

Eagle: Making multi-locus association mapping on a genome-wide scale routine

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We present Eagle, a new method for multi-locus association mapping. The motivation for developing Eagle was to make multi-locus association mapping "easy" and the method-of-choice. Eagle's strengths are that it a. is considerably more powerful than single-locus association mapping b. doesn't suffer from multiple testing issues c. is threshold free d. gives results that are immediately interpretable and e. has a computational footprint comparable to single-locus association mapping. By conducting a large simulation study, we will show that Eagle finds true and avoids false SNP-trait associations better than competing single- and multi-locus methods. We also analyse data from a published mouse study. Eagle found over two times more validated findings than the state-of-the-art single-locus method. Eagle has been implemented as an R package, with a web-based Graphical User Interface (GUI) for users less familiar with R.

Over the past decade, genome-wide association studies (GWASs) have changed considerably in both their analysis and design. Early studies followed a case-control design. Association mapping methods were no more complicated than contingency table tests or simple linear regression. These designs though had a tendency to yield spurious findings if there was unrecognised population stratification ¹. This prompted a shift towards family-based designs and score tests, such as the transmission/disequilibrium test (TDT) and its variants ². Today, instead of by design, it is through statistical modelling that we account for the effects of population stratification ³. This has meant that data can be collected from general populations, even if these populations are highly structured. Analysis via sophisticated association mapping methods, such as linear mixed model based approaches, is now almost routine ^{4,5}.

What has not changed is that it remains common practice to analyse genome-wide association study (GWAS) data on a locus-by-locus basis. This is despite there being several significant problems with analysing data in this way. First, the aim of association mapping is to identify re-

regions of the genome that house genes that are influencing a trait. The identification of these regions from these analyses is not always straightforward. GWAS results are reported, typically, via Manhattan plots that plot the $-\log_{10}$ of the p value for each locus against the map position of the locus. The p value is obtained by testing the statistical significance of a single nucleotide polymorphisms (SNP) when treated as an effect in an appropriate model. The location of peaks in this plot identify genomic regions of interest. Inferring the exact number of regions though can be difficult if the peaks are not well separated. Second, when multiple statistical tests are performed, the probability of wrongly accepting a result (type 1 error) is inflated. This is known as the multiple testing problem. Many different solutions have been offered⁶⁻⁸. Yet, there is still no well accepted way of correcting for multiple testing in the context of genome-wide association mapping. Third, many of the traits whose genetic secrets we are trying to discover are complex. There will be multiple SNPs in linkage disequilibrium with genes that are influencing the trait. Yet, a locus-by-locus mapping approach only assesses the evidence for association between a single marker locus and trait.

It is somewhat surprising then that multi-locus association mapping methods haven't attracted more attention. Methods based on regularisation techniques, such as ridge regression⁹ and lasso¹⁰, measure all locus-trait associations simultaneously. Here, multiple testing is not an issue. These techniques though are computationally demanding. Also, their results can be difficult to interpret. The strength of association is not measured by a p value but by the size of the regression coefficient for the SNP in the model. More recently, associations have started to be mapped with random forests¹¹. Similar to regularisation techniques though, it is not clear how to infer genomic regions of interest from their findings. A multi-locus method that does show promise is the multiple-locus linear mixed model method¹². The best multi-locus model is built with simple forward selection. Results are immediately interpretable but here, computation becomes challenging for large datasets.

In this paper, we present our new multi-locus method for genome-wide association mapping, which we are calling Eagle. Eagle combines the strength of regularisation techniques (being able to fit all SNP-trait associations jointly), with forward selection giving easy-to-interpret threshold-free results. We are able to achieve a computational performance similar to the fastest single-locus linear mixed model implementations through a dimension reduction step. Our aim was to make multi-locus association mapping on a genome-wide scale routine. To this end, we have implemented Eagle within an R package of the same name. Our package accepts marker data of

different formats, can handle data larger than a computer's memory capacity, and makes heavy use of parallel computing for computation when available.

1 Results

Association Mapping Methods We compared Eagle, in terms of computational and statistical performance, against seven other association mapping methods. We chose methods that almost all had been purpose built for genome-wide analysis, that could handle data from quantitative traits, and where the methods had been implemented in freely available computer programs or packages. Two of the methods are based on single-locus (or locus-by-locus) models and five are based on multi-locus models. Of the many ways of performing single-locus association mapping, we chose GEMMA¹³ and FaST-LMM¹⁴ because of their popularity and computational speed. For multi-locus association mapping, we chose bigRR⁹, glmnet¹⁵, LMM-Lasso¹⁰, MLMM¹², and r2VIM¹¹. Each takes a different approach to multi-locus association mapping. A summary of the key attributes of the different computer programs/packages is given in Supplementary Table 1 (see Methods for further details).

Simulation Study A large simulation study was performed where we sought to answer two questions. First, how well does Eagle find true associations (power) and avoid false associations (type 1 errors)? Second, how does Eagle compare, in terms of run time and memory usage, to competing implementations? Data were generated under six different scenarios; a study of size 150 individuals and 5,000 single SNPs (150 x 5K), 350 individuals and 400,000 SNPs (350 x 400K), 1,500 individuals and 50,000 SNPs (1500 x 50K), 2,000 individuals and 500,000 SNPs (2000 x 500K), 4,000 individuals and 1,500,000 SNPs (4000 x 1.5M), and 10,000 individuals and 1,500,000 SNPs (10000 x 1.5M). These scenarios reflect, at least in some cases, the sizes of study being performed in animals, plants, and humans.

For each scenario, 100 replicates were generated. A single replicate consisted of SNP and quantitative trait data. Extra realism was introduced into the simulation study through the drawing of the SNP genotypes from the 1000 Genome Project, phase 3¹⁶. The quantitative trait data were generated by selecting, randomly, a set of SNPs and assigning these loci additive allelic effects. Random errors were then drawn from a normal distribution with variance set to give a heritability of 50% for the trait. For each individual, a quantitative trait value was obtained by summing its

random error and additive allelic effects. The number of randomly selected SNPs follows a Poisson distribution with mean 30. The size of the allelic effects across the selected loci are equal.

Analyses by the eight programs/packages of a replicate proceeded as follows. They were all run at their default settings. Eagle and MLMM were the easiest of the programs/packages to implement. The only parameters requiring specification were the amount of available memory and number of CPUs for Eagle and the number of chunks for MLMM. Their results were also immediately interpretable. Their findings were the set of SNPs in strongest association with the trait. Each SNP in this set identified a separate genomic region of interest, whose position was given by the map location of the SNP.

BigRR, LMM-Lasso, and glmnet required more effort to implement. They are based on regularisation methods and as such, all the SNPs were fitted simultaneously in a regression framework. The difficulty was in calculating the significance of the SNP effects. To do this analytically is challenging. We instead opted for stability selection (see Methods), an empirical approach for calculating significance.

R2VIM is different from the rest in that it is a nonparametric approach for association mapping. It is based on random forests. Three important parameters needed to be set. These were the number of trees, the number of variables for building a tree, and the minimum size of a terminal node. Ideally, these parameters would be "tuned" on a replicate-by-replicate basis¹⁷. However, this was not practical here. We instead used the same settings as in¹¹ where the number of trees was set to 1000, the number of variables was set to 20% of the number of SNPs, and the minimum size of a node was set to 10% of the sample size. A relative importance measure was calculated for each SNP measuring its strength of association with the trait.

FaST-LMM and GEMMA implement single-locus association mapping. FaST-LMM was run in two ways. One way was where a subset of the SNP data were used in calculating the similarity (or relationship) matrix. Here, FaST-LMM is highly efficient, computationally. The other was where calculation of the similarity matrix was based on all the SNP data. The p values of the SNP were reported as their results.

The results from all but Eagle and MLMM required post-processing before the findings were

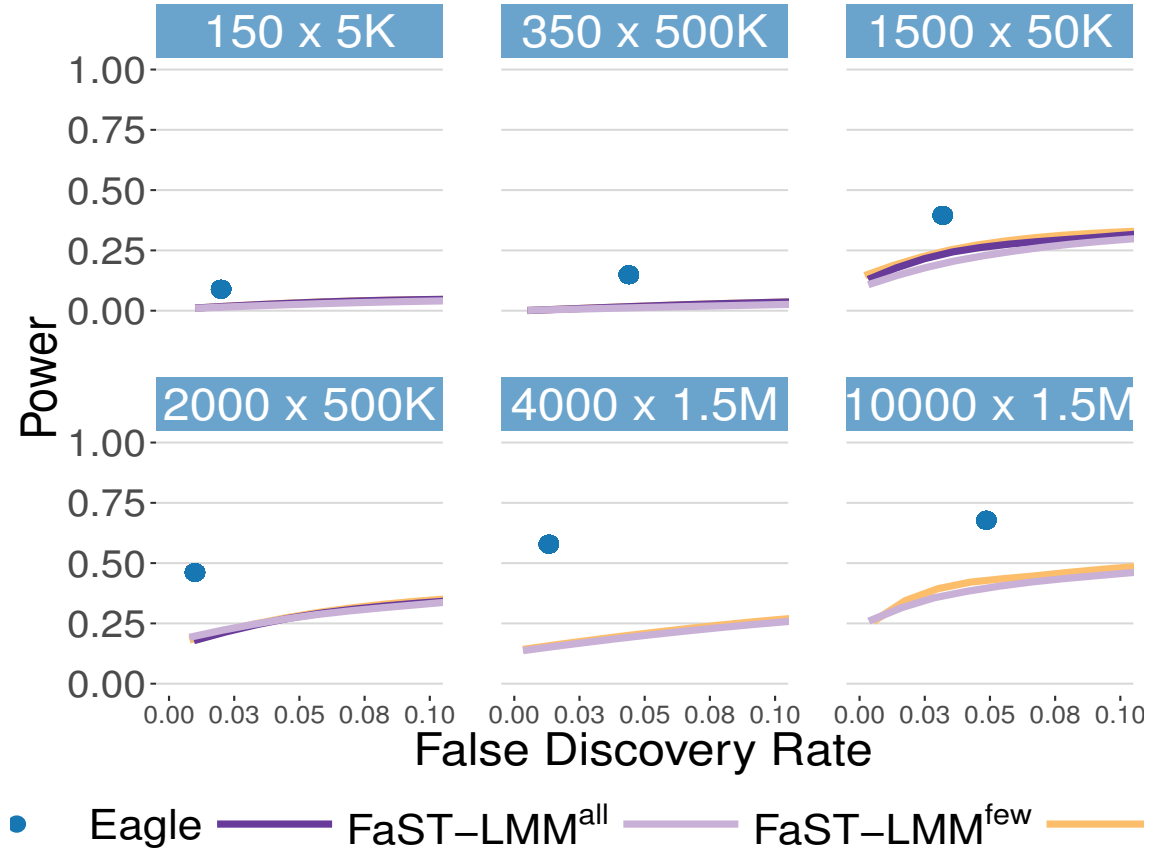
interpretable. The SNPs were placed in map order, a significance threshold was set, peak regions containing SNPs with significance measures above the threshold were identified, and the SNP with the largest significance measure in each of the peak regions was recorded as a finding.

Power and False Discovery Rates Here, we answer the question of how well Eagle finds true SNP-trait associations and avoids false SNP-trait associations. We do this by estimating the power and false discovery rates of Eagle and the other methods for the six scenarios. Since, for a replicate, we knew which SNPs were assigned additive effects, we knew the SNPs that were in true association with the trait. We will refer to these SNPs as being true SNPs. By knowing the true SNPs, we were able to assess the validity of a method's findings. A finding was counted as true if it was positioned within 40 kilobase pairs of the location of a true SNP. When a replicate was analysed, we obtained an estimate of the power of the method by taking the number of findings that were found to be true and dividing by the number of true SNPs. We also obtained an estimate of a method's false discovery rate. It is the number of findings that were found to be false divided by the number of (true and false) findings found by the method. Both these estimates varied with replicate. The power (false discovery rate) of a method, for a scenario, was found by taking the median of the power (false discovery rate) estimates over the 100 replicates.

The power and false discovery rates of Eagle and the other multi-locus methods across the scenarios 150 x 5K, 350 x 500K, 1500 x 50K, and 2000 x 500K are shown in Supplementary Figure 1. We restricted our attention to these scenarios because not all multi-locus methods could cope with the size of data in the other scenarios. Each plot contains single points and power curves. The single points are the power and false discovery rates for Eagle and MLMM. These two methods treat association mapping as a model selection problem. There are no significance thresholds to be set. The power curves are for those methods that treat association mapping as a variable selection problem. Here, the significance of the findings are assessed against a significance threshold. The power curves in the plot show how power changes with the false discovery rate as the significance threshold is adjusted. The power and false discovery rate of Eagle and the two single-locus methods, GEMMA and FaST-LMM, are shown in Figure 1.

In answer to the question of how well Eagle finds true SNP-trait associations and avoids false SNP-trait associations, it does extremely well. Of the multi-locus methods, Eagle had the highest power while keeping its false discovery rate low (Supplementary Figure 1). MLMM also

Figure 1: Power verse false discovery rates for Eagle and the single-locus methods GEMMA and FaST-LMM. FaST-LMM was run where all the SNP data are used to estimate the relationship matrix (FaST-LMM^{all}) and where genotype data from every five-hundredth SNP are used to estimate the relationship matrix (FaST-LMM^{few}). Eagle has substantially higher power than the single-locus methods.

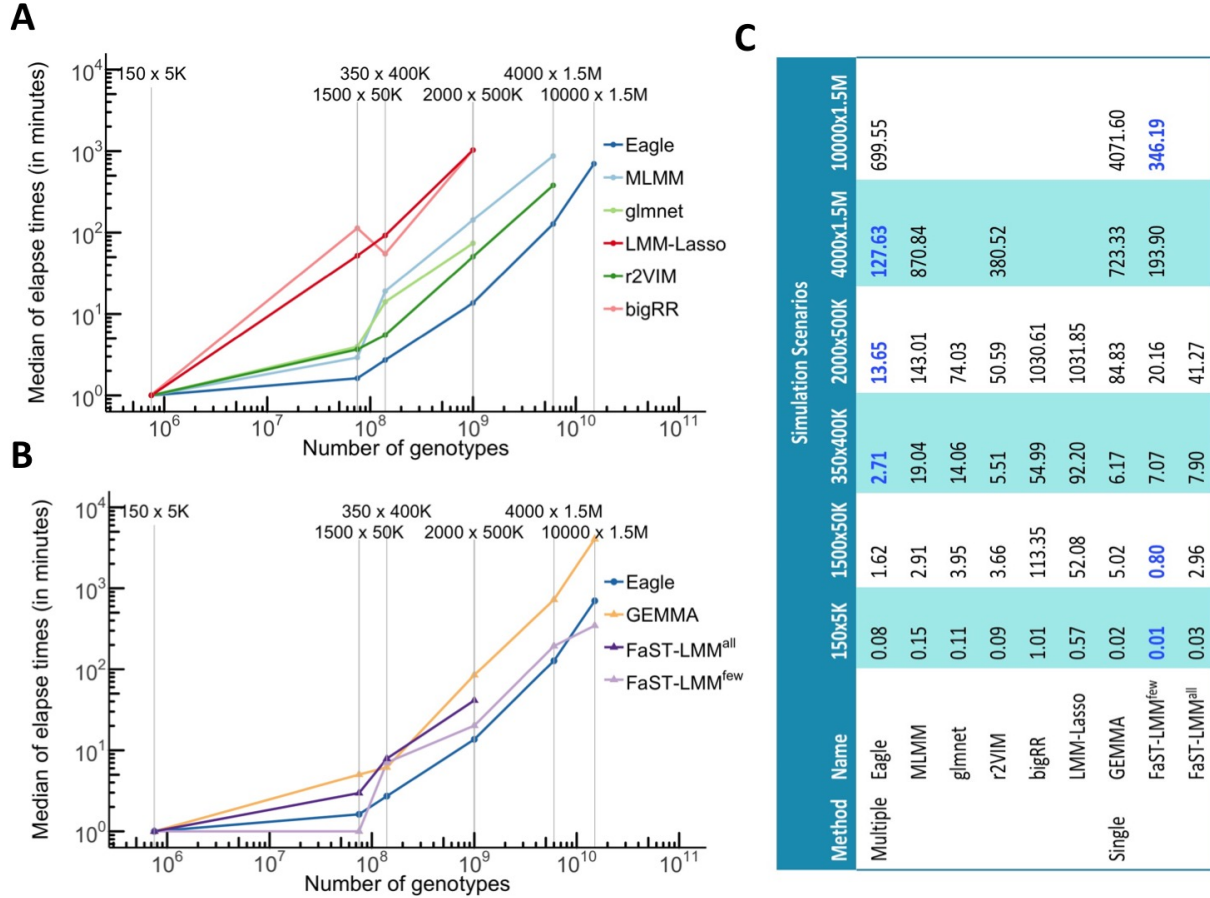


performed well. However, it was when Eagle was compared against single-locus methods that the difference in power was most noticeable. Eagle had much higher power than single-locus methods for finding SNP in true association with a trait while avoiding false associations (Figure 1).

Memory Usage and Run Times Memory usage and run (or elapse) times were recorded for Eagle and the other computer programs/packages across the simulation scenarios. Analyses were performed on a high-end desktop computer with dual 8-core Xeon processors and 128 gigabytes of RAM. Not all data generated under the six scenarios could be analysed by all implementations. Memory usage for many of the computer programs/packages was the limiting factor (see Supplementary Figure 2). The single-locus program GEMMA was by far the most memory efficient. Not surprisingly, the multi-locus programs were memory intensive. Most required in excess of the 128 gigabytes of available RAM for the analysis of data generated under 4000 x 1.5M and 10000 x 1.5M. Even FaST-LMM, when all the SNP data were being used to calculate the similarity matrix, ran out of memory for the larger scenarios. Of the multi-locus programs/packages, only Eagle, with its ability to handle data larger than the memory capacity of the computer, was capable of producing findings for data from our largest scenario, 10000 x 1.5M.

The median run times for Eagle and the other computer programs/packages across the six scenarios are shown in Figure 2. The x- and y-axes are on a log scale. A unit change on the x- or y-axis is equivalent to a change in the order of magnitude. In answer to our question of how does Eagle compare in terms of run time to competing implementations, Eagle was significantly faster, sometimes by orders of magnitude, than the other multi-locus implementations and is comparable to the single-locus implementations. For a simulation study with 150 individuals and 5000 SNPs, Eagle produced results in seconds. For the larger simulation scenarios of 1500 x 50K and 350 x 400K, analyses with Eagle took under two minutes. Even for data from a couple of thousand individuals and half a million SNPs (2000 x 500K), the median run time of Eagle was under 14 minutes. For our scenarios where there were thousands of individuals and 1.5 million SNPs, Eagle took just over two hours for the analysis of data from 4000 x 1.5M and 12 hours for the analysis of data from 10000 x 1.5M. Towards the final stages of writing this paper, we gained access to a high-end server with 14-core Xeon processors and 256 gigabytes of RAM. We reran Eagle on data from the largest scenario 10000 x 1.5M to measure the impact on run time. The median run time dropped by more than 70% from 12 hours to 3.31 hours.

Figure 2: Median run times, in minutes, for the analysis of simulation study data from the six scenarios. Eagle is compared against five other multi-locus programs/packages (A) and two single-locus programs (B). The x- and y-axes are on a log scale for improved aesthetics. Eagle has the lowest run-times of the multi-locus programs/packages, sometimes by orders of magnitude. Eagle can even produce results faster than single-locus programs. The median run times for the programs/packages across the scenarios are given in the table (C). The entries in a blue font correspond to the lowest run-time for a scenario. FaST-LMM^{all} is where calculation of the similarity matrix is based on all the SNP data. FaST-LMM^{few} is where calculation of the similarity matrix is based on a subset of the SNP data.



Mouse Data Analysis We were interested in comparing results from Eagle with those from single-locus association mapping for a real data set. We chose to focus on data from a large outbred mouse study¹⁸. This study was a little unusual in that it collected and analysed SNP dosages (continuous values from zero to one of expected allele counts) instead of the more common SNP genotypes. Analyses based on dosages rather than discrete genotypes have been shown to have greater power for the detection of genes that are influencing a trait¹⁹. By converting the dosages into genotypes and analysing the data with the single-locus program FaST-LMM, we obtained a subset of those findings reported in the original study. We then analysed the data with Eagle. Due to Eagles increased power, we found SNP-trait associations not found with FaST-LMM. However, we were able to confirm the validity of these new findings as they matched what was found in the original study. Having the ability to confirm new findings in a real study was one of the primary motivators for choosing these data for analysis.

We repeated the single-locus analyses, as first performed¹⁸, except that we focused on autosomal SNP and our analyses were based on SNP genotypes rather than SNP dosages. In the original analysis, a genome-wide threshold that gave a false discovery rate of 5%, was found via permutation. We followed the same empirical procedure but increased the number of permutations from 100 to 250.

Eagle was run in three ways. Eagle chooses the best model via the extended Bayesian information criteria (extBIC)²⁰. The conservativeness of the extBIC can be adjusted by a single regularisation parameter γ that ranges from zero to one. In the simulation study, this parameter was set to one, its most conservative and default setting. The mouse data were also analysed under this setting (Eagle^{default}). An alternate²⁰, less conservative way of setting γ is to let $\gamma = 1 - \frac{1}{(2\kappa)}$ with $\kappa = \frac{\log(L)}{\log(n_g)}$ where L is the number of loci that span the genome, and n_g is the number of individuals/groups/lines/strains in the study (Eagle^{alt}). However, our preferred way is to set the γ parameter for each trait via permutation (Eagle^{optimal}). We used 100 permutations to set γ to give a false positive rate of 5%. This only took six times as long as a single analysis of the data. This is because the marker data need only be read once, and only the trait data changes across permutations leading to other computational efficiencies. This permutation method has been implemented within the Eagle package.

The genome wide results from the analyses of the mouse data are shown in Figure 3. The

mouse study recorded measurements on 200 traits. Of these, in the original study, 45 were able to have their findings corroborated by previously published work. We focused our analyses here on these same 45 traits. Overall, FaST-LMM, Eagle^{default}, Eagle^{alt}, and Eagle^{optimal} found 50, 37, 67, and 106, SNP-trait findings, respectively, across 39 traits. No associations were found by FaST-LMM and Eagle for the other six traits. Eagle^{alt} and Eagle^{optimal} also found SNP-trait associations not found in the original study. This is despite their analyses being based on the SNP genotype data and the original study being based on SNP dosage data. Eagle^{alt} found two and Eagle^{optimal} found seven new findings (Supplementary Table 2). These new findings all involved SNPs whose association had been confirmed for other related traits in the original study.

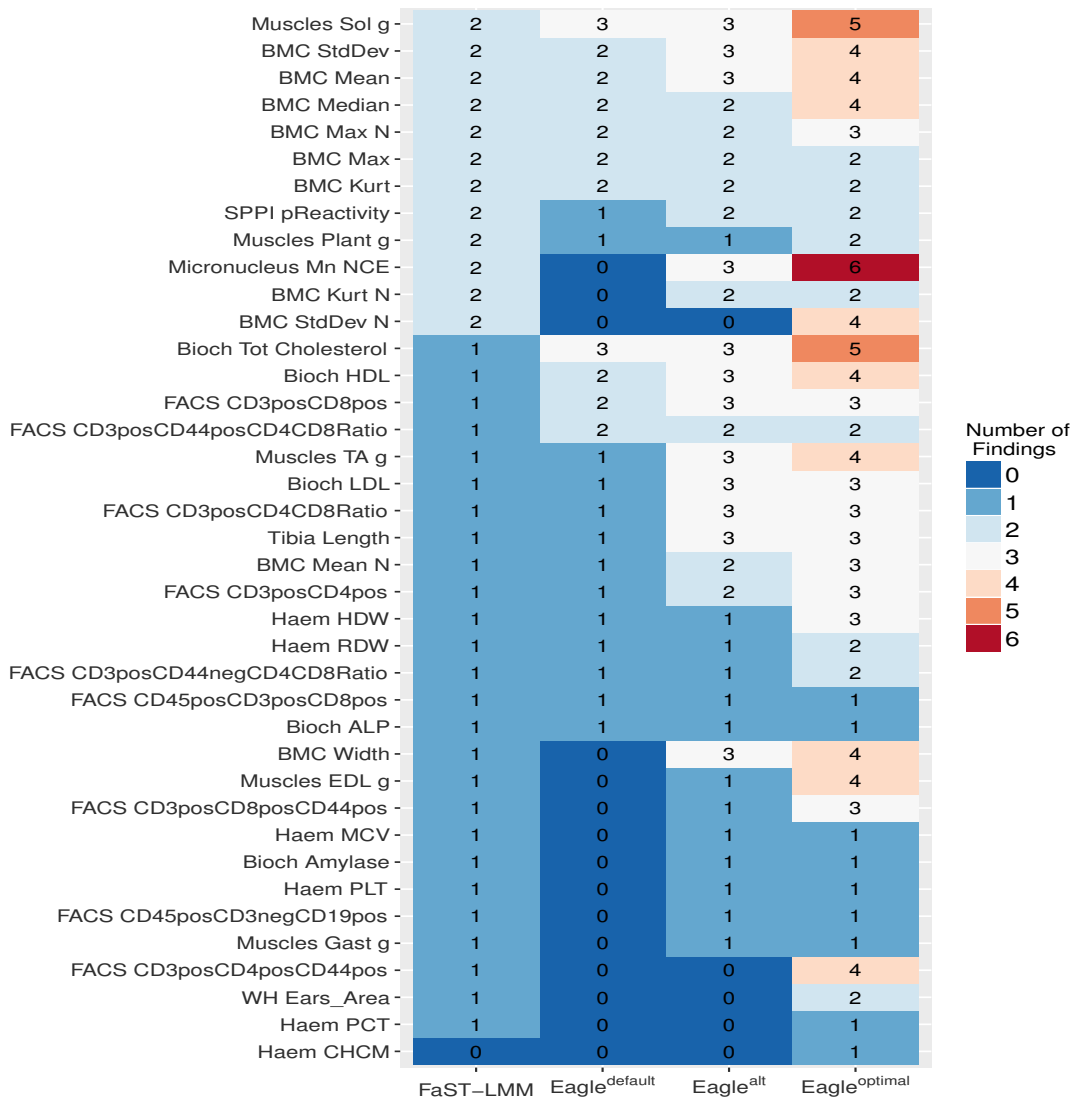
It is worth noting that the mouse data is similar in size to data generated under the simulation scenario 2000x500K. In the simulation study,

2 Discussion

Eagle is a new linear mixed model based method (and R package) for multi-locus association mapping. It advances the state of association mapping in several ways. First, its computational footprint is much smaller than other multi-locus implementations. Eagle makes multi-locus analysis practical, even when the datasets are large. Second, the results from Eagle are immediately interpretable. They are the set of SNPs in strongest association with the trait where each SNP identifies a separate genomic region of interest. Third, it treats association mapping as a model selection problem, avoiding multiple testing issues and the need for significance thresholds. As we saw in the simulation study, Eagle has considerably higher power than single-locus methods but is comparable in run time. Also, when analysing the mouse data, Eagle found more than double the SNP-trait associations than with single-locus association mapping, the method of choice. Furthermore, these extra findings were all true.

Eagle outperformed the other multi-locus methods in our simulation study. However, we are cognisant of the fact that we made several implementation choices that impact our conclusions. For instance, we chose to calculate the significance of the SNP effects from bigRR, LMM-Lasso, and glmnet via stability selection. Permutation and its variants^{21,22} are also equally valid empirical approaches. Stability selection though has the advantage of being based on repeated sampling of only a proportion (50% in our case) of the data. Also, when analysing the (sub)samples, it was not

Figure 3: Genome-wide association mapping results from analyses of the mouse data for the single-locus method FaST-LMM and the multi-locus method Eagle. Eagle was run under three settings; its default setting (Eagle^{default}), an alternate less conservative setting based on the number of SNPs and sample size (Eagle^{alt}), and where the model selection had been optimised for a false positive rate of 5% (Eagle^{optimal}). The number of SNP-trait associations found are reported in the cells.



necessary to calculate the entire solution path for a method. Instead, analyses are performed for a fixed value of the regularisation parameter, greatly reducing the amount of computation required. For r2VIM, an R package implementing random forests, we had to decide on the minimum size of a terminal node, the number of trees, and number of potential variables. The setting of these parameters greatly affects performance. We acknowledge that in the hands of an expert, r2VIM could be fine-tuned for a better balance of computational and statistical performance. However, we would like to think that the parameter settings we used are sensible since they match the values in the original r2VIM publication ¹¹.

Eagle's computational speed does come at a cost. It is a weakness shared by all of the methods considered here, although in different ways. Eagle cannot handle extra random effects which are sometimes needed when more advanced study designs are employed. One solution is to adopt a two-stage analysis procedure. In the first stage, a single linear mixed model is fitted to the data. Much of the modelling complexity, including the extra random effects, is captured in this first-stage model. In the second stage, Eagle is run not on the original trait data but adjusted trait data which are obtained from the first stage analysis. Even though this is a well accepted practice, it is approximate ²³. A better solution is to fit a single model to the data. Although not specifically designed for association mapping, WGAIM ²⁴, upon which Eagle is based, and RWGAIM ²⁵ are two R packages where this is possible. The difficulty is that for large datasets and/or complex models, run time and memory usage can become limiting factors for analysis.

Over the coming years, computationally, the demand placed upon association mapping methods is going to increase. High-throughput array-based technologies continue to decrease the cost of genotyping, permitting ever larger GWASs to be performed. Whole-genome sequencing is also now a reality. Already sequence across entire genomes are being collected for GWASs ^{26,27} culminating in data on millions of SNPs. It is because of this growing demand that we have purposely structured the Eagle package for continued development. We are already experimenting with a GPU-based version of Eagle. Early results suggest that for small to moderate sized datasets (<10,000 samples), there is little improvement in performance over CPU-based computation. However, for larger study sizes, we are seeing up to a 40% decrease in run times. We also have plans for Eagle to run on computer clusters. Structuring Eagle for larger-than-memory calculations was a preemptive step in this direction. GWASs have changed significantly in the past decade but the size and complexity of GWASs is expected to change even more in the coming decade.

3 Methods

Mouse Data The data were obtained from a large genome-wide association study that was performed in outbred mice¹⁸. Phenotypic and genotypic data were available on 1,887 adult mice. The phenotypic data included raw and adjusted (for fixed effects) measurements from 200 behavioural, tissue, and physiological traits. Of these traits, 43 yielded SNP-trait associations that could be corroborated through other independent published work. It was these 43 traits that were the focus of our real data analyses. As in the original study¹⁸, our analyses were based on the adjusted traits. Genotypic data were available on 359,559 (353,697 autosomal) SNPs in the form of marker dosages (expected allele counts that ranged from zero to one). All missing data had been imputed. We converted the dosages into discrete genotypes by clustering around 0, 0.5, and 1, corresponding to SNP genotypes AA, AB, and BB, respectively. We focused our analyses on the autosomal SNPs.

Eagle Approach for Multi-locus Association Mapping Eagle is a method for multi-locus association mapping on a genome-wide scale. It is based on linear mixed models. It differs from most other single- and multi-locus association mapping methods. Eagle treats association mapping as a model selection instead of variable selection problem. Consequently, we do not have to contend with multiple testing issues or having to construct significance thresholds. Eagle also reports as its findings only those SNPs that are in strongest linkage disequilibrium, and hence closest to the genes influencing a trait. The methodological foundation for Eagle comes from a whole-genome linkage analysis method that was developed for mapping quantitative trait loci in experimental crosses²⁴.

Let $S = \{S_1, S_2, \dots, S_s\}$ be a set of s ordinal numbers where S_k is the S_k th ordered SNP that was selected in the k th iteration of the model building process. Suppose three iterations ($s = 3$) have been performed and say the 500023rd, 15th, and 420th SNP were selected. Then $S = \{500023, 15, 420\}$. Let $\mathbf{y}^{(n \times 1)}$ be a vector containing n measurements of the quantitative trait. Let $\mathbf{M}^{(n_g \times L)} = [\mathbf{m}_1 \mathbf{m}_2 \dots \mathbf{m}_L]$ be a matrix containing the genotype data which have been collected from L loci that span the genome on n_g groups/lines/strains. Here, $n \geq n_g$ meaning that a single or several trait measurements may be taken of the same group/line/strain. It is common for the columns of \mathbf{M} to be in map order but this is not a requirement. The vector $\mathbf{m}_j^{(n_g \times 1)}$ contains the genotypes for the j th SNP. The genotypes are coded as -1, 0, and 1 corresponding to SNP

298 genotypes AA, AB, and BB, respectively.

299 The specifics of the Eagle method are as follows. Eagle builds the "best" model iteratively,
 300 via forward selection. Suppose s iterations of our model building process have already been per-
 301 formed. This means s SNP-trait associations have been identified. It also means that s separate
 302 genomic regions of interest have been found. To perform the $s + 1$ th iteration, we first fit the current
 303 model to the data. The (current) model is of the form

$$\mathbf{y} = \mathbf{X}\boldsymbol{\tau} + \mathbf{Z}\mathbf{u}_g + \mathbf{e} \quad (1)$$

304 where $\mathbf{X}^{(n \times p)}$ and $\mathbf{Z}^{(n \times n_g)}$ are known design matrices with \mathbf{X} being of full rank and \mathbf{Z} con-
 305 taining zeros and ones that assign the appropriate genetic effect to each measurement. The vec-
 306 tor $\boldsymbol{\tau}^{(p \times 1)}$ has p fixed effects parameters including the intercept. The vector $\mathbf{u}_g^{(n_g \times 1)}$ contains
 307 the genetic effects. The vector of residuals is $\mathbf{e}^{(n \times 1)}$ whose distribution is assumed to follow
 308 $N(\mathbf{0}, \sigma_e^2 \mathbf{I}^{(n \times n)})$. So far, this model differs little from standard linear mixed models for association
 309 mapping^{28,29} However, it is how we specify \mathbf{u}_g that distinguishes our model from the others.

310 The genetic effects \mathbf{u}_g are modelled as

$$\mathbf{u}_g = \sum_{k=1}^s \mathbf{m}_{S_k} a_{S_k} + \mathbf{M}_{-S} \mathbf{a}_{-S} \quad (2)$$

311 where $\mathbf{m}_{S_k}^{(n_g \times 1)}$ is the vector of genotypes for the k th selected SNP, a_{S_k} is the additive effect of
 312 the k th selected SNP, $\mathbf{M}_{-S}^{(b \times L-s)}$ is the matrix of SNP genotypes with the data for the SNP in S
 313 removed, and $\mathbf{a}_{-S}^{(L-s \times 1)}$ is a random effect whose distribution is $\mathbf{a}_{-S} \sim N(\mathbf{0}, \sigma_a^2 \mathbf{I}^{(L-s \times L-s)})$. The
 314 terms in the summation on the left hand side are fixed effects. They account for the additive effects
 315 of those SNPs that have been found to be in association with the trait. The other term is a random
 316 effect. It accounts for the joint effect of the yet-to-be-identified SNP that are in association with
 317 the trait. This is a simple genetic model but it is effective for discovering SNP-trait associations.

318 Second, we estimate the parameters of (1) and (2) via residual maximum likelihood (REML).
 319 For complex models, REML can be computationally demanding. However, our model only con-
 320 tains a single random effect (\mathbf{a}_{-S}). Here, highly efficient single-dimension optimisation via spec-
 321 tral decomposition is possible³⁰.

322 Third, we identify the $(s + 1)$ th SNP that is in strongest association with the trait, based on

the maximum score statistic $t_j^2 = \frac{\tilde{a}_j^2}{\text{var}(\tilde{a}_j)}$ where \tilde{a}_j is the best linear unbiased predictor (BLUP), and $\text{var}(\tilde{a}_j)$ is its variance. This statistic is not only appealing intuitively, where we identify a SNP based on its (random) effect size and accuracy, but is justifiable, theoretically ²⁵.

Fourth, we determine the importance of the $(s + 1)$ th selected SNP via a model selection strategy ²⁴. We begin by reforming (2) where S now contains the $s + 1$ selected SNP. We then fit this new model to the data via maximum likelihood and calculate its extended Bayesian information criteria (extBIC) ²⁰. The extBIC is a model selection measure that takes into account the number of unknown parameters and the complexity of the model space. It is well suited to the model selection problem in genome-wide association studies ²⁰. It is different to the model selection measure used in ²⁴. If this new model has a larger extBIC than the current model, then the $s + 1$ th selected SNP is added to the current model and the above process is repeated. If this new model has a smaller extBIC than the current model, then the model building process is complete. The set of SNP in strongest association with the trait is the s SNPs previously identified.

Reducing the dimension of the model

In practice, estimating the parameters of (2) can be demanding, computationally. The vector \mathbf{a}_{-S} has $L - s$ random effects where in modern genome-wide association studies, L , the number of SNPs, can be extremely large. An alternative model is given by Verbyla ^{25,31}. They show how to reformulate (2) to be a model with a random effect with only n elements

$$\mathbf{u}_g = \sum_{k=1}^s \mathbf{m}_{S_k} a_{S_k} + (\mathbf{M}_{-S} \mathbf{M}_{-S}^T)^{1/2} \mathbf{a}_{-S}^* \quad (3)$$

where $\mathbf{a}^* \sim N(\mathbf{0}, \sigma_a^2 \mathbf{I}^{(n_g \times n_g)})$, and $(\mathbf{M}_{-S} \mathbf{M}_{-S}^T)^{1/2}$ can be calculated via singular value decomposition ³². Although it may not be obvious, the two models are equivalent, having identical variance structures. Yet, the computational cost of model (3) compared to model (2) is much less, due to the random term in model (3) having only n instead of $L - s$ effects needing estimating.

Verbyla ^{25,31} go on to show how to recover $\tilde{\mathbf{a}}$ from estimates from model (3) with

$$\tilde{\mathbf{a}} = \left[\mathbf{M}_{-S}^T (\mathbf{M}_{-S} \mathbf{M}_{-S}^T)^{-1/2} \right] \tilde{\mathbf{a}}^* \quad (4)$$

346 where its variance matrix is

$$\text{var}(\tilde{\mathbf{a}}) = \mathbf{M}_{-S}^T (\mathbf{M}_{-S} \mathbf{M}_{-S}^T)^{-1/2} \text{var}(\tilde{\mathbf{a}}^*) (\mathbf{M}_{-S} \mathbf{M}_{-S}^T)^{-1/2} \mathbf{M}_{-S} \quad (5)$$

347 These values are needed in order to calculate the score statistic t_j^2 for identifying the SNP in
348 strongest association with the trait. Fortunately, when calculating t_j^2 , only the diagonal elements of
349 the variance matrix are needed which simplifies the calculation of (5).

350 **Comparison Methods**

351 **Multi-locus methods**

352 We compare the computational and statistical performance of Eagle against five multi-locus meth-
353 ods. They are bigRR, LMM-Lasso, glmnet, MLMM, and r2VIM. All but glmnet have been pur-
354 posely designed for genome-wide association mapping. BigRR, LMM-Lasso, and glmnet are
355 regression-based regularisation methods. BigRR is based on generalised ridge regression, LMM-
356 Lasso is based on lasso, and glmnet is based on elastic net. Regularisation methods make param-
357 eter estimation possible in models where the number of predictors is far greater than the number
358 of samples. They allow the strength of association between all the SNPs and trait to be measured
359 within a single model, simultaneously. A limitation of these methods though is that the statistical
360 significance of the SNP effects cannot be easily determined. Due to the adaptive nature of the
361 estimation procedures, to do this analytically is challenging and is an area of active research³³.
362 Instead, we calculate significance empirically via stability selection (see below).

363 MLMM is closest in philosophy to Eagle. It too is based on building the best model via
364 forward selection, within a linear mixed model framework, using the extBIC. However, there are
365 differences between the two methods. MLMM does not make use of dimension reduction. Also,
366 it builds its "best" model differently to Eagle. Eagle uses a score statistic in which to identify
367 SNPs to enter the model. MLMM uses the statistical significance of the SNPs. This involves
368 fitting a separate linear mixed model for each unidentified SNP. Both are R packages but there is a
369 significant difference in computational performance (see Results).

370 R2VIM differs to the other four methods in that it is a non-parametric model-free approach.

It implements random forests but where multiple parallel runs are performed. Each run leads to different random forests being created. A relative importance score is calculated, within a run, for each SNP. This is done by dividing a SNP's importance score by the minimum importance score observed across all the SNPs within a run. Only those SNPs with relative importance scores above a certain threshold across all the runs are deemed to be significant. Unfortunately, the relationship between threshold value and false positive rate is unknown. The threshold could be found empirically via permutation but the computational cost is high, restricting the size of data that can be analysed.

Single-locus methods

We also compare the performance of Eagle against two single-locus methods, GEMMA¹³ and FaST-LMM¹⁴. Both are based on linear mixed models. The models have a single fixed effect for the SNP, other fixed effects, a single random effect to account for familial relatedness (or polygenic background), and an error. The significance of the SNP effect in the model is a measure of the strength of association. They are of the same computational complexity¹³, and produce exact results. Both perform a single spectral decomposition of the relationship (or similarity) matrix K , use an eigenvector matrix to rotate the data, and reformulate the (residual) log likelihood for easier computation. They do differ in their estimation procedure. GEMMA implements Newton-Raphson. FaST-LMM implements Brent's algorithm. Newton-Raphson is more complicated but has better convergence properties than Brent's algorithm. Both methods are state-of-the-art and have been implemented in highly efficient computer programs.

Stability Selection Stability selection³⁴ is a subsampling strategy with a range of applications, including the estimation of the significance of effects in regression models where the number of parameters (p) can be much larger than the number of samples (n). We chose stability selection over permutation and other sampling procedures because of its low computational cost. We employ stability selection to estimate, empirically, the significance of the SNP effects from the regularisation method's analyses of the simulated data.

To obtain significance estimates via stability selection, we do the following. For LMM-Lasso and glmnet, we begin by performing a preliminary analysis of a replicate where we tune the reg-

ularisation parameter to yield 20 to 30 non-zero SNP effects. This tuning of the regularisation parameter is done for data under each simulation scenario but it is not necessary to do it for every replicate within a scenario. It is sufficient to pick a single replicate at random upon which to tune the regularisation parameter. Here, we have the luxury of knowing the genetic conditions under which the simulated trait data are generated. We know that 20 to 30 SNP-trait associations is a reasonable number of findings to expect. However, stability selection estimates are robust³⁴ and the regularisation parameter can be tuned to any reasonable number of non-zero effects. We sample repeatedly, without replacement, from the replicate. We draw 100 replicate subsets of size $n/2$. Each replicate subset is then analysed with LMM-Lasso and glmnet, with their regularisation parameter set to the value found in the preliminary analysis. A (probability) estimate of the statistical significance of a SNP effect is obtained by counting the number of times the SNP have a non-zero effect size over all the replicates divided by the number of replicates (which was 100).

For bigRR, we modify our stability selection procedure slightly. There is no need to tune the regularisation parameter for bigRR as an optimal value is found as part of its analysis. As described above, we draw 100 replicate subsets of size $n/2$ and analyse these data with bigRR. We then order the SNPs according to the absolute size of their SNP effects and record the top 20 SNPs. A (probability) estimate of the statistical significance of a SNP effect is then obtained by counting the number of times the SNP is recorded divided by the number of replicates.

Generation of Simulation Data The data are generated via data perturbation²⁹. Data perturbation amalgamates real with simulated data to generate replicates. It is a way of introducing greater realism into a simulation study. Here, the genotype data are real, the quantitative trait data are simulated. The SNP genotypes are drawn, according to the specifications of the scenario, from data collected from the 1000 Genome Project, version 3¹⁶. Across scenarios, the SNP data differs. Across replicates within a scenario, the SNP data are the same.

To generate the trait data \mathbf{y} , first, q , the number of SNPs that are to be assigned a quantitative value is drawn from a Poisson distribution with mean 30. Second, q SNP are selected randomly. Third, we assume an additive model for the SNPs. The SNP genotypes AA, AB, and BB are assigned the values -1, 0, and 1, respectively. Fourth, the SNP effects are summed across the q selected loci, for each individual, to generate a $\mathbf{g}^{(n \times 1)}$ vector of genetic values where n is the number of individuals. Fifth, $\mathbf{e}^{(n \times 1)}$, a vector of residuals, is drawn from a normal distribution

where $e_i \sim N(0, \sigma_e^2)$ and σ_e^2 is the residual variance that has been set to yield a trait with heritability 0.5. Sixth, the trait data are formed as $\mathbf{y} = \mathbf{g} + \mathbf{e}$. In forming \mathbf{y} , we have purposely not included any other environmental variables such as age, sex, or experimental design effects. This is because not all the methods were implemented to handle the inclusion of additional fixed effects. A two-stage modelling approach is often adopted to deal with this situation, but we chose not to introduce this complexity into the analyses.

Implementation Eagle has been implemented as an R package of the same name. Much of the computation though is performed outside of R via C++ functions that utilise Eigen C++ routines. Eagle has been purpose built to rely heavily on calls to BLAS and LAPACK, mathematical libraries common to most computer systems. By making use of multi-threaded BLAS and LAPACK libraries, many of the calculations in Eagle are parallelised. We have gone to great lengths to make Eagle easy-to-use. Tutorials, videos, How-To guides, and a link to our server for demonstrating Eagle on some test data are available on the Eagle website. Eagle is available for download from the CRAN website.

Data Availability

The input files for Eagle containing the mouse GWAS data are available for download from <https://doi.org/10.25919/5bc08287717dd>. The original data were obtained from the Heterogeneous Stock Mice website <http://wp.cs.ucl.ac.uk/outbredmice/heterogeneous-stock-mice/>.

References

1. Cardon, L. R. & Palmer, L. J. Population stratification and spurious allelic association. *The Lancet* **361**, 598–604 (2003).
2. Spielman, R. S. & Ewens, W. J. The TDT and other family-based tests for linkage disequilibrium and association. *American Journal of Human Genetics* **59**, 983 (1996).
3. Price, A. L., Zaitlen, N. A., Reich, D. & Patterson, N. New approaches to population stratification in genome-wide association studies. *Nature Reviews Genetics* **11**, 459 (2010).
4. Purcell, S. *et al.* PLINK: a tool set for whole-genome association and population-based linkage analyses. *The American Journal of Human Genetics* **81**, 559–575 (2007).

5. Bradbury, P. J. *et al.* TASSEL: software for association mapping of complex traits in diverse samples. *Bioinformatics* **23**, 2633–2635 (2007).
6. Storey, J. D. & Tibshirani, R. Statistical significance for genomewide studies. *Proceedings of the National Academy of Sciences* **100**, 9440–9445 (2003).
7. Li, J. & Ji, L. Adjusting multiple testing in multilocus analyses using the eigenvalues of a correlation matrix. *Heredity* **95**, 221 (2005).
8. de Bakker, P. I. *et al.* Efficiency and power in genetic association studies. *Nature Genetics* **37**, 1217 (2005).
9. Shen, X., Alam, M., Fikse, F. & Rönnegård, L. A novel generalized ridge regression method for quantitative genetics. *Genetics* **193**, 1255–1268 (2013).
10. Rakitsch, B., Lippert, C., Stegle, O. & Borgwardt, K. A Lasso multi-marker mixed model for association mapping with population structure correction. *Bioinformatics* **29**, 206–214 (2013).
11. Szymczak, S. *et al.* r2VIM: A new variable selection method for random forests in genome-wide association studies. *BioData Mining* **9**, 7 (2016).
12. Segura, V. *et al.* An efficient multi-locus mixed-model approach for genome-wide association studies in structured populations. *Nature Genetics* **44**, 825–830 (2012).
13. Zhou, X. & Stephens, M. Genome-wide efficient mixed-model analysis for association studies. *Nature Genetics* **44**, 821–824 (2012).
14. Lippert, C. *et al.* FaST linear mixed models for genome-wide association studies. *Nature Methods* **8**, 833–835 (2011).
15. Friedman, J., Hastie, T. & Tibshirani, R. Regularization Paths for Generalized Linear Models via Coordinate Descent. *Journal of Statistical Software* **33**, 1–22 (2010). URL <http://www.jstatsoft.org/v33/i01/>.
16. Consortium, . G. P. *et al.* A map of human genome variation from population-scale sequencing. *Nature* **467**, 1061 (2010).

17. Boulesteix, A.-L., Janitza, S., Kruppa, J. & König, I. R. Overview of random forest methodology and practical guidance with emphasis on computational biology and bioinformatics. *Wiley Interdisciplinary Reviews: Data Mining and Knowledge Discovery* **2**, 493–507 (2012).
18. Nicod, J. *et al.* Genome-wide association of multiple complex traits in outbred mice by ultra-low-coverage sequencing. *Nature Genetics* (2016).
19. Zheng, J., Li, Y., Abecasis, G. R. & Scheet, P. A comparison of approaches to account for uncertainty in analysis of imputed genotypes. *Genetic Epidemiology* **35**, 102–110 (2011).
20. Chen, J. & Chen, Z. Extended Bayesian information criteria for model selection with large model spaces. *Biometrika* **95**, 759–771 (2008).
21. Browning, B. L. PRESTO: rapid calculation of order statistic distributions and multiple-testing adjusted P-values via permutation for one and two-stage genetic association studies. *BMC Bioinformatics* **9**, 309 (2008).
22. Pahl, R. & Schafer, H. PERMORY: an LD-exploiting permutation test algorithm for powerful genome-wide association testing. *Bioinformatics* **26**, 2093–2100 (2010).
23. Gogel, B., Smith, A. & Cullis, B. Comparison of a one-and two-stage mixed model analysis of Australia's National Variety Trial Southern Region wheat data. *Euphytica* **214**, 44 (2018).
24. Verbyla, A. P., Cullis, B. R. & Thompson, R. The analysis of QTL by simultaneous use of the full linkage map. *Theoretical and Applied Genetics* **116**, 95 (2007).
25. Verbyla, A. P., Taylor, J. D. & Verbyla, K. L. RWGAIM: an efficient high-dimensional random whole genome average (QTL) interval mapping approach. *Genetics Research* **94**, 291–306 (2012).
26. Gudbjartsson, D. F. *et al.* Large-scale whole-genome sequencing of the Icelandic population. *Nature Genetics* **47**, 435 (2015).
27. Long, T. *et al.* Whole-genome sequencing identifies common-to-rare variants associated with human blood metabolites. *Nature Genetics* **49**, 568 (2017).
28. Yu, J. *et al.* A unified mixed-model method for association mapping that accounts for multiple levels of relatedness. *Nature Genetics* **38**, 203 (2006).

- 509 29. Zhao, K. *et al.* An Arabidopsis example of association mapping in structured samples. *PLoS*
510 *Genetics* **3**, e4 (2007).
- 511 30. Kang, H. M. *et al.* Efficient control of population structure in model organism association
512 mapping. *Genetics* **178**, 1709–1723 (2008).
- 513 31. Verbyla, A. P., Cavanagh, C. R. & Verbyla, K. L. Whole-genome analysis of multienvironment
514 or multitrait QTL in MAGIC. *G3: Genes, Genomes, Genetics* **4**, 1569–1584 (2014).
- 515 32. Golub, G. H. & Van Loan, C. F. *Matrix Computations*, vol. 3 (JHU Press, 2012).
- 516 33. Lockhart, R., Taylor, J., Tibshirani, R. J. & Tibshirani, R. A significance test for the Lasso.
517 *Annals of Statistics* **42**, 413 (2014).
- 518 34. Meinshausen, N. & Bühlmann, P. Stability Selection. *Journal of the Royal Statistical Society:*
519 *Series B (Statistical Methodology)* **72**, 417–473 (2010).

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