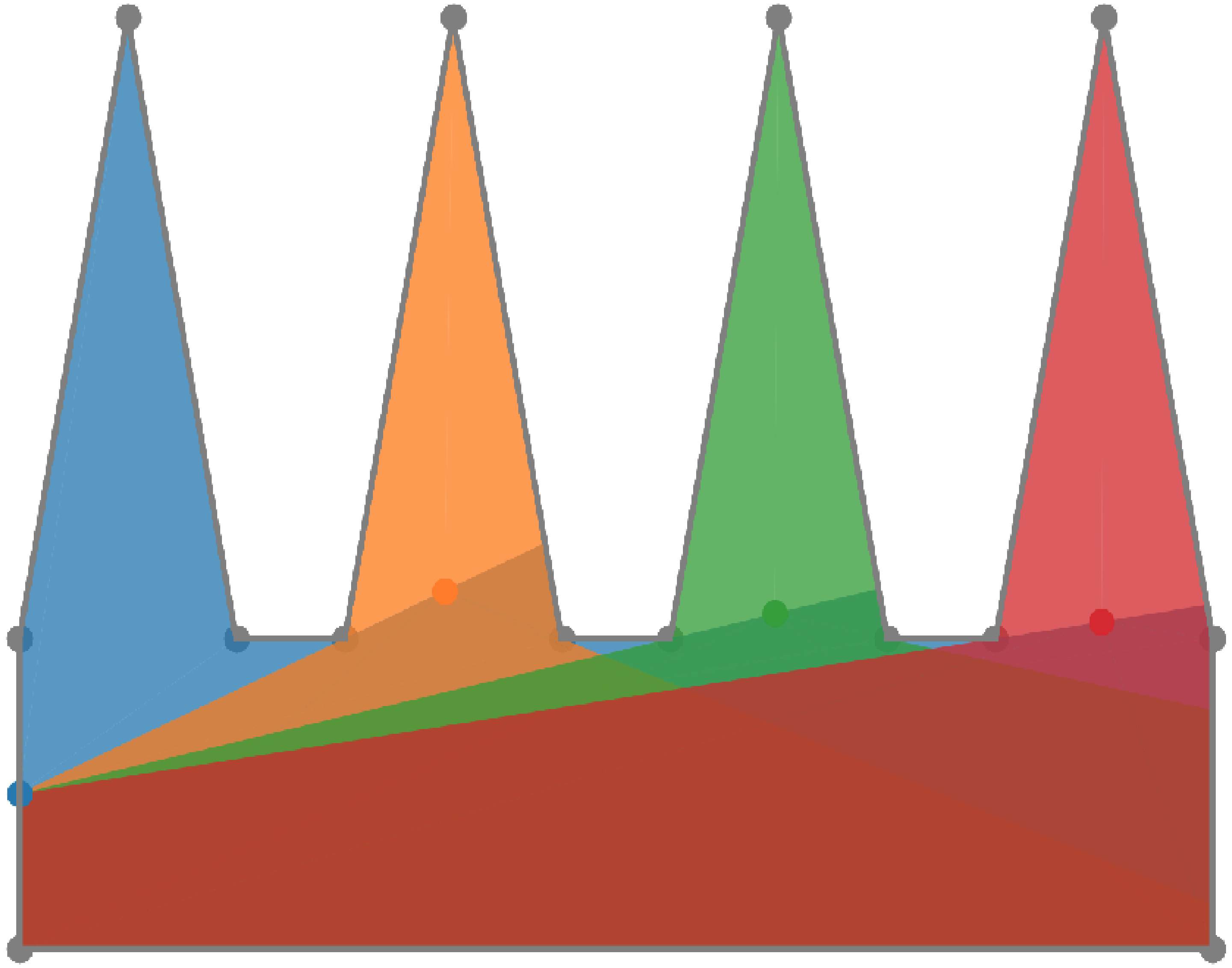
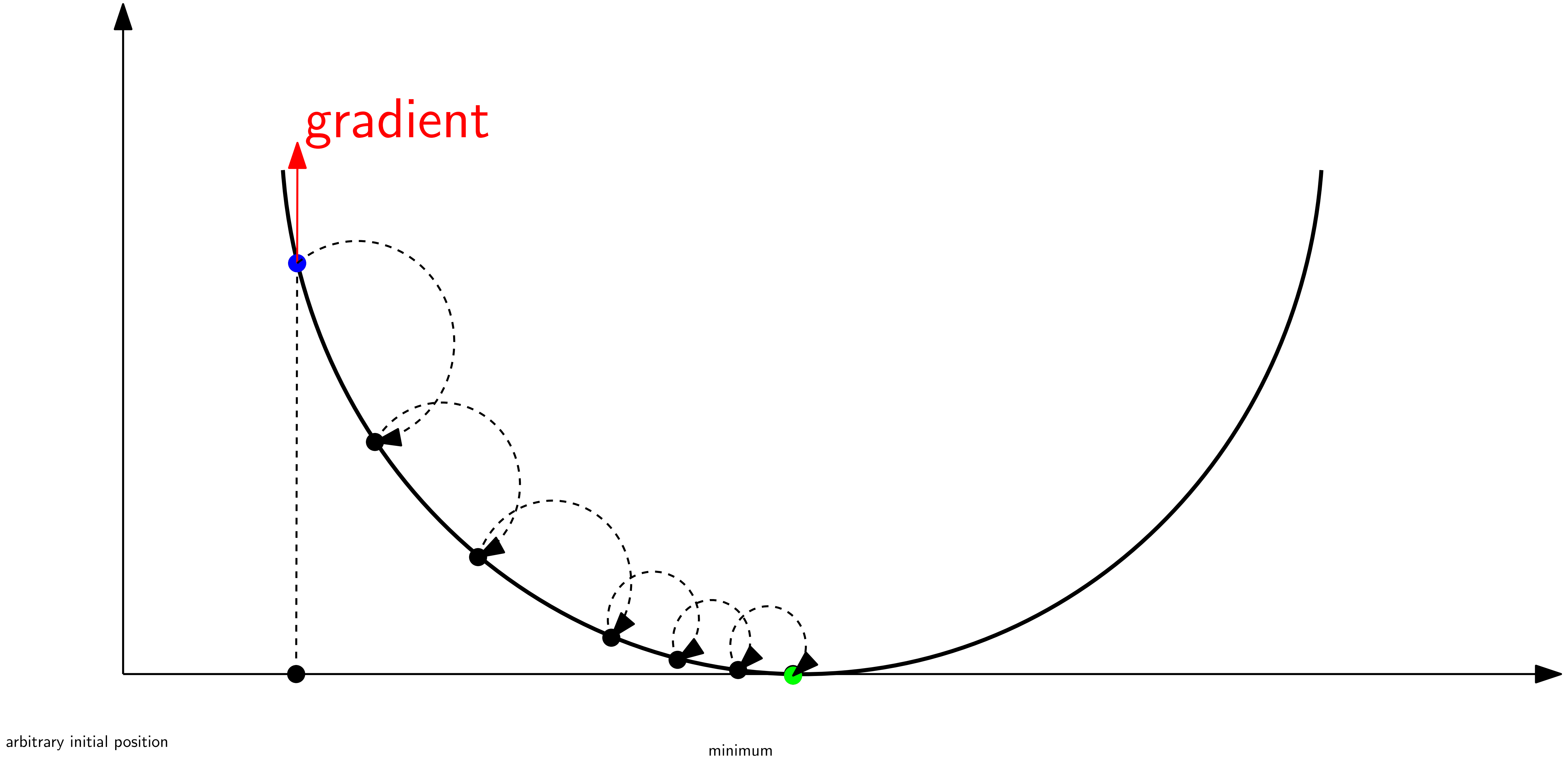
Solving the Art Gallery Problem Using Gradient Descent

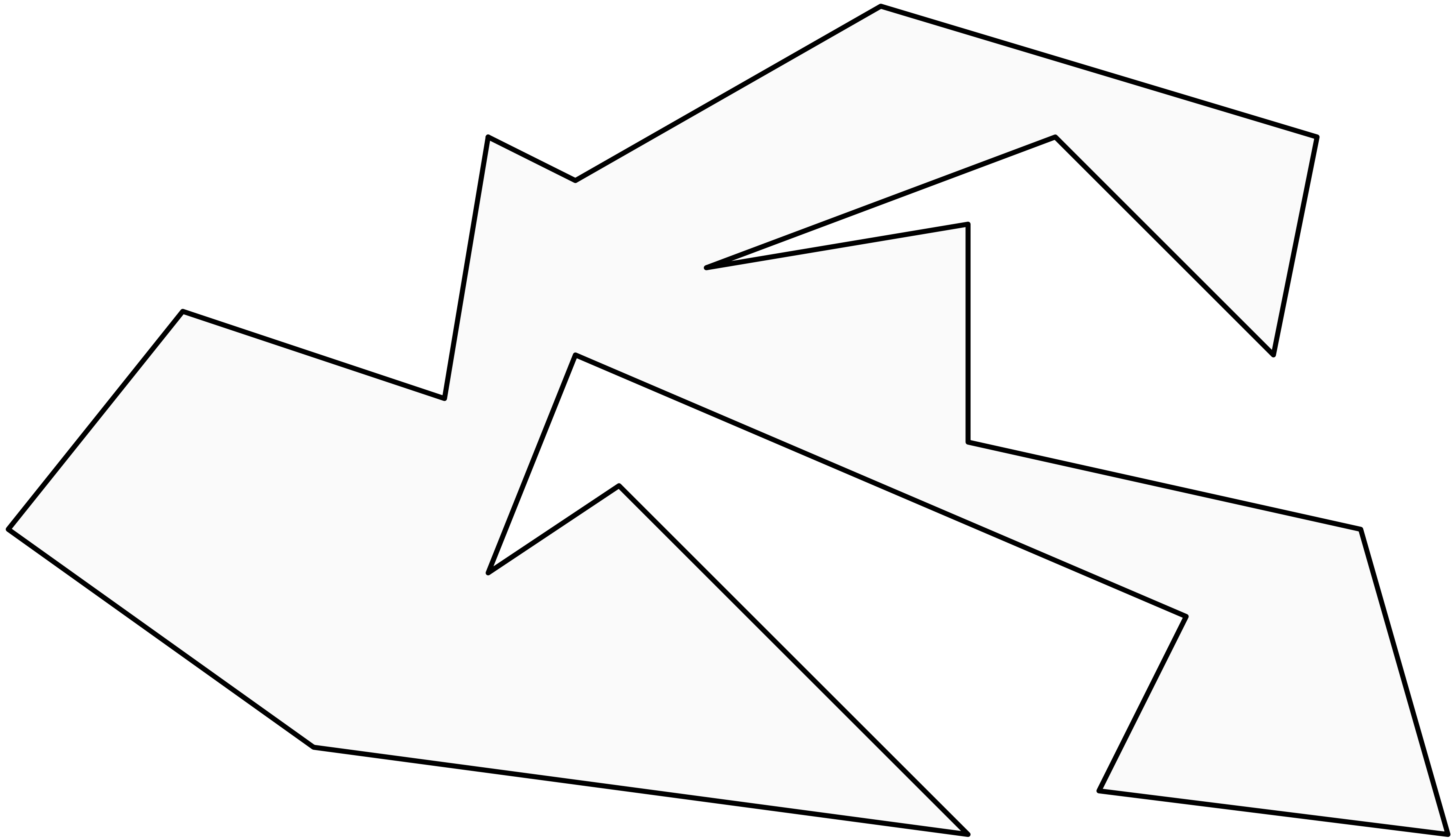
Geo Juglan



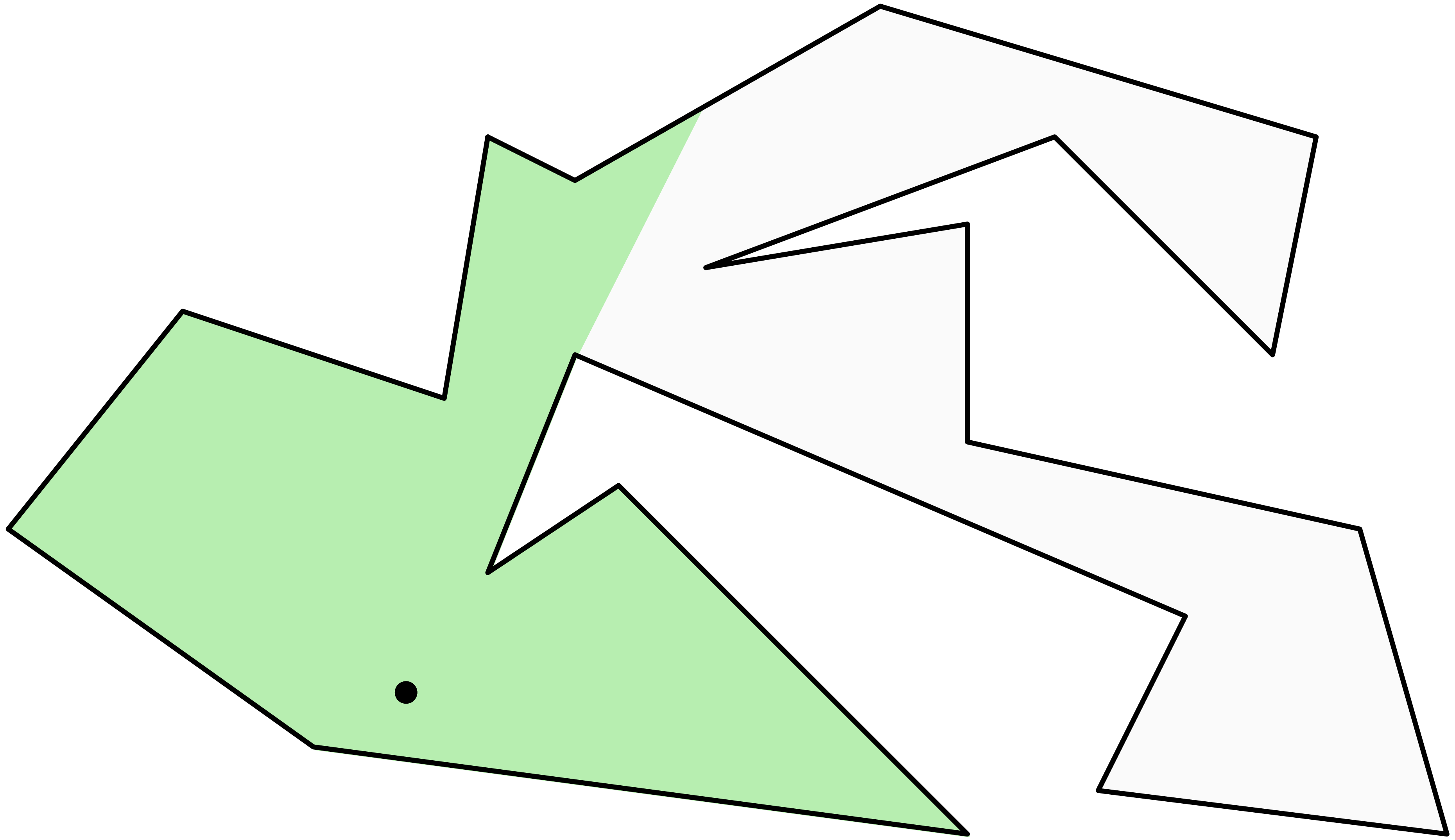
Gradient Descent



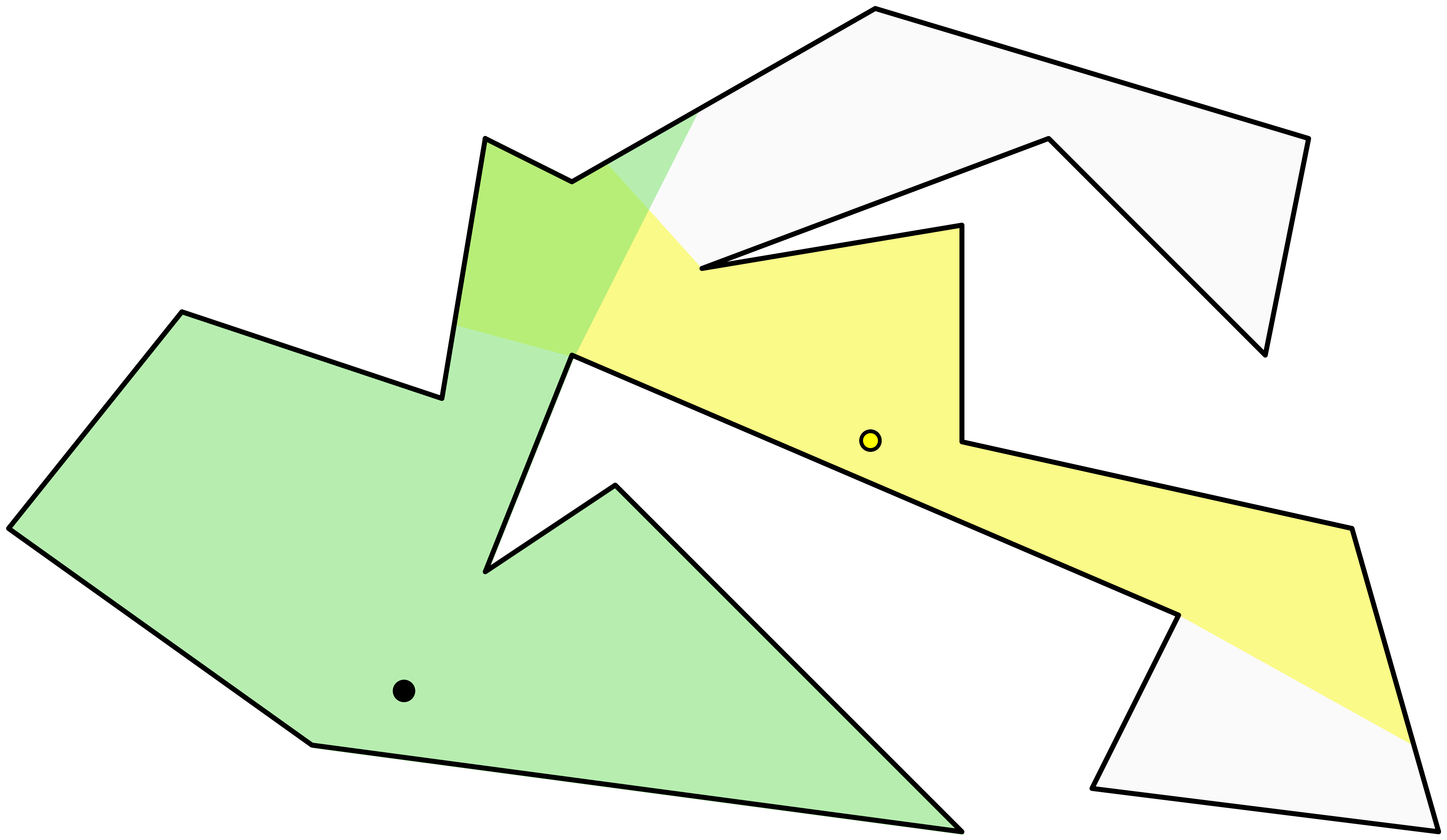
The Art Gallery Problem



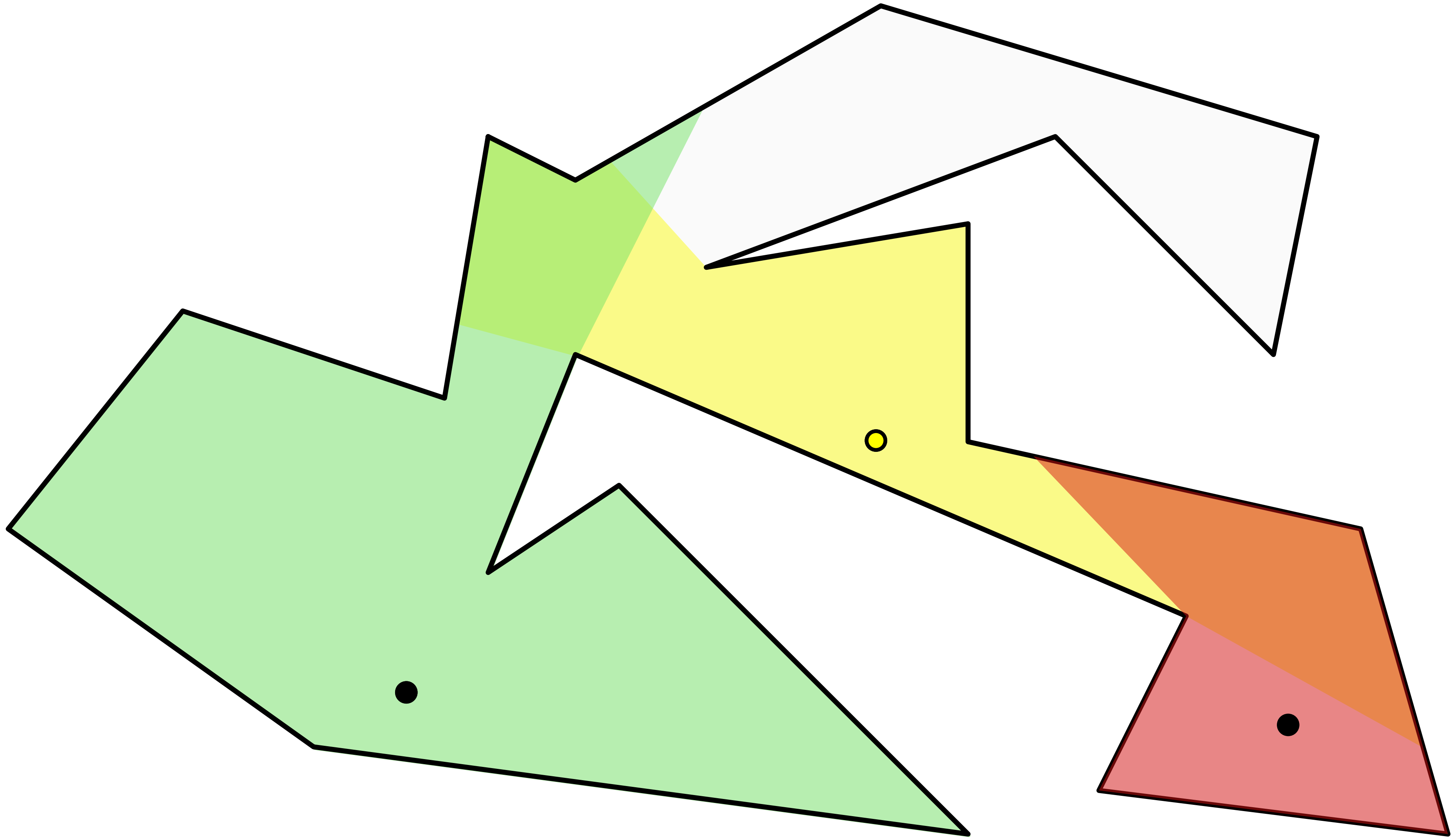
The Art Gallery Problem



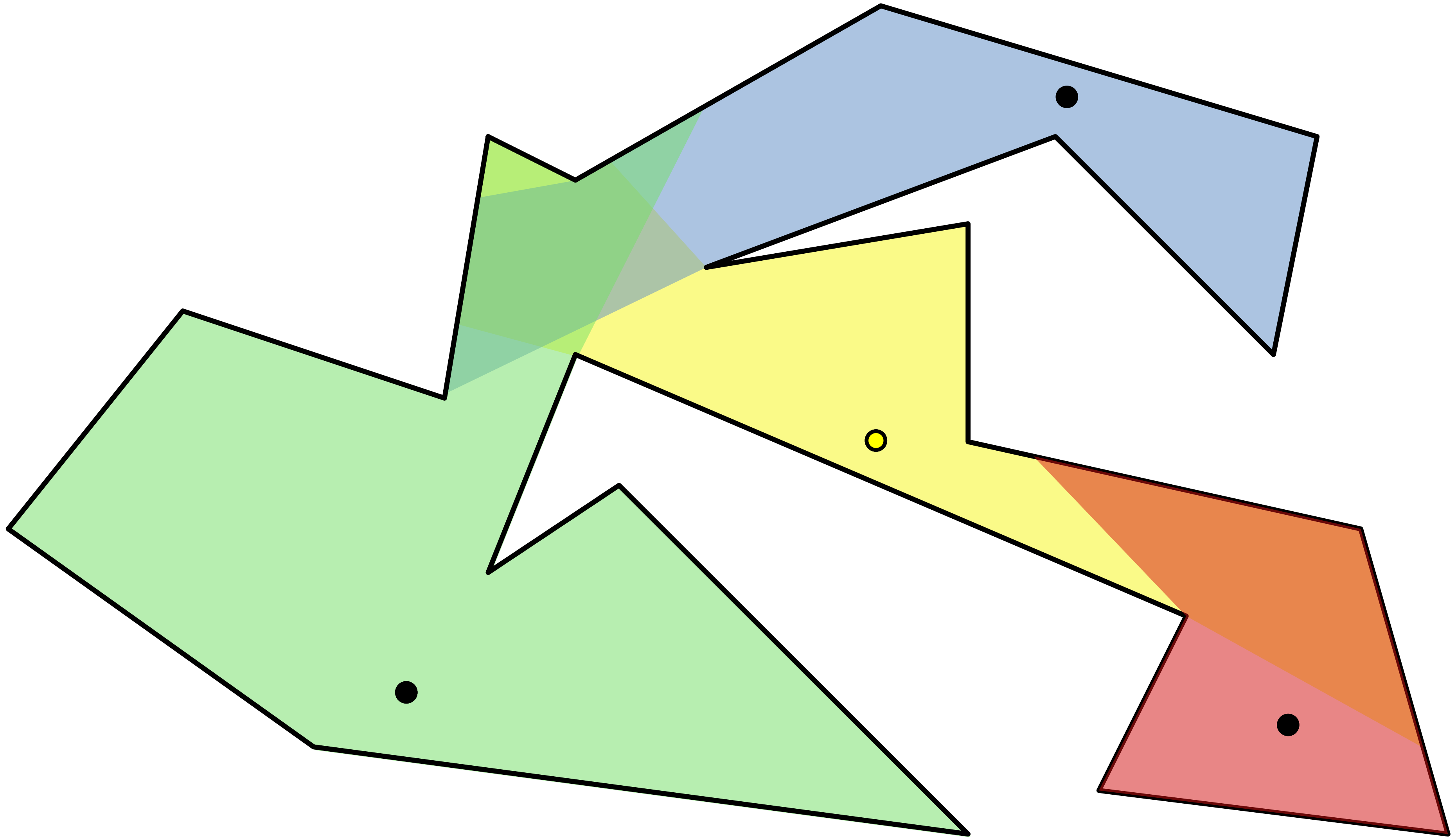
The Art Gallery Problem



The Art Gallery Problem

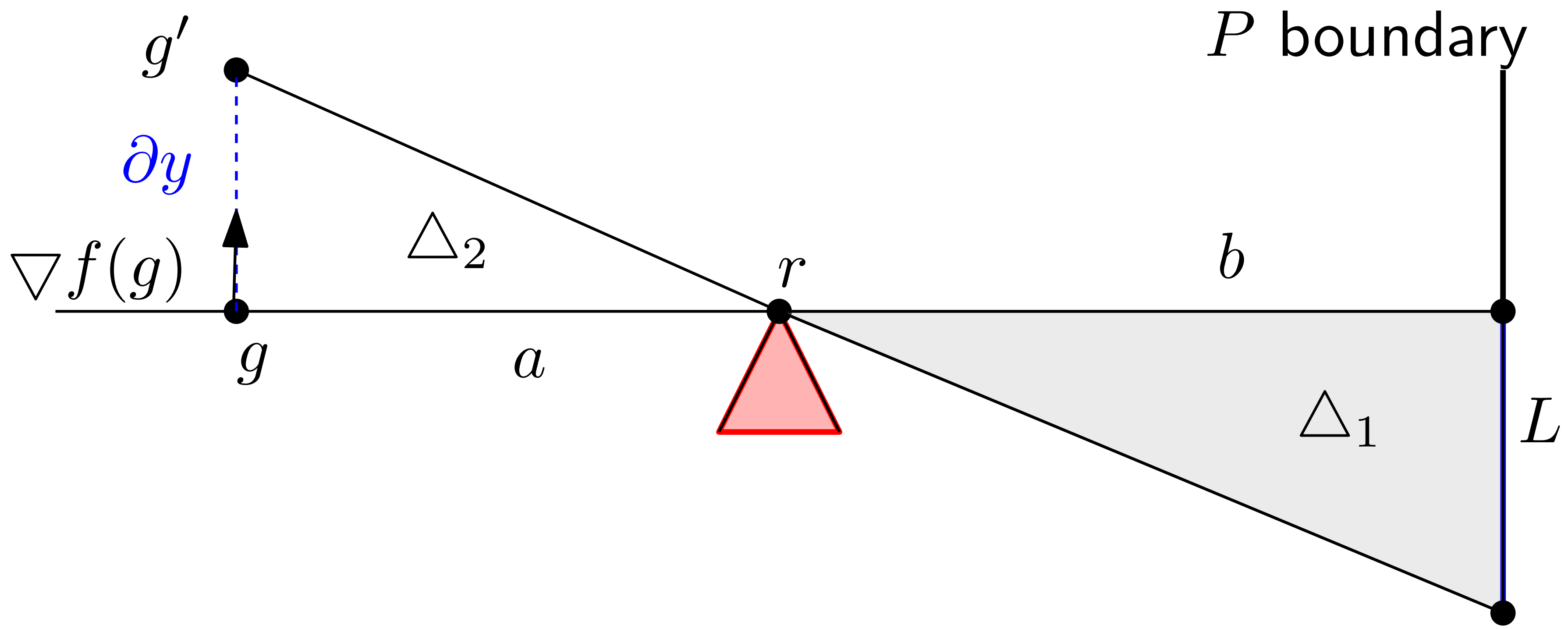


The Art Gallery Problem

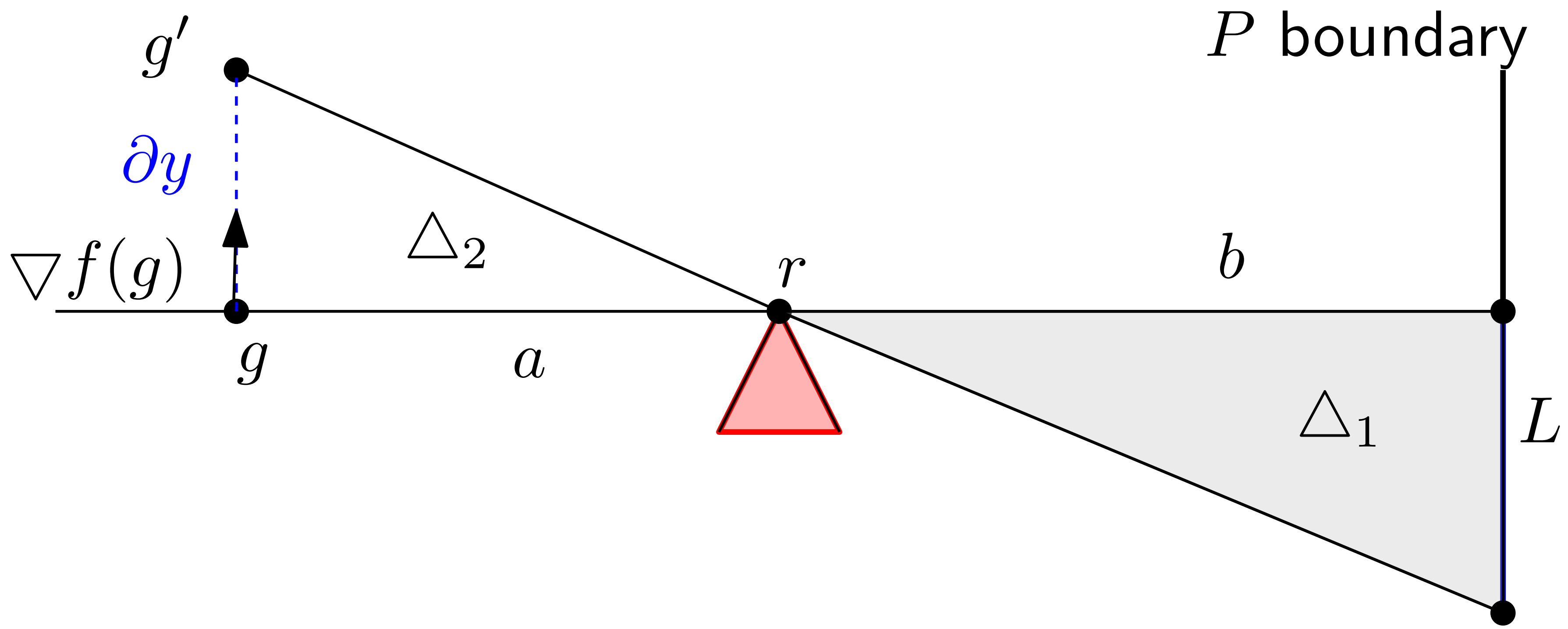


Computing the gradient for one guard

$$\triangledown f(g) = \triangledown \text{ Area}_{\triangle_1}(g)$$
$$\triangledown f(g) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)^{\mathsf{T}}$$



Computing the gradient for one guard

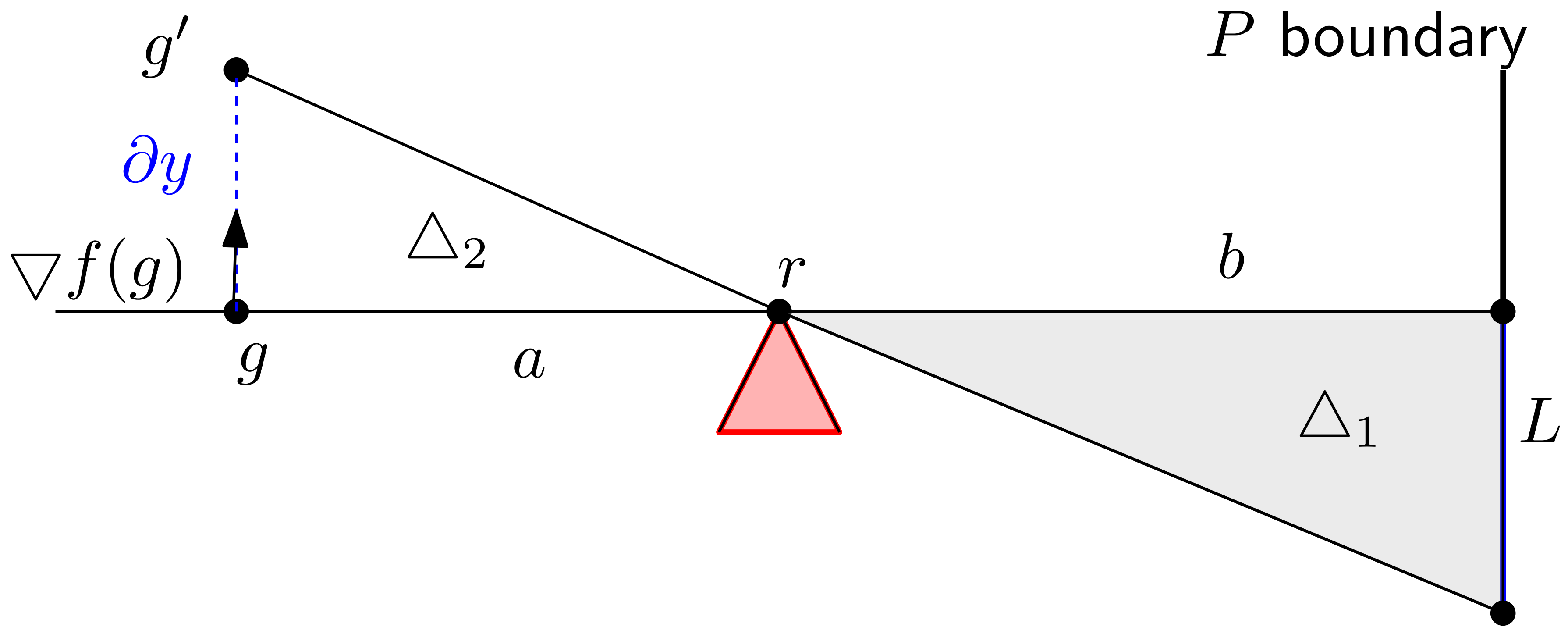


$$\bigtriangledown f(g) = \bigtriangledown \text{ Area}_{\triangle_1}(g)$$

$$\bigtriangledown f(g) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)^{\mathsf{T}}$$

$$\bigtriangledown f(g) = \left(0, \frac{b^2}{2a}\right)^{\mathsf{T}}$$

Computing the gradient for one guard



$$\bigtriangledown f(g) = \bigtriangledown \text{Area}_{\triangle_1}(g)$$

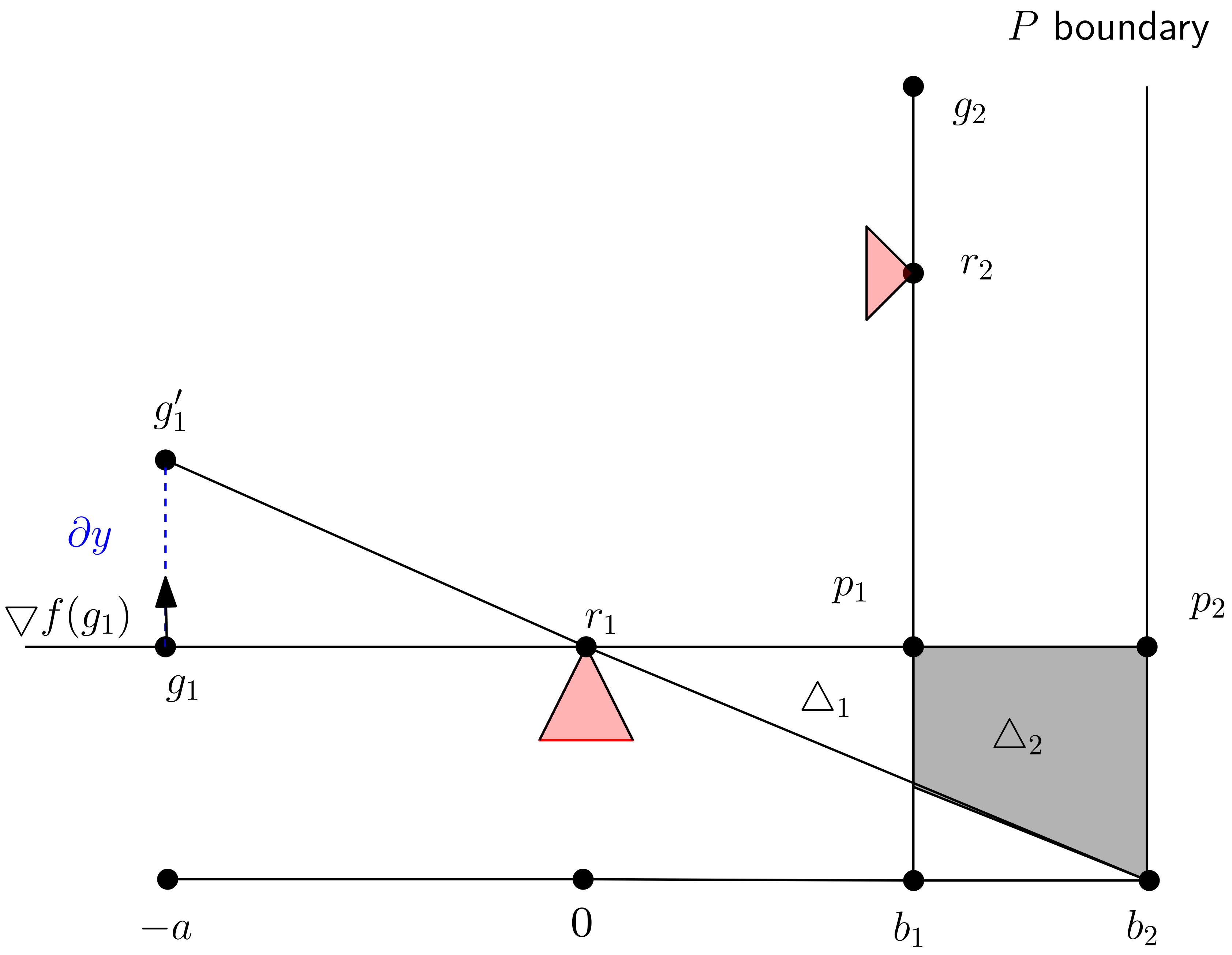
$$\bigtriangledown f(g) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)^{\mathsf{T}}$$

$$\bigtriangledown f(g) = \left(0, \frac{b^2}{2a}\right)^{\mathsf{T}}$$

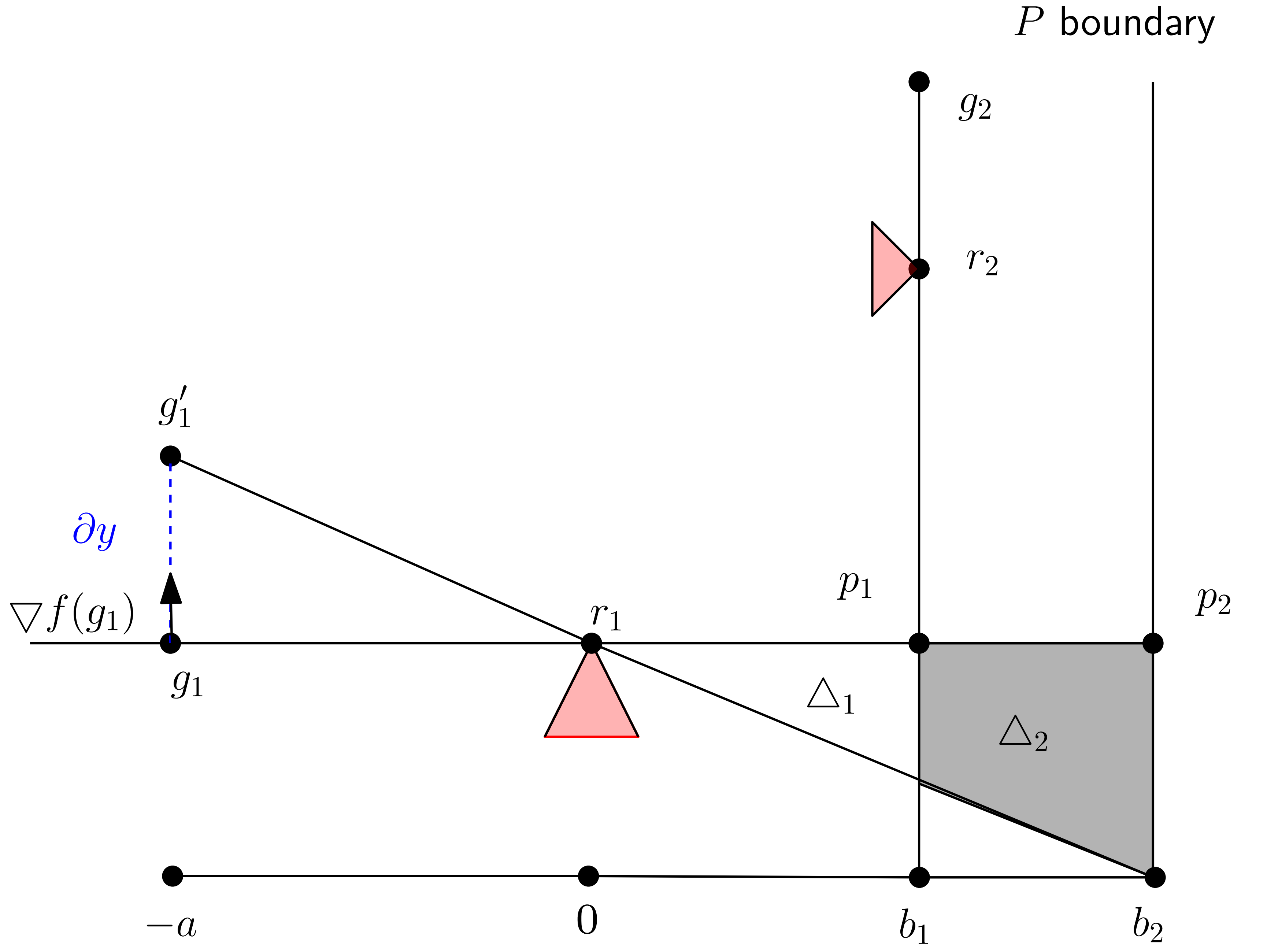
$$g' = g + \alpha \bigtriangledown f(g)$$

α - learning rate

Computing the gradient for multiple guards

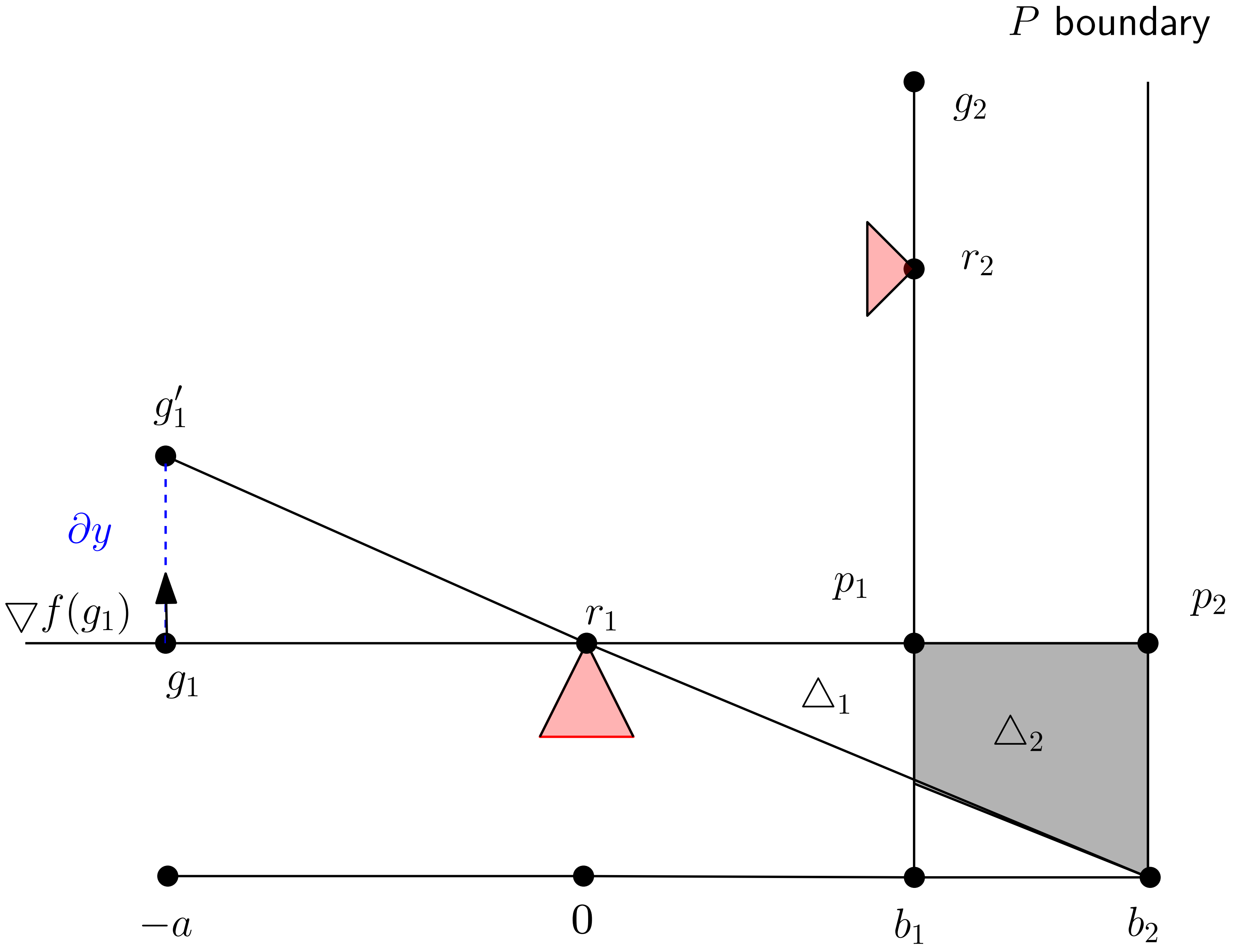


Computing the gradient for multiple guards



$$\text{Area}_{\triangle_1+\triangle_2}(g_1) = (b_1 + b_2)^2 \frac{\partial y}{2a}$$

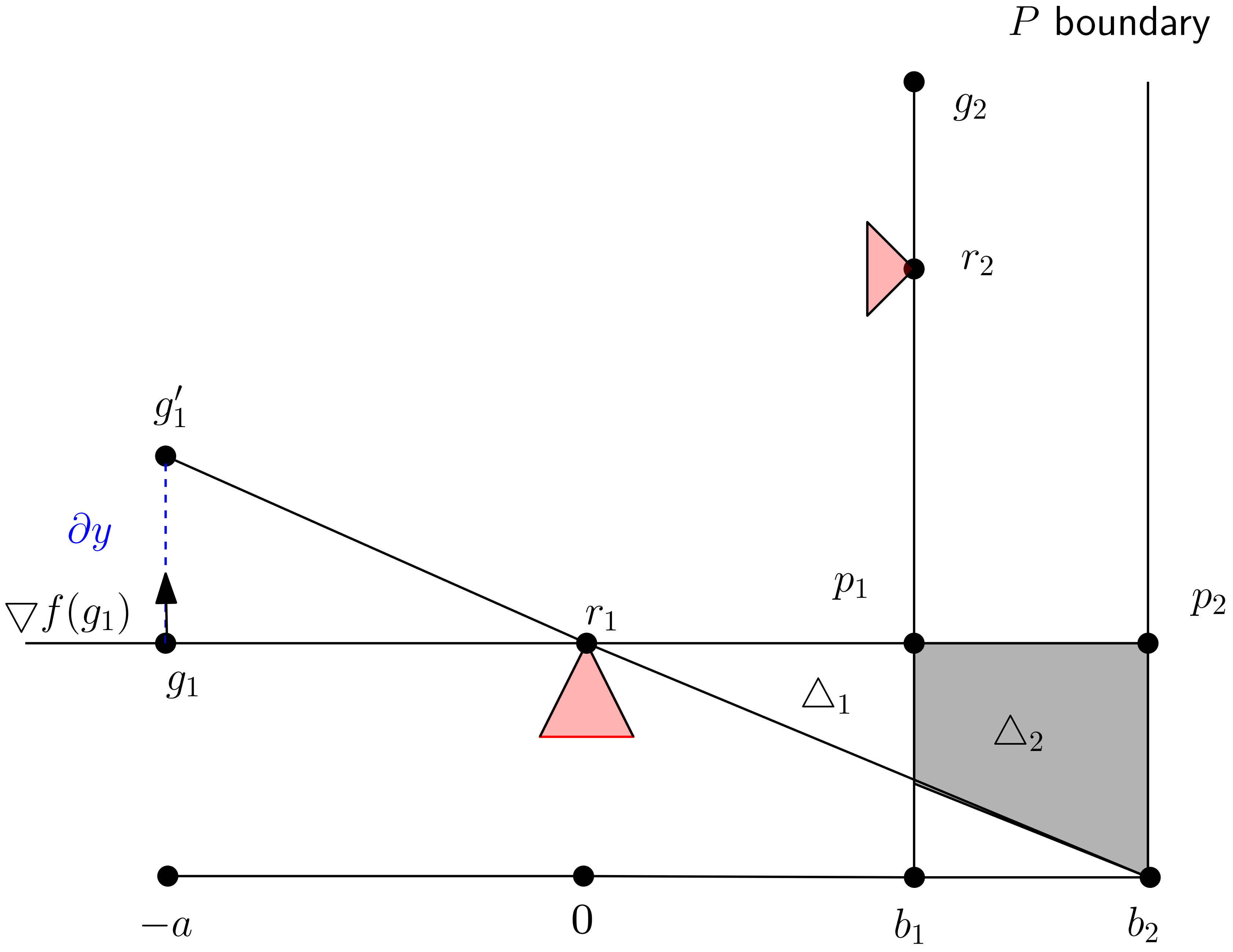
Computing the gradient for multiple guards



$$\text{Area}_{\triangle_1+\triangle_2}(g_1) = (b_1 + b_2)^2 \frac{\partial y}{2a}$$

$$\text{Area}_{\triangle_1}(g_1) = b_1^2 \frac{\partial y}{2a}$$

Computing the gradient for multiple guards

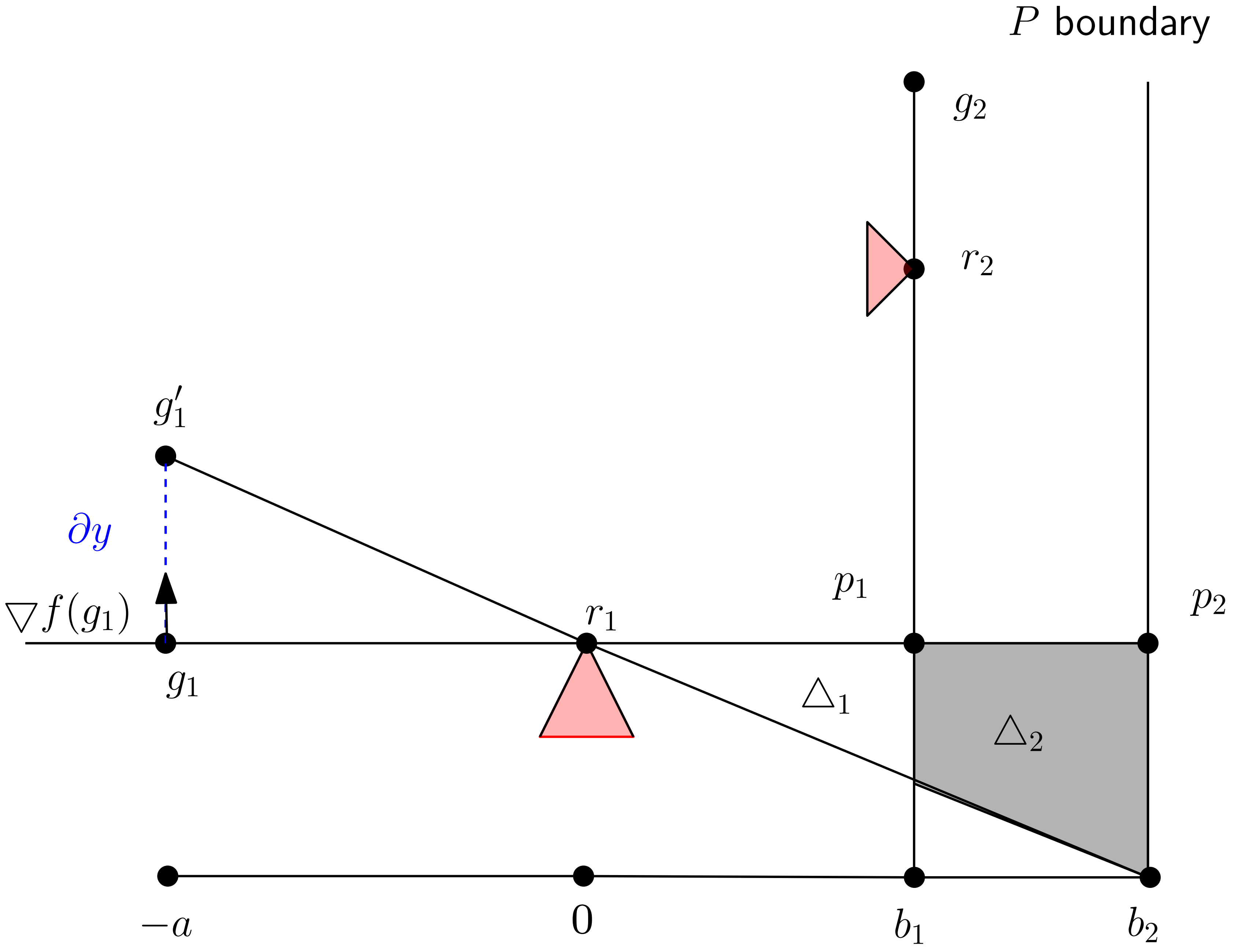


$$\text{Area}_{\triangle_1+\triangle_2}(g_1) = (b_1 + b_2)^2 \frac{\partial y}{2a}$$

$$\text{Area}_{\triangle_1}(g_1) = b_1^2 \frac{\partial y}{2a}$$

$$\text{Area}_{\triangle_2}(g_1) = \text{Area}_{\triangle_1+\triangle_2}(g_1) - \text{Area}_{\triangle_1}(g_1)$$

Computing the gradient for multiple guards

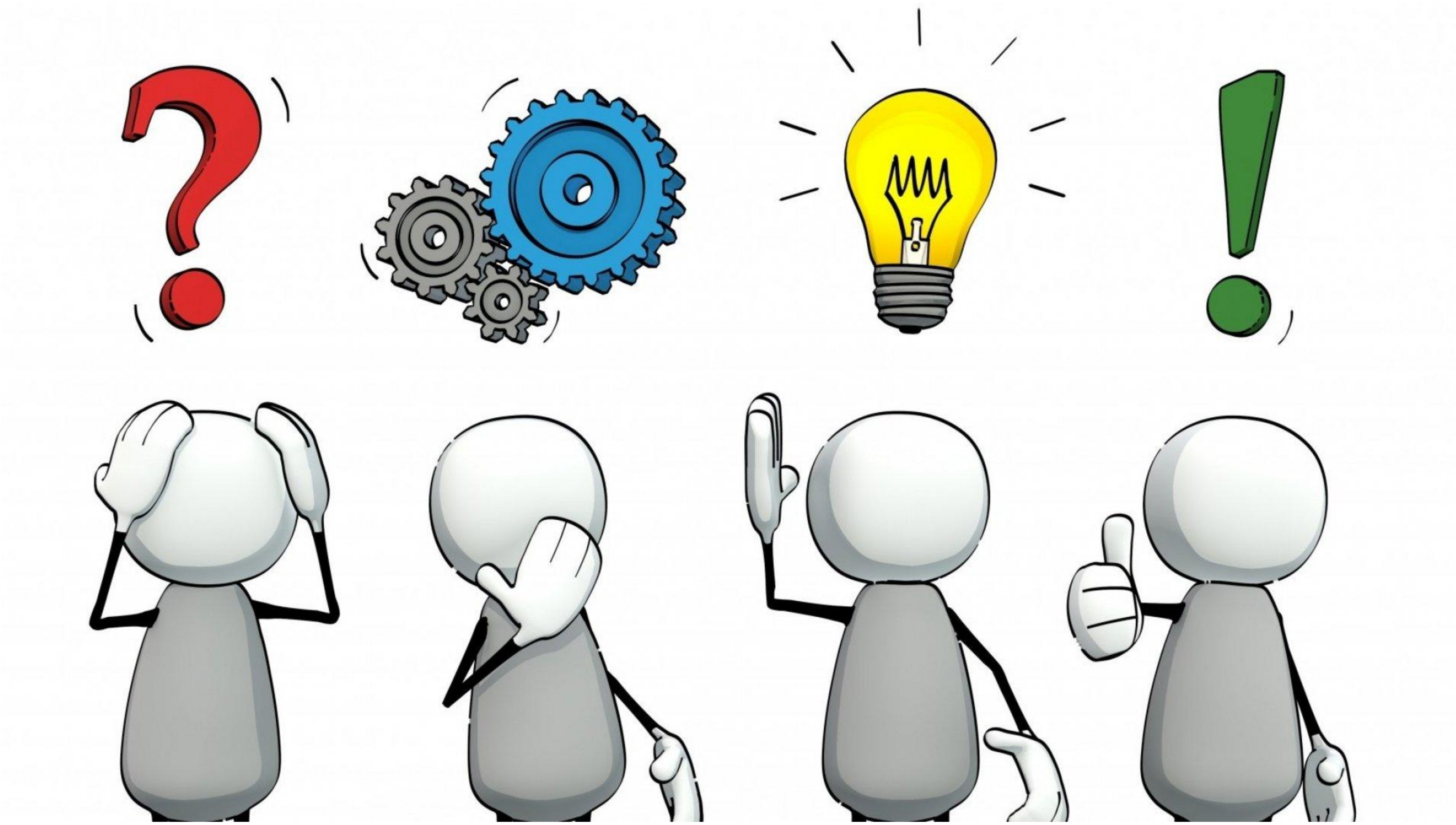


$$\text{Area}_{\triangle_1+\triangle_2}(g_1) = (b_1 + b_2)^2 \frac{\partial y}{2a}$$

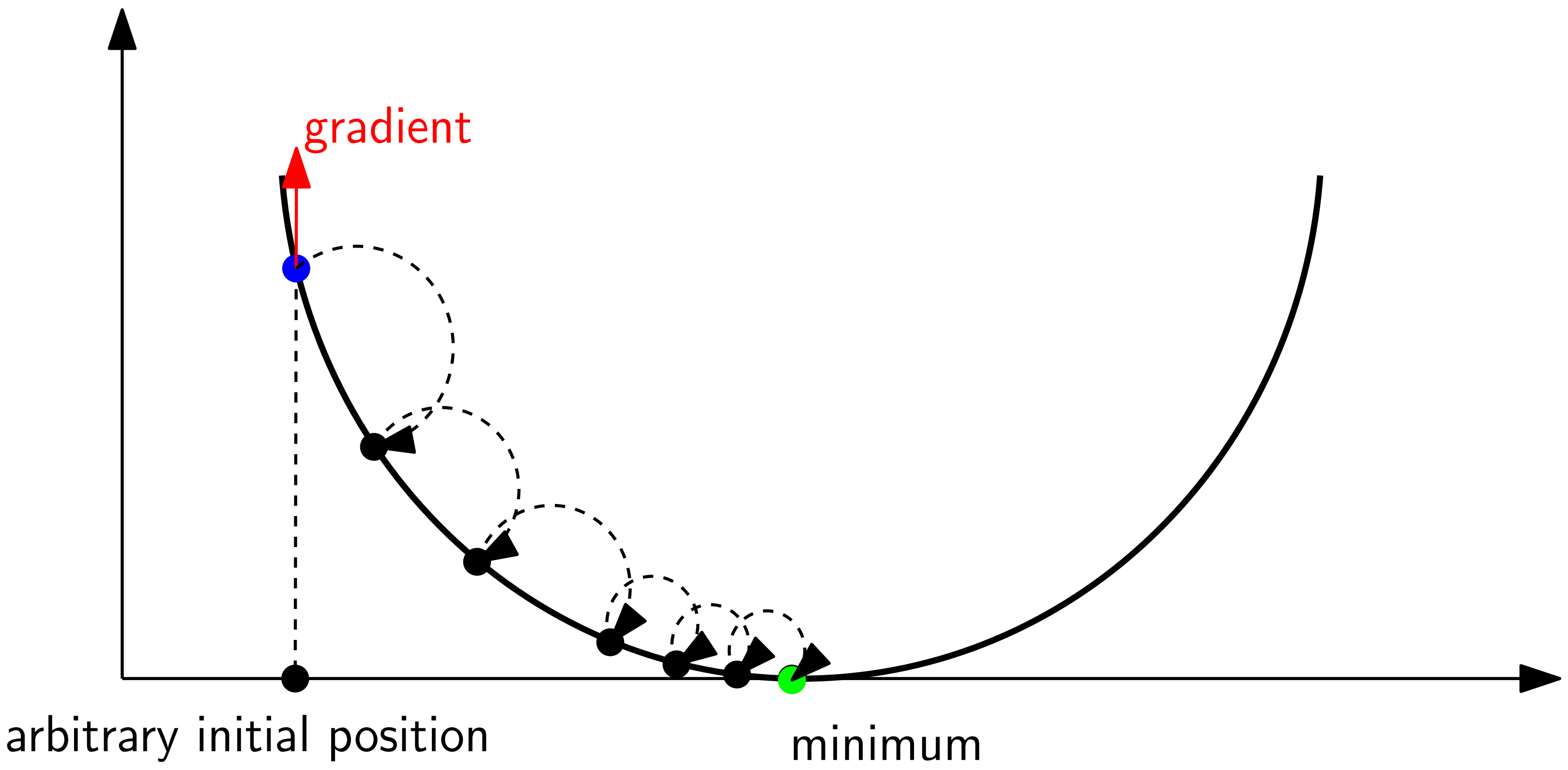
$$\text{Area}_{\triangle_1}(g_1) = b_1^2 \frac{\partial y}{2a}$$

$$\begin{aligned}\text{Area}_{\triangle_2}(g_1) &= \text{Area}_{\triangle_1+\triangle_2}(g_1) - \text{Area}_{\triangle_1}(g_1) \\ &= \left[(b_1 + b_2)^2 - b_1^2\right] \frac{\partial y}{2a}\end{aligned}$$

Heuristics

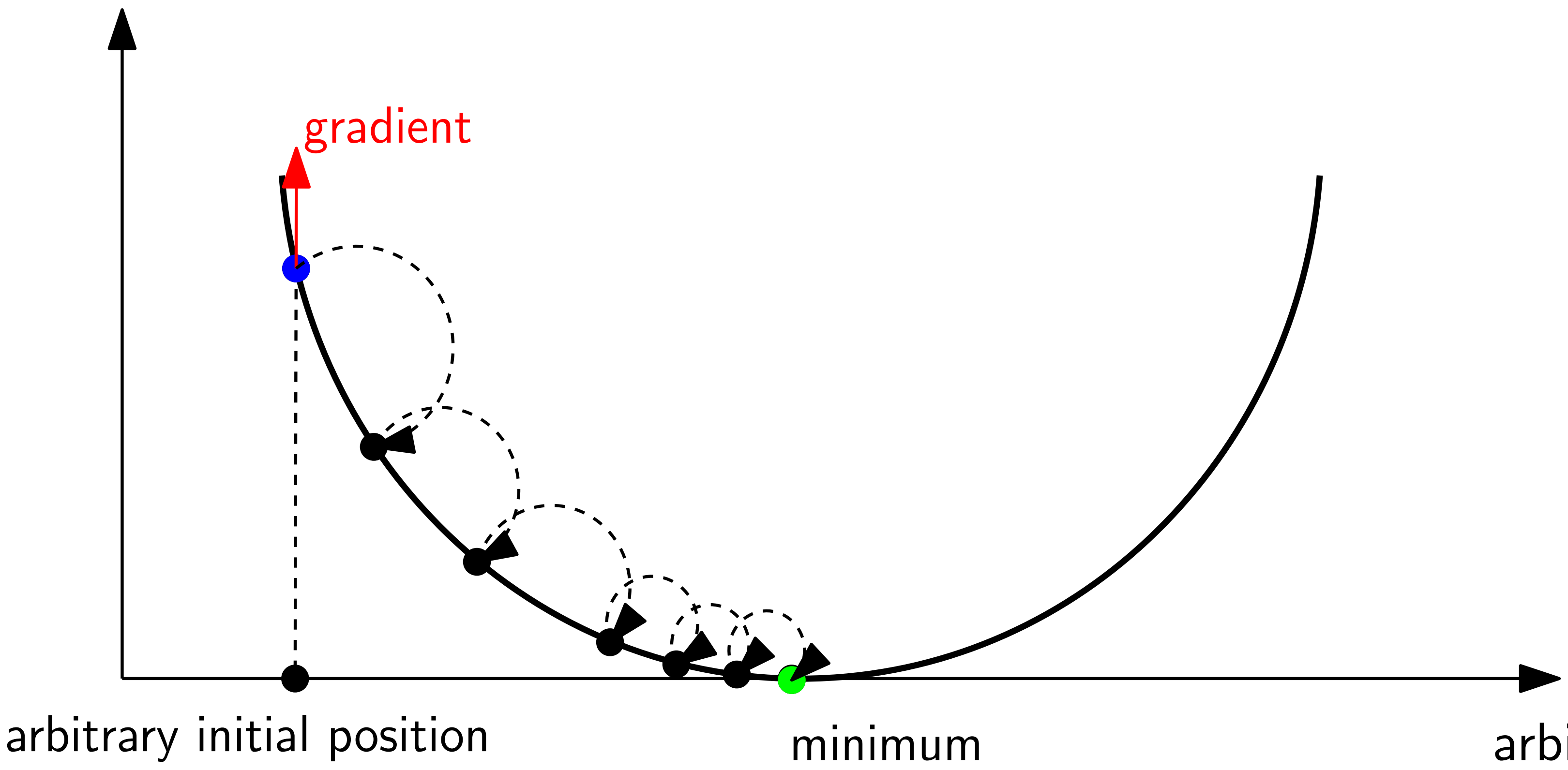


Heuristics: Momentum

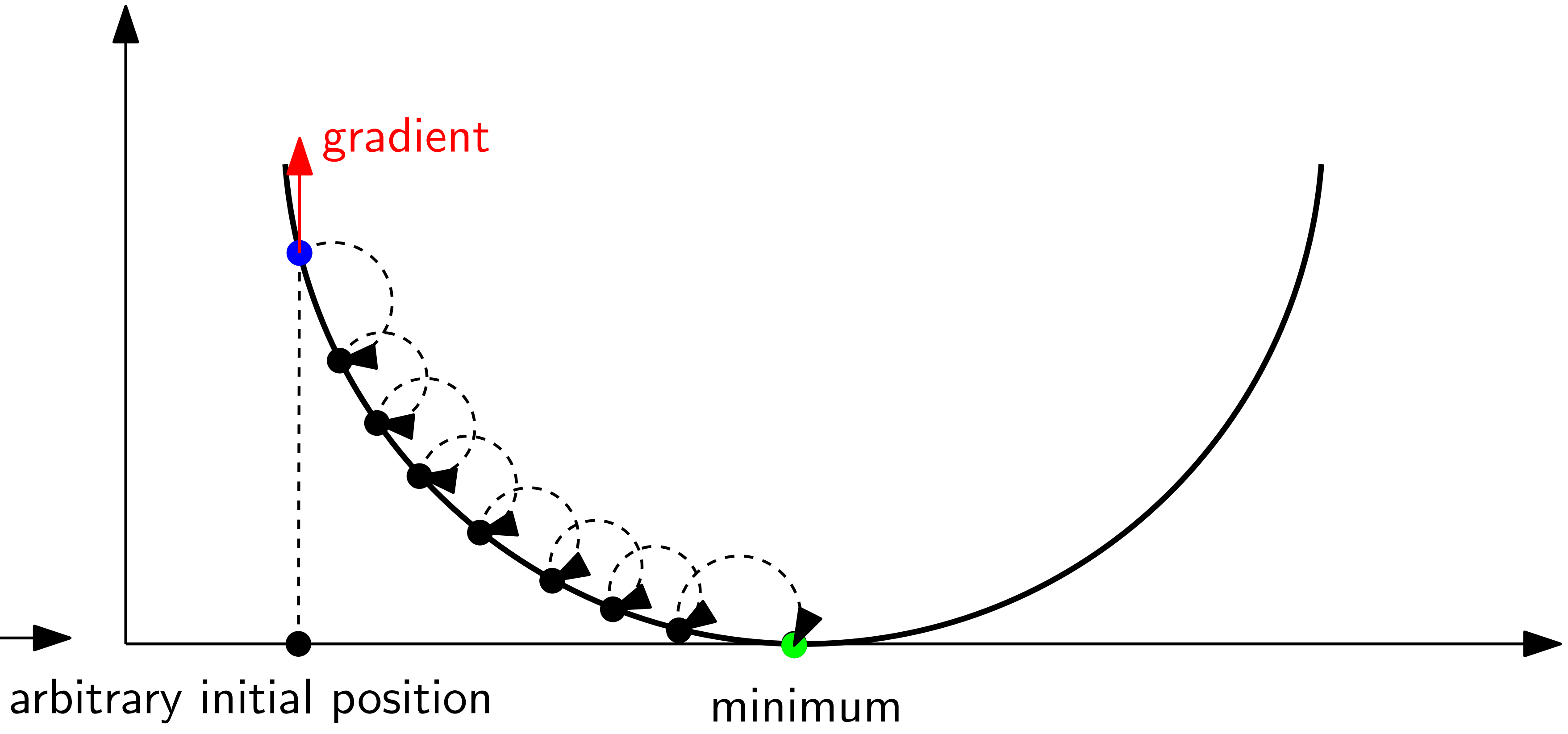


No Momentum

Heuristics: Momentum



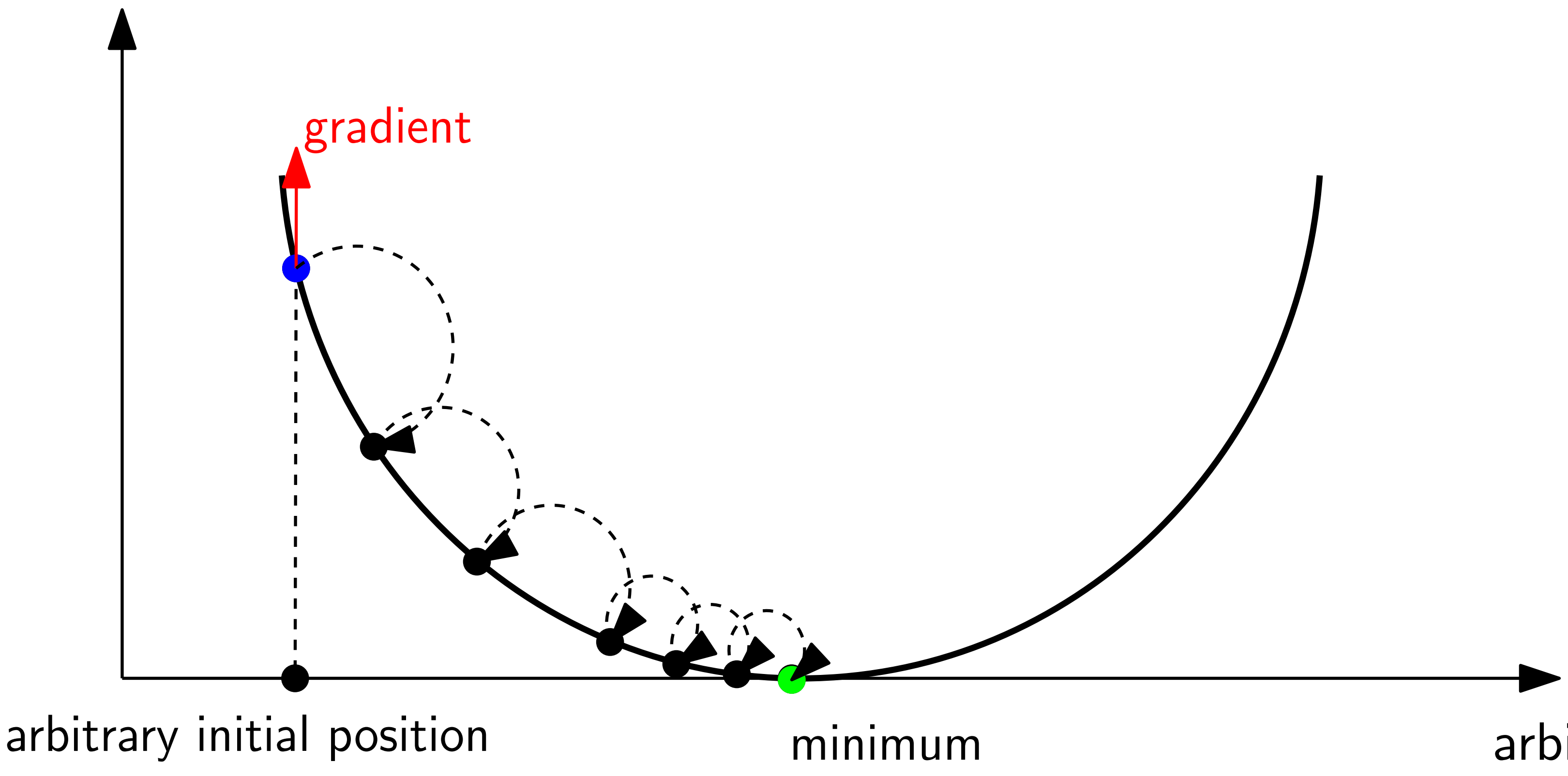
No Momentum



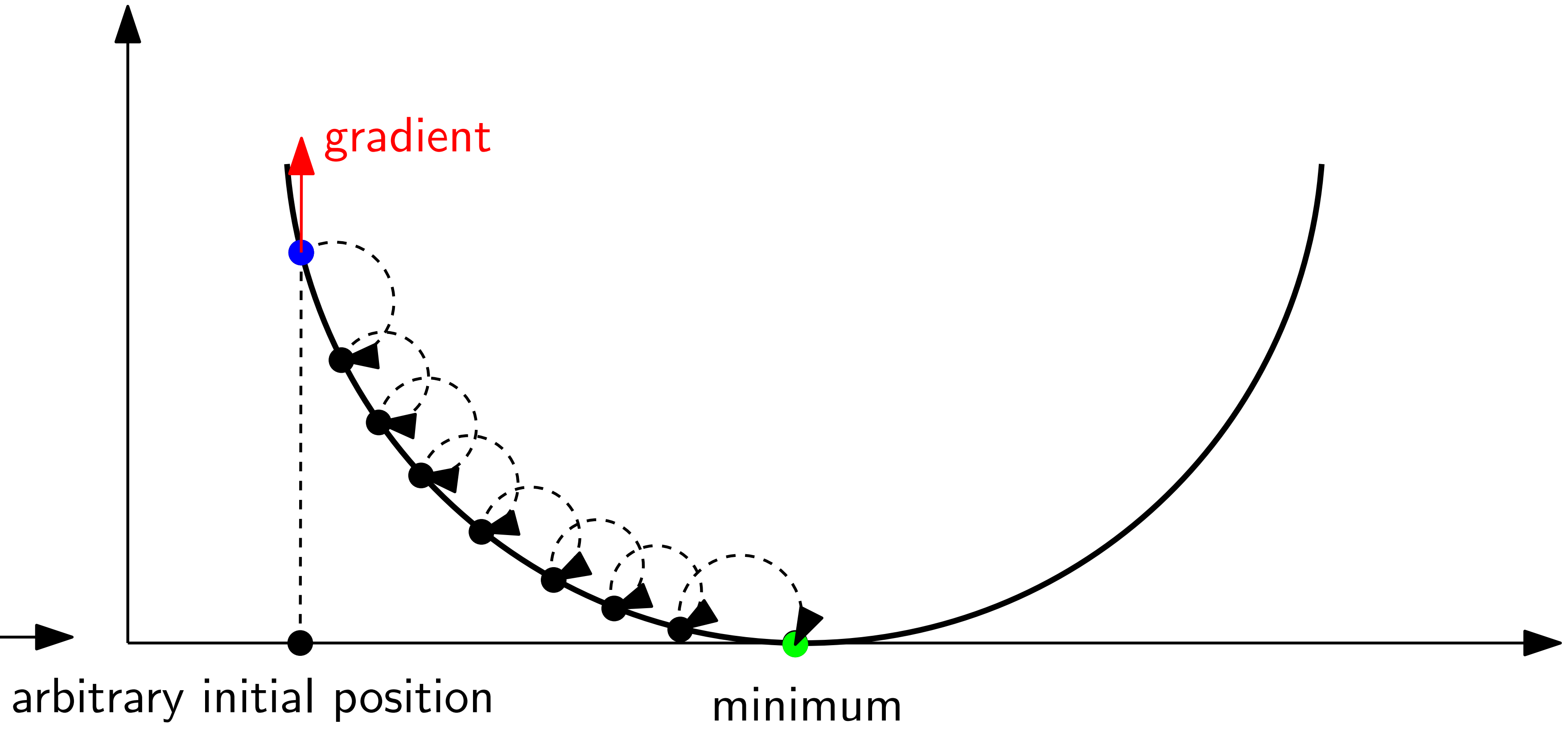
With Momentum

Heuristics: Momentum

$$M_i(g_i) = \gamma M_{i-1}(g_{i-1}) + (1-\gamma) \bigtriangledown f_i(g_i)$$



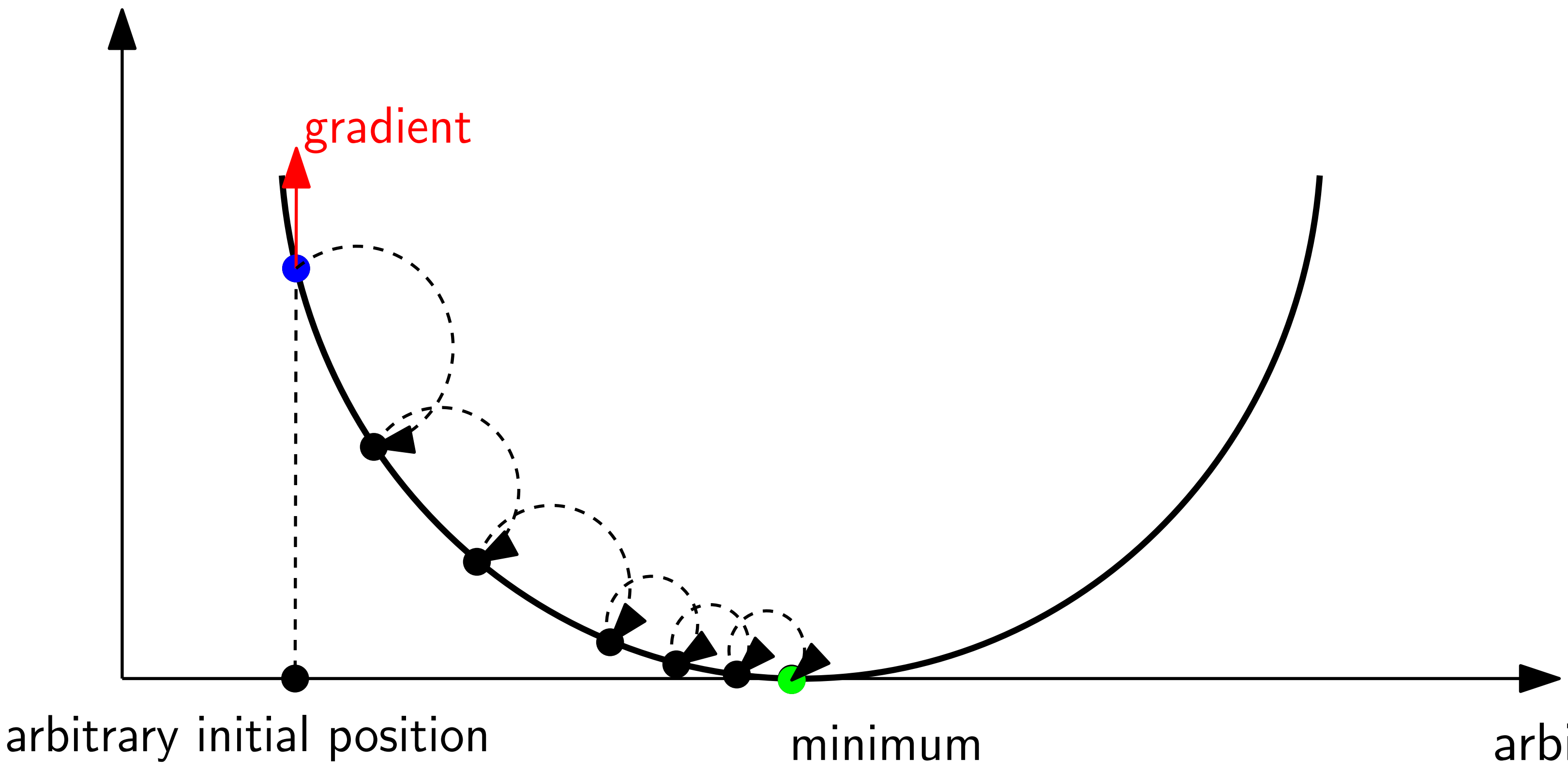
No Momentum



With Momentum

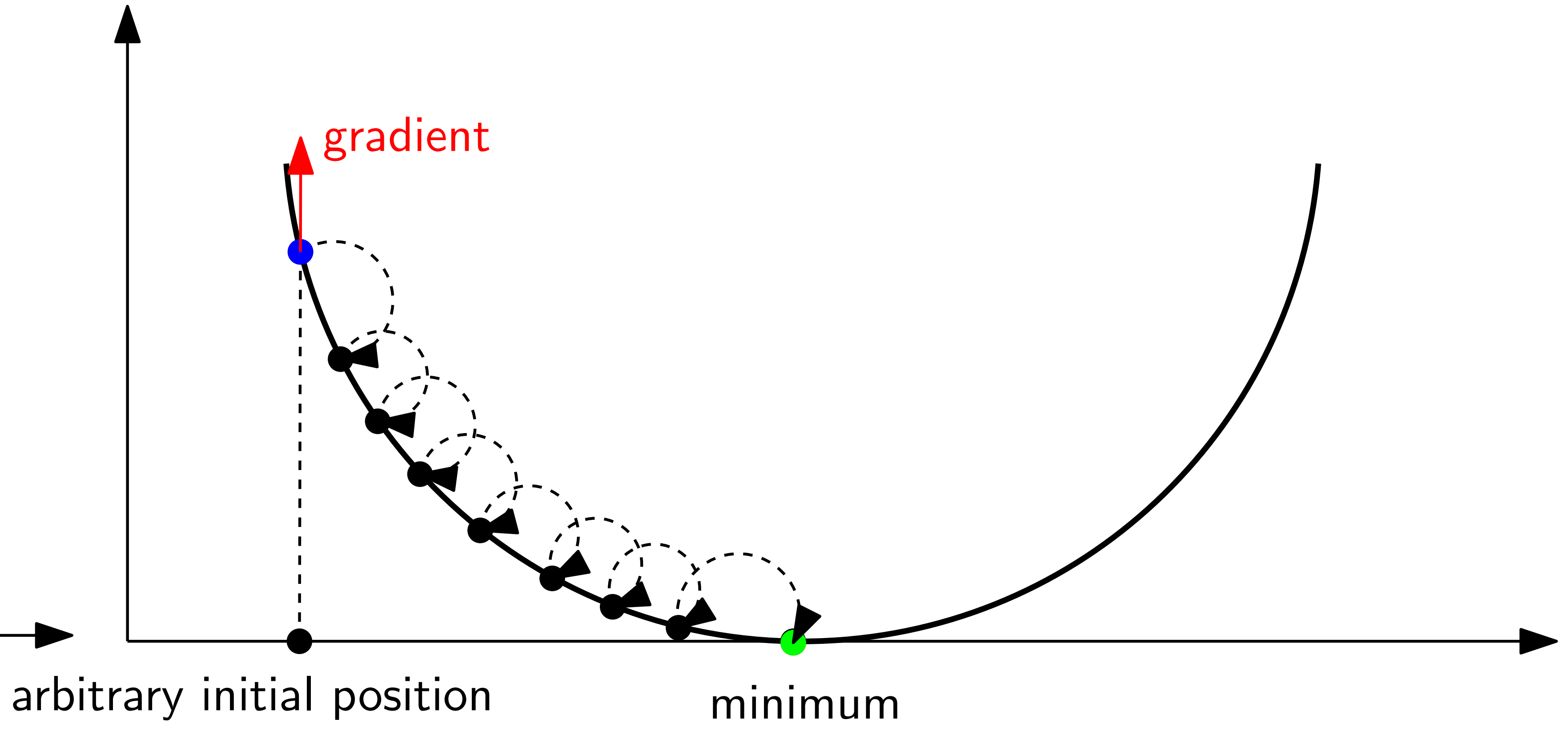
Heuristics: Momentum

$$g_i = g_{i-1} + \alpha \bigtriangledown f_i(g_i)$$



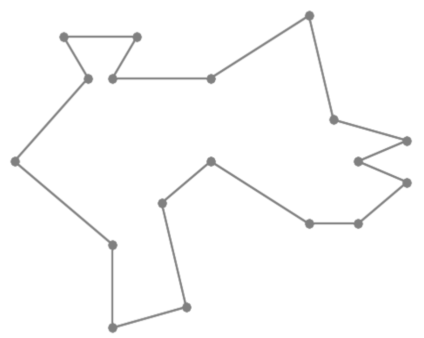
No Momentum

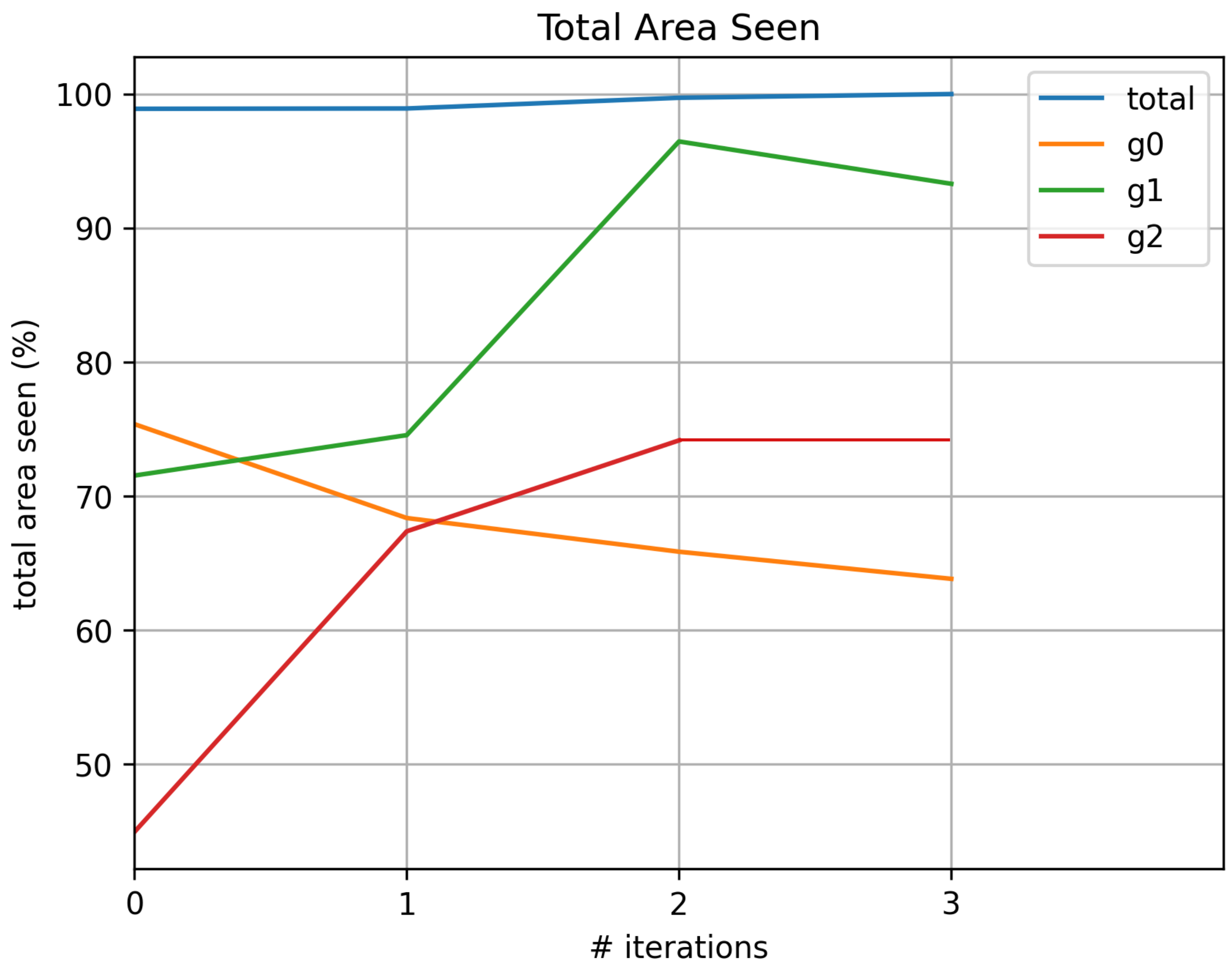
$$M_i(g_i) = \gamma M_{i-1}(g_{i-1}) + (1-\gamma) \bigtriangledown f_i(g_i)$$
$$g_i = g_{i-1} + \alpha M_i(g_{i-1})$$



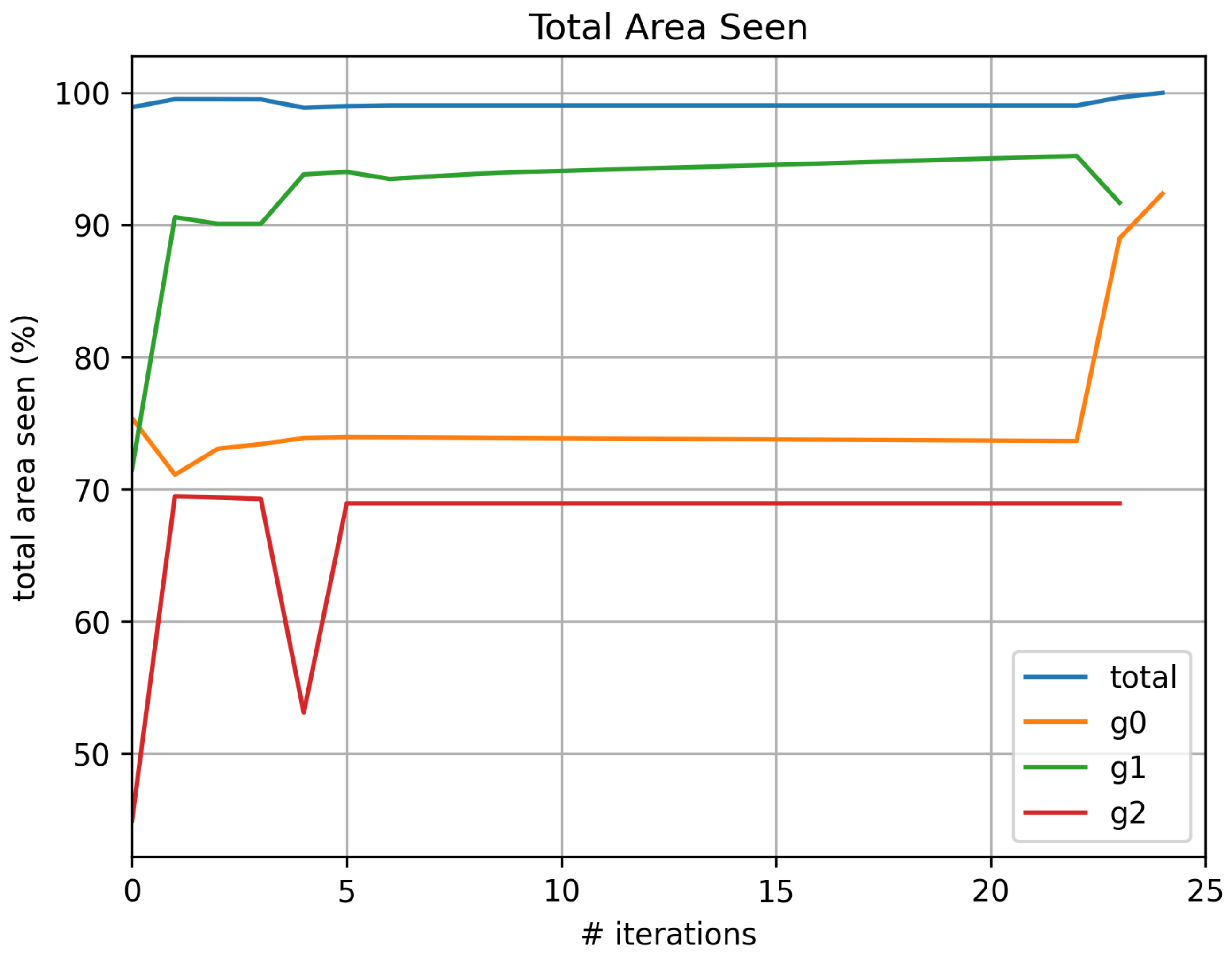
With Momentum

Heuristics: Momentum



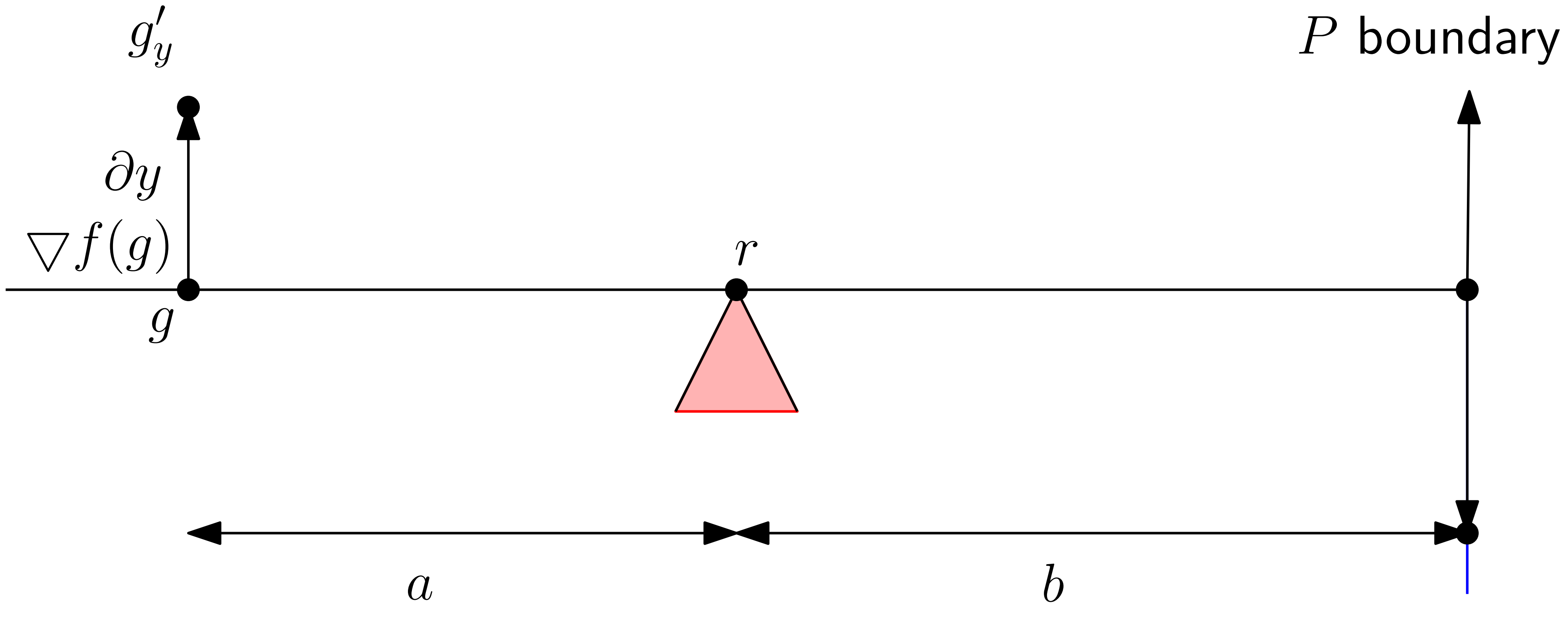


All heuristics

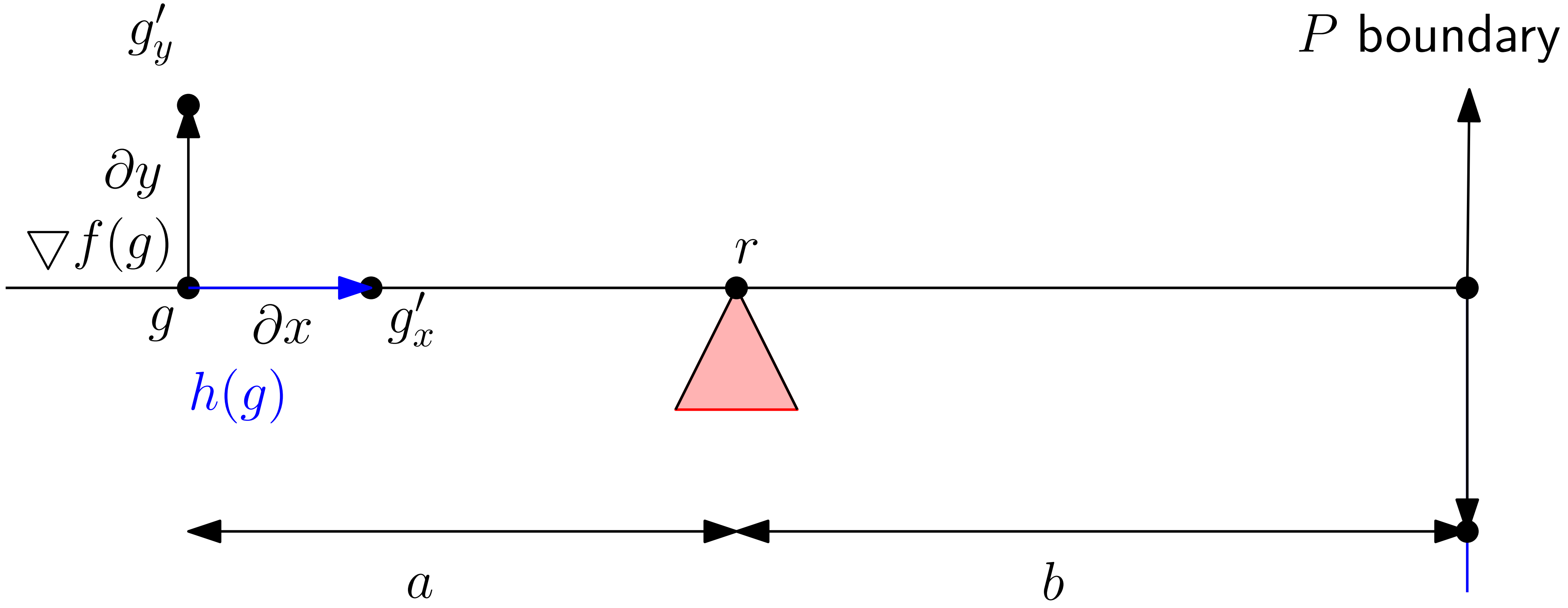


No momentum

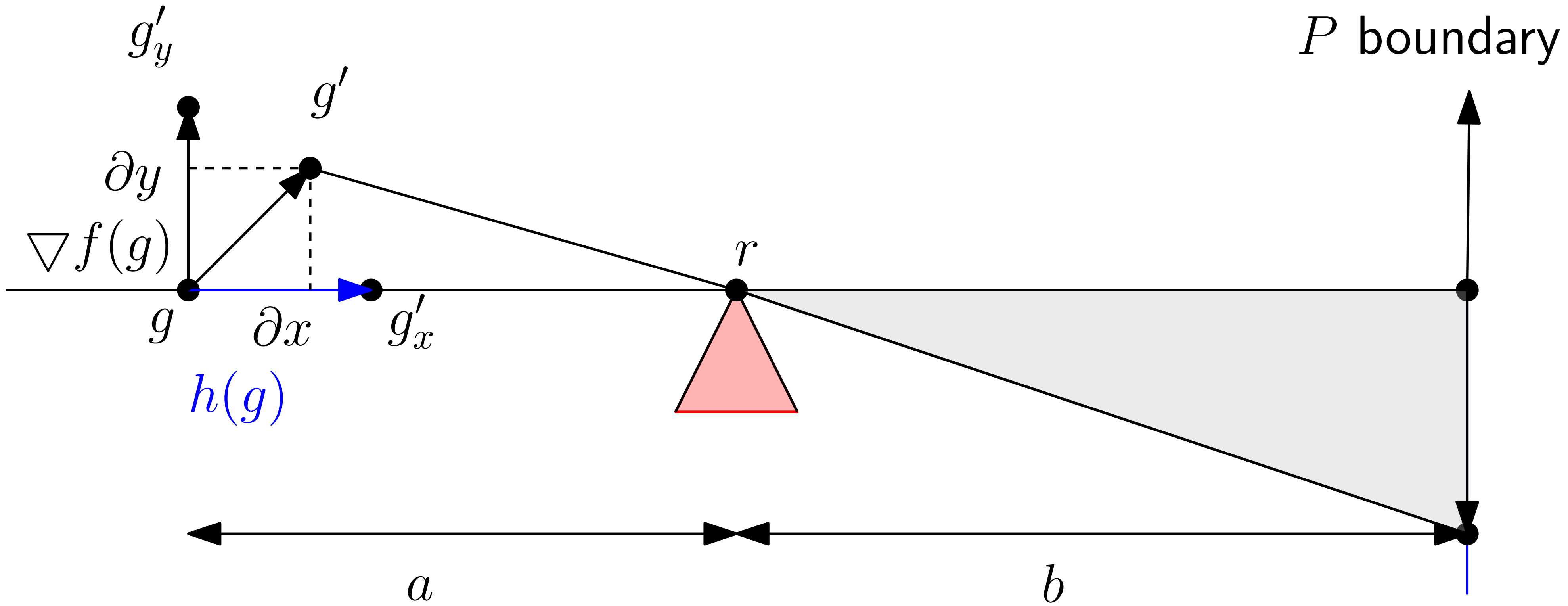
Heuristics: Pull towards reflex vertex



Heuristics: Pull towards reflex vertex

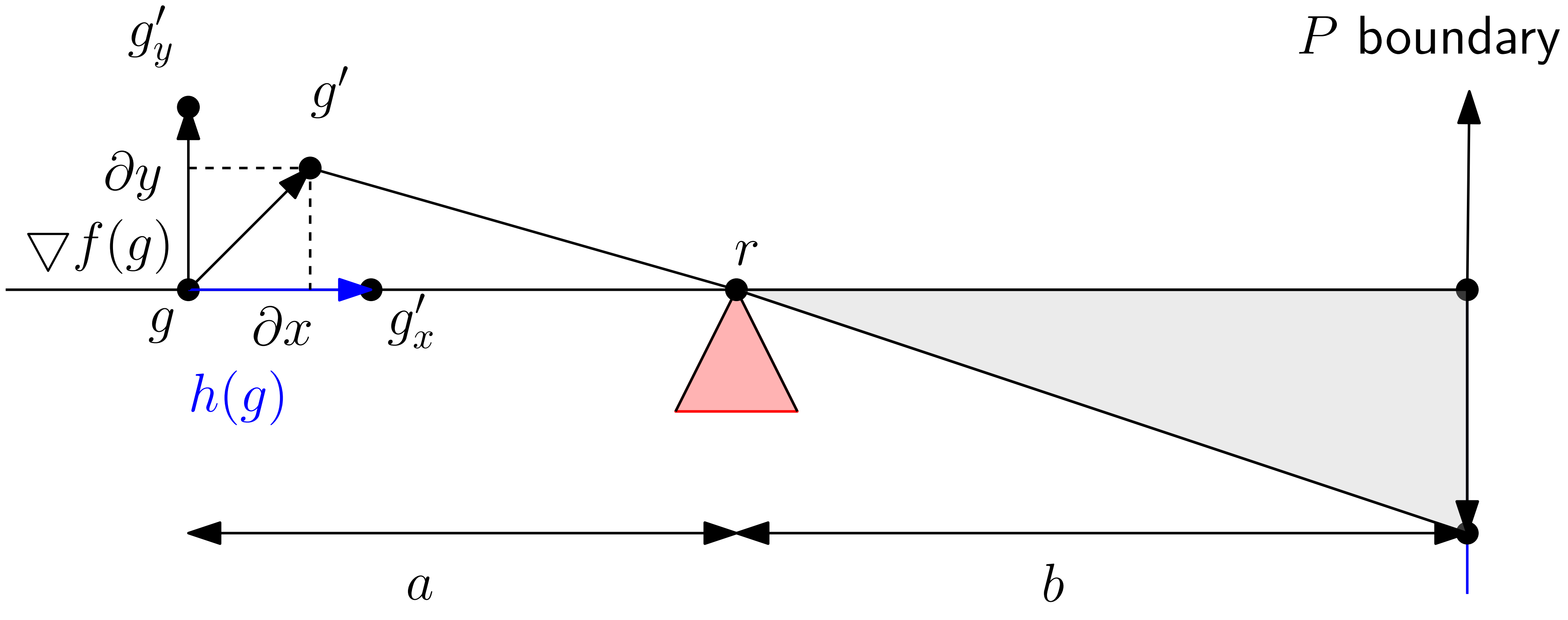


Heuristics: Pull towards reflex vertex

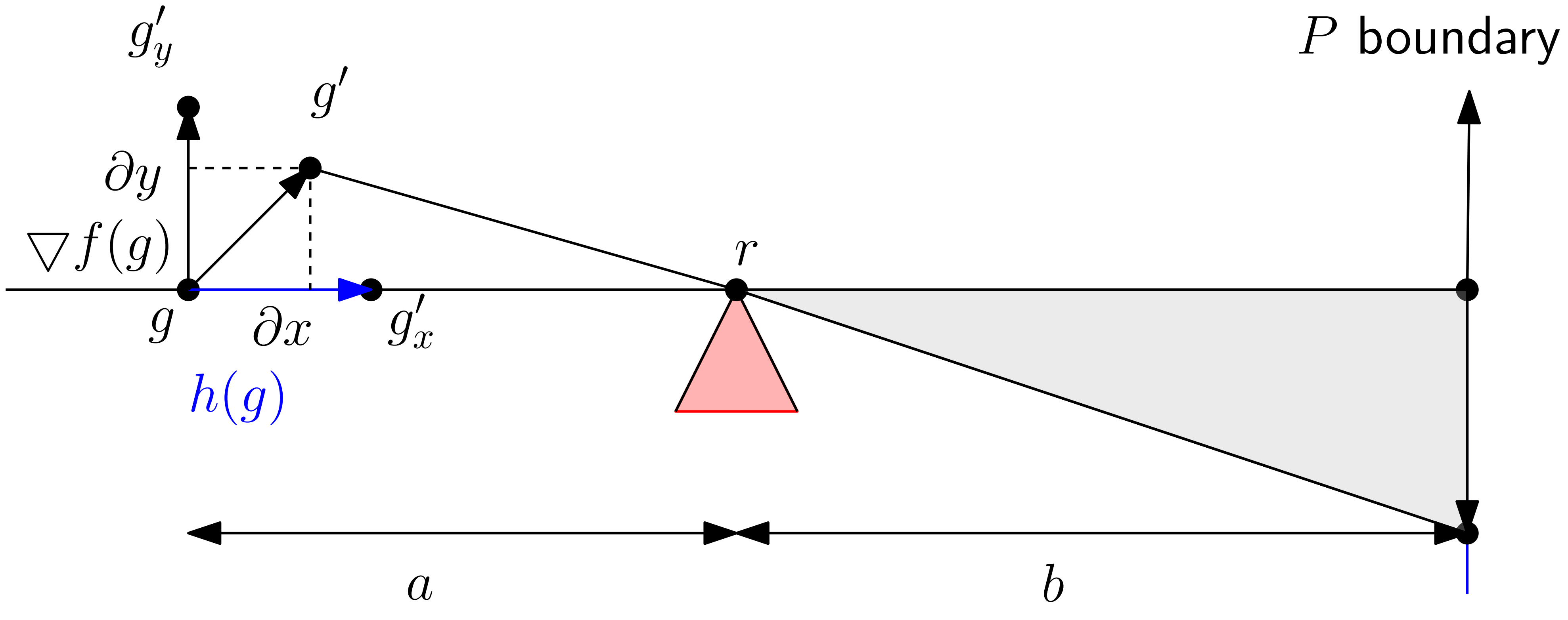


Heuristics: Pull towards reflex vertex

$$h(g) = \bigtriangledown || \bigtriangledown f(g)||$$



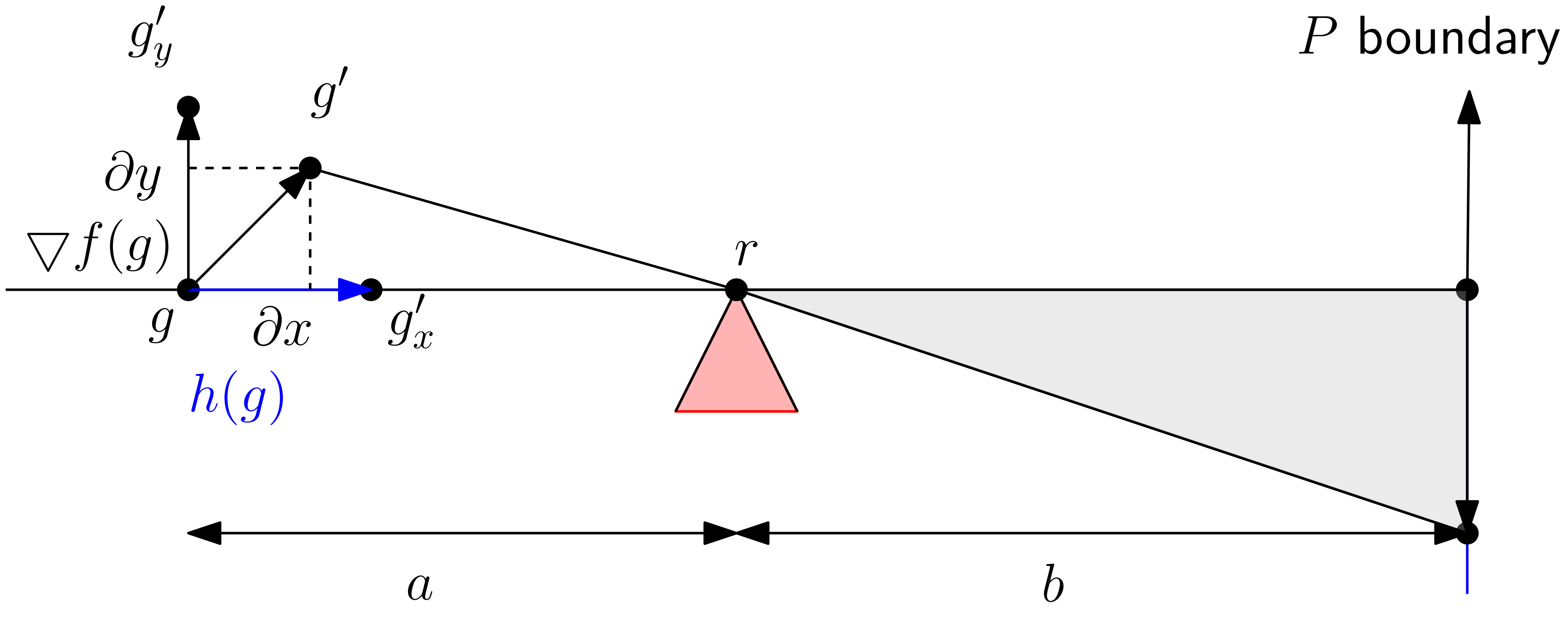
Heuristics: Pull towards reflex vertex



$$h(g) = \bigtriangledown || \bigtriangledown f(g)||$$

$$h(g) = \left(\frac{\partial \bigtriangledown f(g)}{\partial x}, \frac{\partial \bigtriangledown f(g)}{\partial y} \right)^{\mathsf{T}}$$

Heuristics: Pull towards reflex vertex

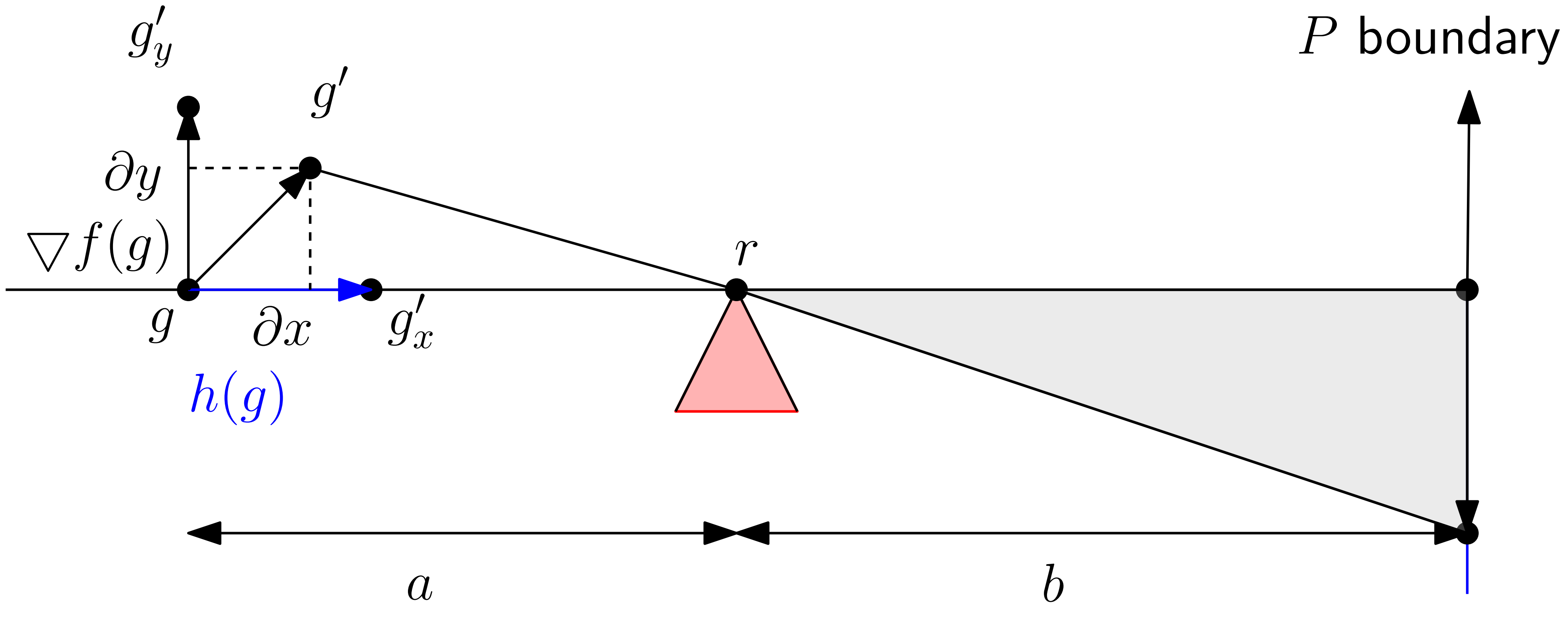


$$h(g) = \triangledown || \triangledown f(g)||$$

$$h(g) = \left(\frac{\partial \triangledown f(g)}{\partial x}, \frac{\partial \triangledown f(g)}{\partial y}\right)^{\mathsf{T}}$$

$$h(g) = \left(\frac{-b^2}{2a^3}, 0\right)^{\mathsf{T}}$$

Heuristics: Pull towards reflex vertex



$$h(g) = \bigtriangledown || \bigtriangledown f(g)||$$

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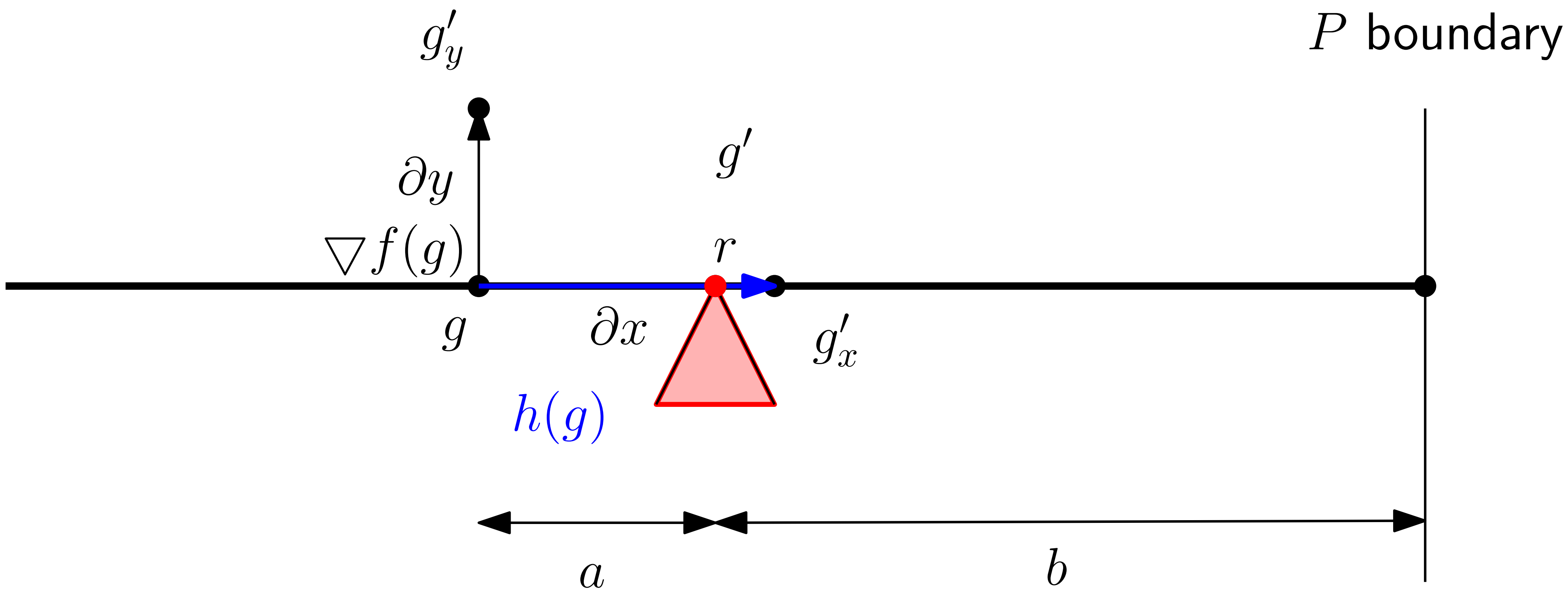
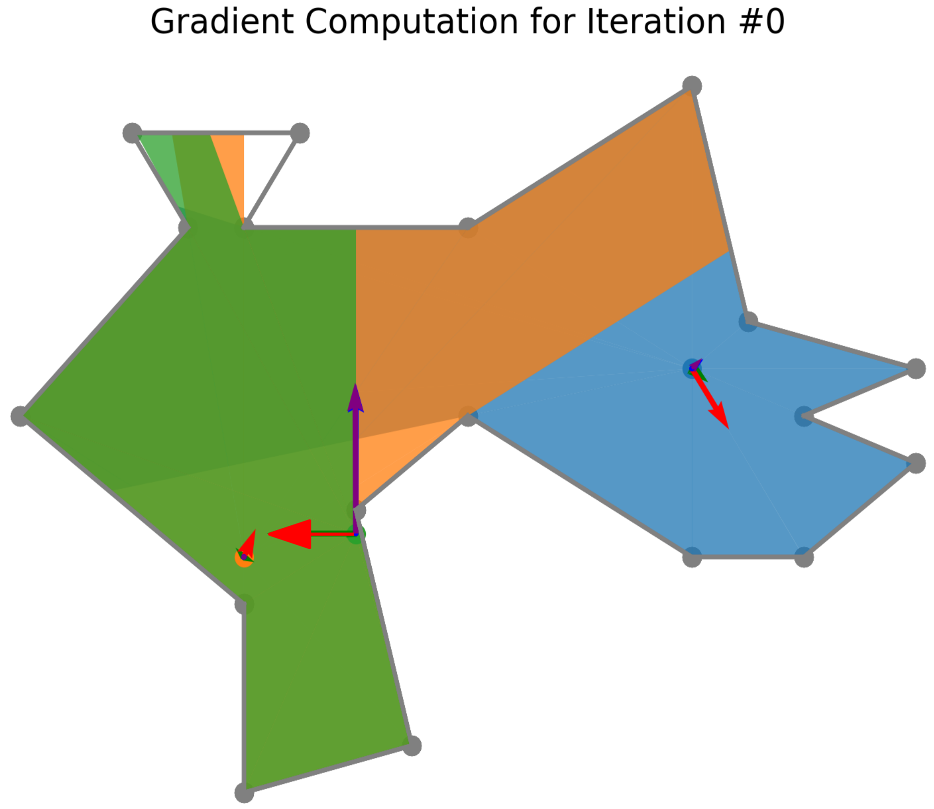
$$h(g) = \left(\frac{-b^2}{2a^3}, 0\right)^{\mathsf{T}}$$

$$g' = g + \alpha(\bigtriangledown f(g) + h(g))$$

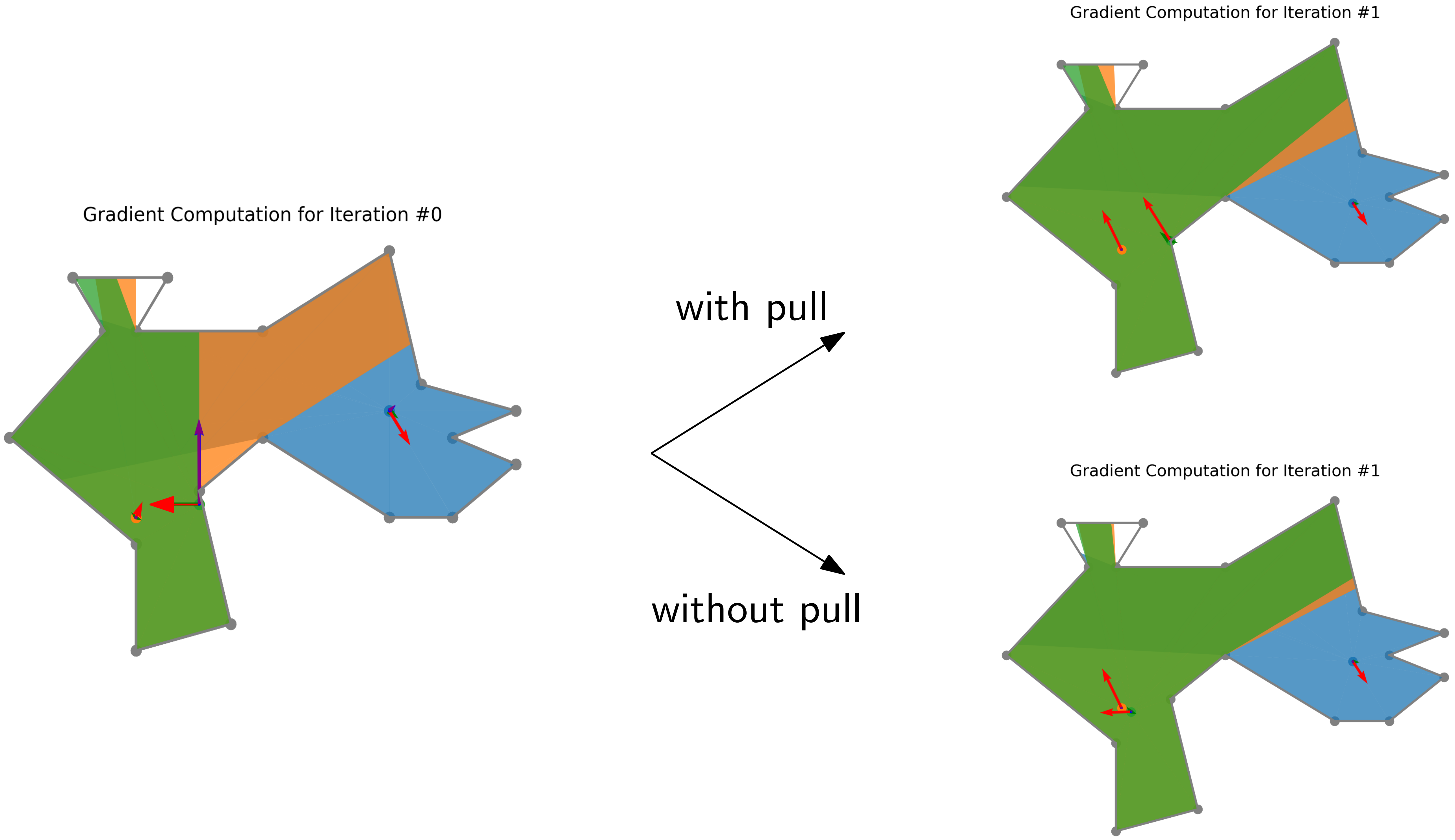
Heuristics: Pull towards reflex vertex



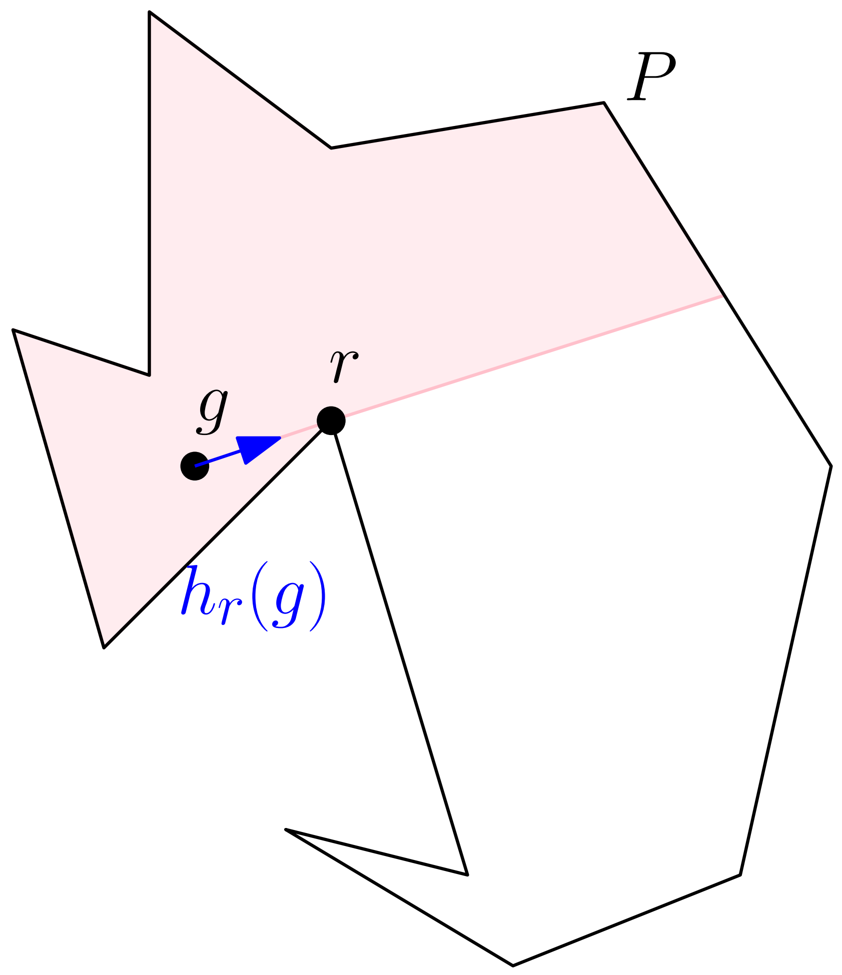
Heuristics: Pull towards reflex vertex



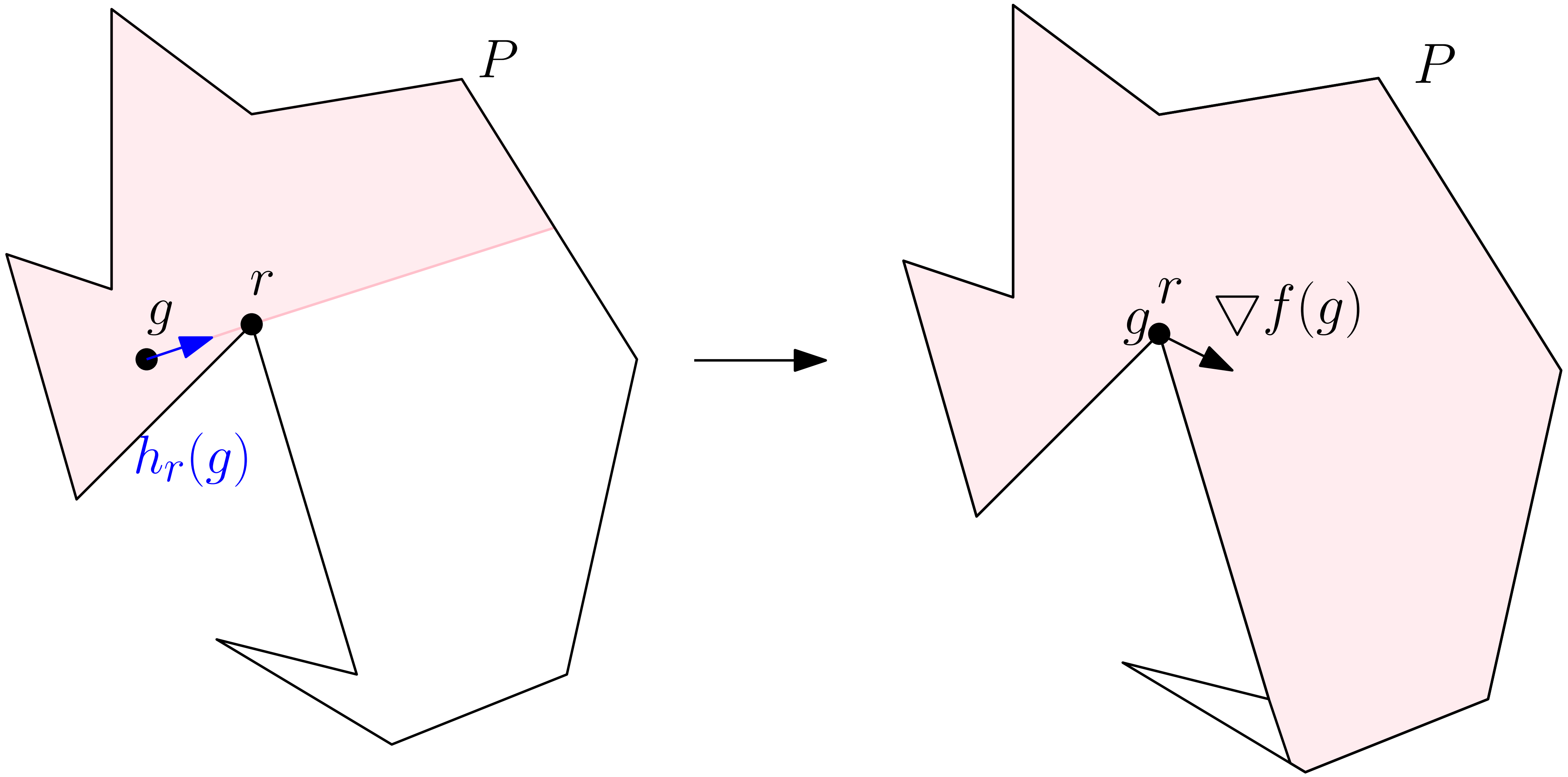
Heuristics: Pull towards reflex vertex



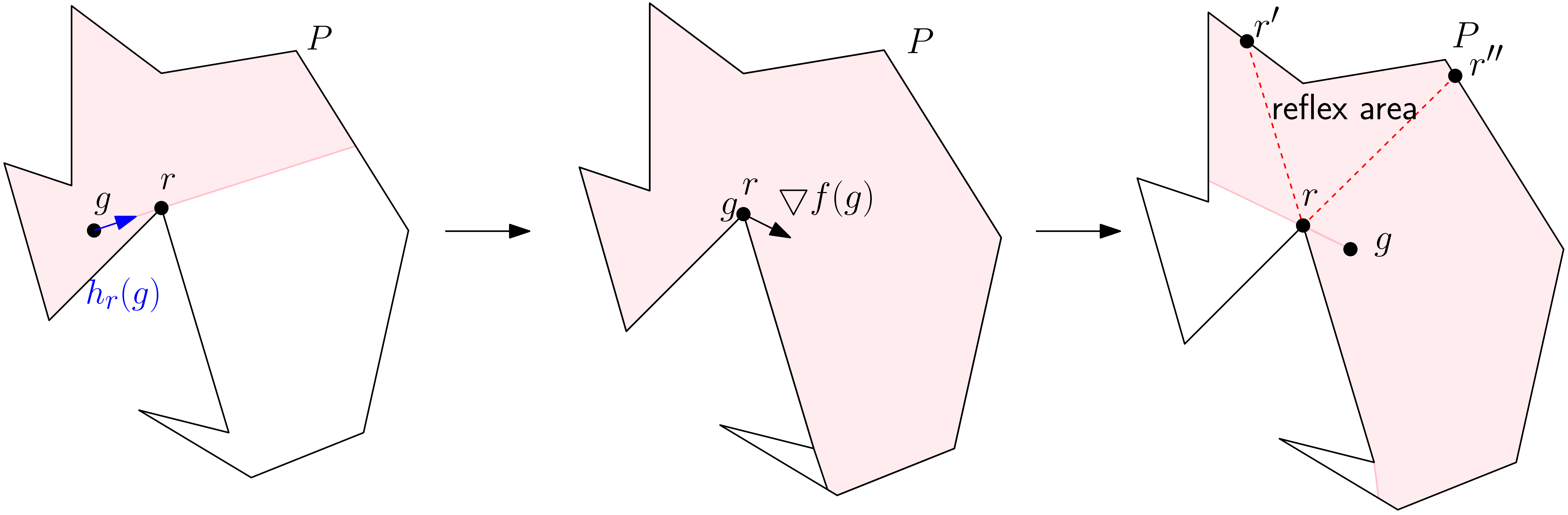
Heuristics: Reflex area



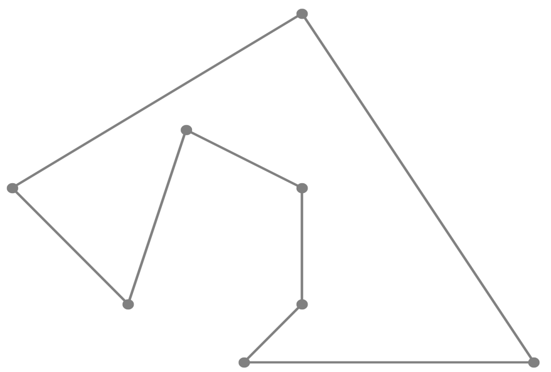
Heuristics: Reflex area

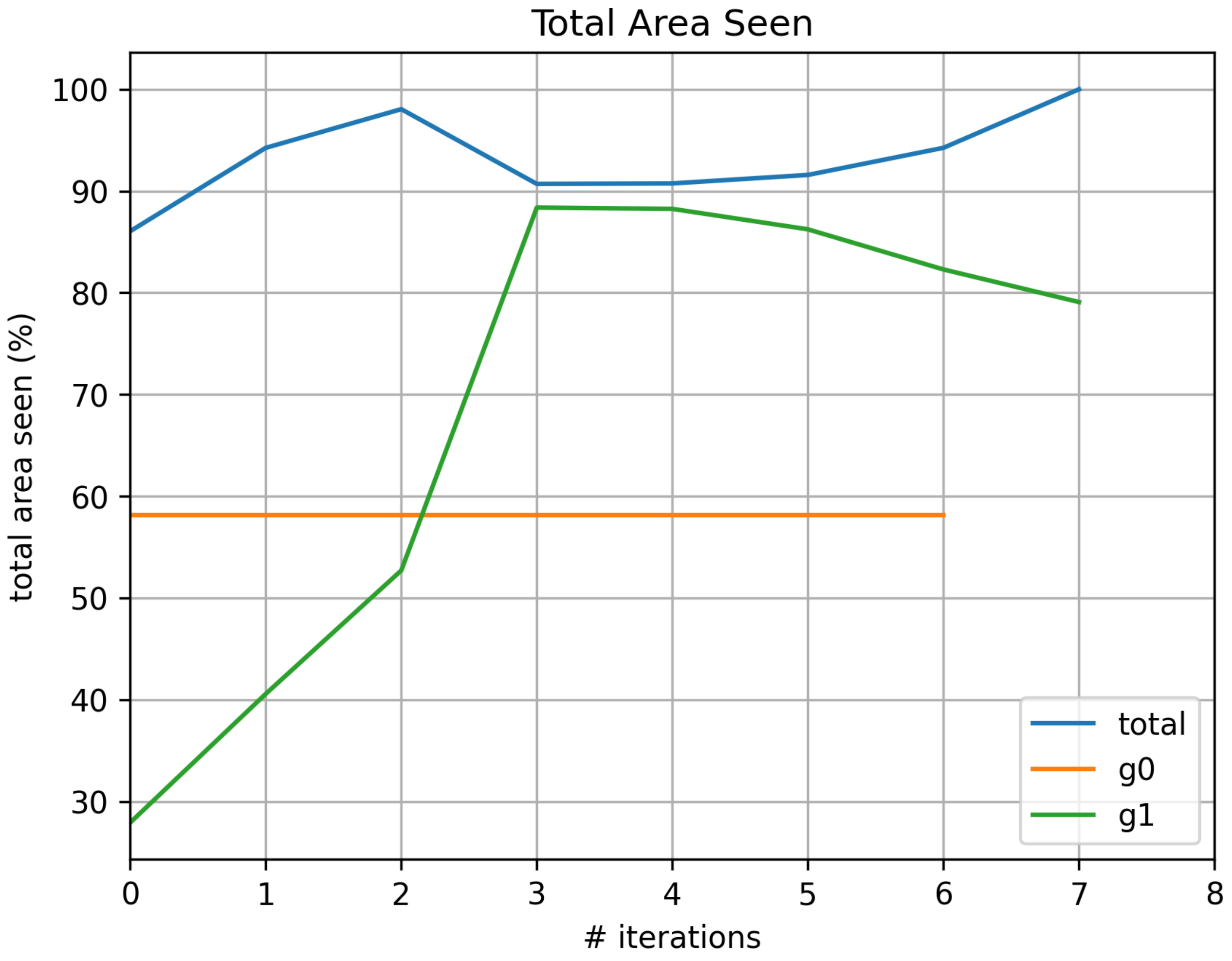


Heuristics: Reflex area

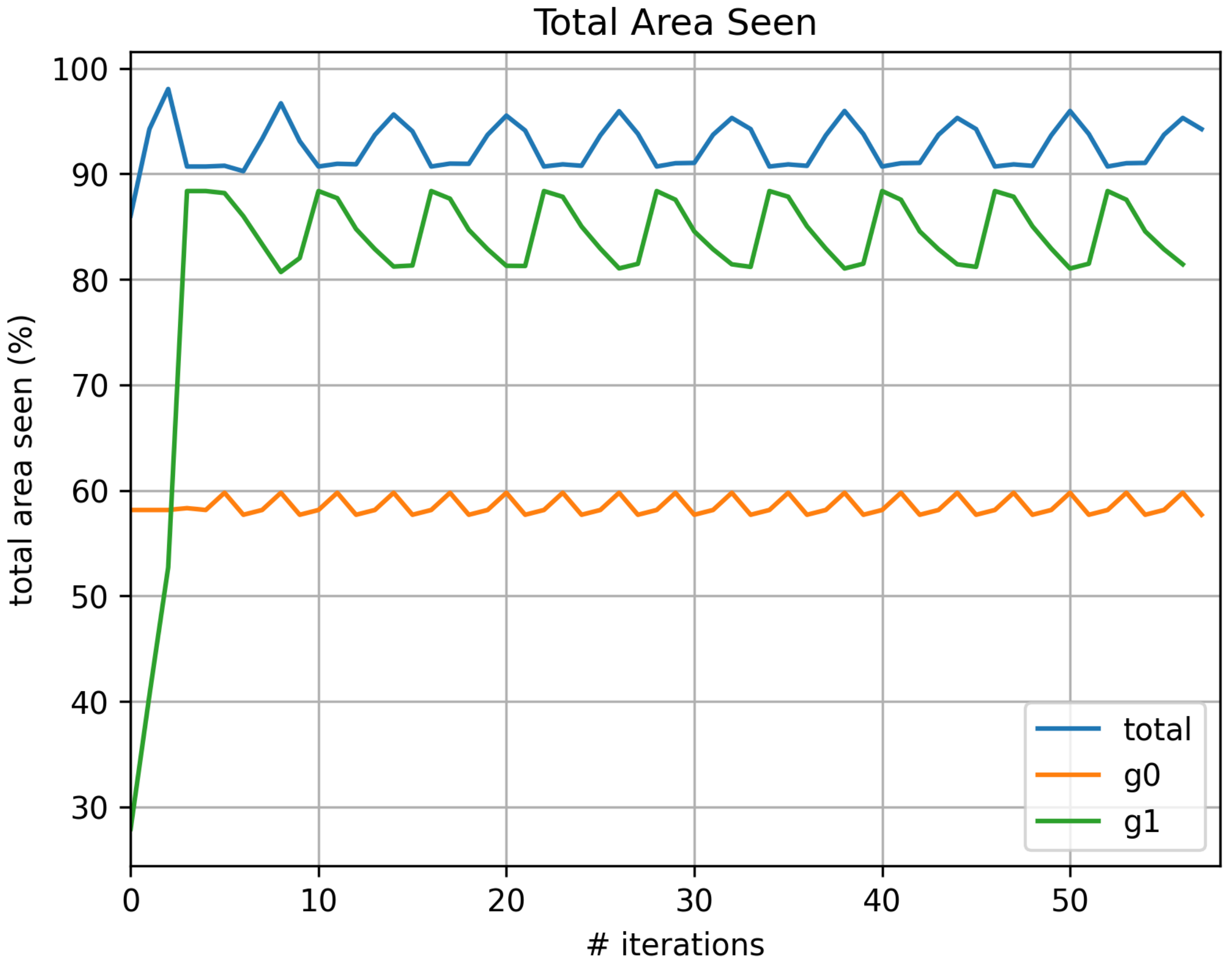


Heuristics: Reflex area





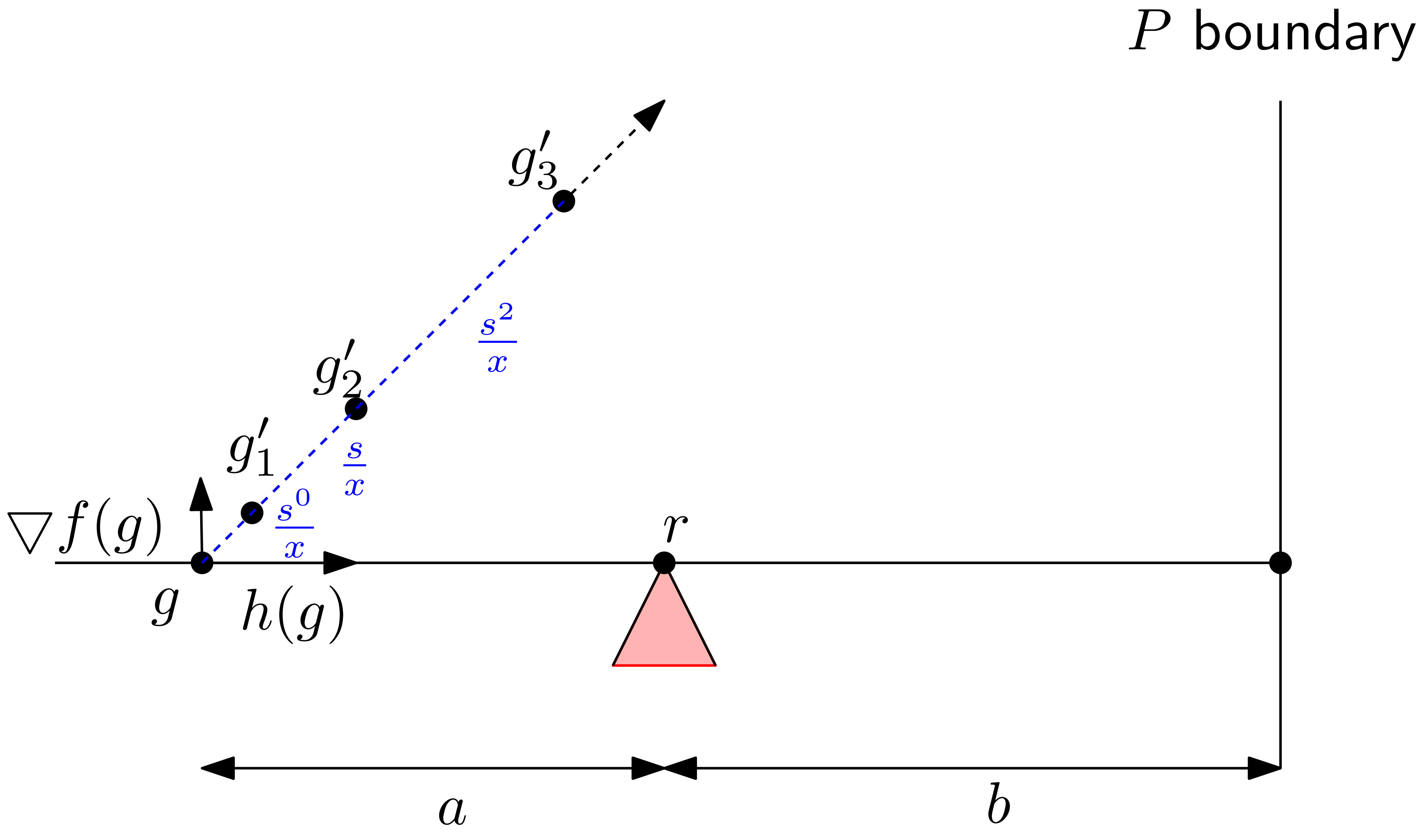
All heuristics



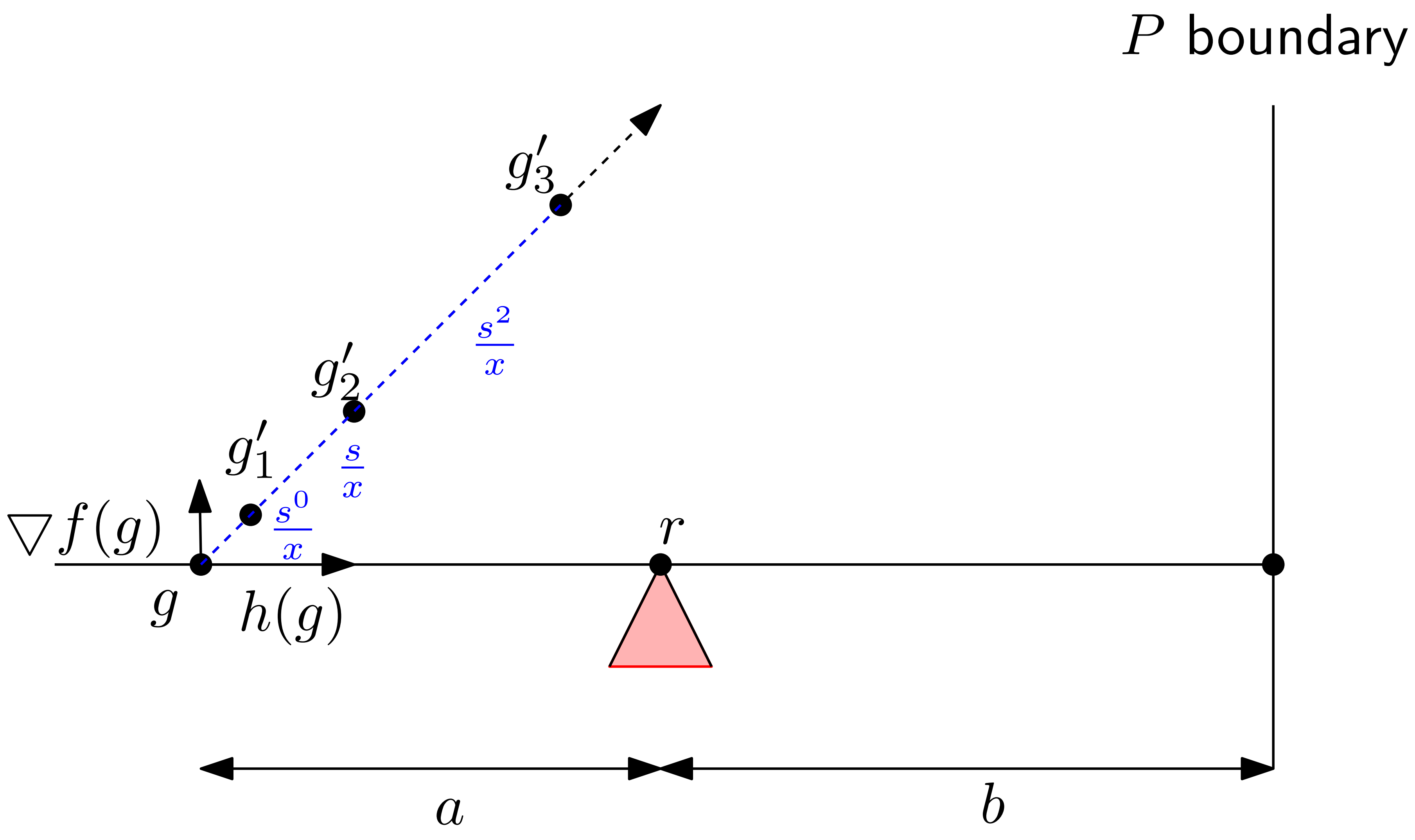
No reflex area

Heuristics: Line Search

s - step size
x - search factor



Heuristics: Line Search



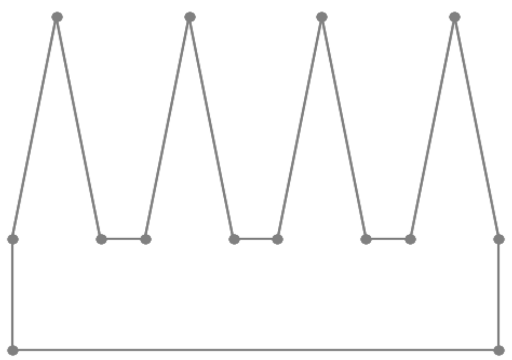
s - step size
x - search factor

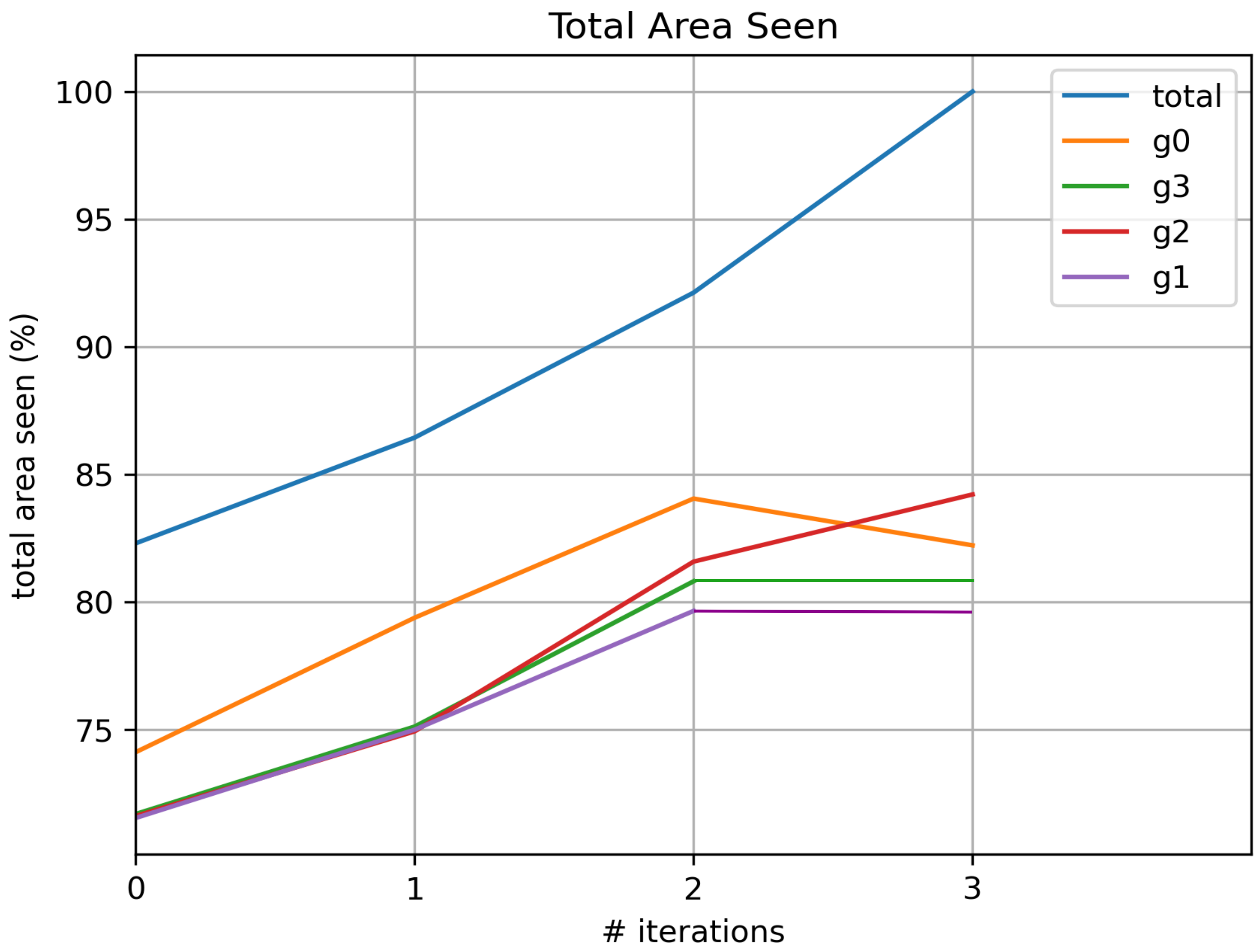
$$g'_1 = g + \frac{1}{x}M(g)$$

$$g'_2 = g + \frac{s}{x}M(g)$$

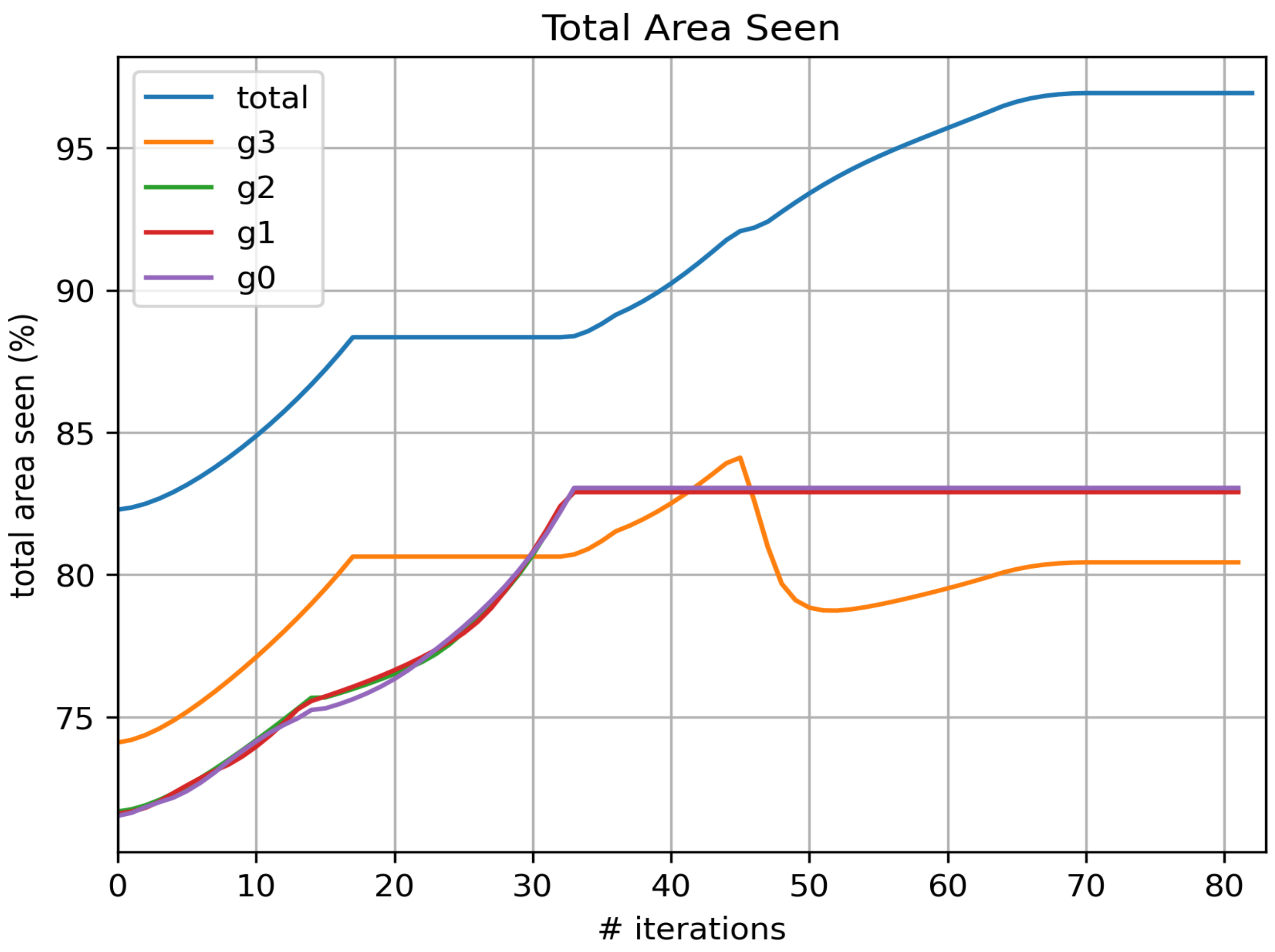
$$g'_3 = g + \frac{s^2}{x}M(g)$$

Heuristics: Line Search



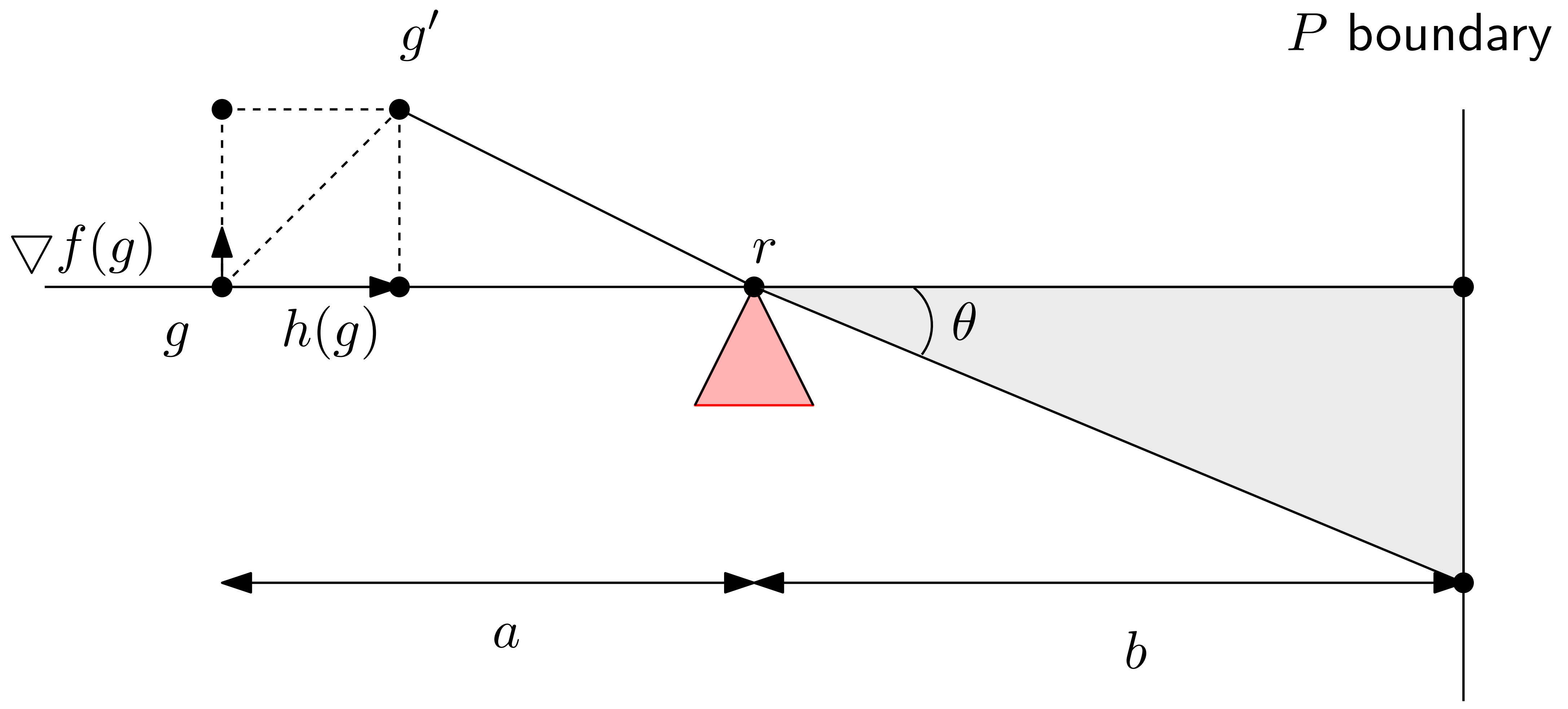


All heuristics



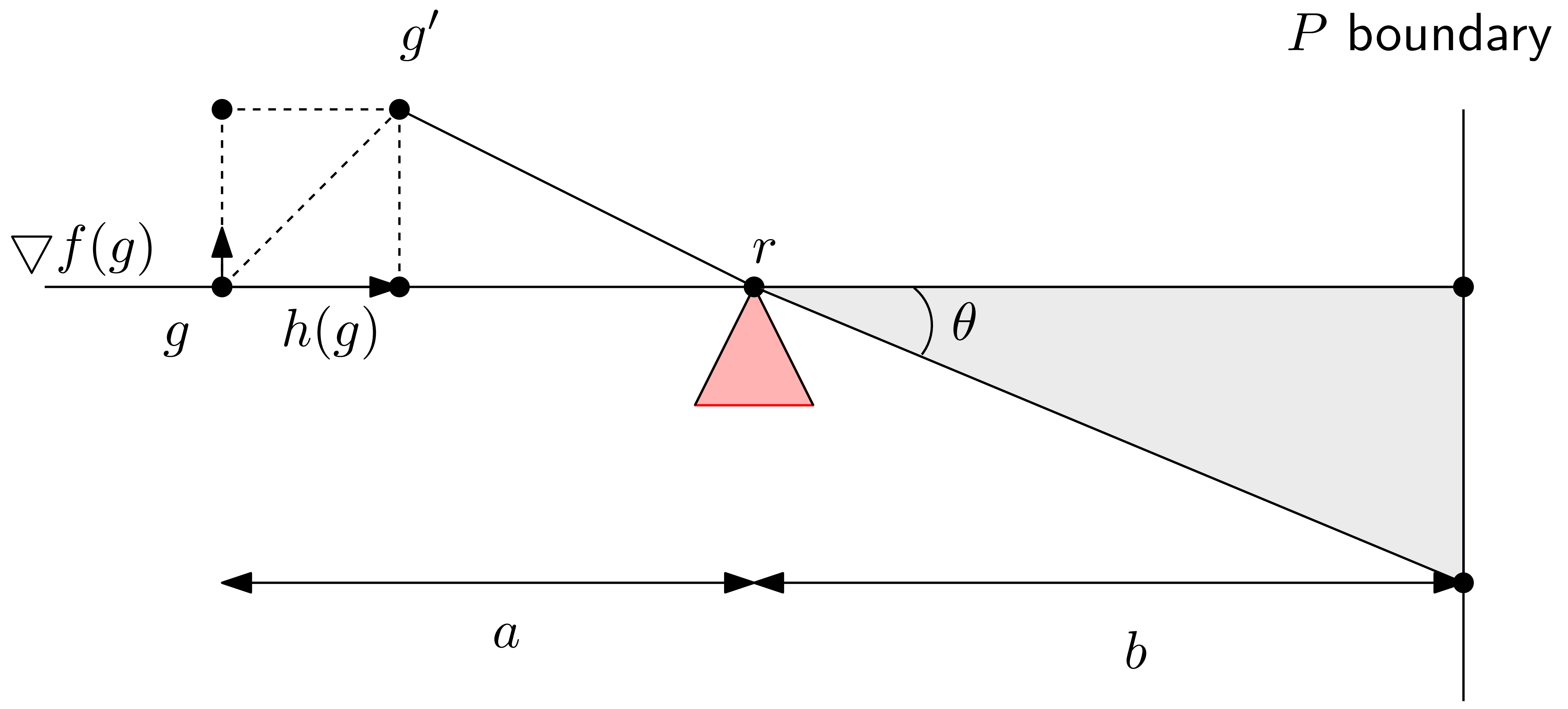
No line search

Heuristics: Angle behind reflex vertex

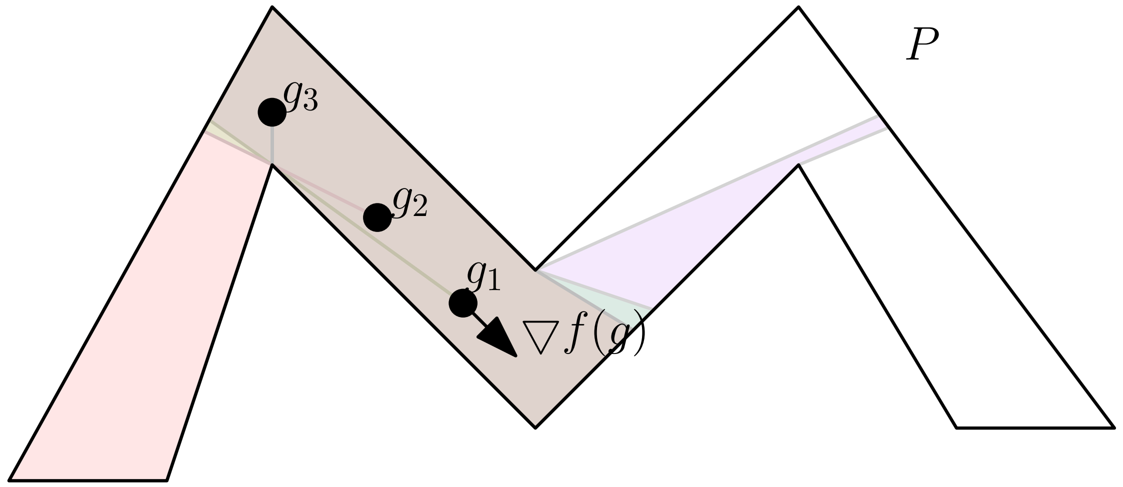


Heuristics: Angle behind reflex vertex

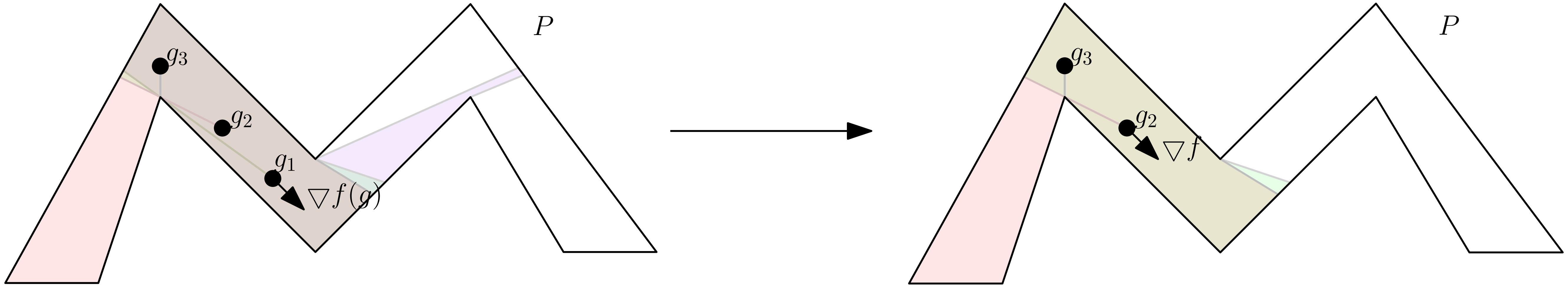
$$g' = g + (\tfrac{\theta}{2\pi} + c)(\nabla f(g) + h(g))$$



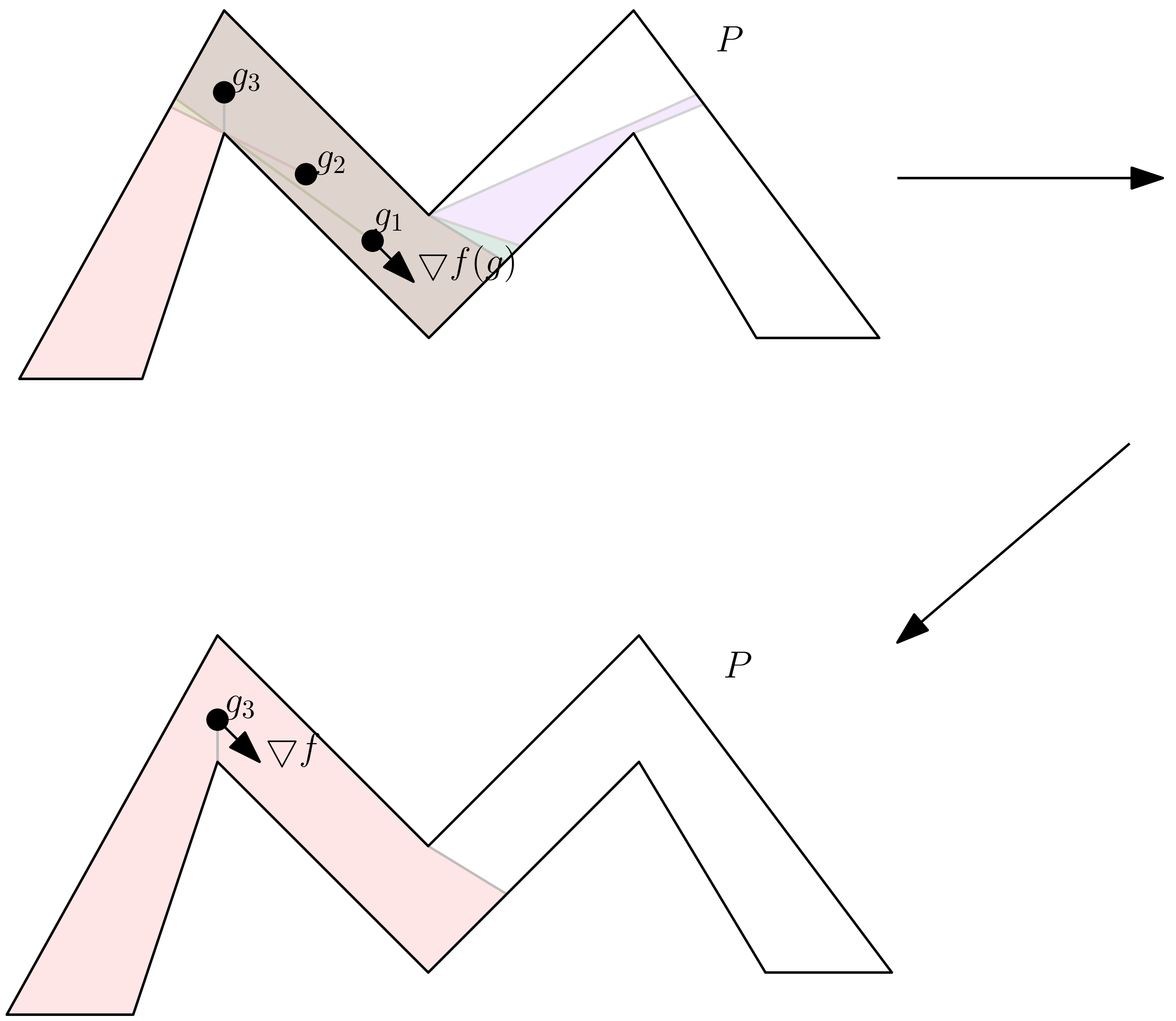
Heuristics: Hidden movement



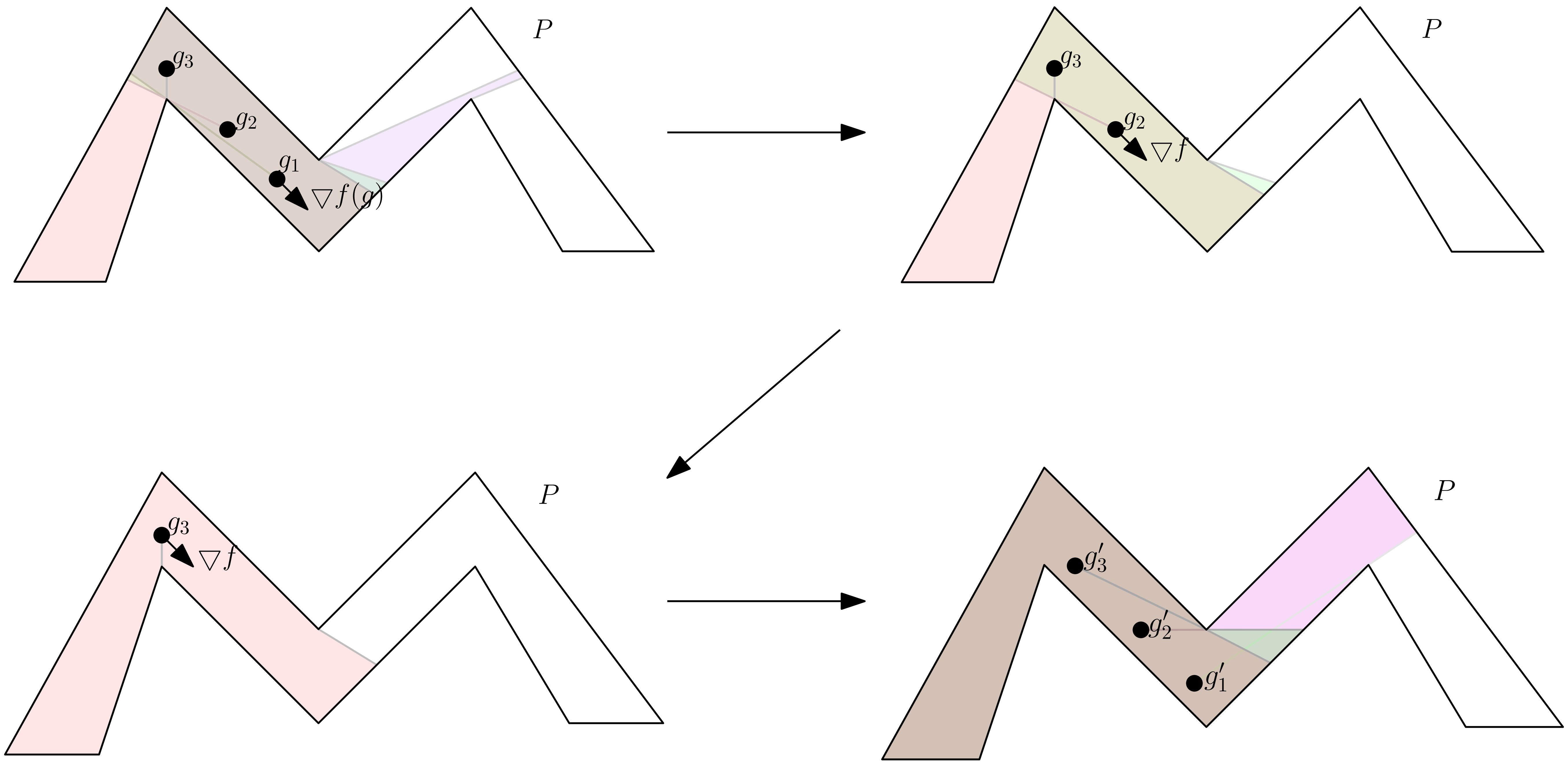
Heuristics: Hidden movement



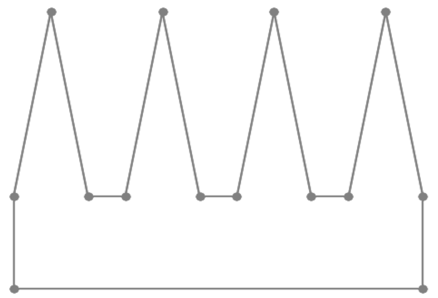
Heuristics: Hidden movement

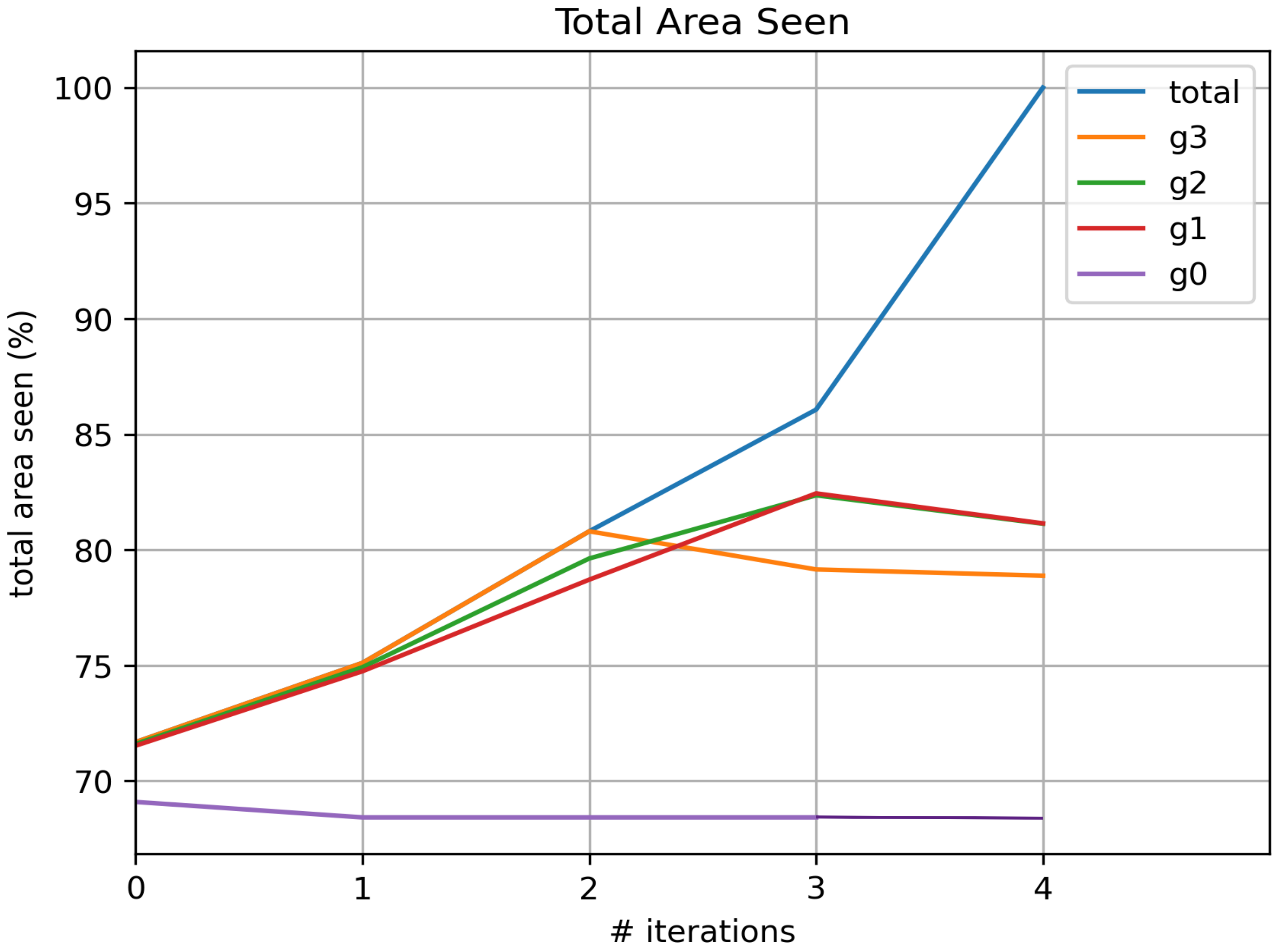


Heuristics: Hidden movement

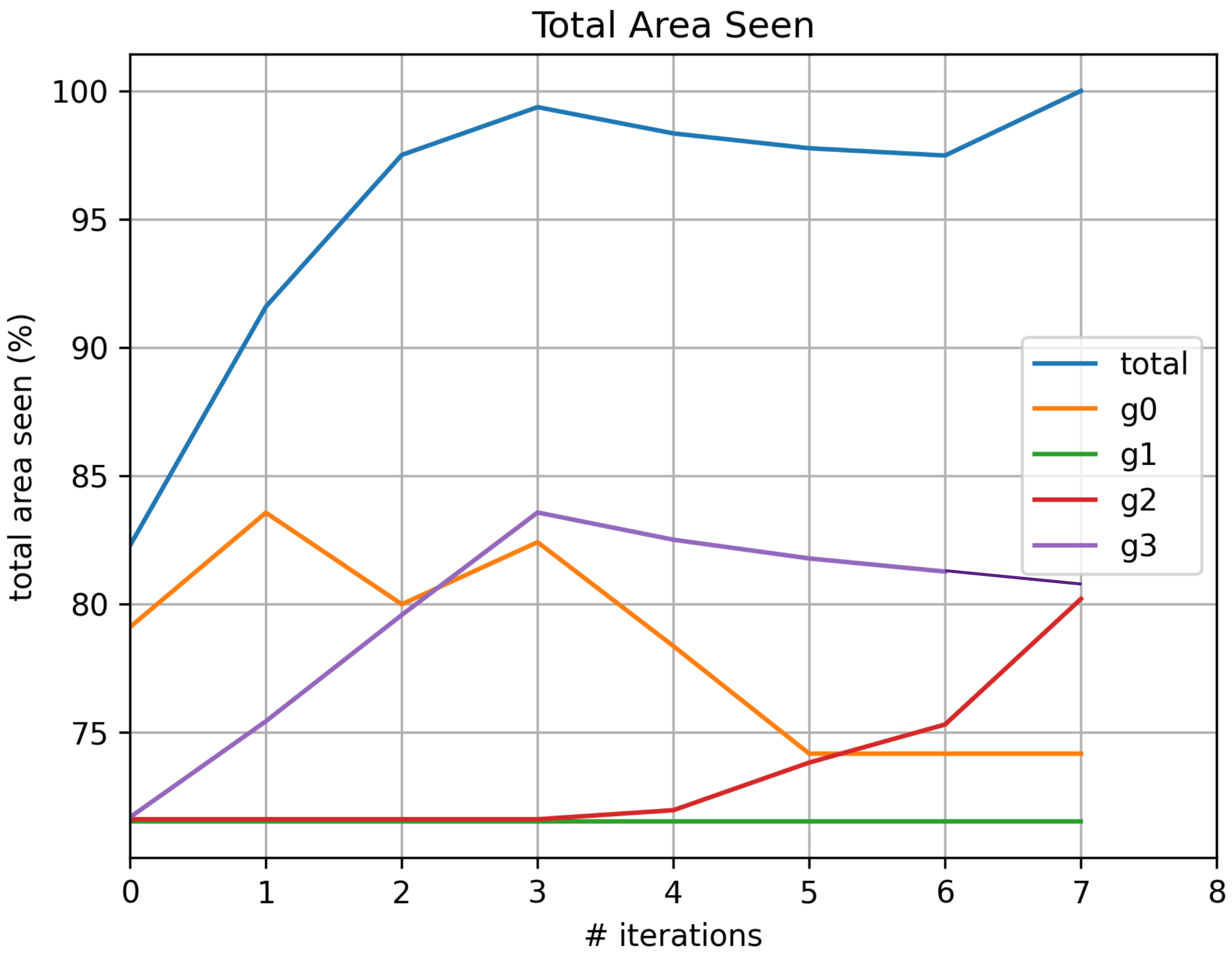


Heuristics: Hidden movement



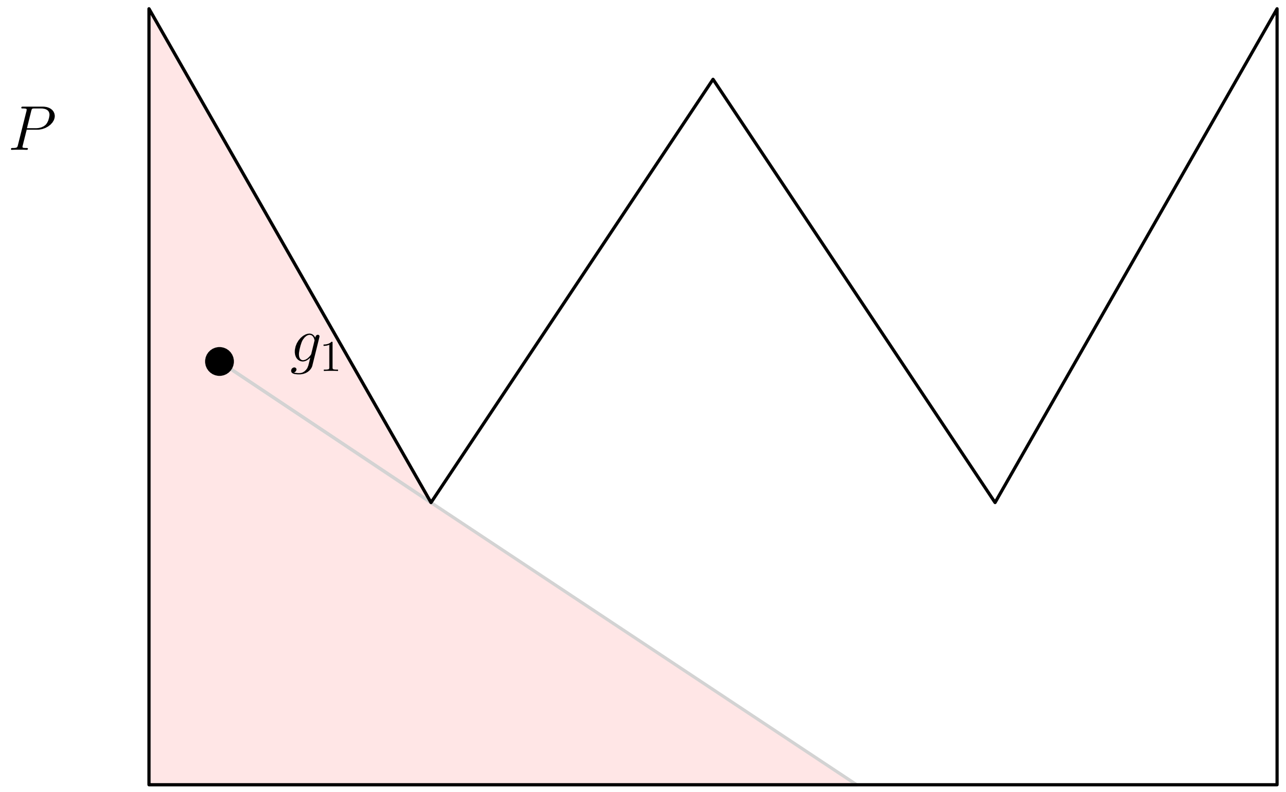


All heuristics

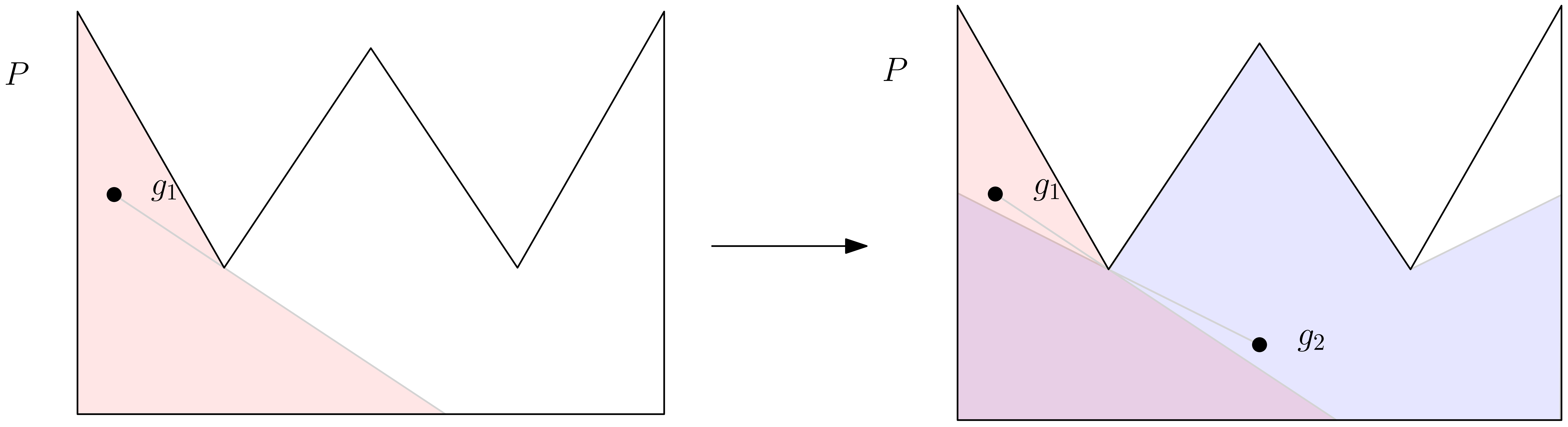


No hidden movement

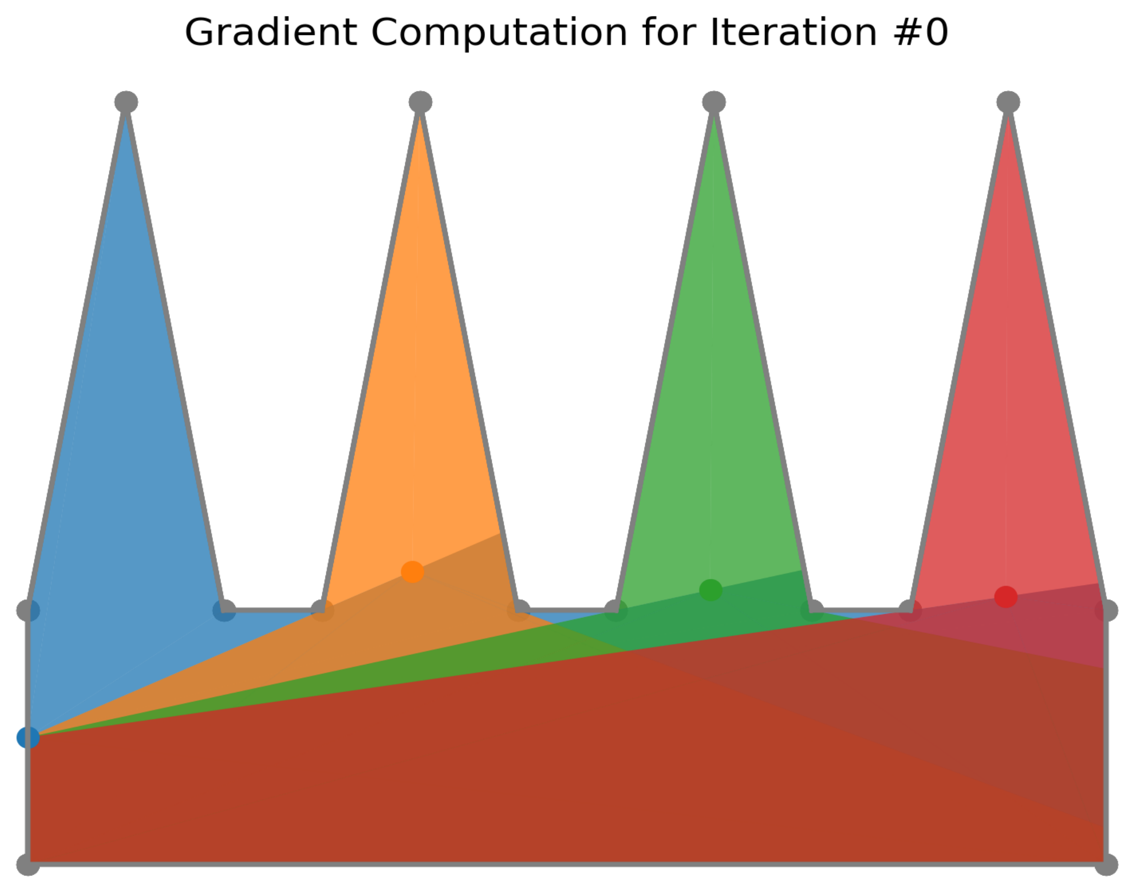
Heuristics: Greedy initialisation



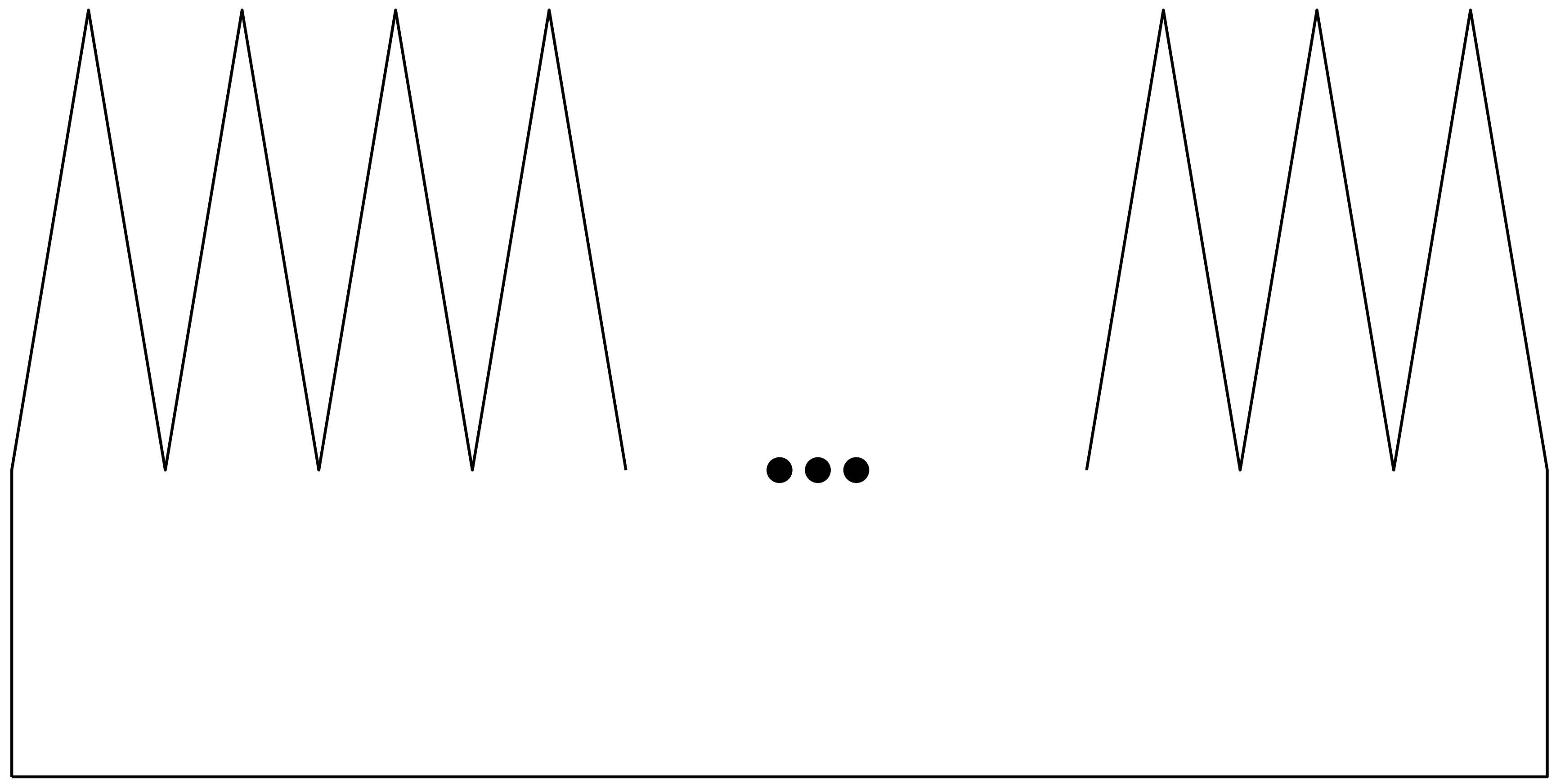
Heuristics: Greedy initialisation



Heuristics: Greedy initialisation

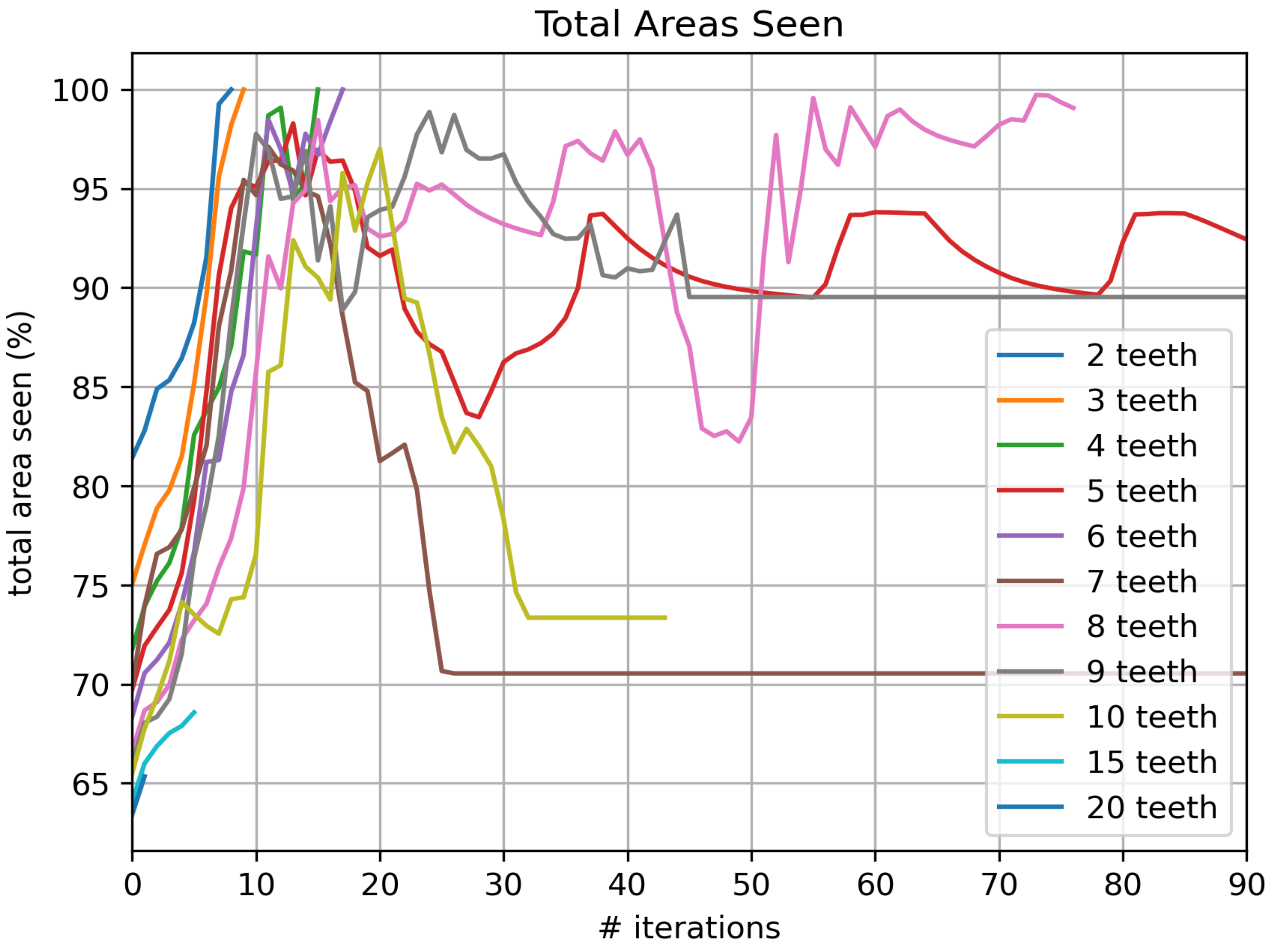


Scalability for the comb polygon

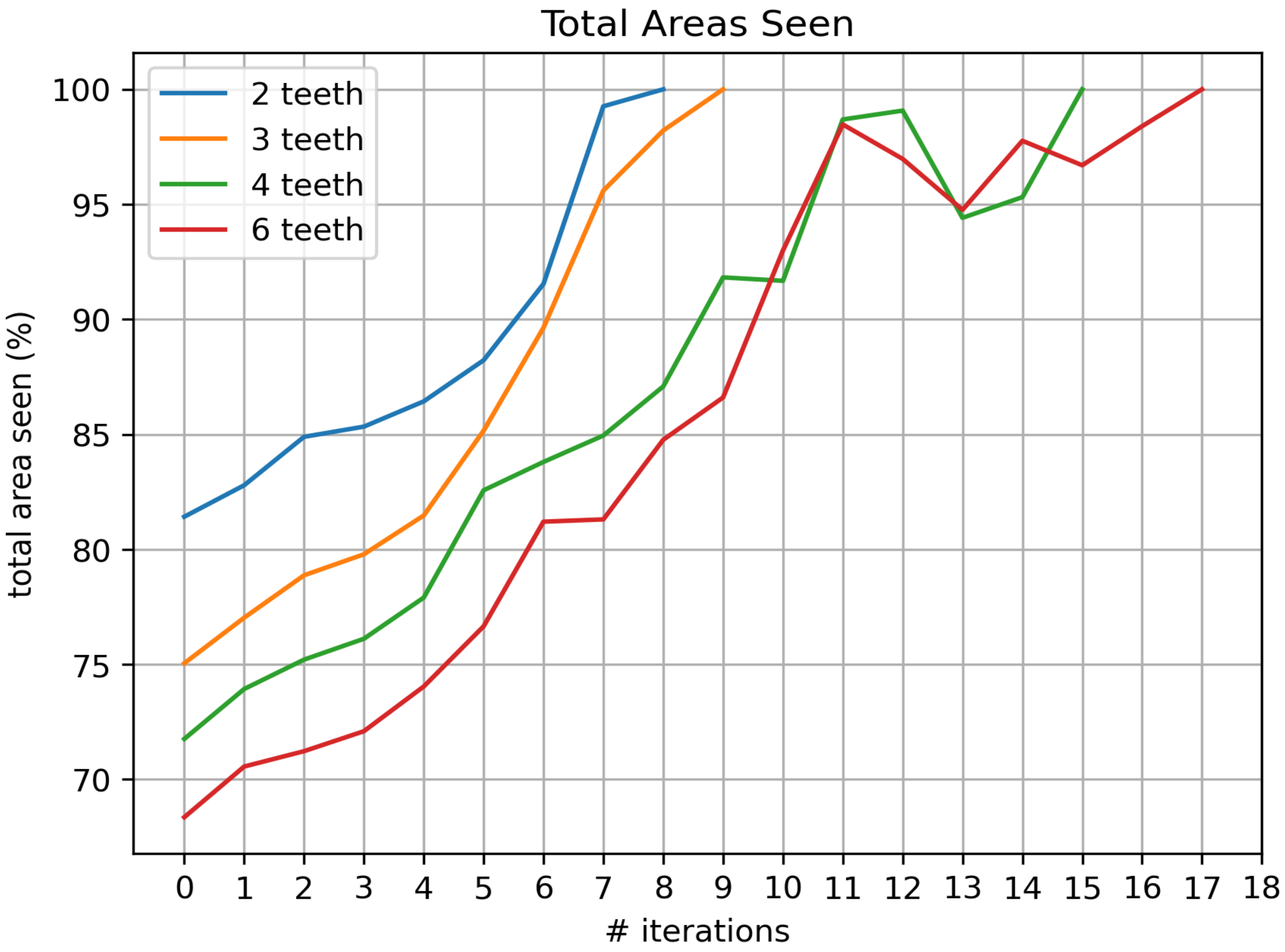


2, 3, ..., 10, 15, 20 teeth

Scalability for the comb polygon



Scalability for the comb polygon



Problems encountered

Future work

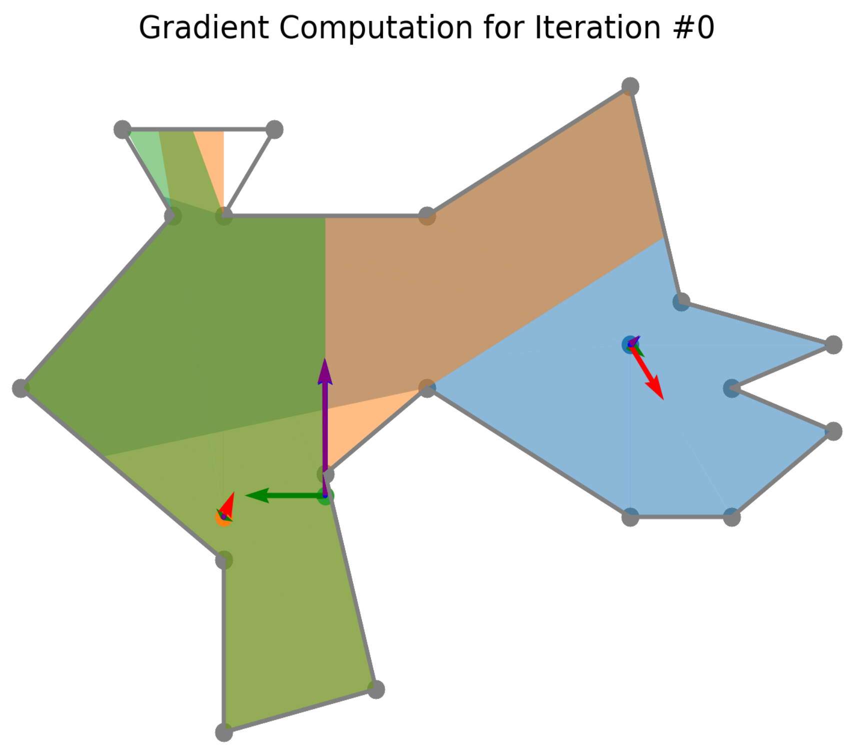
improve the algorithm's robustness, performance and scalability

implement other heuristics

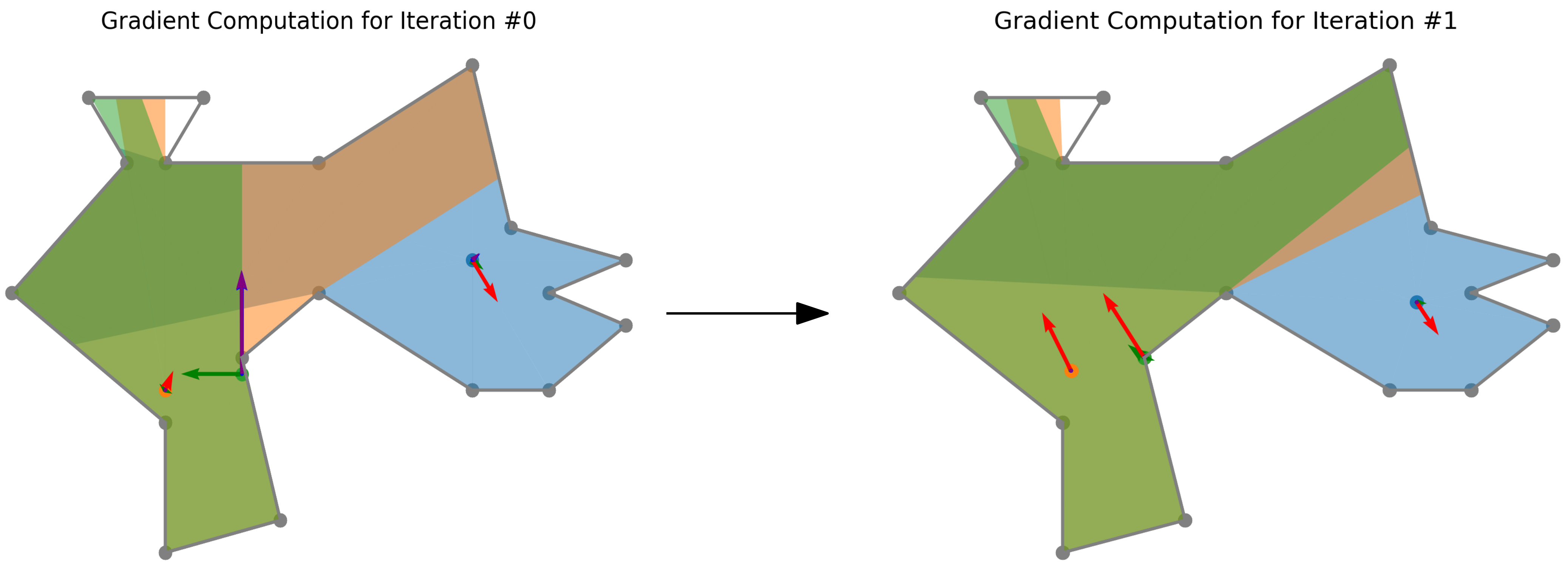
test the algorithm on larger polygons with more guards

solve existing bugs

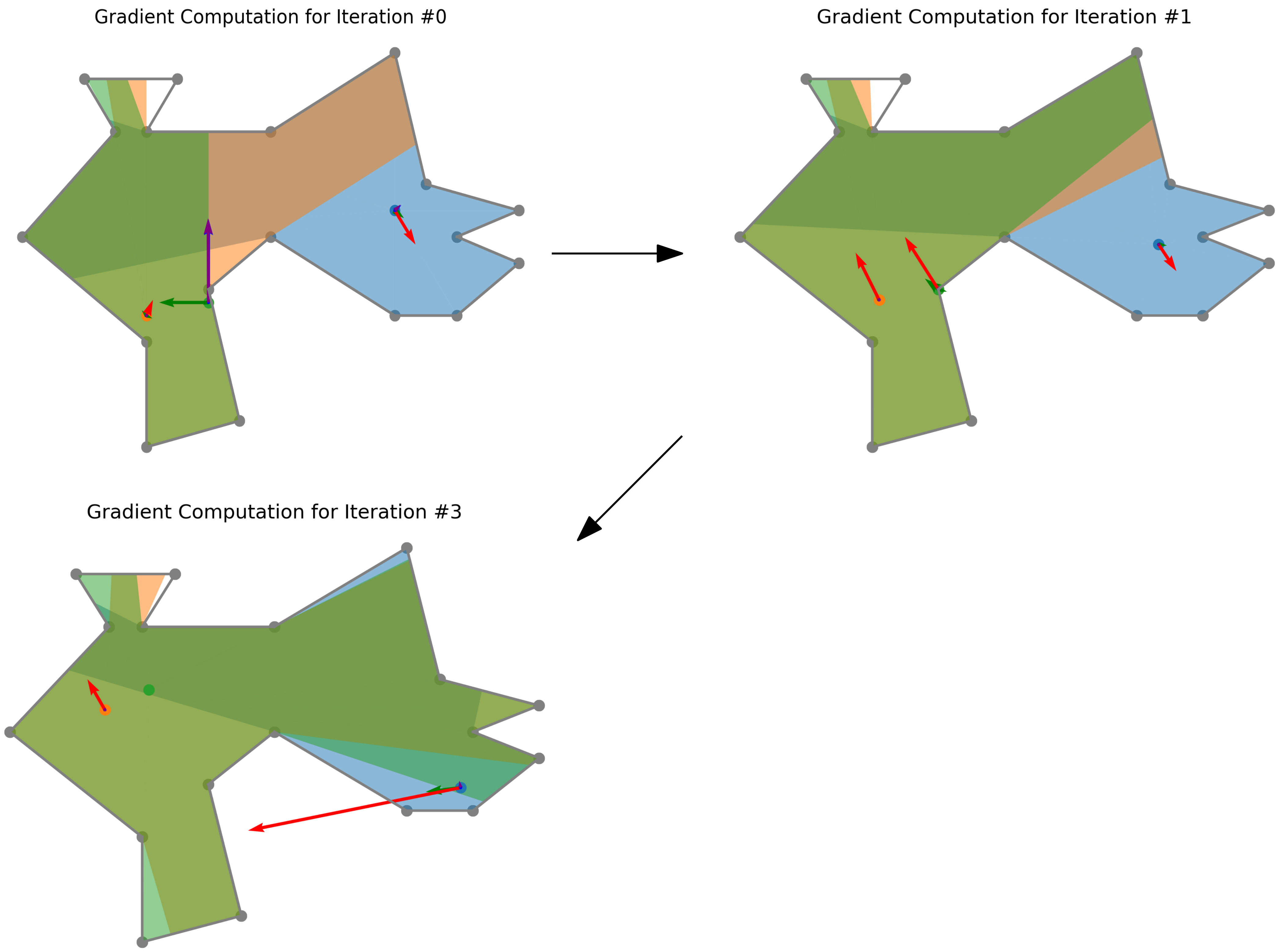
The Art Gallery Problem



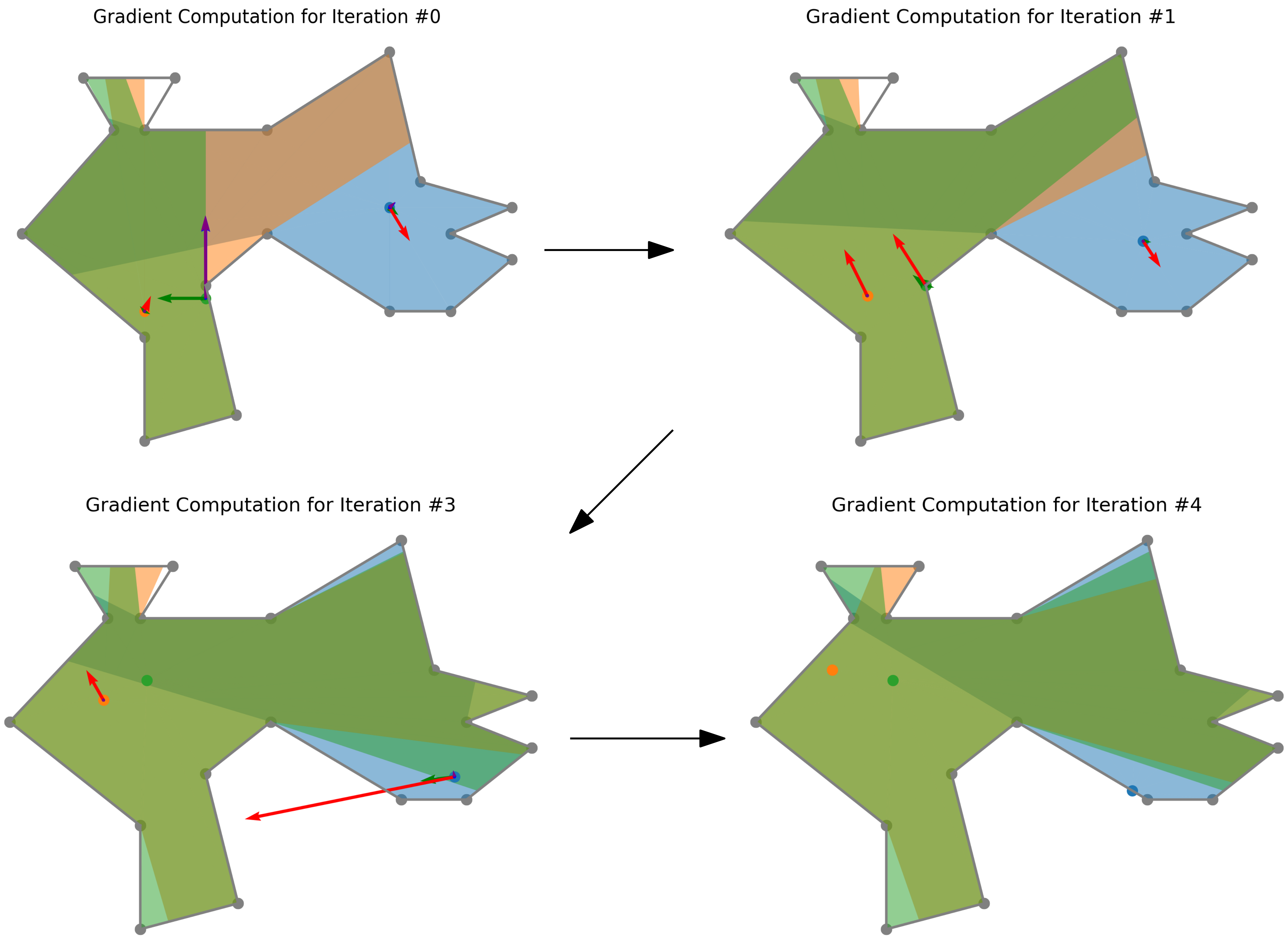
The Art Gallery Problem



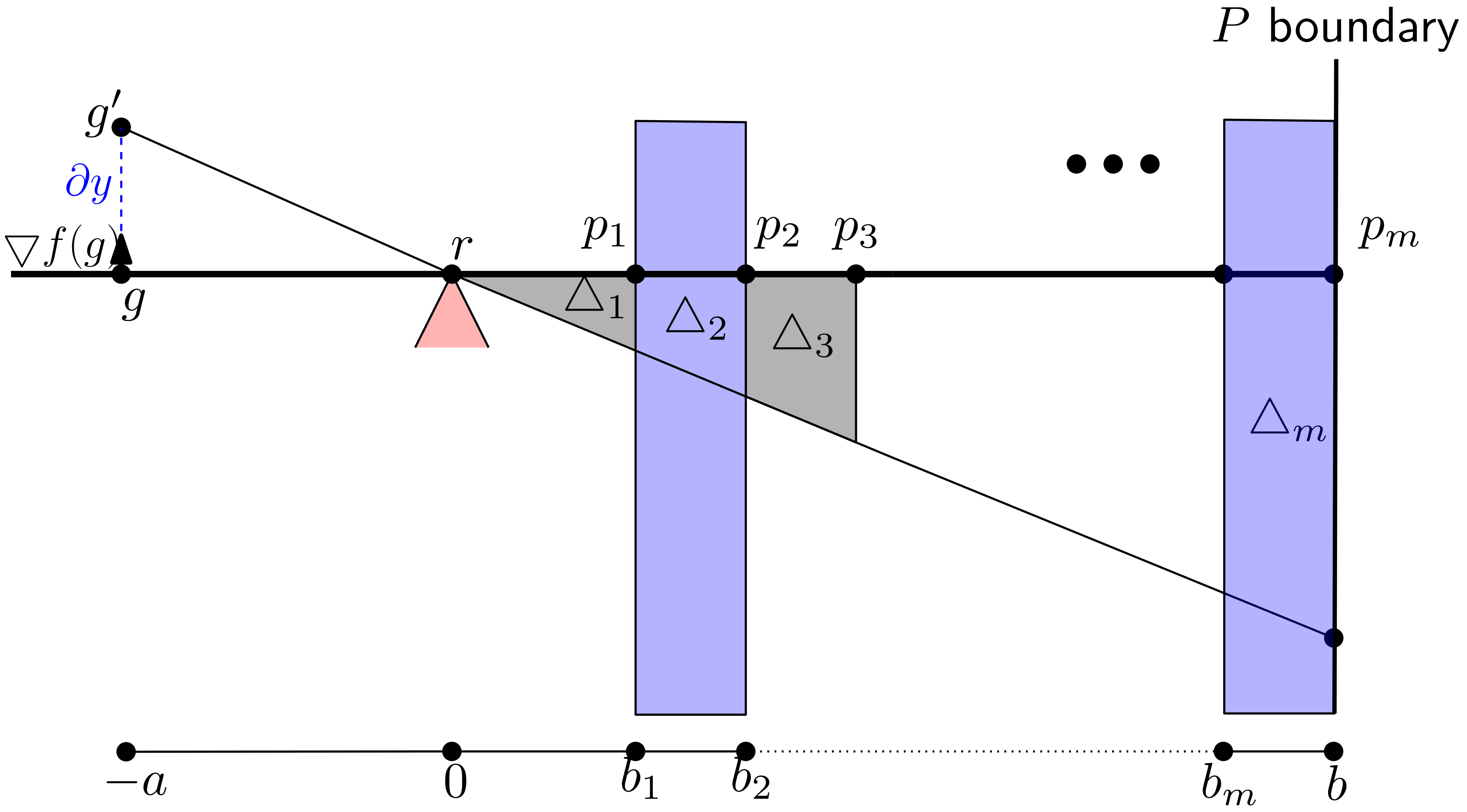
The Art Gallery Problem



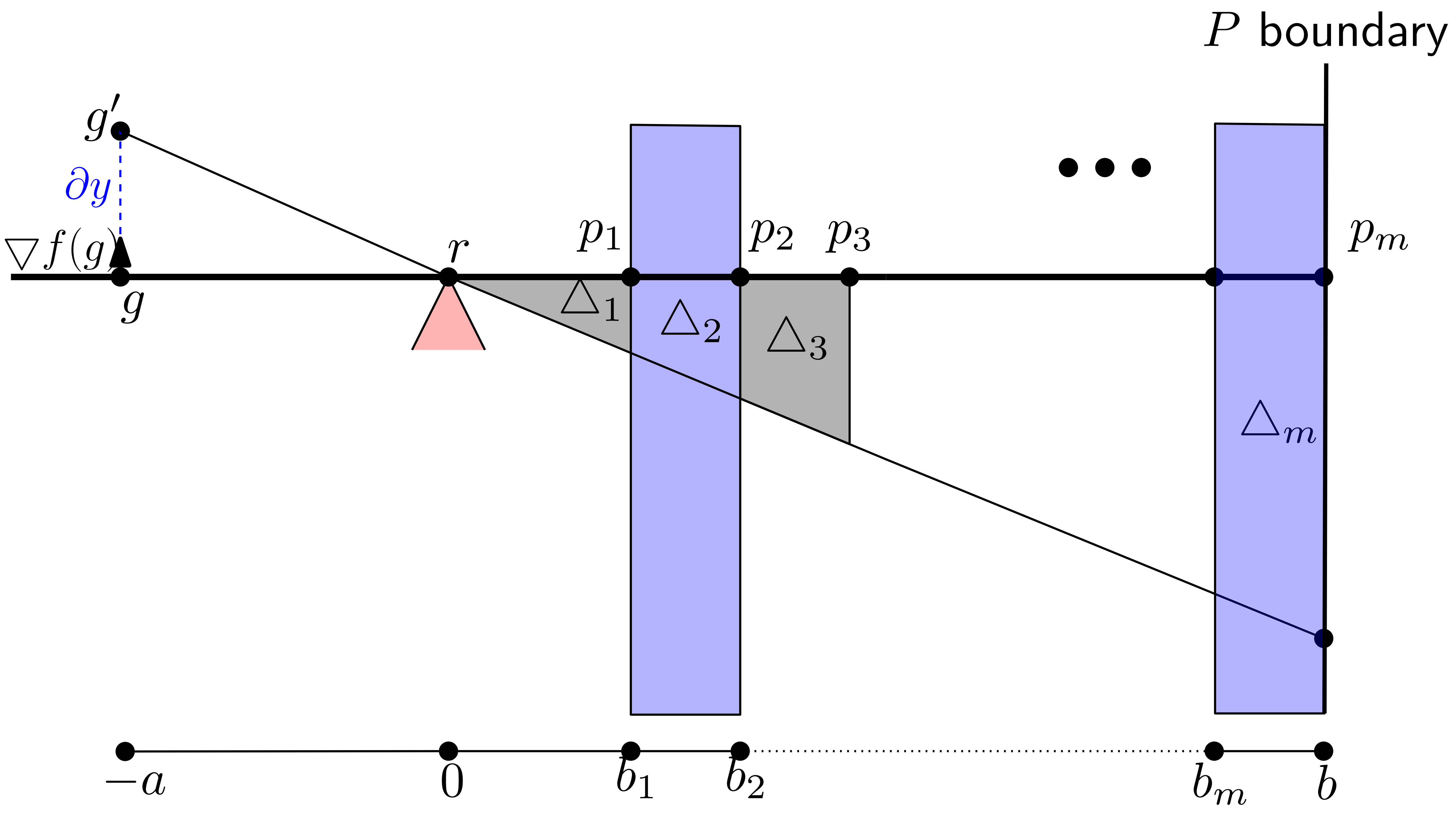
The Art Gallery Problem



Computing the gradient for multiple guards

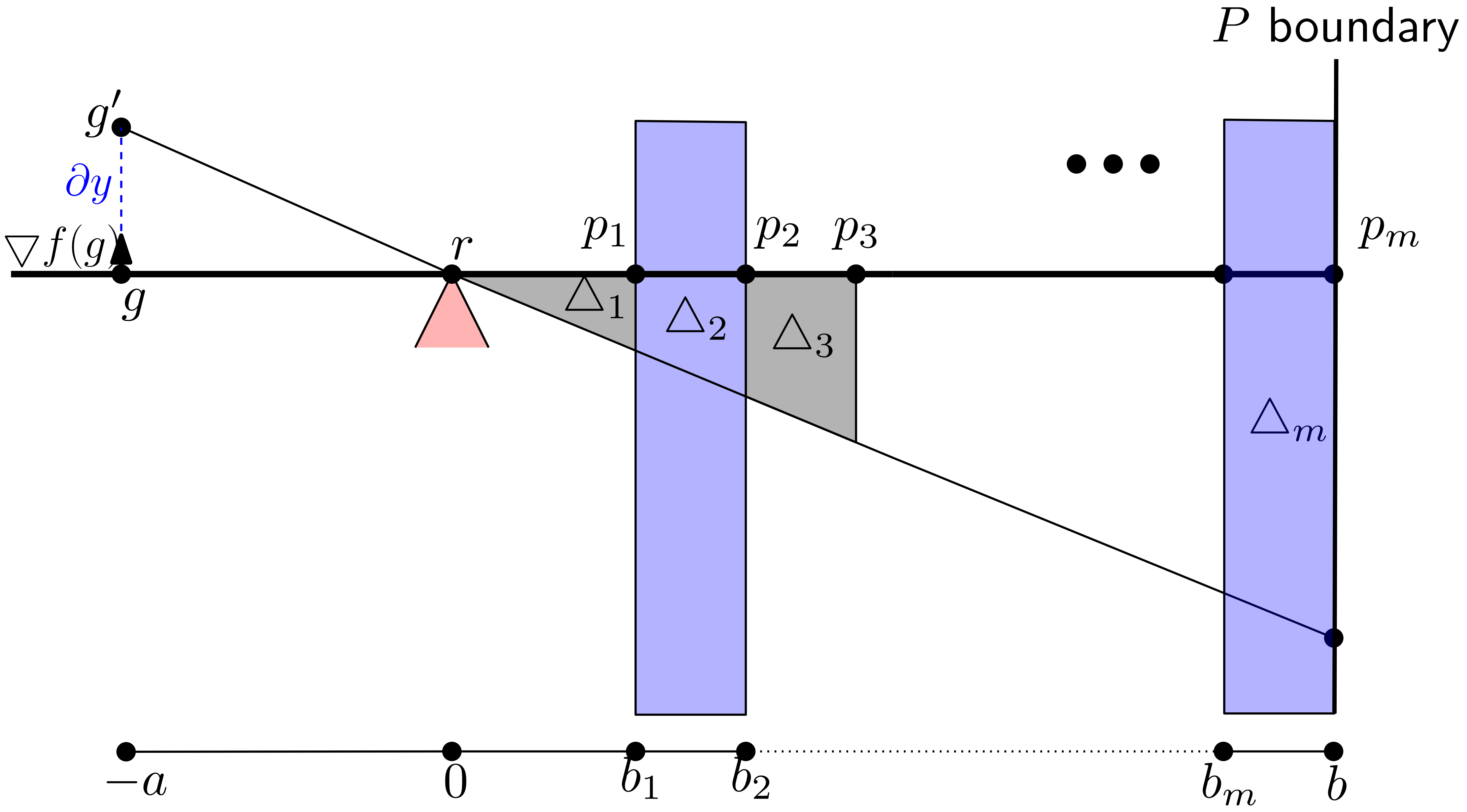


Computing the gradient for multiple guards



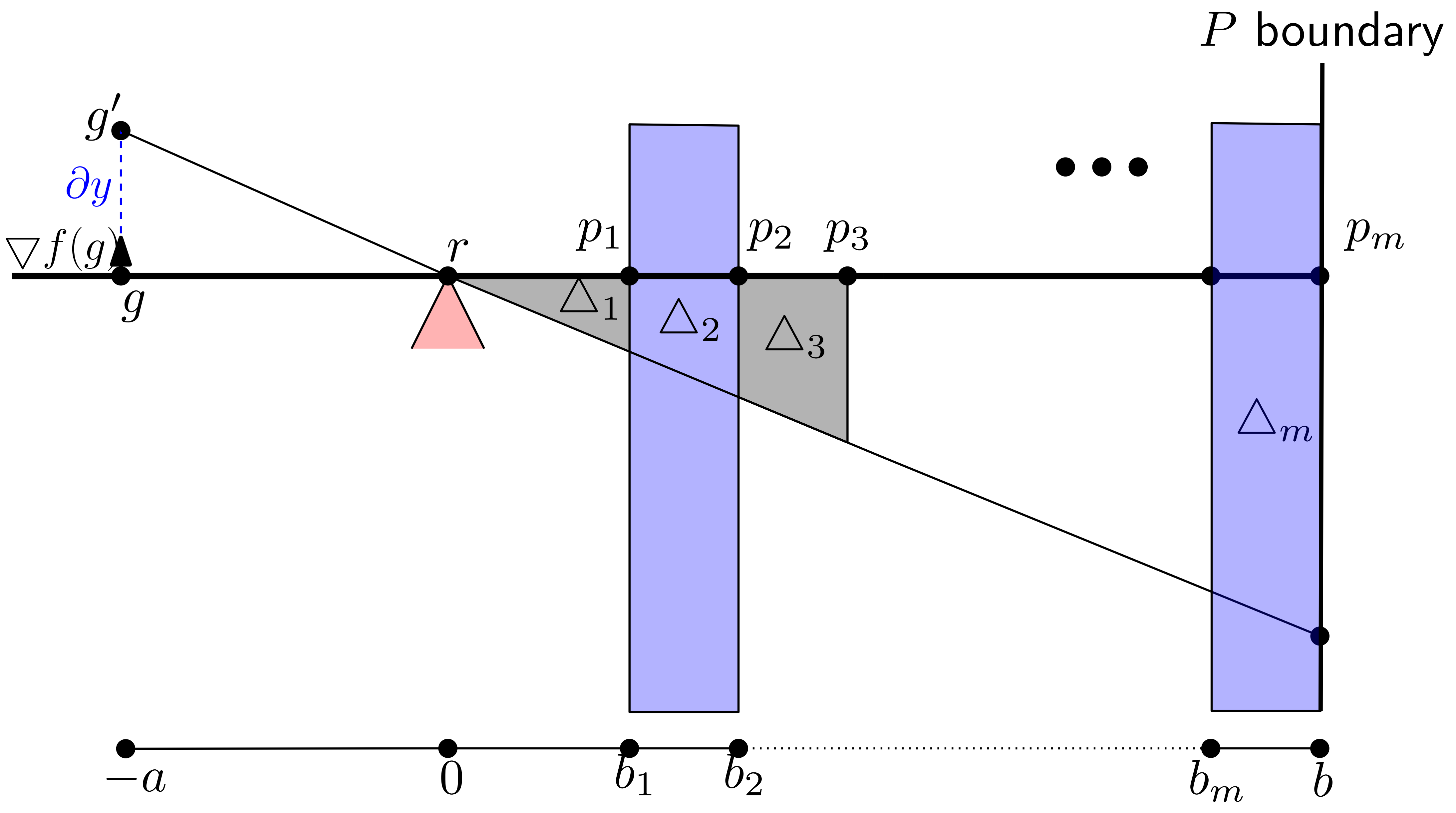
Area$_{\triangle_1+\triangle_3+\ldots+\triangle_{m-1}}(g)$

Computing the gradient for multiple guards



$$\text{Area}_{\triangle_1+\triangle_3+\ldots+\triangle_{m-1}}(g) = \text{Area}_{\triangle_1+\ldots+\triangle_m}(g) - \text{Area}_{\triangle_{m-1}}(g) + \text{Area}_{\triangle_{m-2}}(g) - \ldots - \text{Area}_{\triangle_2}(g) + \text{Area}_{\triangle_1}(g)$$

Computing the gradient for multiple guards



$$\text{Area}_{\triangle_1+\triangle_3+\ldots+\triangle_{m-1}}(g) = \text{Area}_{\triangle_1+\ldots+\triangle_m}(g) - \text{Area}_{\triangle_{m-1}}(g) + \text{Area}_{\triangle_{m-2}}(g) - \ldots - \text{Area}_{\triangle_2}(g) + \text{Area}_{\triangle_1}(g)$$

$$= \left(b^2 - b_m^2 + b_{(m-1)}^2 - \ldots - b_2^2 + b_1^2\right) \frac{\partial y}{2a}$$