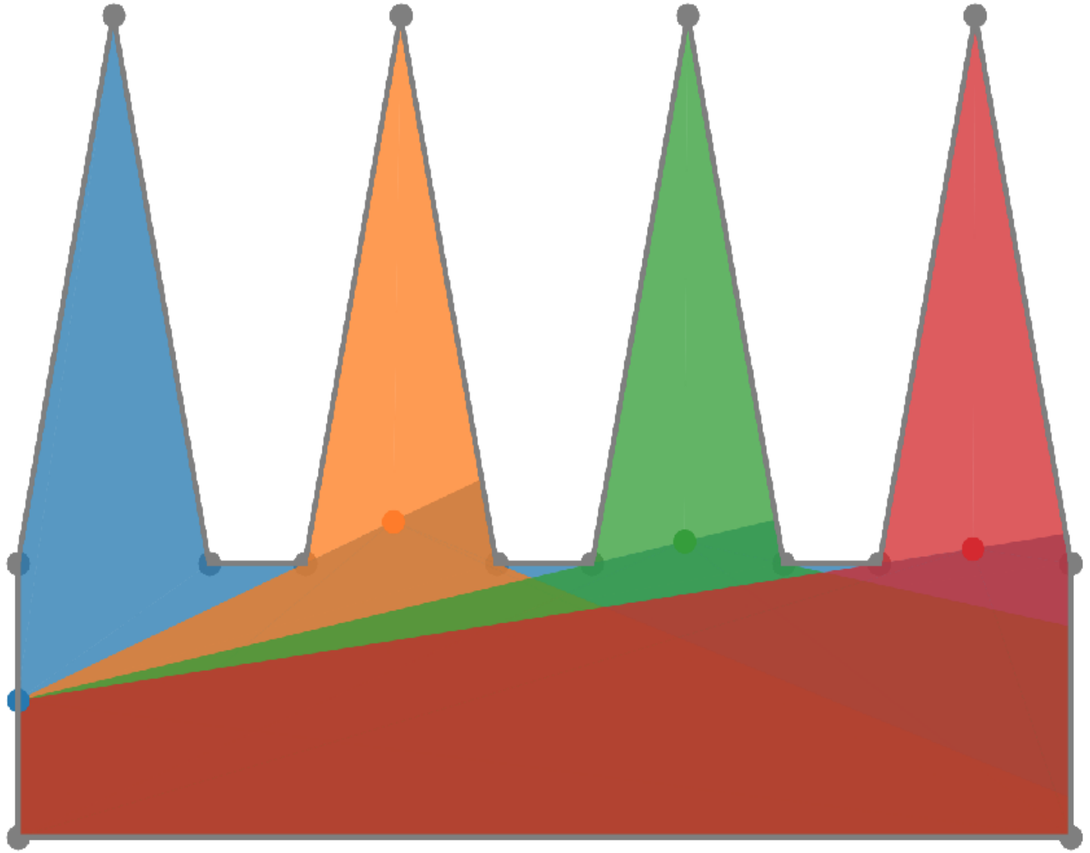
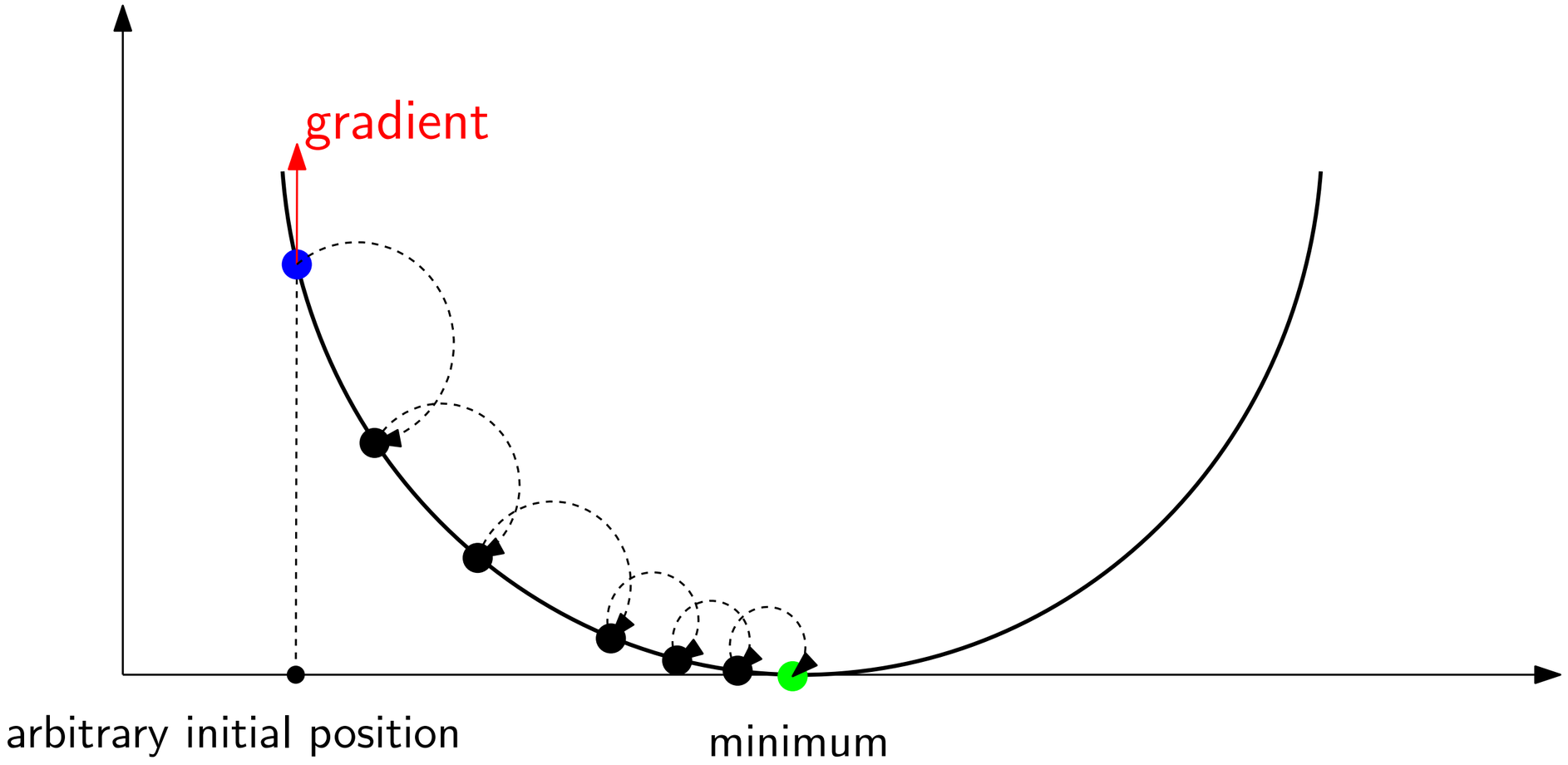
Solving the Art Gallery Problem Using Gradient Descent

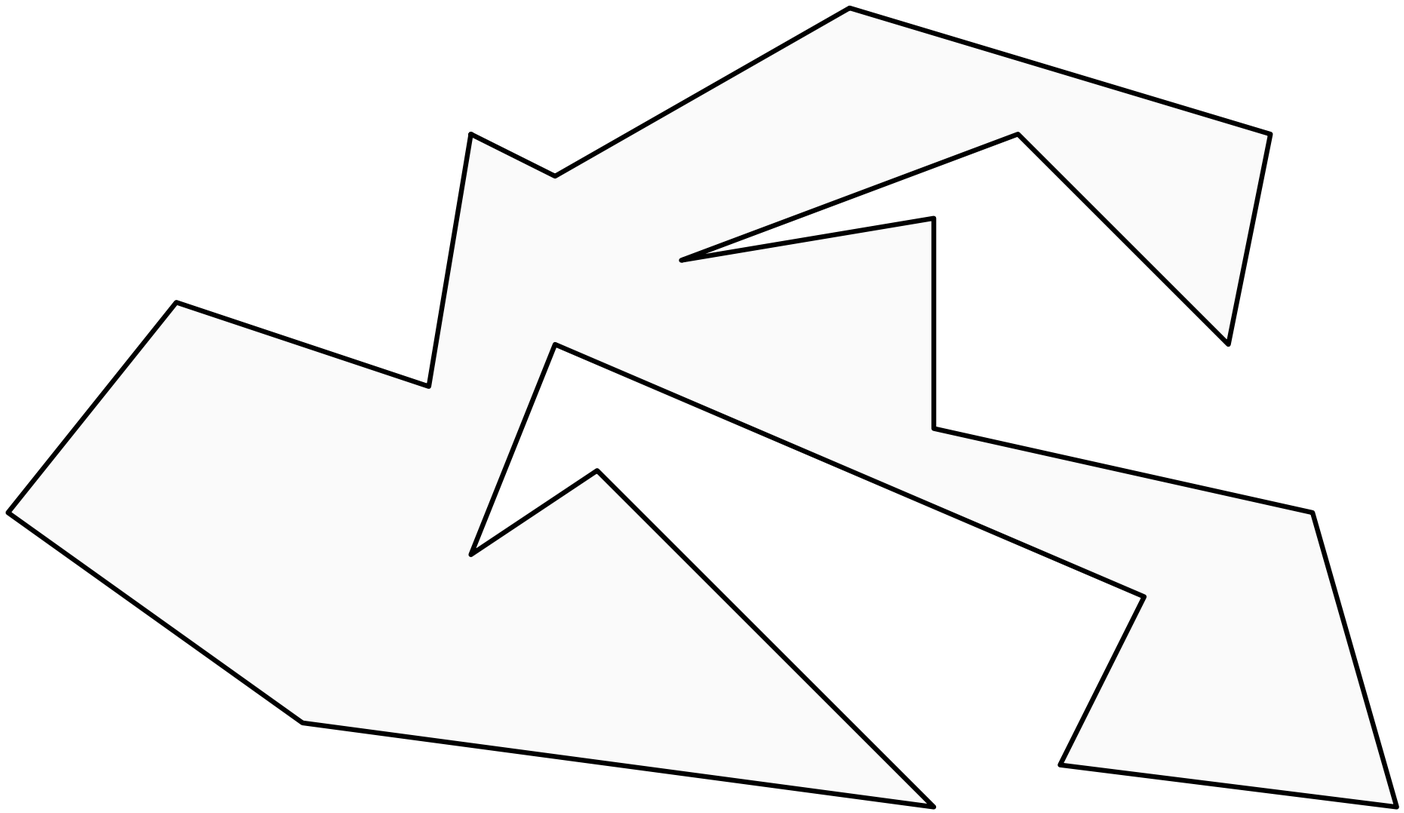
Geo Juglan



Gradient Descent



The Art Gallery Problem



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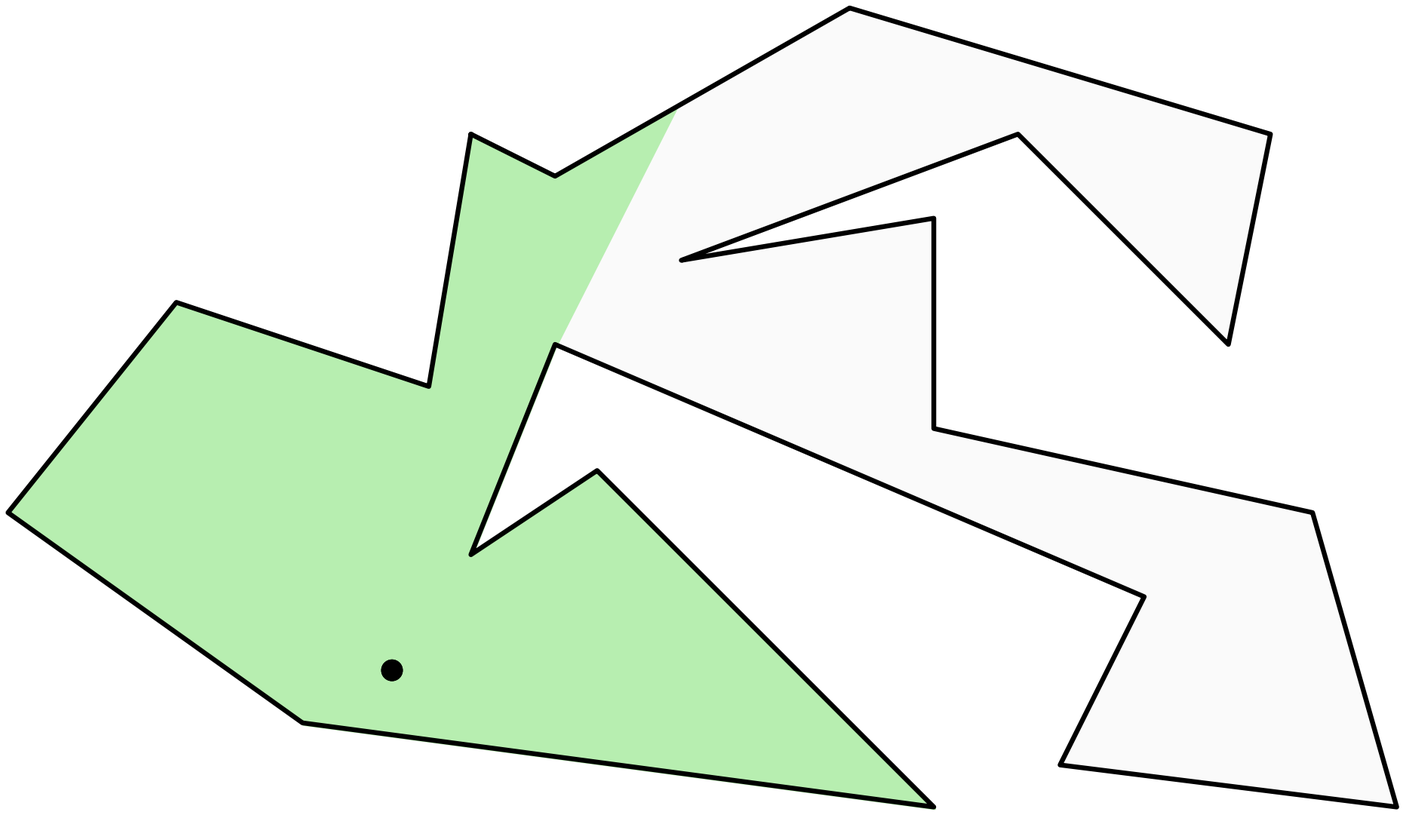

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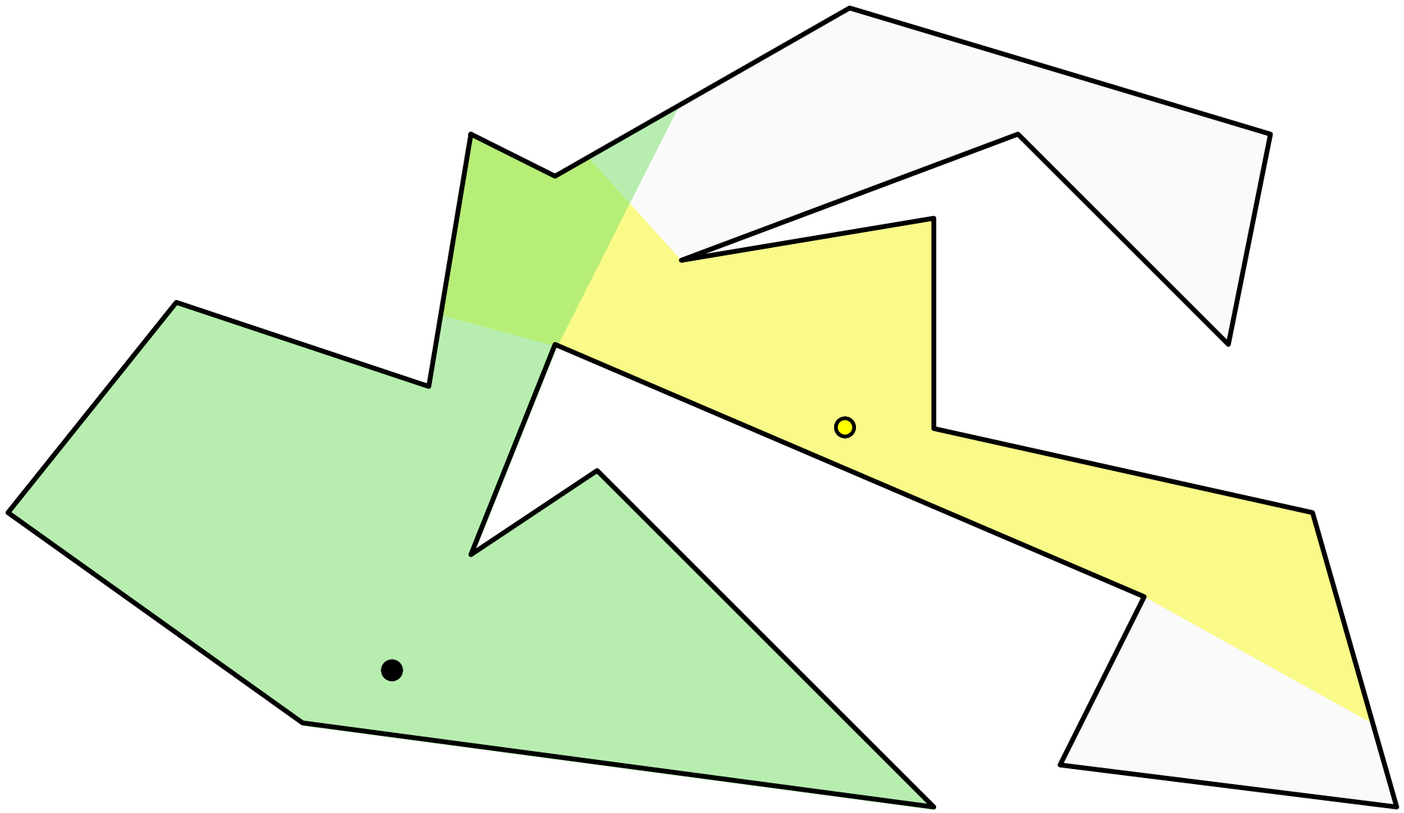

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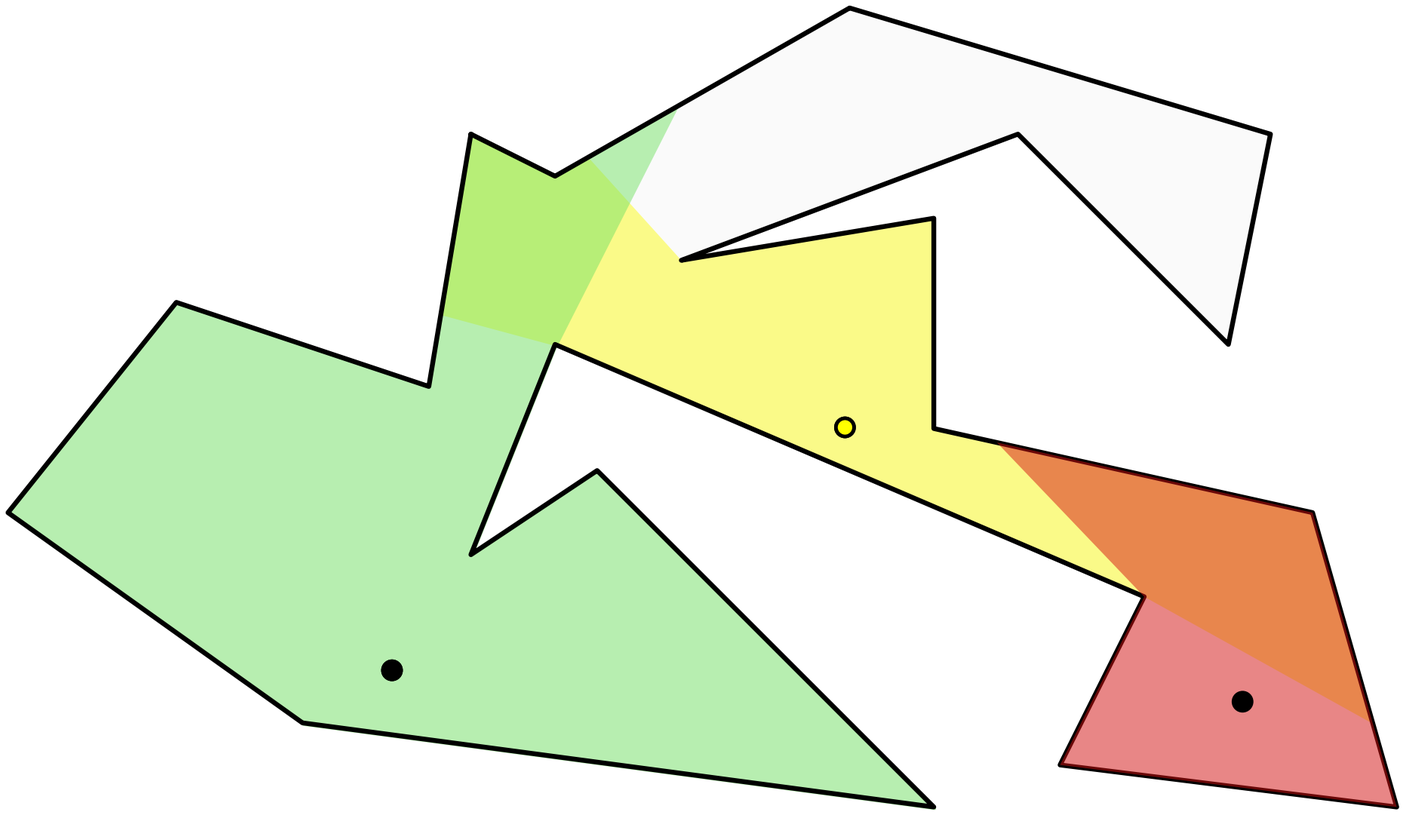

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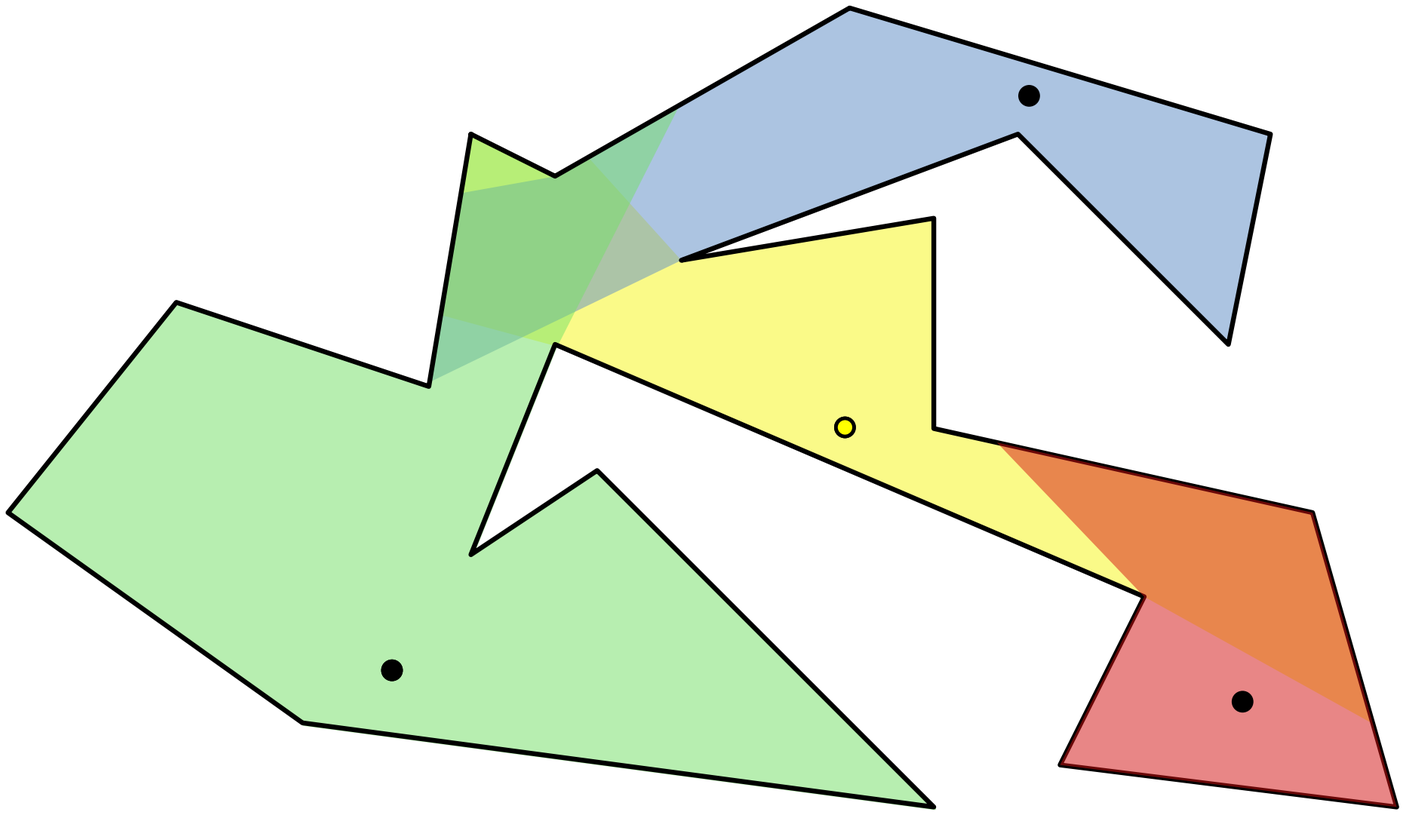
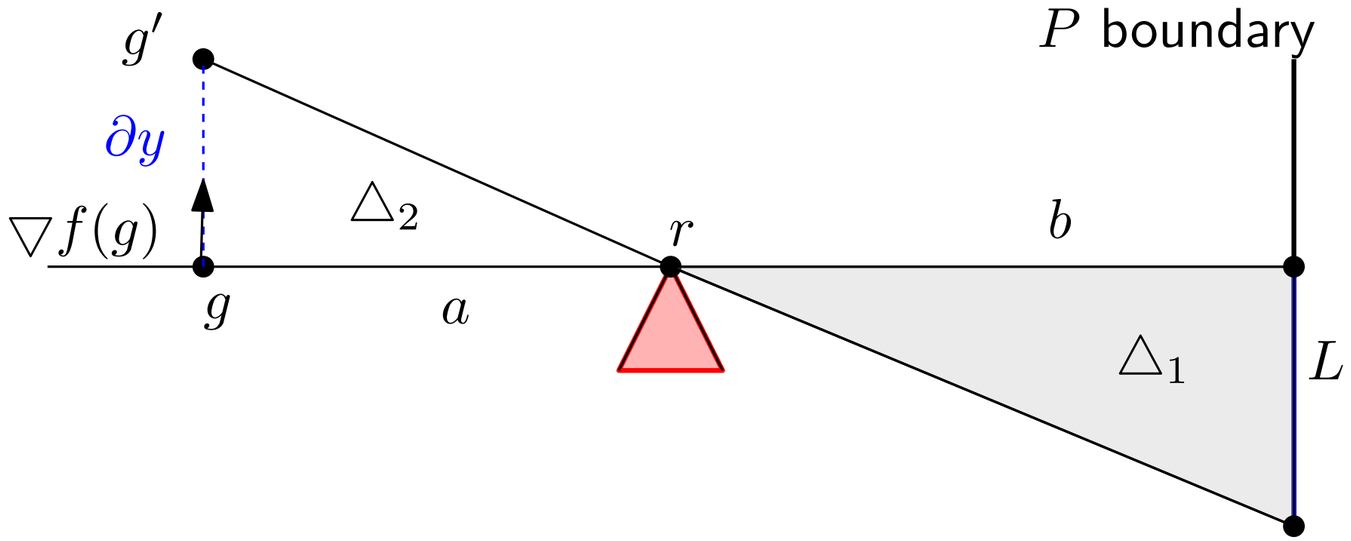
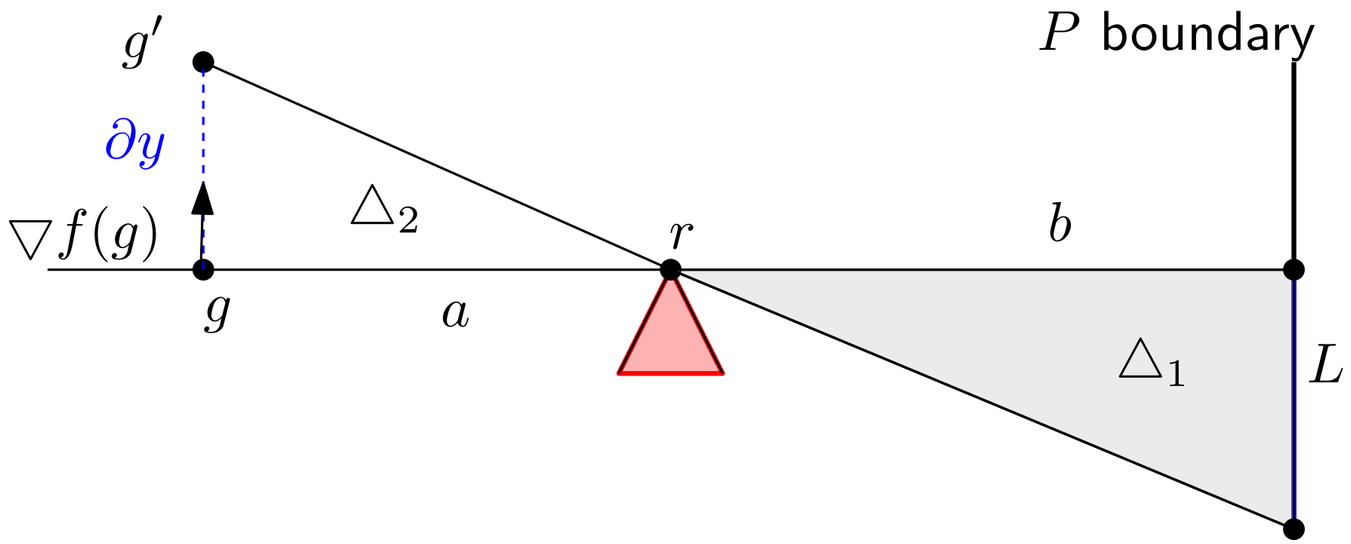


Computing the gradient for one guard

$$\nabla f(g) = \nabla \text{ Area}_{\triangle_1}(g)$$
$$\nabla f(g) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)^{\mathsf{T}}$$



Computing the gradient for one guard



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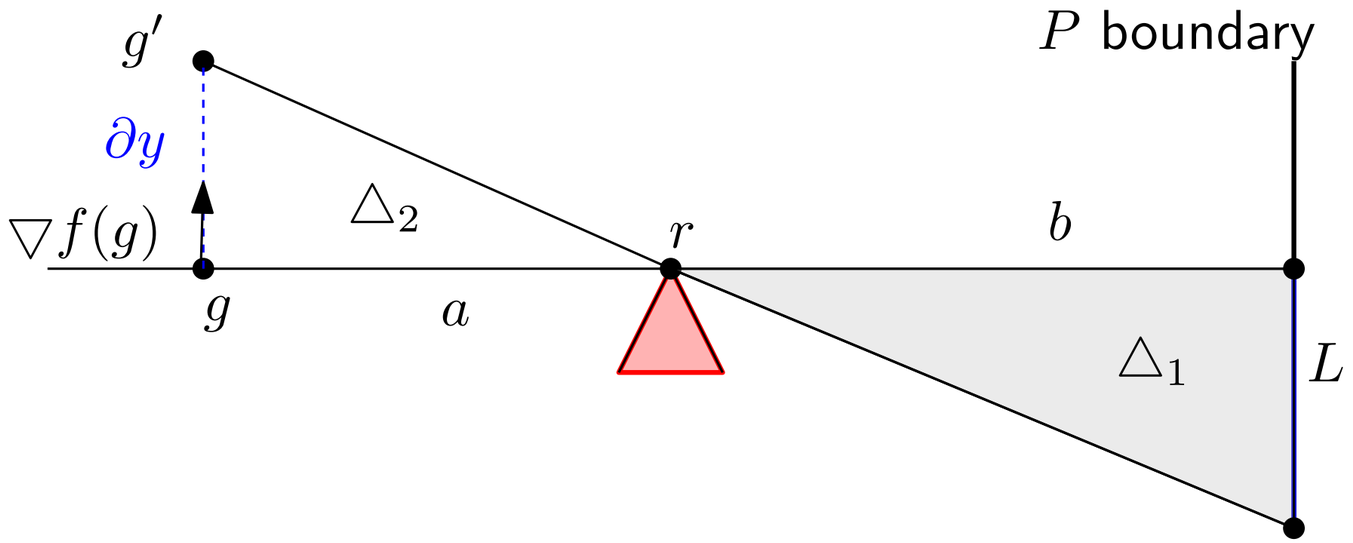
$$\nabla f(g) = \left(0, \frac{b^2}{2a}\right)^{\mathsf{T}}$$

Computing the gradient for one guard



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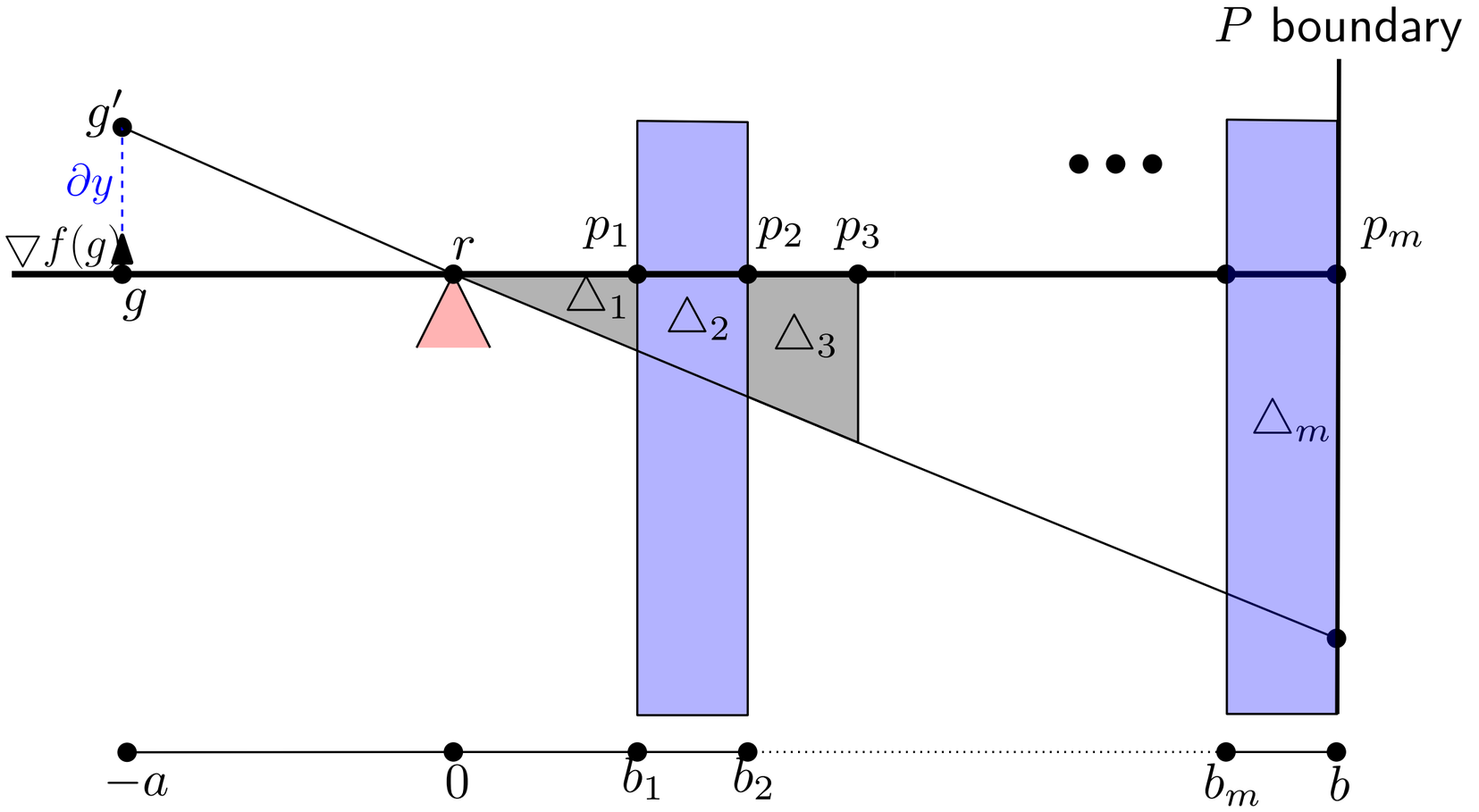
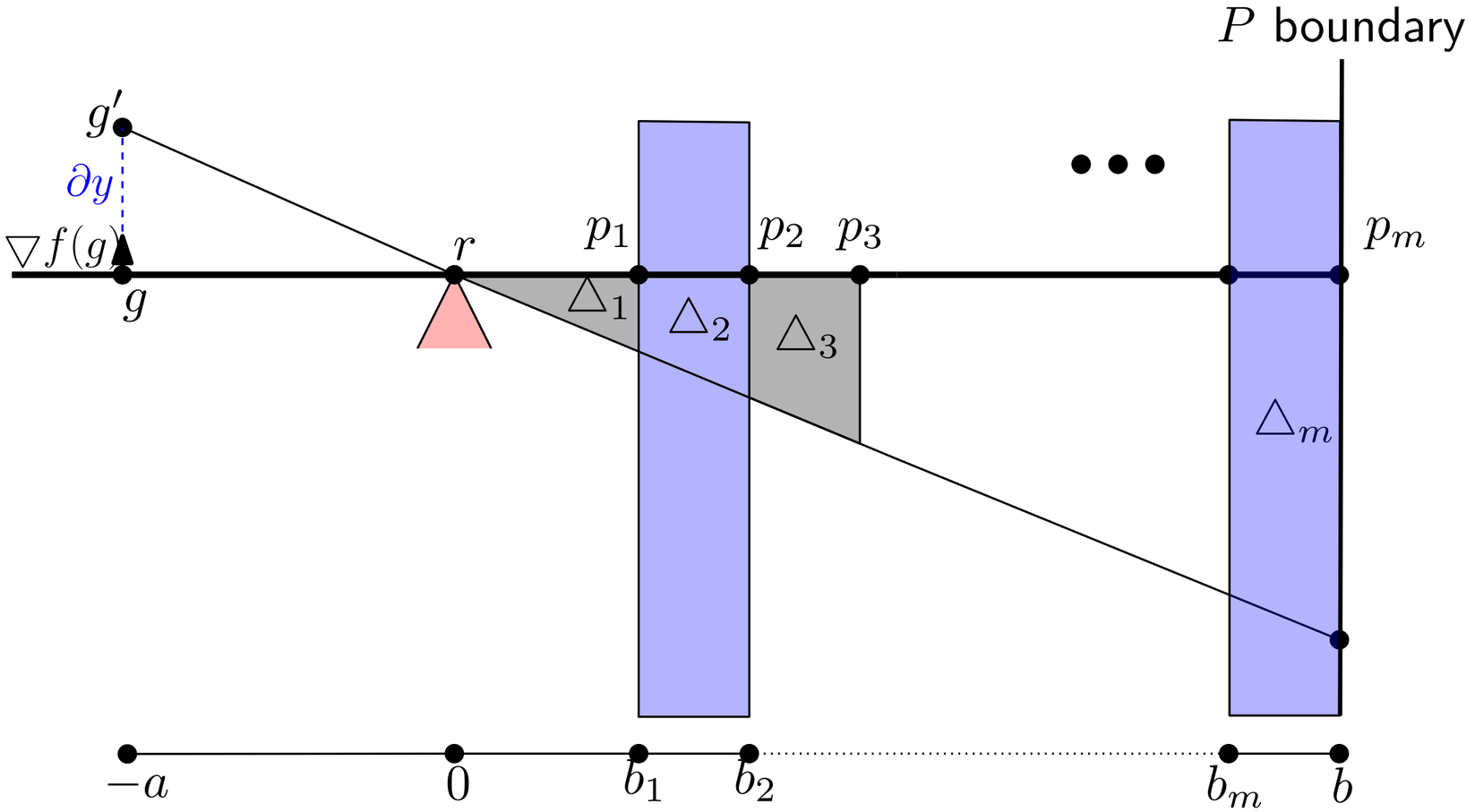
$$g' = g + \alpha \nabla f(g)$$

α - learning rate

Computing the gradient for multiple guards

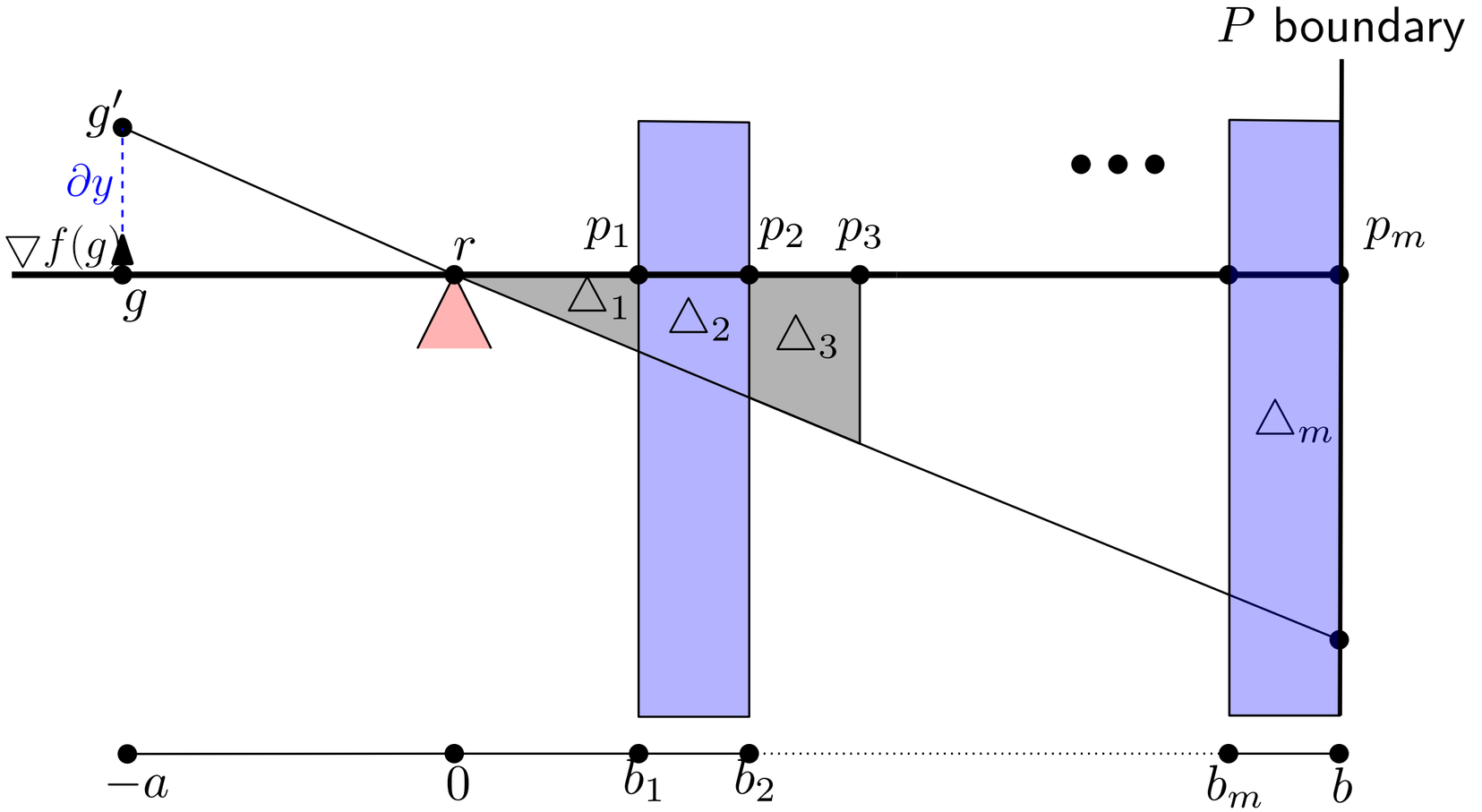


Computing the gradient for multiple guards



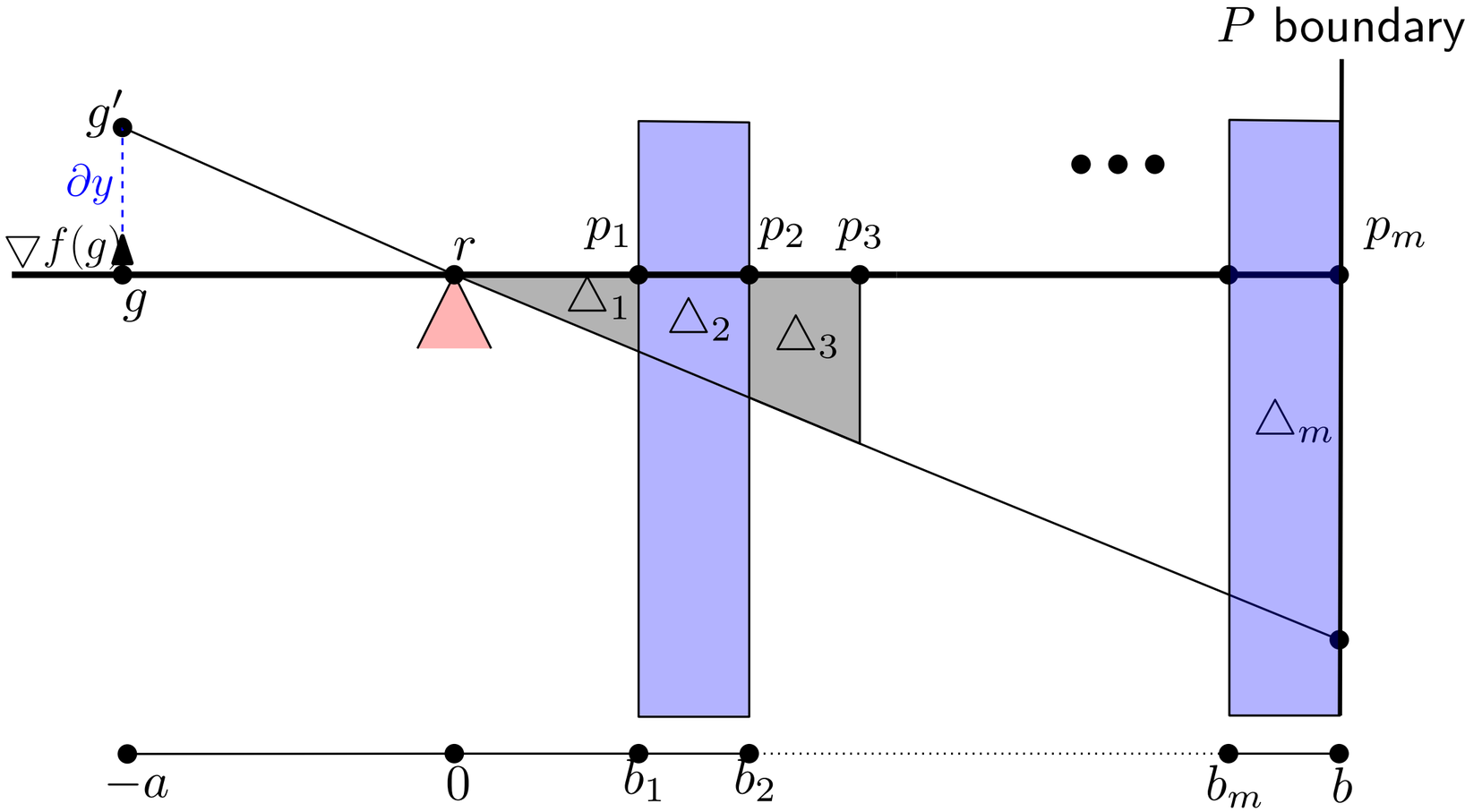
$\text{Area}_{\triangle_1+\triangle_3+\ldots+\triangle_{m-1}}(g) = \text{Area}_{\triangle_1+\ldots+\triangle_m}(g)$

Computing the gradient for multiple guards



$$\text{Area}_{\triangle_1+\triangle_3+\ldots+\triangle_{m-1}}(g) = \text{Area}_{\triangle_1+\ldots+\triangle_m}(g) - \text{Area}_{\triangle_{m-1}}(g) + \text{Area}_{\triangle_{m-2}}(g)$$

Computing the gradient for multiple guards



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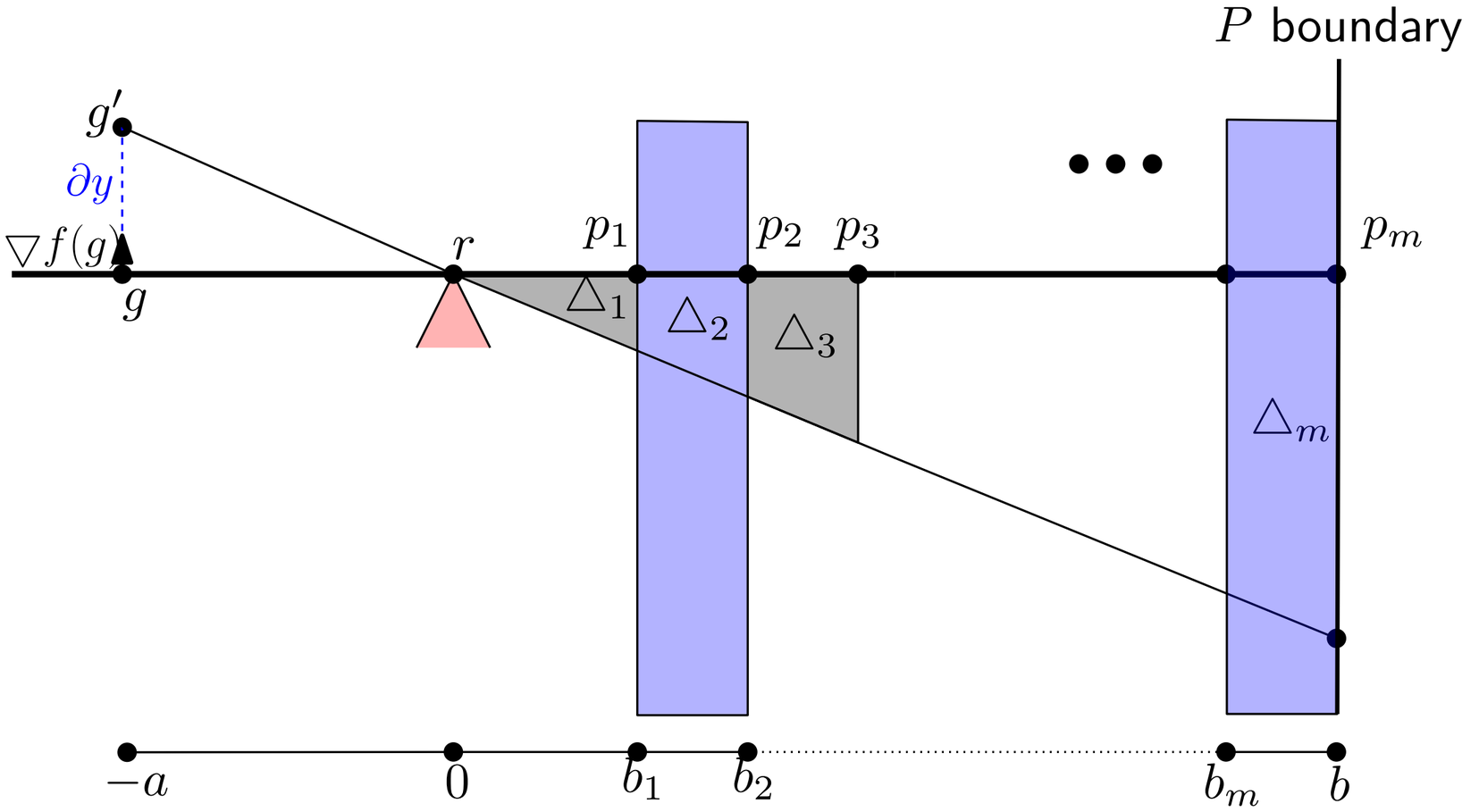
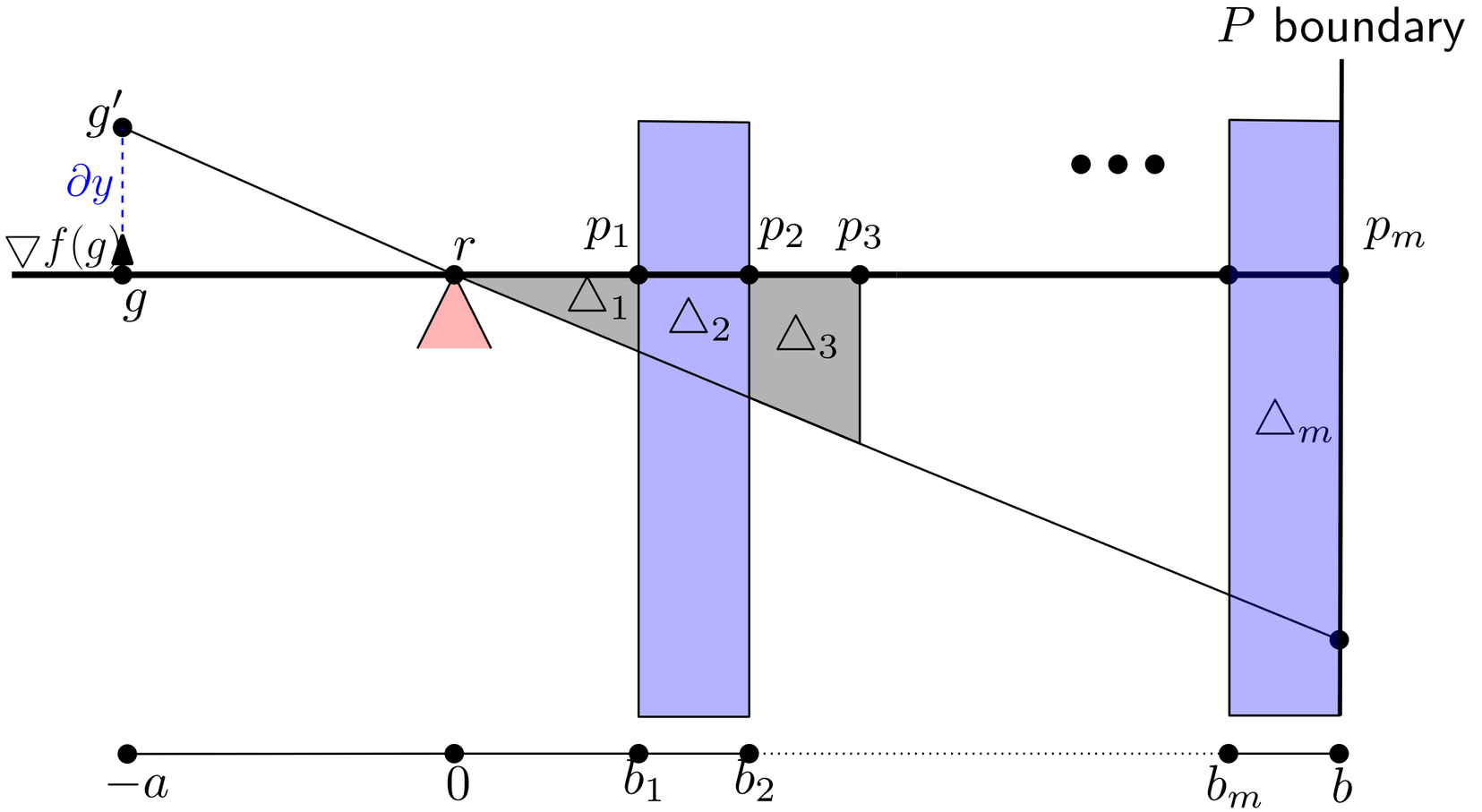
Computing the gradient for multiple guards



$$\text{Area}_{\triangle_1+\triangle_3+\ldots+\triangle_{m-1}}(g) = \text{Area}_{\triangle_1+\ldots+\triangle_m}(g) - \text{Area}_{\triangle_{m-1}}(g) + \text{Area}_{\triangle_{m-2}}(g) - \ldots - \text{Area}_{\triangle_2}(g) + \text{Area}_{\triangle_1}(g)$$

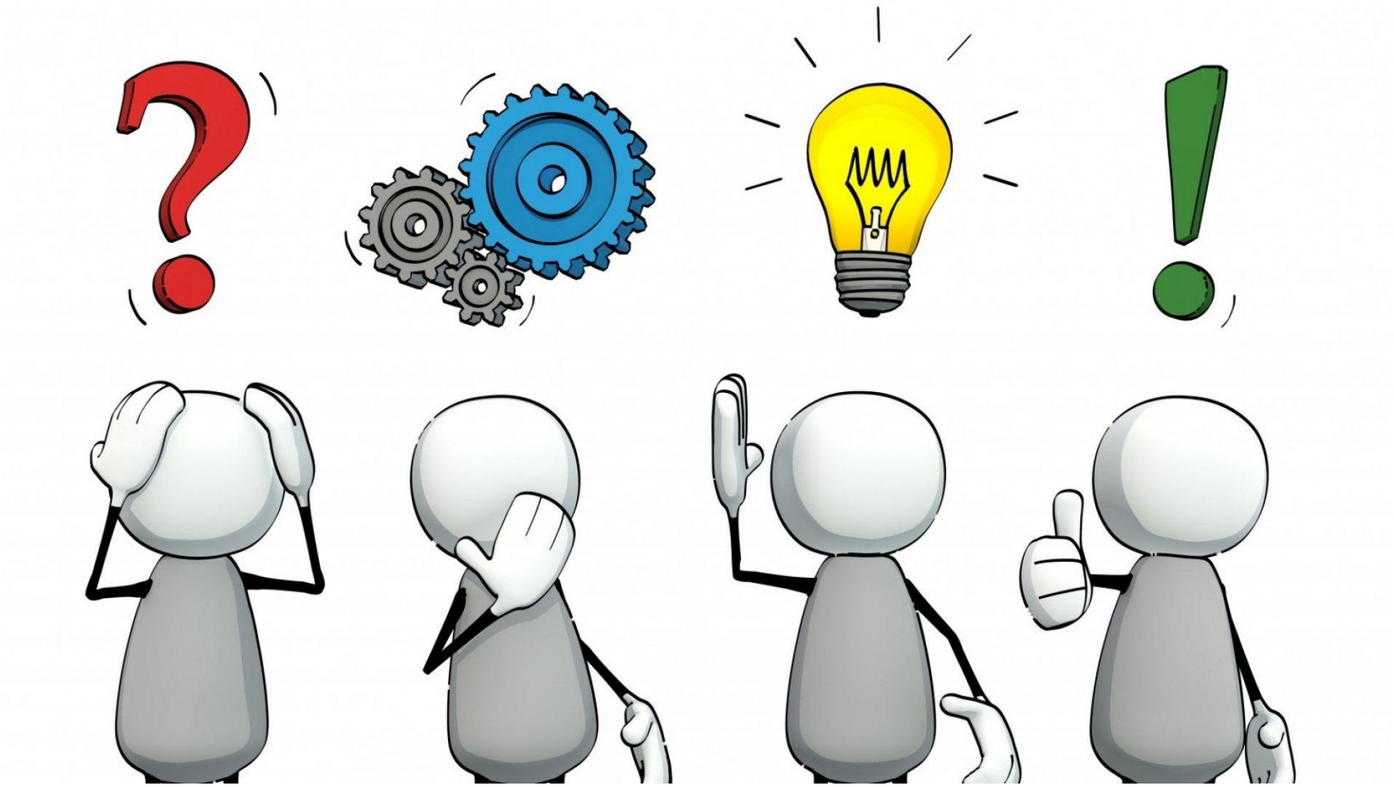
Computing the gradient for multiple guards



$$\text{Area}_{\triangle_1+\triangle_3+\ldots+\triangle_{m-1}}(g) = \text{Area}_{\triangle_1+\ldots+\triangle_m}(g) - \text{Area}_{\triangle_{m-1}}(g) + \text{Area}_{\triangle_{m-2}}(g) - \ldots - \text{Area}_{\triangle_2}(g) + \text{Area}_{\triangle_1}(g)$$

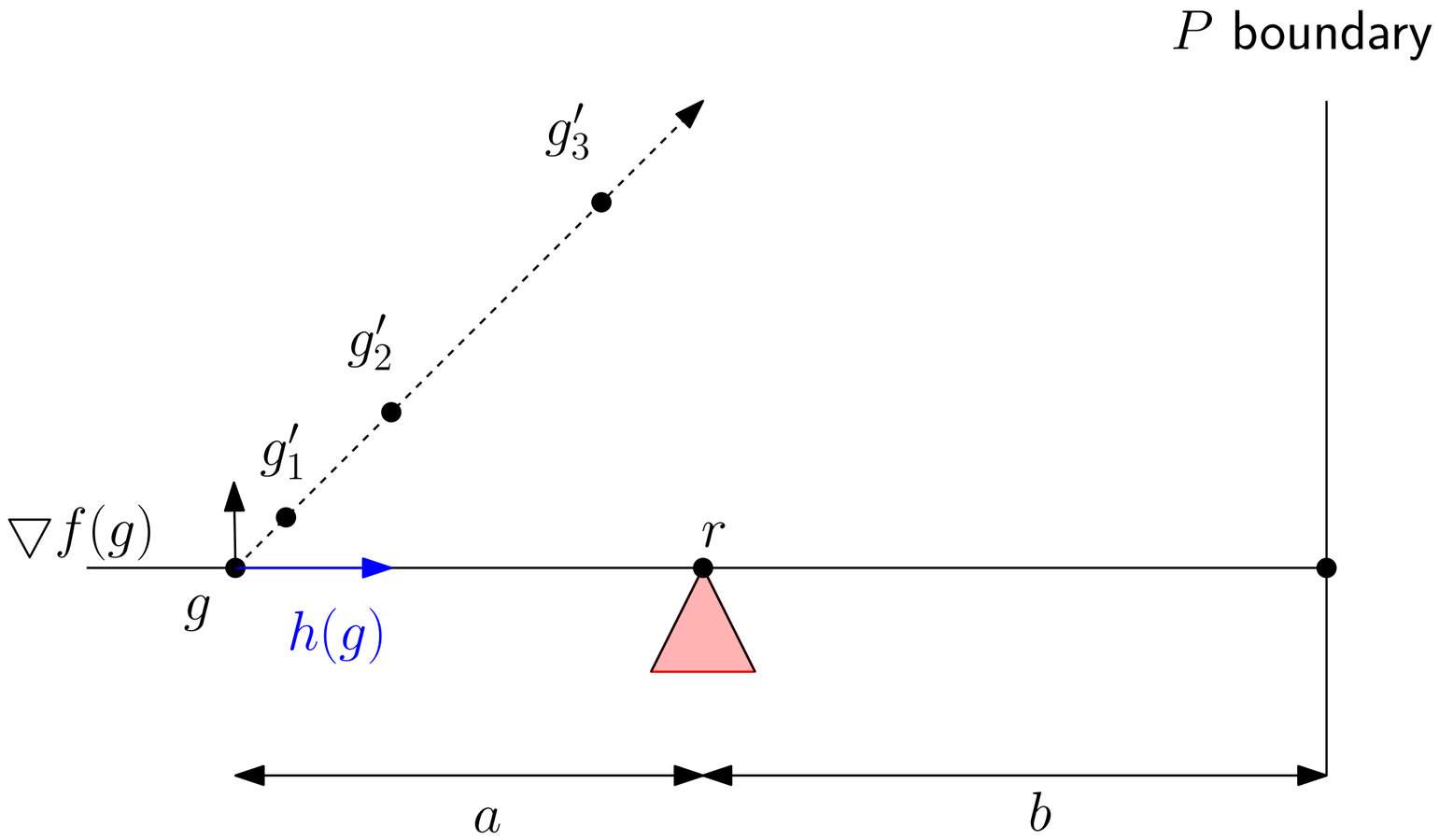
$$= \left(b^2 - b_m^2 + b_{(m-1)}^2 - \ldots - b_2^2 + b_1^2\right) \frac{\partial y}{2a}$$

Heuristics

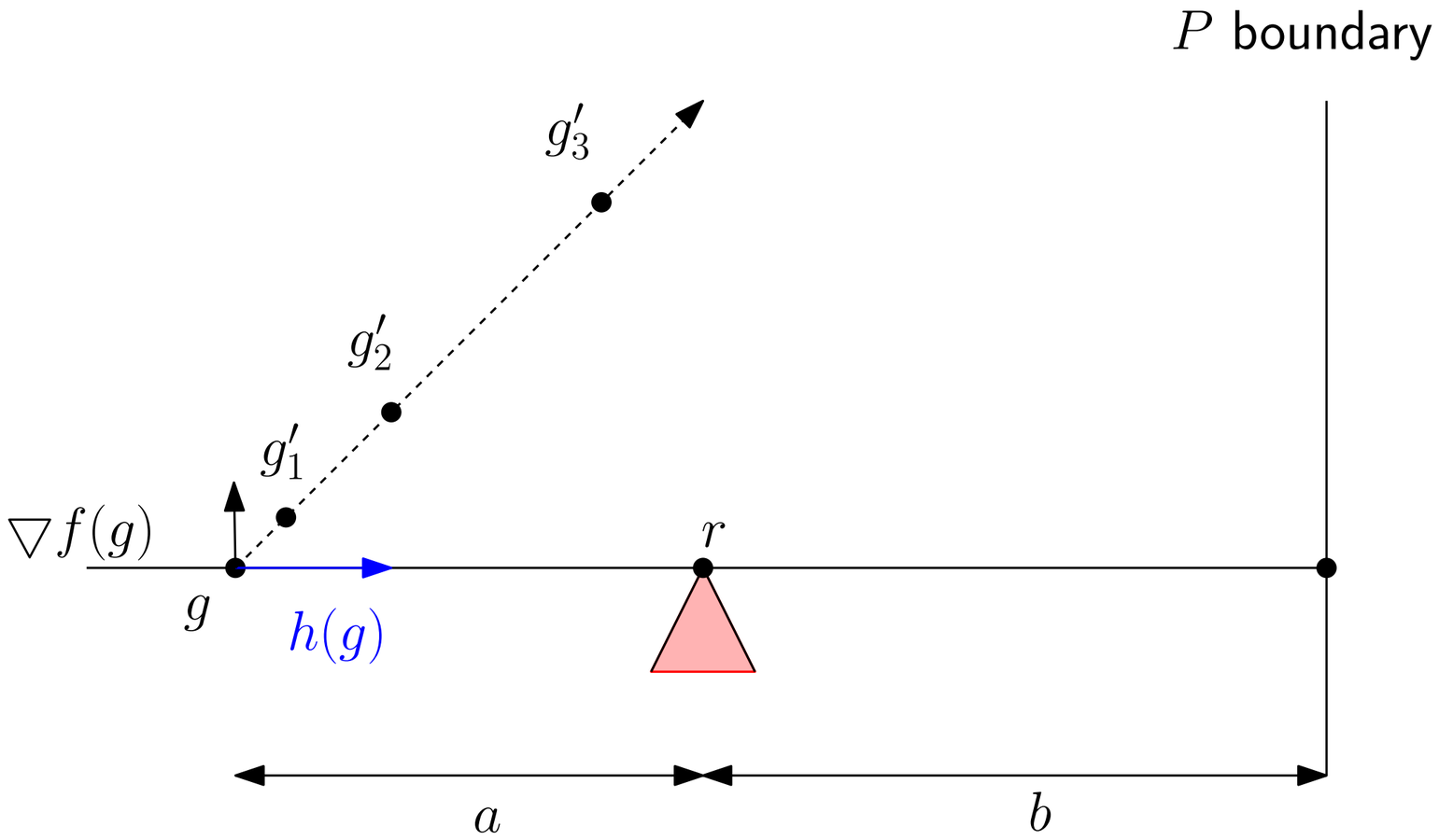


Heuristics: Momentum

Heuristics: Line Search



Heuristics: Line Search



Heuristics: Pull towards reflex vertex

Heuristics: Pull onto reflex vertex

Heuristics: Pull capping

Heuristics: Reflex area

Heuristics: Angle behind reflex vertex

Heuristics: Hidden movement

Heuristics: Greedy initialisation

Hyperparameters

Scalability

Problems encountered

Future work

improve the algorithm's robustness, performance and scalability

implement other heuristics

test the algorithm on larger polygons with more guards

solve existing bugs

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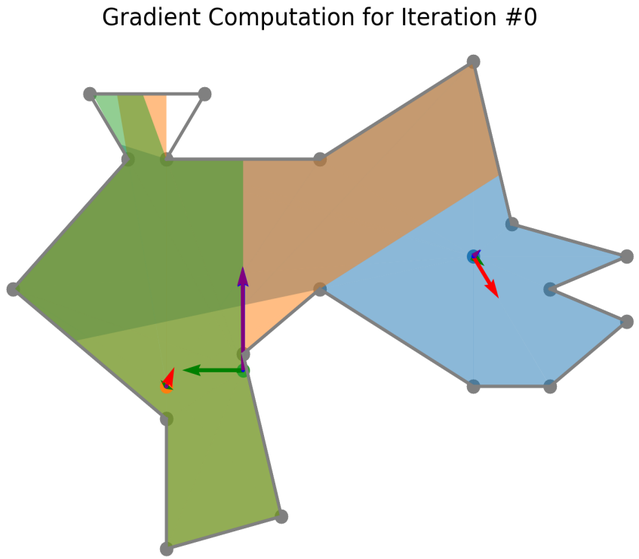
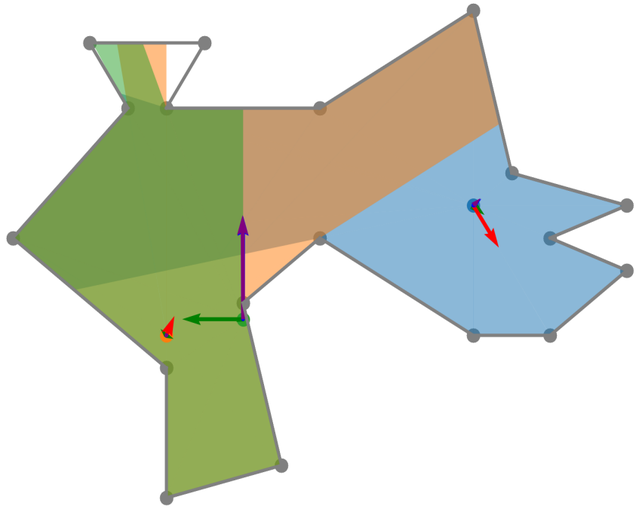
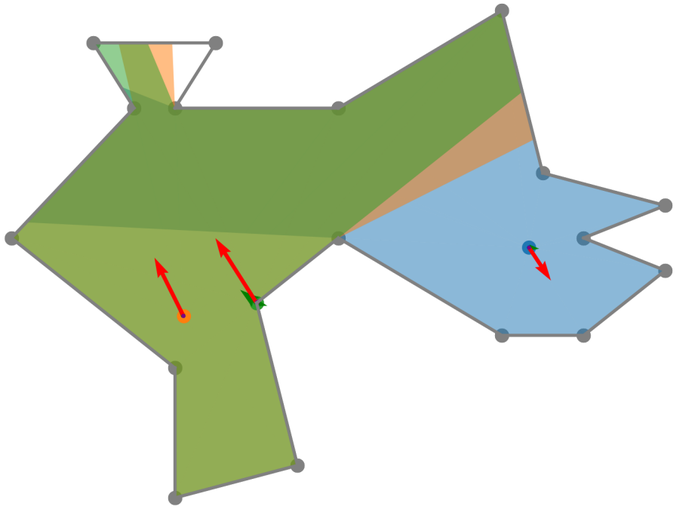


The Art Gallery Problem

Gradient Computation for Iteration #0

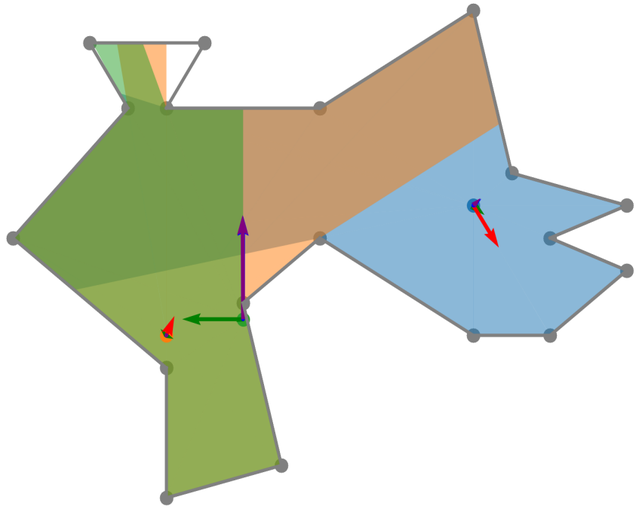


Gradient Computation for Iteration #1

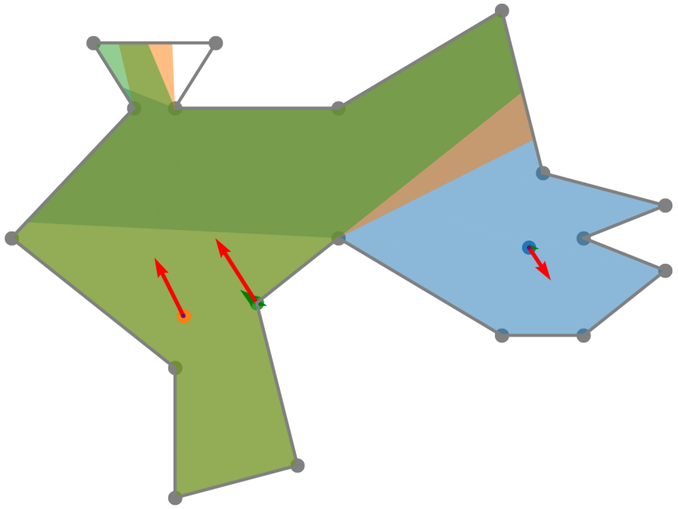


The Art Gallery Problem

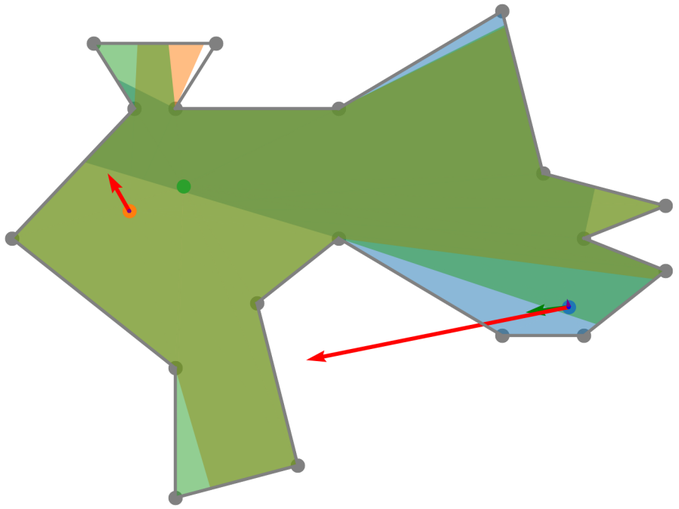
Gradient Computation for Iteration #0



Gradient Computation for Iteration #1



Gradient Computation for Iteration #3



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