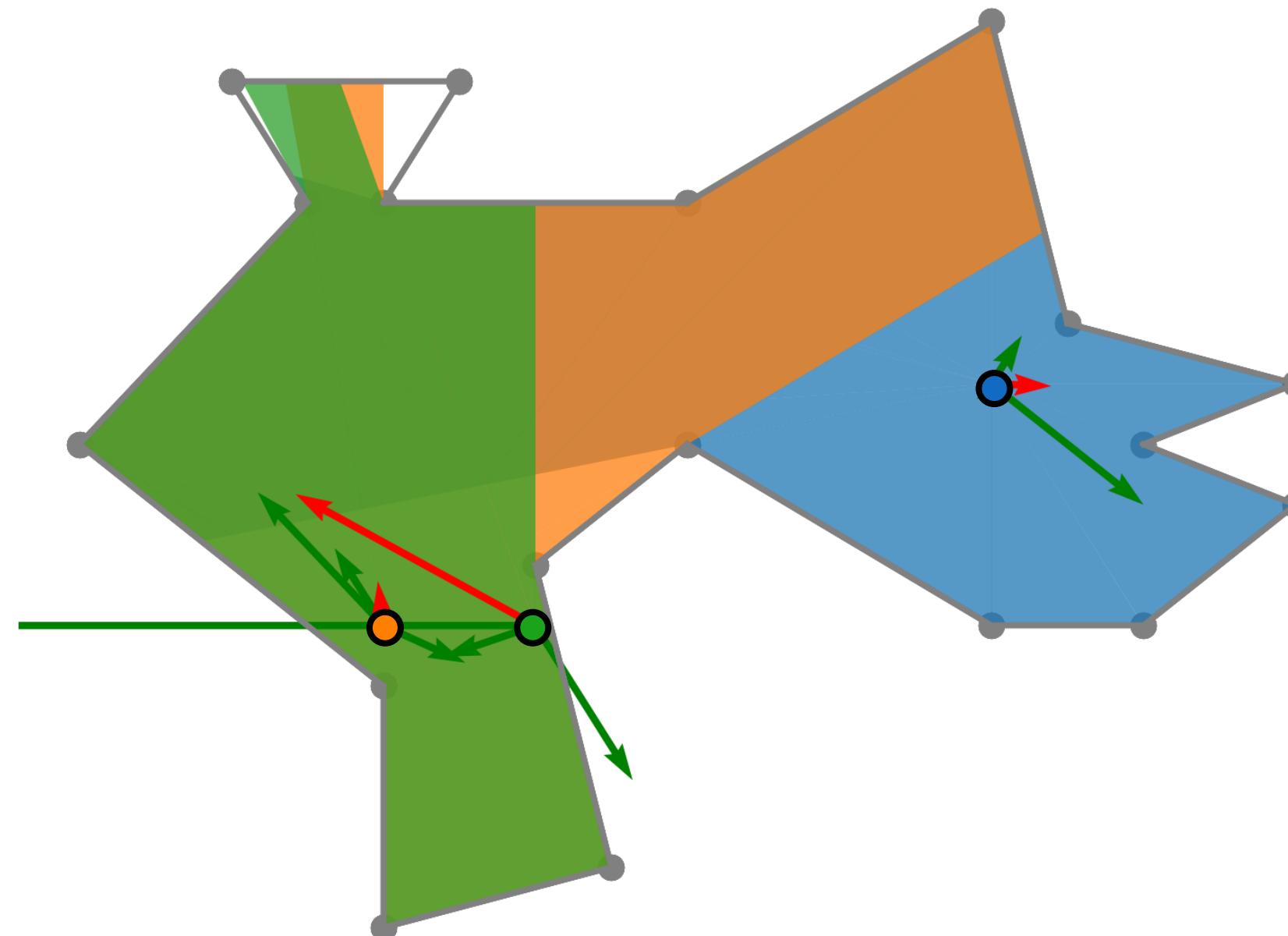
No Momentum



With Momentum

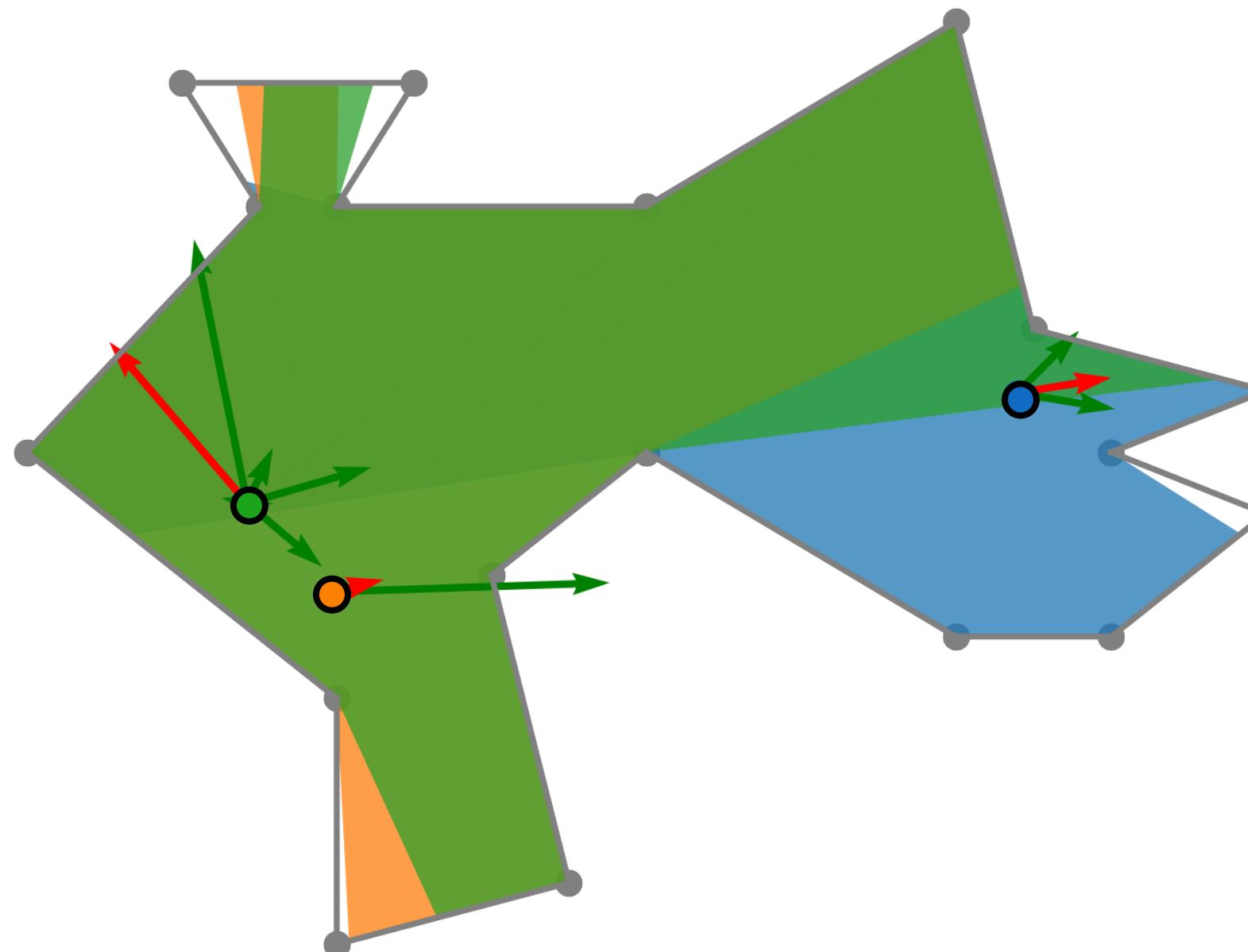
Heuristics: Momentum

Gradient Computation for Iteration #0



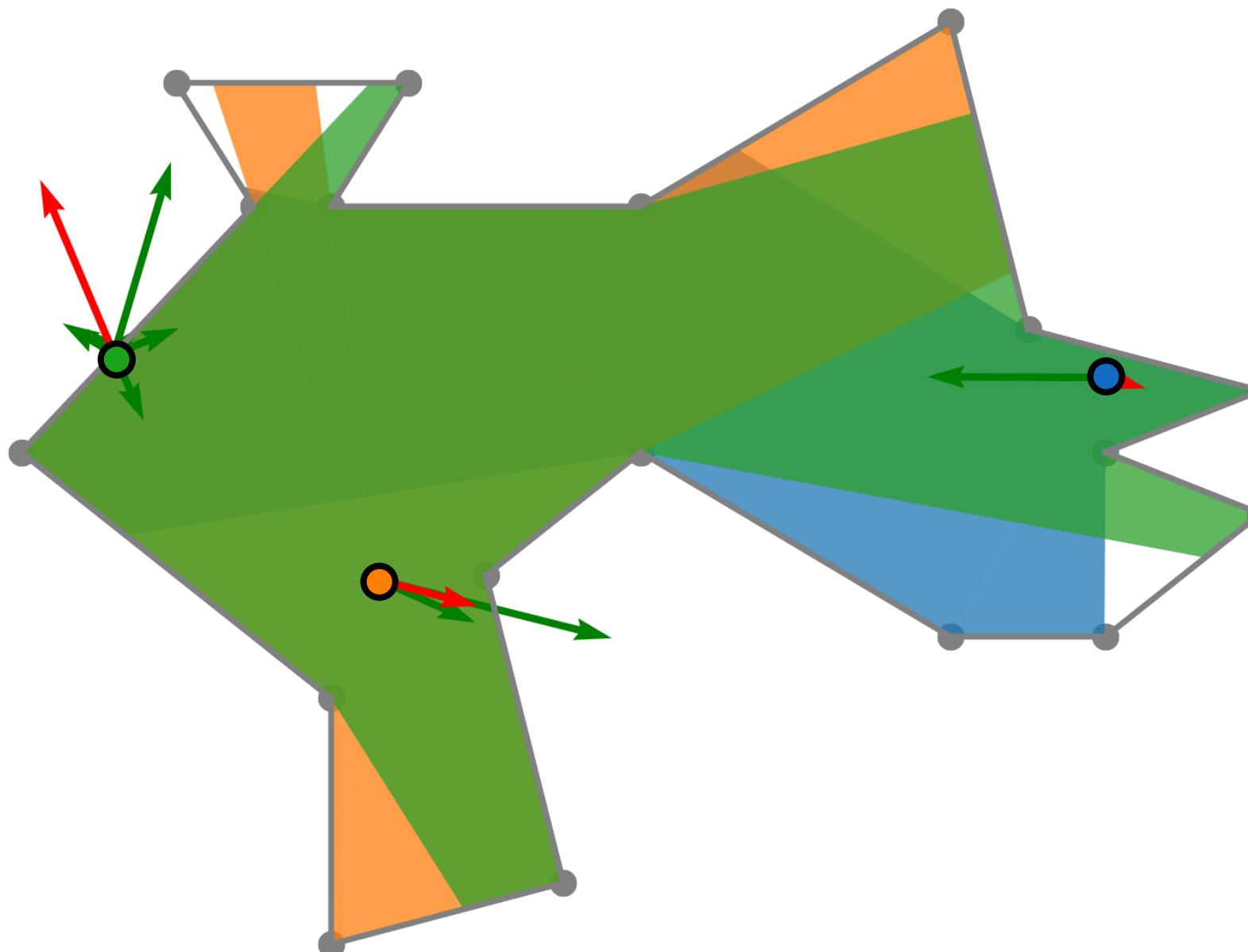
Heuristics: Momentum

Gradient Computation for Iteration #1



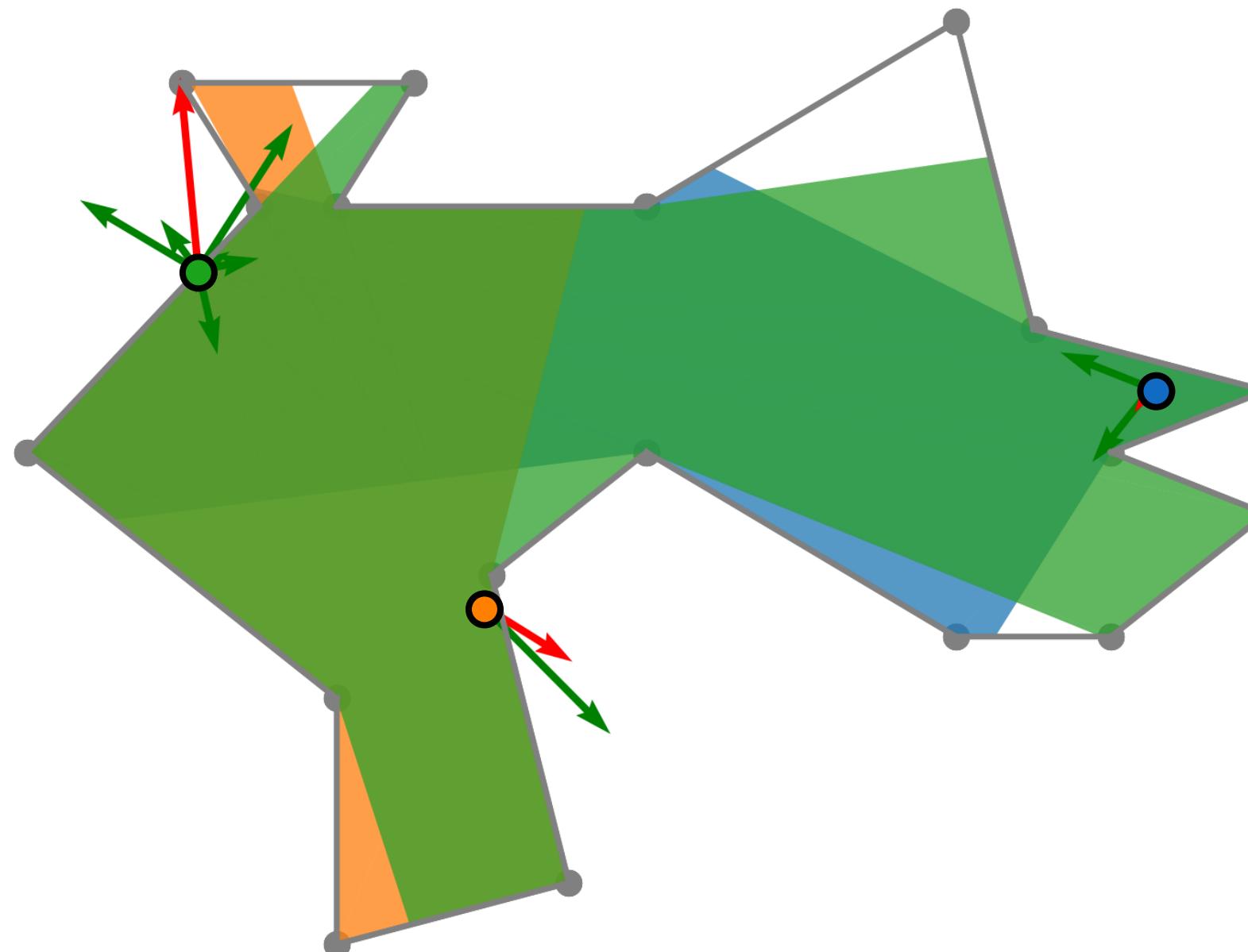
Heuristics: Momentum

Gradient Computation for Iteration #2



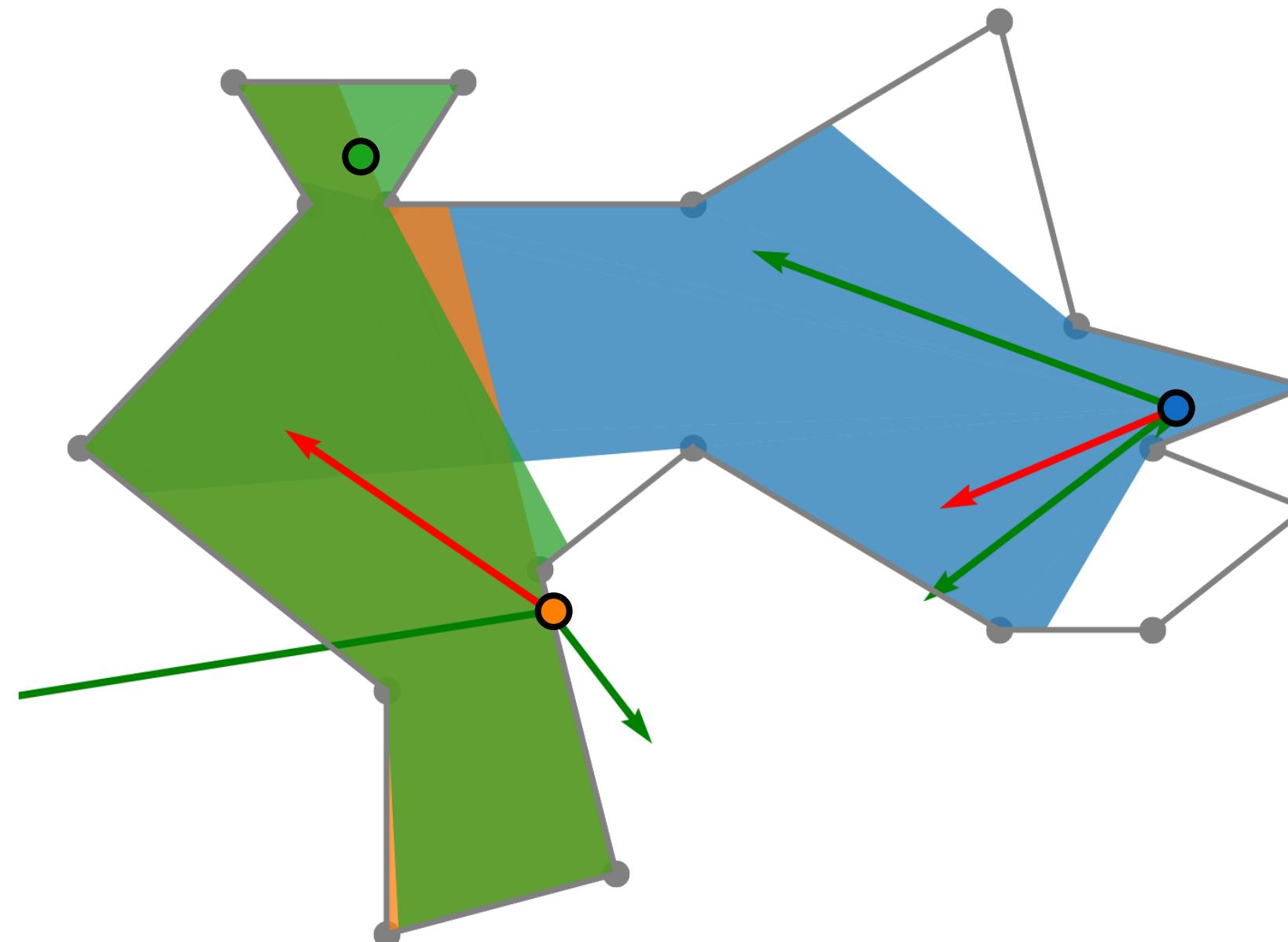
Heuristics: Momentum

Gradient Computation for Iteration #3



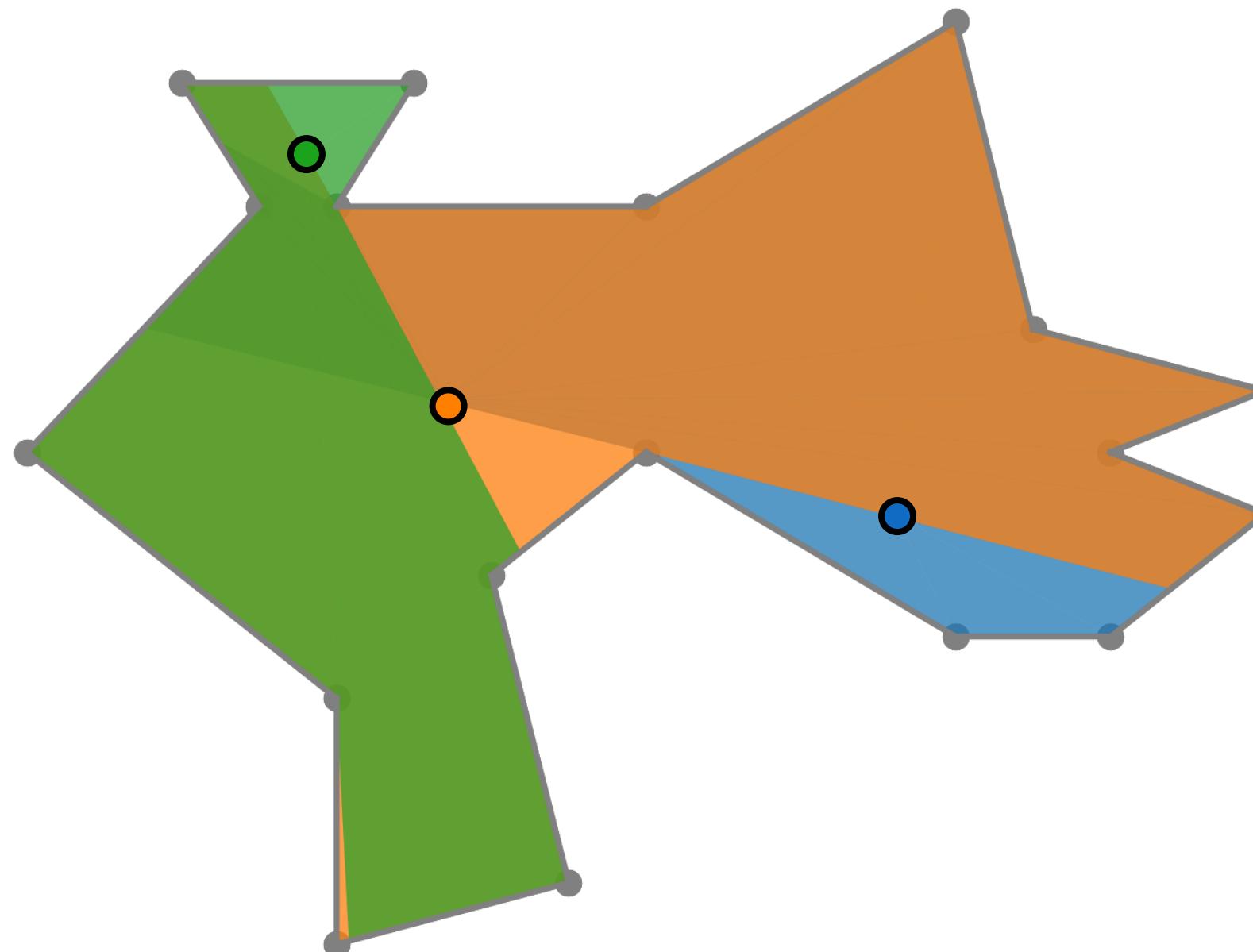
Heuristics: Momentum

Gradient Computation for Iteration #4



Heuristics: Momentum

Gradient Computation for Iteration #5



Heuristics

momentum

pull towards reflex vertex

pull onto the reflex vertex

pull capping

reflex area

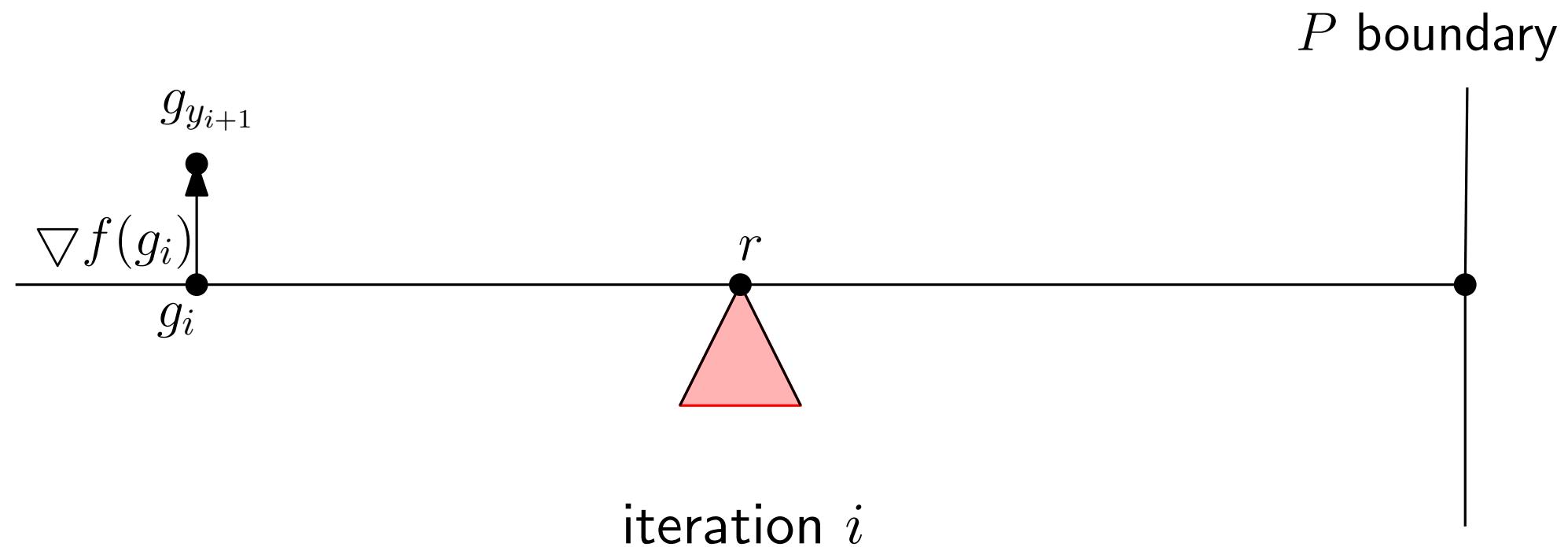
line search

angle behind reflex vertex

hidden movement

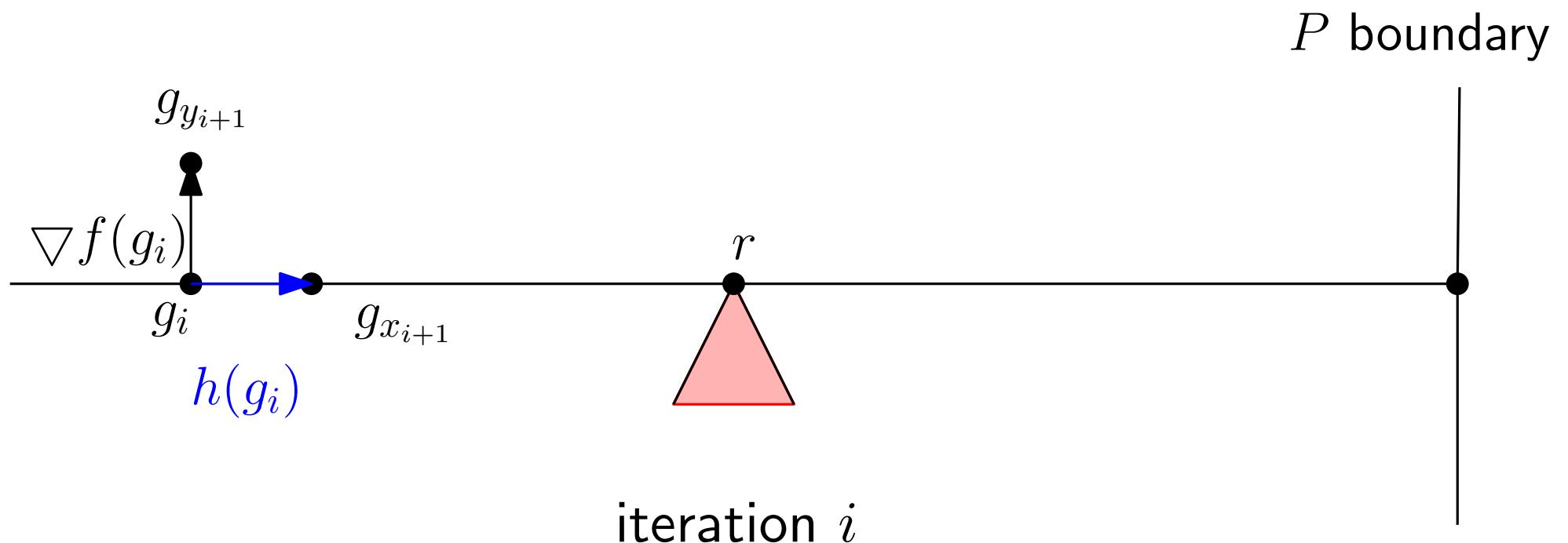
greedy initialisation

Heuristics: Pull towards reflex vertex



Heuristics: Pull towards reflex vertex

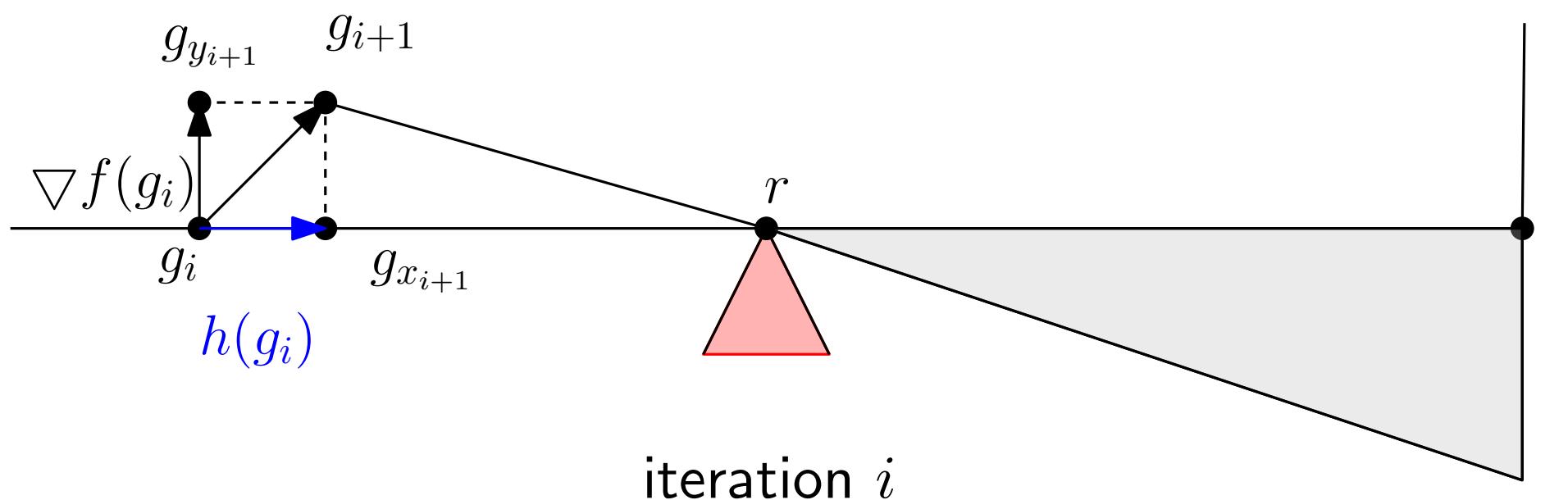
$$h(g_i) = \nabla||\nabla f(g_i)||$$



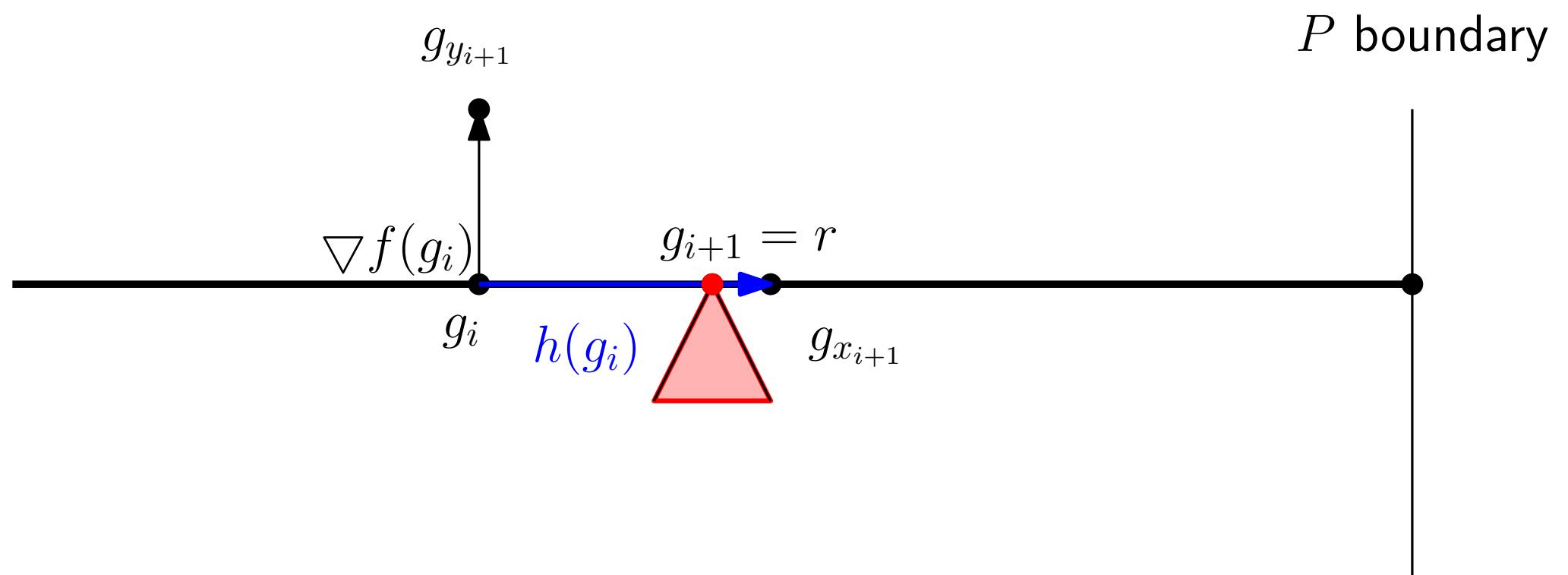
Heuristics: Pull towards reflex vertex

$$h(g_i) = \nabla ||\nabla f(g_i)||$$

$$g_{i+1} = g_i + \alpha(\nabla f(g_i) + h(g_i))$$

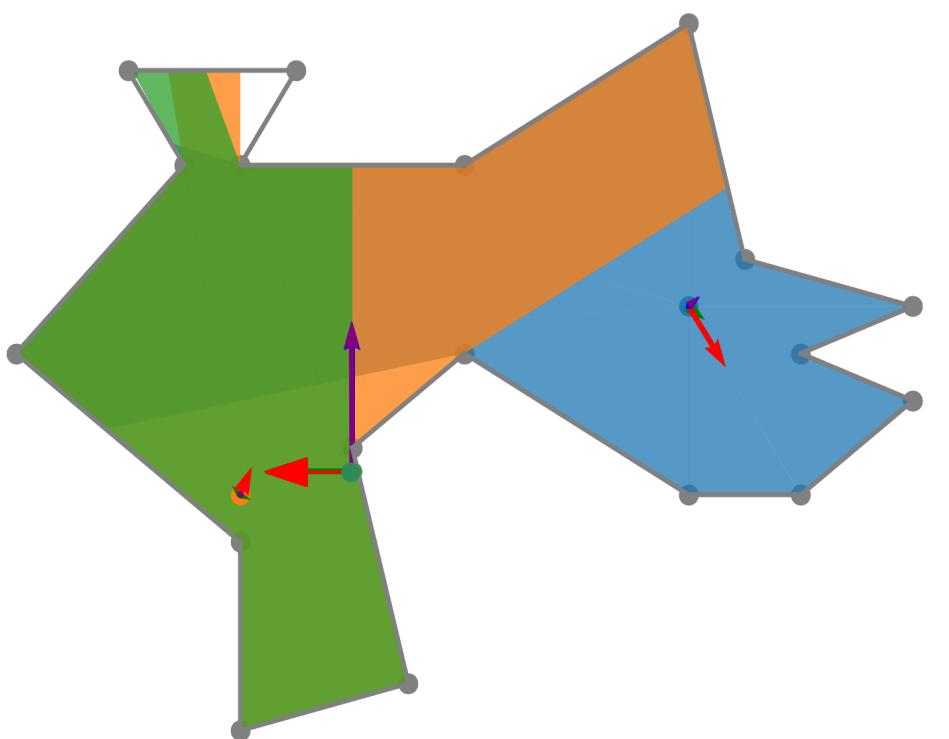


Heuristics: Pull towards reflex vertex

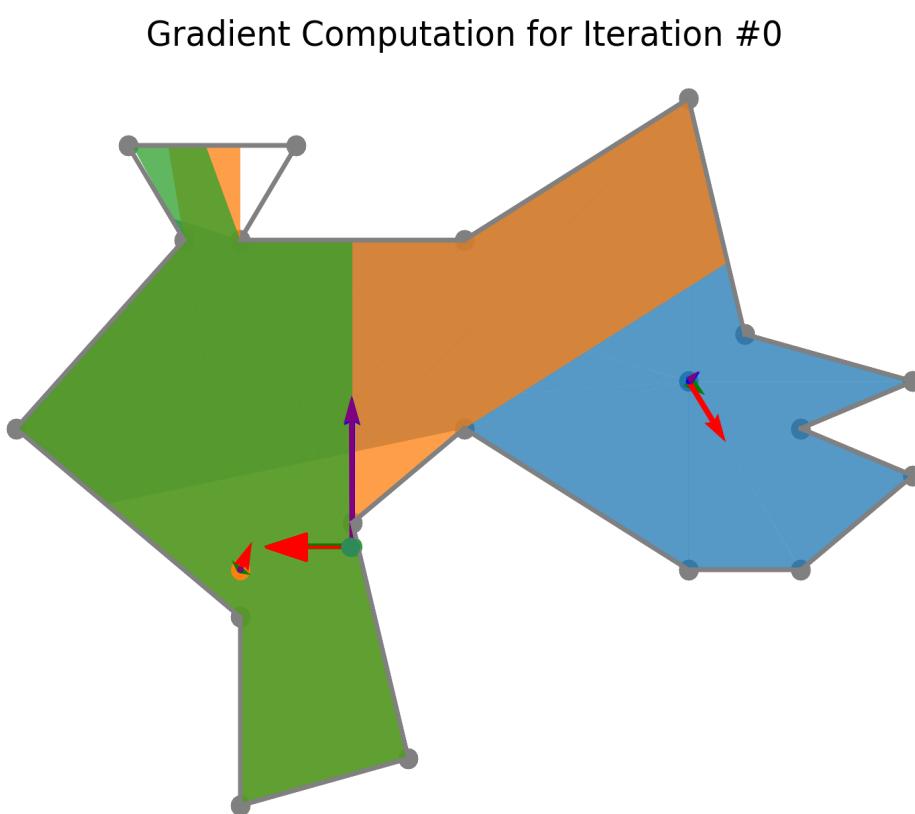


Heuristics: Pull towards reflex vertex

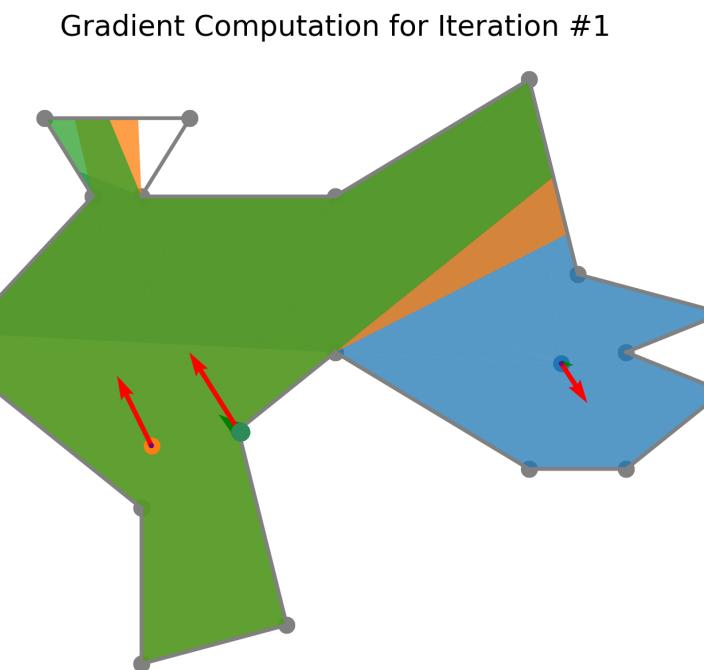
Gradient Computation for Iteration #0



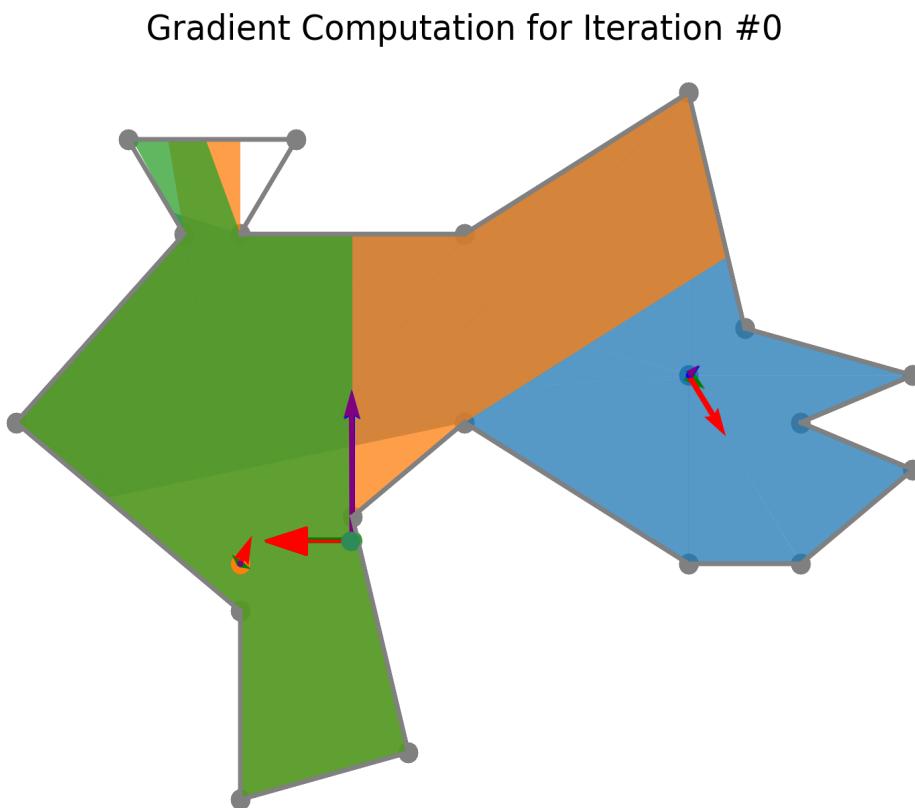
Heuristics: Pull towards reflex vertex



with pull

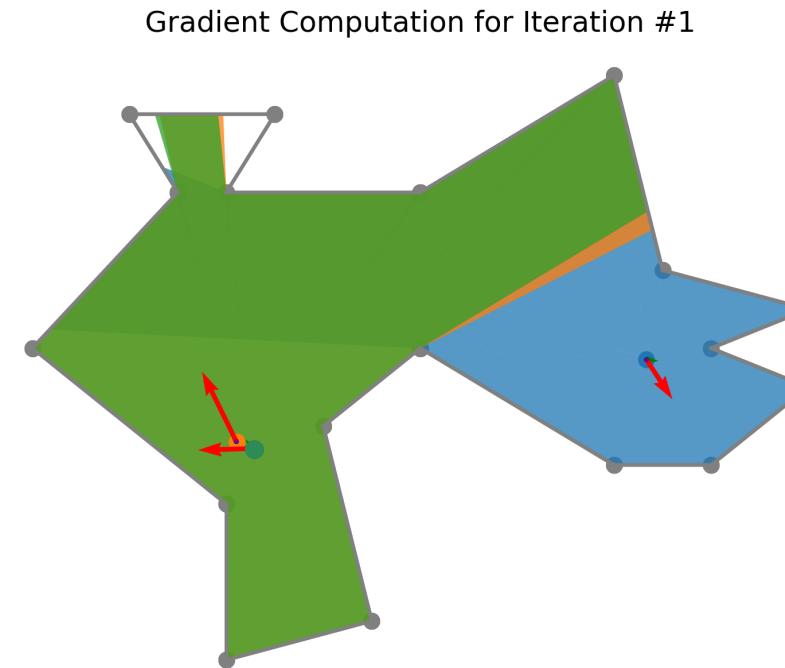
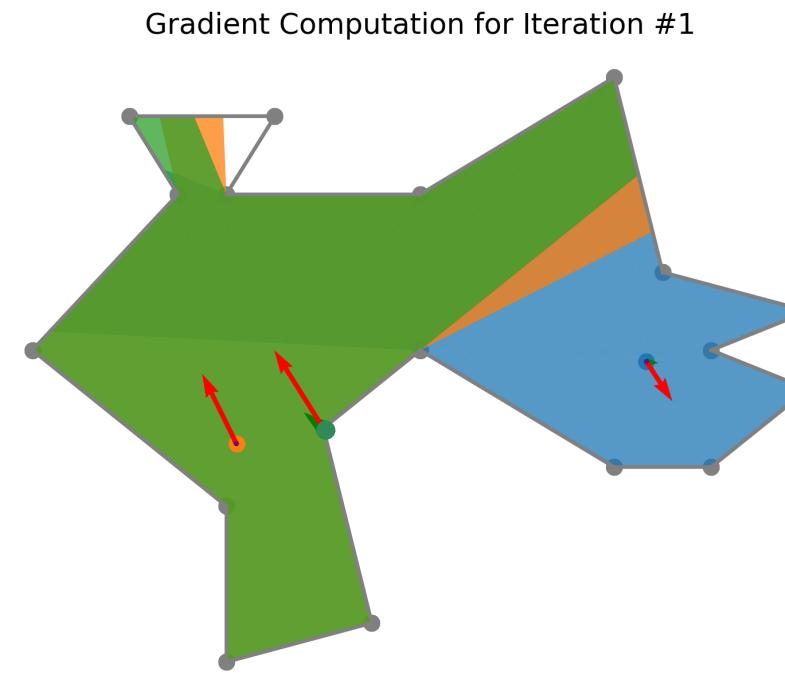


Heuristics: Pull towards reflex vertex

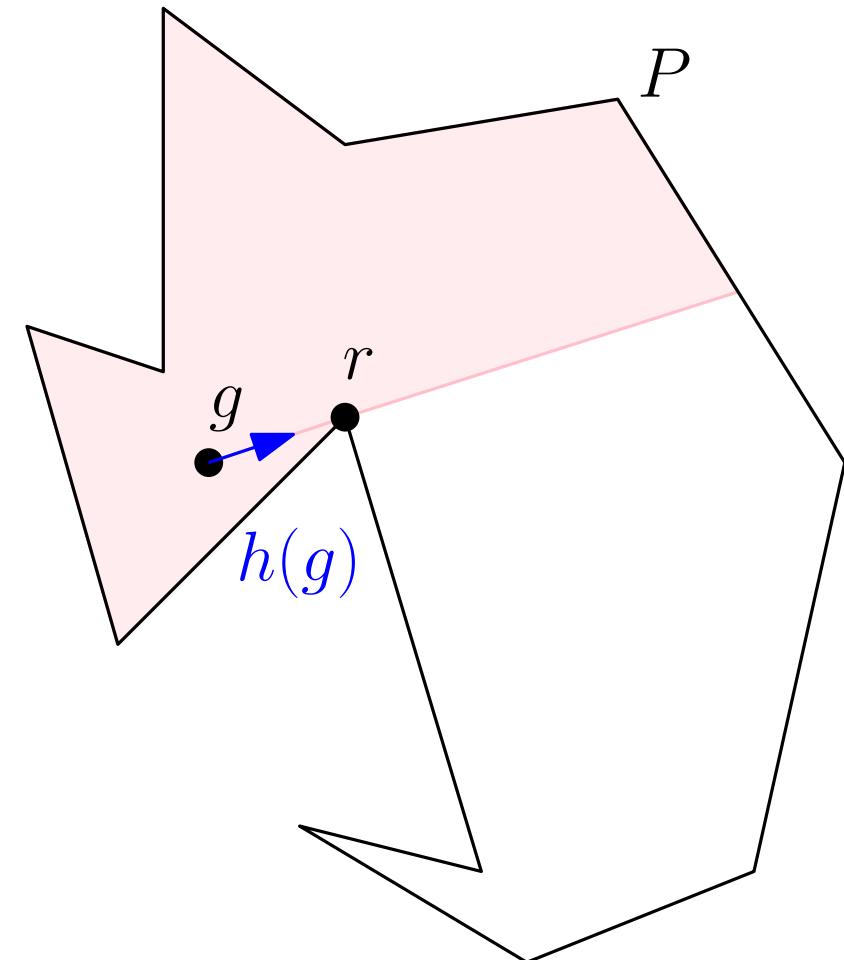


with pull
without pull

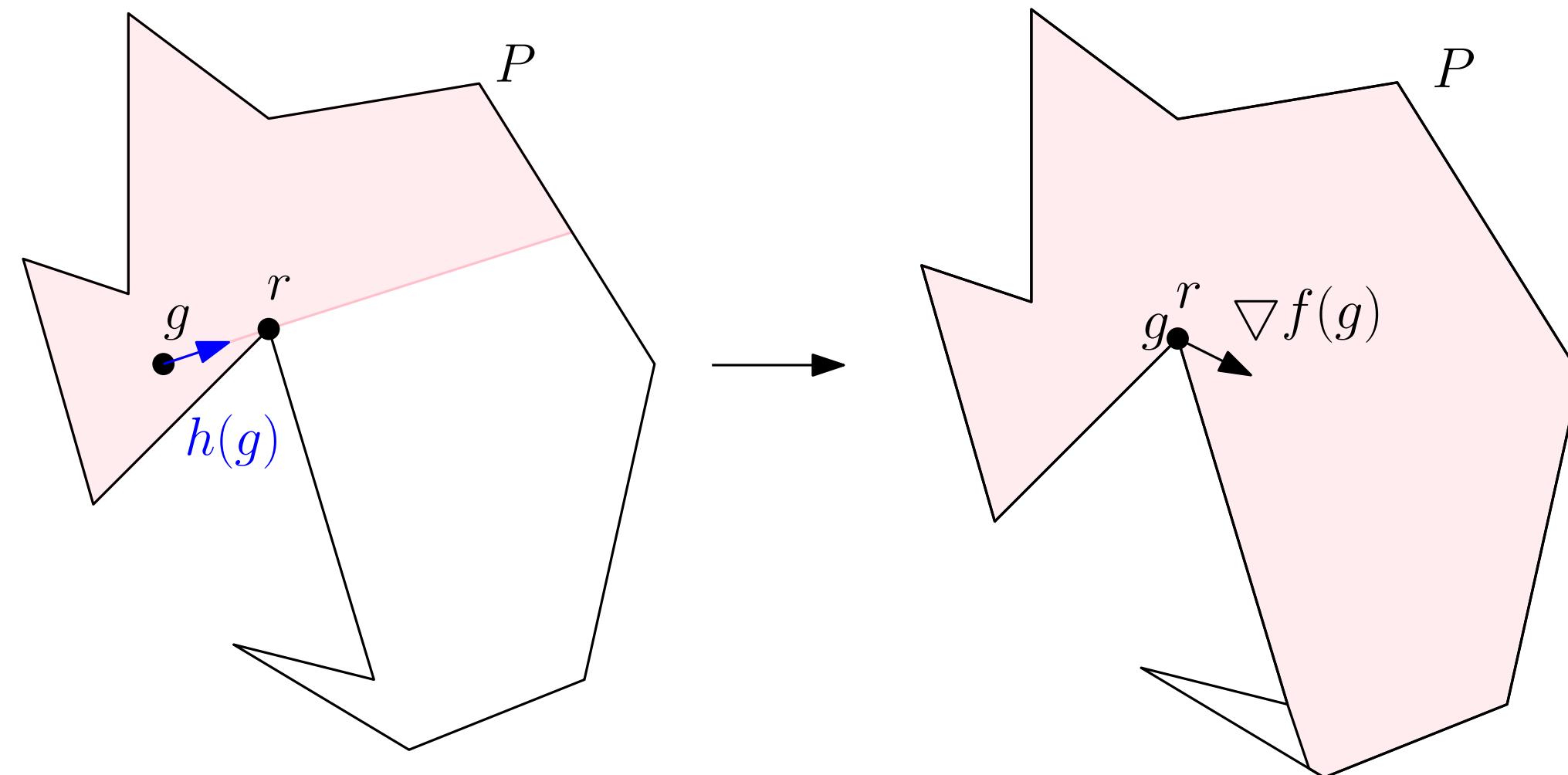
A large black arrow pointing downwards, separating the 'with pull' and 'without pull' diagrams.



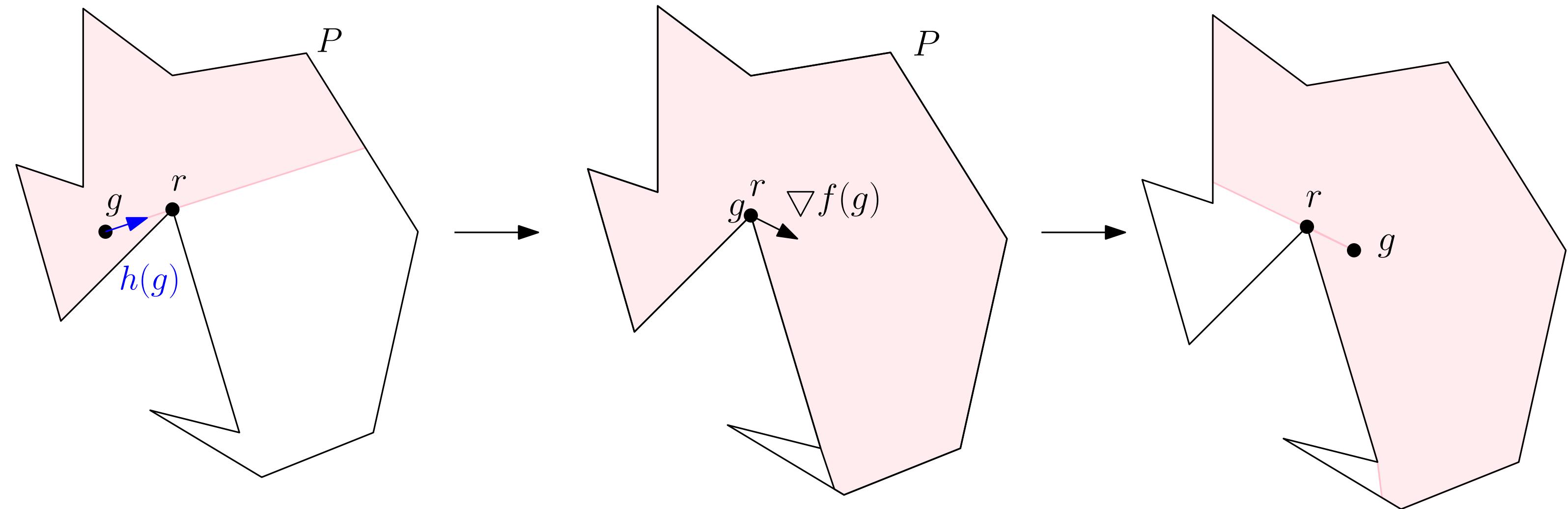
Heuristics: Reflex area



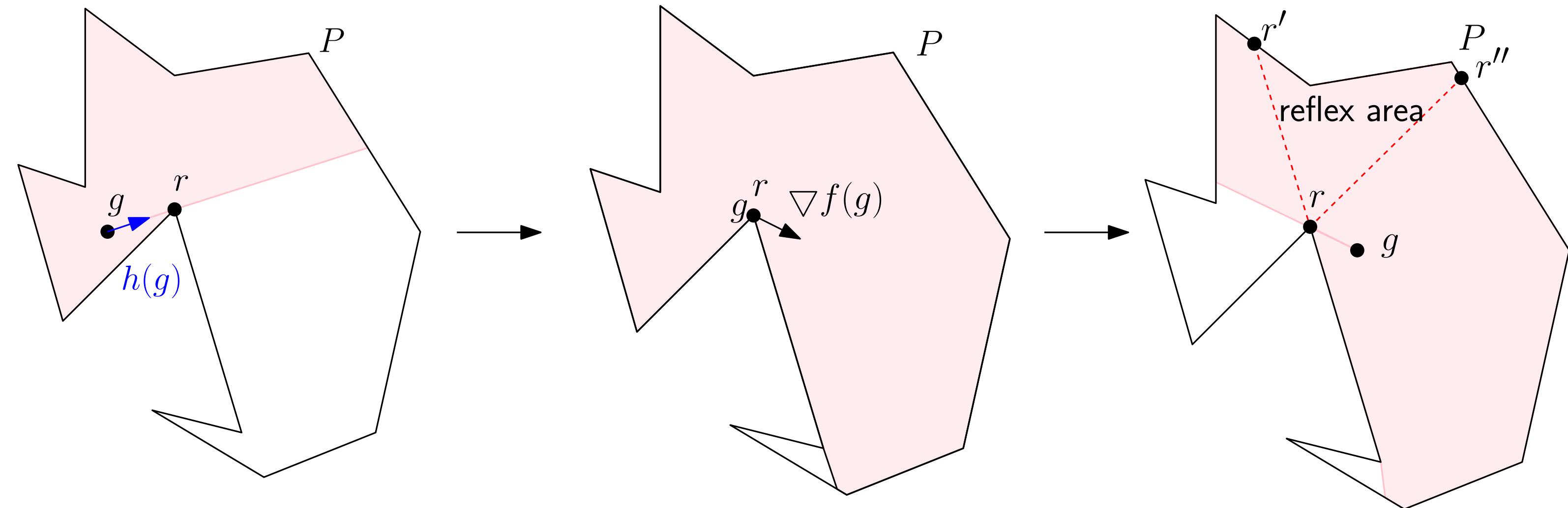
Heuristics: Reflex area



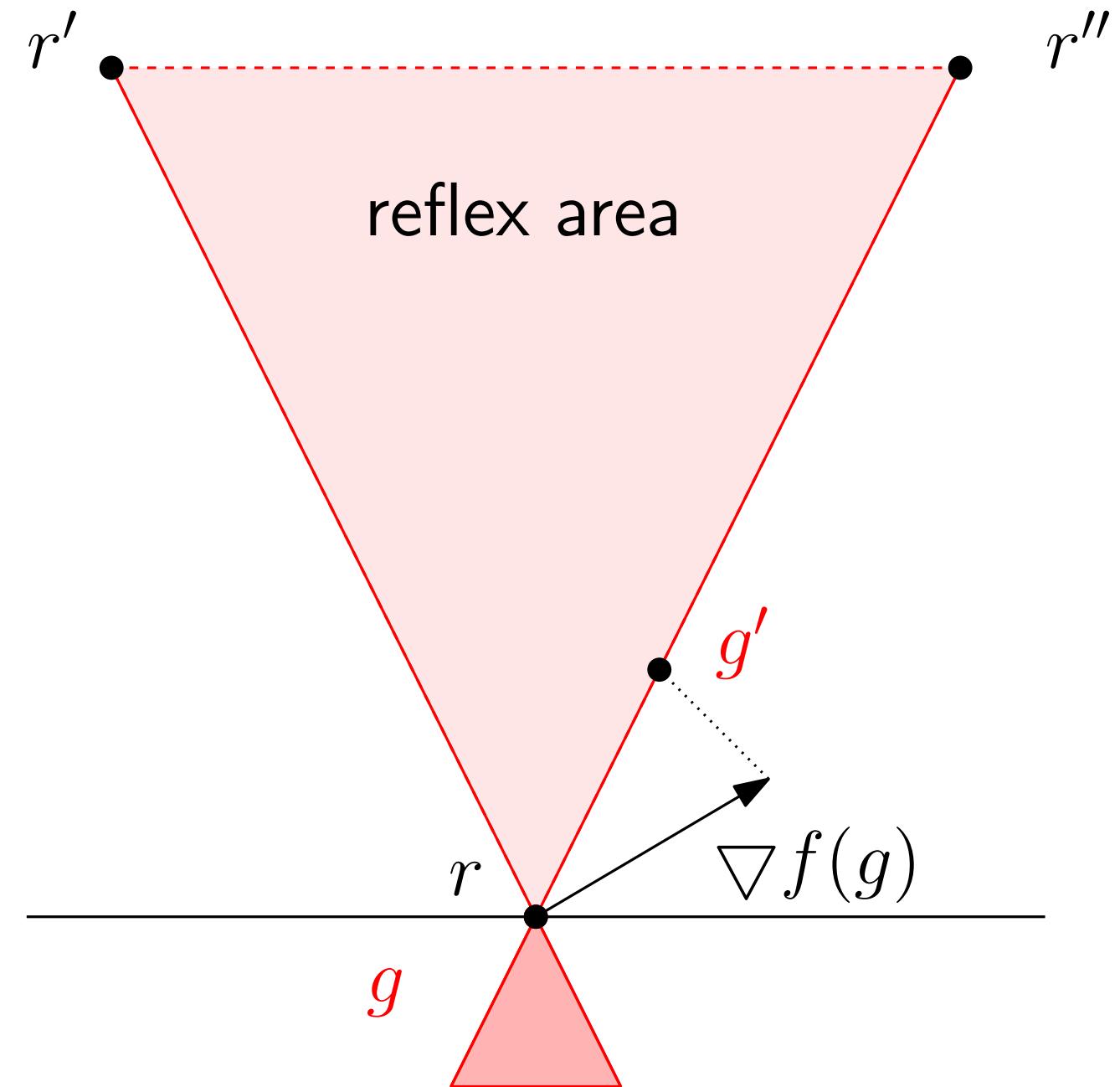
Heuristics: Reflex area



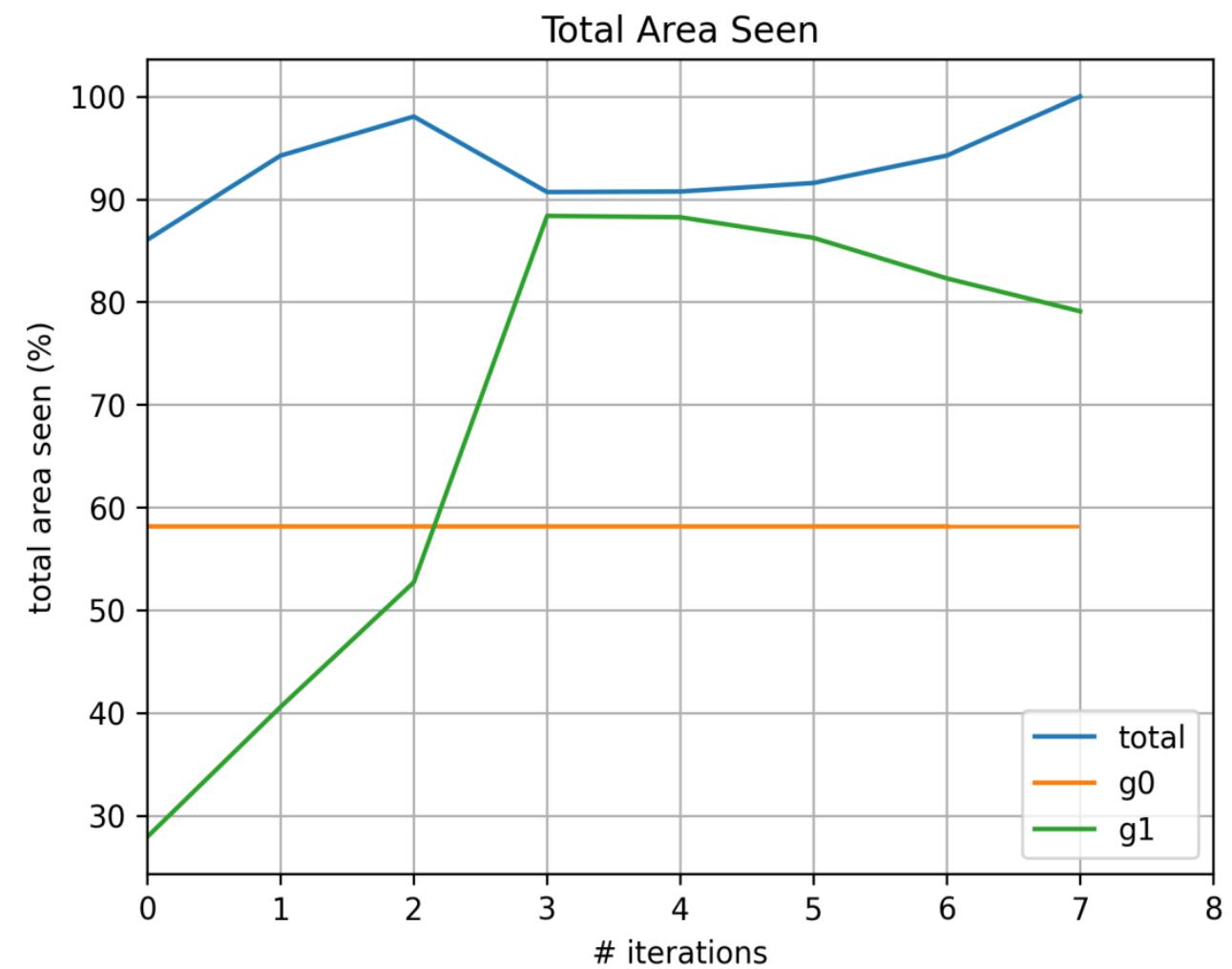
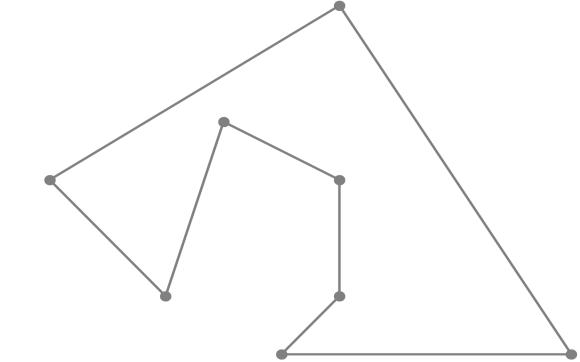
Heuristics: Reflex area



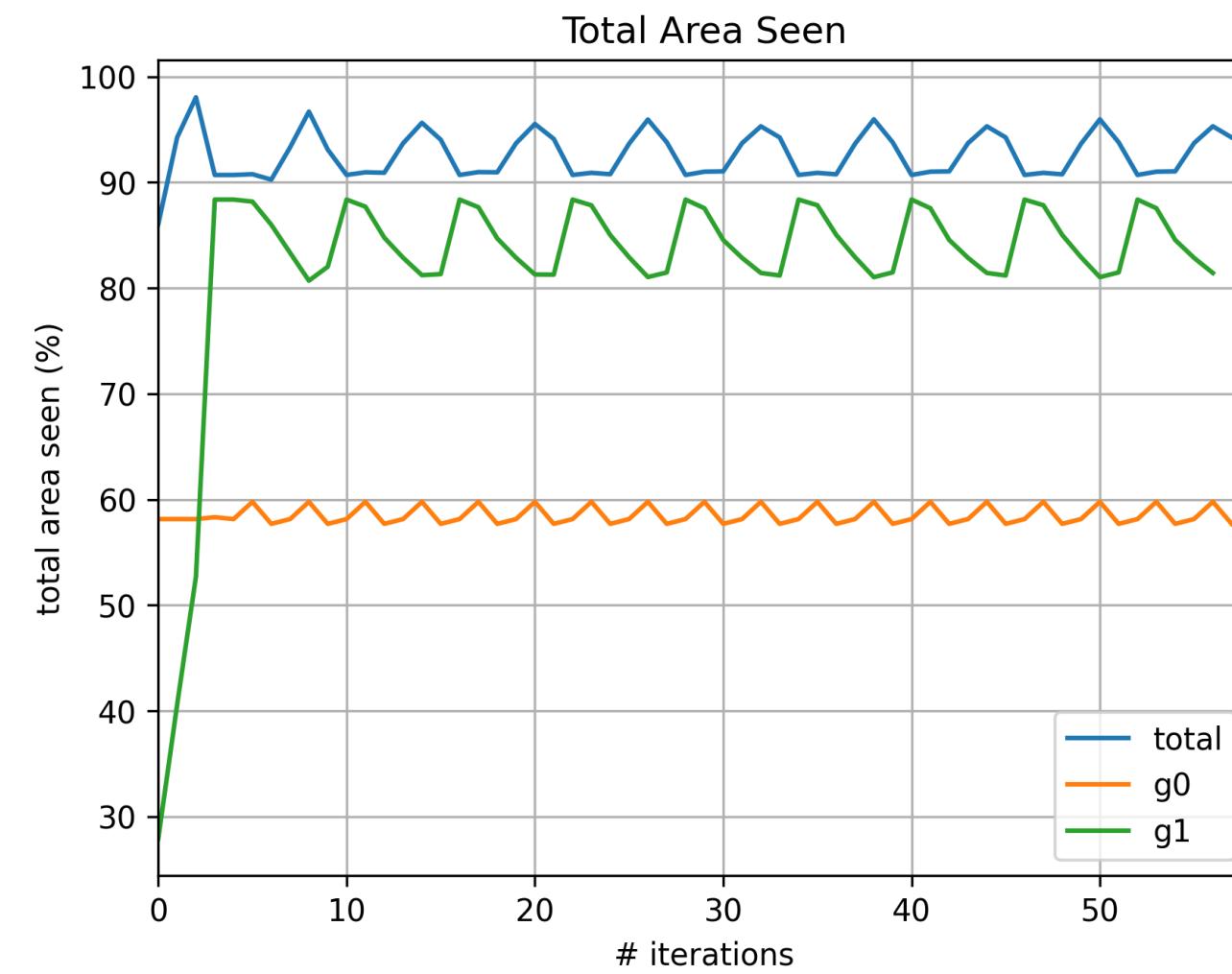
Heuristics: Reflex area



Heuristics: Reflex area

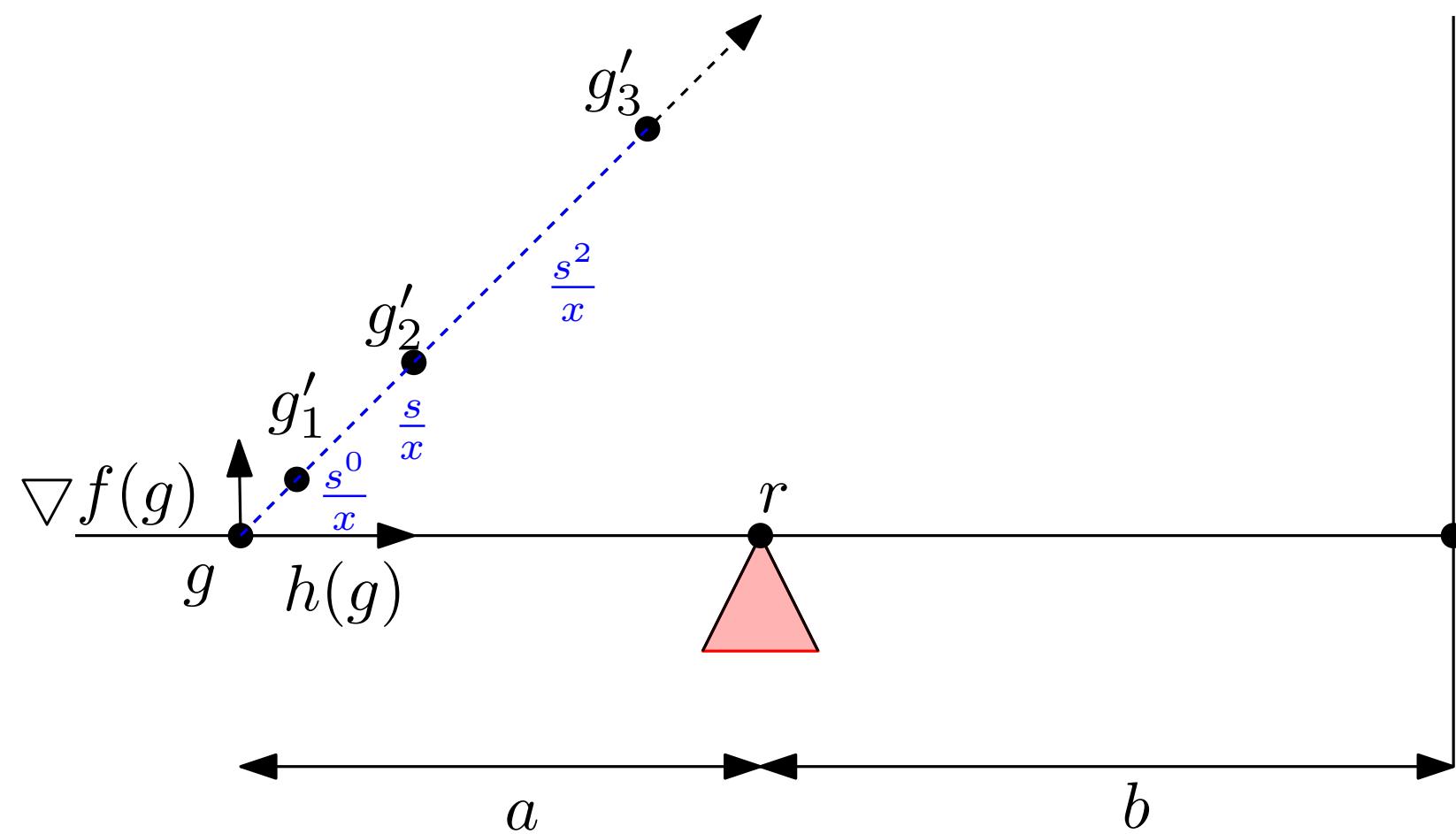


All heuristics



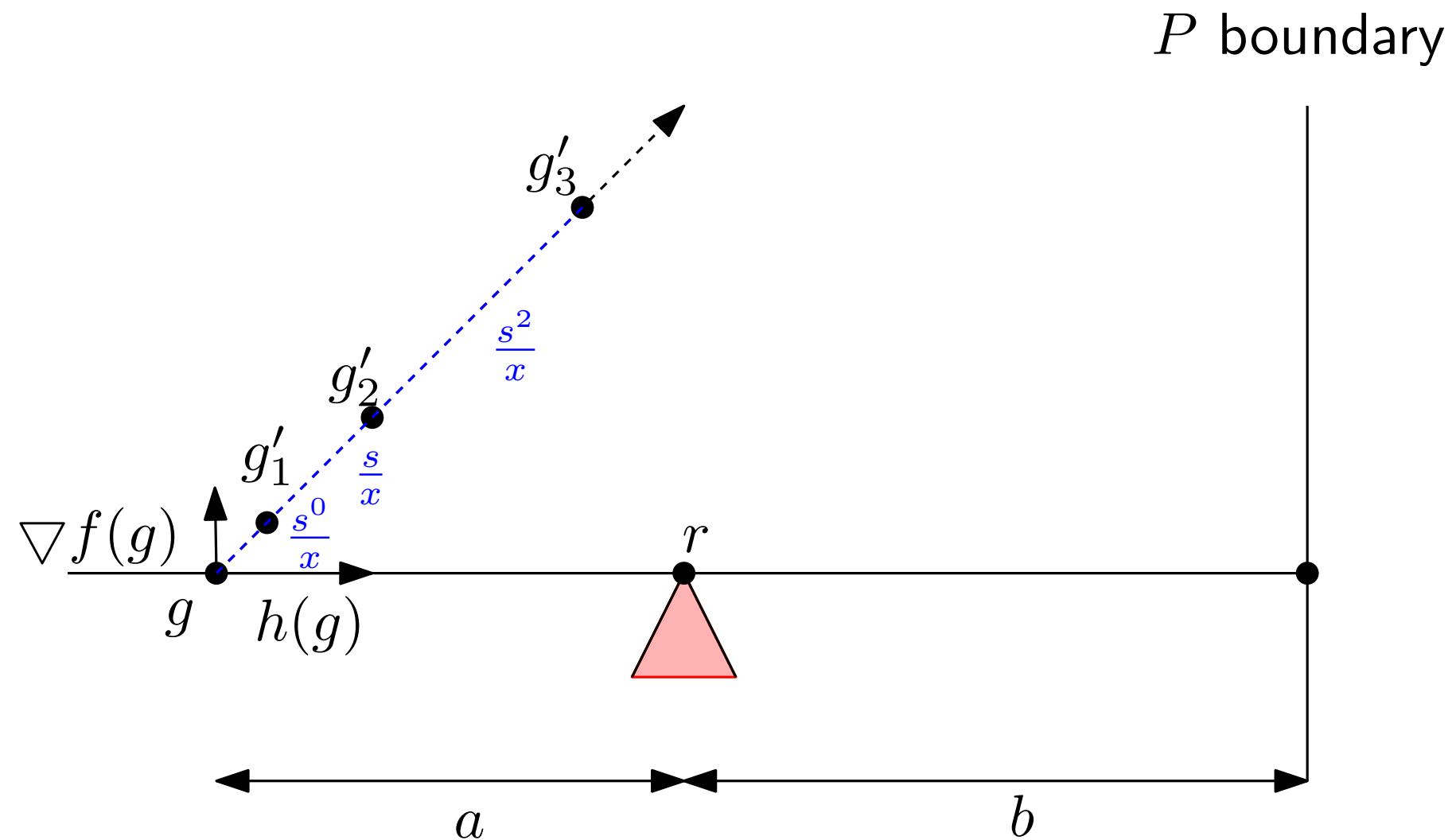
No reflex area

Heuristics: Line Search



s - step size
 x - search factor

Heuristics: Line Search



s - step size

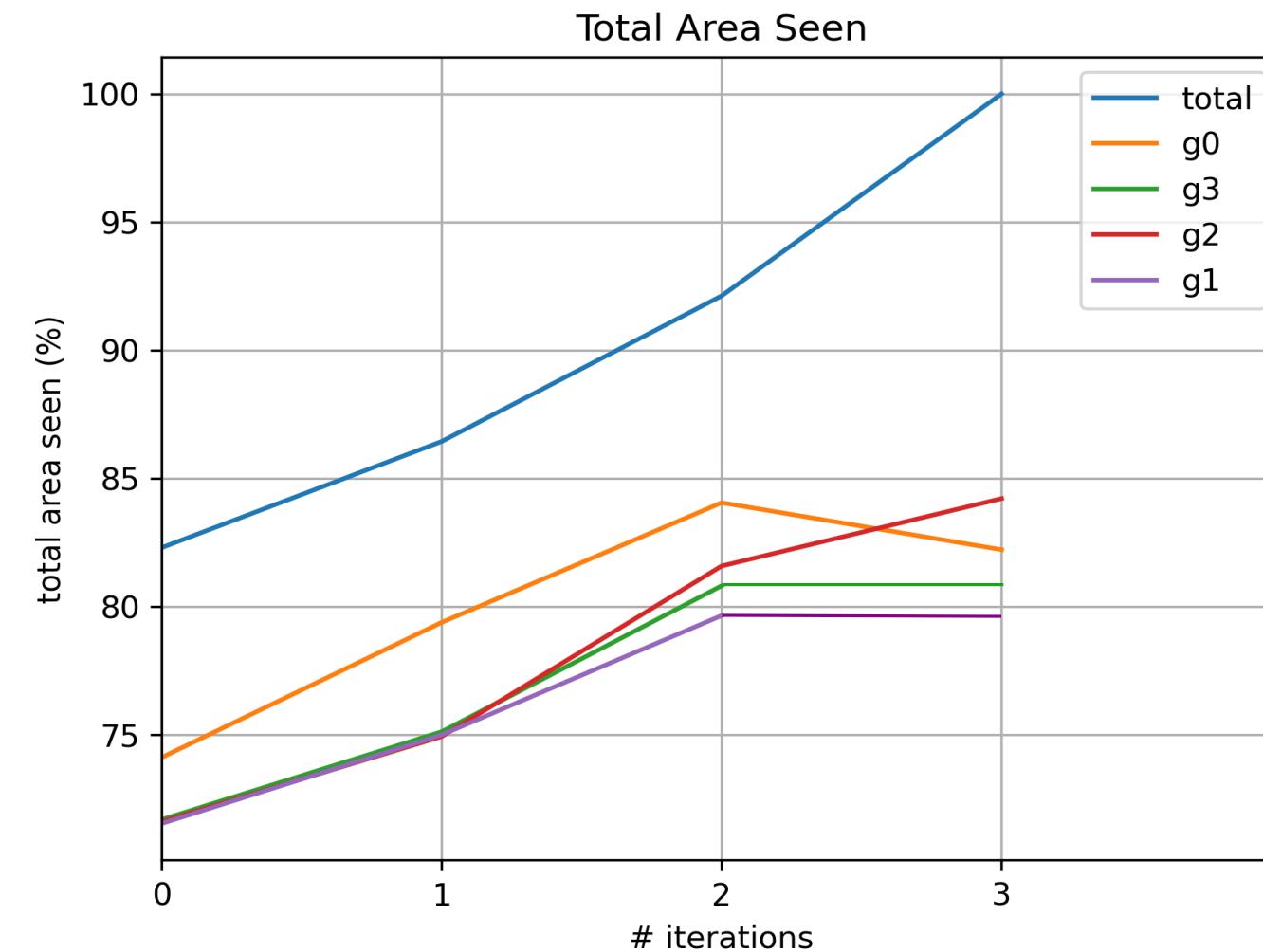
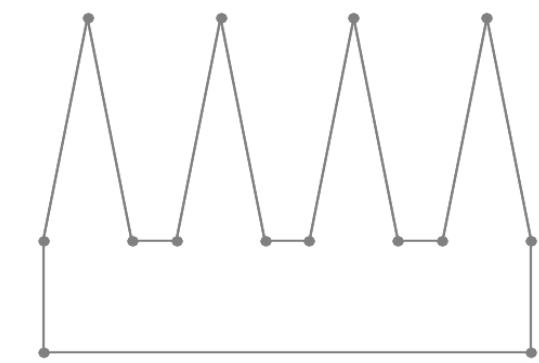
x - search factor

$$g'_1 = g + \frac{1}{x} M(g)$$

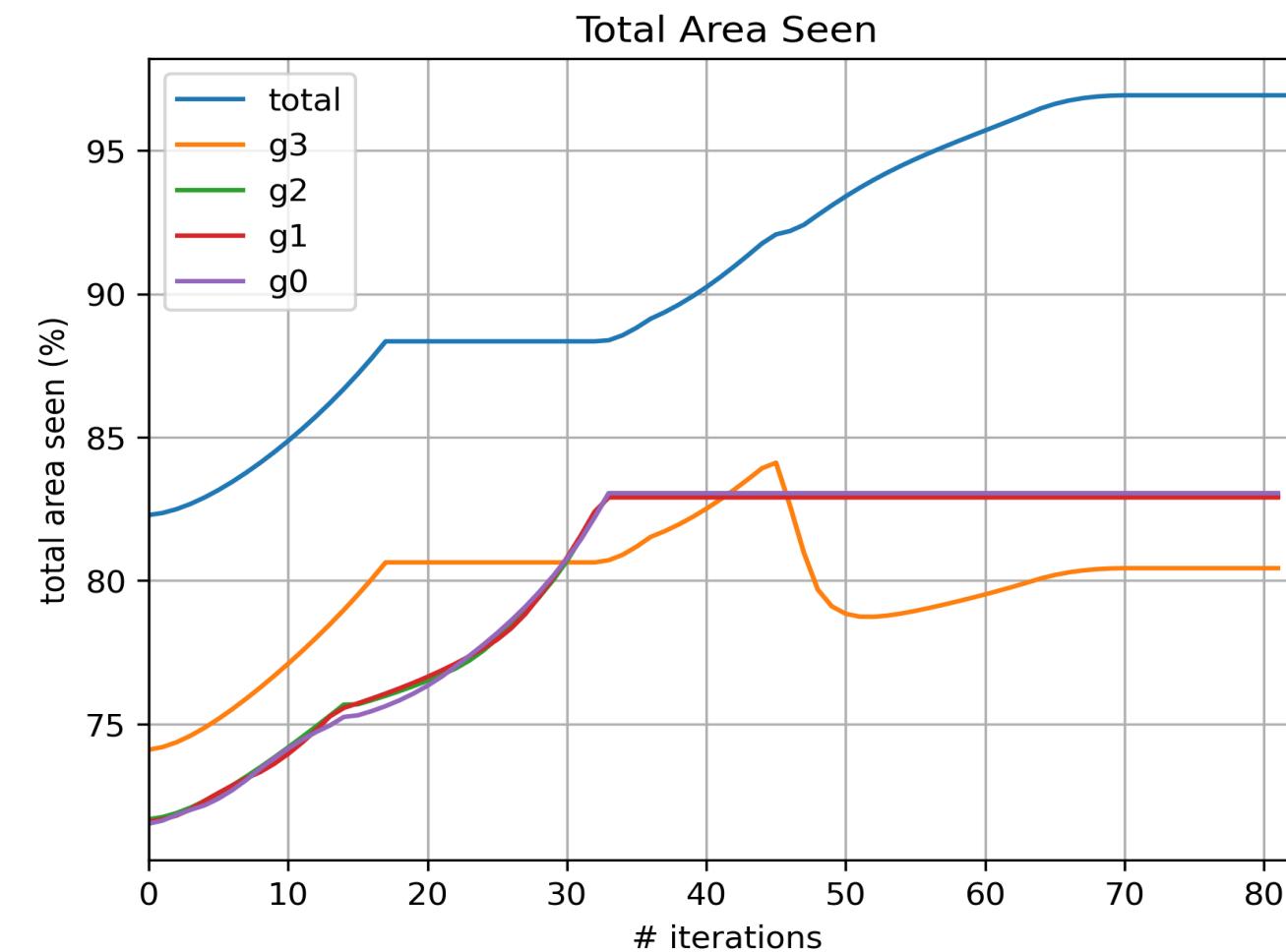
$$g'_2 = g + \frac{s}{x} M(g)$$

$$g'_3 = g + \frac{s^2}{x} M(g)$$

Heuristics: Line Search

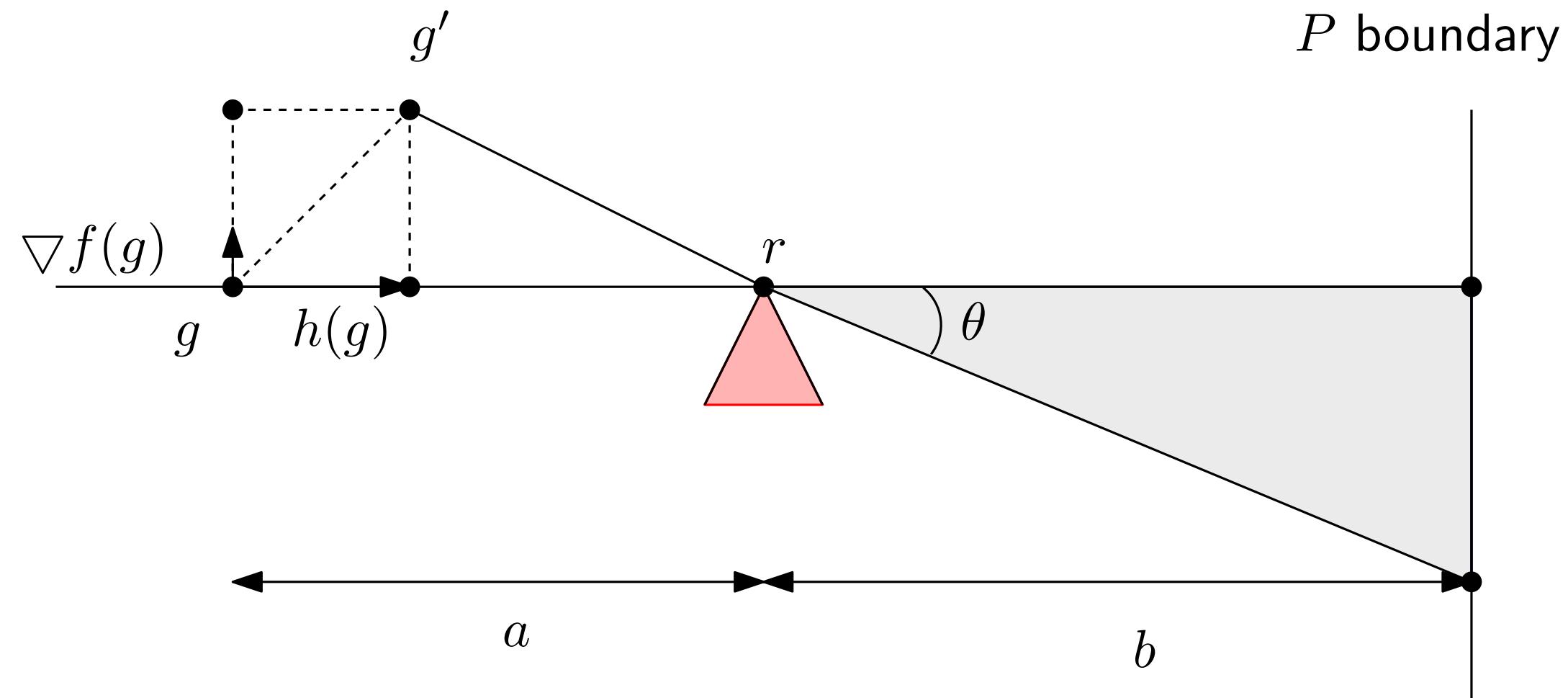


All heuristics



No line search

Heuristics: Angle behind reflex vertex

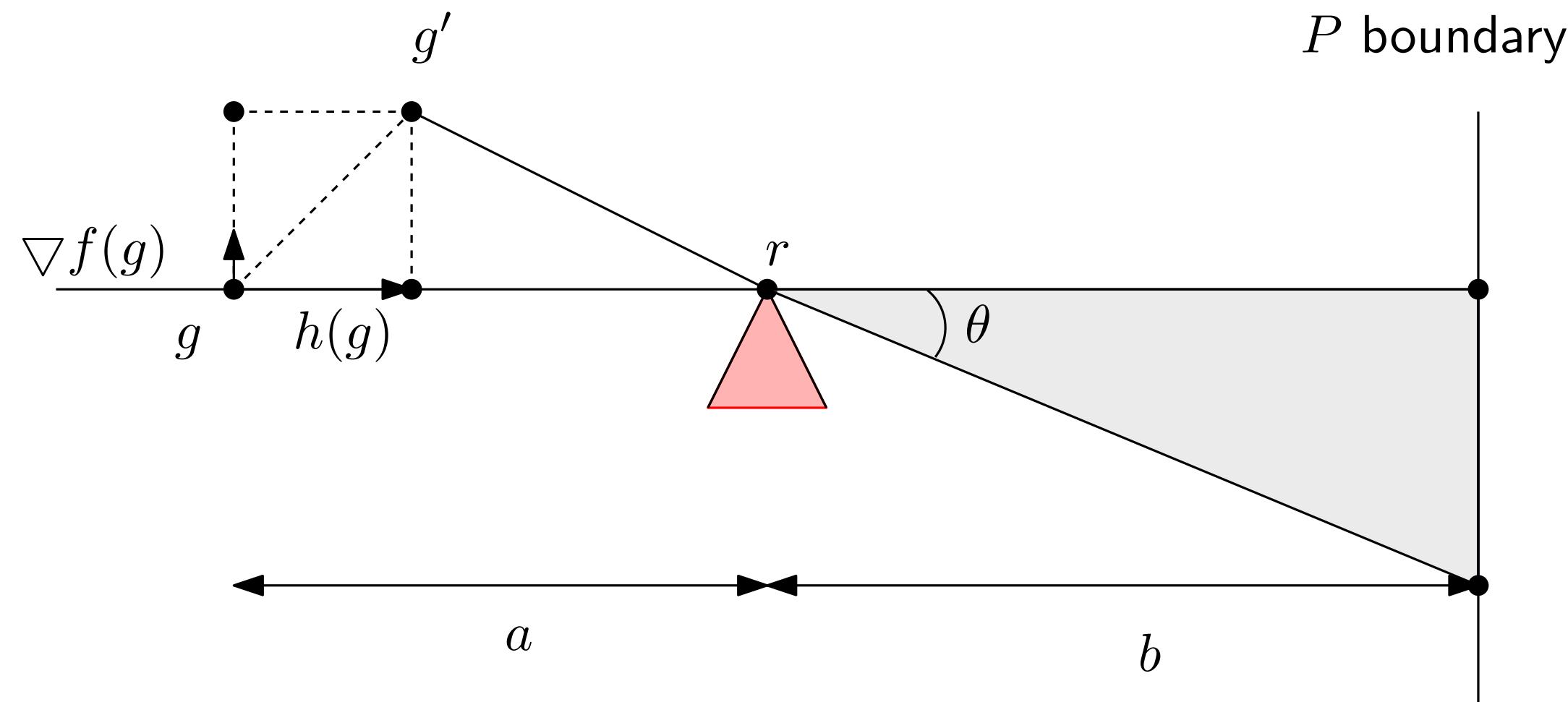


Heuristics: Angle behind reflex vertex

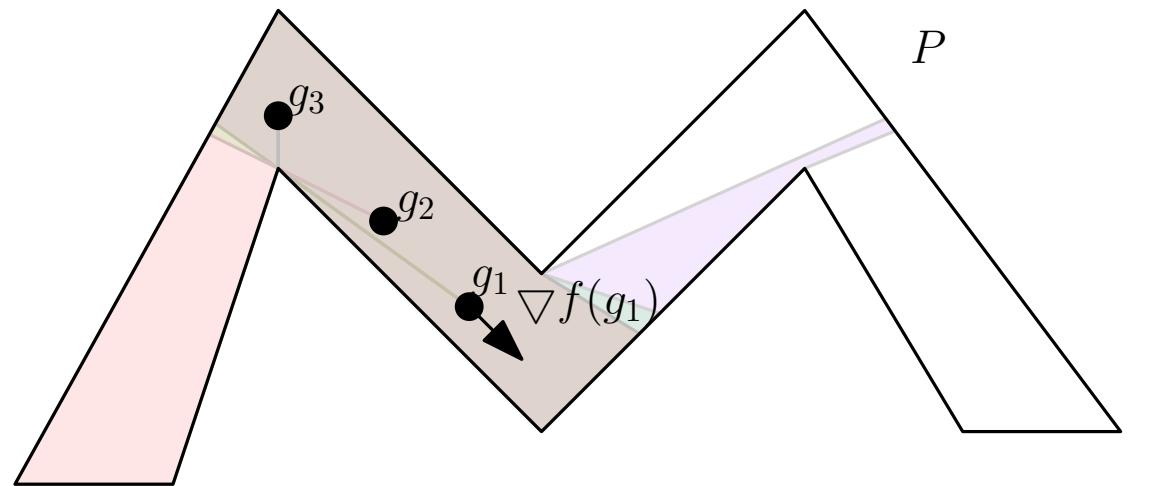
$$g' = g + \frac{s^t}{x} \left(\frac{\theta}{2\pi} + c \right) M(g)$$

c - constant

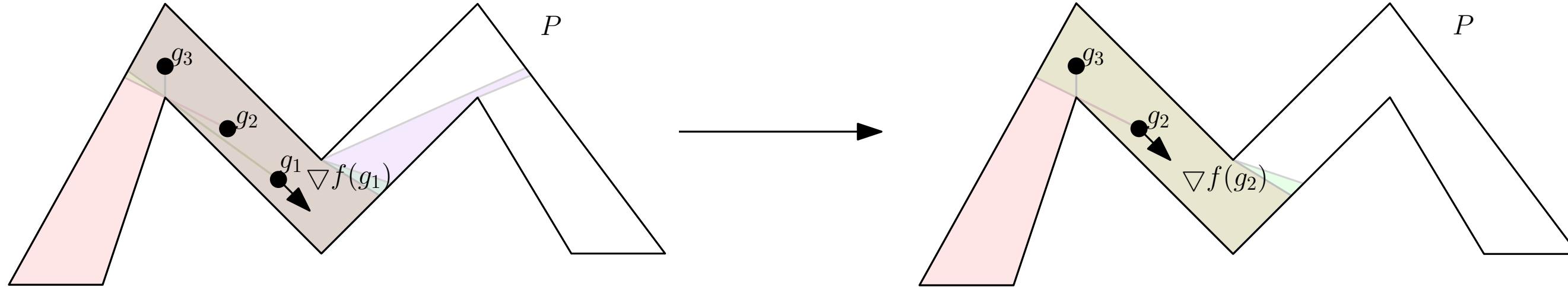
t - optimum step factor



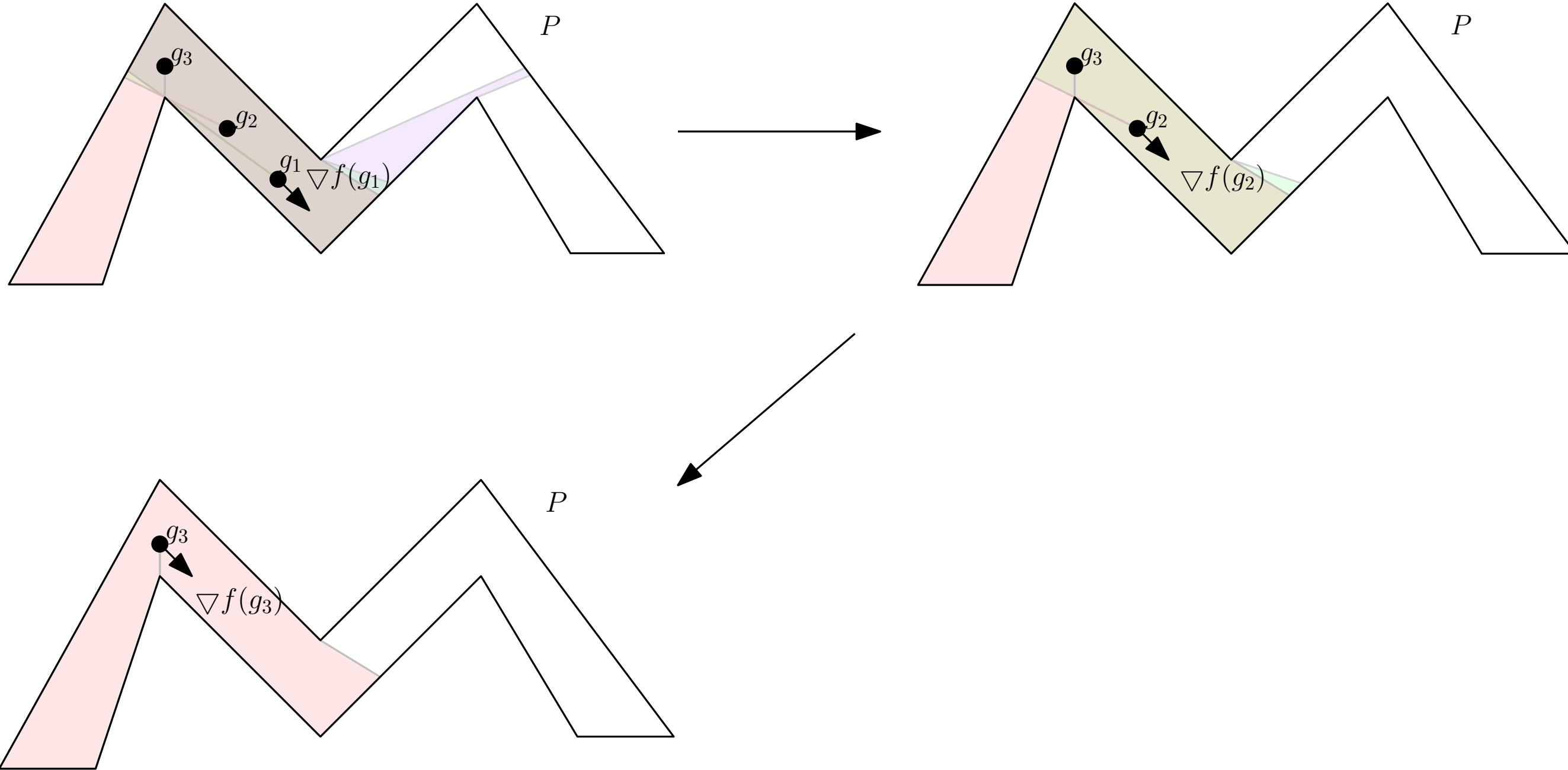
Heuristics: Hidden movement



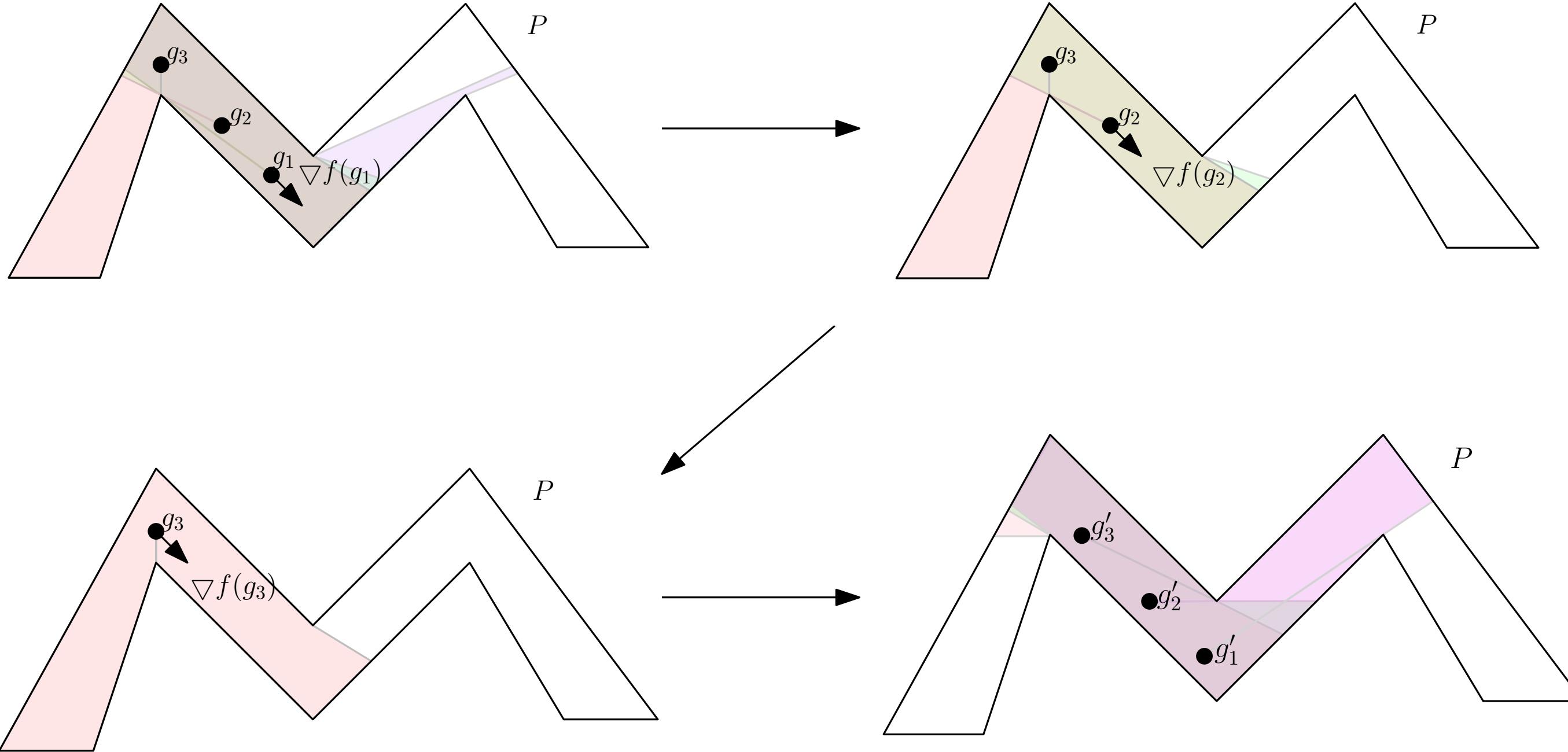
Heuristics: Hidden movement



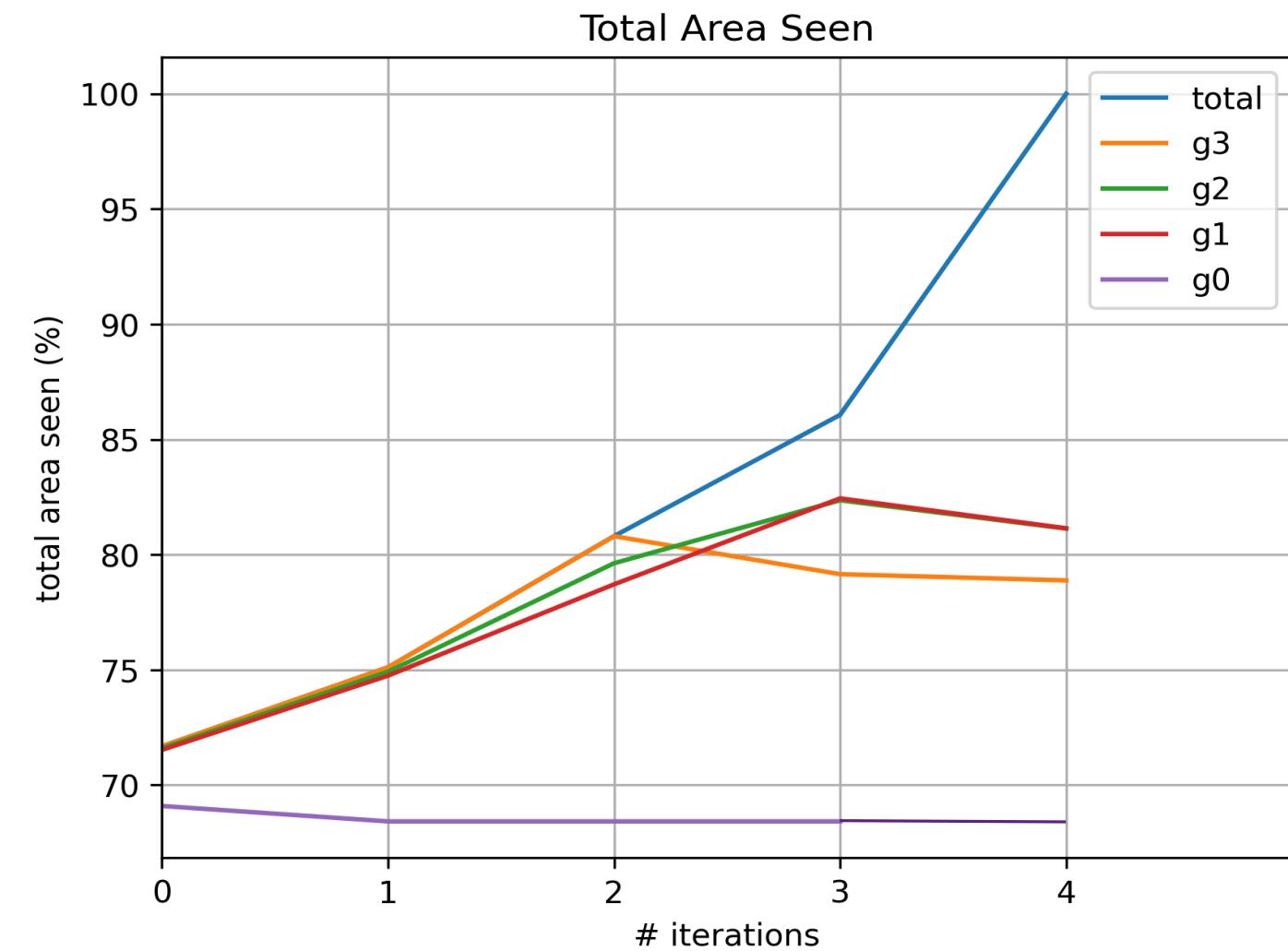
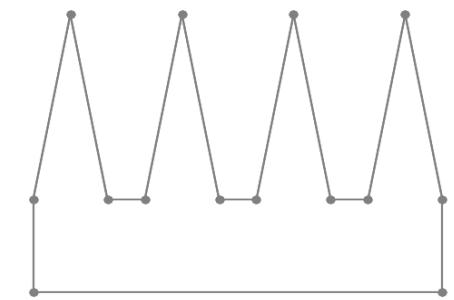
Heuristics: Hidden movement



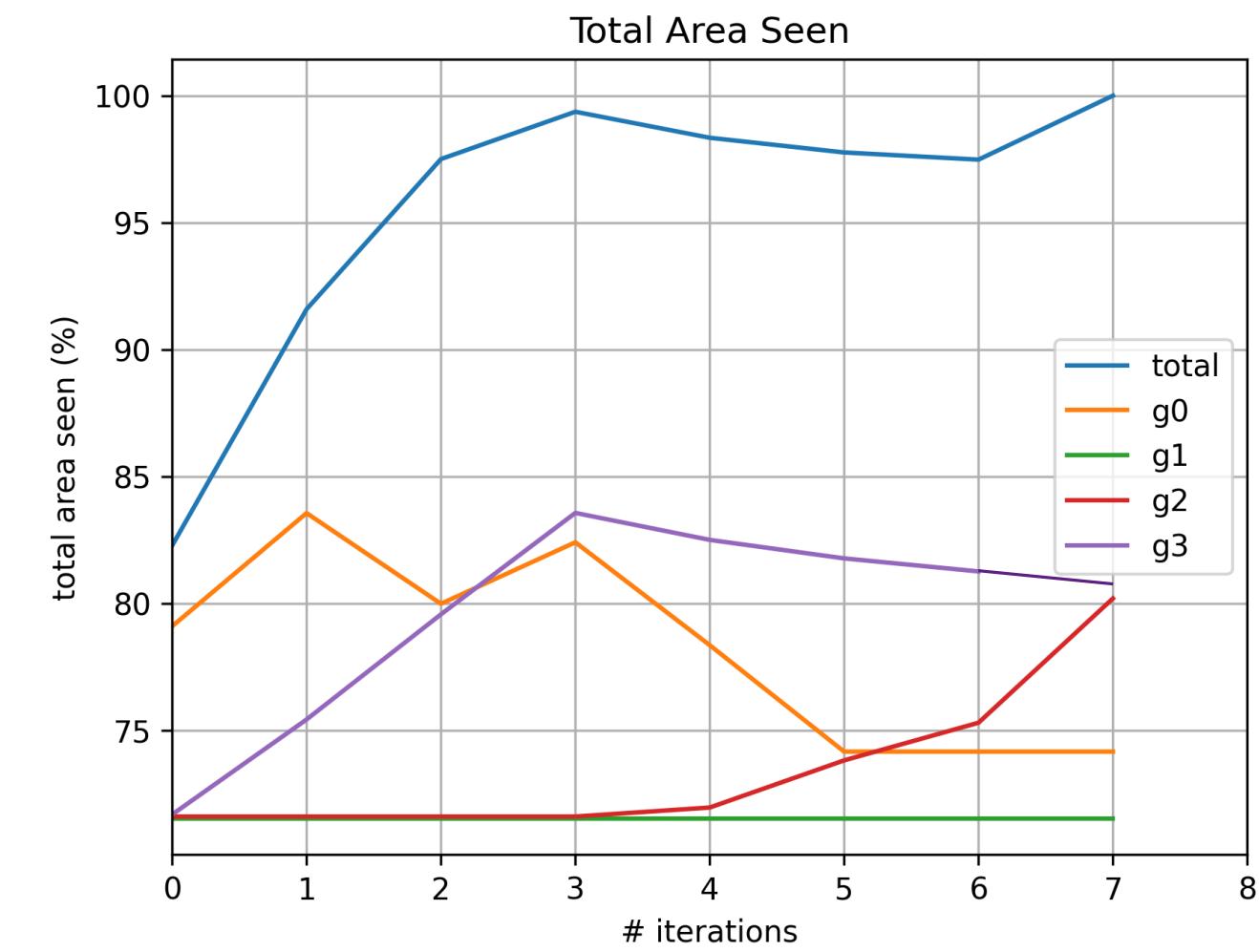
Heuristics: Hidden movement



Heuristics: Hidden movement

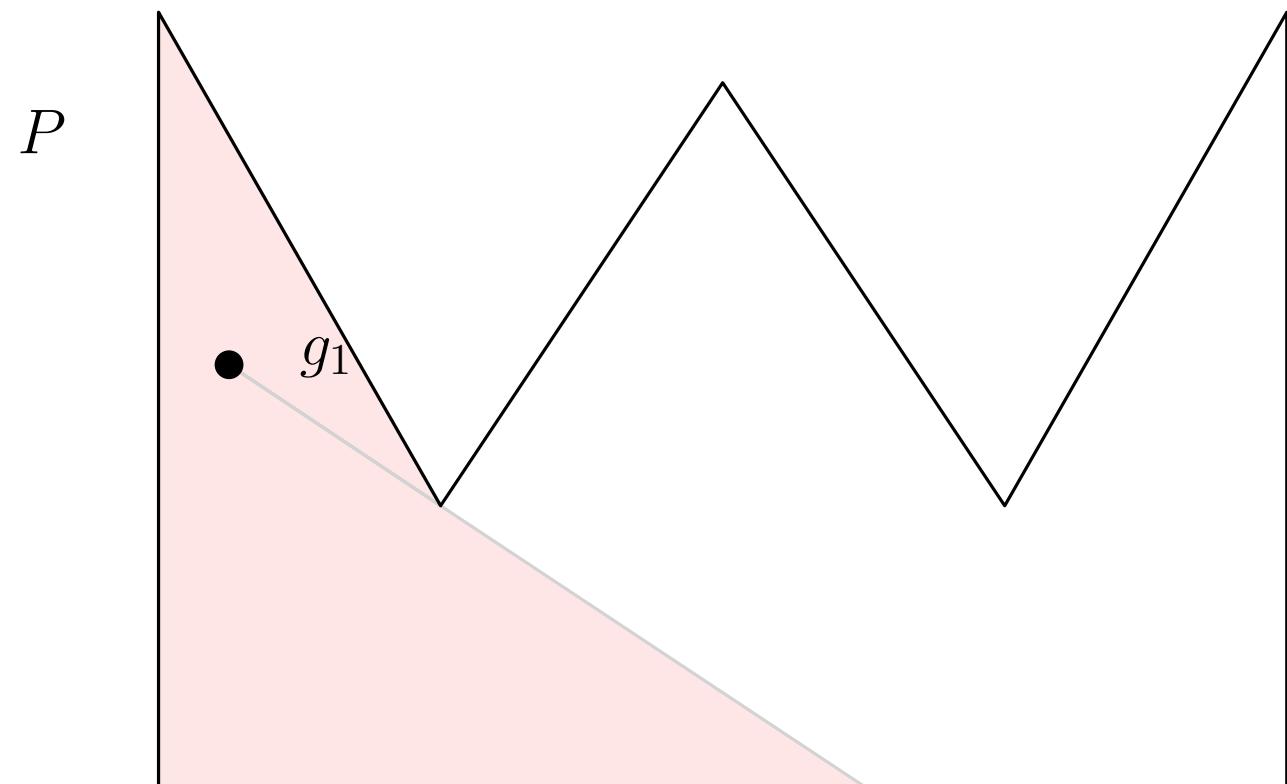


All heuristics

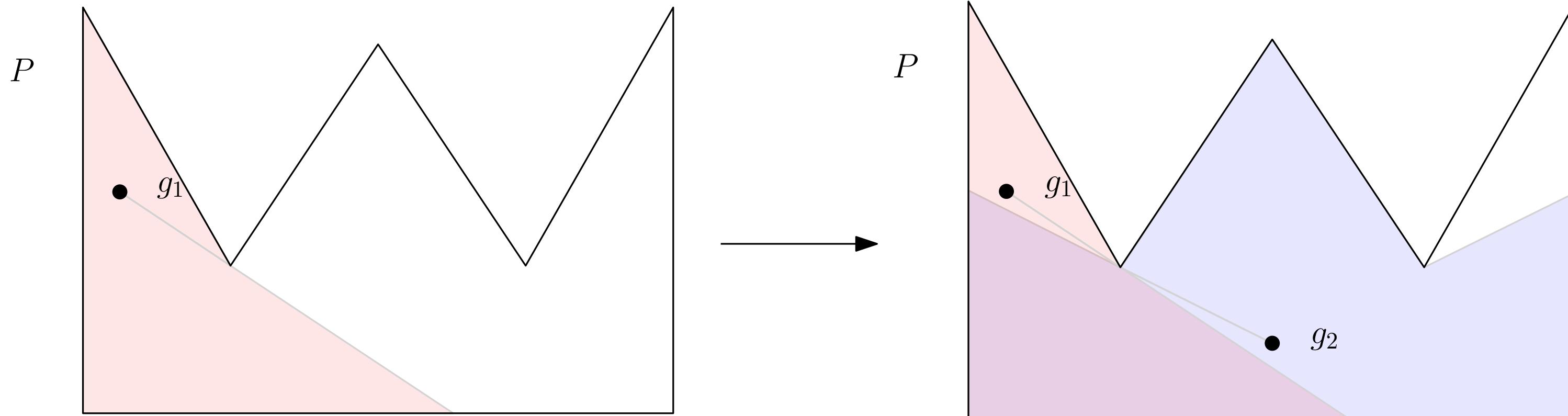


No hidden movement

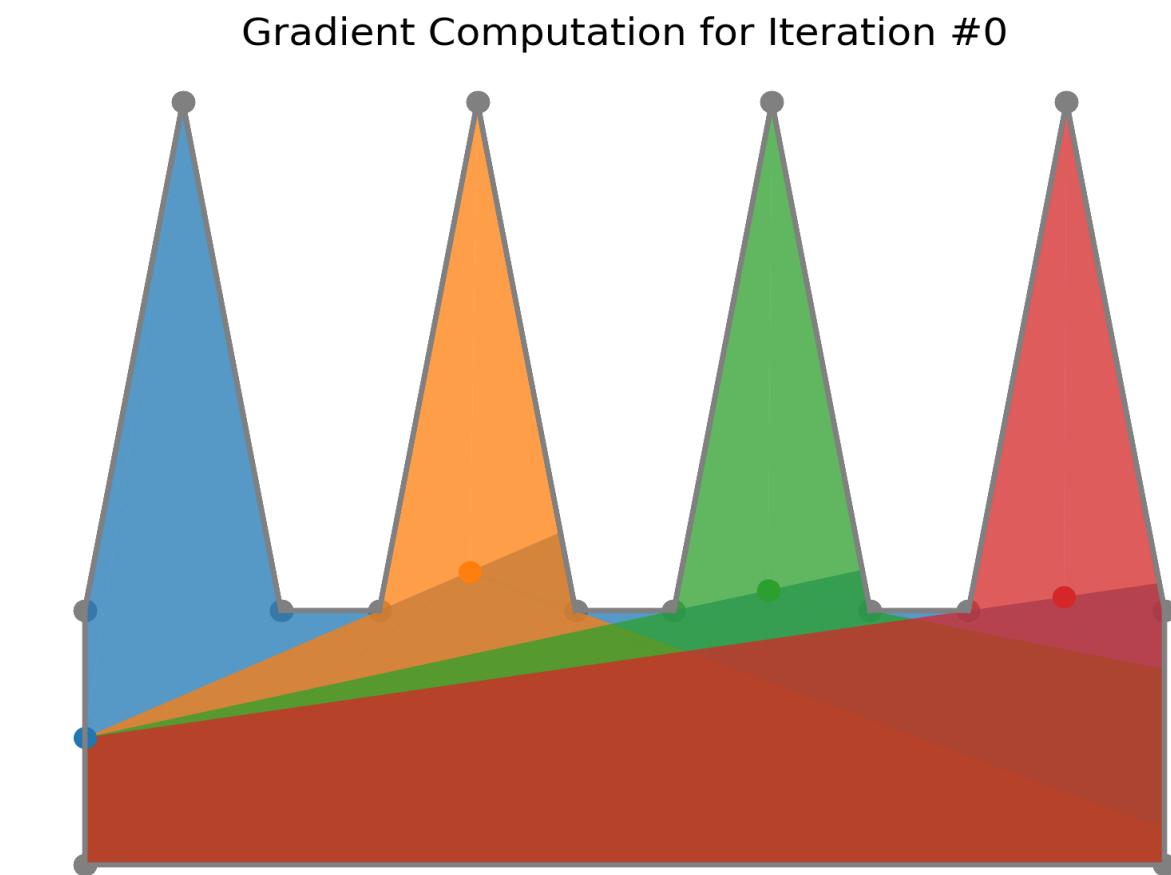
Heuristics: Greedy initialisation



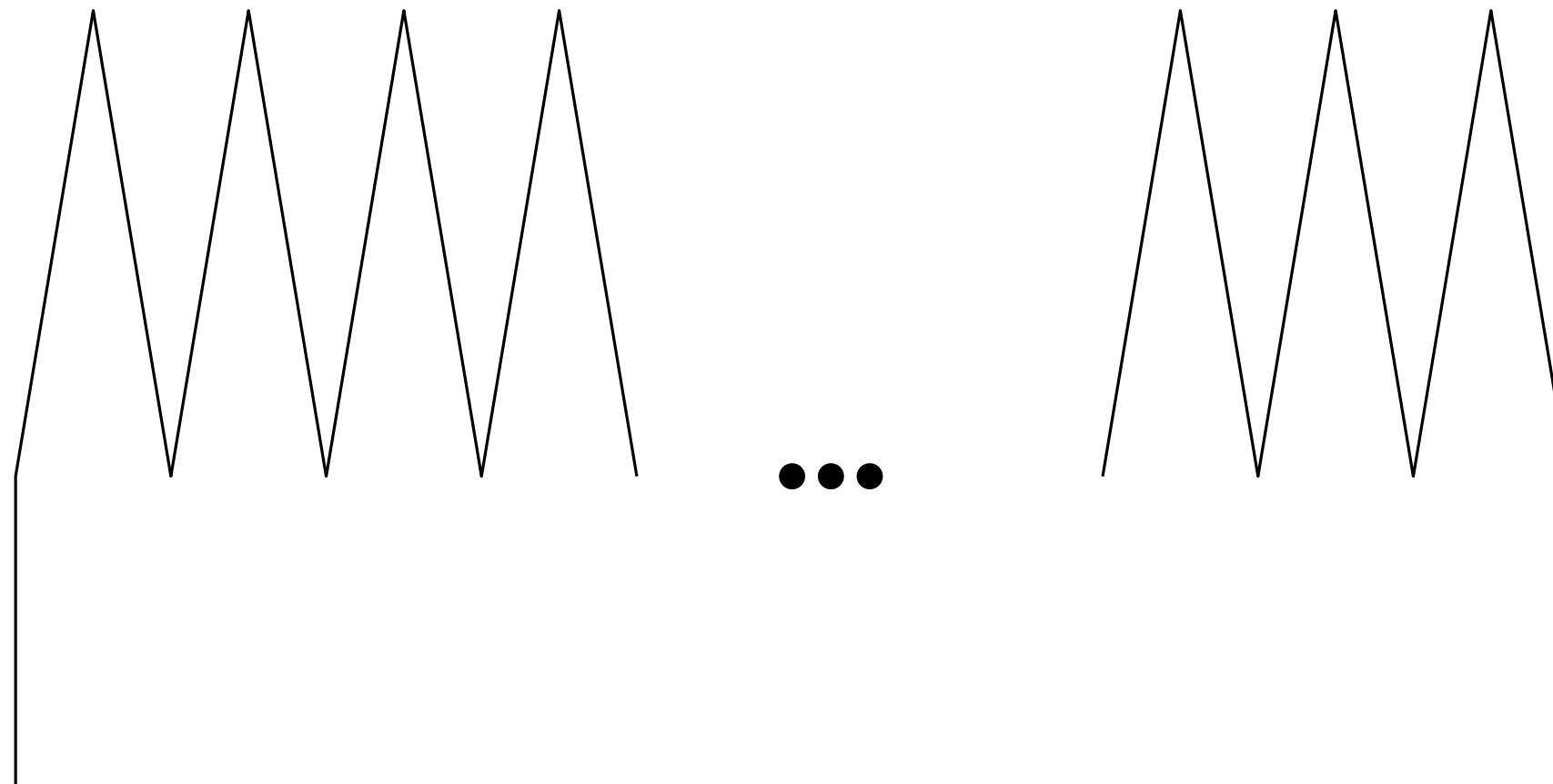
Heuristics: Greedy initialisation



Heuristics: Greedy initialisation

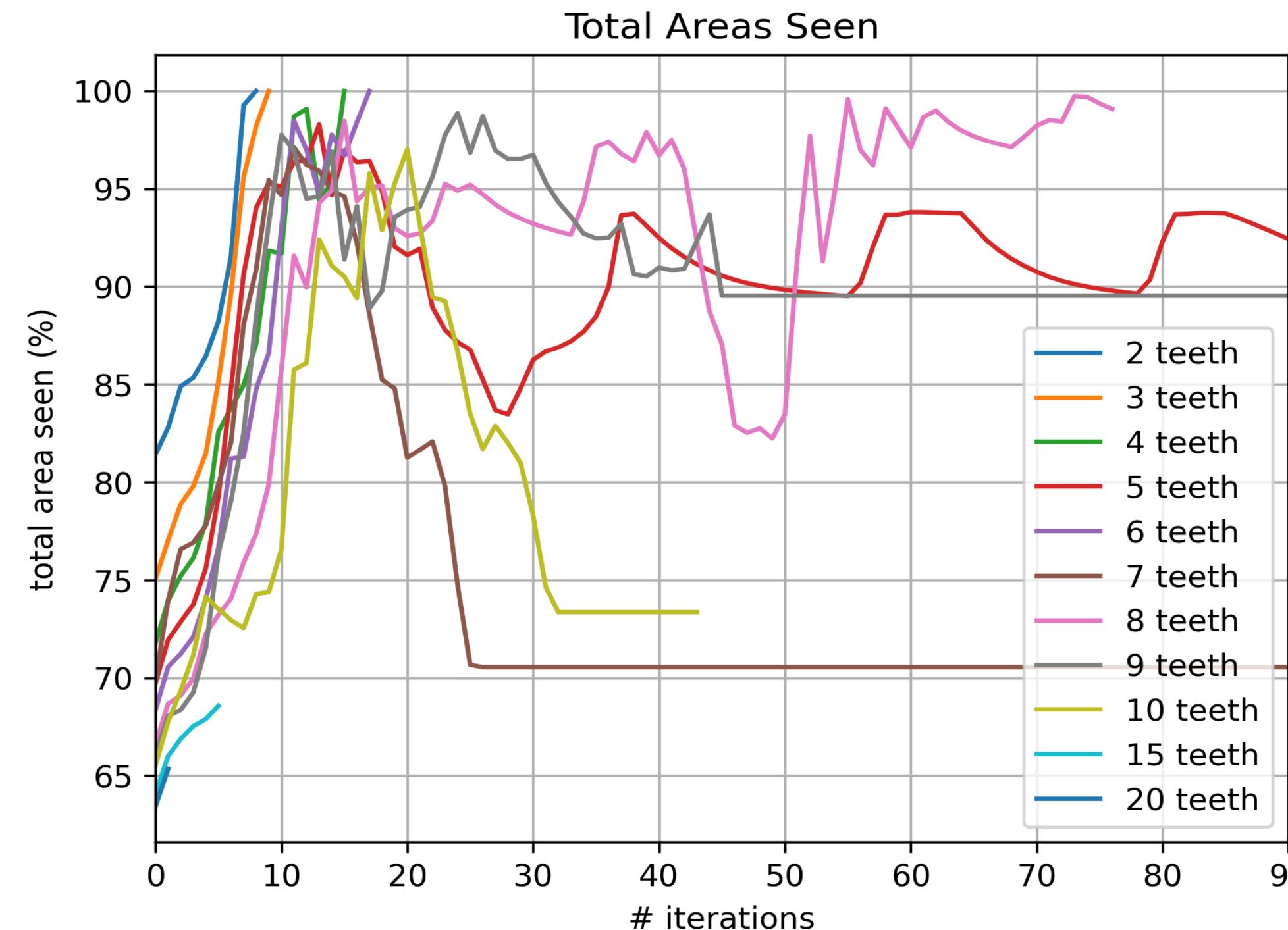


Scalability for the comb polygon

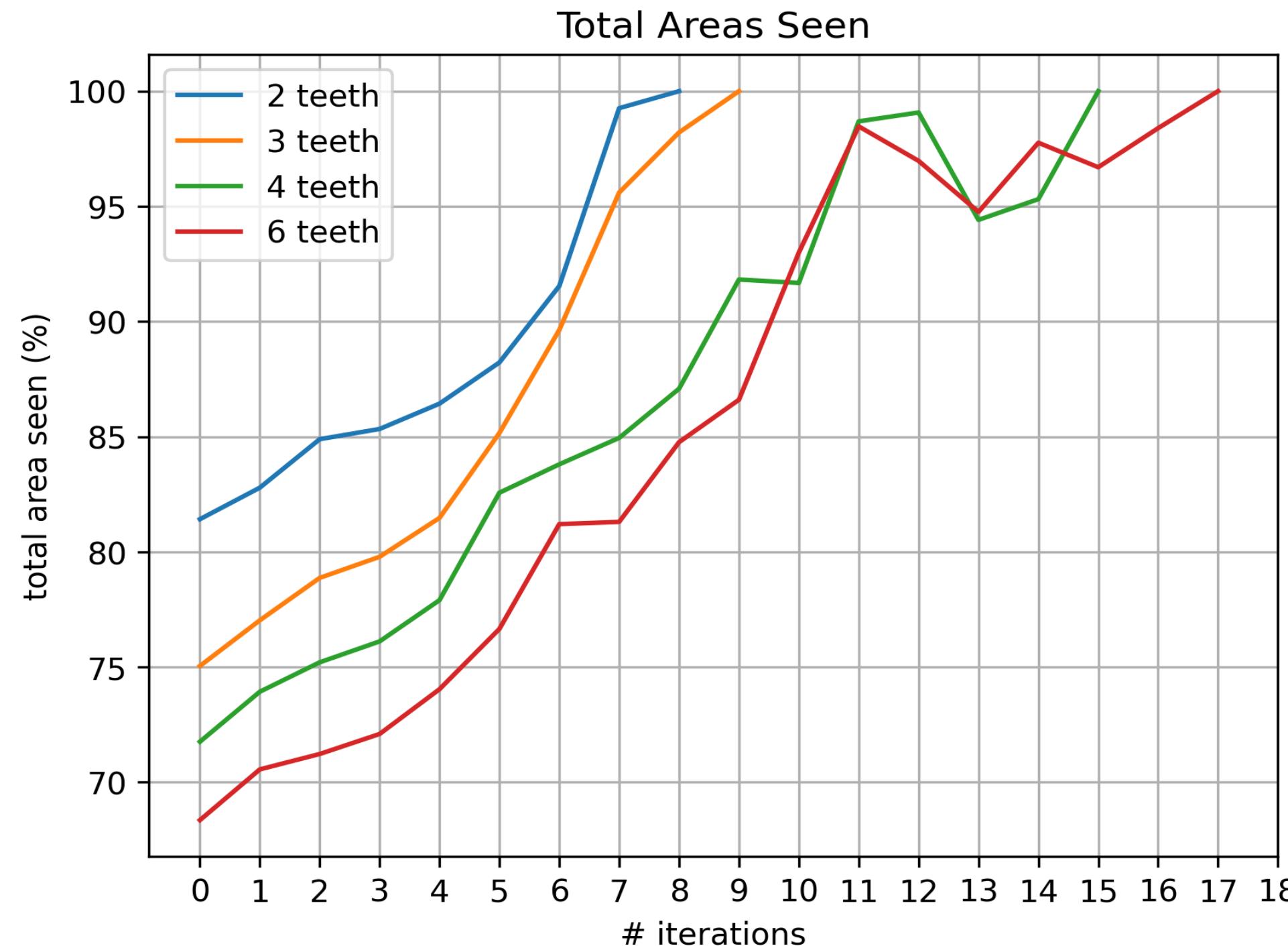


2, 3, ..., 10, 15, 20 teeth

Scalability for the comb polygon



Scalability for the comb polygon



Problems encountered

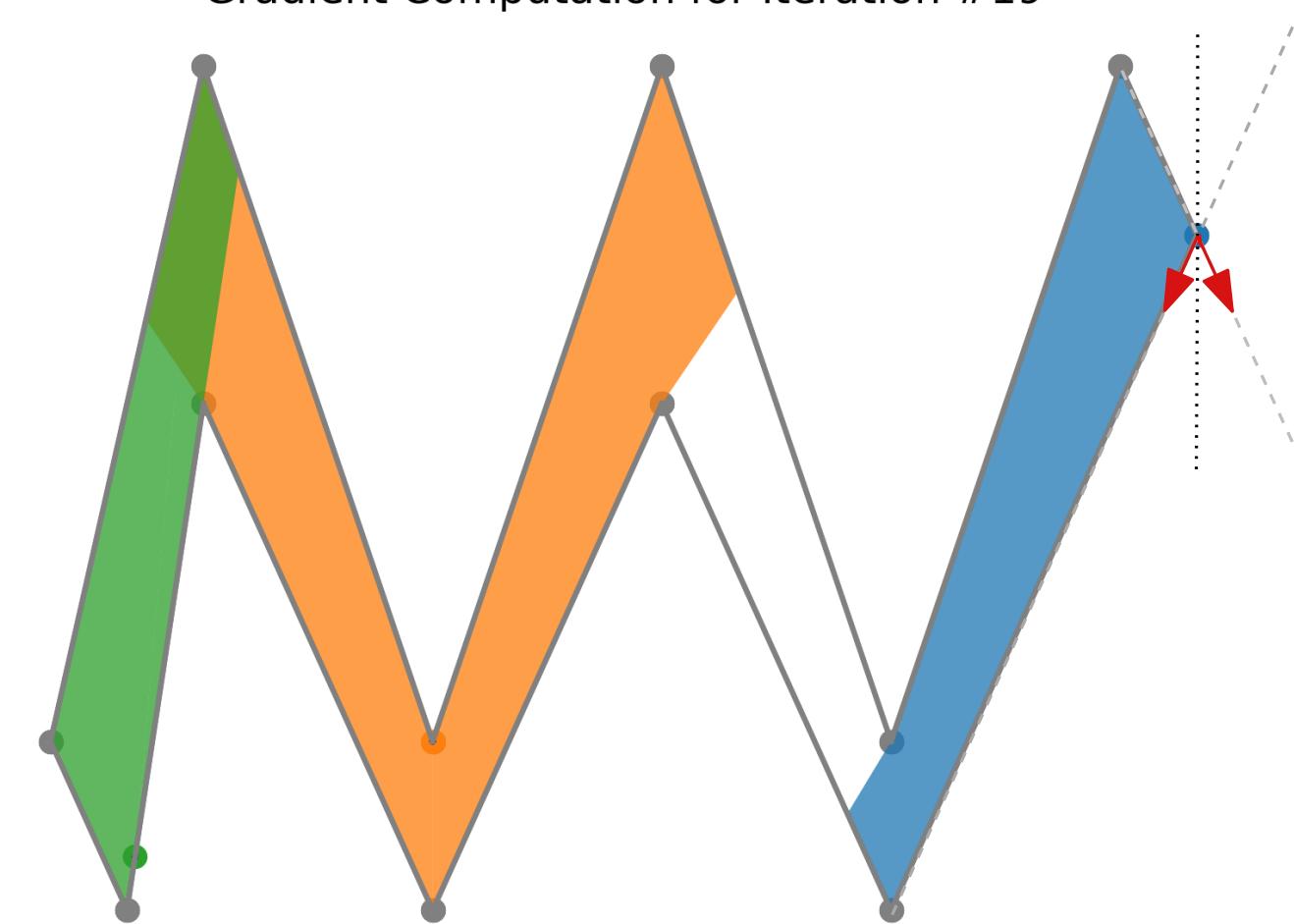
hyperparameter sensitivity

Problems encountered

hyperparameter sensitivity

edge-cases

Gradient Computation for Iteration #19



Problems encountered

hyperparameter sensitivity

edge-cases

CGAL errors

```
terminate called after throwing an instance of 'CGAL::Assertion_exception'
  what(): CGAL ERROR: assertion violation!
Expr: is_finite(d)
File: /usr/include/CGAL/Interval_nt.h
Line: 133
fish: "./../build/main < inputs/love.i..." terminated by signal SIGABRT (Abort)
```

Future work

solve existing bugs



Future work

solve existing bugs

improve the algorithm's robustness, performance and scalability



Future work

solve existing bugs

improve the algorithm's robustness, performance and scalability

implement other heuristics



Future work

solve existing bugs

improve the algorithm's robustness, performance and scalability

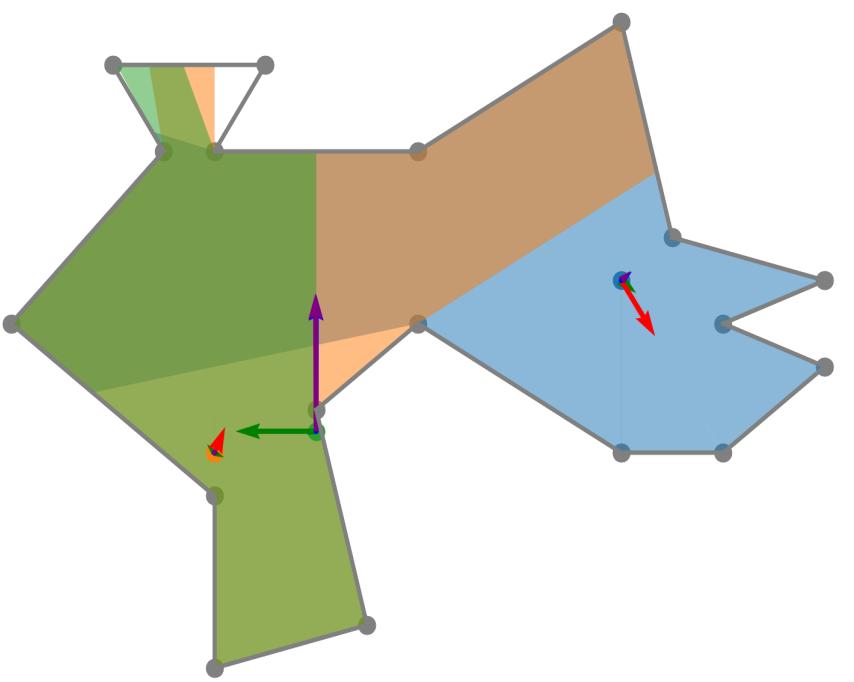
implement other heuristics

test the algorithm on larger polygons with more guards

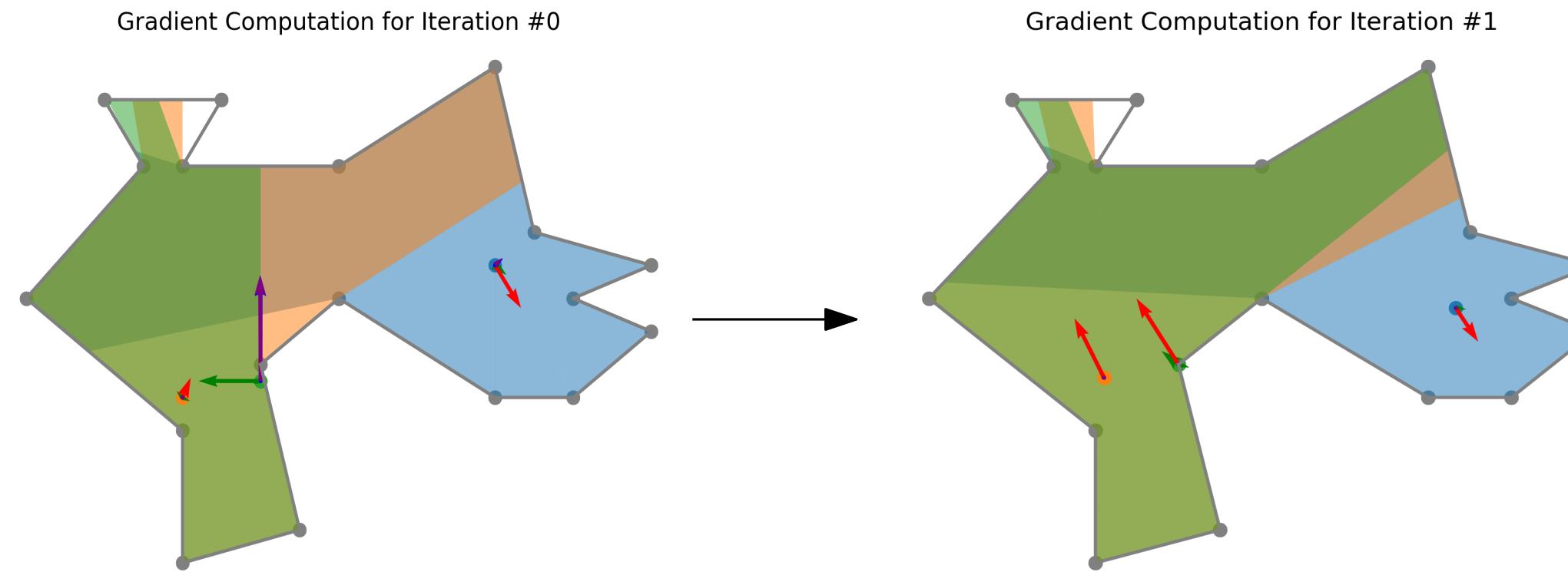


The Art Gallery Problem

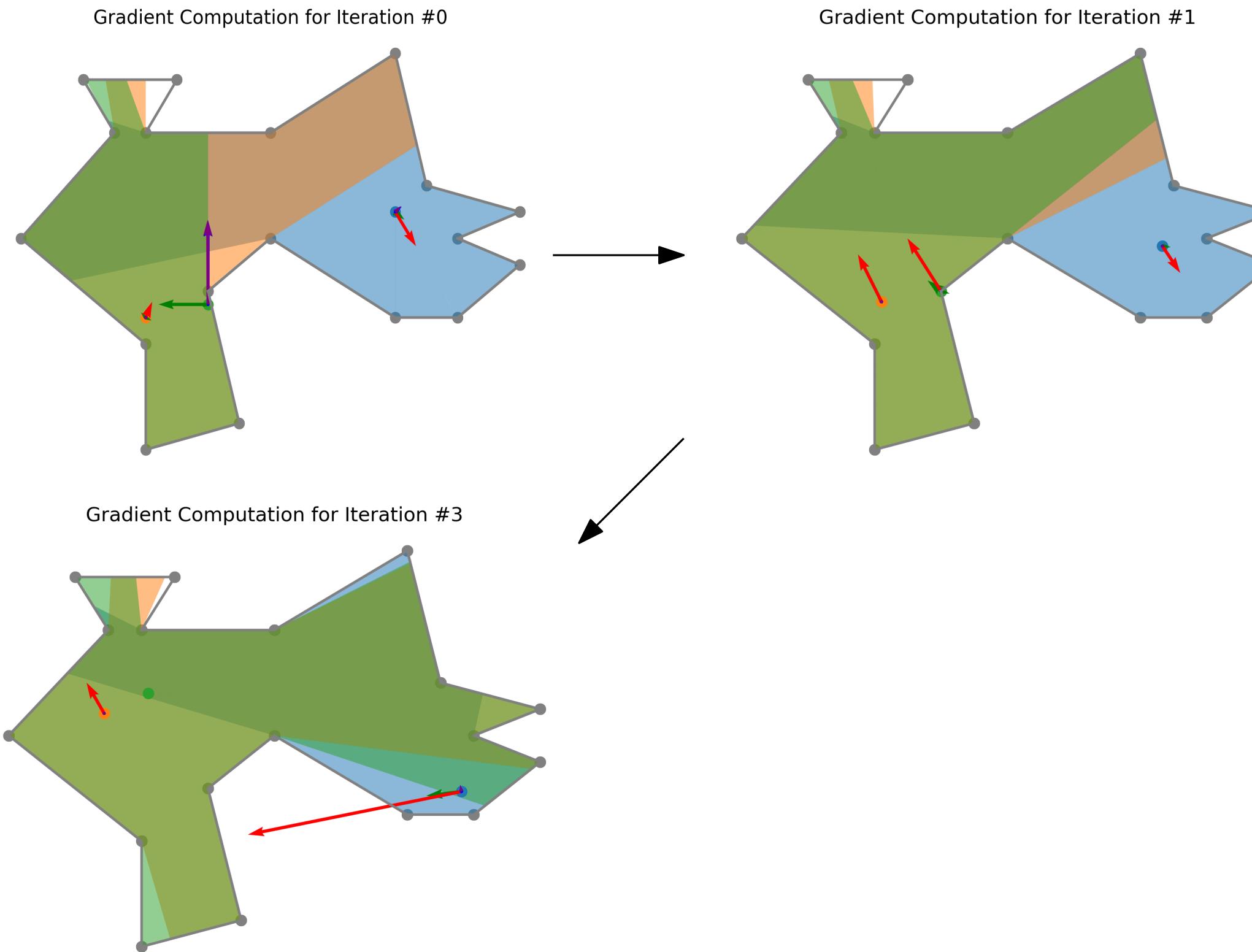
Gradient Computation for Iteration #0



The Art Gallery Problem



The Art Gallery Problem



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