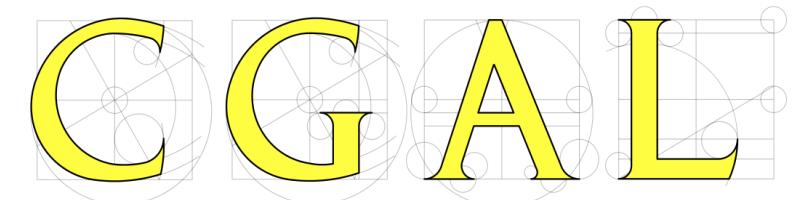
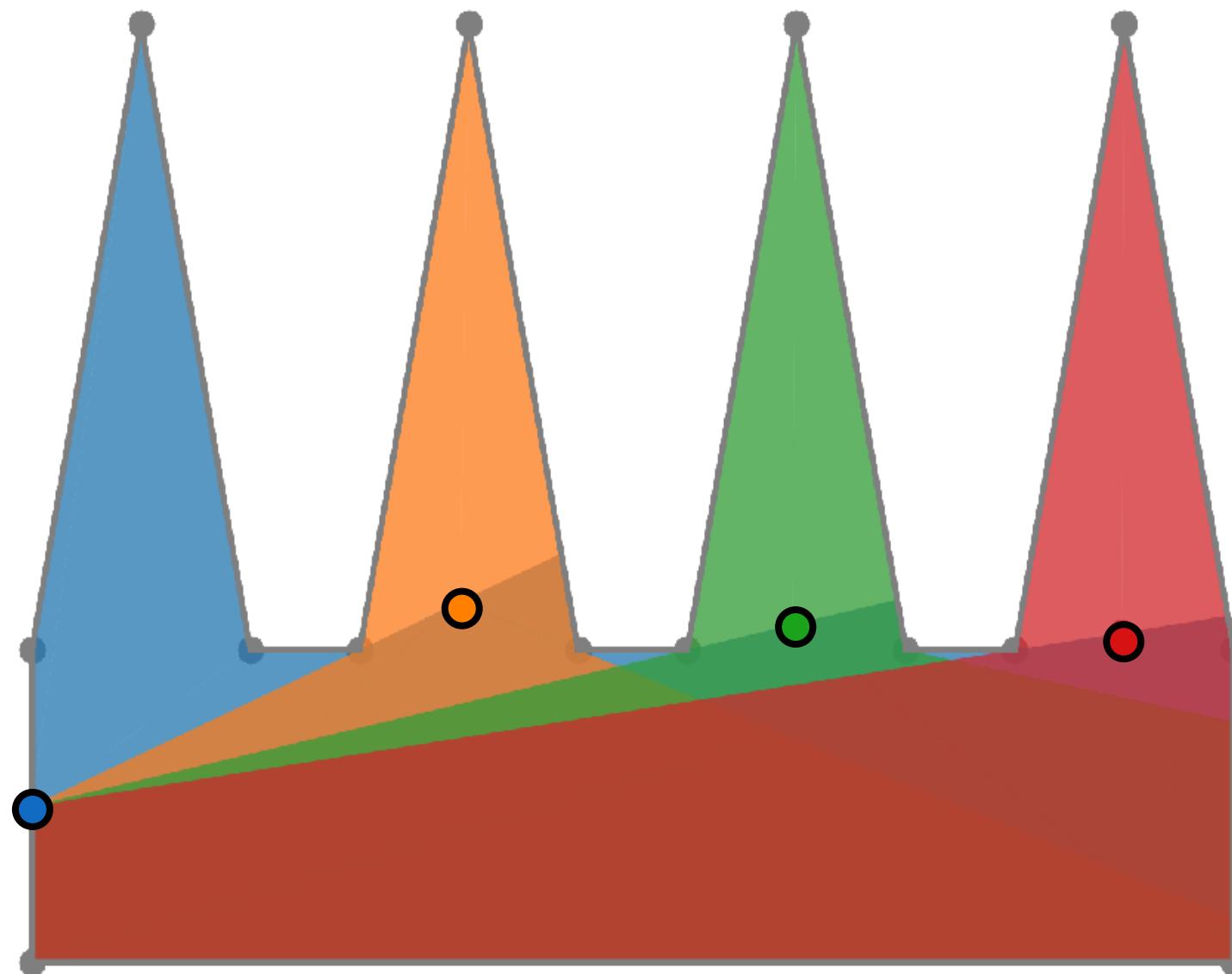


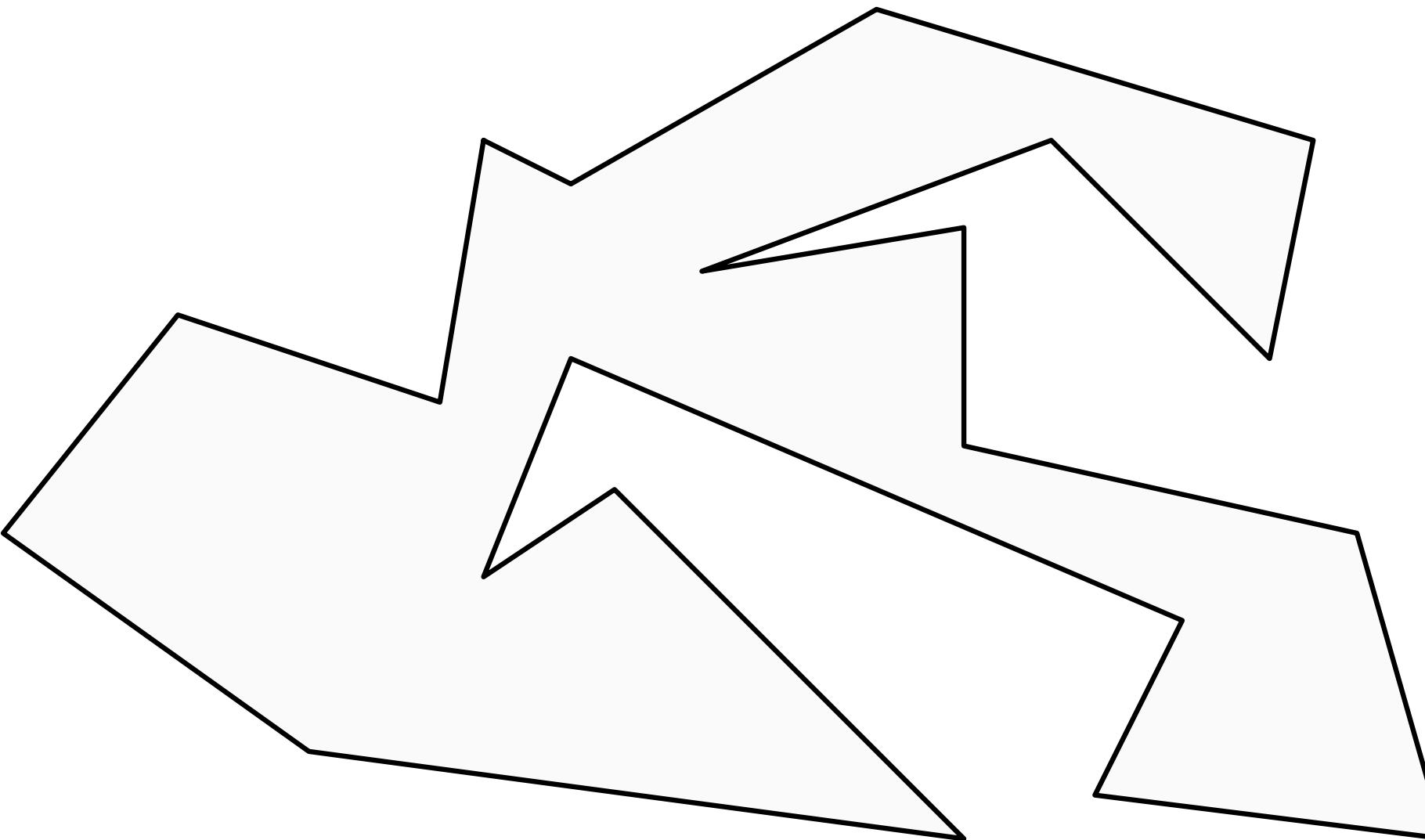
Solving the Art Gallery Problem Using Gradient Descent

Master's Thesis Defense

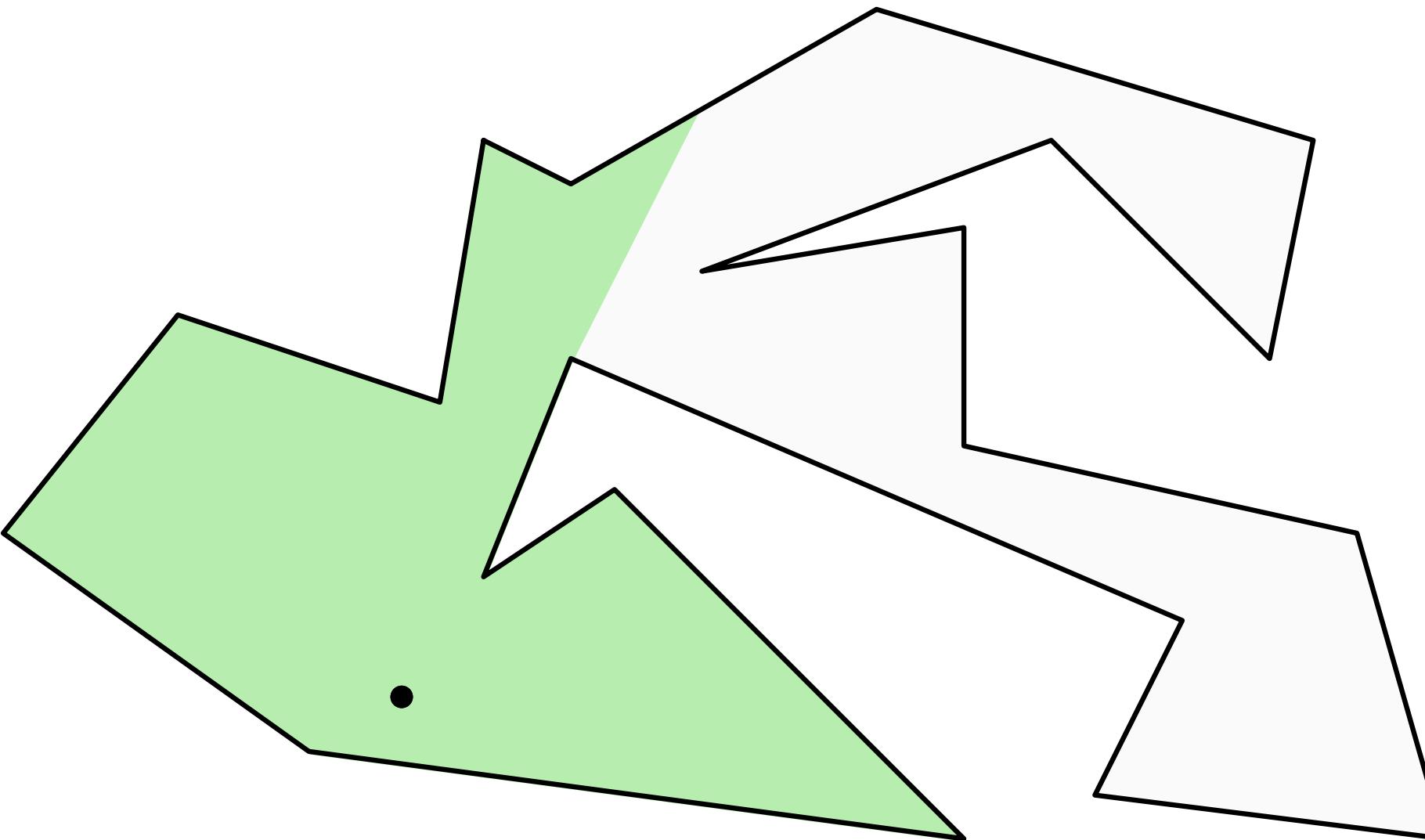
Geo Juglan



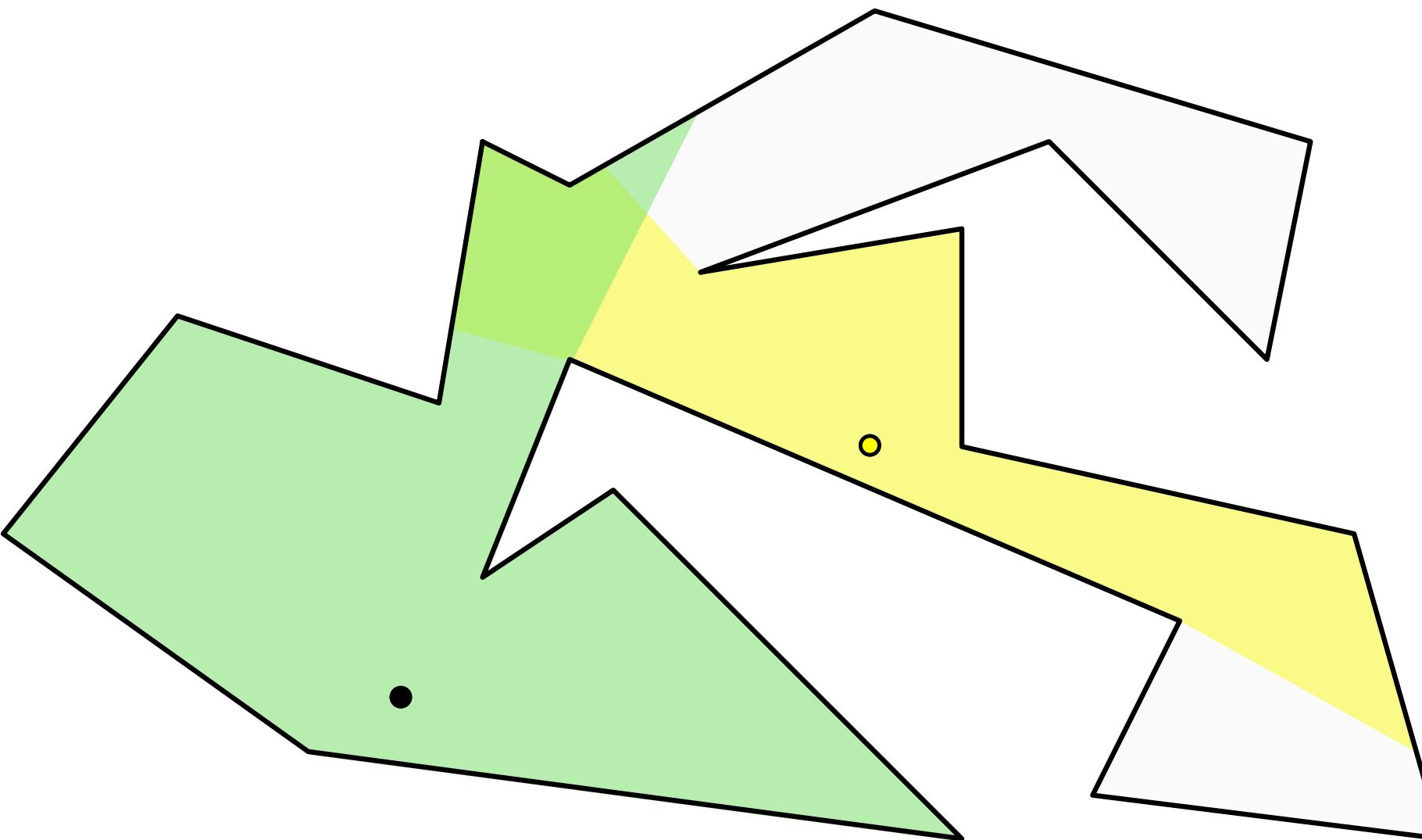
The Art Gallery Problem



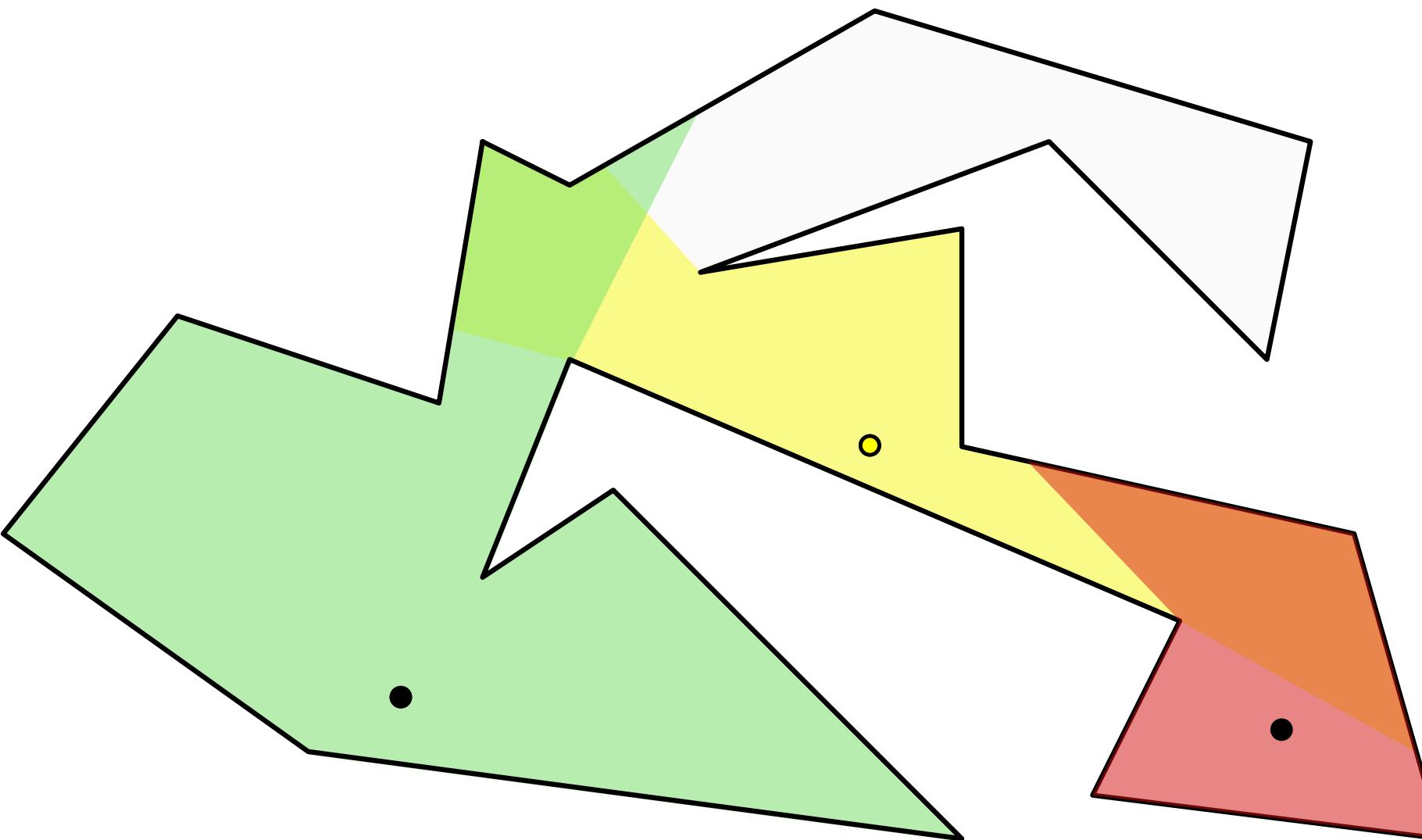
The Art Gallery Problem



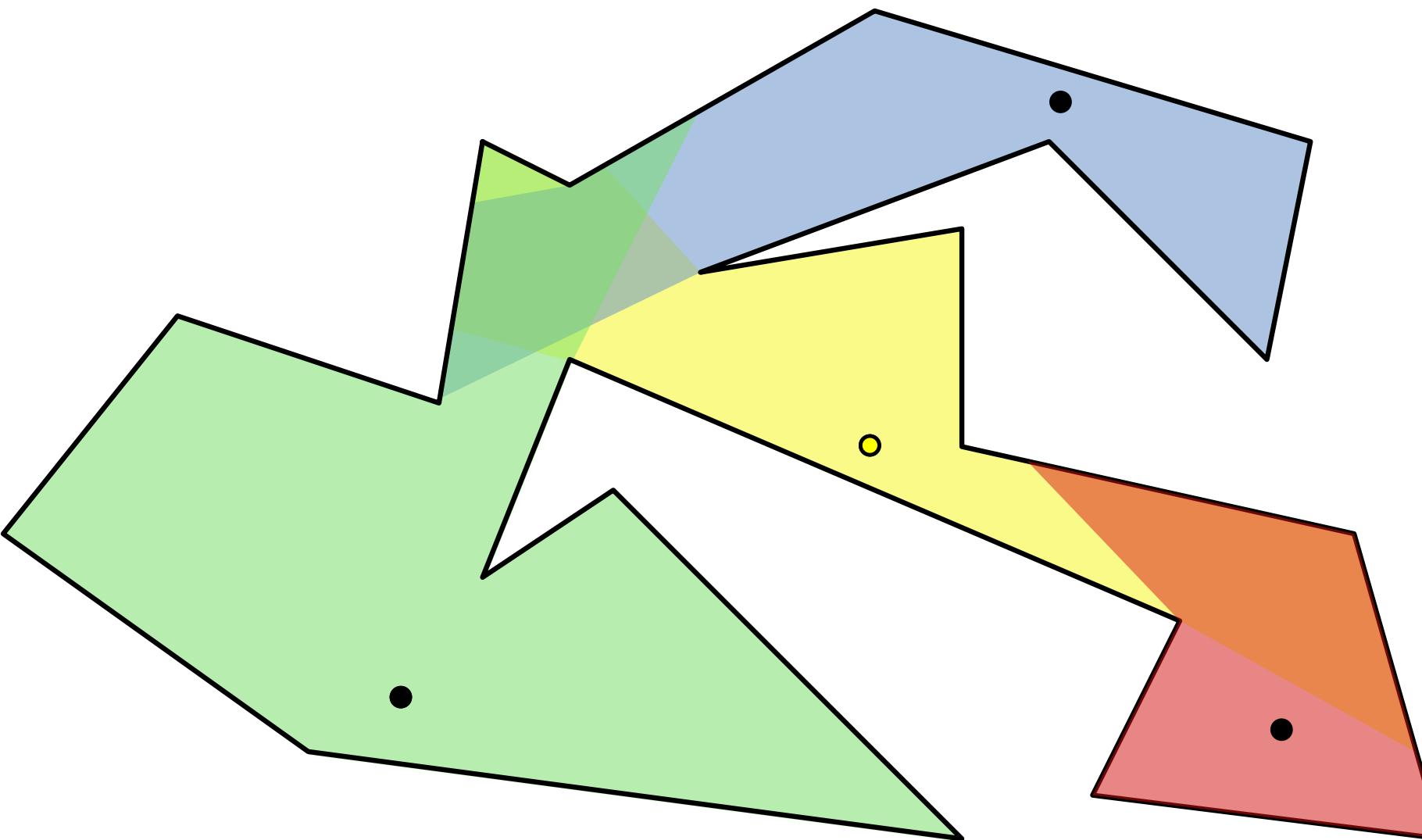
The Art Gallery Problem



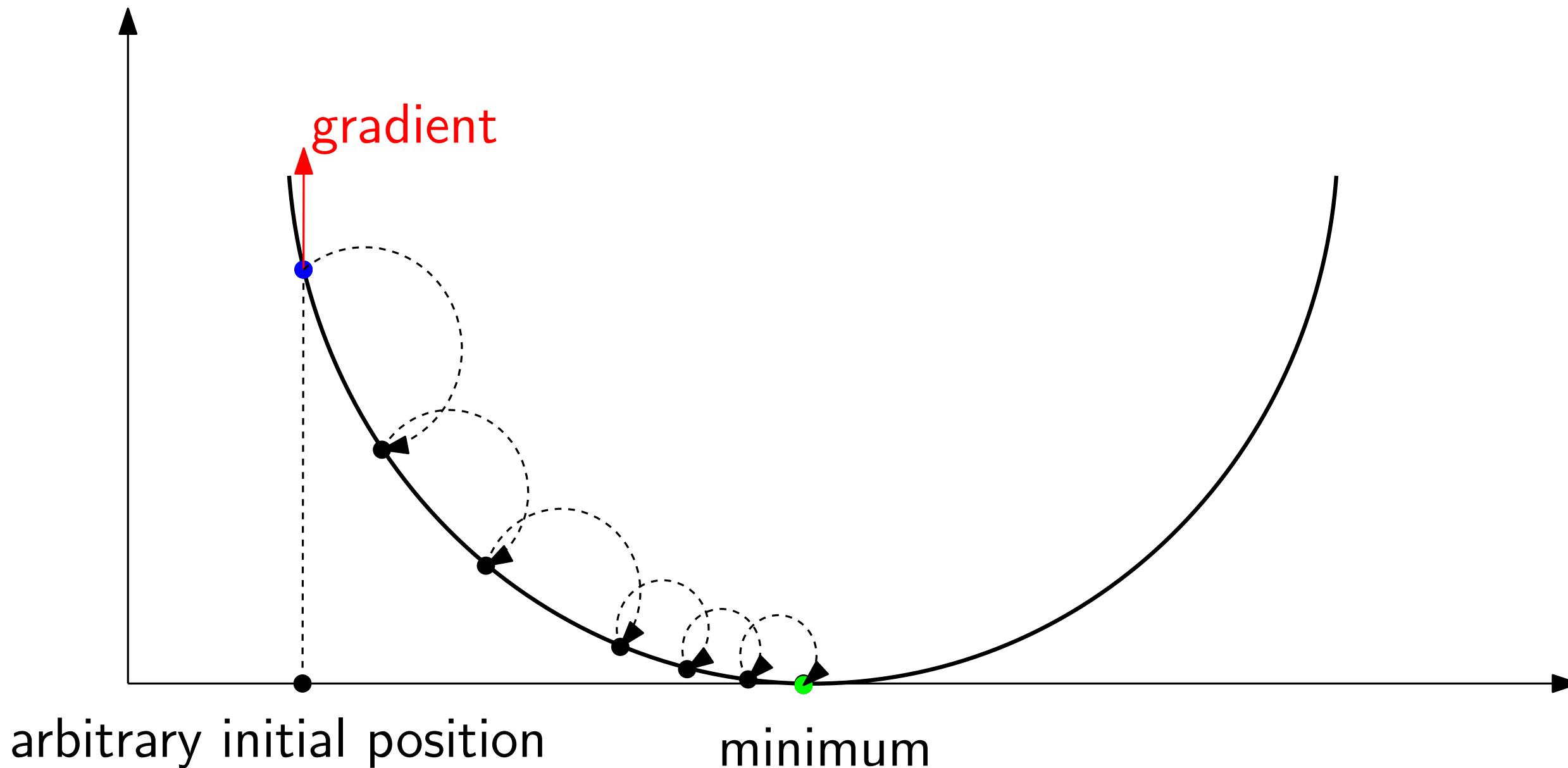
The Art Gallery Problem



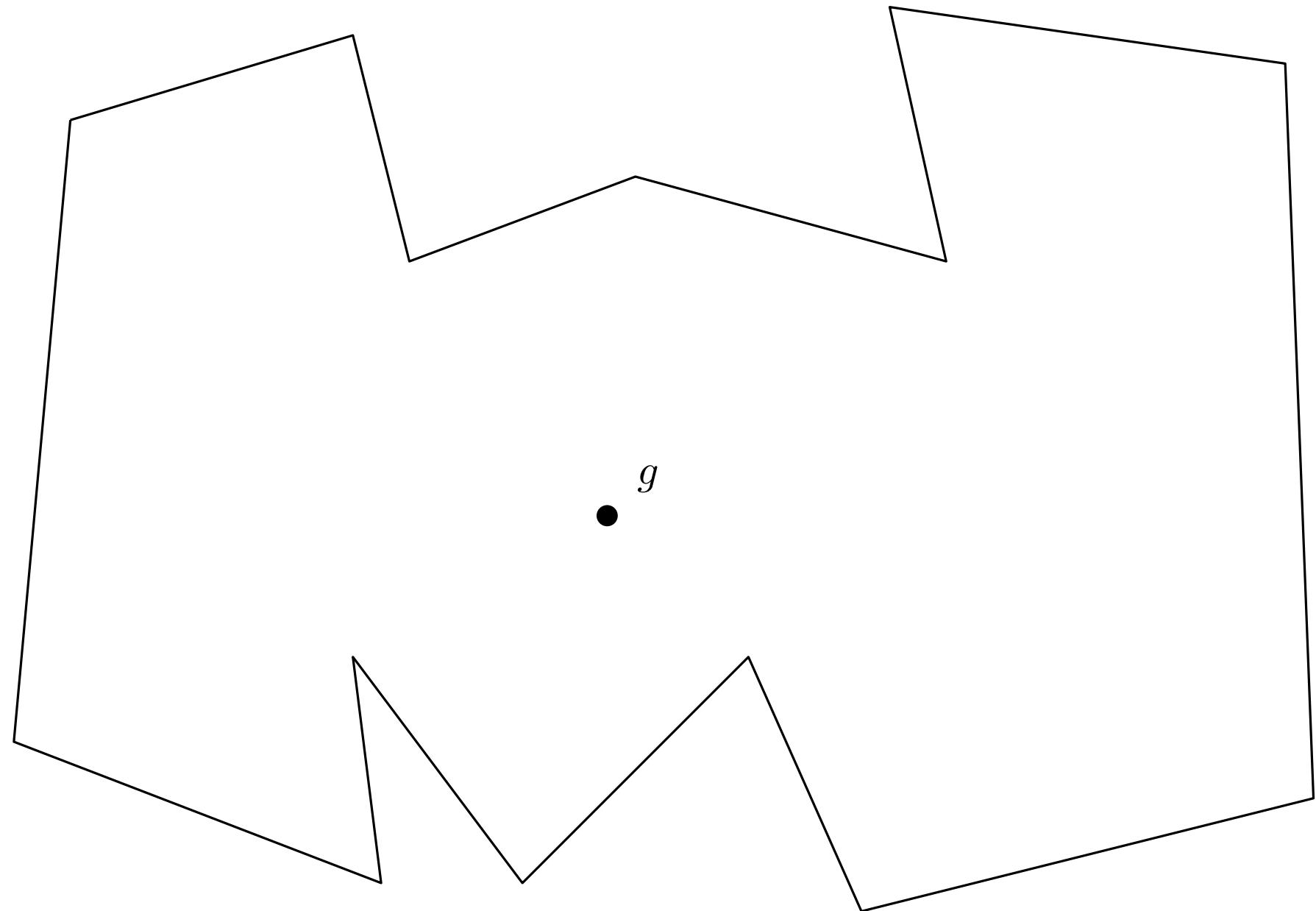
The Art Gallery Problem



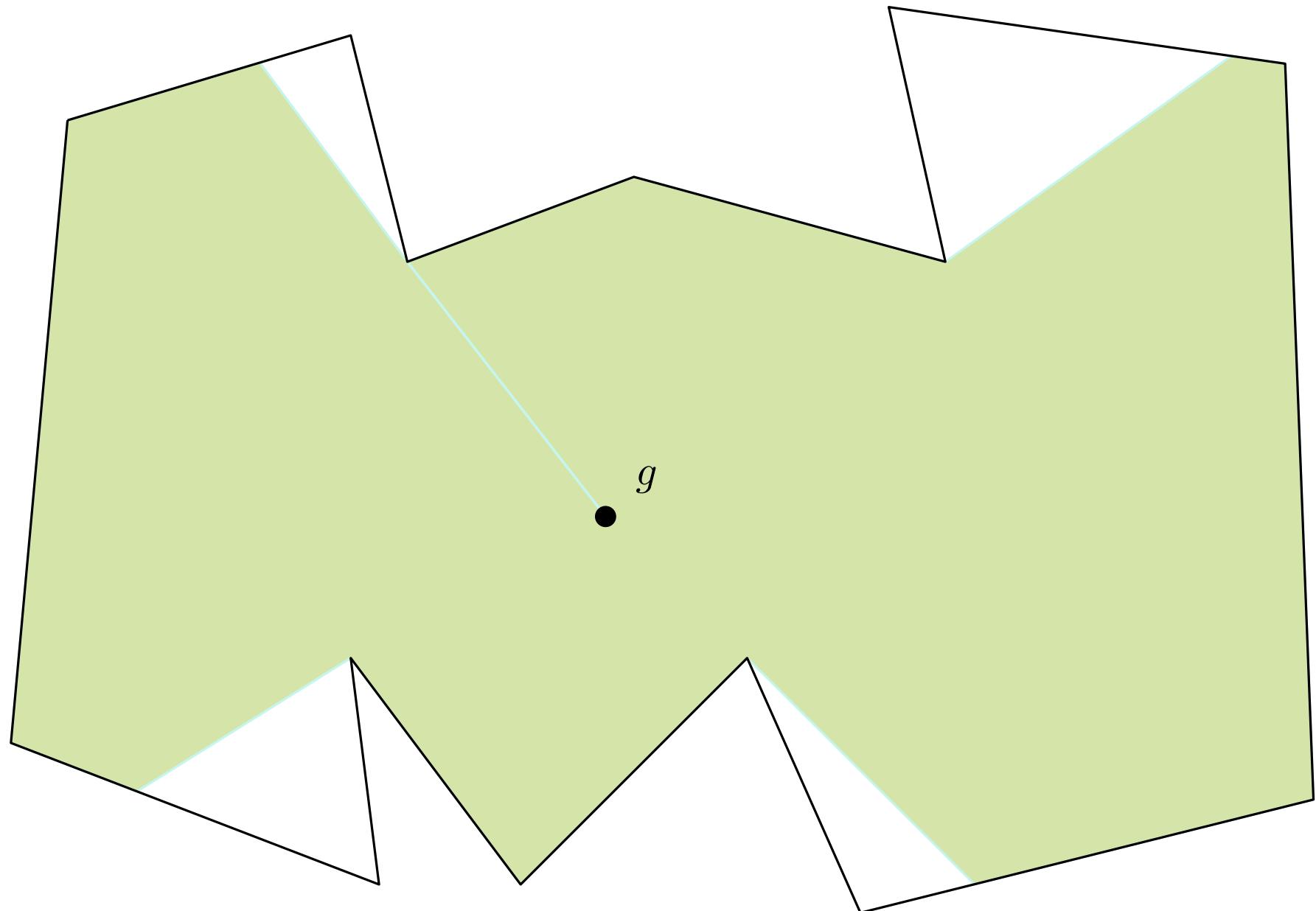
Gradient Descent



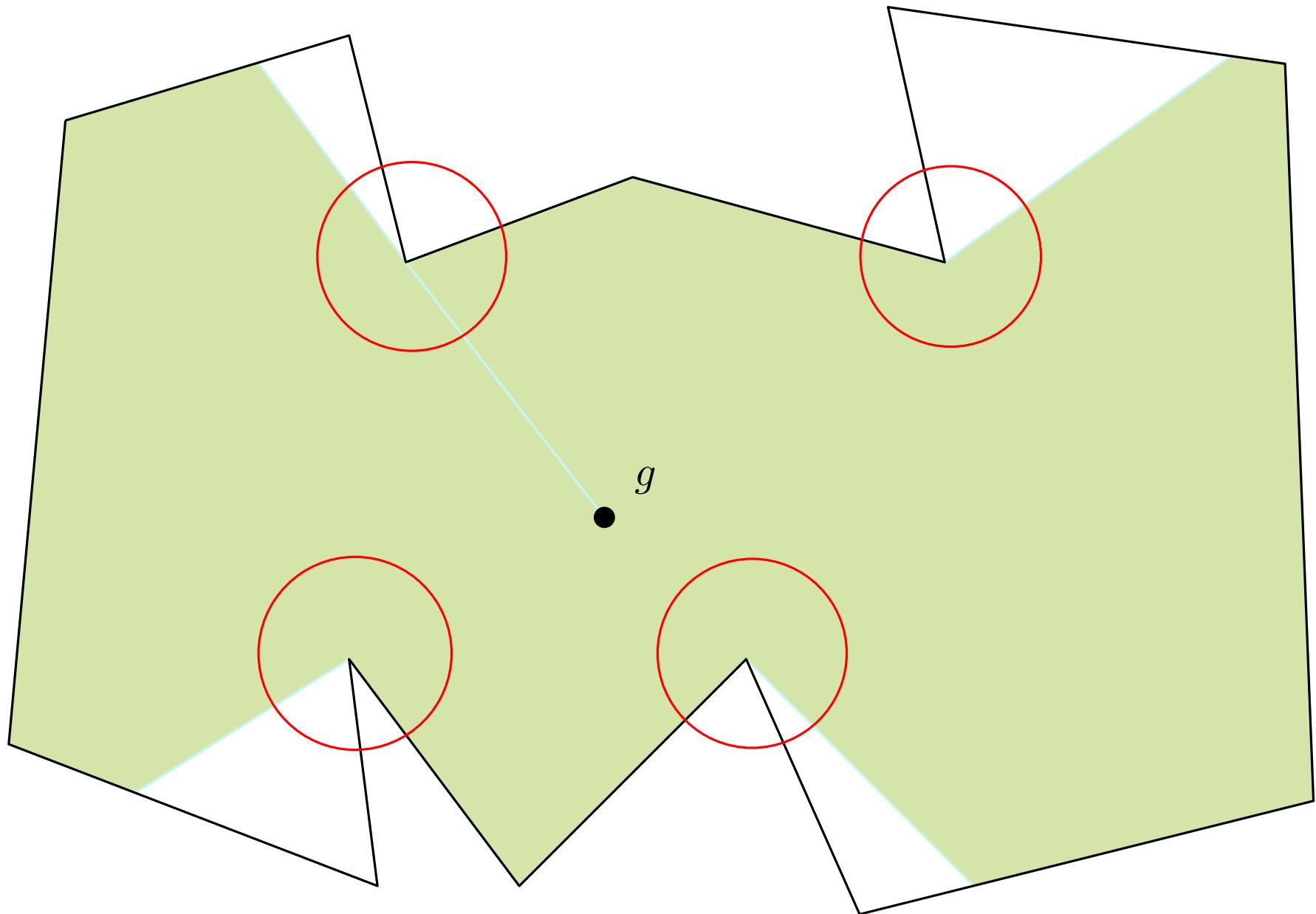
Computing the gradient



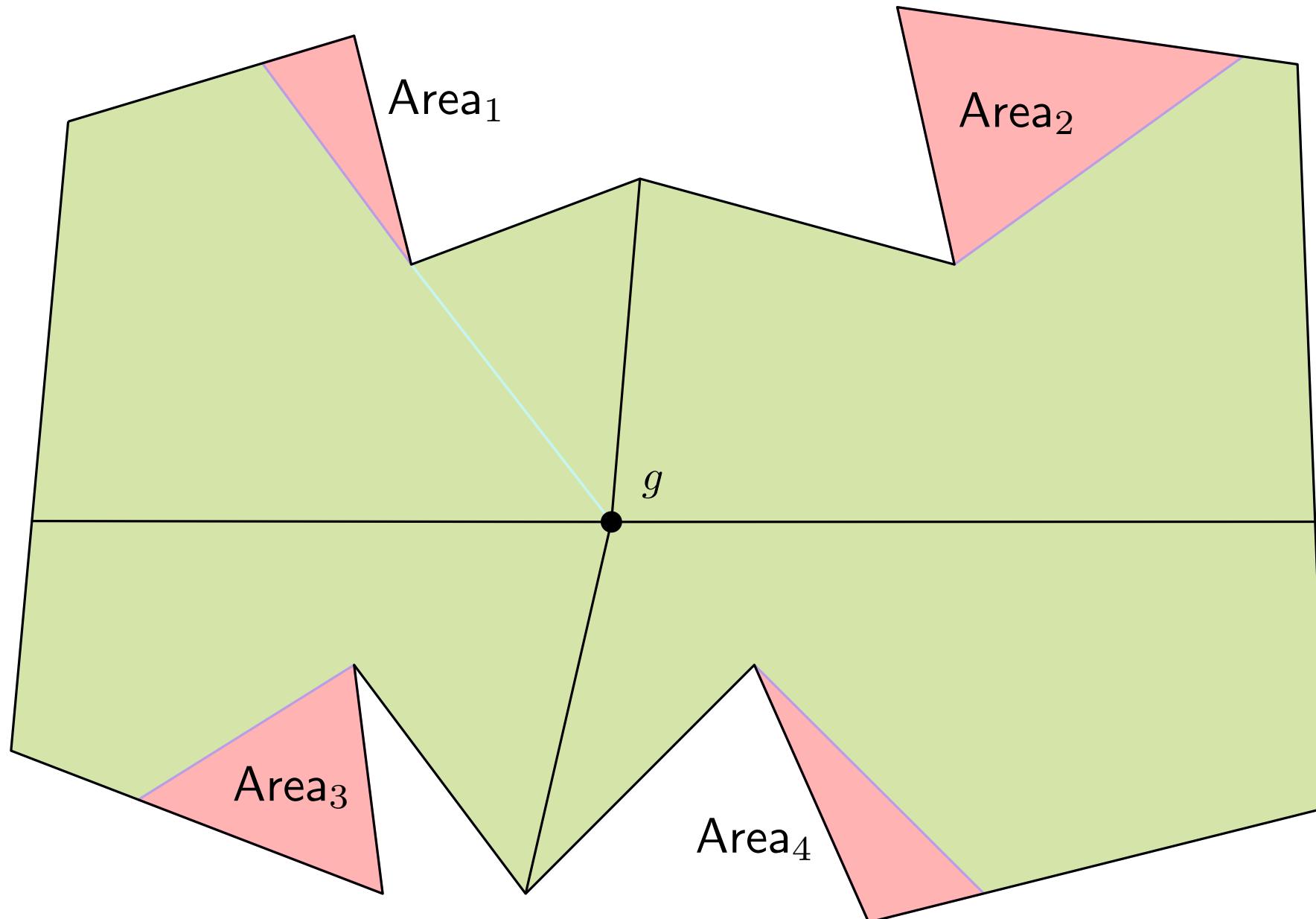
Computing the gradient



Computing the gradient

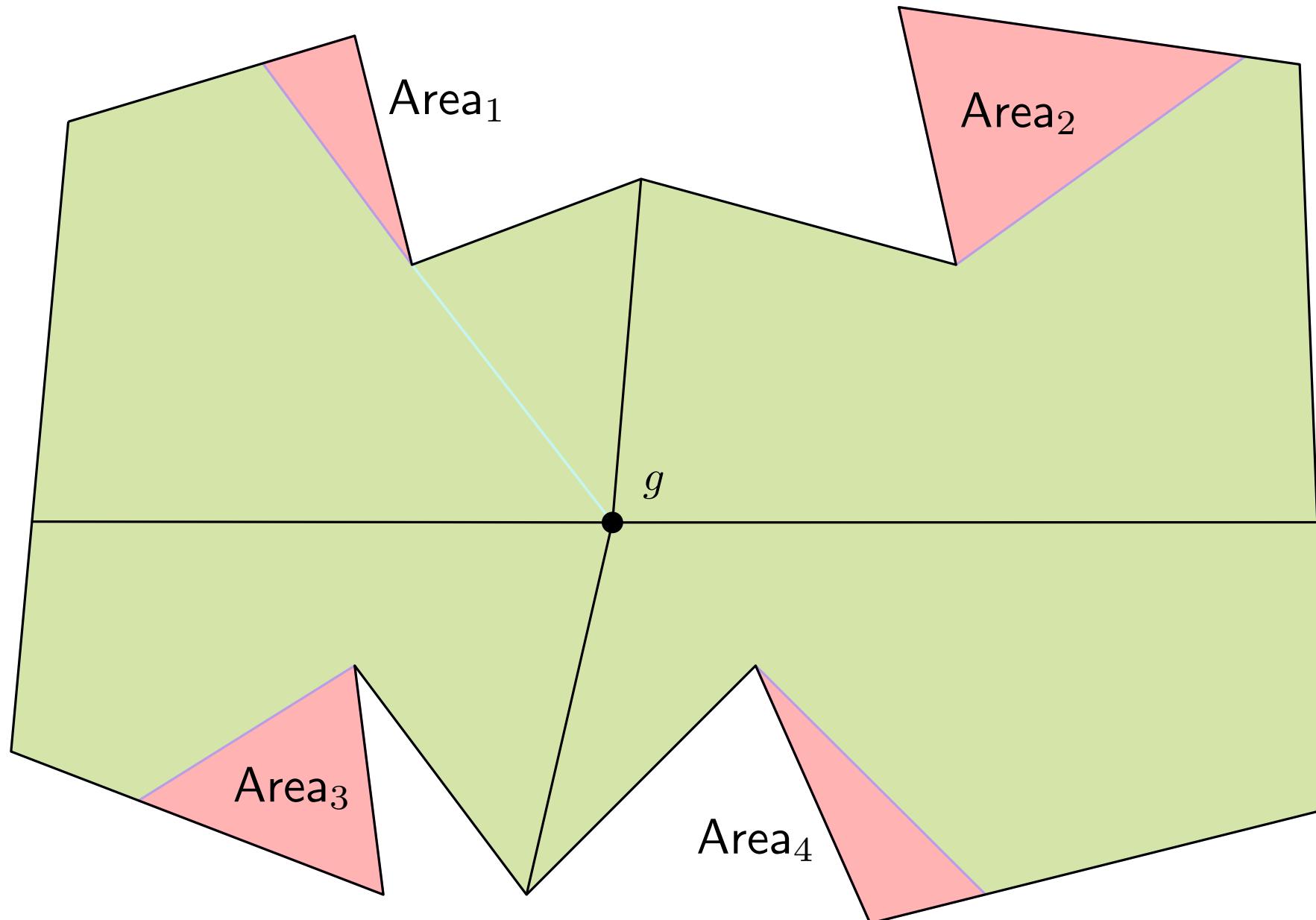


Computing the gradient



$$f(g) = \text{Area}_1 + \text{Area}_2 + \text{Area}_3 + \text{Area}_4$$

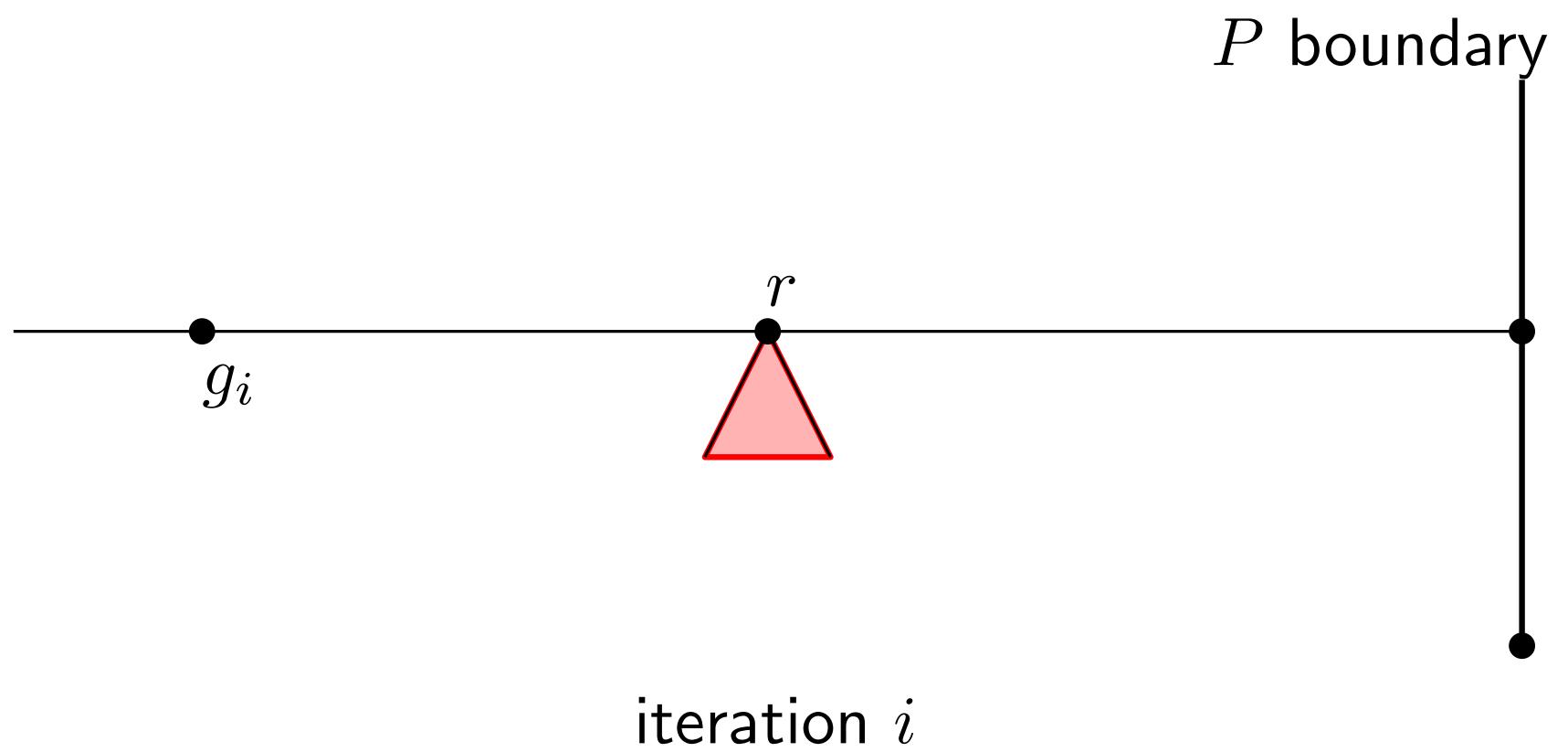
Computing the gradient



$$f(g) = \text{Area}_1 + \text{Area}_2 + \text{Area}_3 + \text{Area}_4$$

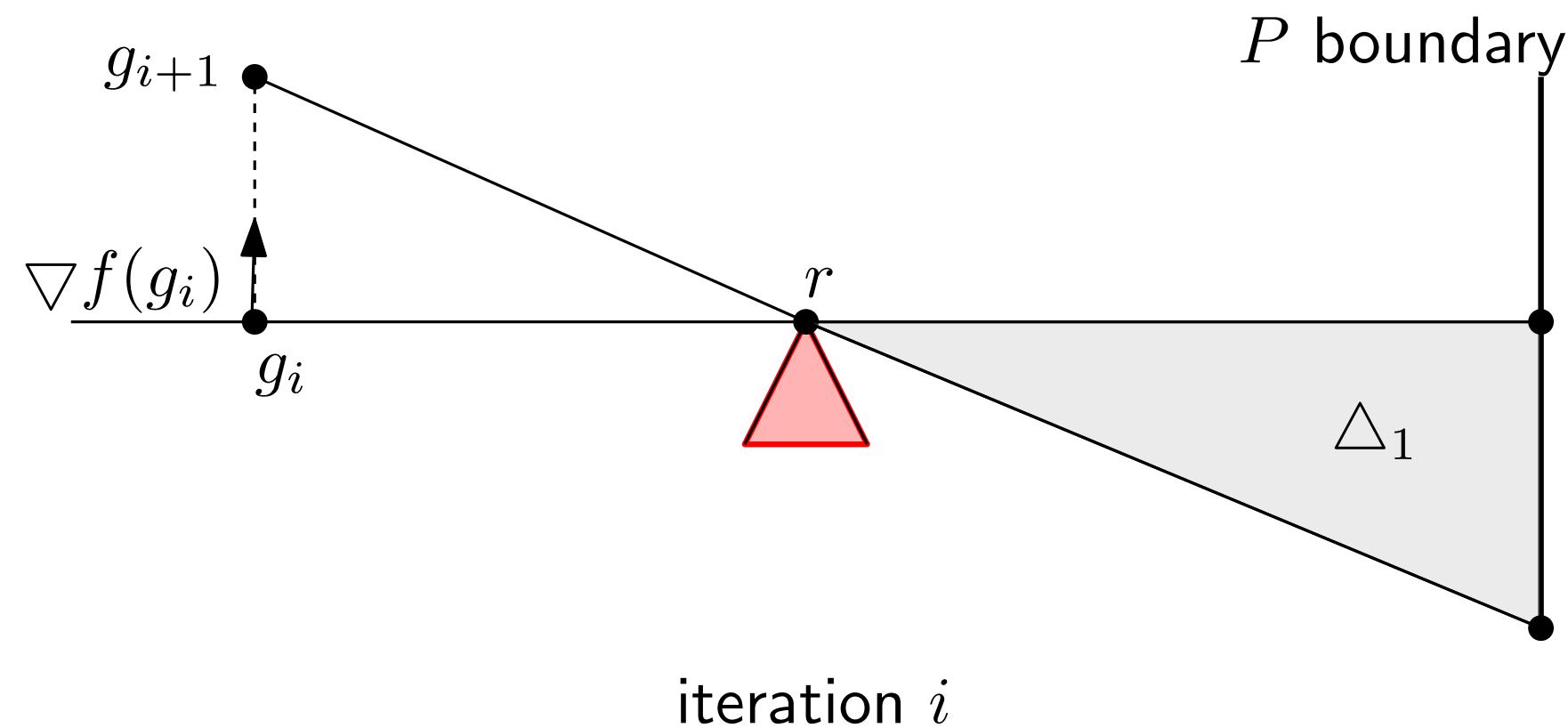
$$\nabla f(g) = \nabla \sum_i \text{Area}_i = \sum_i \nabla \text{Area}_i$$

Computing the gradient



Computing the gradient

$$\nabla f(g_i) = \nabla \text{Area}_{\triangle_1}(g_i)$$

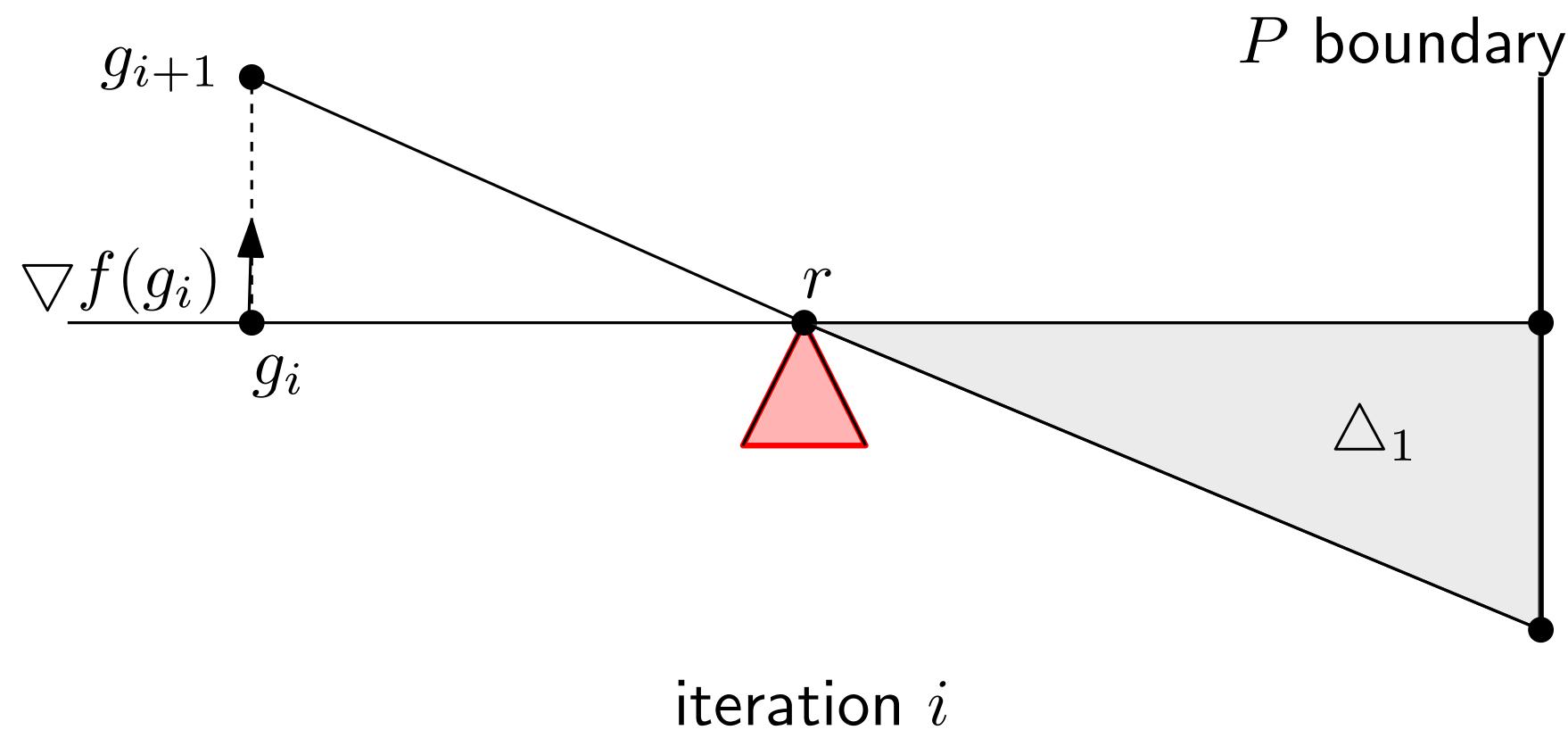


Computing the gradient

$$\nabla f(g_i) = \nabla \text{Area}_{\triangle_1}(g_i)$$

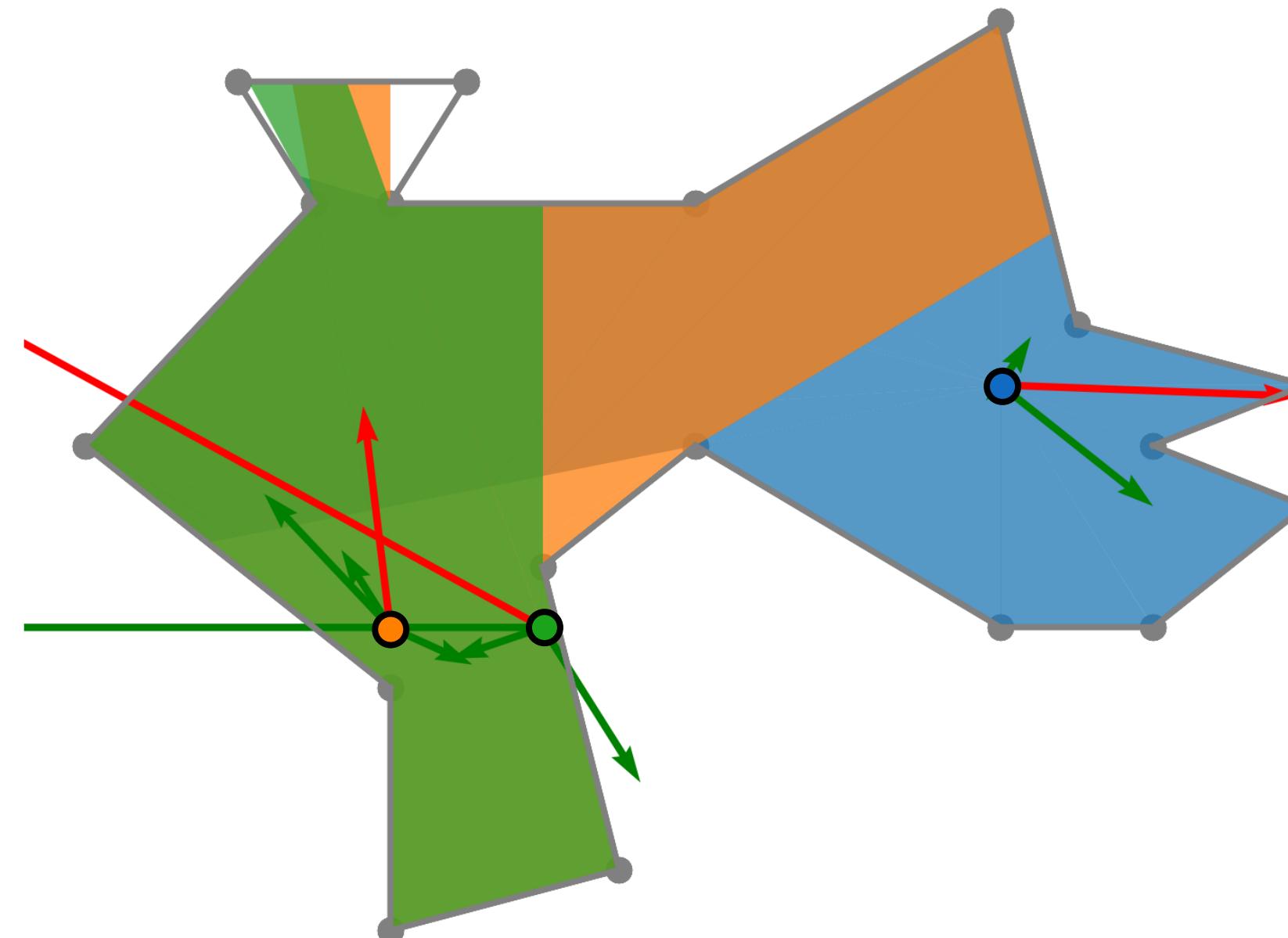
$$g_{i+1} = g_i + \alpha \nabla f(g_i)$$

α - learning rate



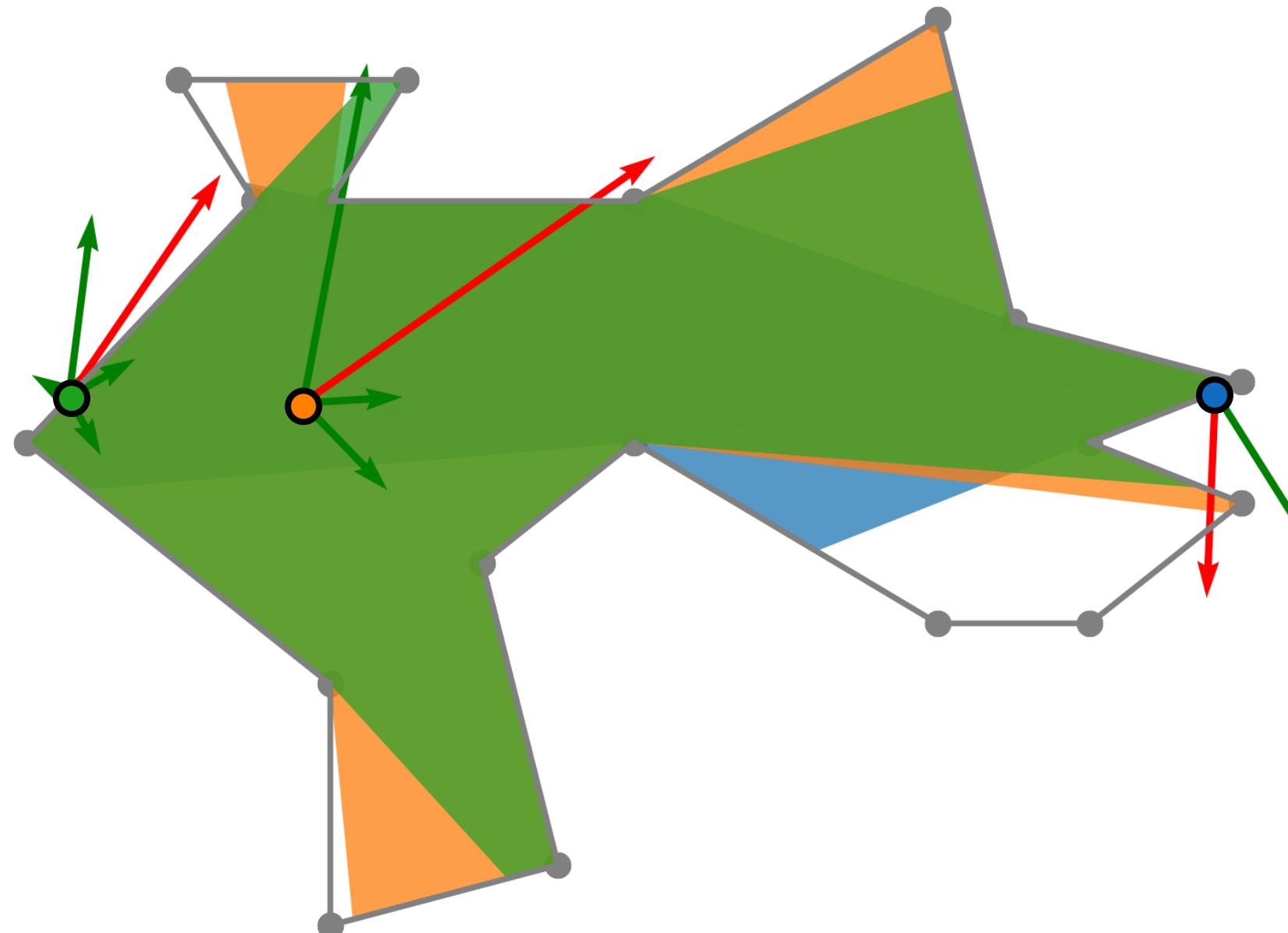
Solving the art gallery problem using the gradient

Gradient Computation for Iteration #0



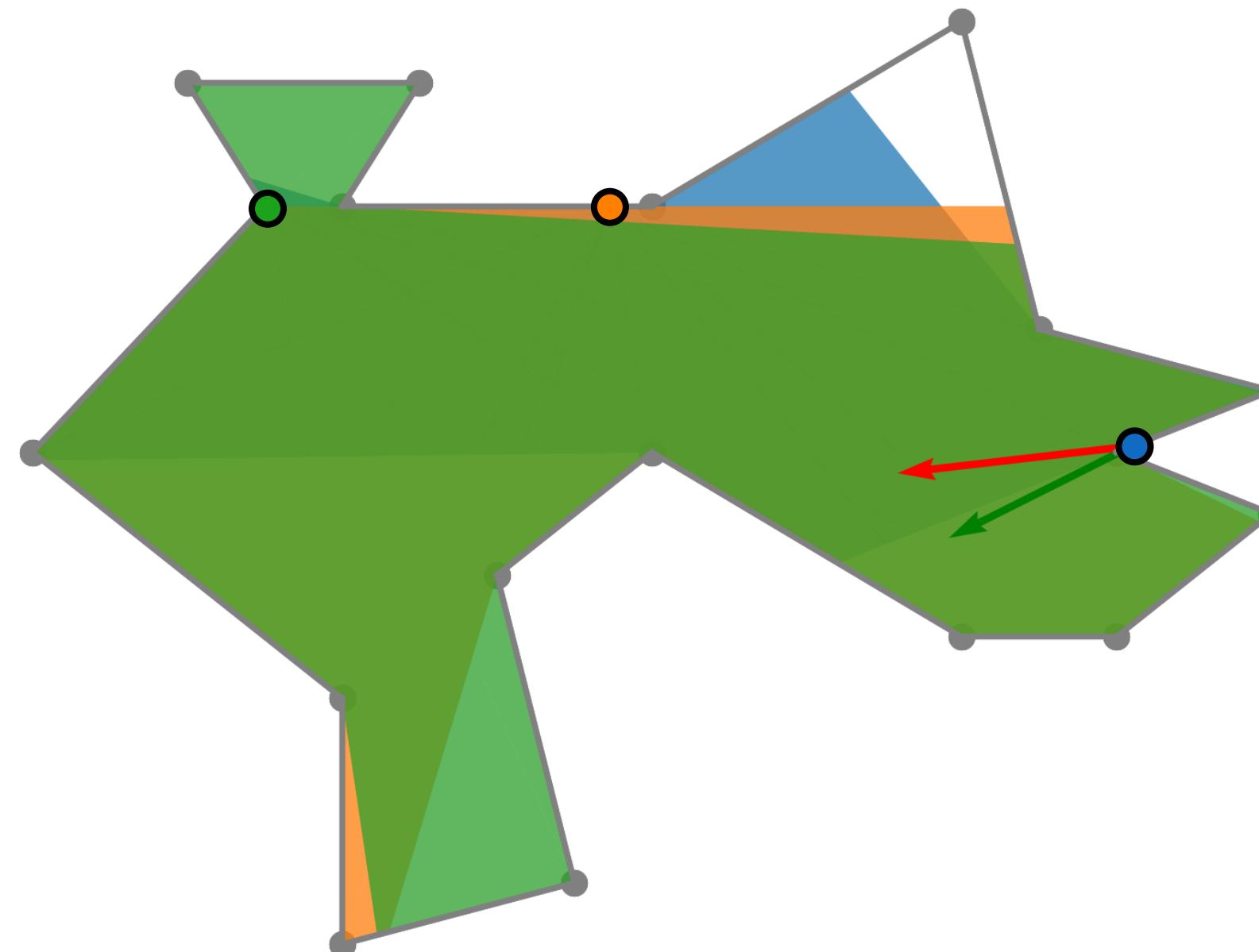
Solving the art gallery problem using the gradient

Gradient Computation for Iteration #1



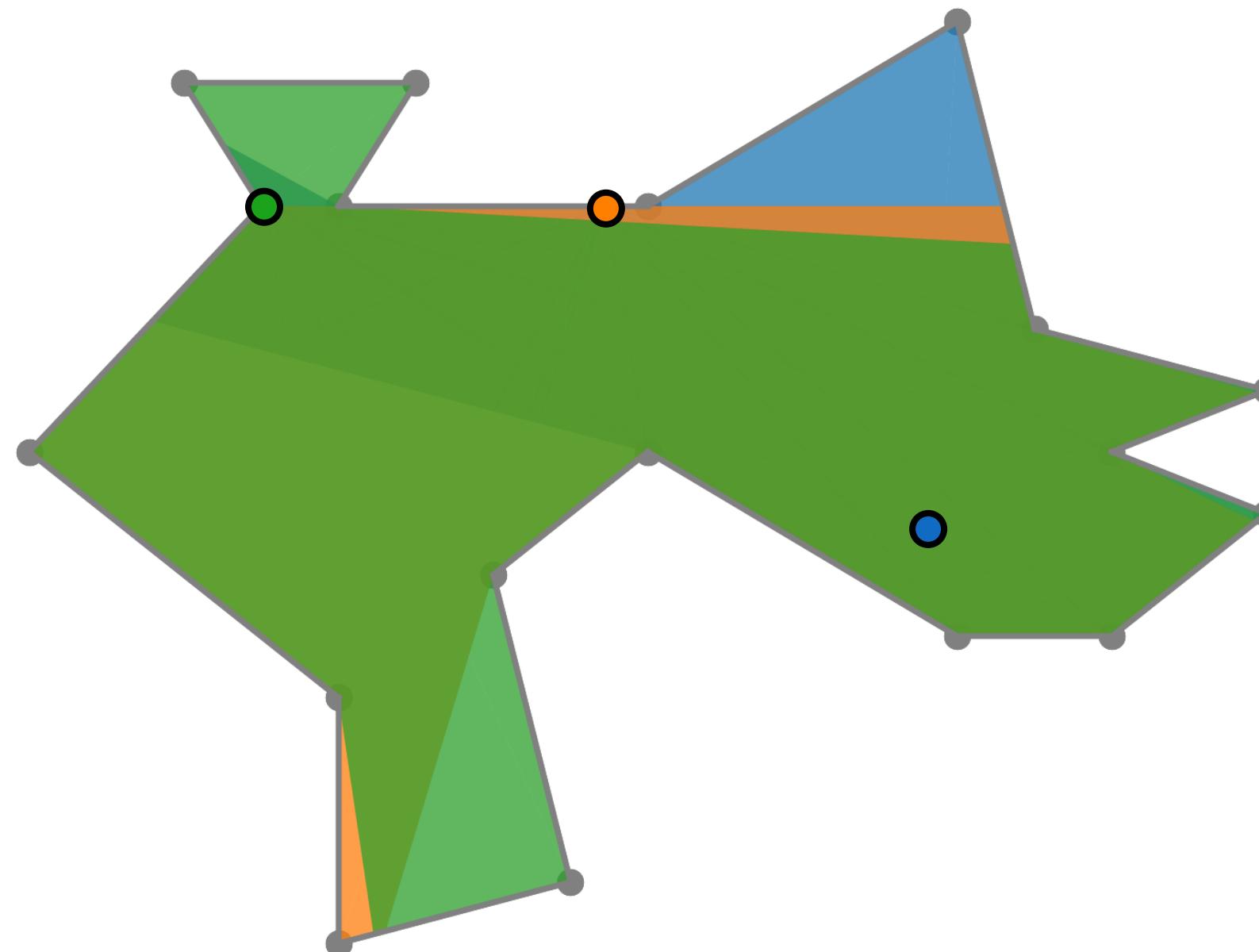
Solving the art gallery problem using the gradient

Gradient Computation for Iteration #2



Solving the art gallery problem using the gradient

Gradient Computation for Iteration #3



Heuristics

momentum

pull towards reflex vertex

pull onto the reflex vertex

pull capping

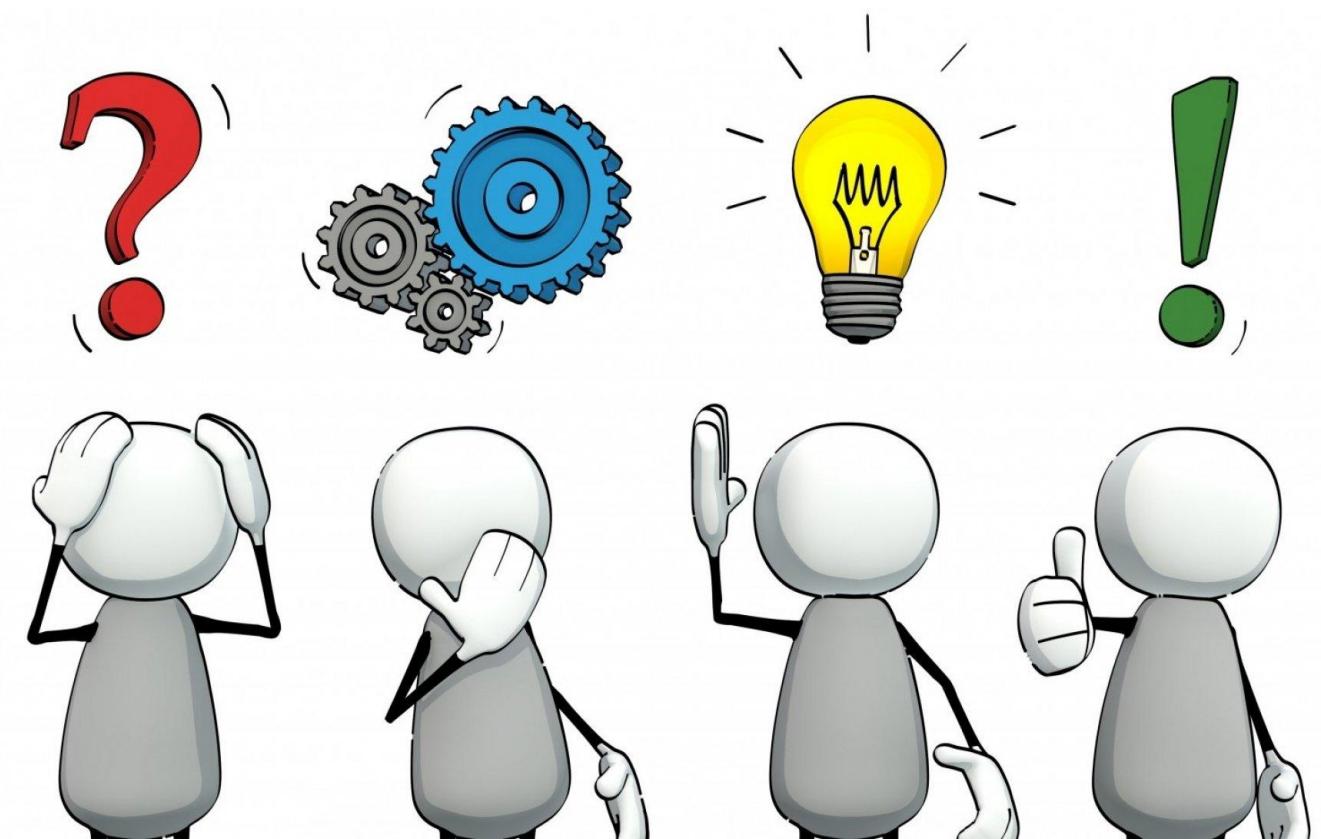
reflex area

line search

angle behind reflex vertex

hidden movement

greedy initialisation



Heuristics

momentum

pull towards reflex vertex

pull onto the reflex vertex

pull capping

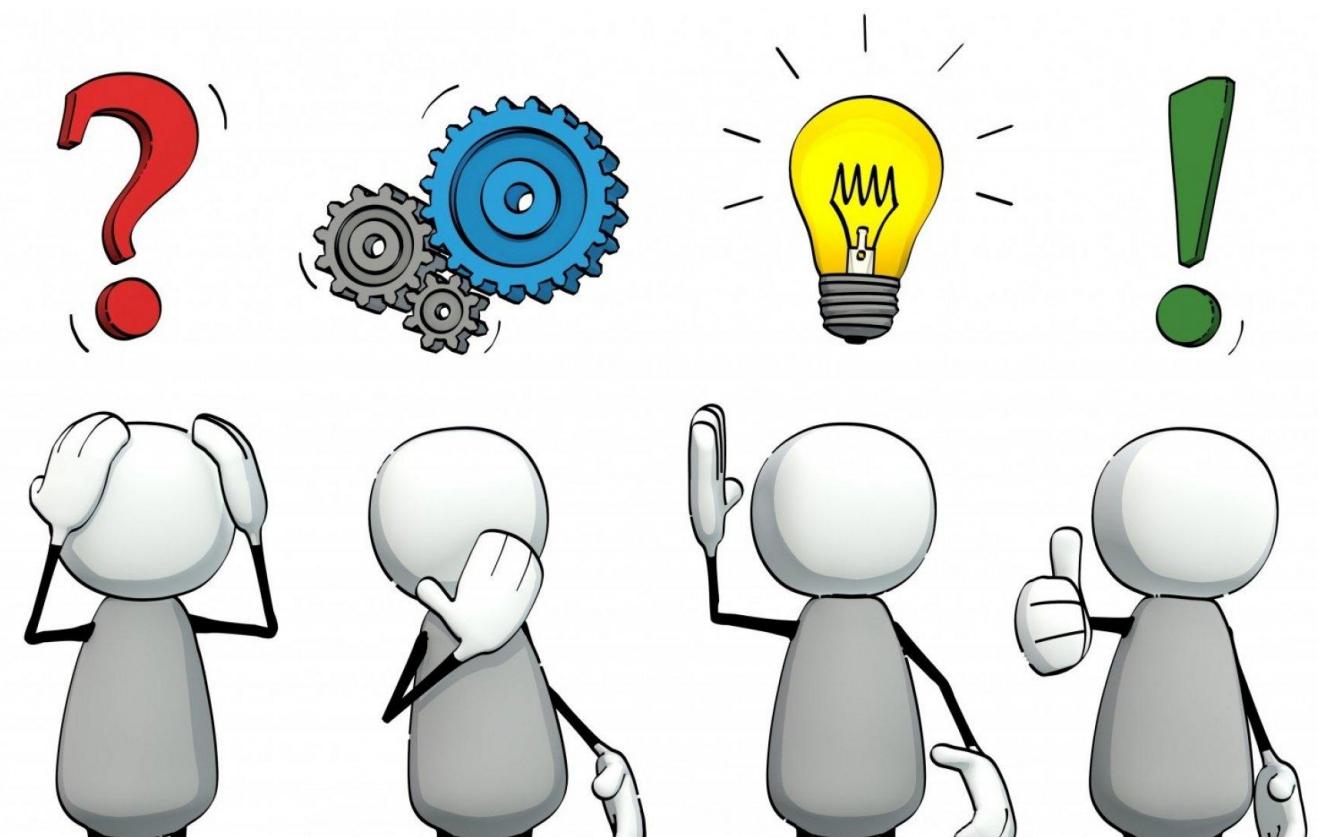
reflex area

line search

angle behind reflex vertex

hidden movement

greedy initialisation



Heuristics

momentum

pull towards reflex vertex

pull onto the reflex vertex

pull capping

reflex area

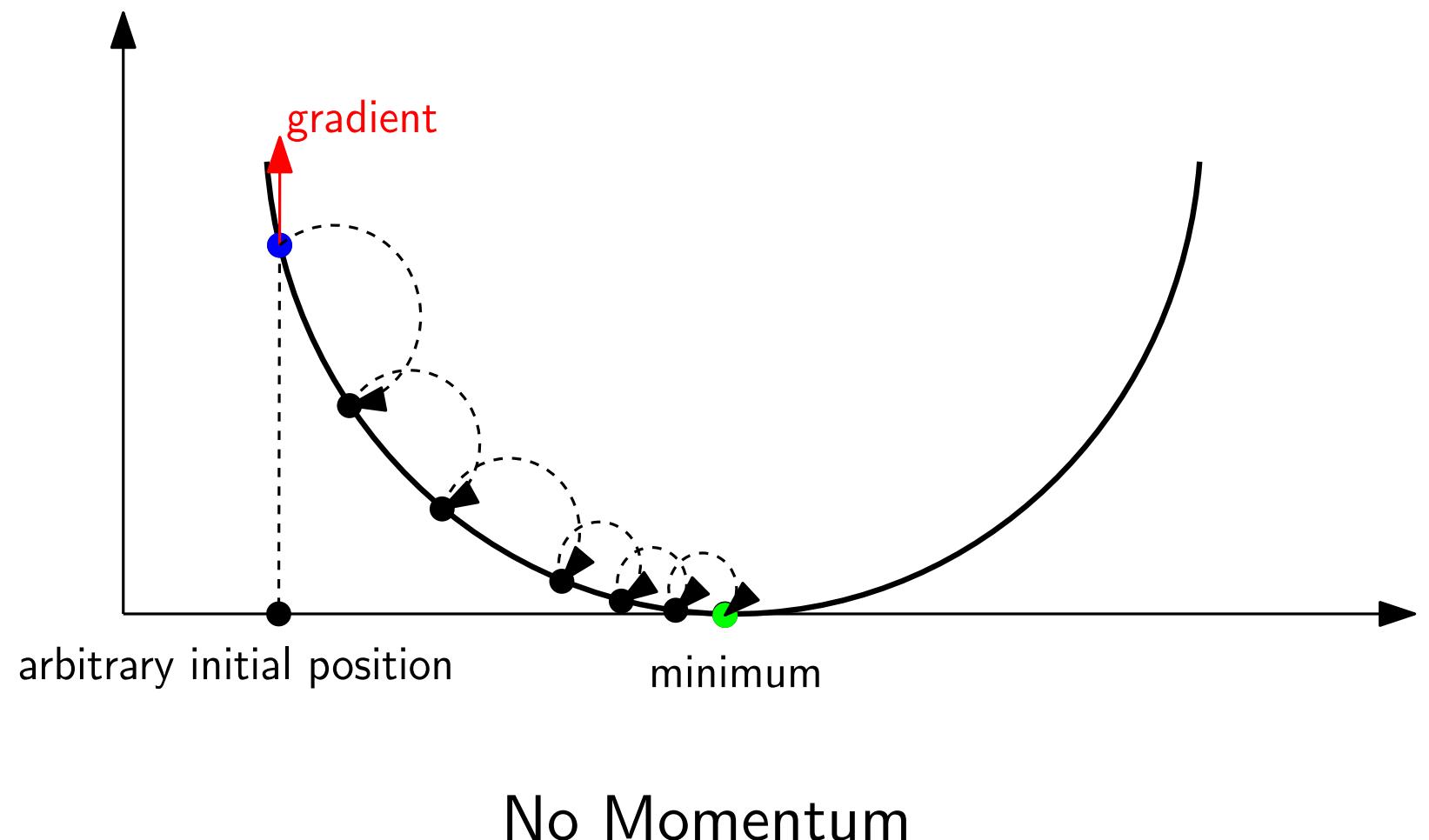
line search

angle behind reflex vertex

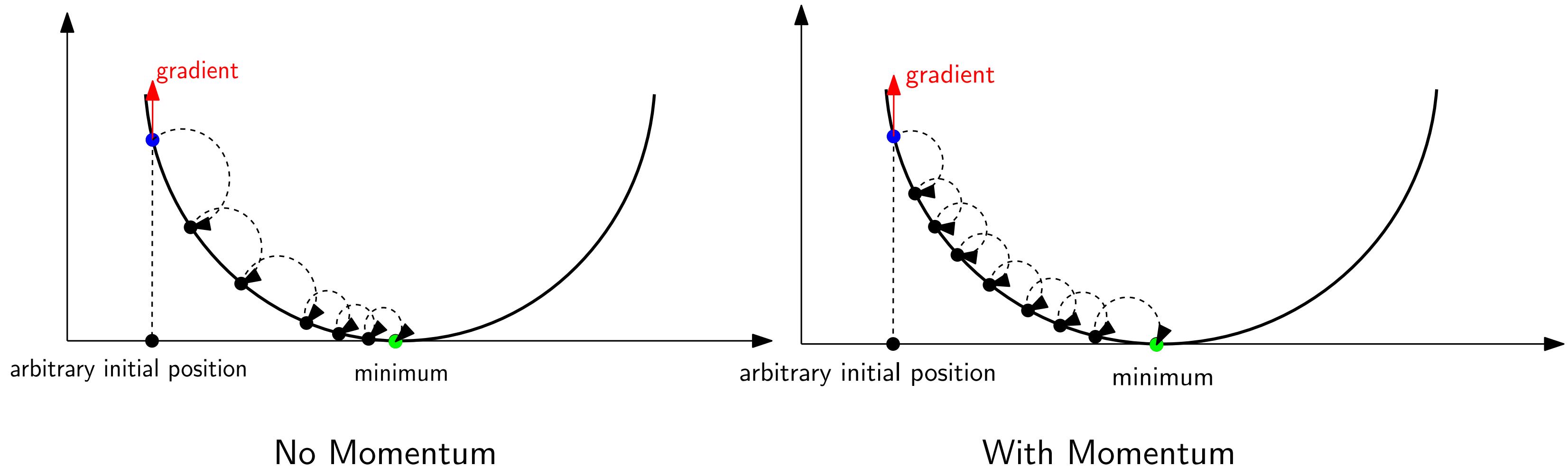
hidden movement

greedy initialisation

Heuristics: Momentum



Heuristics: Momentum

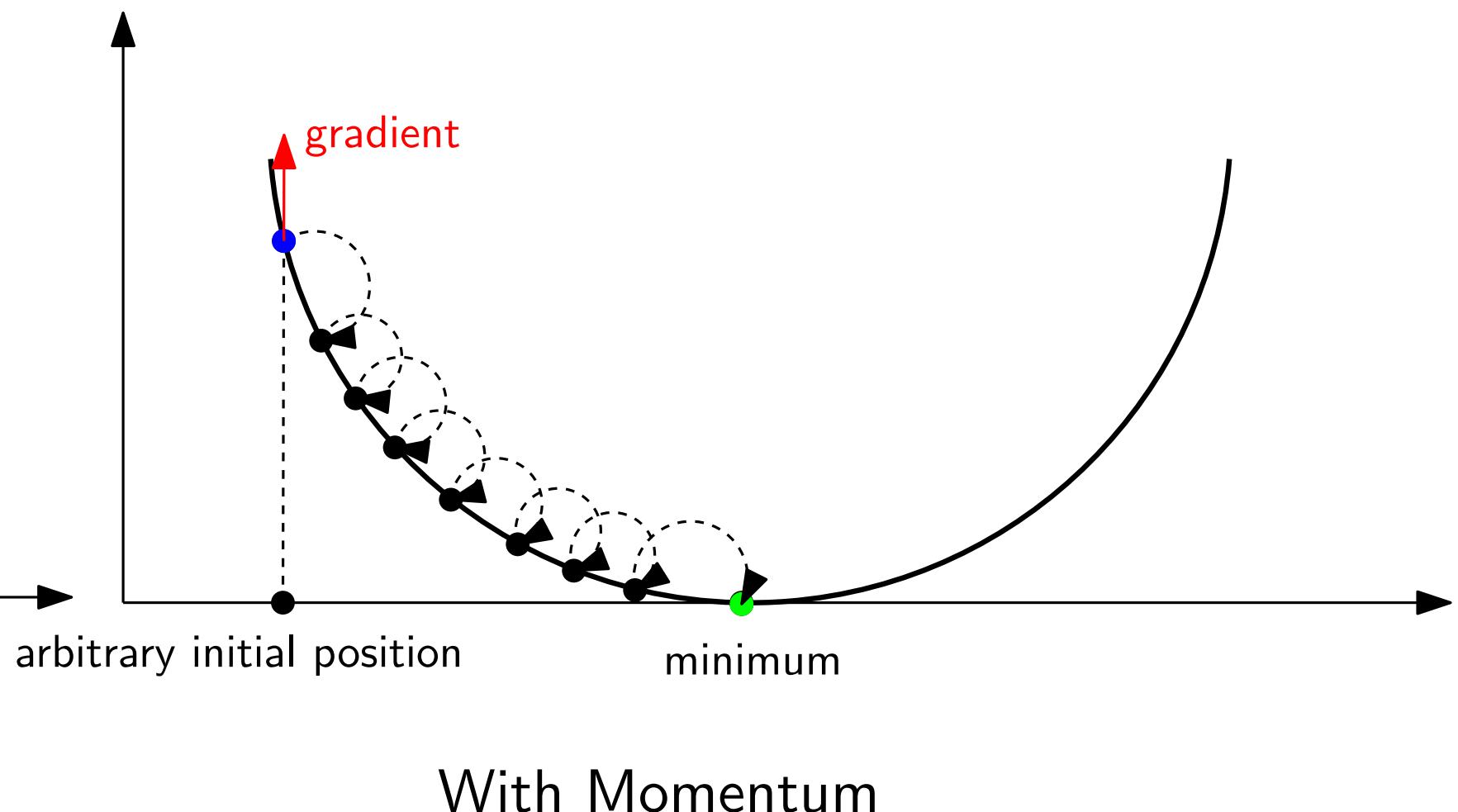
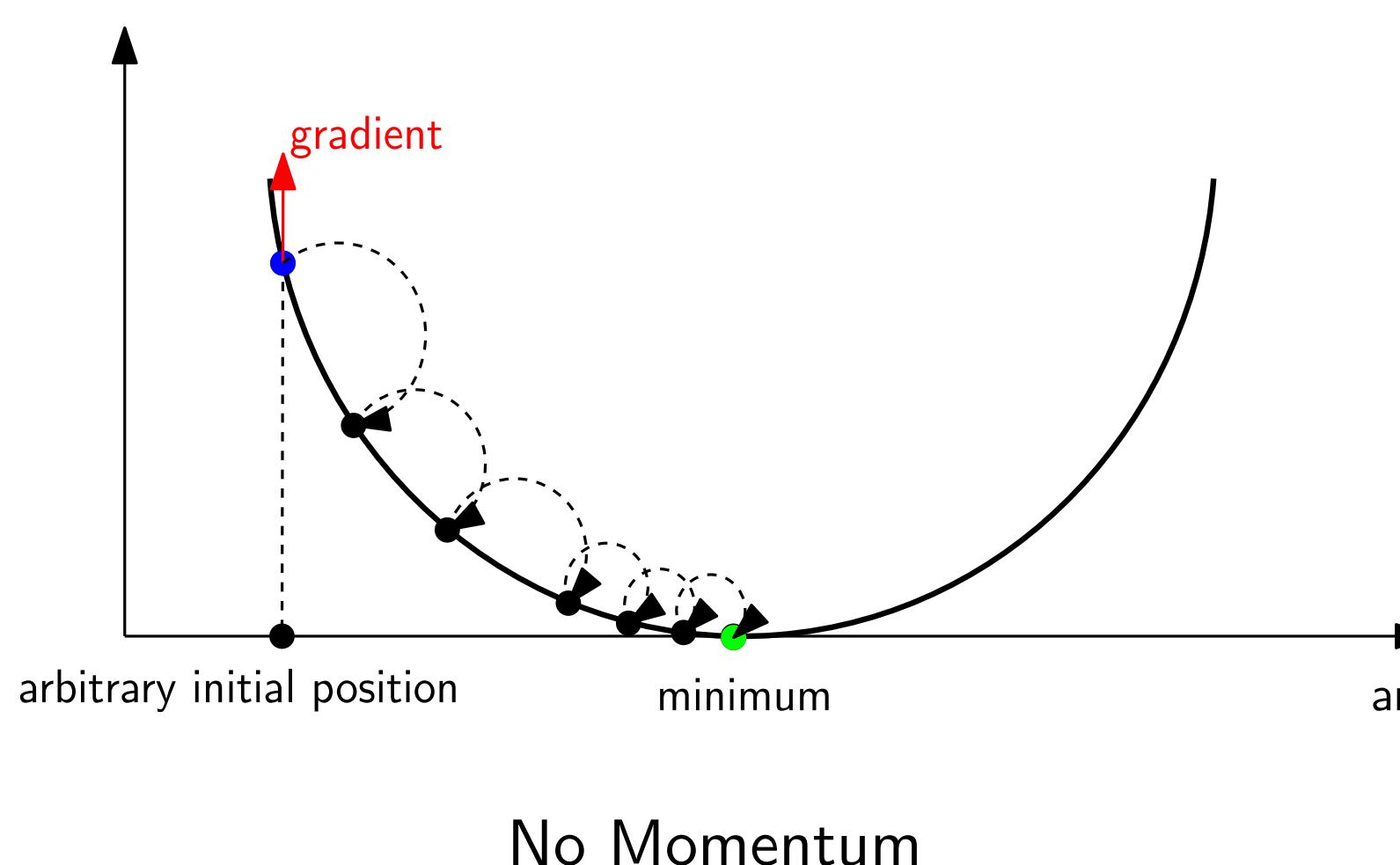


Heuristics: Momentum

iteration i :

γ - past state weight ("inertia")

$$M(g_{i+1}) = \gamma M(g_i) + (1 - \gamma) \nabla f(g_i)$$



Heuristics: Momentum

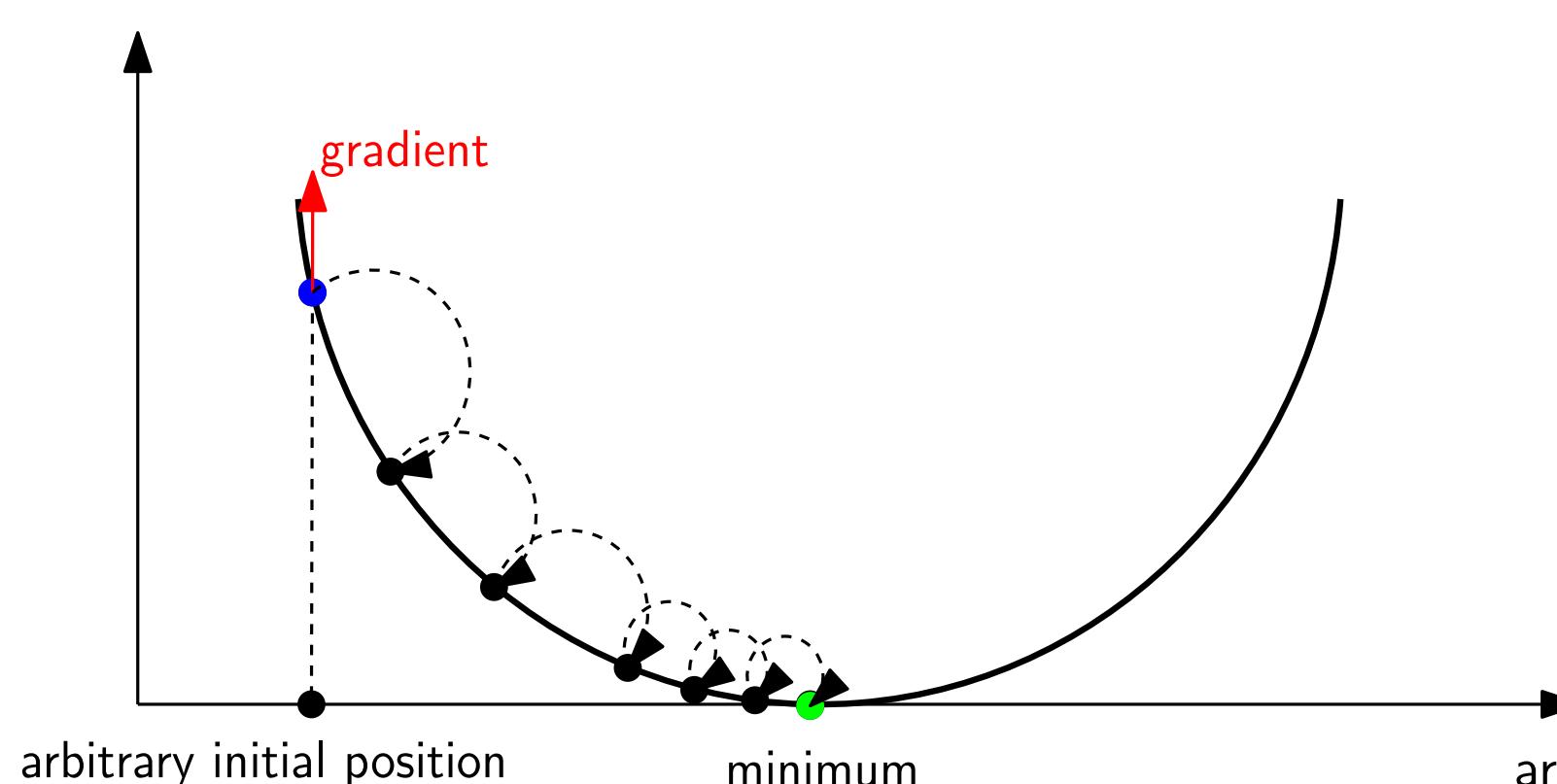
$$g_{i+1} = g_i + \alpha \nabla f(g_i)$$

iteration i :

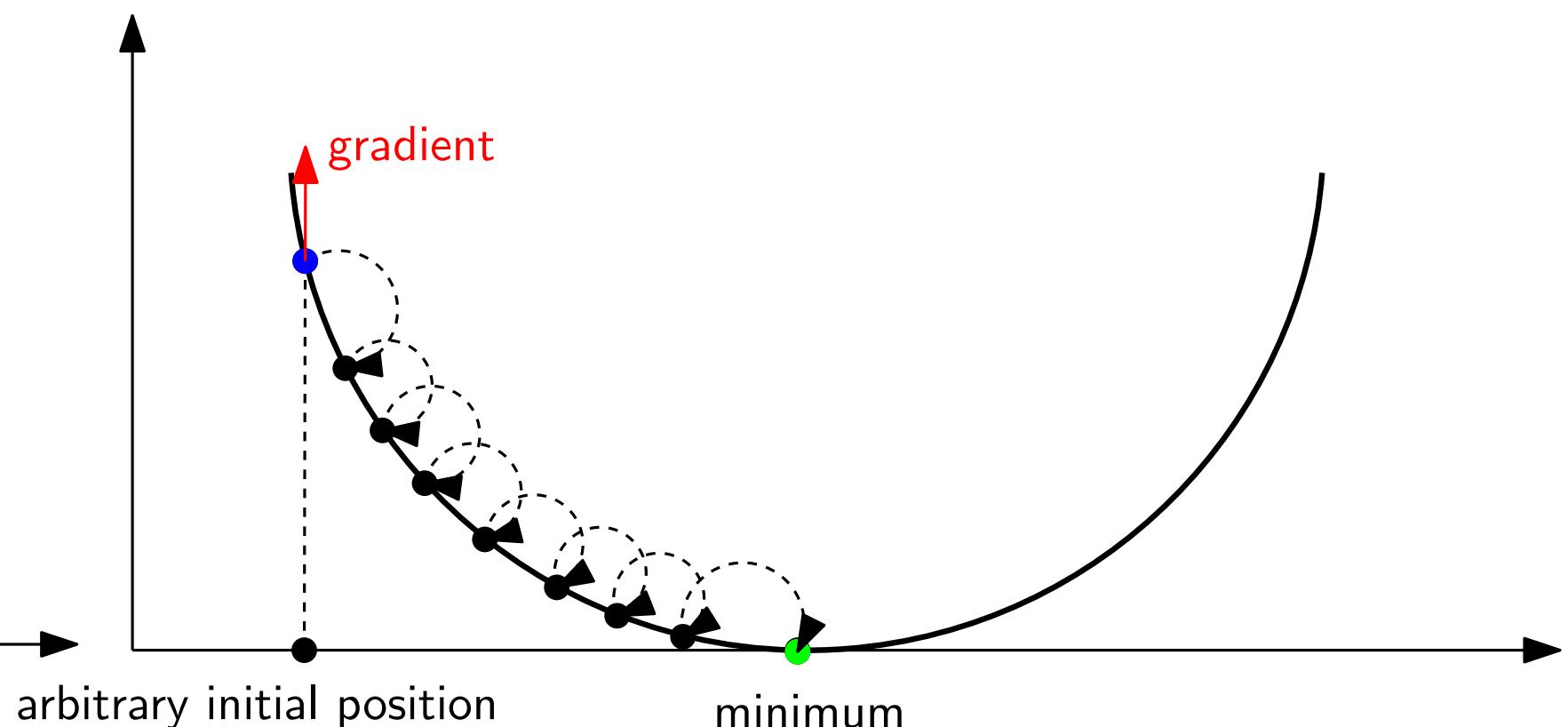
γ - past state weight ("inertia")

$$M(g_{i+1}) = \gamma M(g_i) + (1 - \gamma) \nabla f(g_i)$$

$$g_{i+1} = g_i + \alpha M(g_i)$$



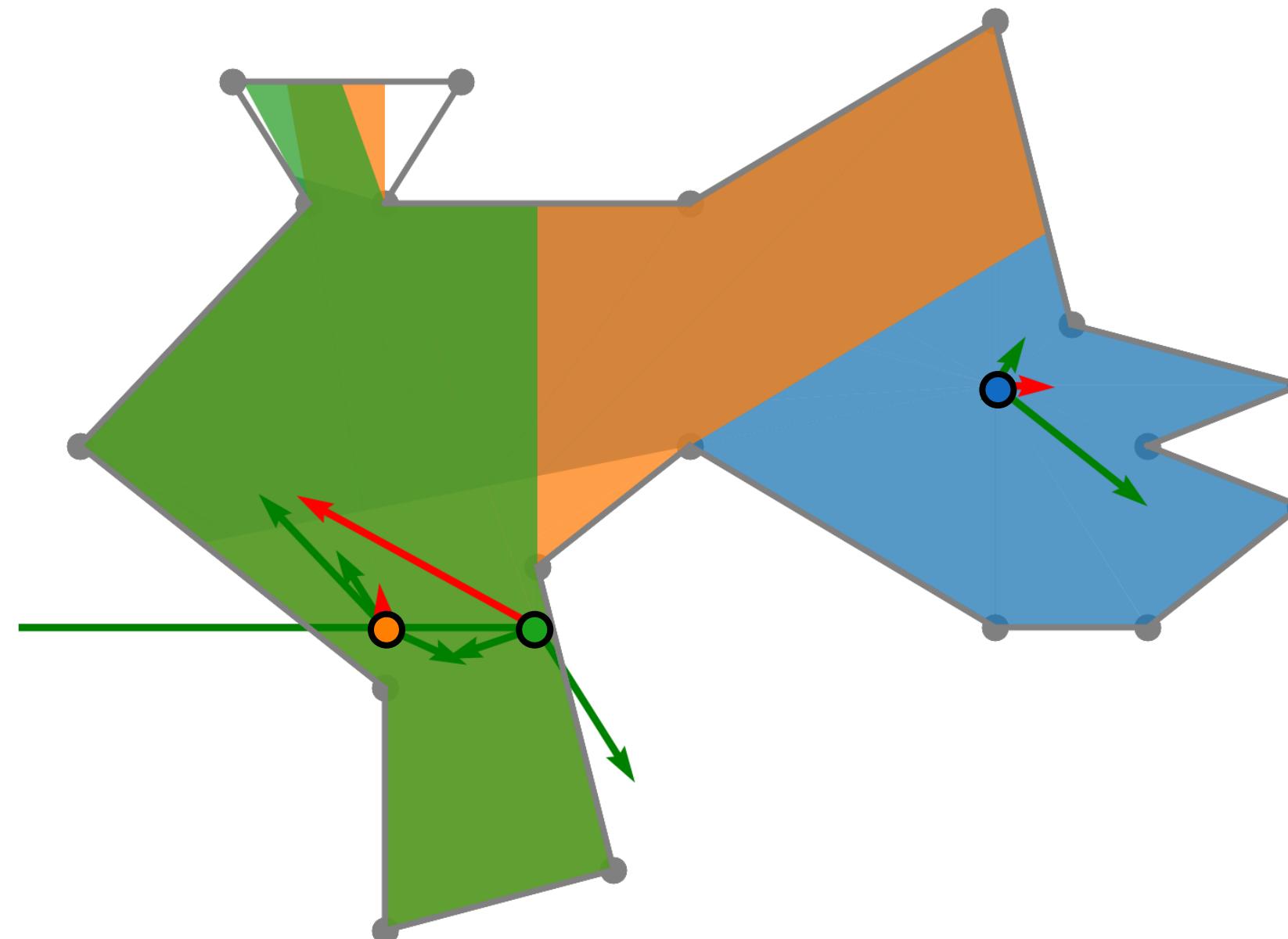
No Momentum



With Momentum

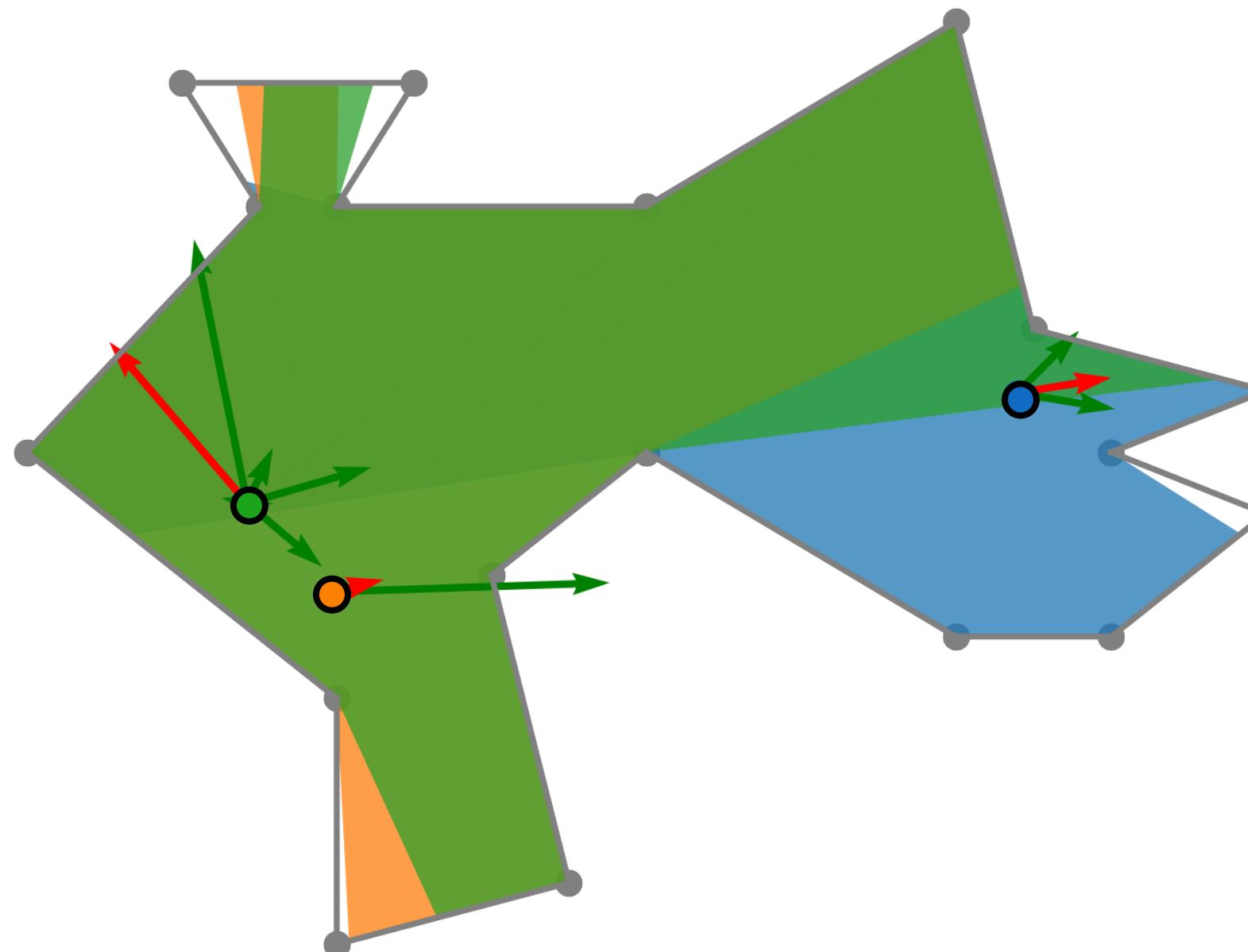
Heuristics: Momentum

Gradient Computation for Iteration #0



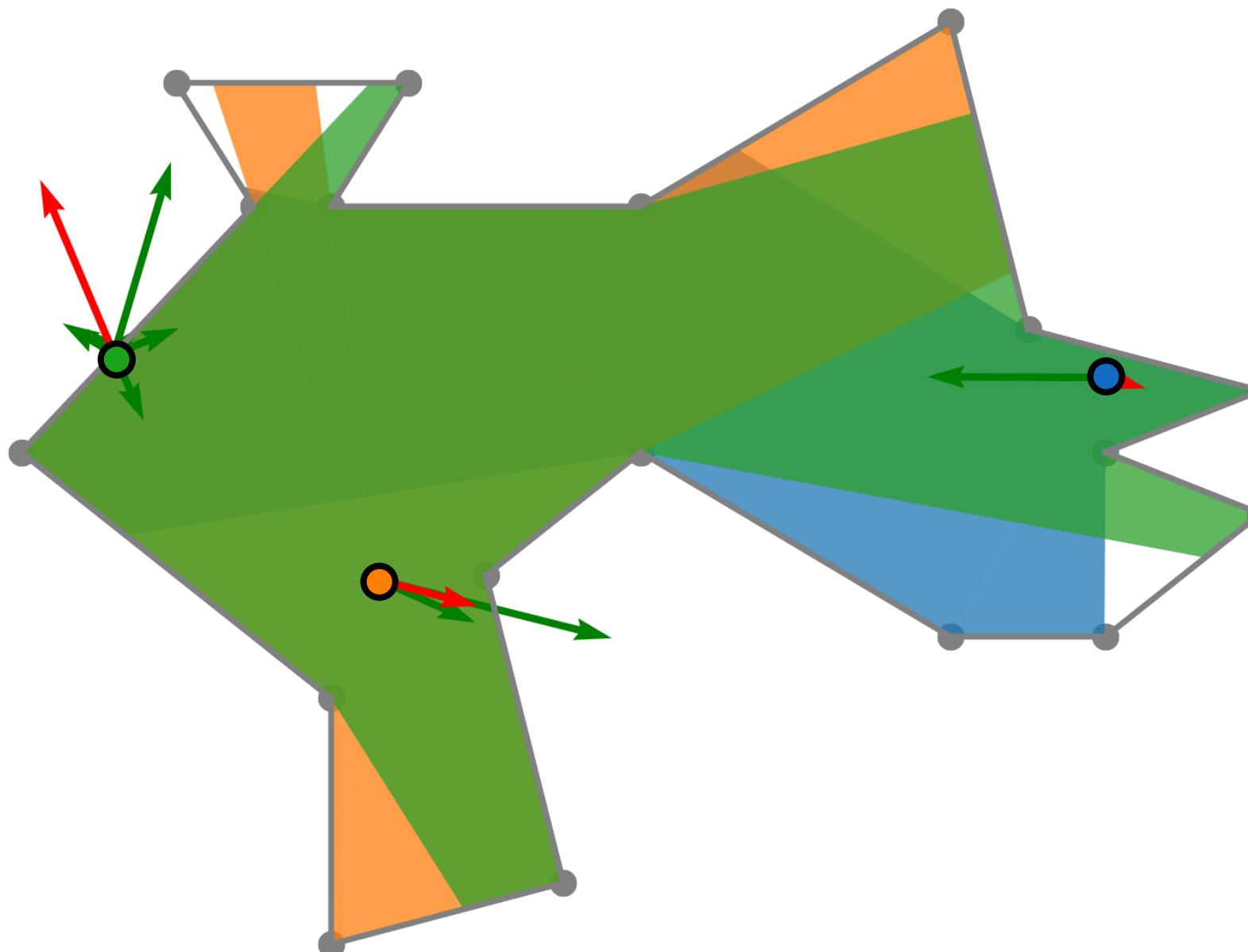
Heuristics: Momentum

Gradient Computation for Iteration #1



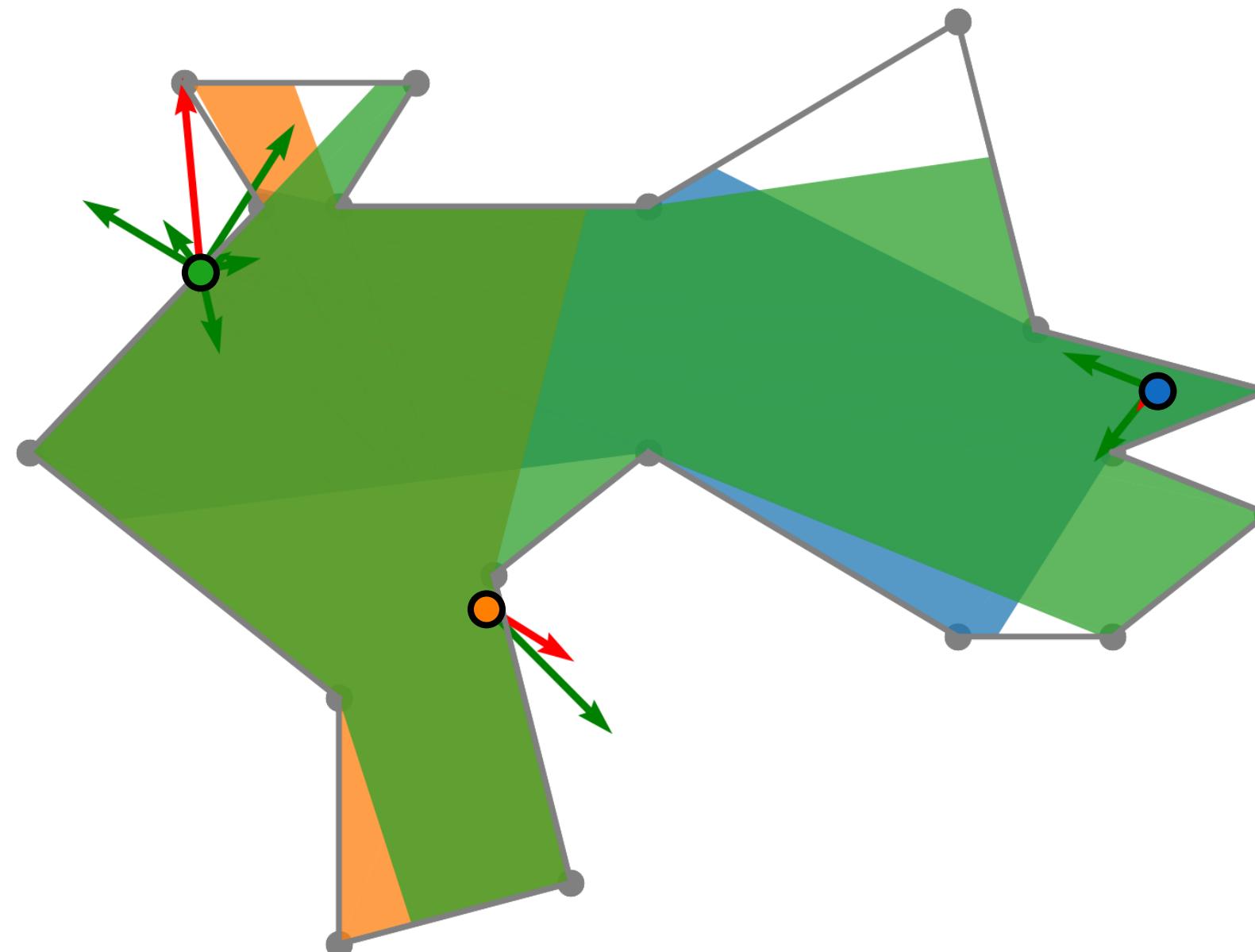
Heuristics: Momentum

Gradient Computation for Iteration #2



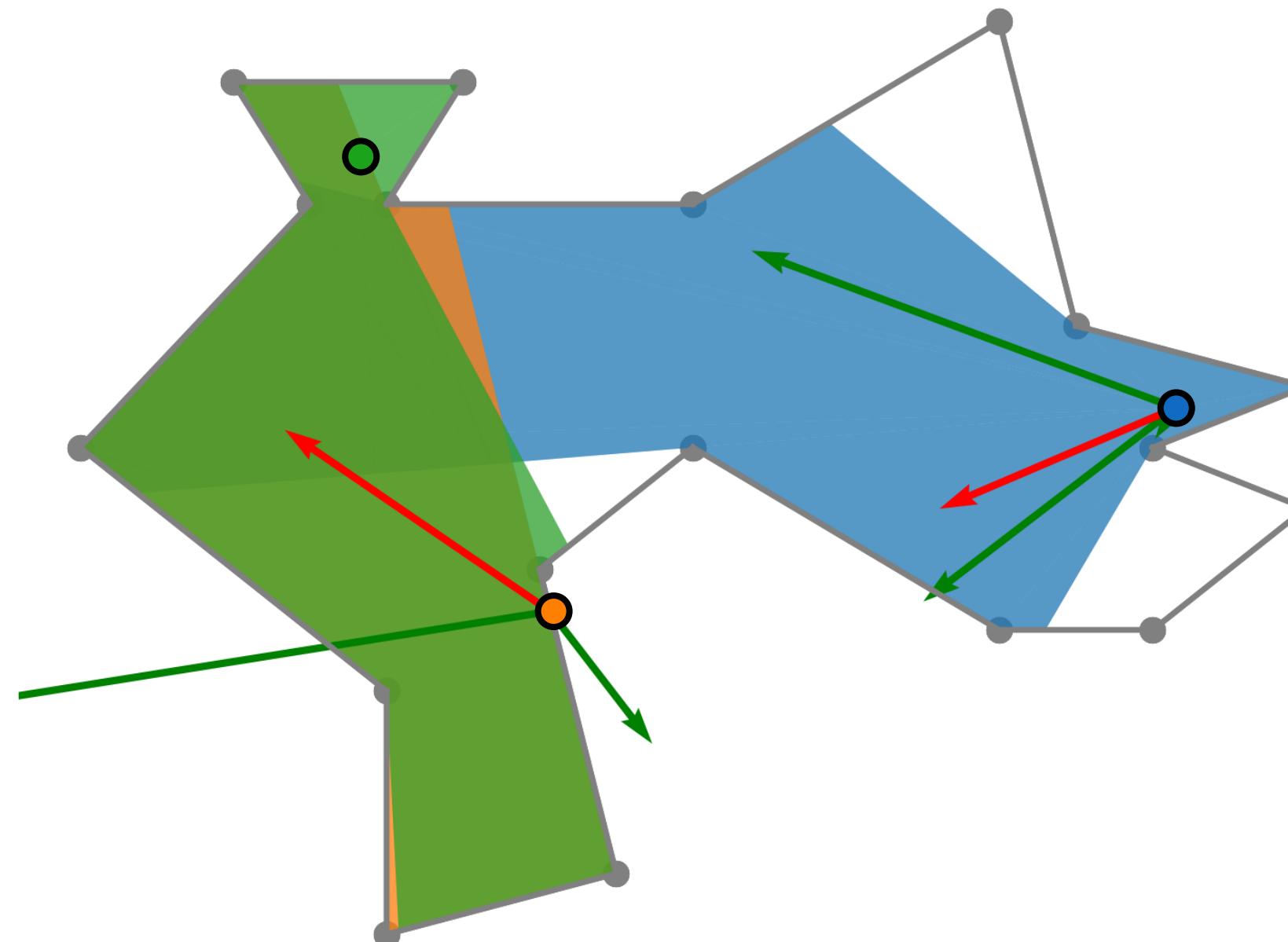
Heuristics: Momentum

Gradient Computation for Iteration #3



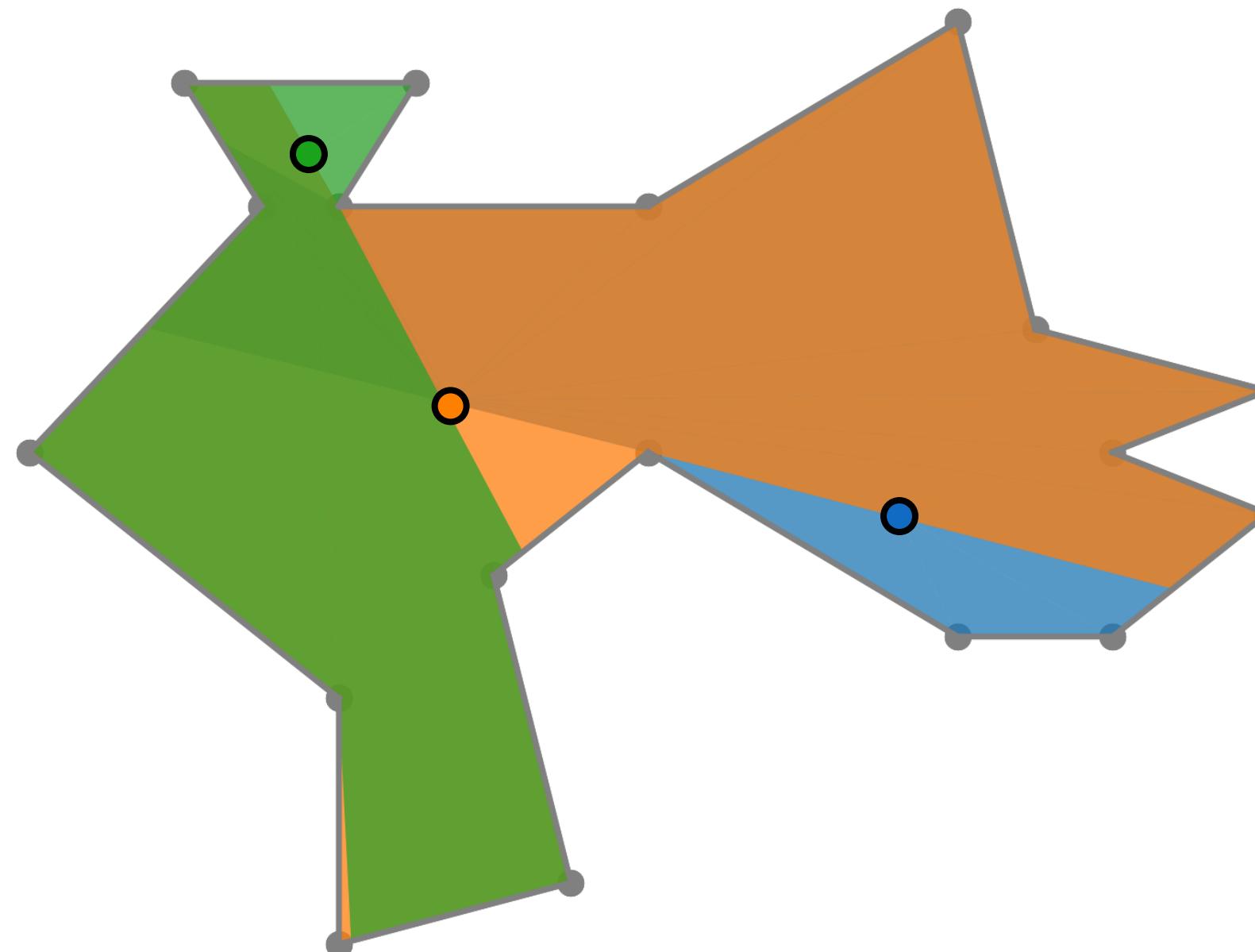
Heuristics: Momentum

Gradient Computation for Iteration #4



Heuristics: Momentum

Gradient Computation for Iteration #5



Heuristics

momentum

pull towards reflex vertex

pull onto the reflex vertex

pull capping

reflex area

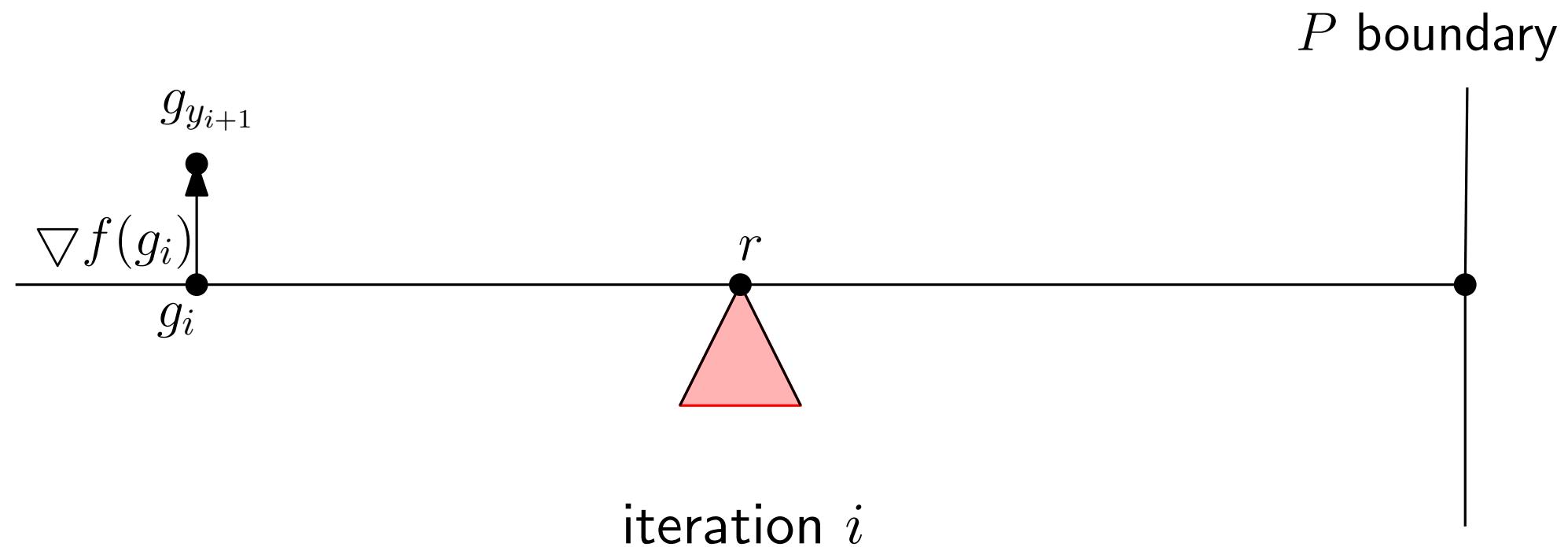
line search

angle behind reflex vertex

hidden movement

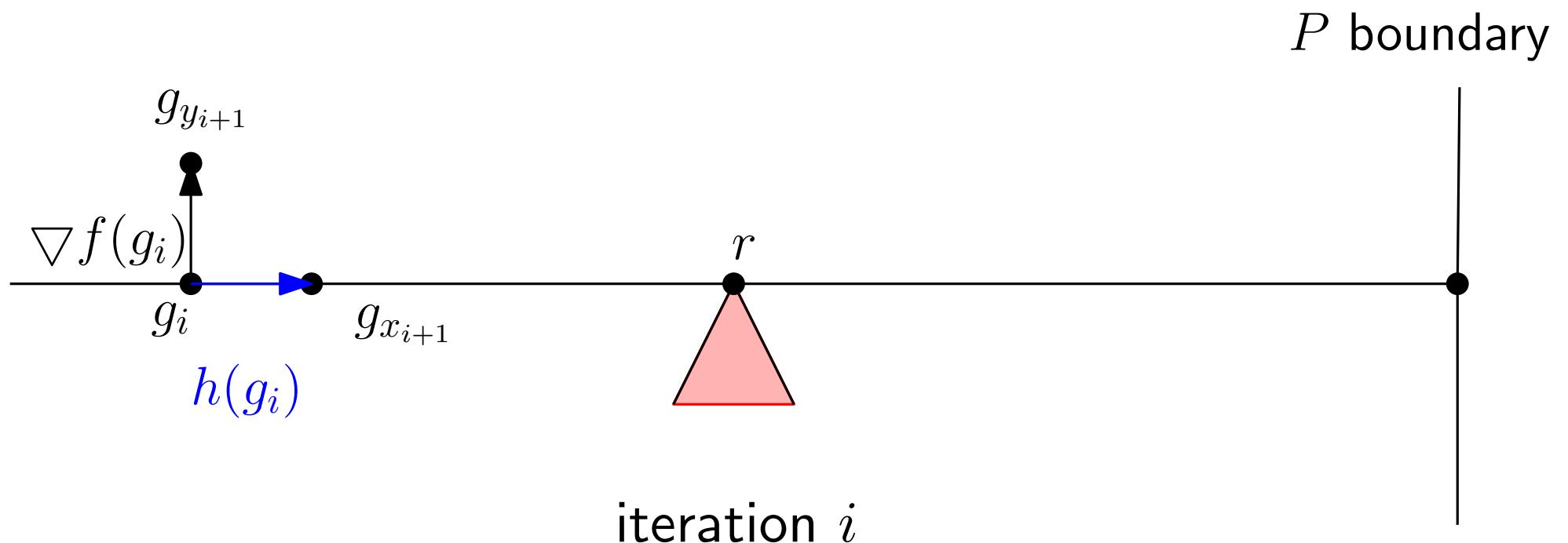
greedy initialisation

Heuristics: Pull towards reflex vertex



Heuristics: Pull towards reflex vertex

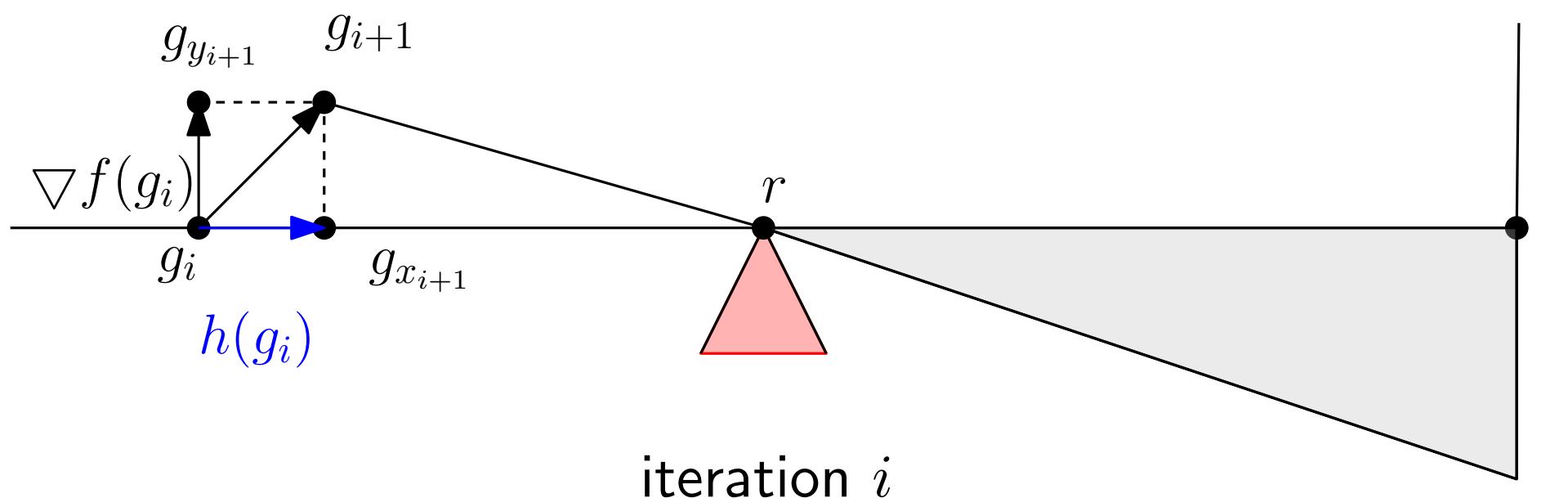
$$h(g_i) = \nabla||\nabla f(g_i)||$$



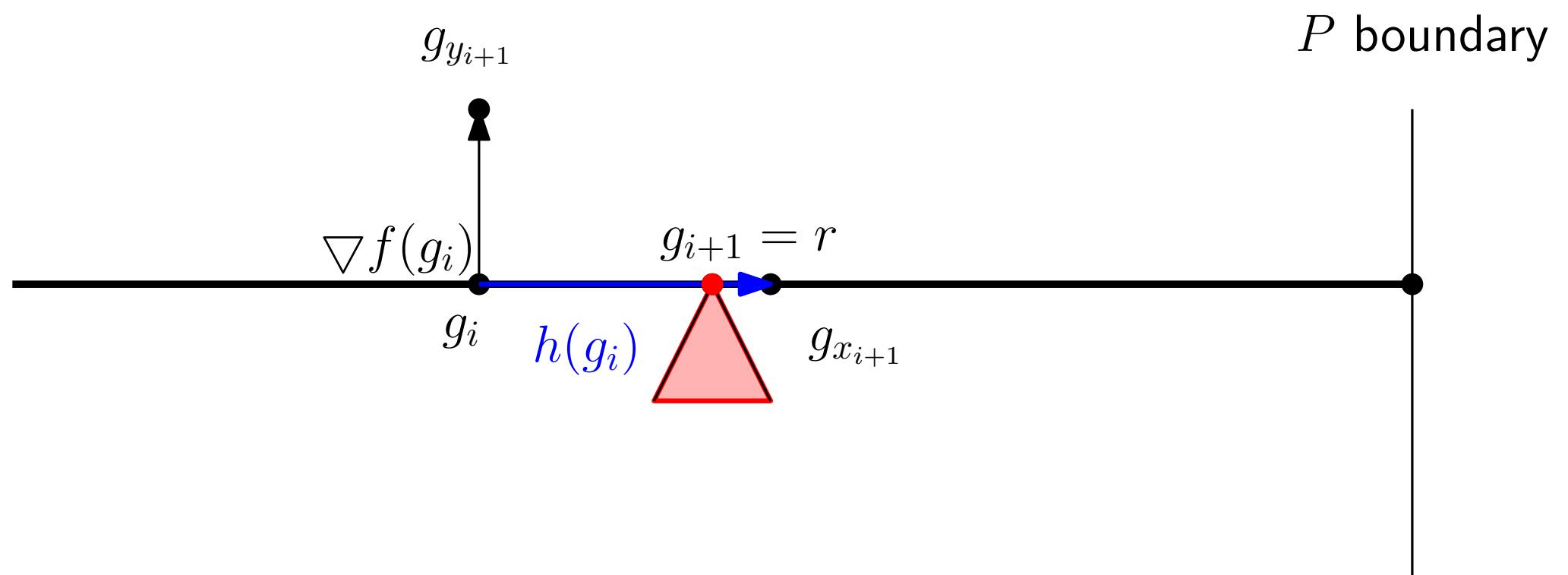
Heuristics: Pull towards reflex vertex

$$h(g_i) = \nabla || \nabla f(g_i) ||$$

$$g_{i+1} = g_i + \alpha(\nabla f(g_i) + h(g_i))$$

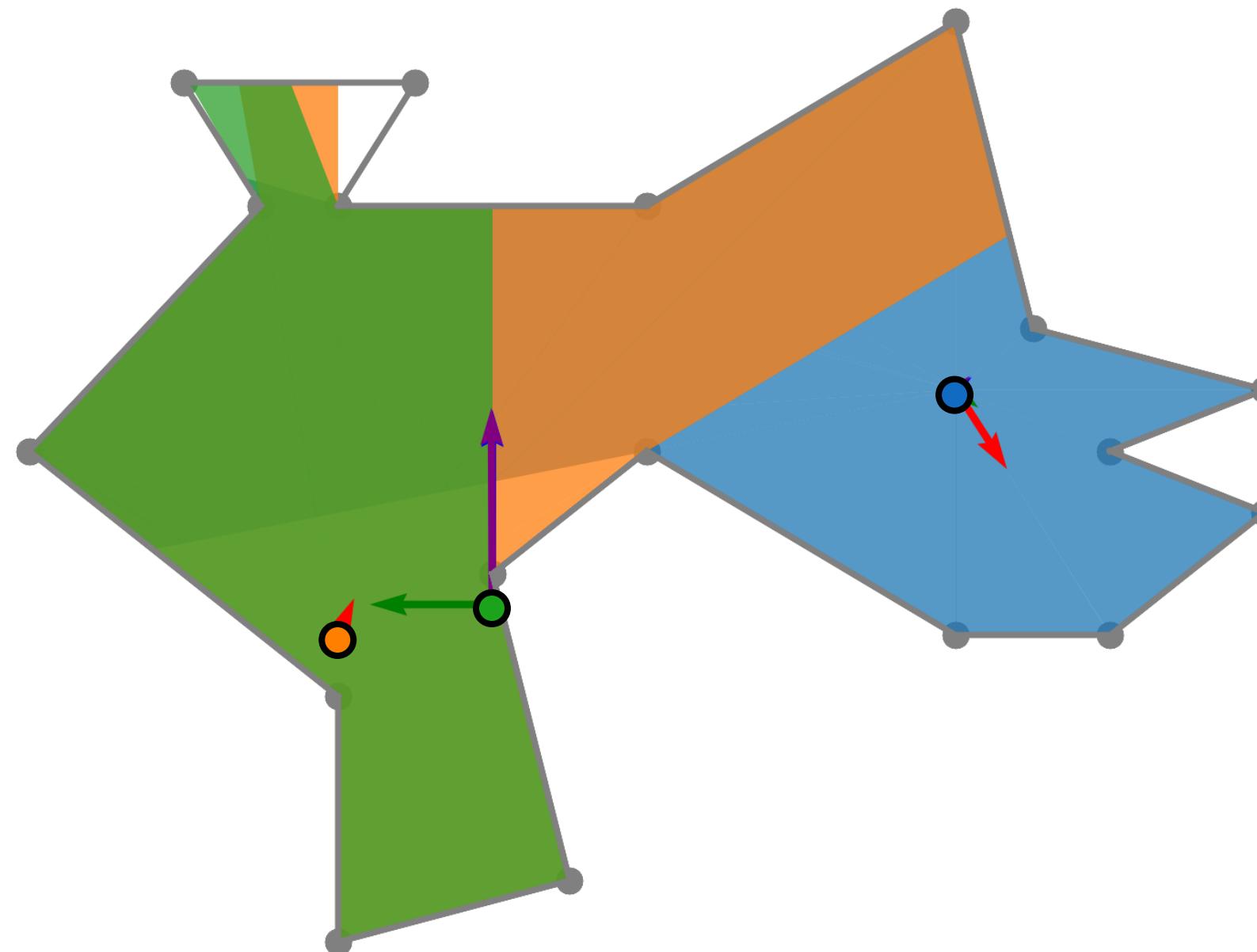


Heuristics: Pull towards reflex vertex



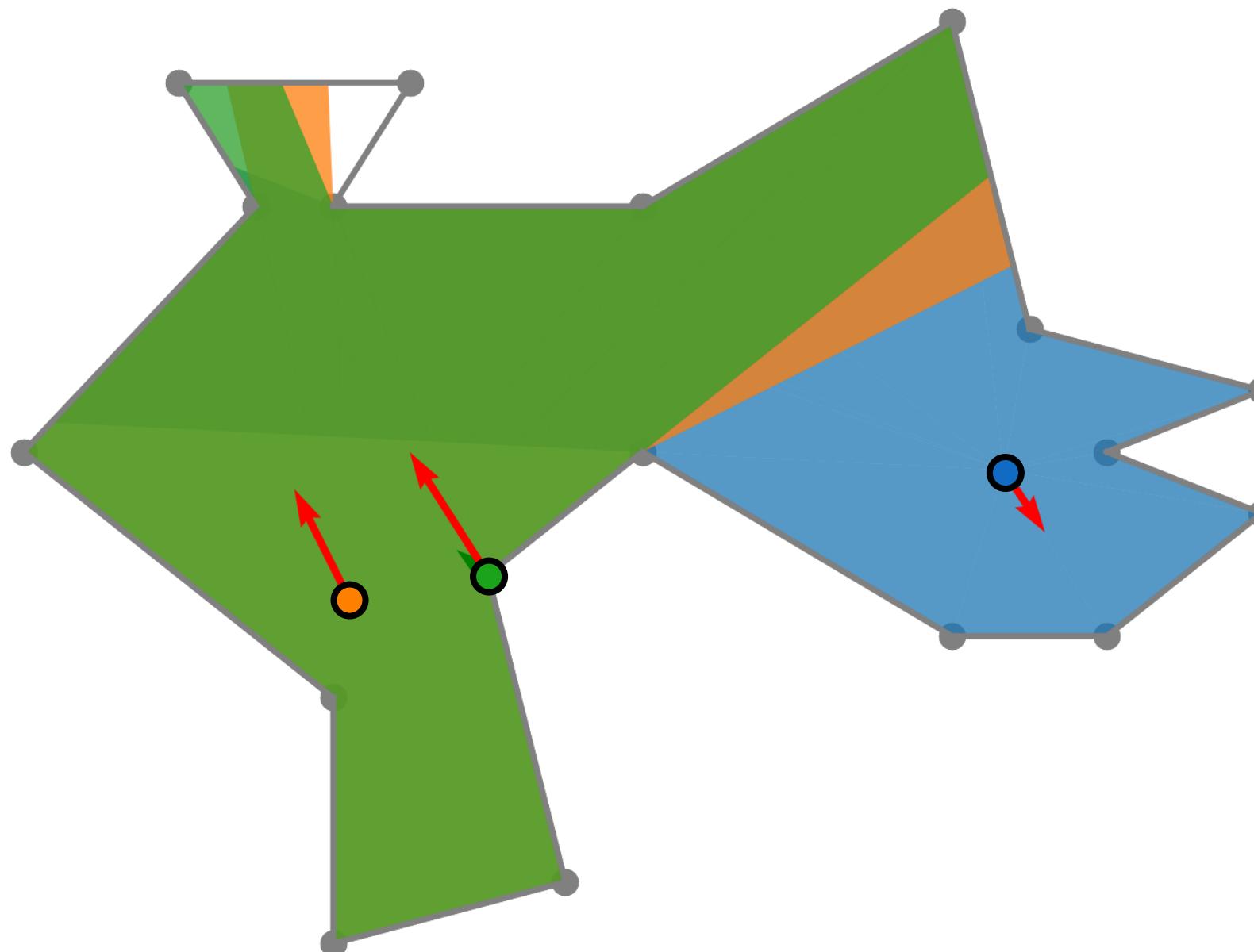
Heuristics: Pull towards reflex vertex

Gradient Computation for Iteration #0



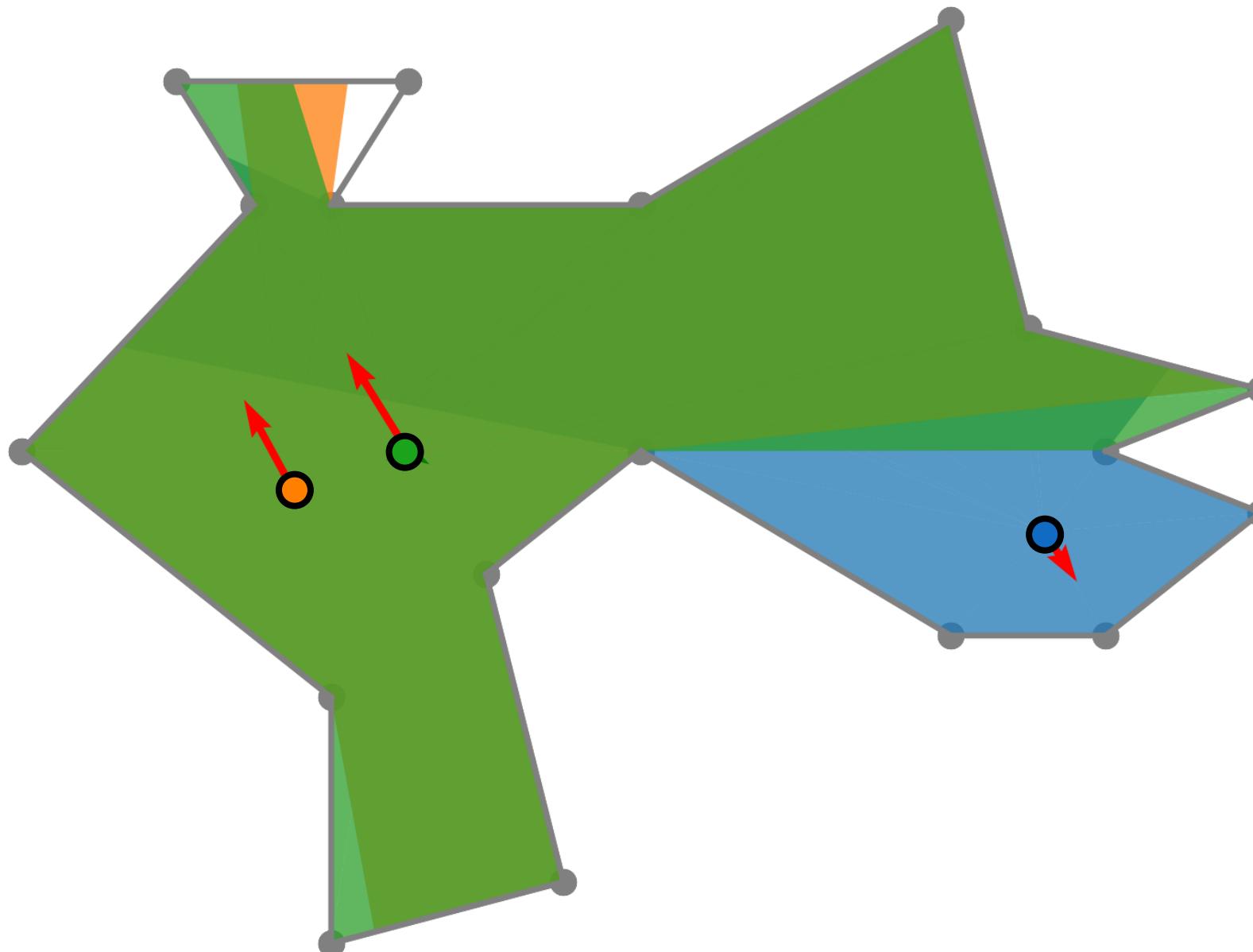
Heuristics: Pull towards reflex vertex

Gradient Computation for Iteration #1



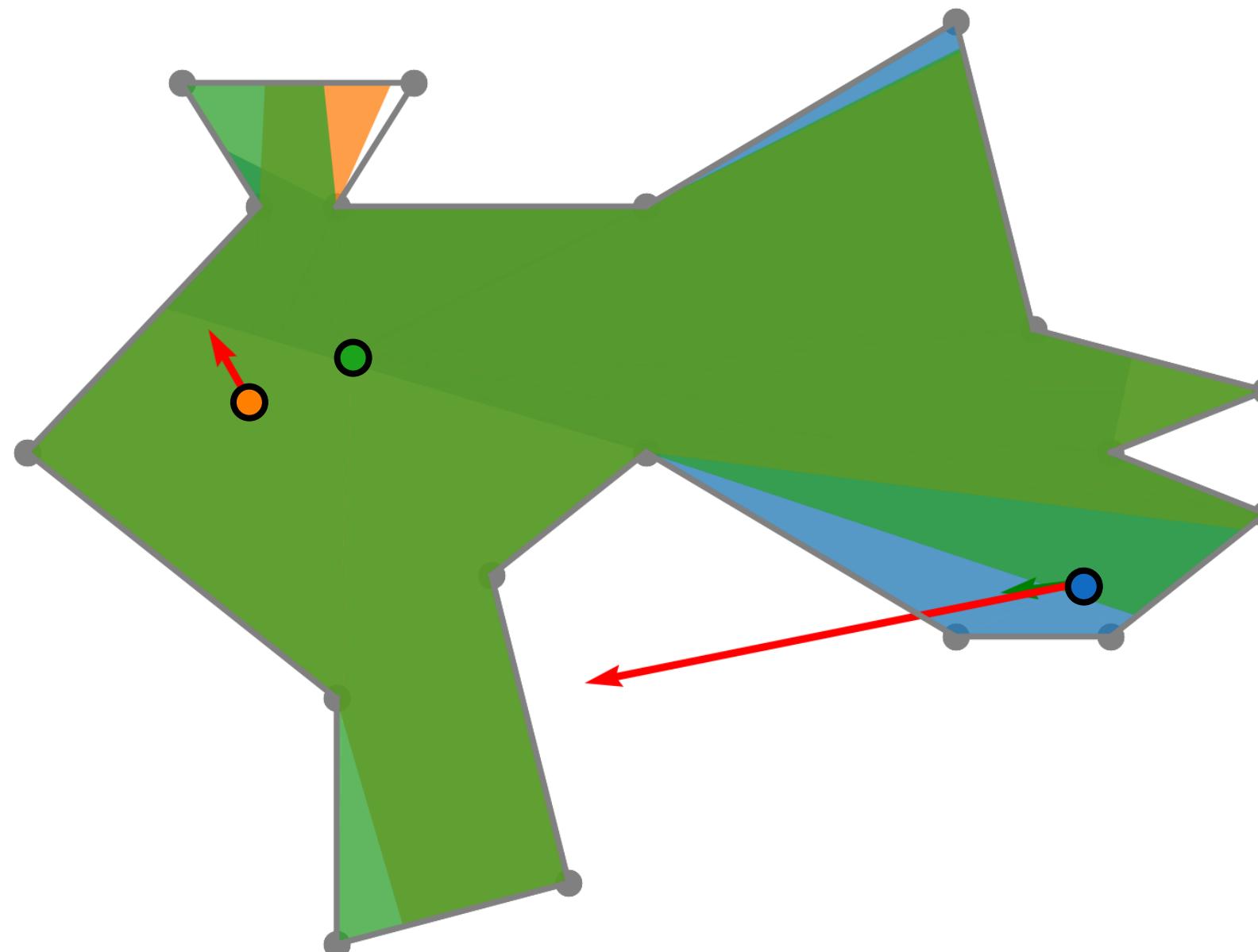
Heuristics: Pull towards reflex vertex

Gradient Computation for Iteration #2



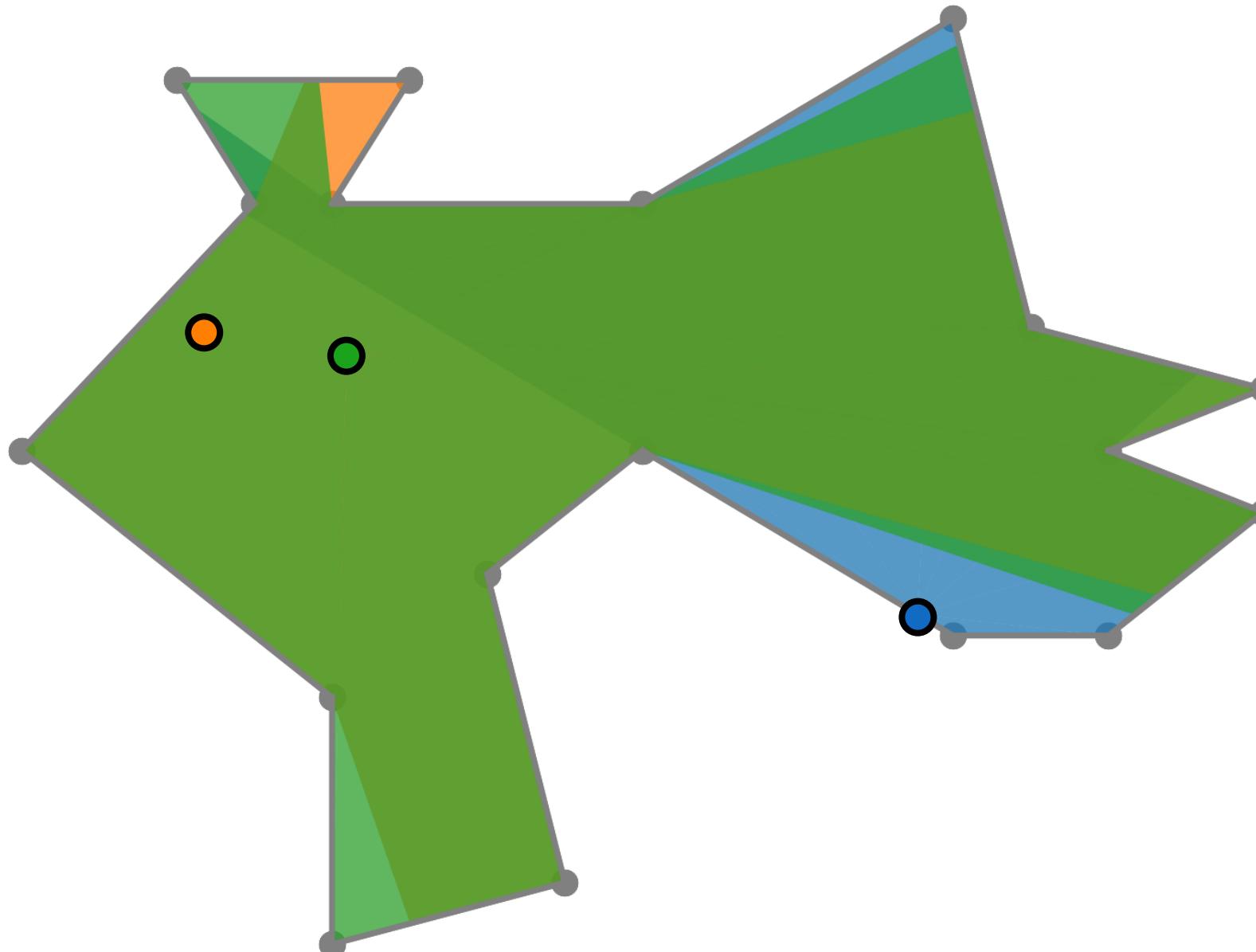
Heuristics: Pull towards reflex vertex

Gradient Computation for Iteration #3



Heuristics: Pull towards reflex vertex

Gradient Computation for Iteration #4



Heuristics

momentum

pull towards reflex vertex

 pull onto the reflex vertex

 pull capping

reflex area

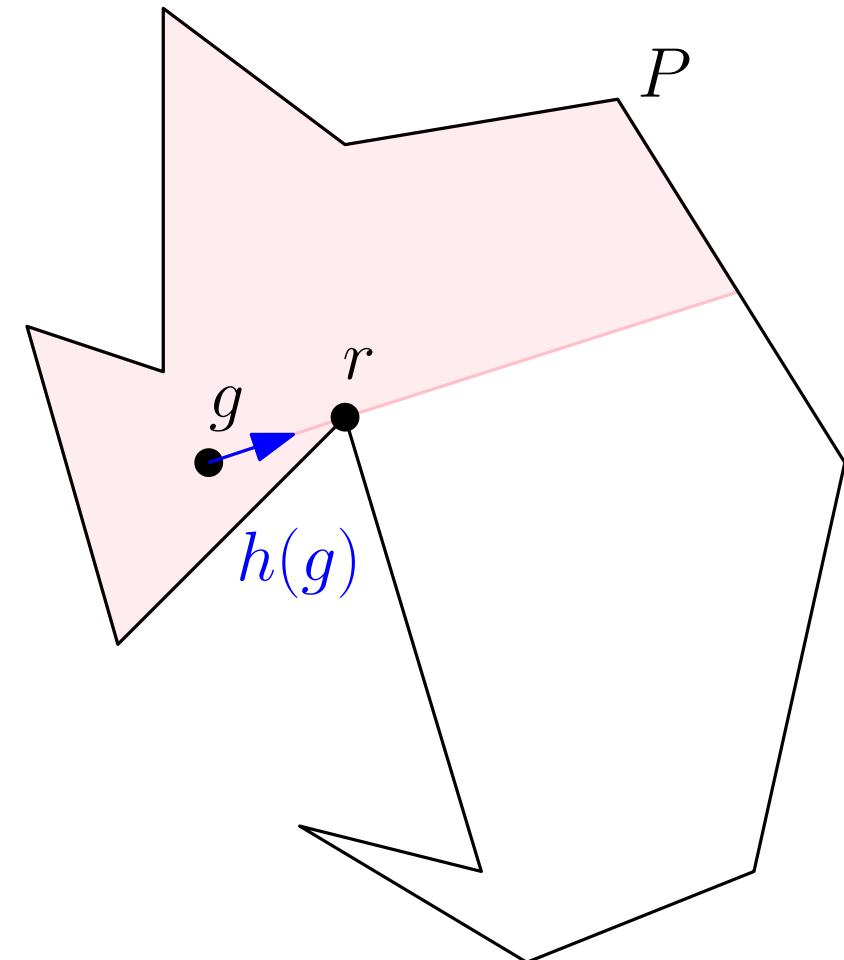
line search

angle behind reflex vertex

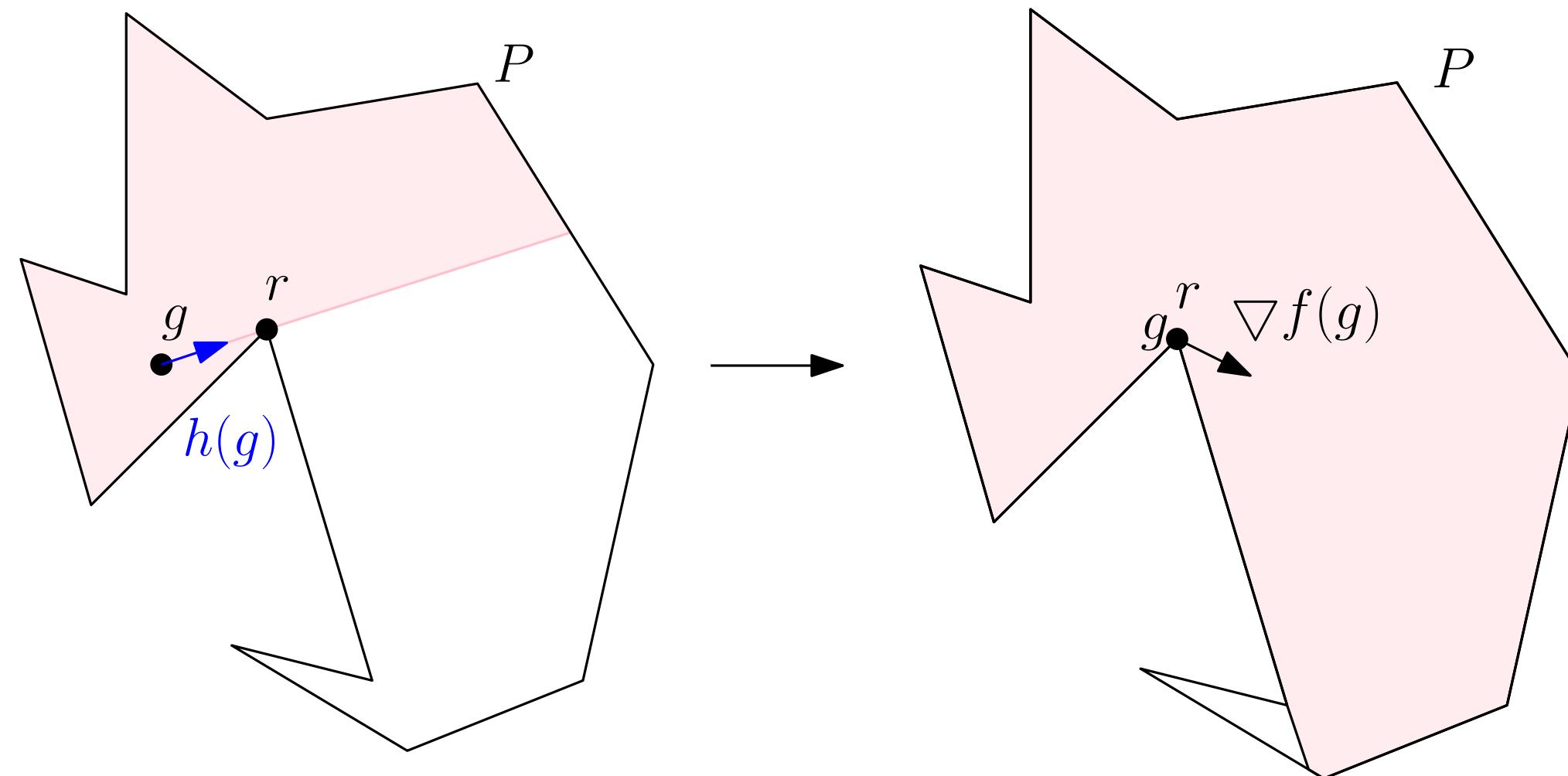
hidden movement

greedy initialisation

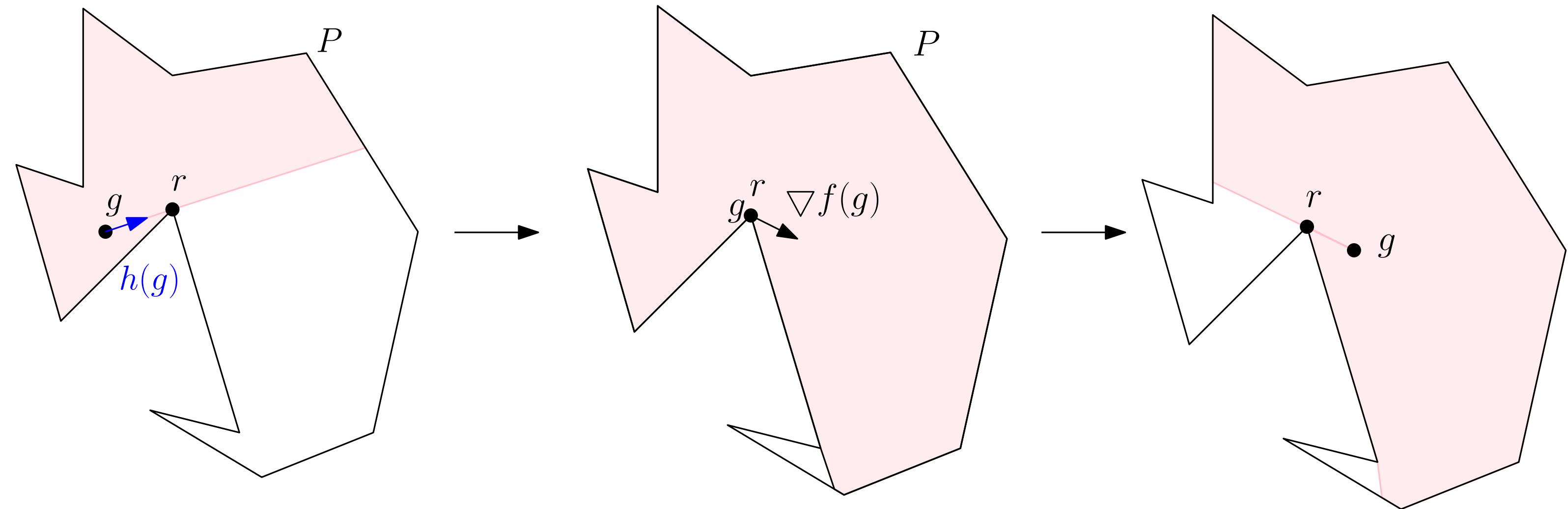
Heuristics: Reflex area



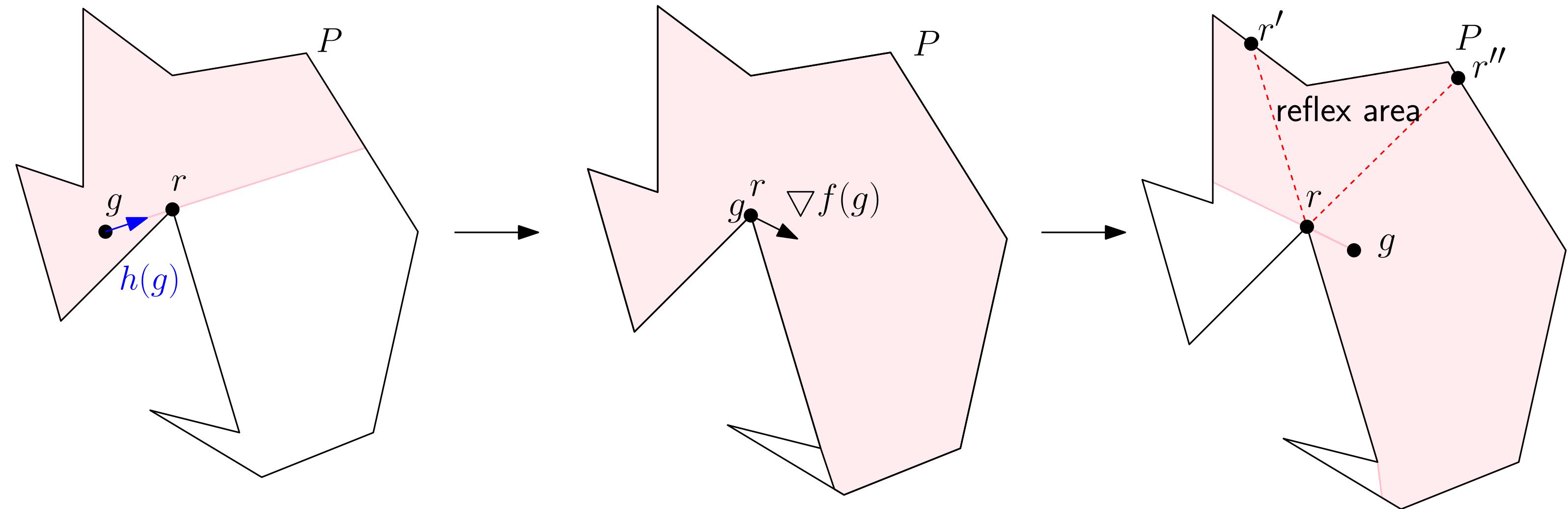
Heuristics: Reflex area



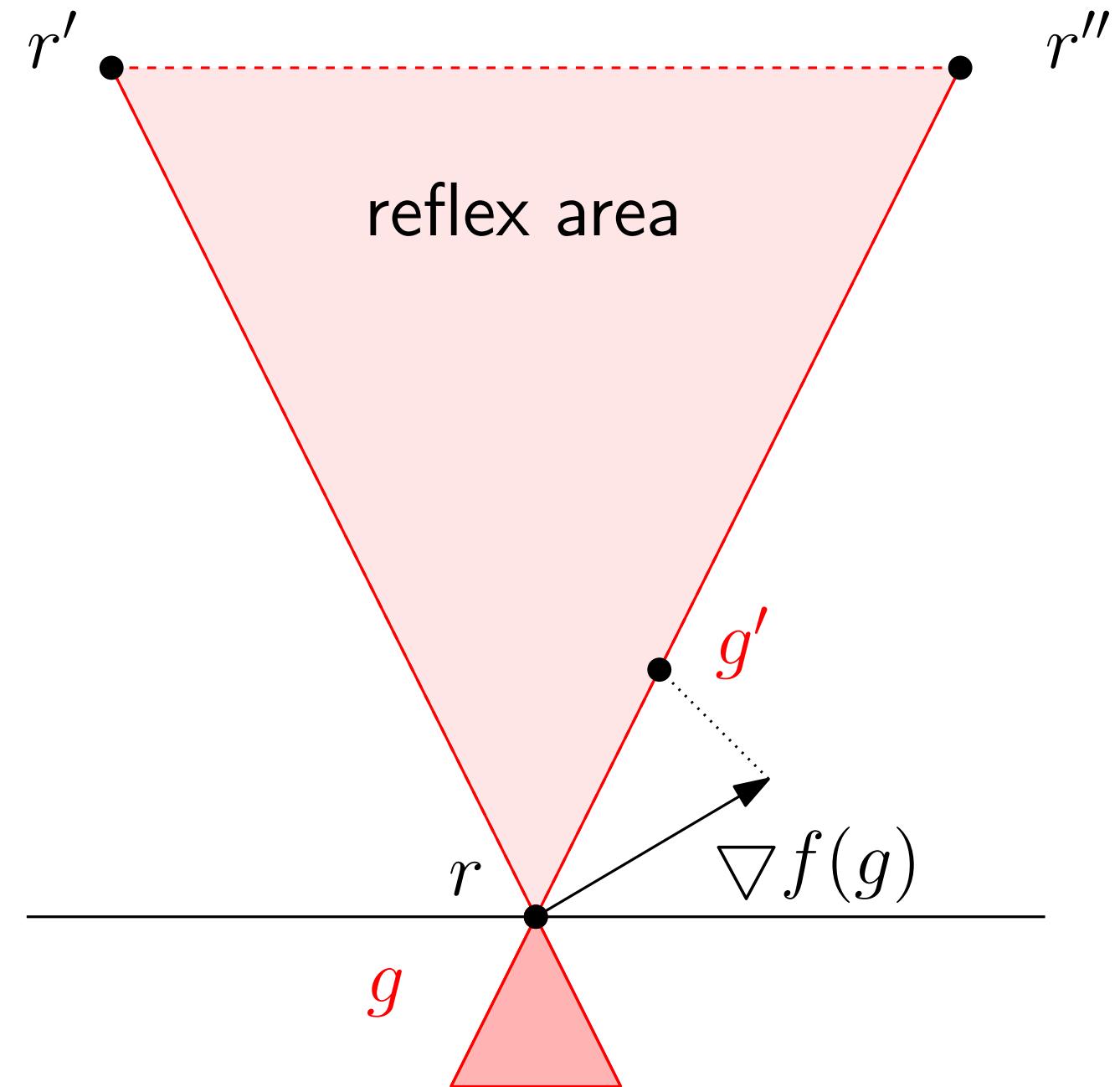
Heuristics: Reflex area



Heuristics: Reflex area

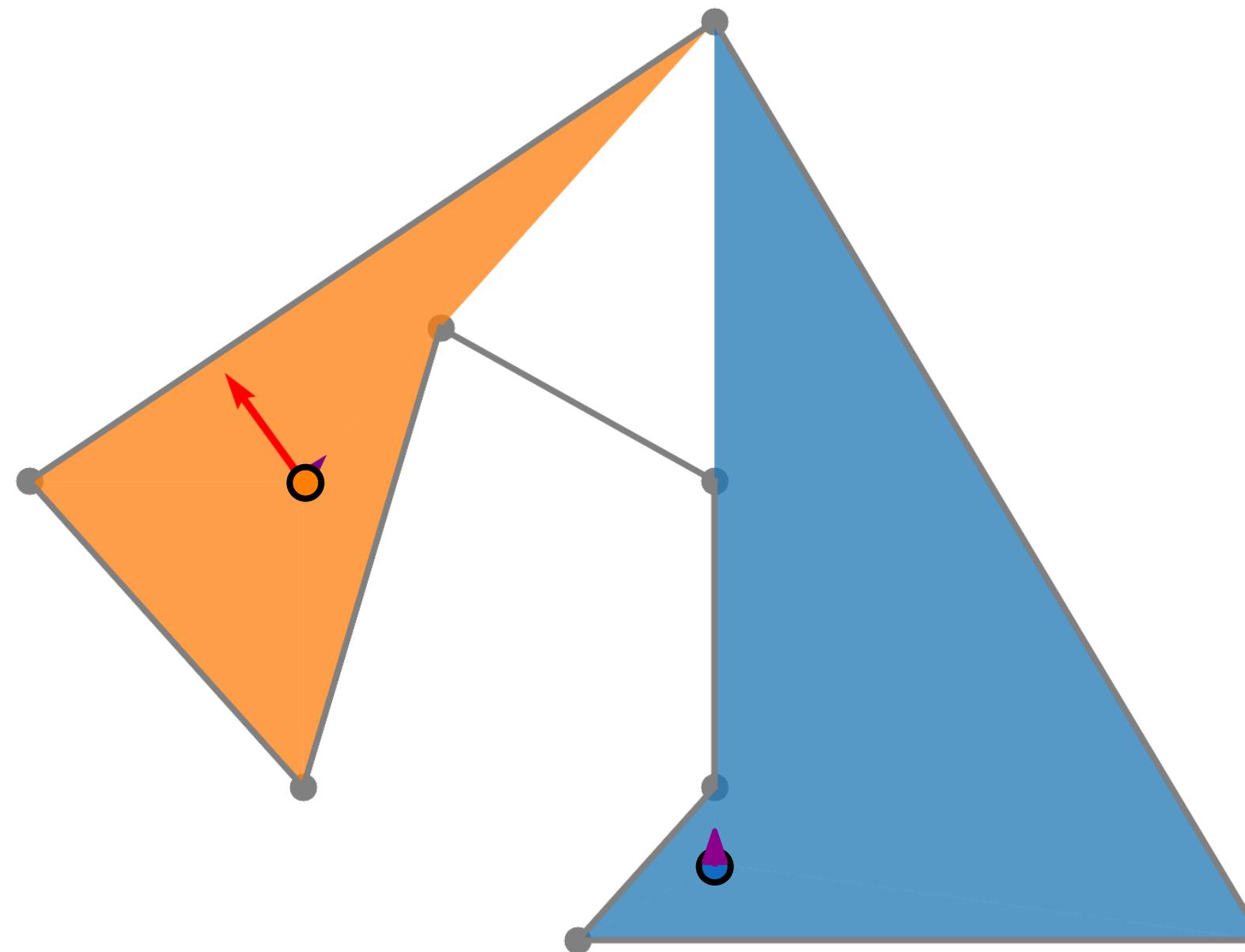


Heuristics: Reflex area



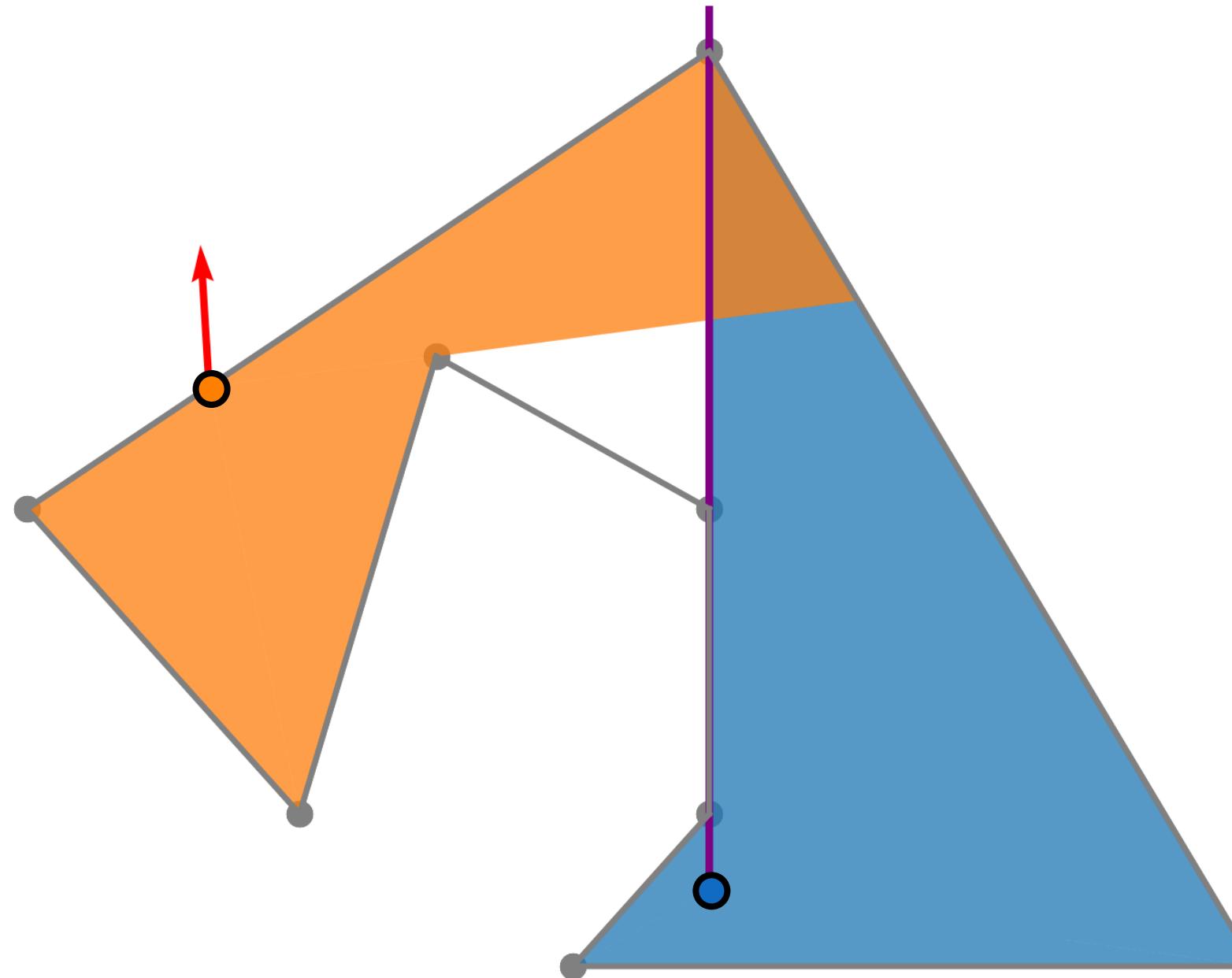
Heuristics: Reflex area

Gradient Computation for Iteration #0



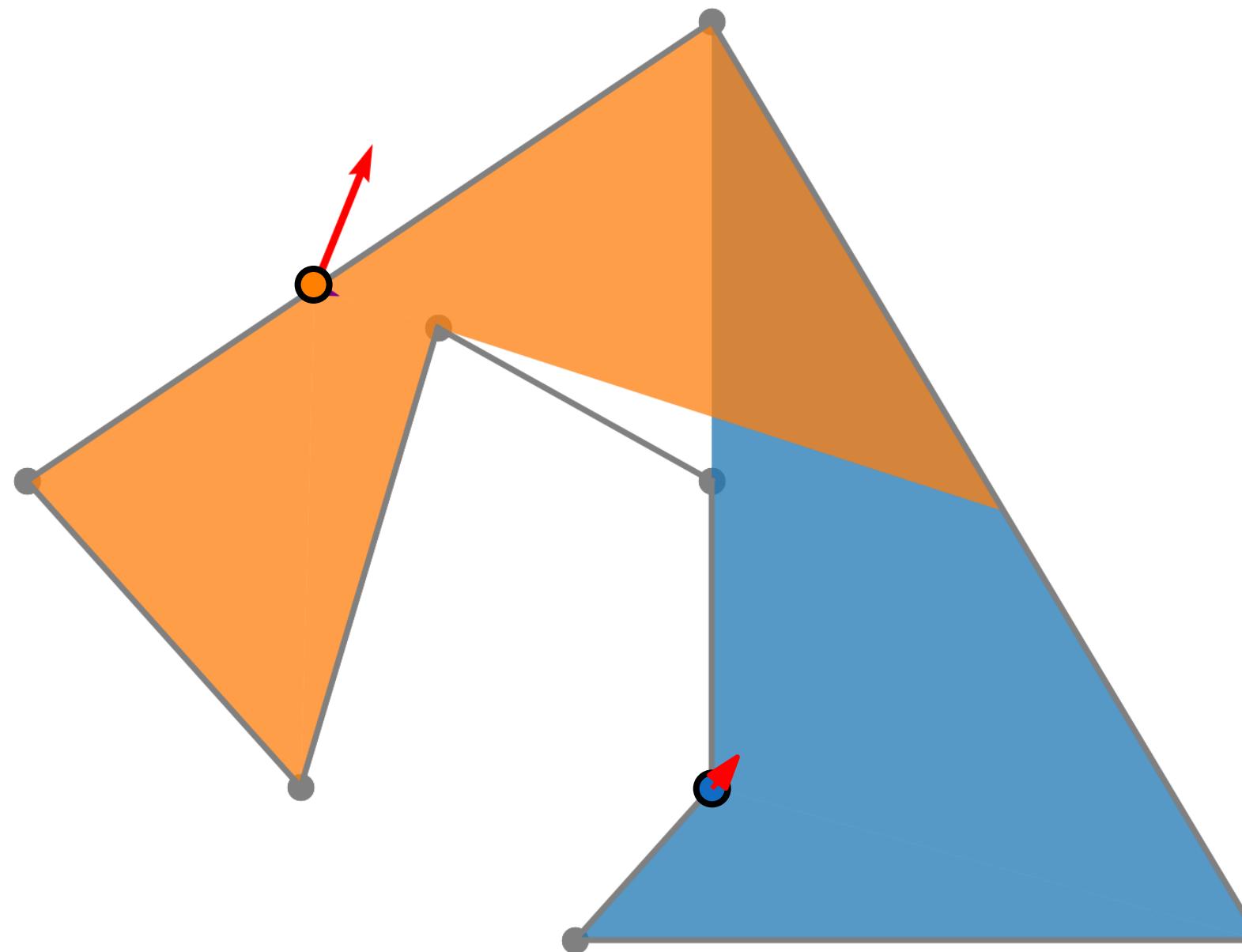
Heuristics: Reflex area

Gradient Computation for Iteration #1



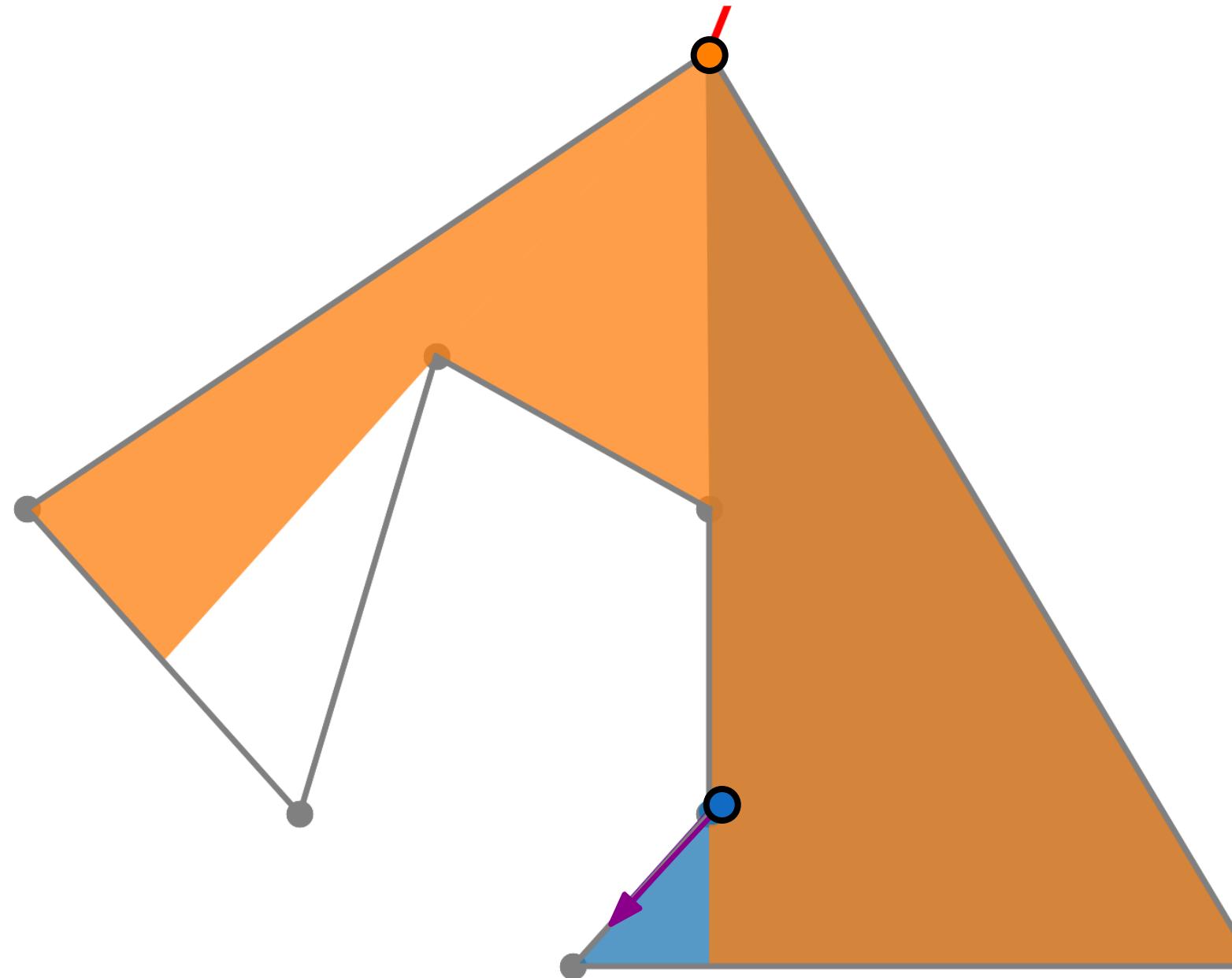
Heuristics: Reflex area

Gradient Computation for Iteration #2



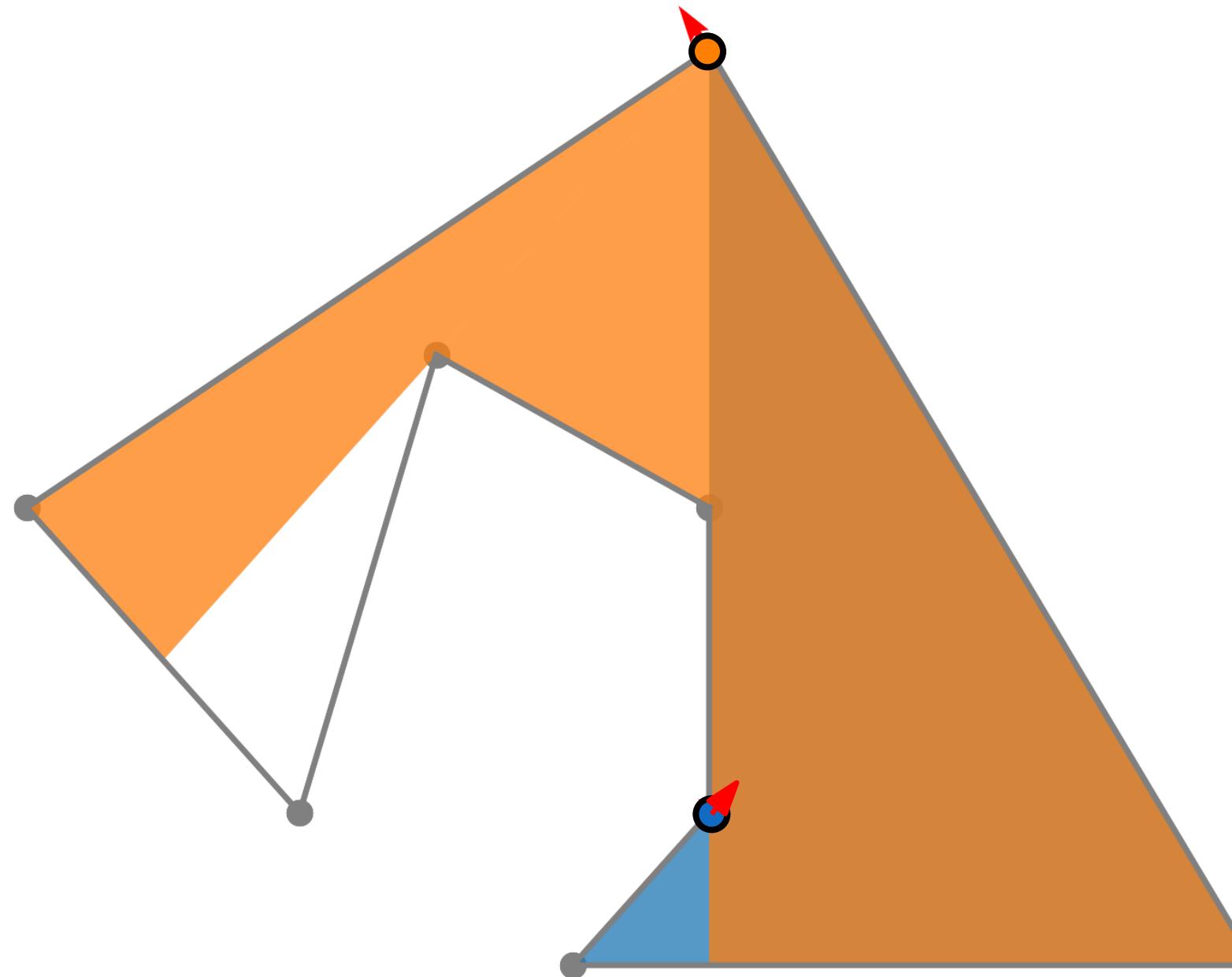
Heuristics: Reflex area

Gradient Computation for Iteration #3



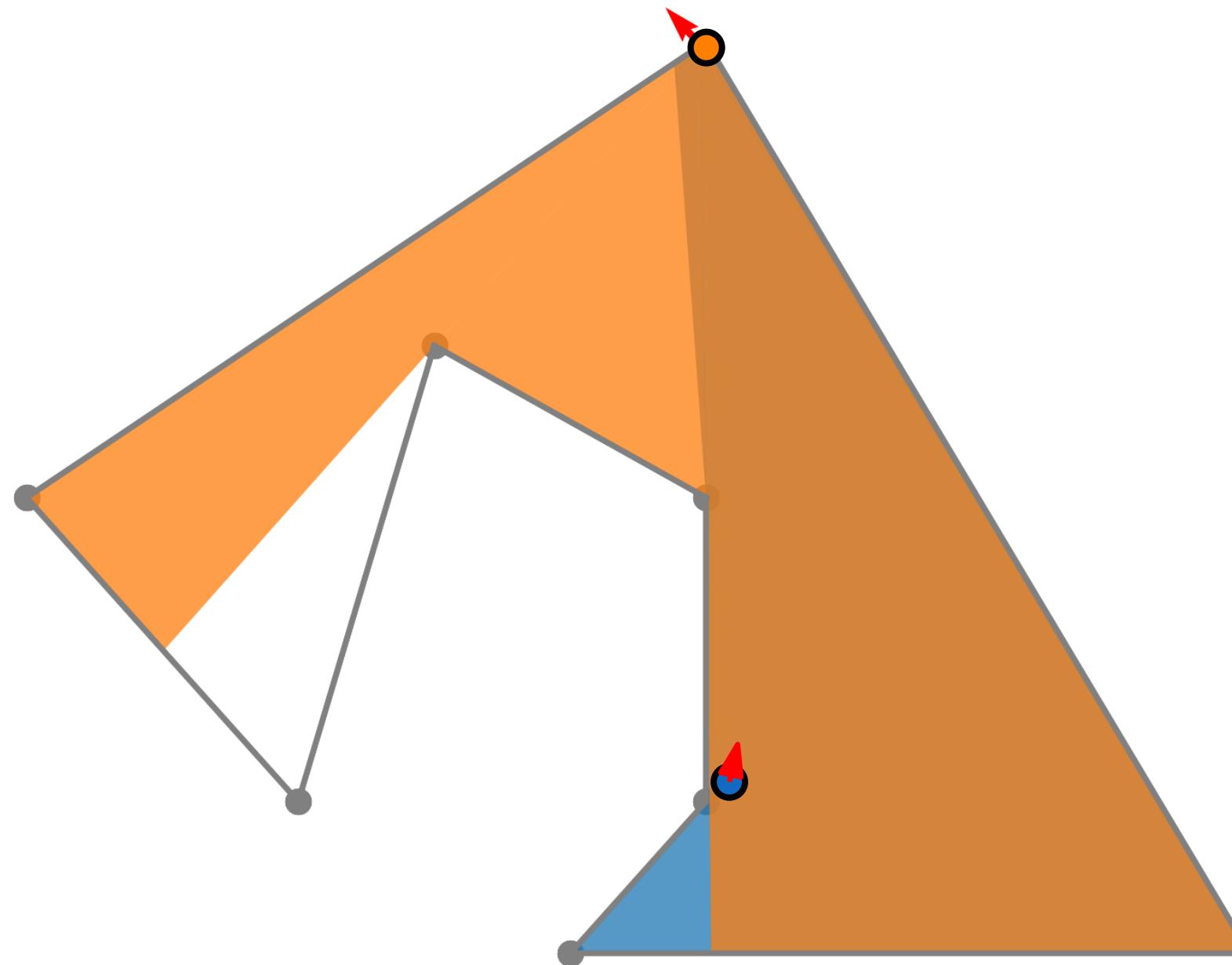
Heuristics: Reflex area

Gradient Computation for Iteration #4



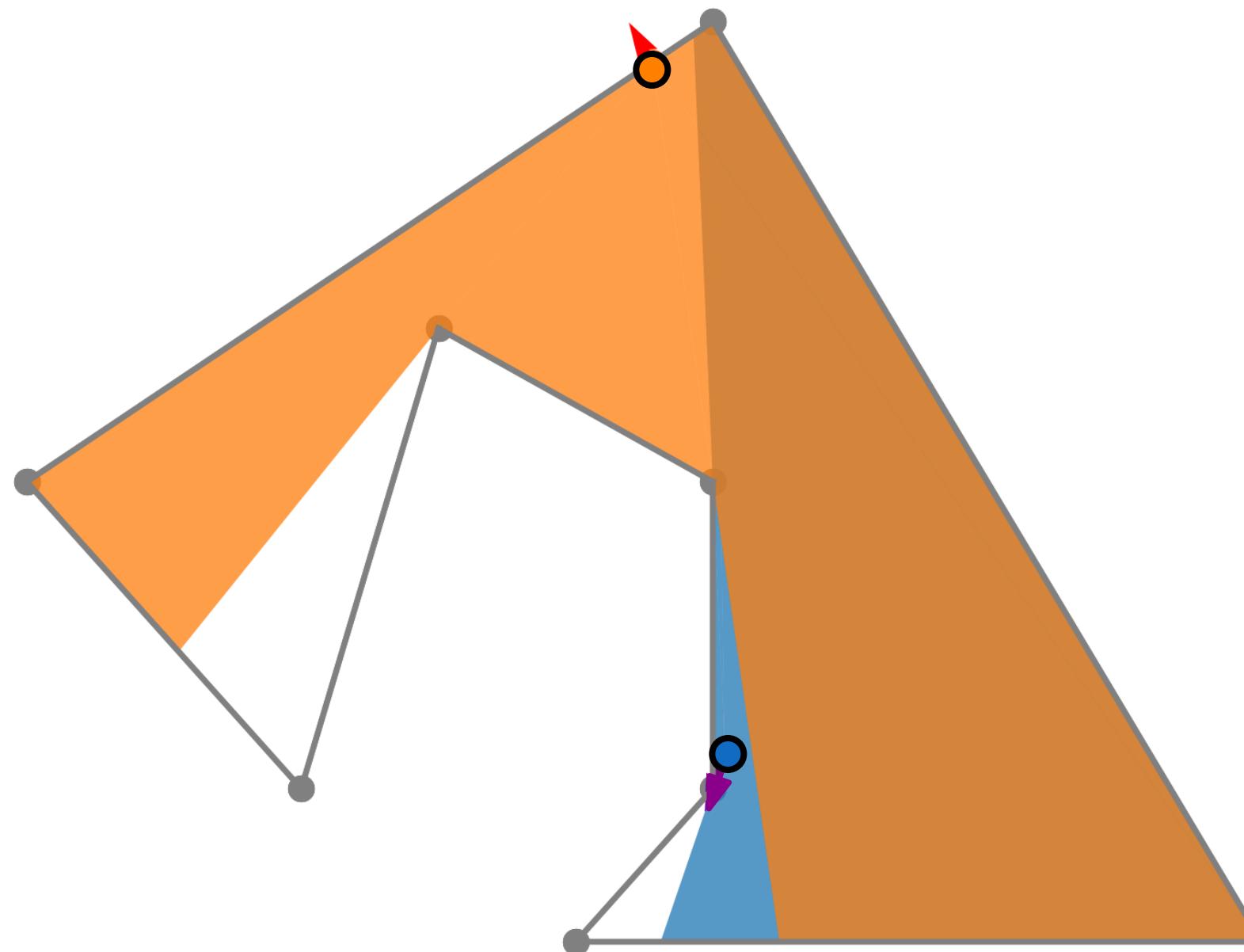
Heuristics: Reflex area

Gradient Computation for Iteration #5



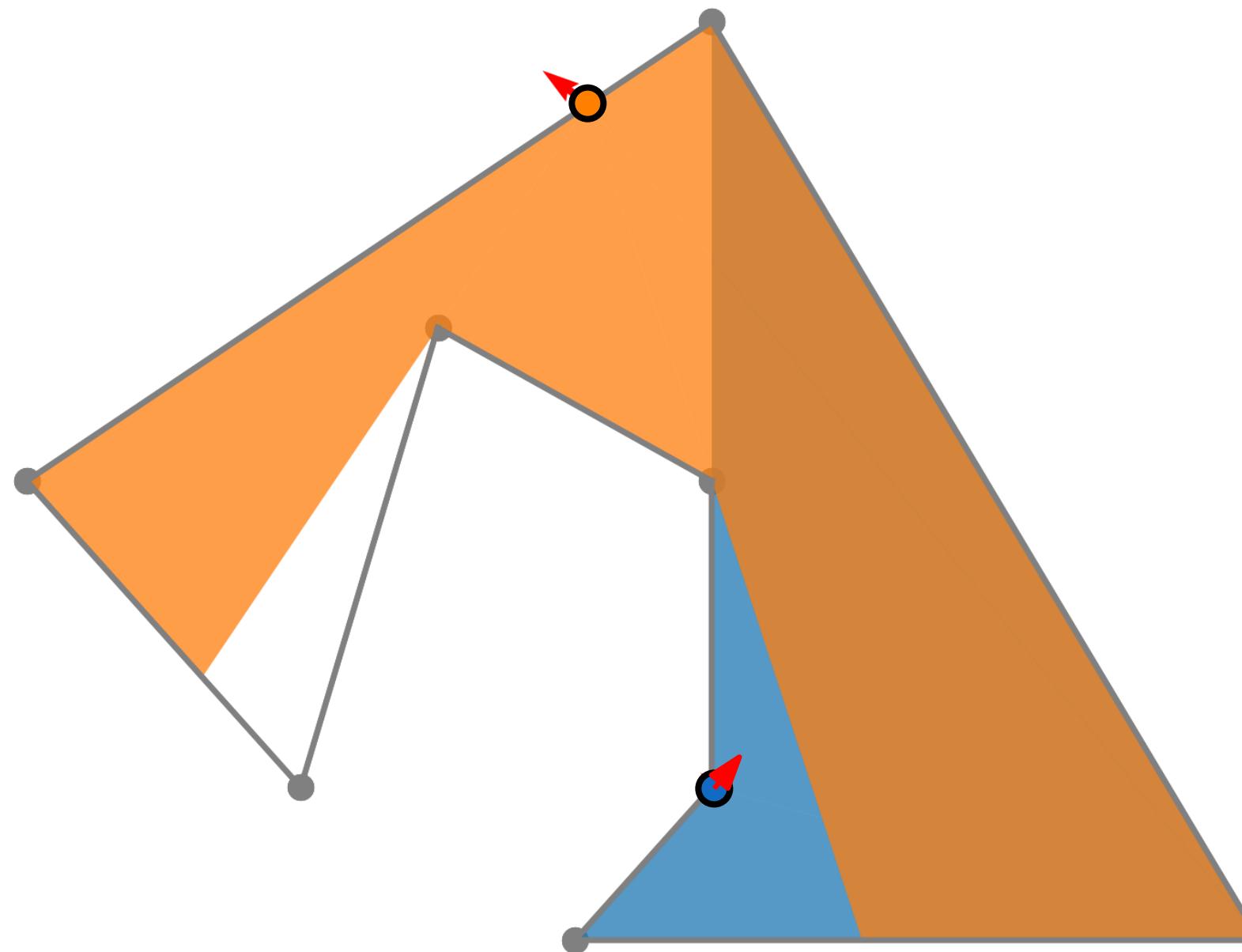
Heuristics: Reflex area

Gradient Computation for Iteration #6



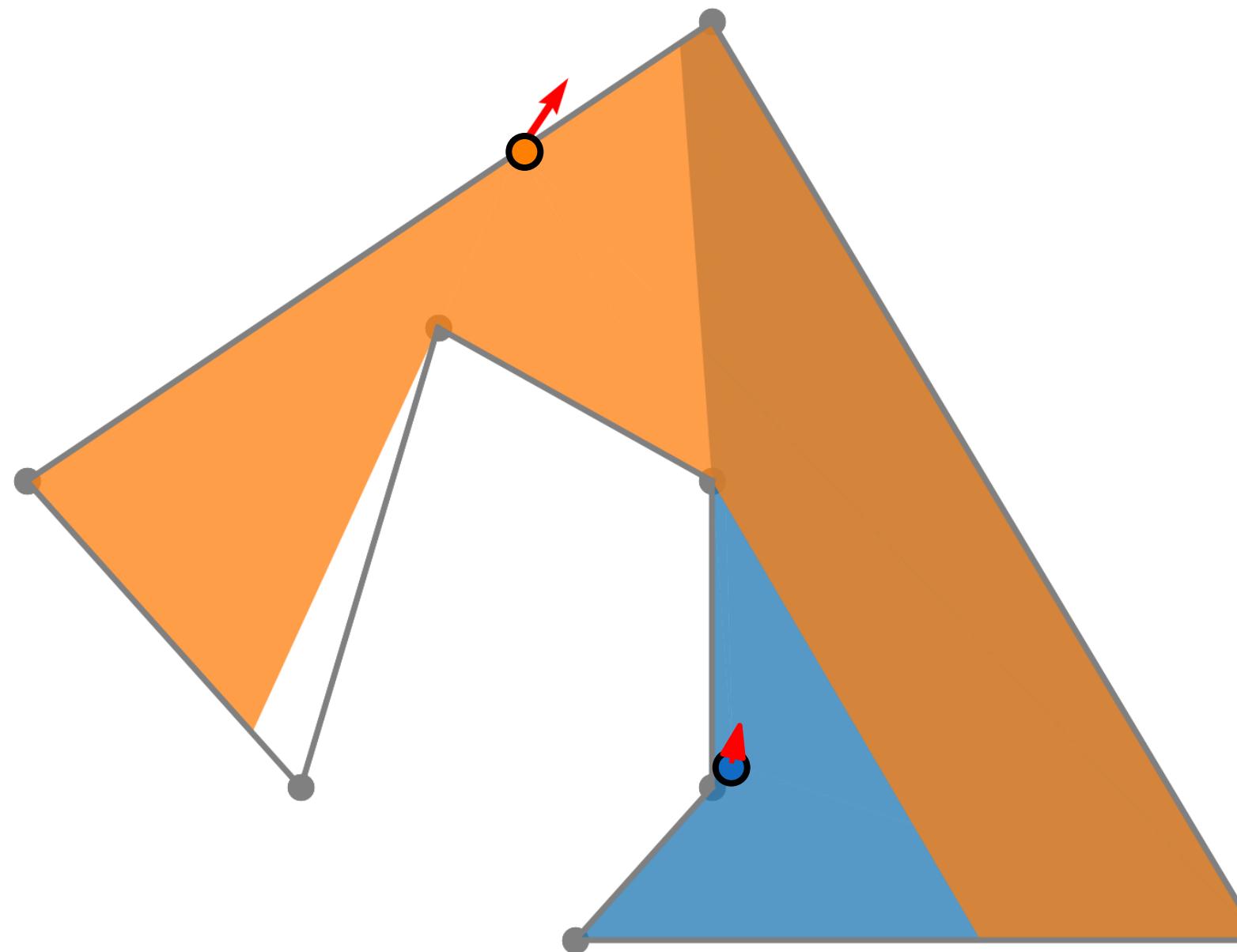
Heuristics: Reflex area

Gradient Computation for Iteration #7



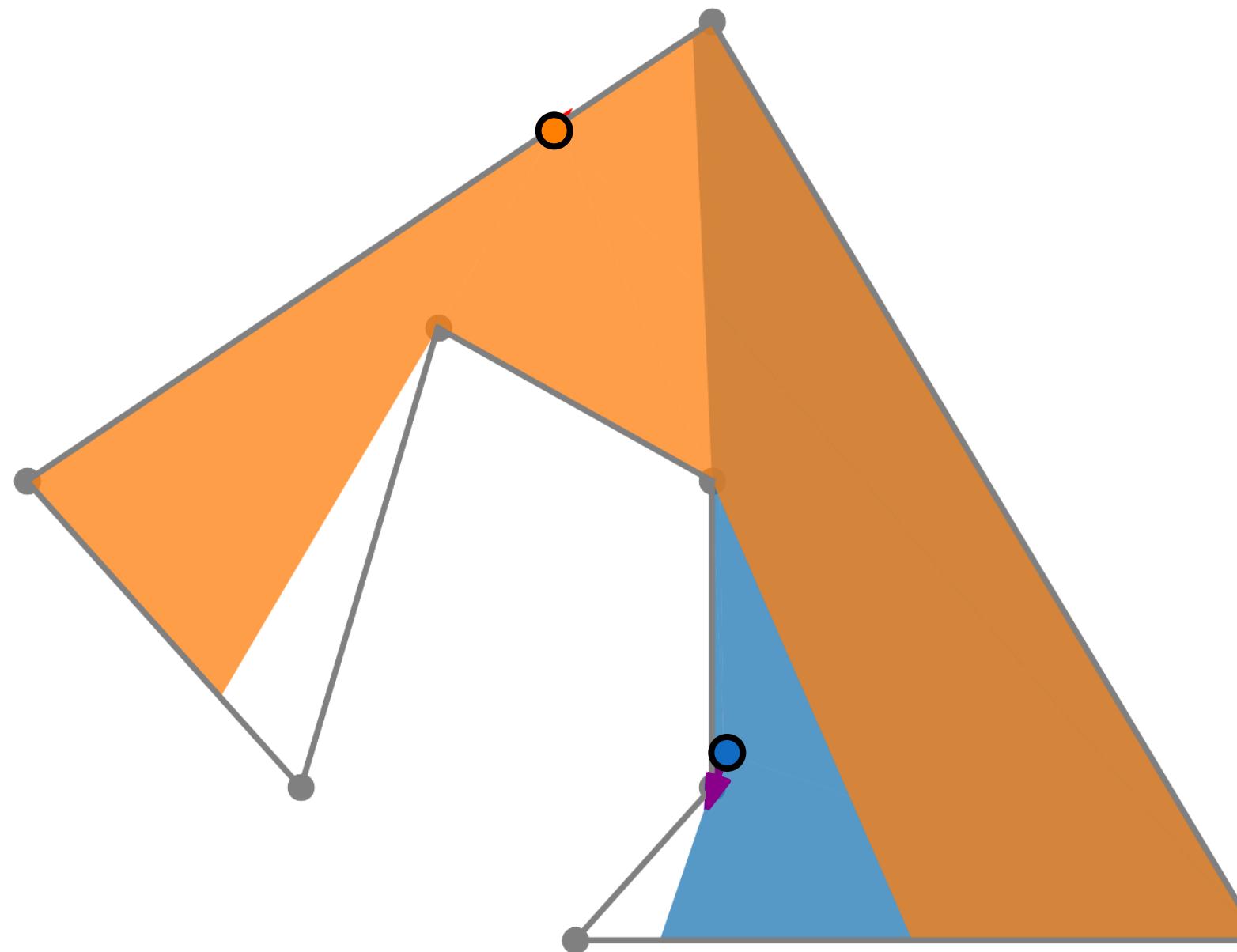
Heuristics: Reflex area

Gradient Computation for Iteration #8



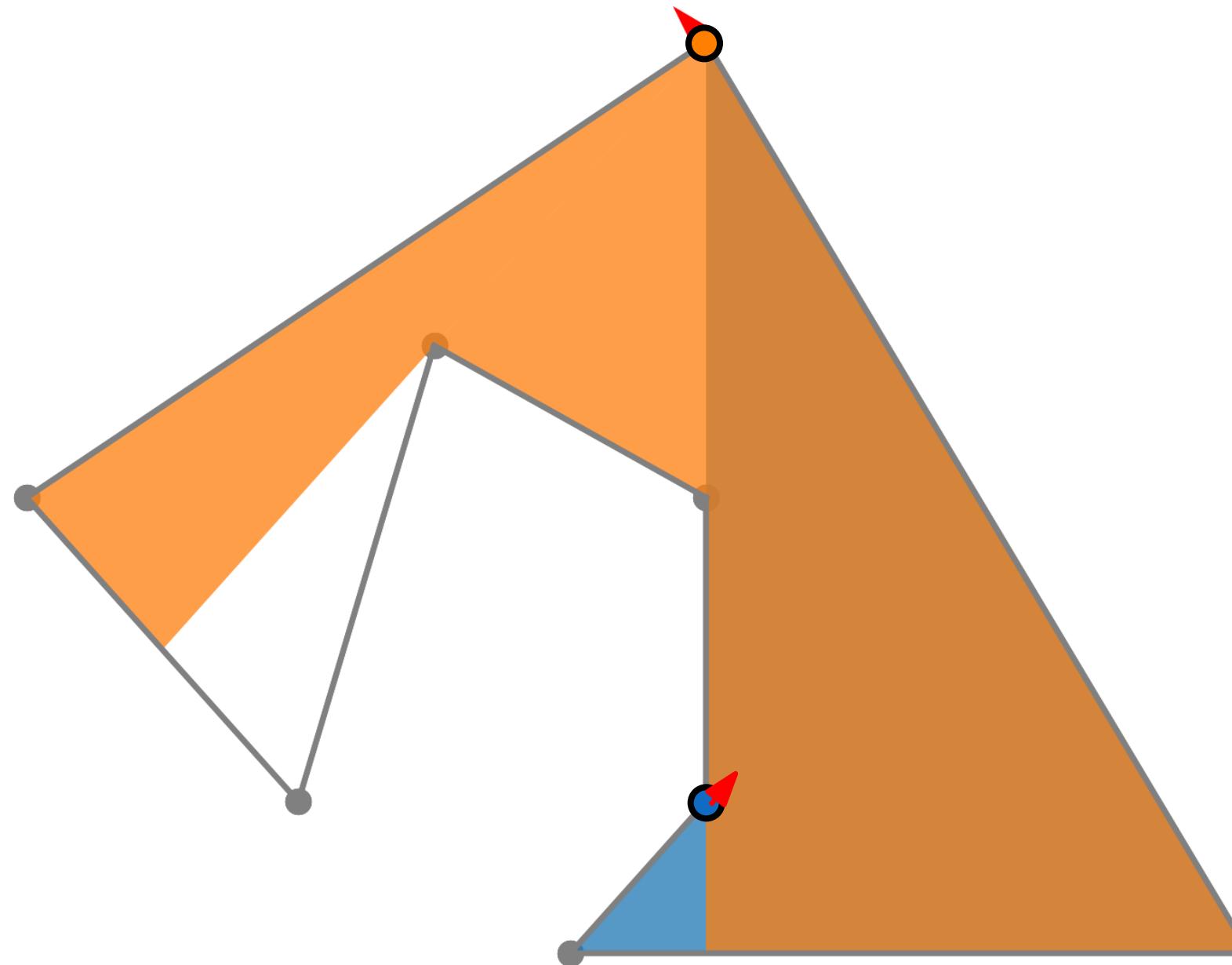
Heuristics: Reflex area

Gradient Computation for Iteration #9



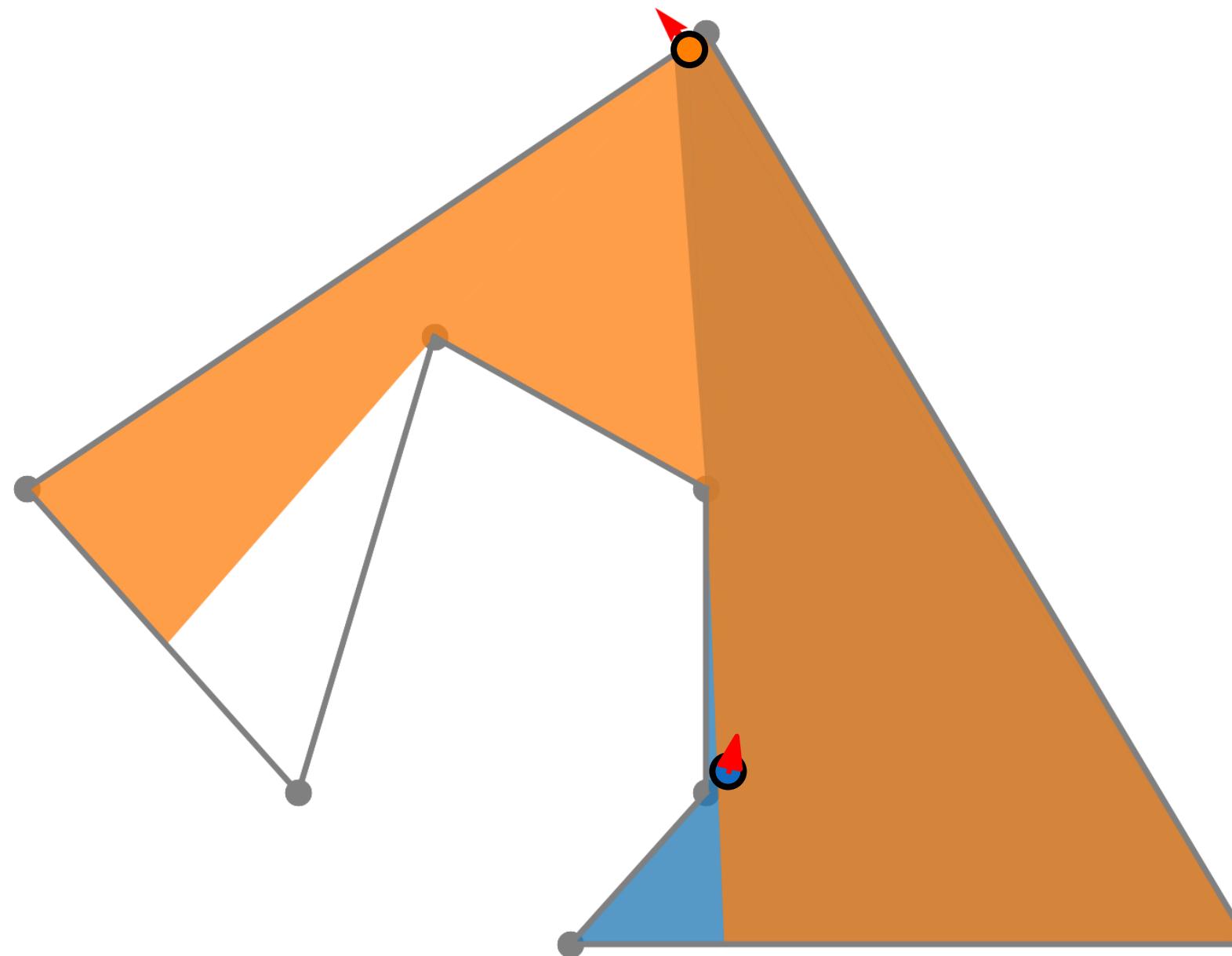
Heuristics: Reflex area

Gradient Computation for Iteration #10



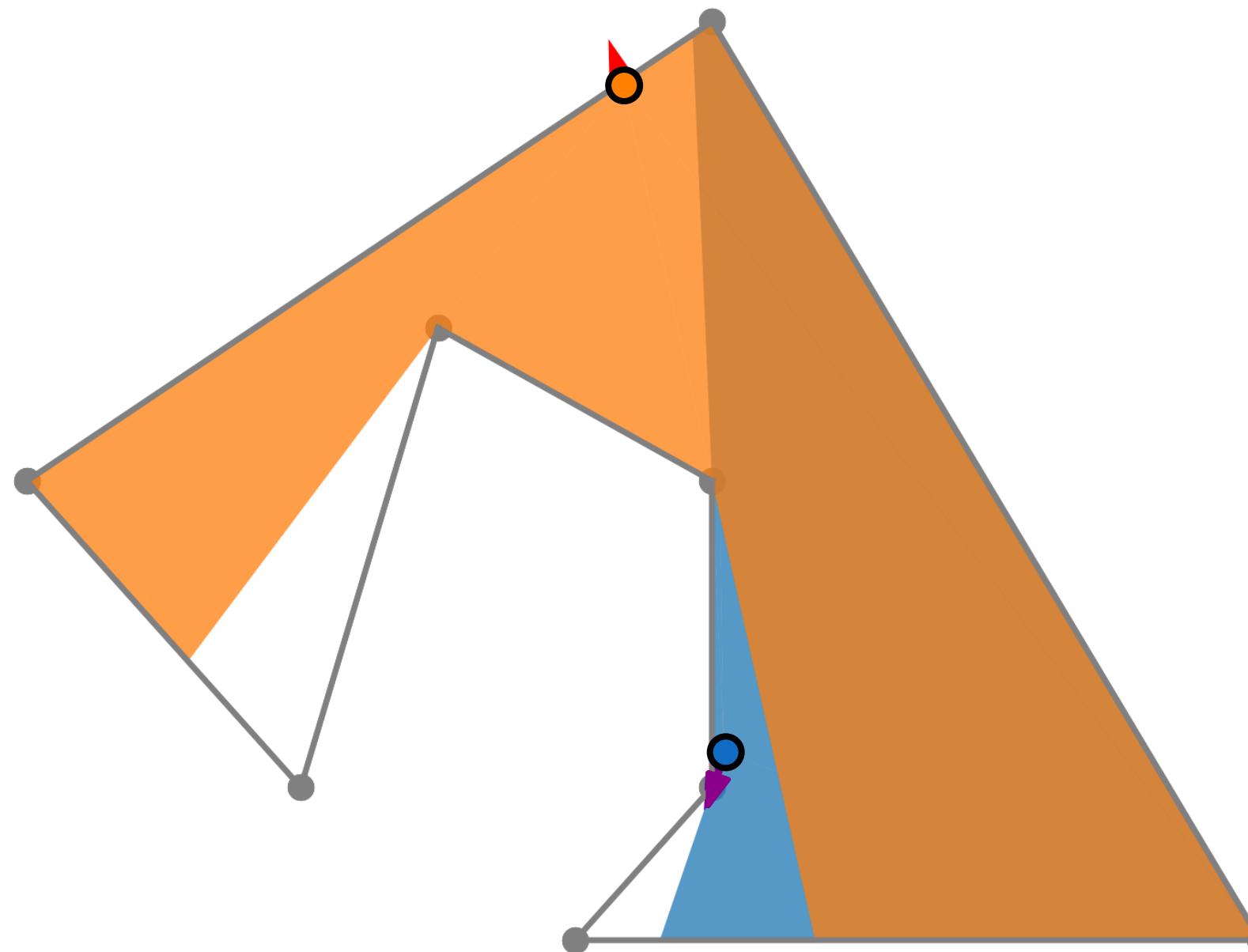
Heuristics: Reflex area

Gradient Computation for Iteration #11



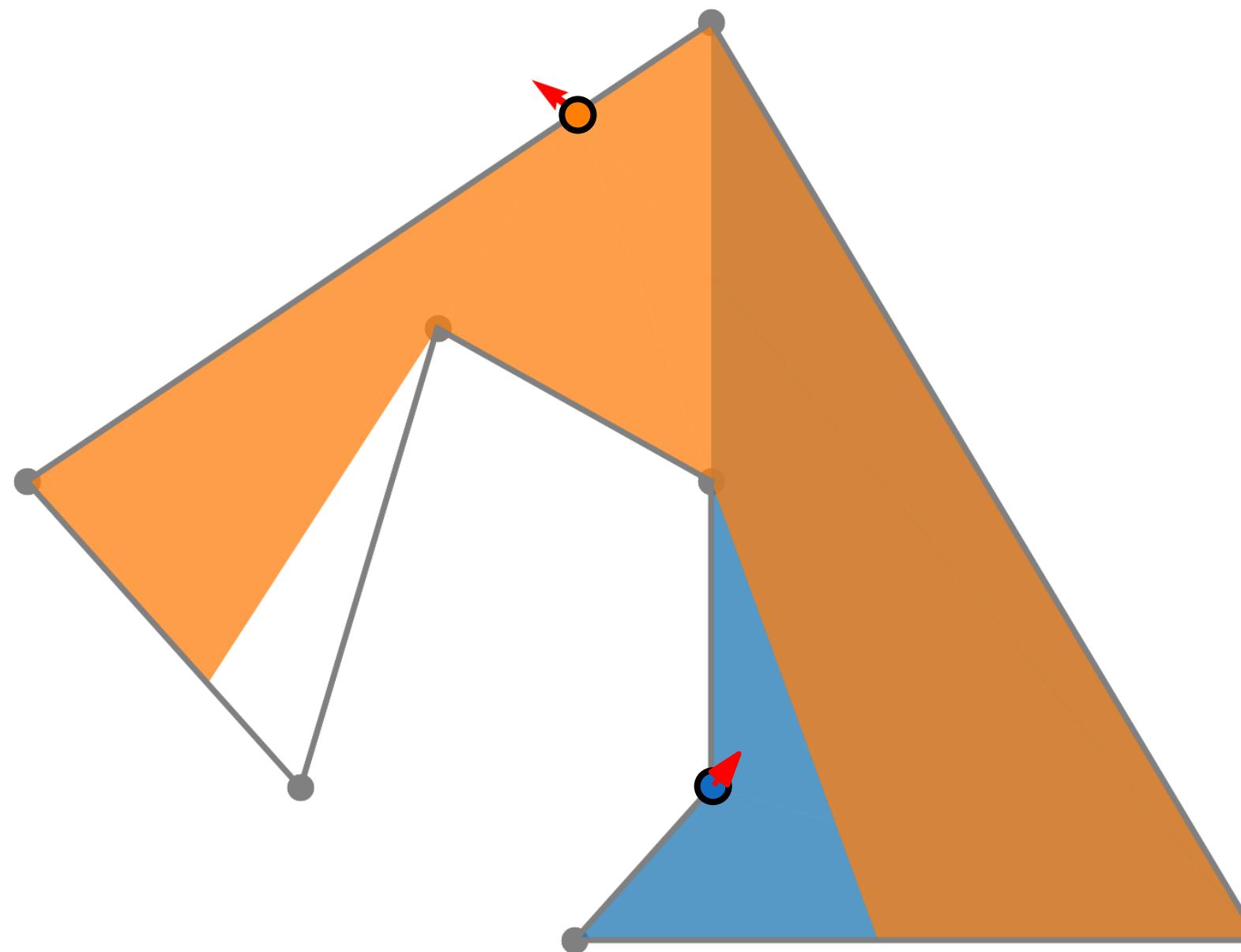
Heuristics: Reflex area

Gradient Computation for Iteration #12



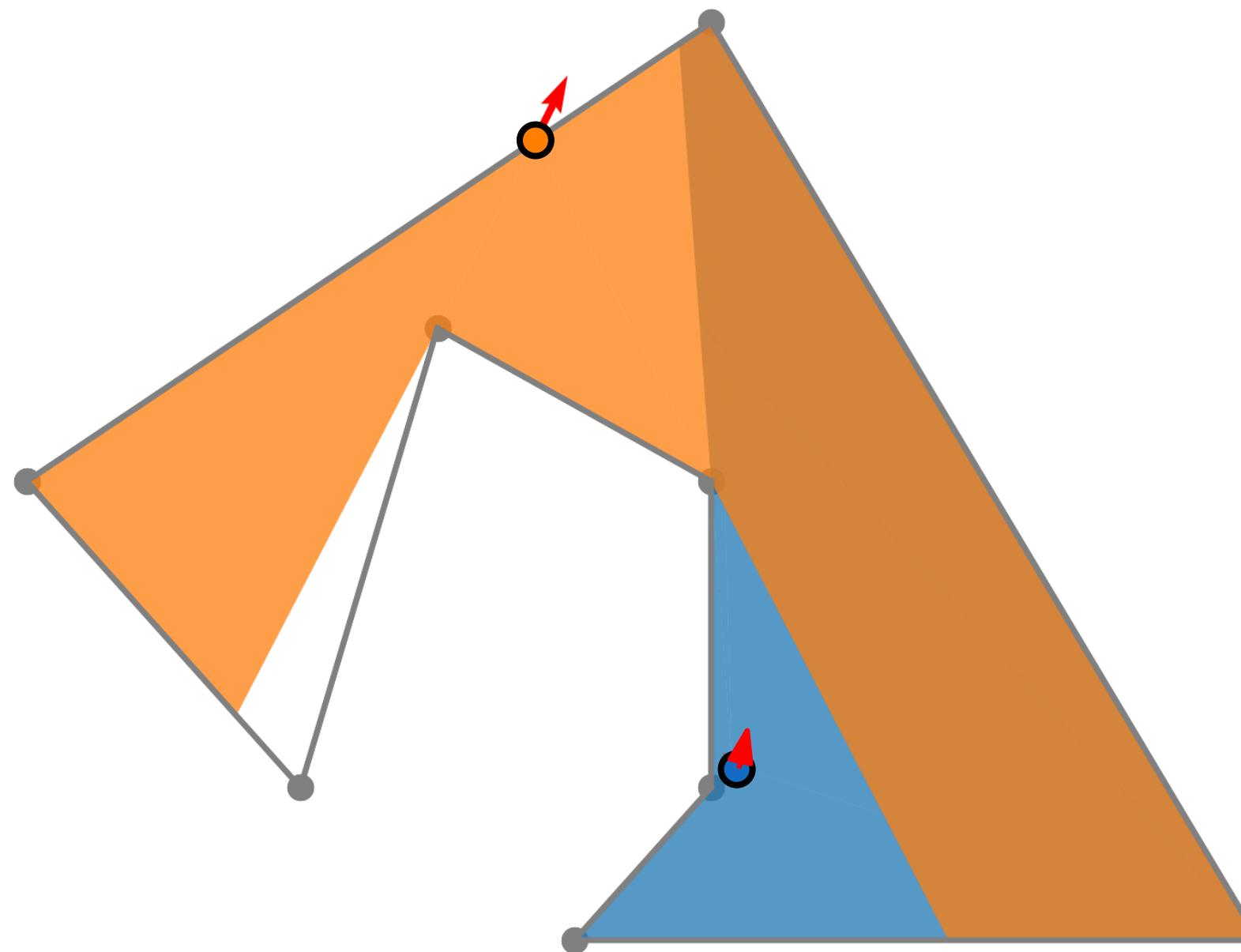
Heuristics: Reflex area

Gradient Computation for Iteration #13

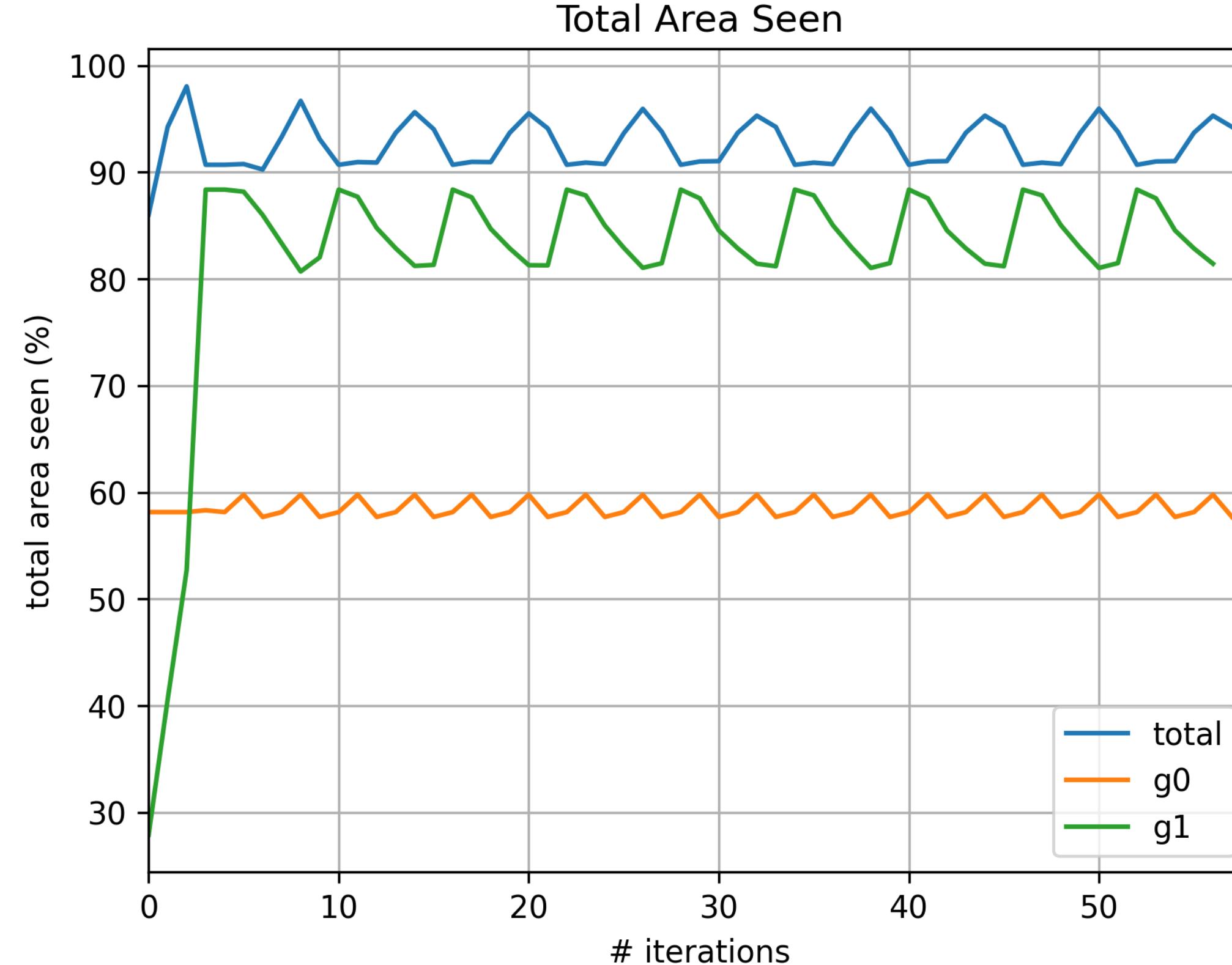


Heuristics: Reflex area

Gradient Computation for Iteration #14

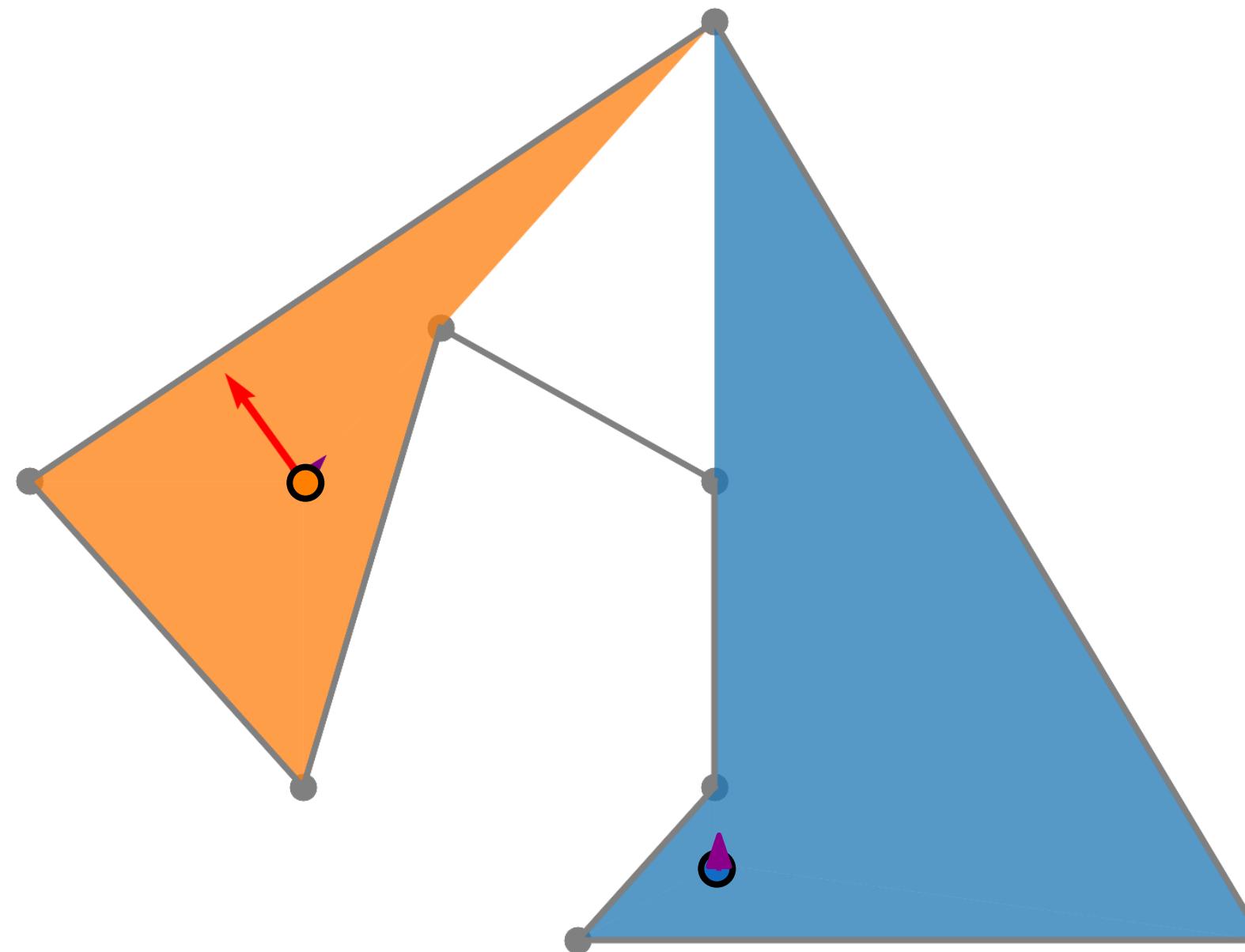


Heuristics: Reflex area



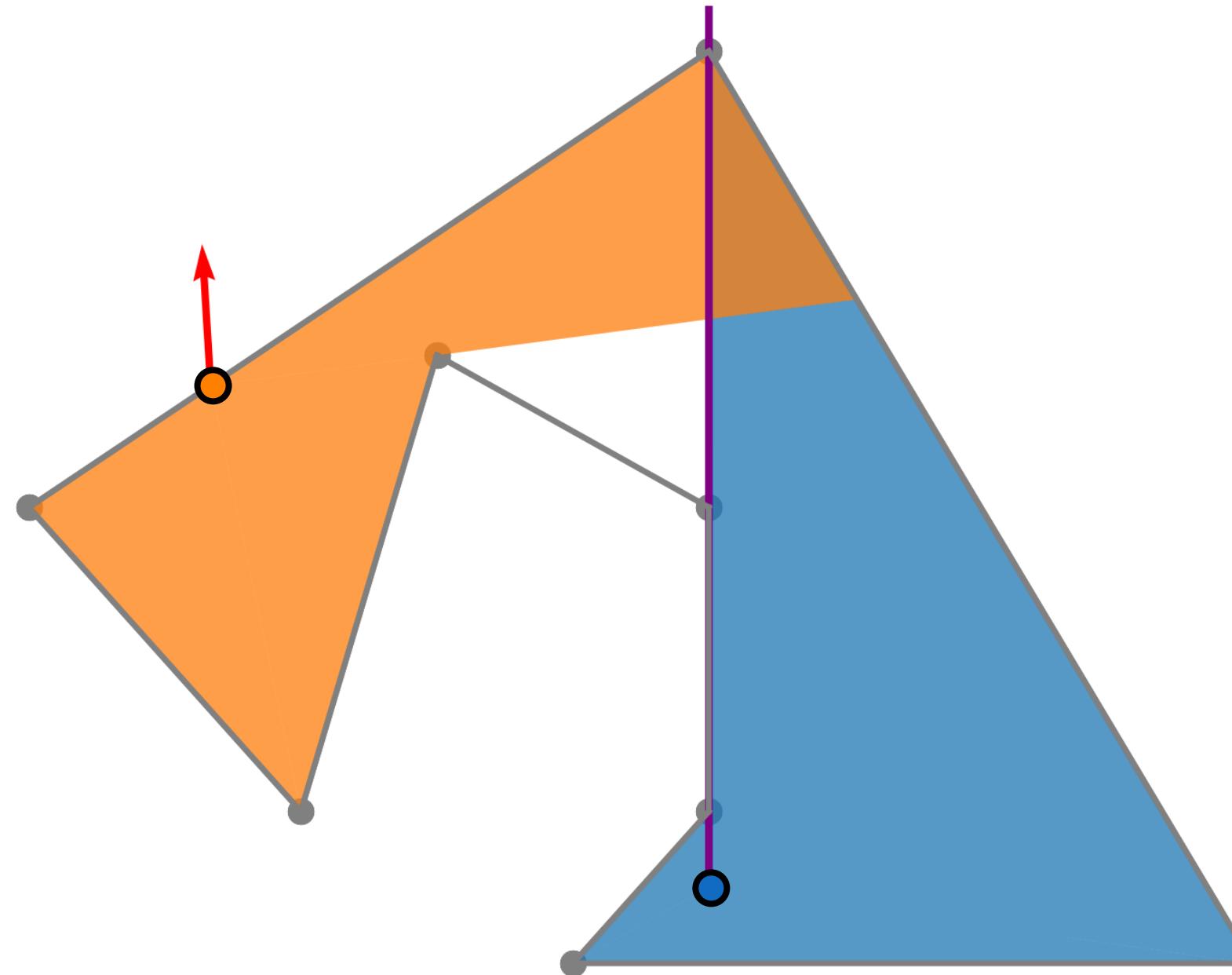
Heuristics: Reflex area

Gradient Computation for Iteration #0



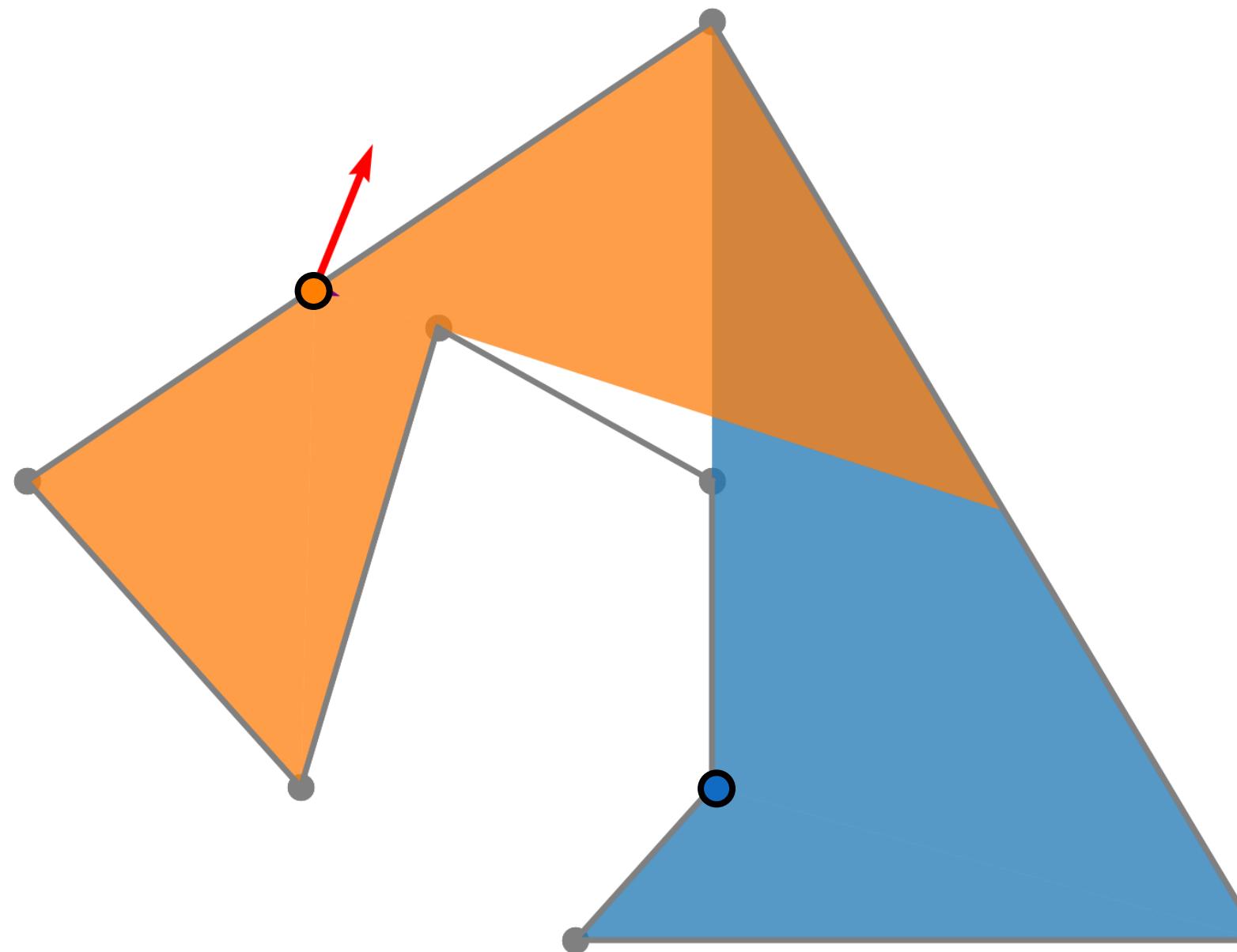
Heuristics: Reflex area

Gradient Computation for Iteration #1



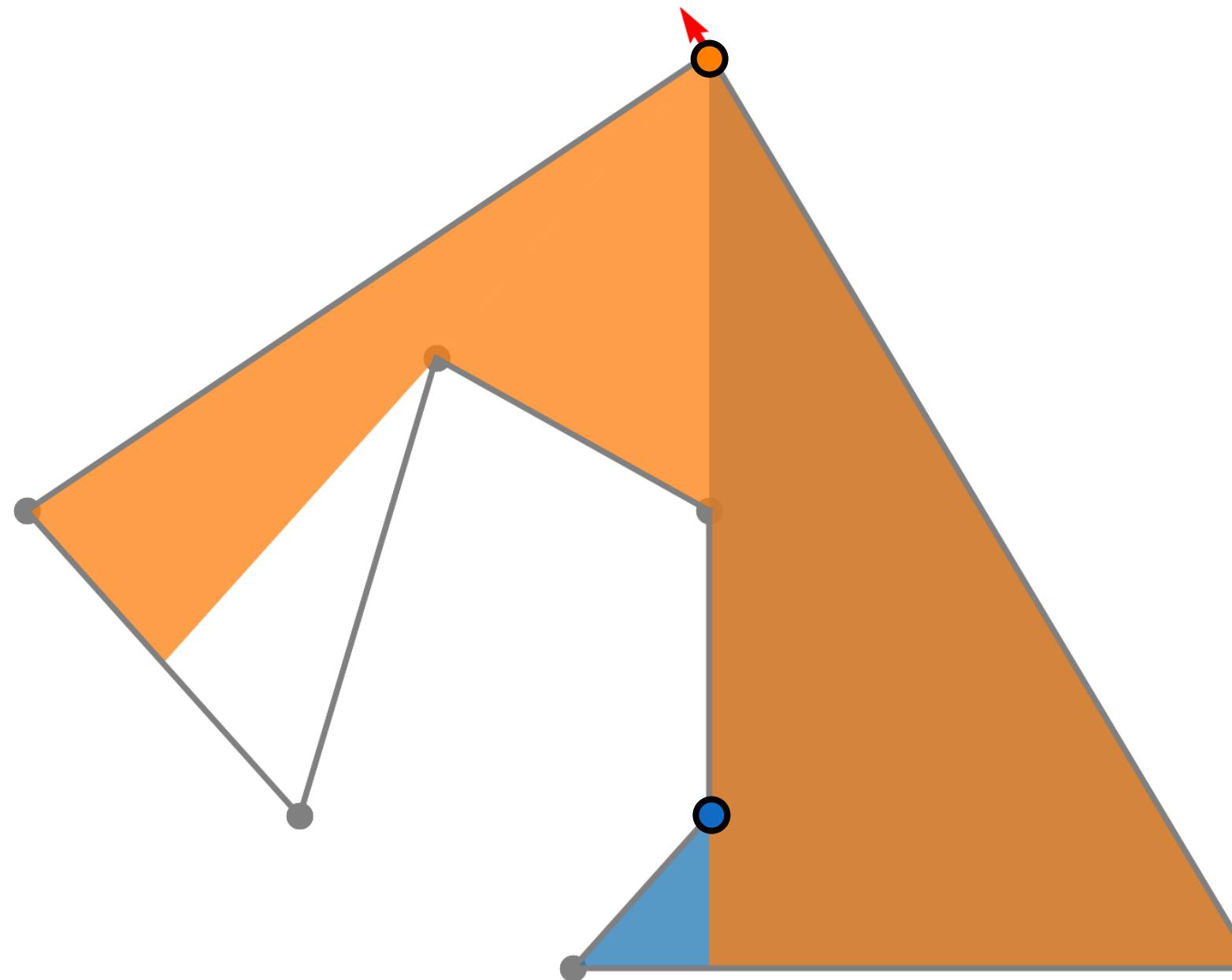
Heuristics: Reflex area

Gradient Computation for Iteration #2



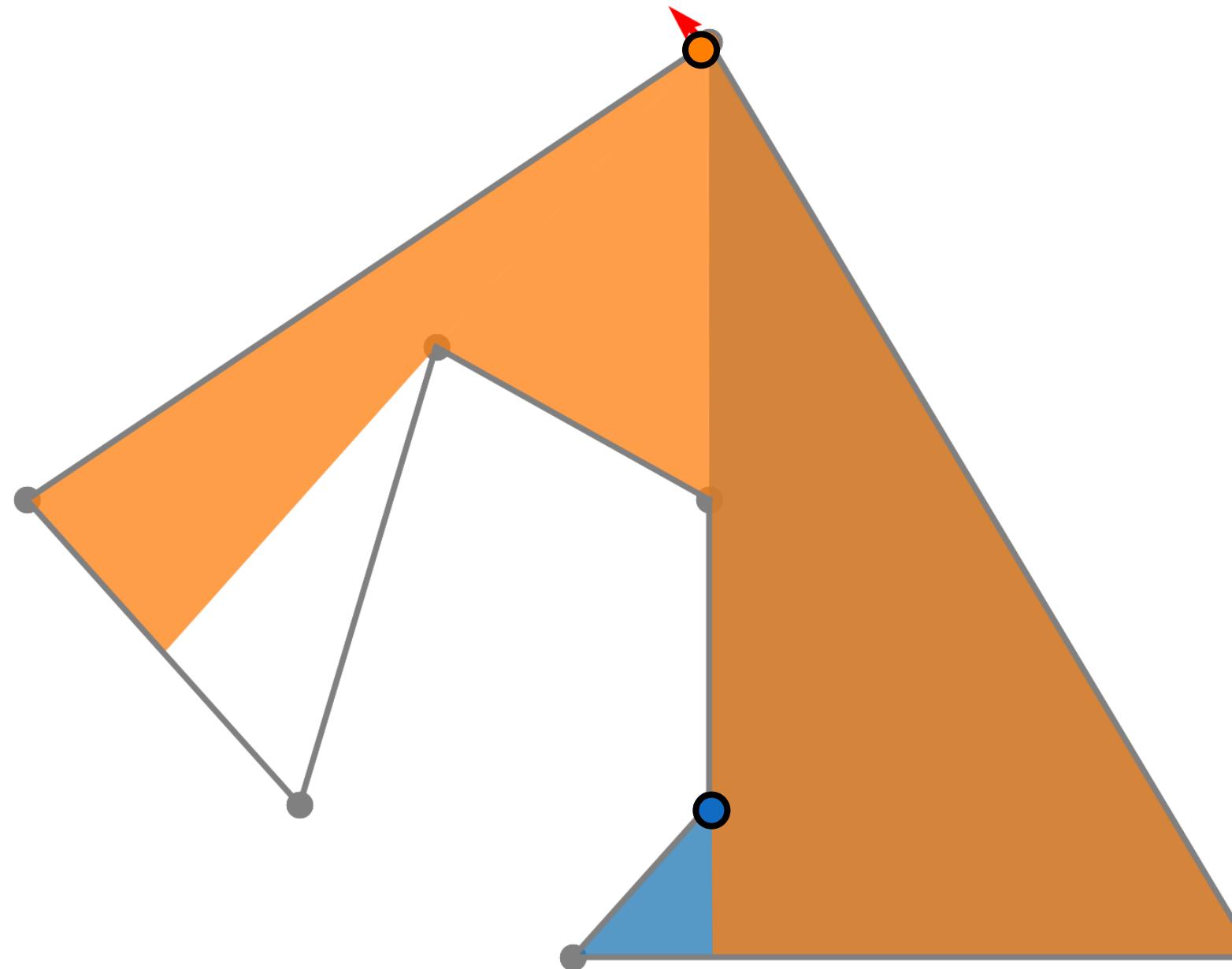
Heuristics: Reflex area

Gradient Computation for Iteration #3



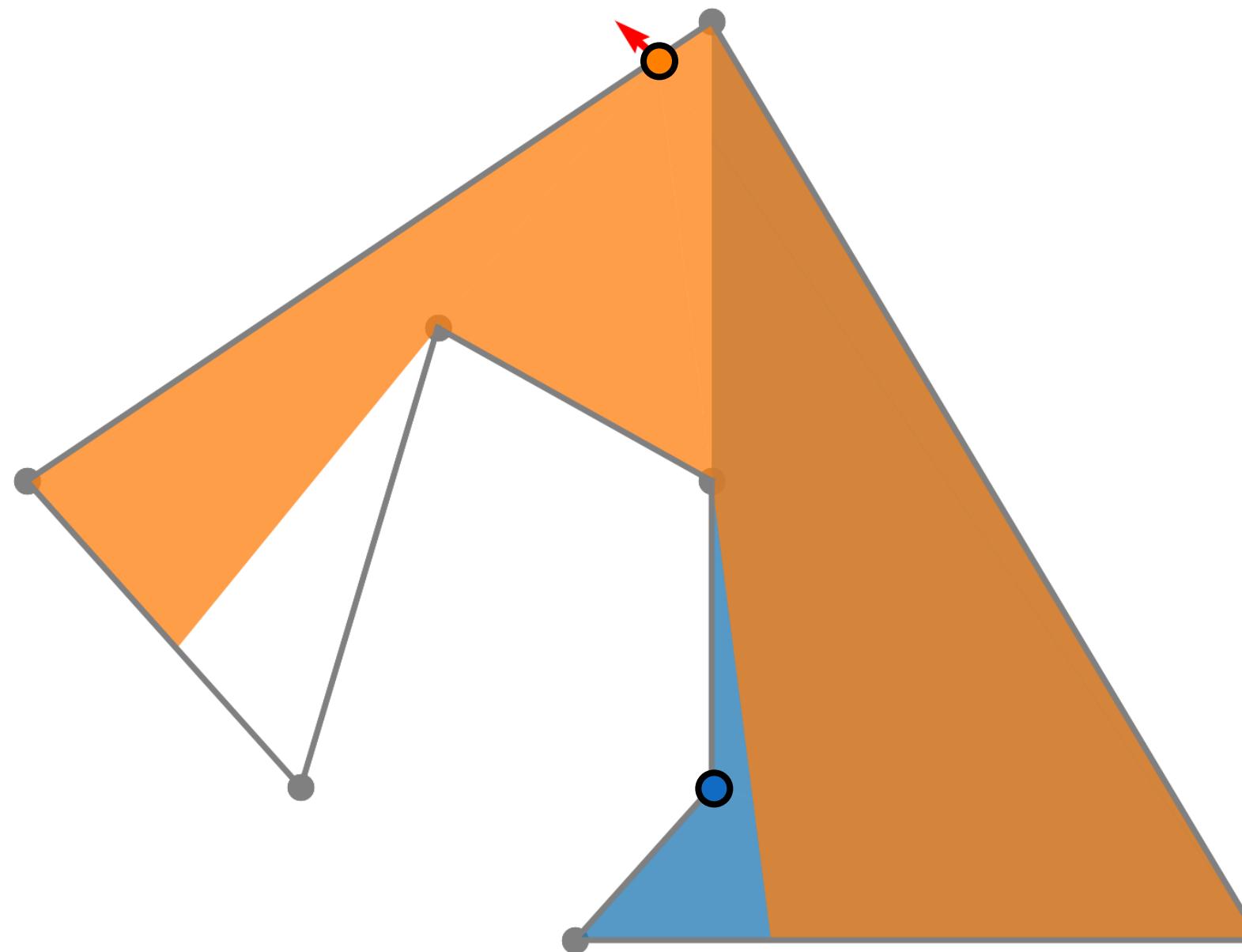
Heuristics: Reflex area

Gradient Computation for Iteration #4



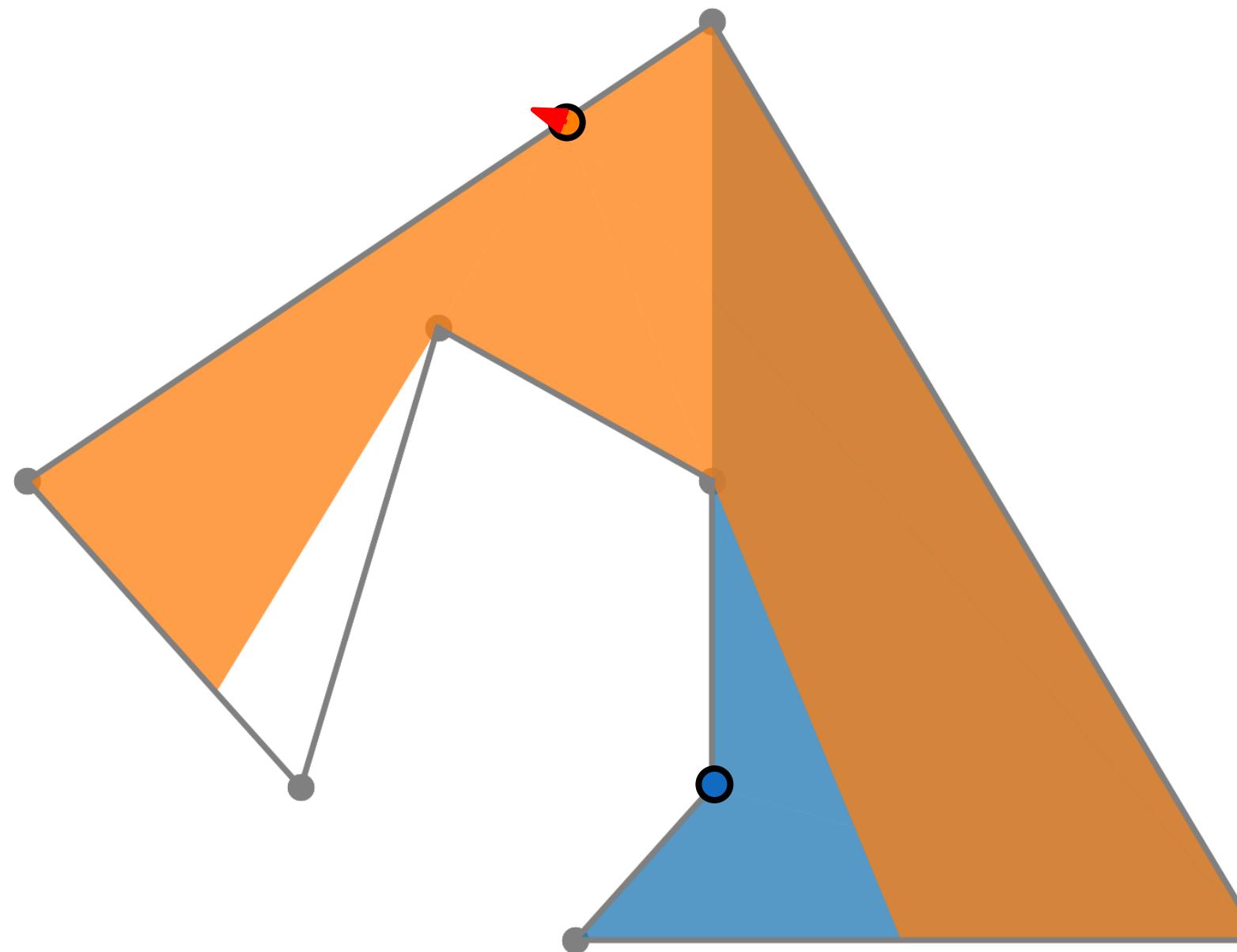
Heuristics: Reflex area

Gradient Computation for Iteration #5



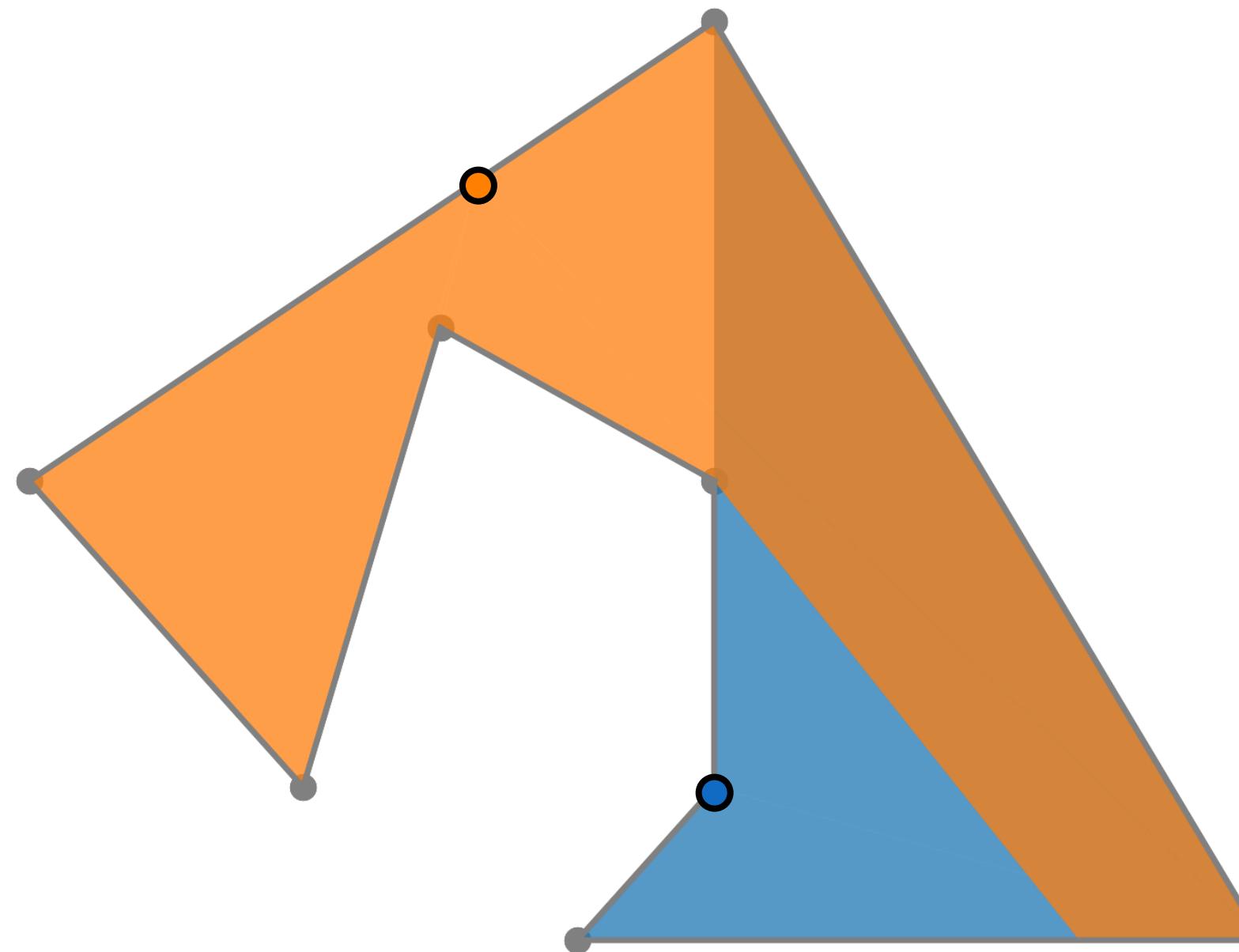
Heuristics: Reflex area

Gradient Computation for Iteration #6



Heuristics: Reflex area

Gradient Computation for Iteration #7



Heuristics

momentum

pull towards reflex vertex

pull onto the reflex vertex

pull capping

reflex area

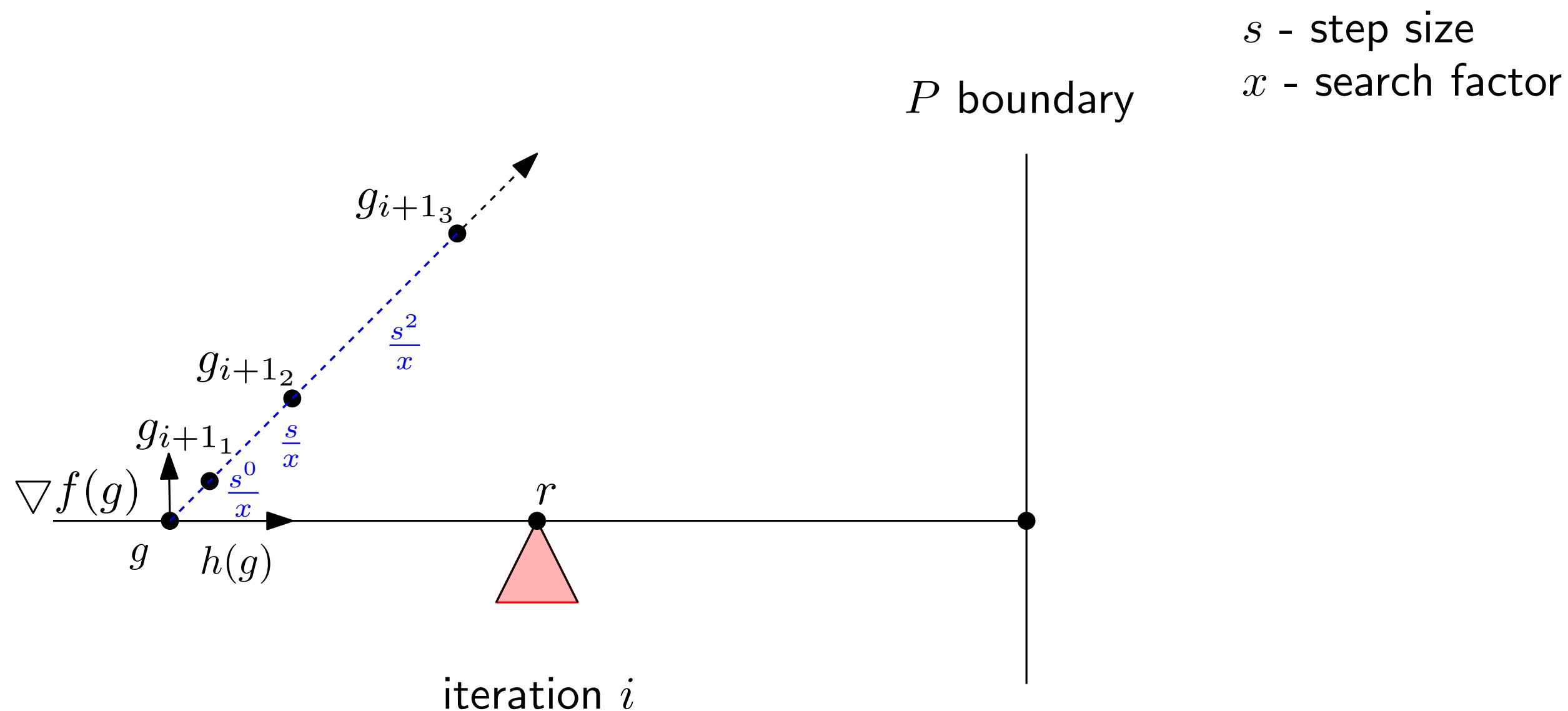
line search

angle behind reflex vertex

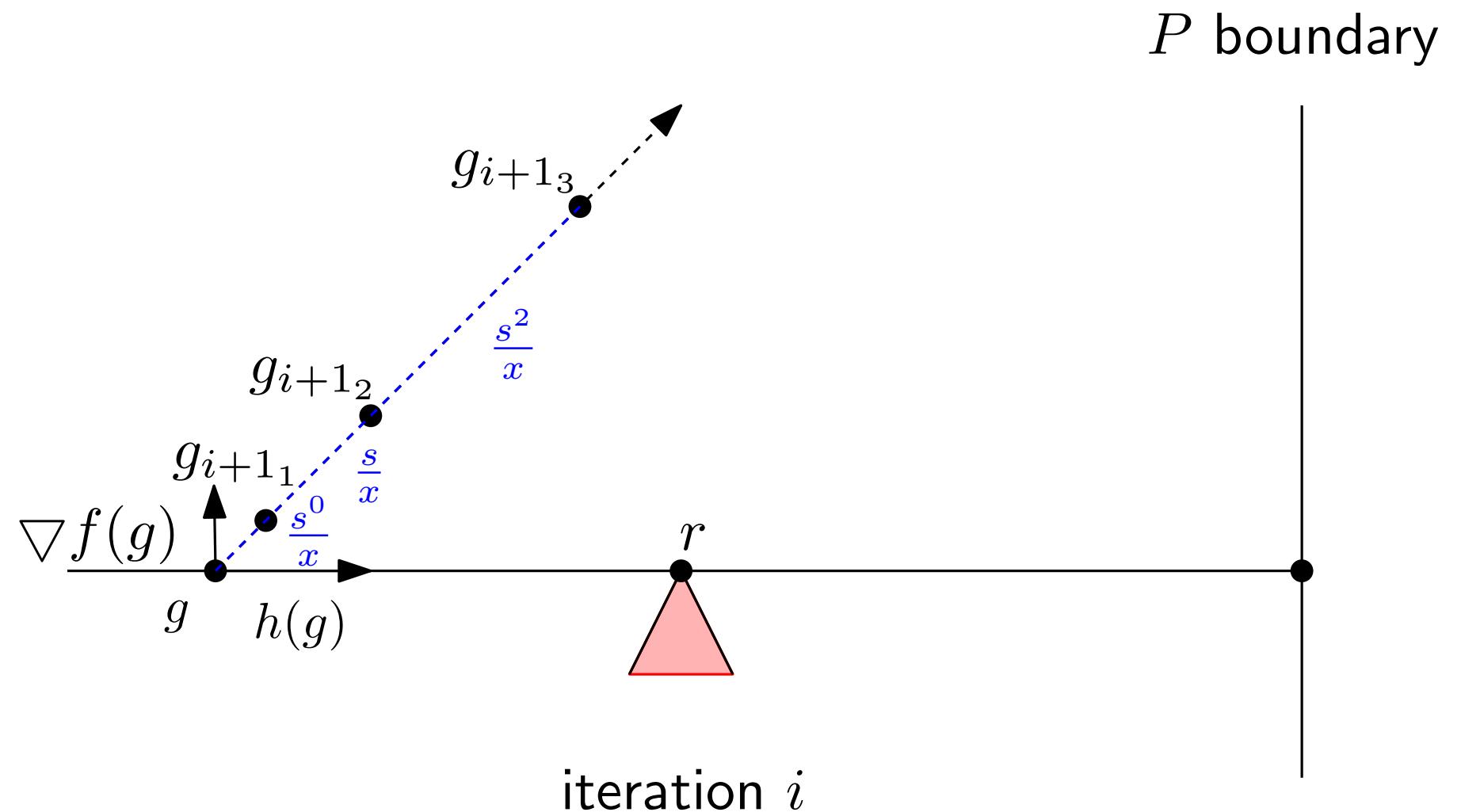
hidden movement

greedy initialisation

Heuristics: Line Search



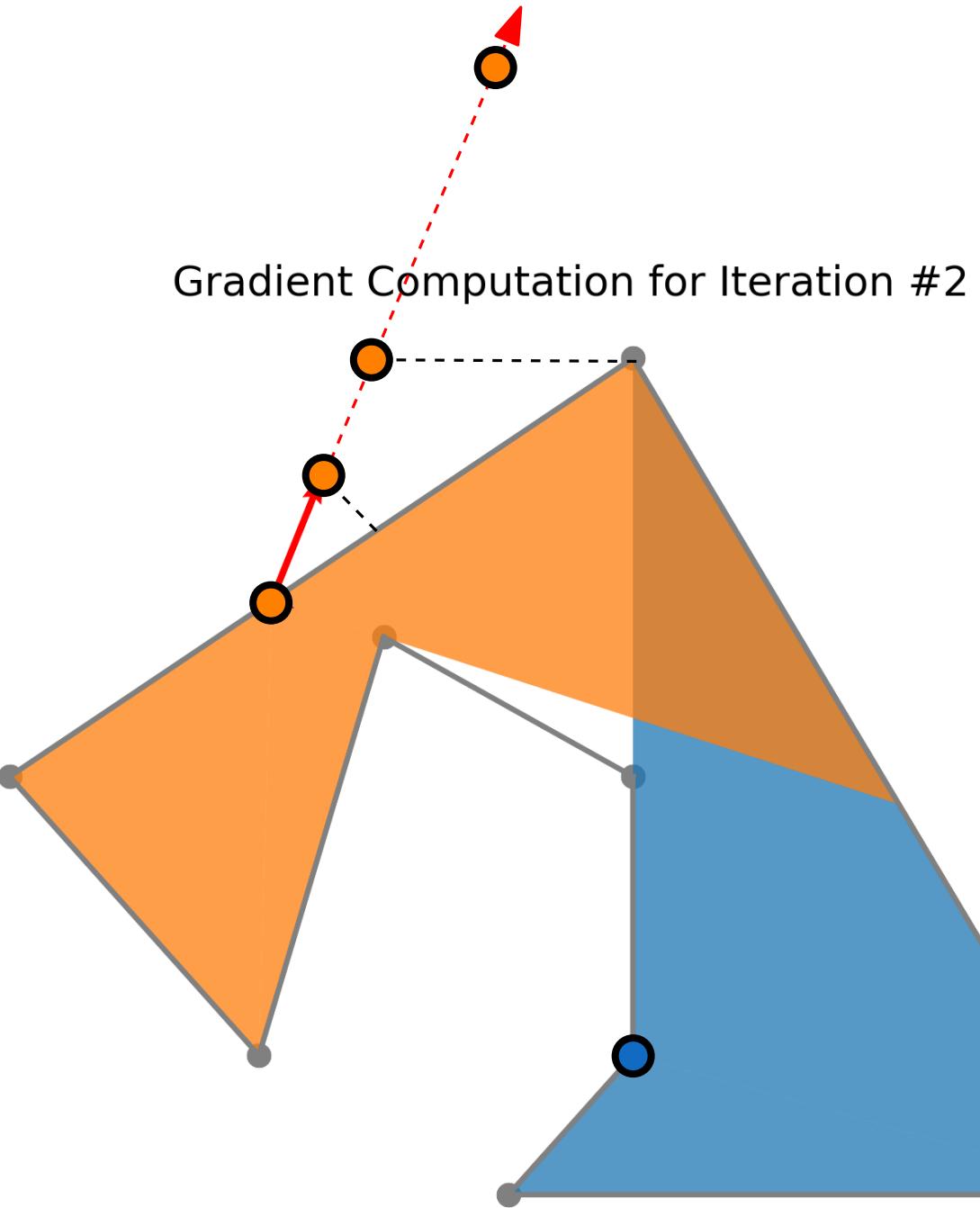
Heuristics: Line Search



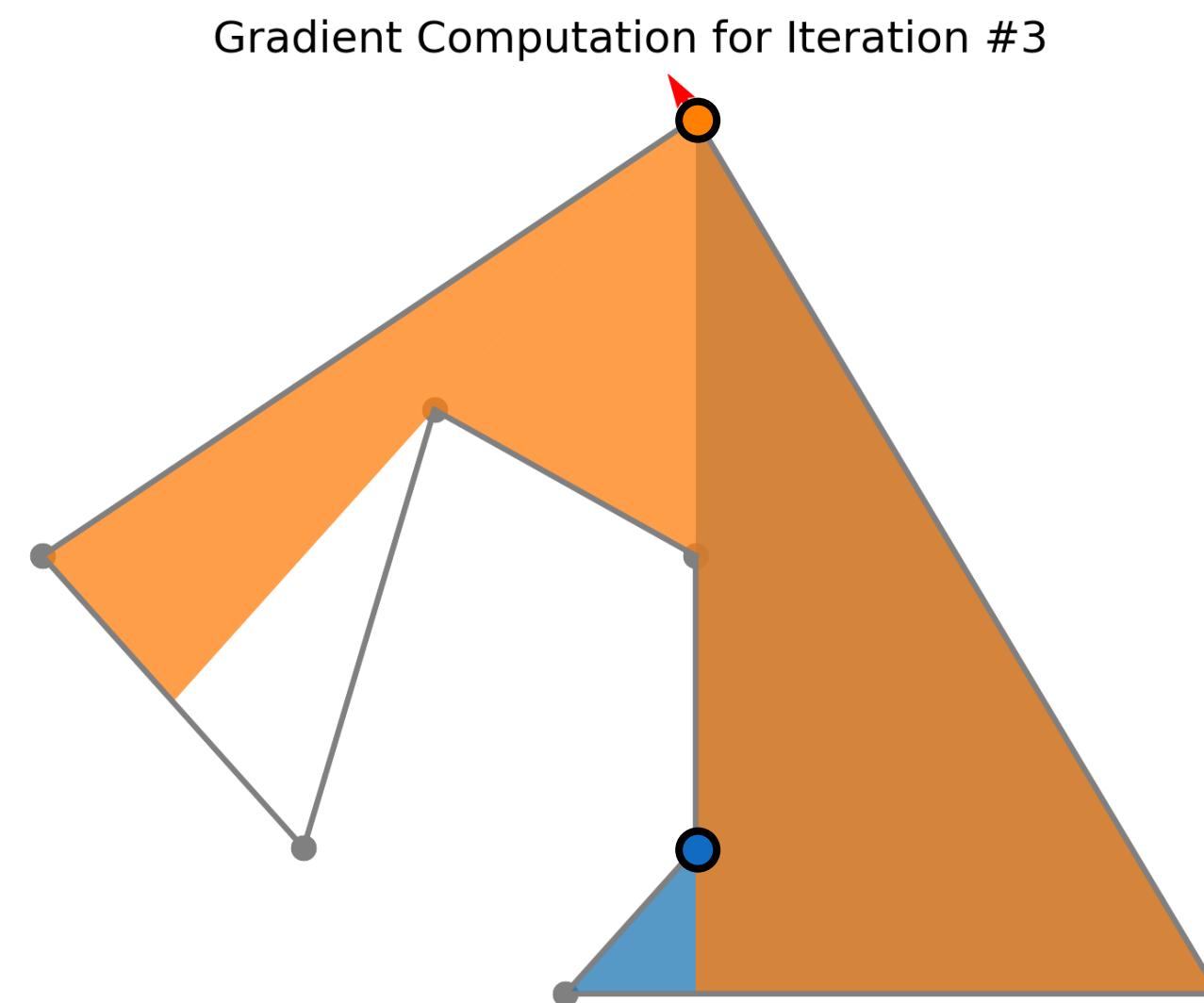
s - step size
 x - search factor

$$g_{i+1_1} = g_i + \frac{1}{x} M(g_i)$$
$$g_{i+1_2} = g_i + \frac{s}{x} M(g_i)$$
$$g_{i+1_3} = g_i + \frac{s^2}{x} M(g_i)$$

Heuristics: Line Search



Heuristics: Line Search



Heuristics

momentum

pull towards reflex vertex

- pull onto the reflex vertex

- pull capping

reflex area

line search

angle behind reflex vertex

hidden movement

greedy initialisation

Heuristics

momentum

pull towards reflex vertex

pull onto the reflex vertex

pull capping

reflex area

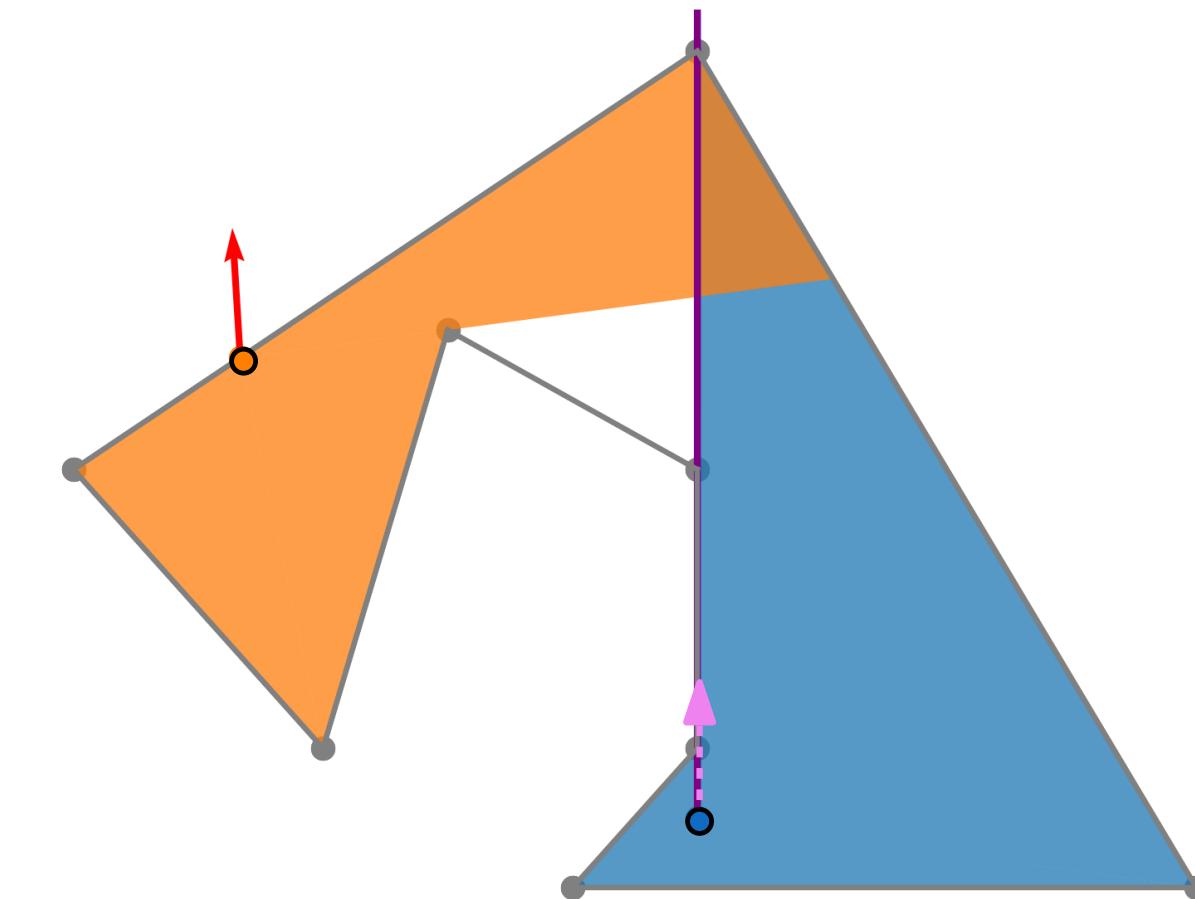
line search

angle behind reflex vertex

hidden movement

greedy initialisation

Gradient Computation for Iteration #1



Heuristics

momentum

pull towards reflex vertex

pull onto the reflex vertex

pull capping

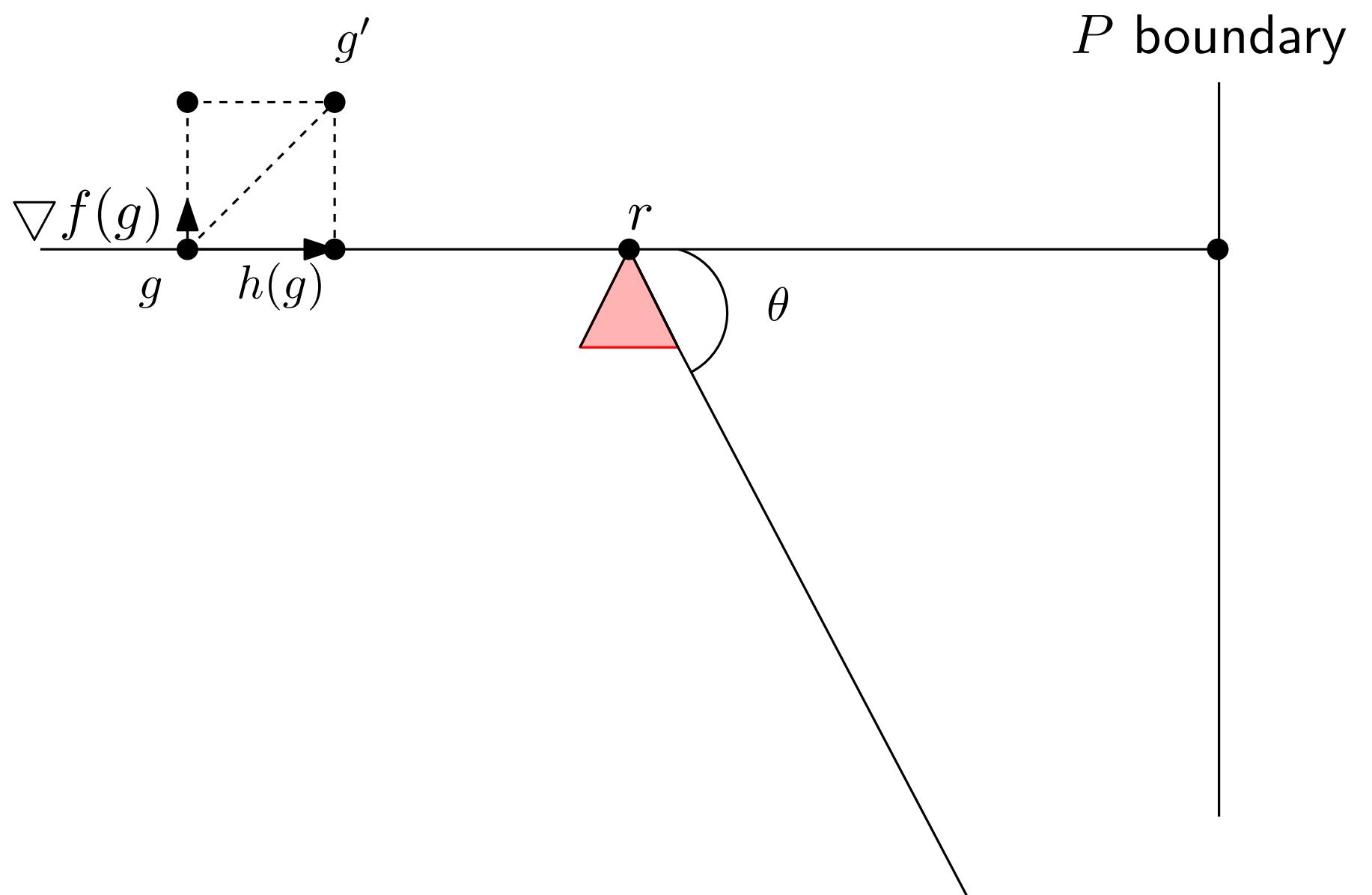
reflex area

line search

angle behind reflex vertex

hidden movement

greedy initialisation



Heuristics

momentum

pull towards reflex vertex

pull onto the reflex vertex

pull capping

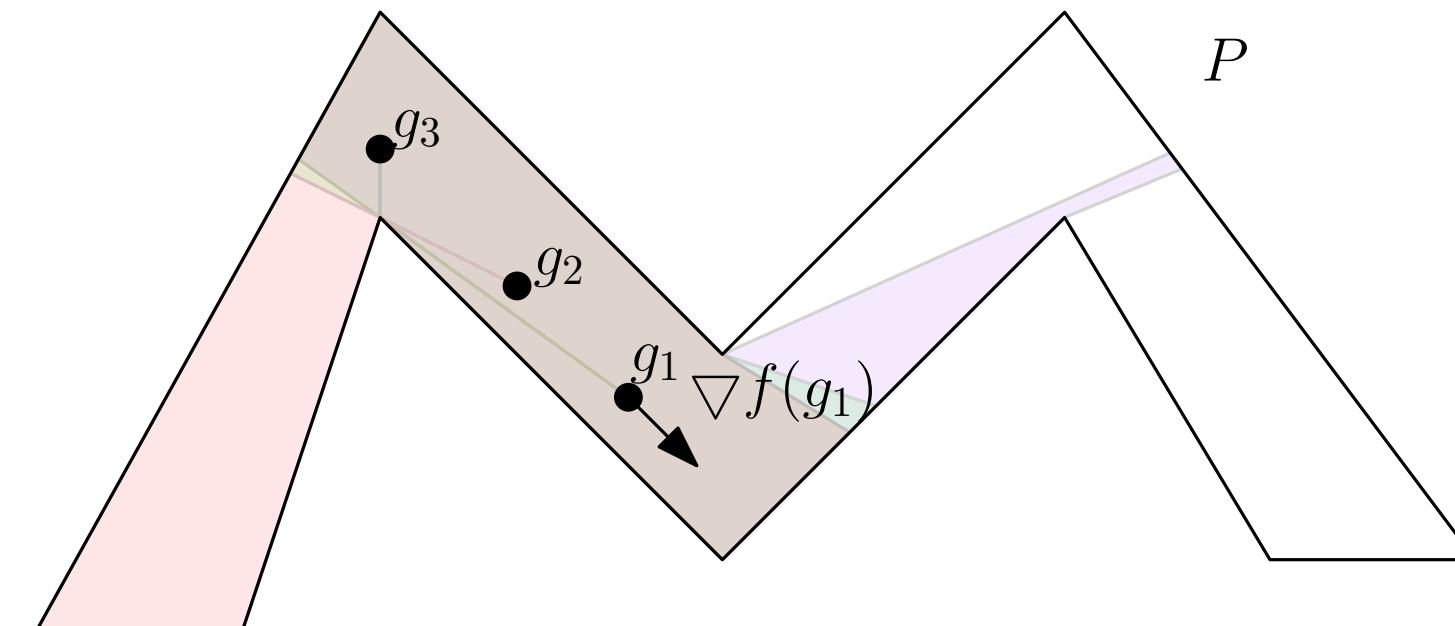
reflex area

line search

angle behind reflex vertex

hidden movement

greedy initialisation



Heuristics

momentum

pull towards reflex vertex

pull onto the reflex vertex

pull capping

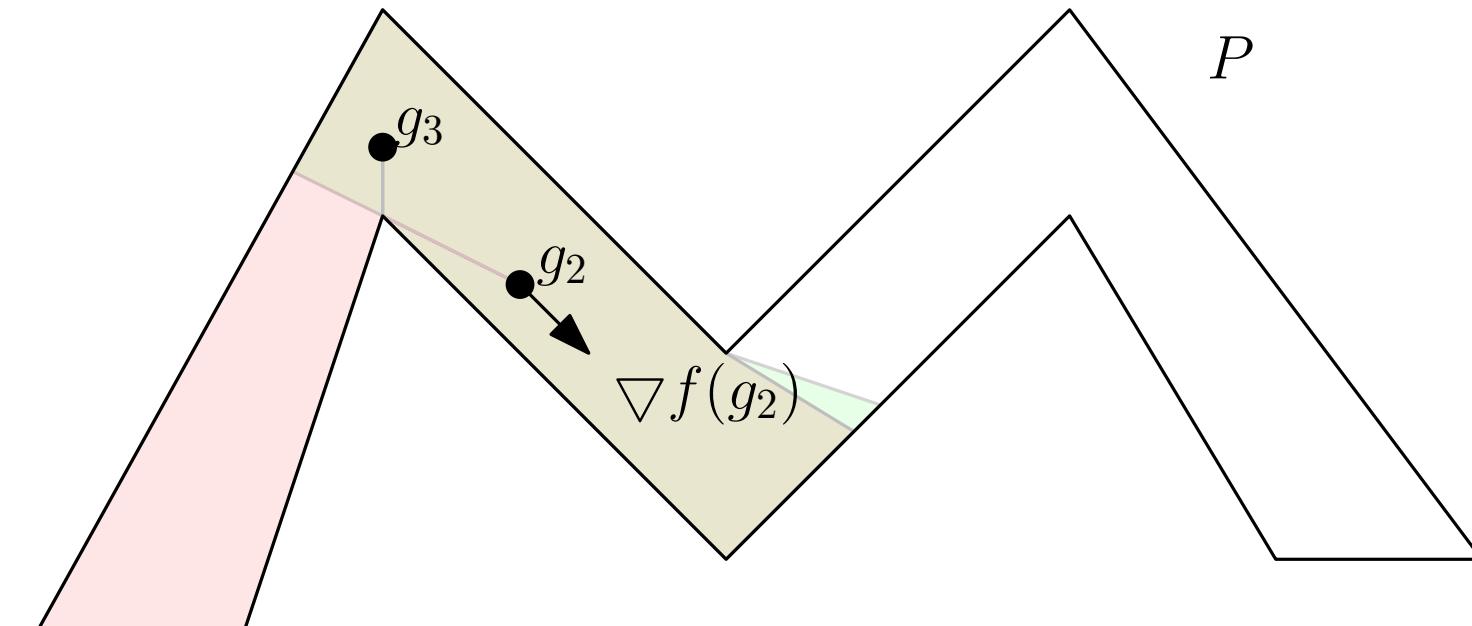
reflex area

line search

angle behind reflex vertex

hidden movement

greedy initialisation



Heuristics

momentum

pull towards reflex vertex

pull onto the reflex vertex

pull capping

reflex area

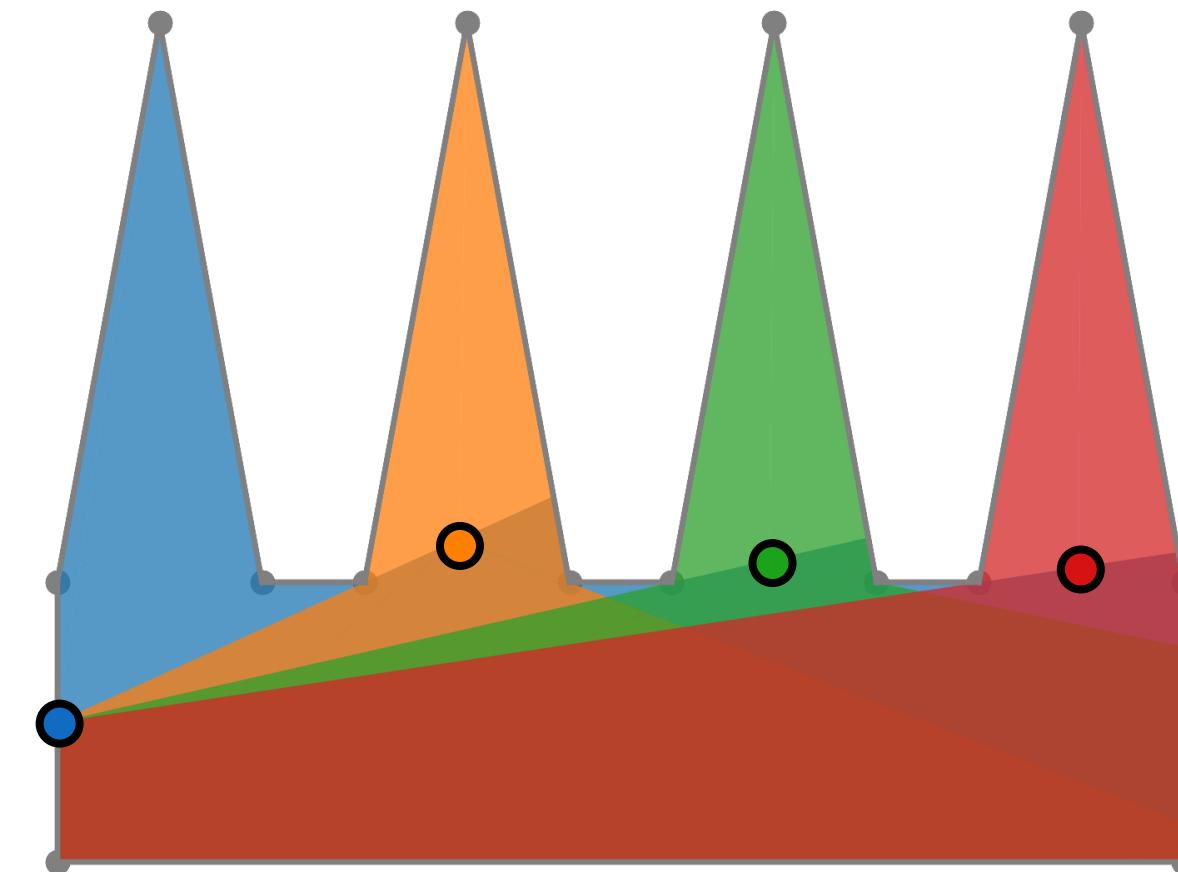
line search

angle behind reflex vertex

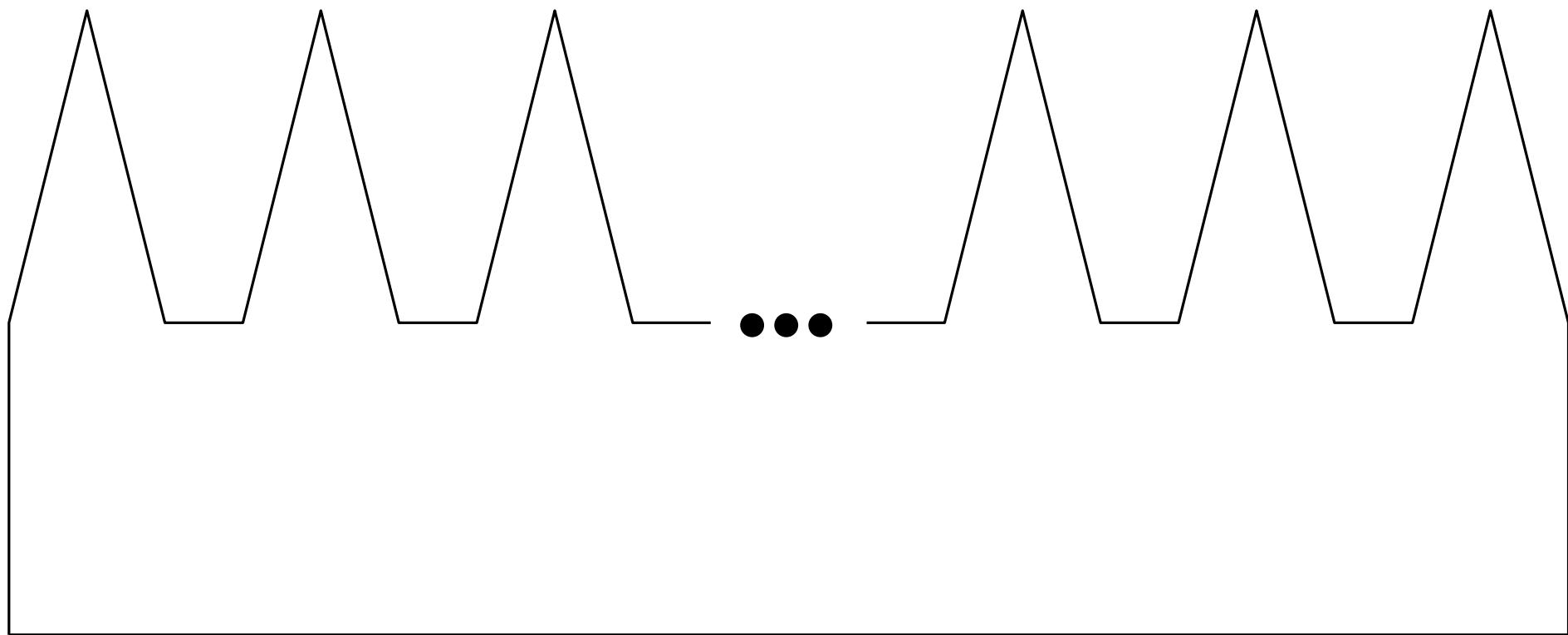
hidden movement

greedy initialisation

Gradient Computation for Iteration #0



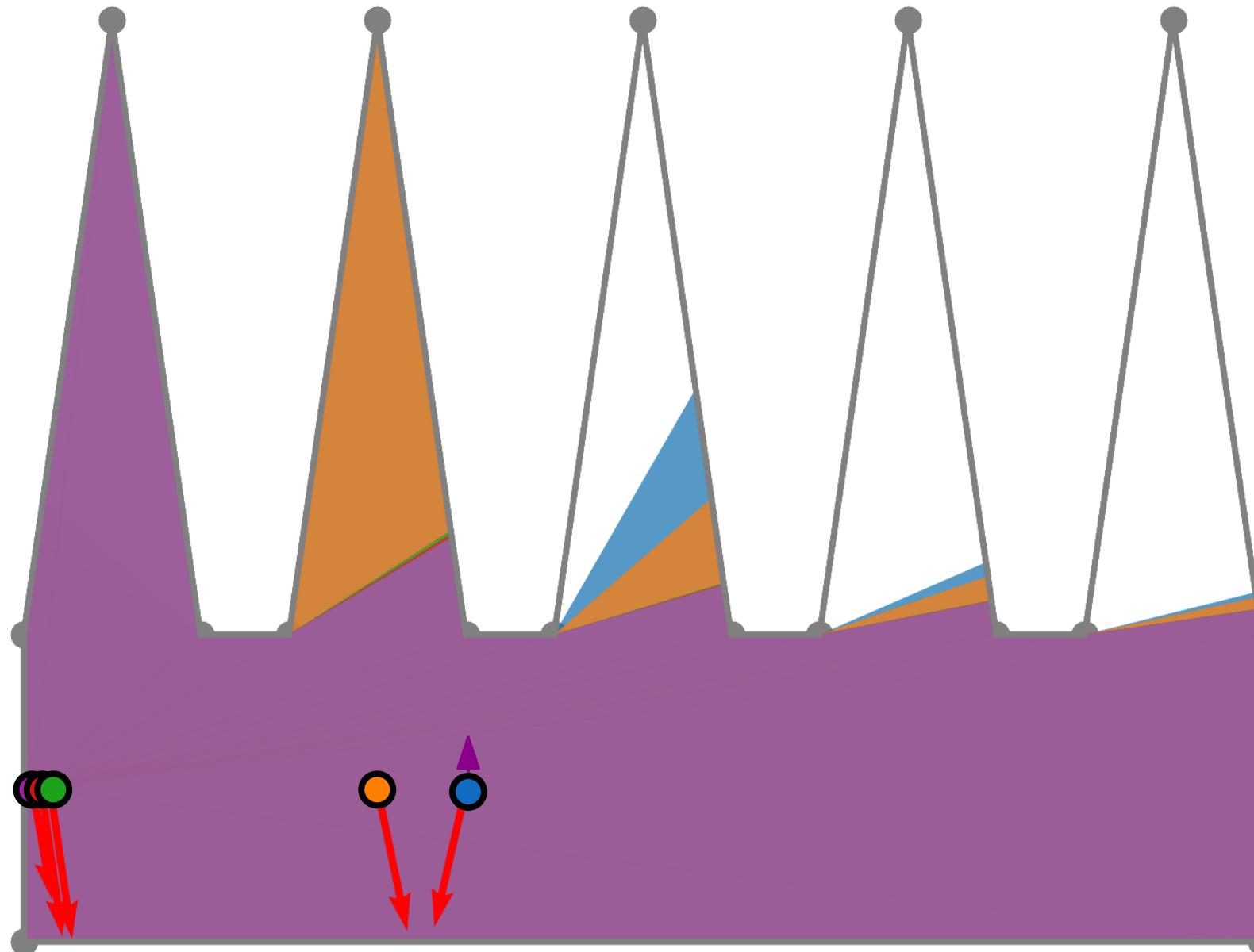
Scalability for the comb polygon



2, 3, ..., 10, 15, 20 teeth

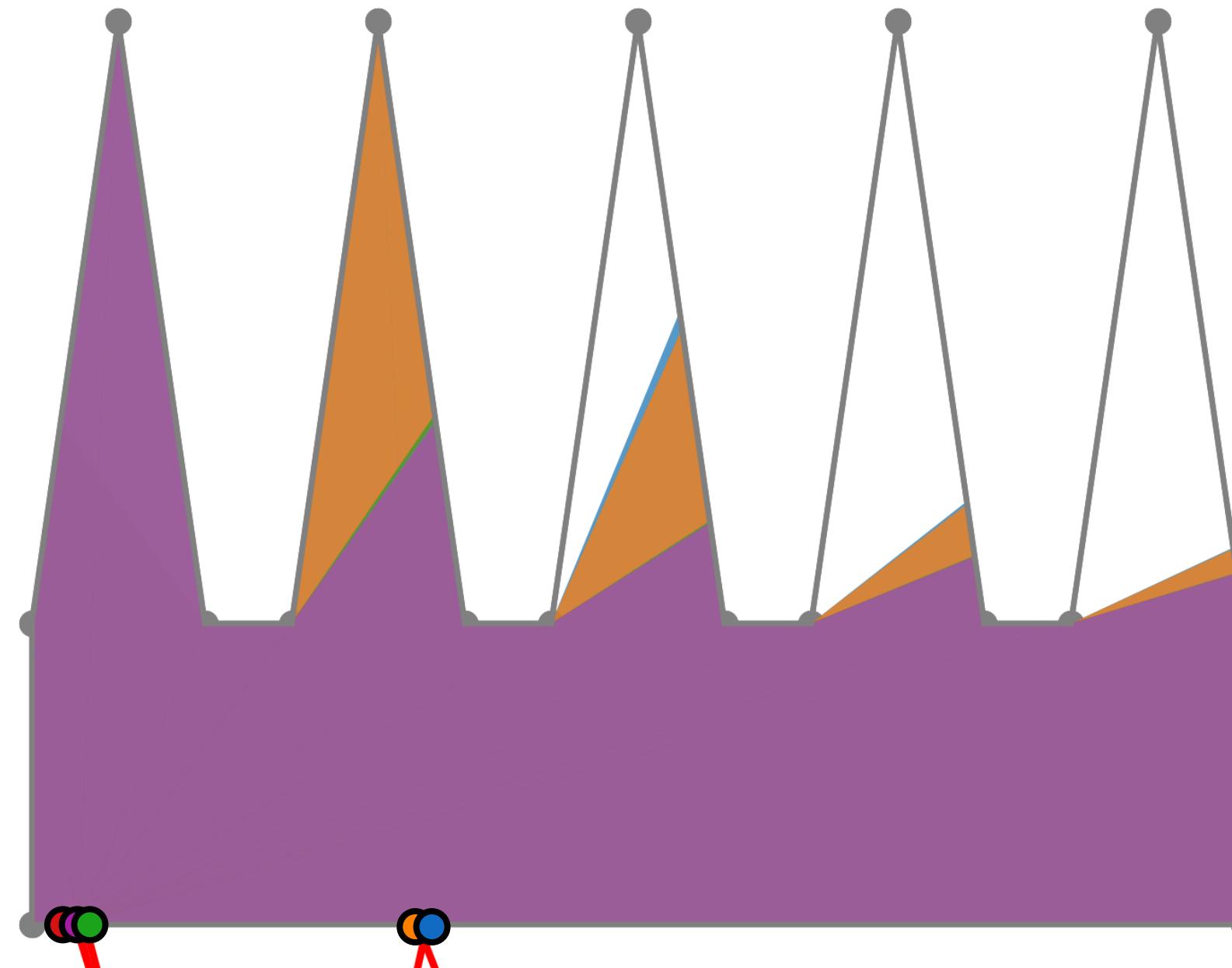
Scalability for the comb polygon

Gradient Computation for Iteration #0



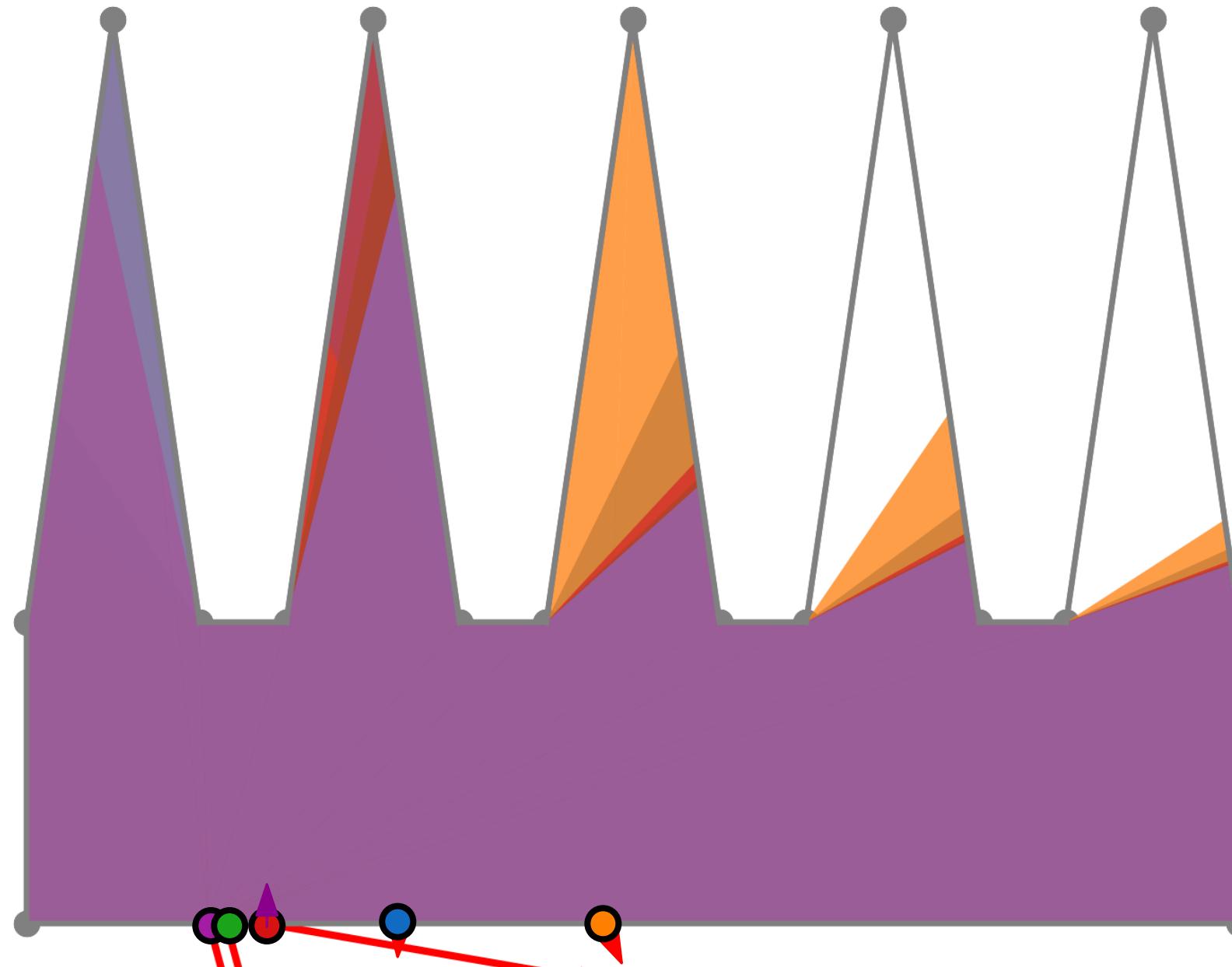
Scalability for the comb polygon

Gradient Computation for Iteration #1



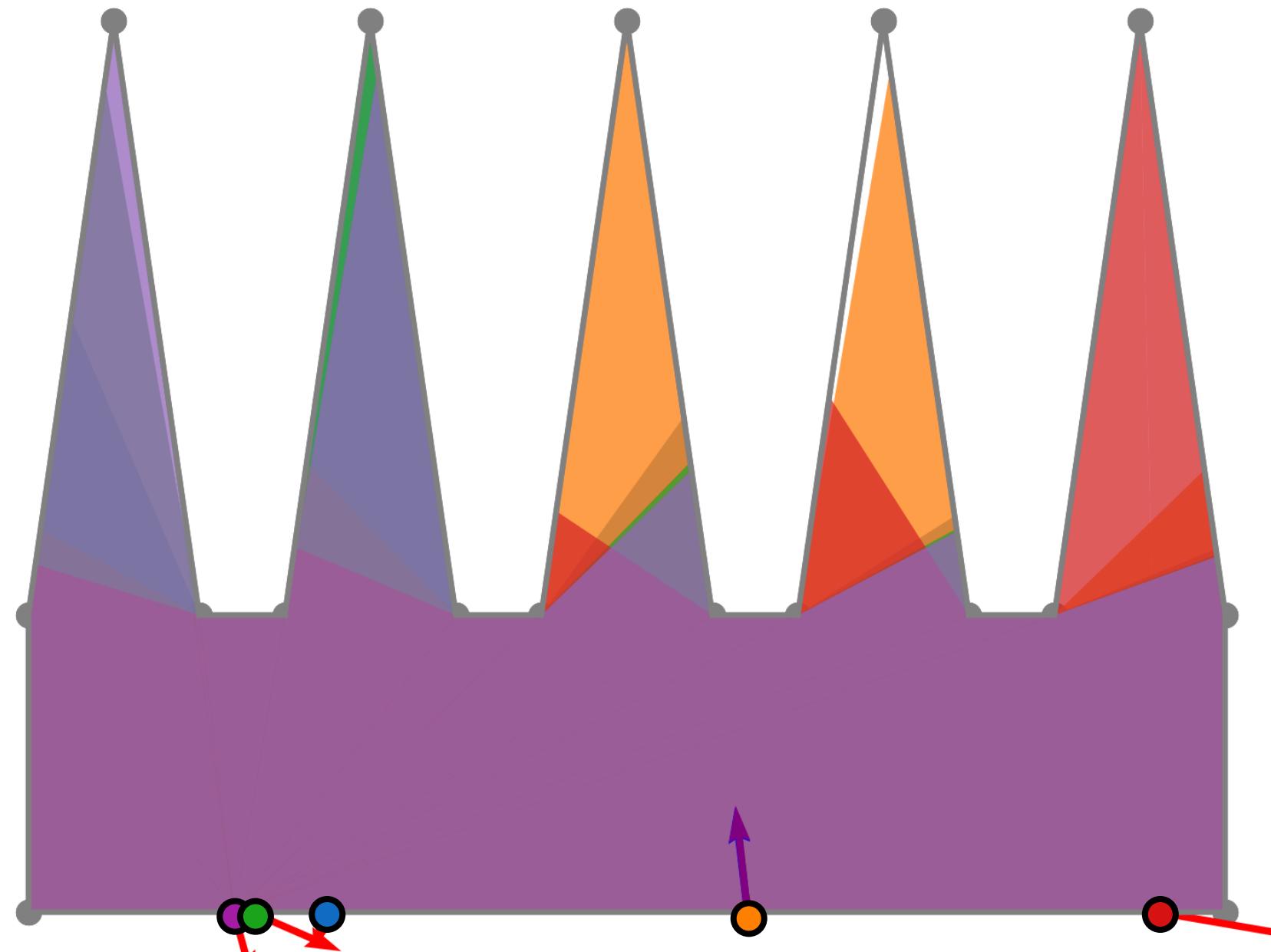
Scalability for the comb polygon

Gradient Computation for Iteration #2



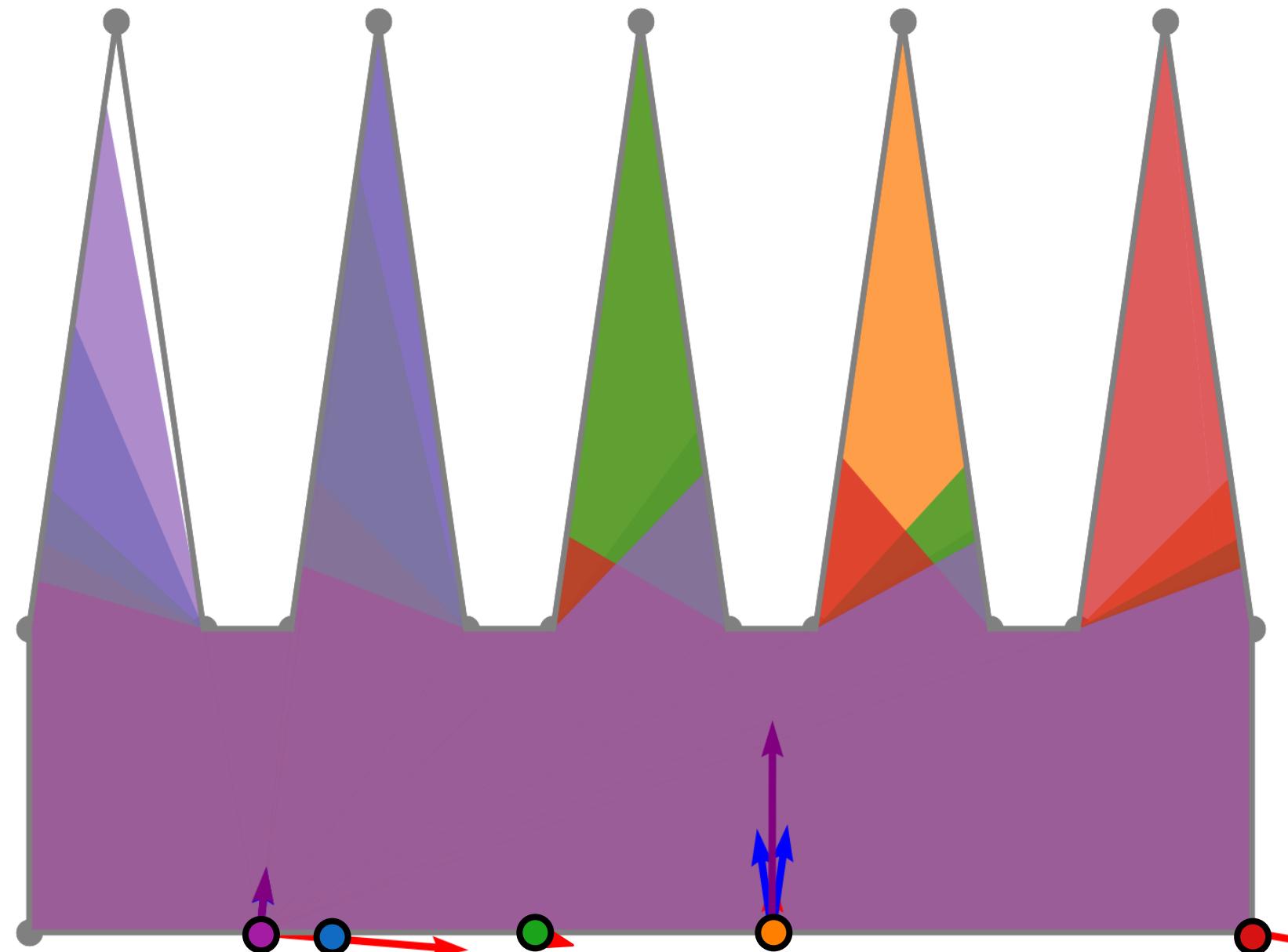
Scalability for the comb polygon

Gradient Computation for Iteration #3



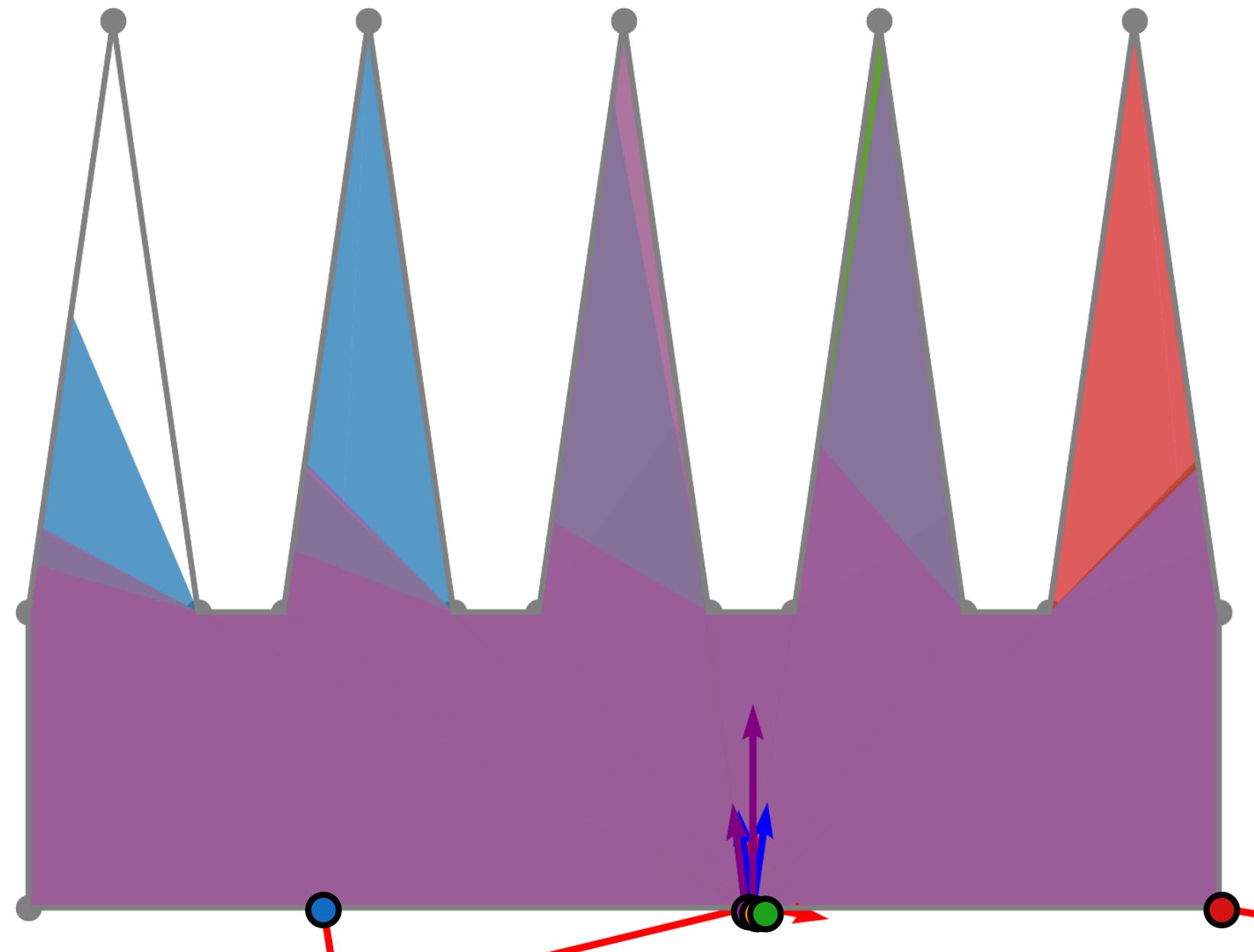
Scalability for the comb polygon

Gradient Computation for Iteration #4



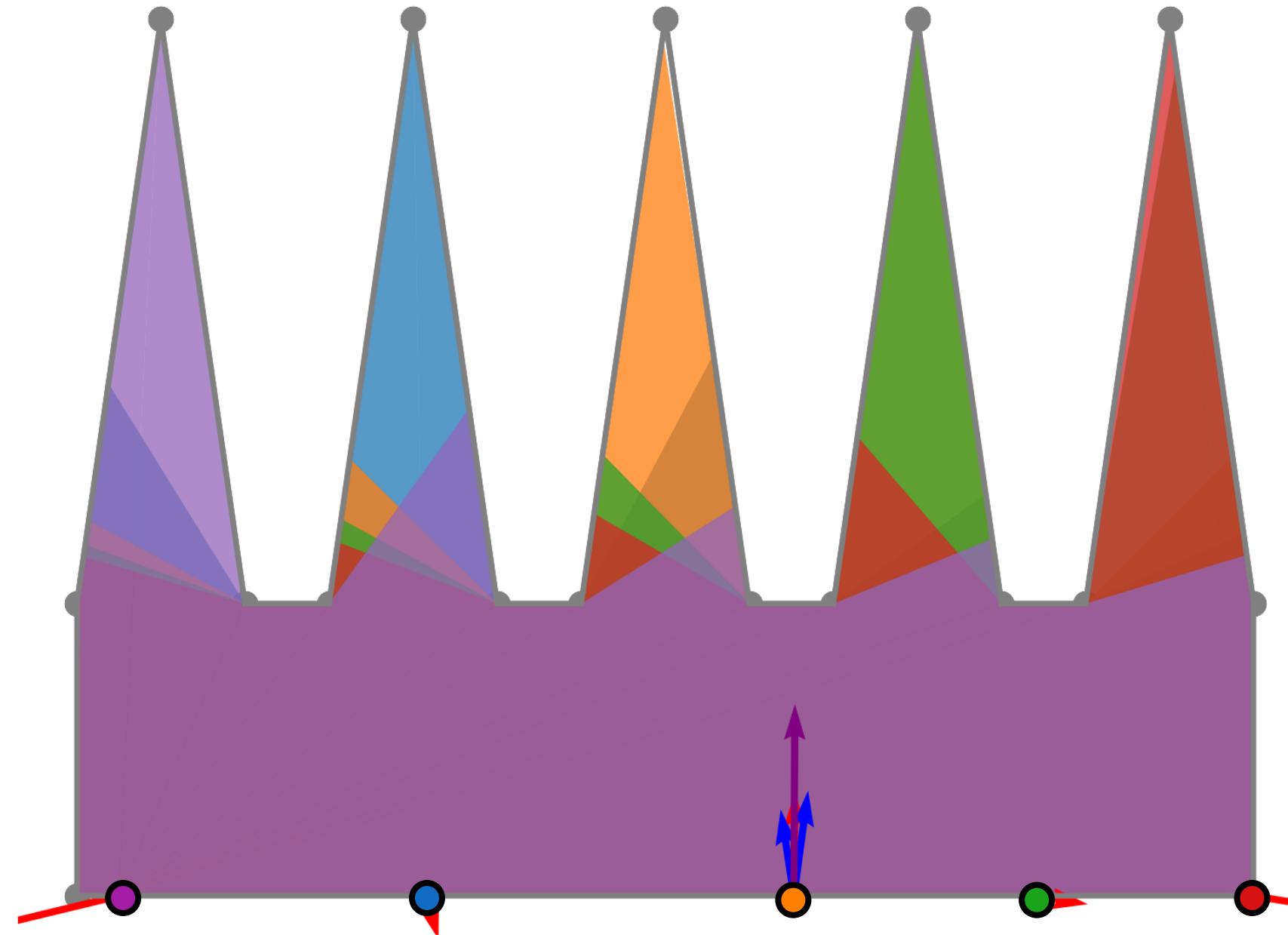
Scalability for the comb polygon

Gradient Computation for Iteration #5



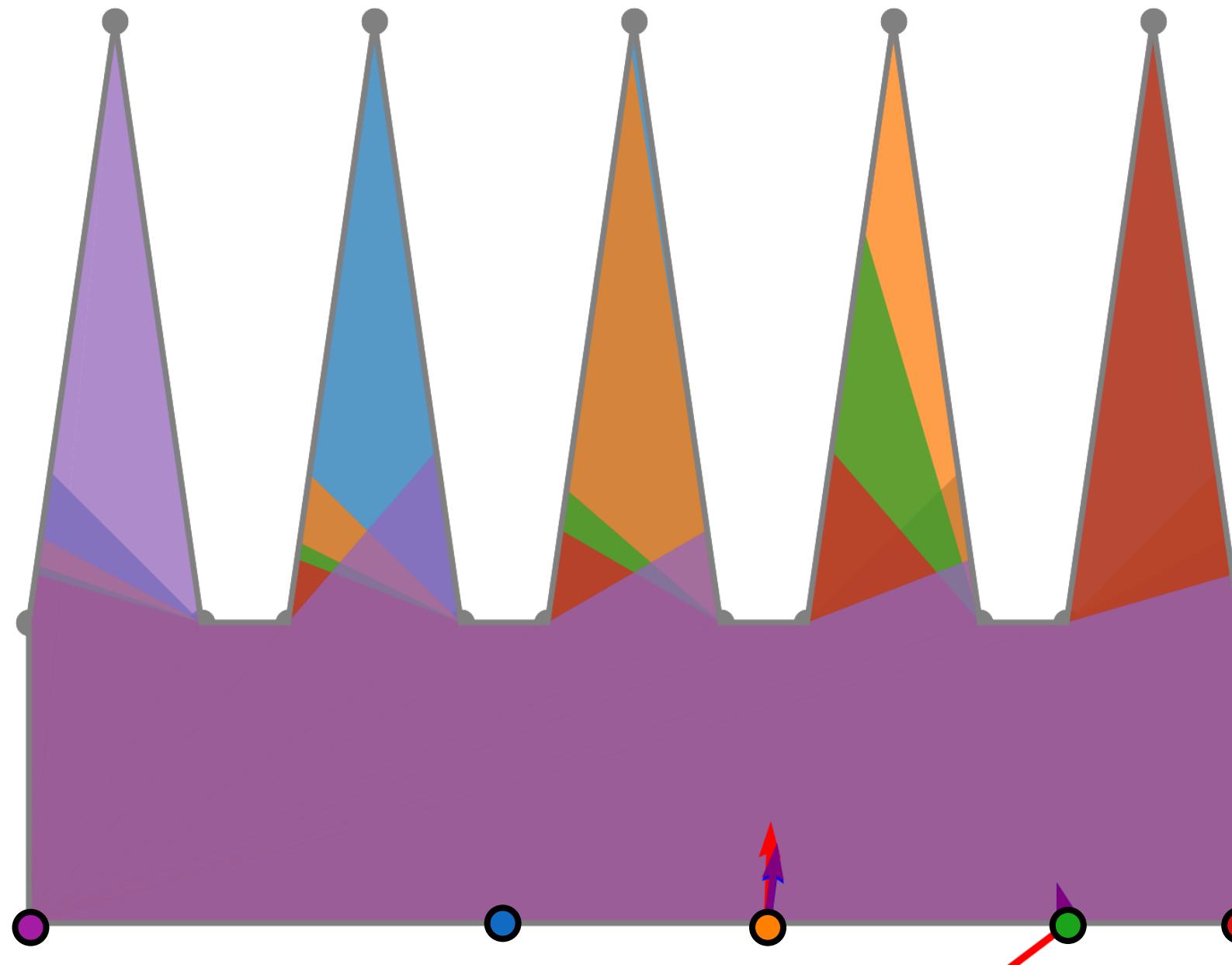
Scalability for the comb polygon

Gradient Computation for Iteration #6



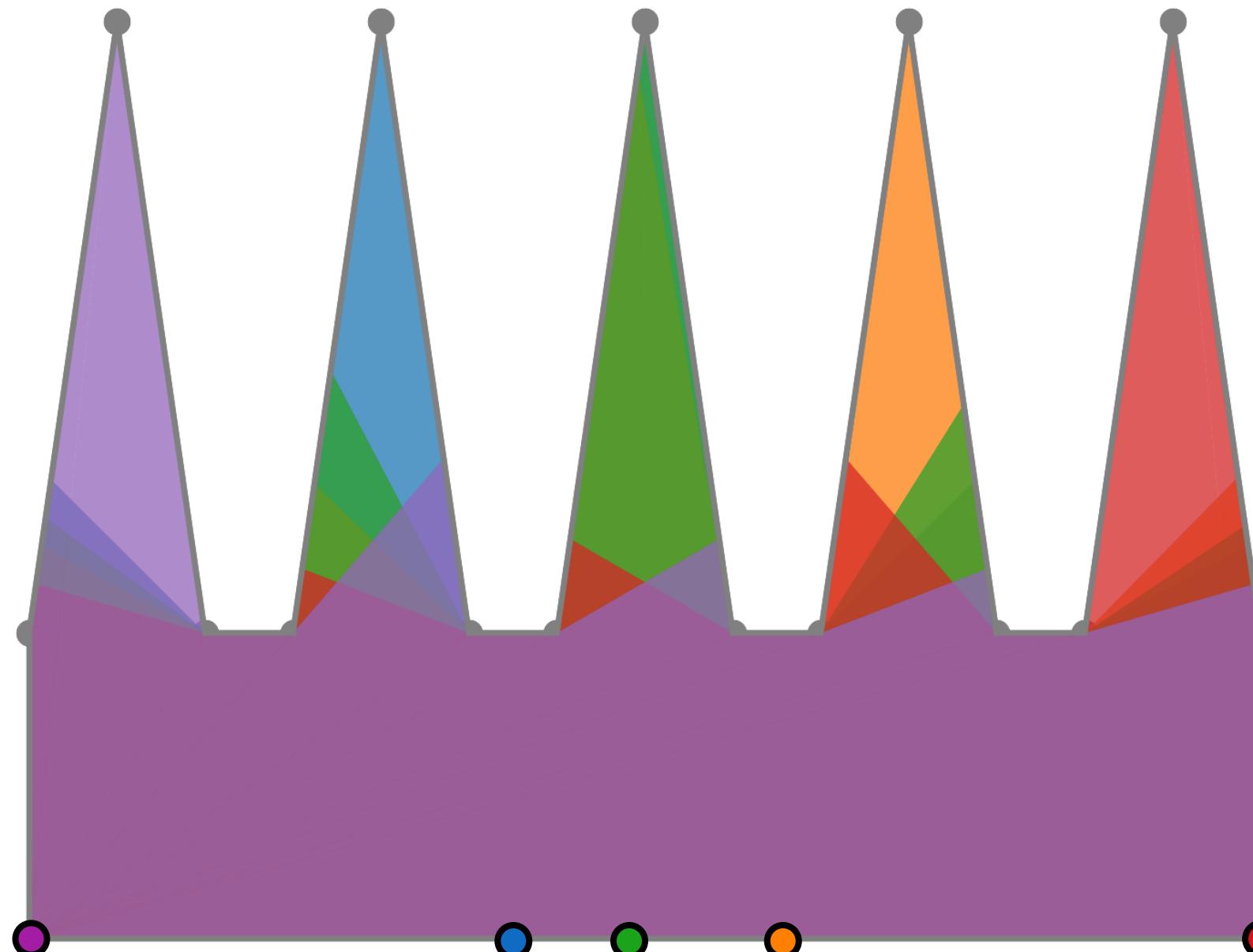
Scalability for the comb polygon

Gradient Computation for Iteration #7

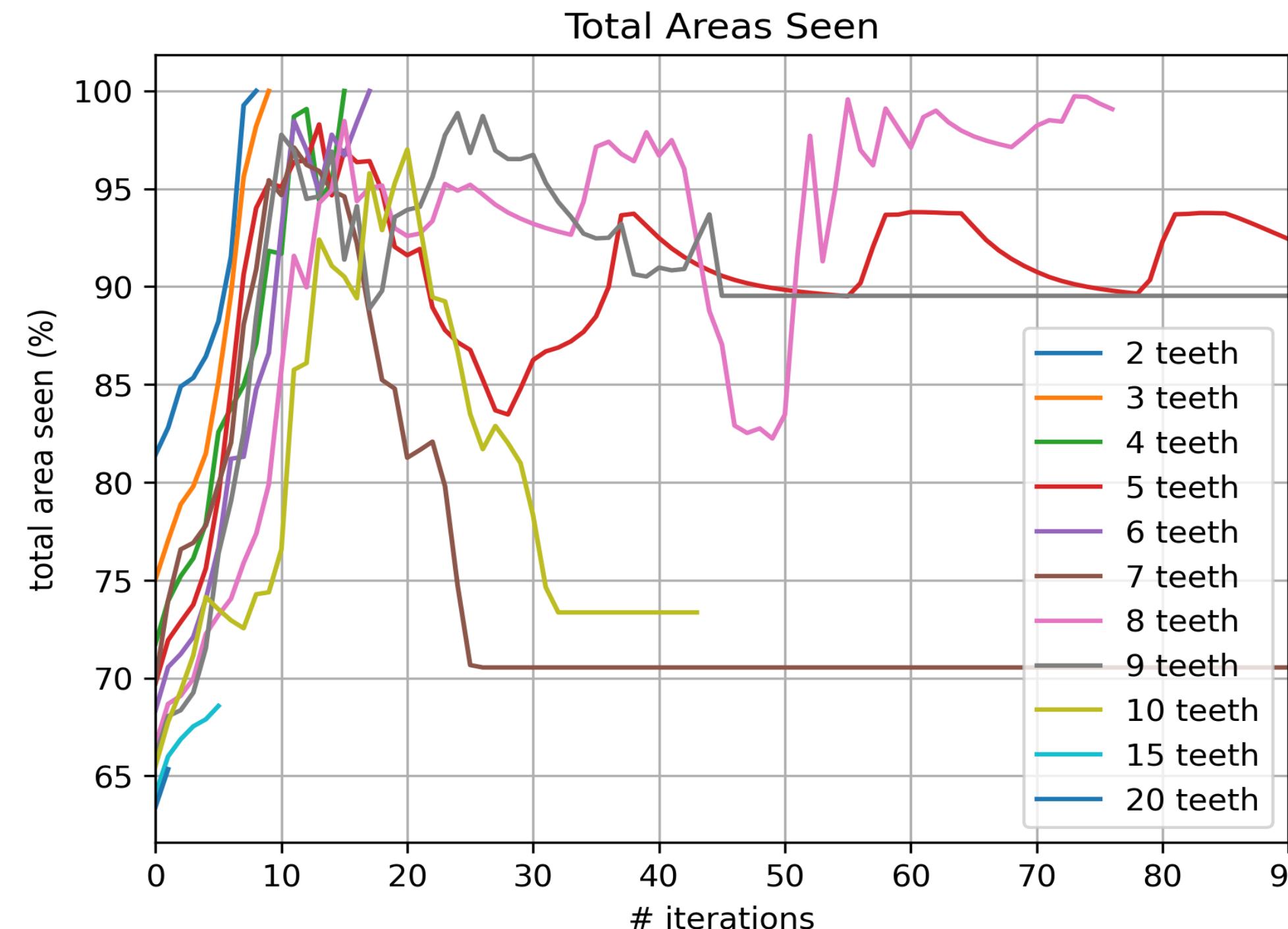


Scalability for the comb polygon

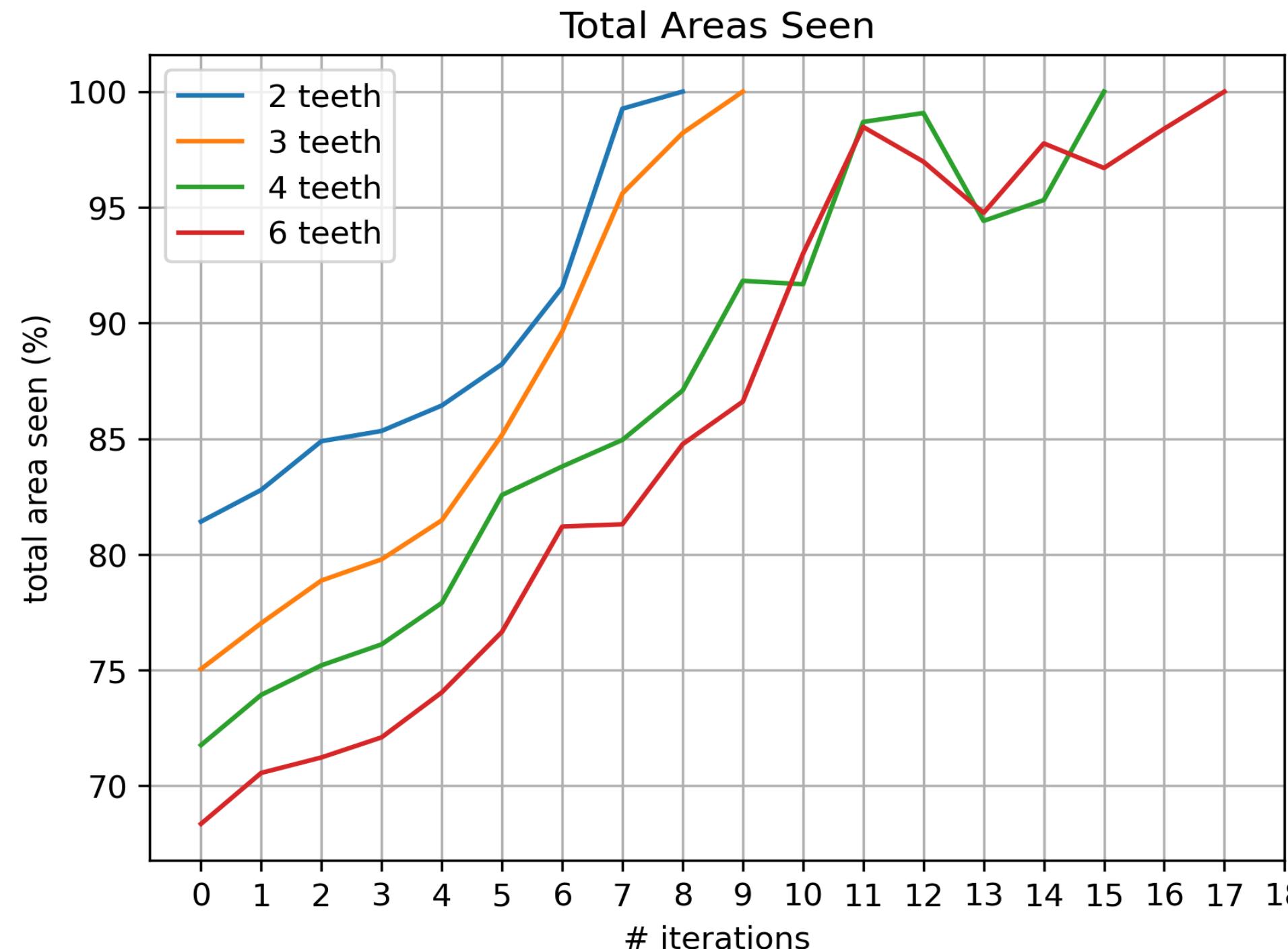
Gradient Computation for Iteration #8



Scalability for the comb polygon



Scalability for the comb polygon



Problems encountered

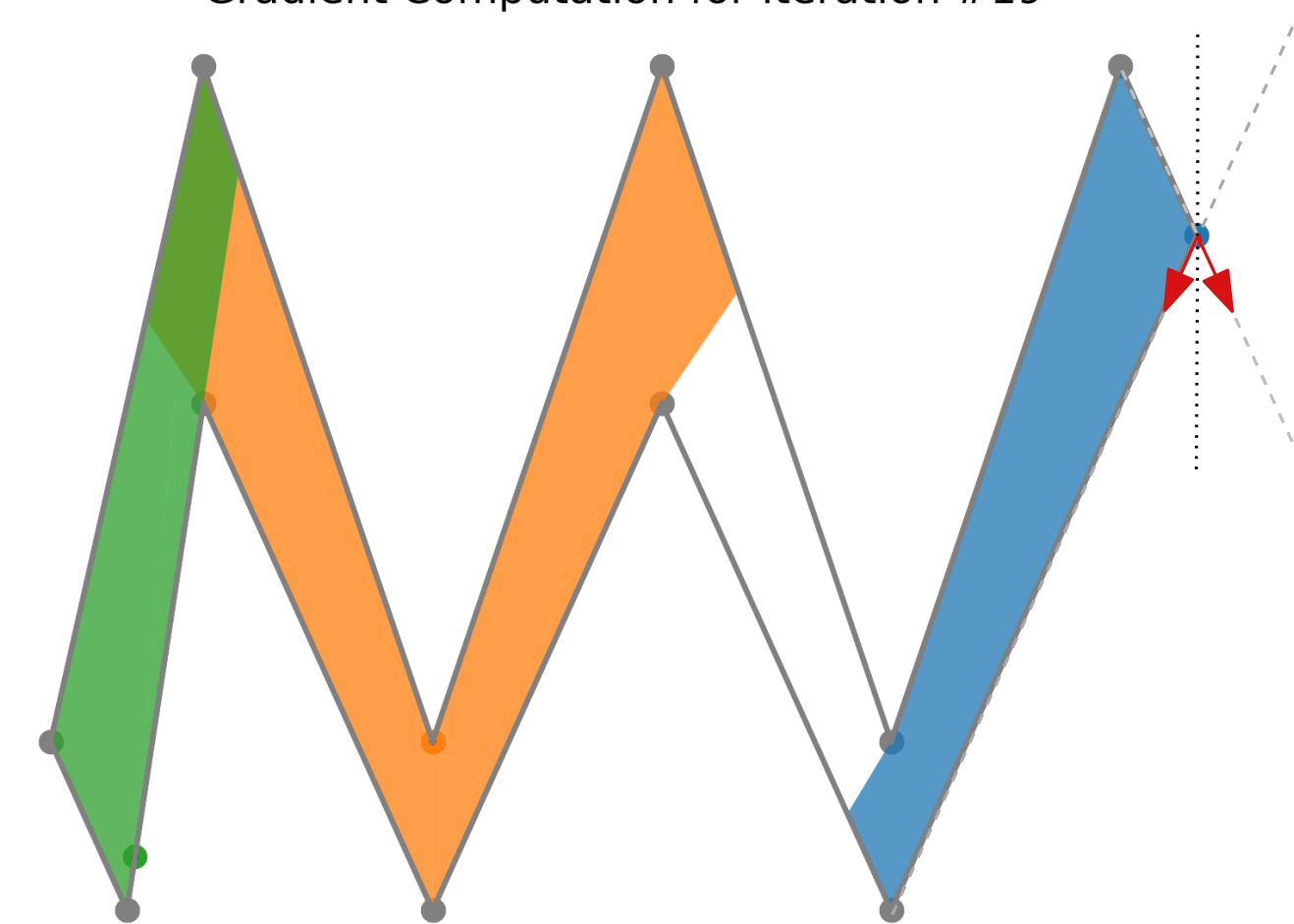
hyperparameter sensitivity

Problems encountered

hyperparameter sensitivity

edge-cases

Gradient Computation for Iteration #19



Problems encountered

hyperparameter sensitivity

edge-cases

CGAL errors

```
terminate called after throwing an instance of 'CGAL::Assertion_exception'
  what(): CGAL ERROR: assertion violation!
Expr: is_finite(d)
File: /usr/include/CGAL/Interval_nt.h
Line: 133
fish: "./../build/main < inputs/love.i..." terminated by signal SIGABRT (Abort)
```

Future work

solve existing bugs



Future work

solve existing bugs

improve the algorithm's robustness, performance and scalability



Future work

solve existing bugs

improve the algorithm's robustness, performance and scalability

implement other heuristics



Future work

solve existing bugs

improve the algorithm's robustness, performance and scalability

implement other heuristics

test the algorithm on larger polygons with more guards

