Using the Rips complex for topologically-certified manifold reconstruction

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Séminaire Geometrica-Titane

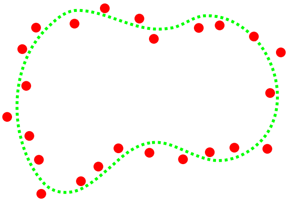
Manifold reconstruction

Data A finite point cloud $P \subset \mathbb{R}^D$ of a d-dimensional manifold M

- D : ambient dimension
- d : intrinsic dimension

Goal Find a simplicial complex K that approximates M in $O(D)$

Hypothesis $d << D$



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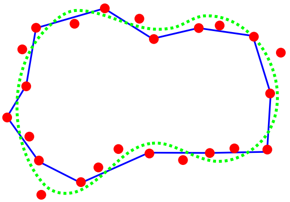
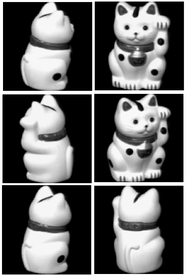


Manifold reconstruction

An example

Data A set of images 128×128 of a toy cat placed on a turntable and observed by a fixed camera.



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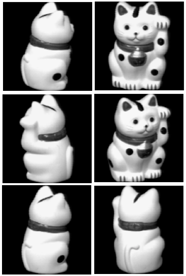

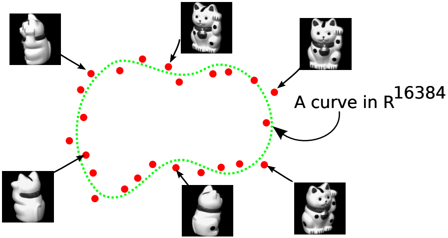

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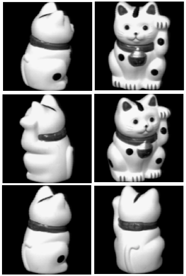
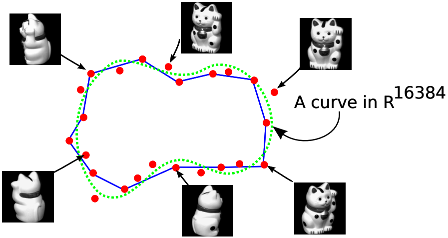
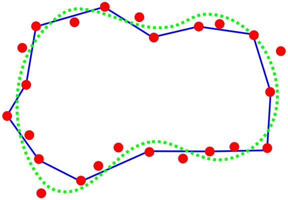




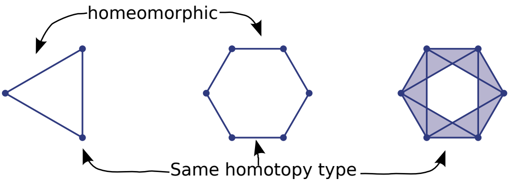
Data A finite point cloud $P \subset \mathbb{R}^D$ of a d-dimensional manifold M

Goal
1. Find conditions on the density of P such that an approximation K has the same topology as M
2. Compute efficiently K (in $O(D)$)



- K is **homeomorphic** to M if there exists $f : M \to K$ one-to-one such that f and f^{-1} are continuous.
  When this is the case, we denote it by $K \approx M$.
- K has the same **homotopy type** as M if M can be continuously deformed to K.
  When this is the case, we denote it by $K \simeq M$.

We say that K has the same *topology* as M if K is *homeomorphic* to M or if K *has the same homotopy type* as M (weaker condition).



Manifold reconstruction

Previous work

- Many applications in machine learning, data analysis, ...
- Existing algorithms : ISOMAP, LLE, Laplacian eigenmaps, . . .

  $\rightarrow$ few theoritical guarantees on the topology

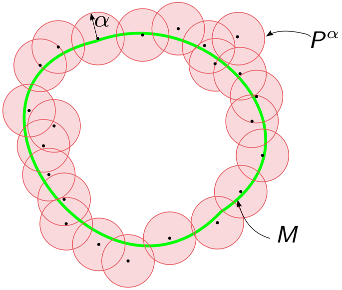
Methods that guaranties homeomorphism

- First approach : using the Delaunay complex [Cheng Dey Ramos 05]
- With the witness complex [Boissonat Guibas Oudot 09]
- Tangential Delaunay complex [Boissonat Ghosh 10]

Manifold reconstruction

Offset of points

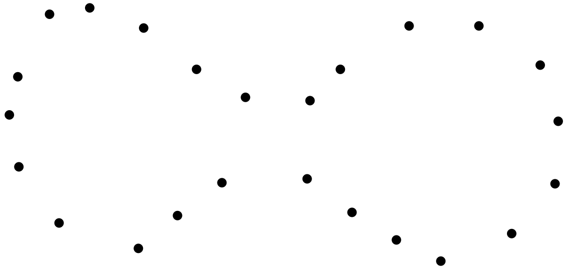
Given $P \subset \mathbb{R}^D$ we denote $P^\alpha = \bigcup B(p, \alpha)$ the α-offset of P.



Homotopy type of the offset of points

- $P^\alpha \simeq M$ when P is dense enough and reach$(M) > 0$ [Niyogi Smale Weinberger 04]
- $P^\alpha \simeq M$ when P is dense enough and μ-reach$(M) > 0$ [Chazal Cohen-Steiner Lieutier 06]

Manifold reconstruction
Reach



Manifold reconstruction

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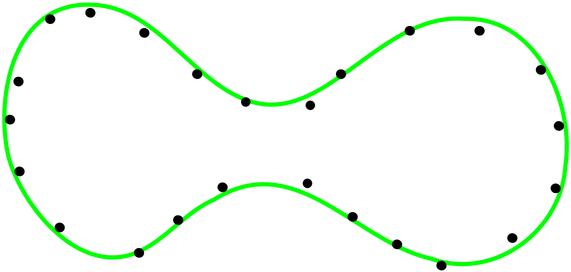

Manifold reconstruction
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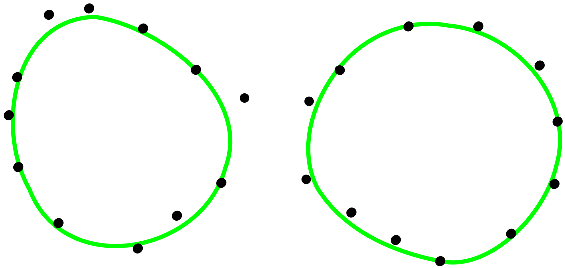

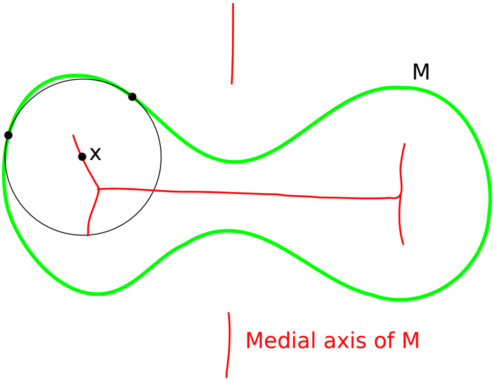

- $\mathrm{MedialAxis}(M) = \{x \in \mathbb{R}^D \mid x \text{ has at least two closest points on } M\}$
- $\mathrm{Reach}(M) = d(M, \mathrm{MedialAxis}(M))$

Theorem [Niyogi Smale Weinberger 08]

The α-offset P^α has the same homotopy type as M i.e. $P^\alpha \simeq M$ when

$$\begin{cases} d_H(P, M) \leq \varepsilon < (3 - \sqrt{8})\ \text{reach}(M) \\ \alpha = (2 + \sqrt{2})\varepsilon \end{cases}$$

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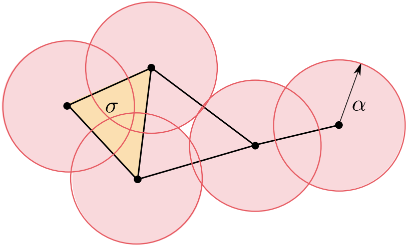
$$\left\{ \begin{array}{c} d_H(P, M) \leq \varepsilon < (3 - \sqrt{8})\ \text{reach}(M) \\ \alpha = (2 + \sqrt{2})\varepsilon \end{array} \right.$$

1. Find conditions on the density of P such that an approximation K has the same topology as M $\rightarrow$ OK
2. Compute efficiently K (in $O(D)$) $\rightarrow$?

Cech complex

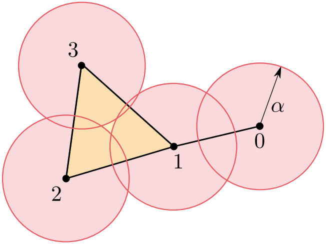
- Nerve of a family : $\mathrm{Nrv}\, F = \{\sigma \subset F \mid \bigcap \sigma \neq \emptyset\}$
- Cech complex $\mathcal{C}(P, \alpha) = \mathrm{Nrv}\{B(p, \alpha) \mid p \in P\}$



Nerve theorem

$P^\alpha \simeq \mathcal{C}(P, \alpha)$

Rips complex



Simplices of $\mathcal{R}(P, \alpha)$:

- cliques in the proximity graph $G(P, 2\alpha)$
- $\{0\}, \{1\}, \{2\}, \{3\}, \{0,1\}, \{1,2\}, \{2,3\}, \{3,1\}, \{1,2,3\}$

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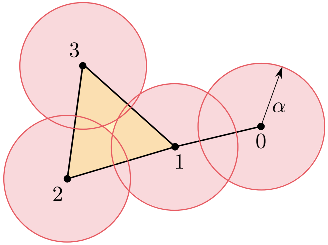

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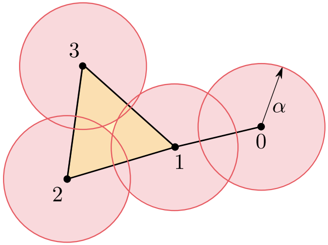

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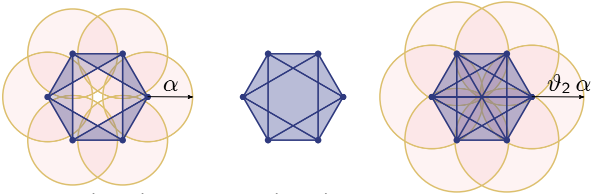
☺ Computation and storage in $O(|P|^2)$

☹ $\mathcal{R}(P, \alpha)$ may not have the same topology as P^α

Fundamental interleaving

$\mathcal{C}(P,\alpha) \subset \mathcal{R}(P,\alpha) \subset \mathcal{C}(P,\theta_D\alpha)$ where $\theta_D = \sqrt{\frac{2D}{D+1}}$



$\mathcal{C}(P,\alpha) \underset{\not\supset}{\subset} \mathcal{R}(P,\alpha) \underset{\not\supset}{\subset} \mathcal{C}(P,\theta_2\alpha)$

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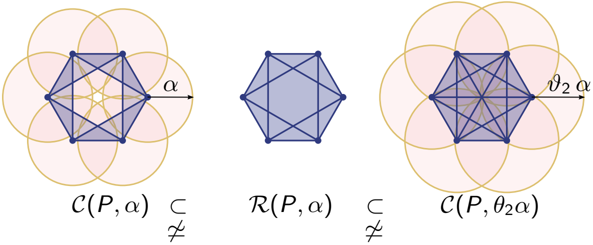
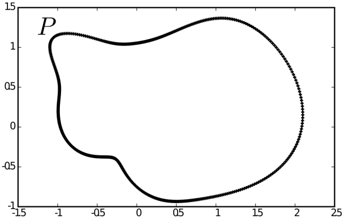
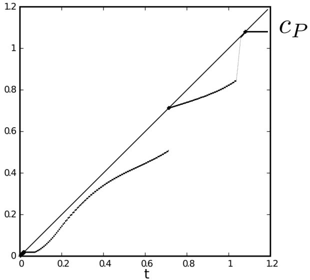


Question: Is it possible to find conditions on P such that $\mathcal{R}(P,\alpha) \simeq \mathcal{C}(P,\alpha)$?

Convexity defect function



$$\mathrm{Centers}(P,t) = \bigcup_{\substack{\emptyset \neq \sigma \subset X \\ \mathrm{Rad}\ \sigma \leq t}} \{\mathrm{Center}(\sigma)\}.$$



$$c_P(t) = d_H(\mathrm{Centers}(P,t), P)$$

Properties of the convexity defect function $c_P(t)$ for a compact set P

- $c_P(t) = 0 \Leftrightarrow$ P is convex
- c_P non decreasing
- $c_P(t) \leq t$
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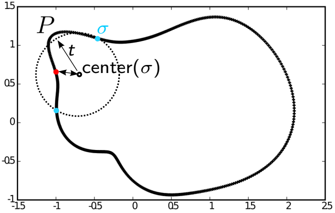

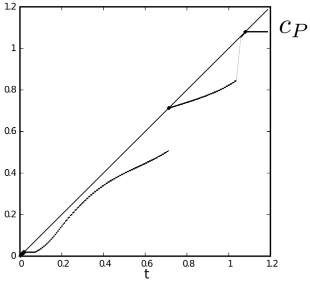

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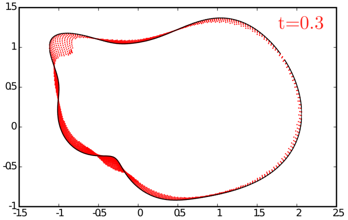

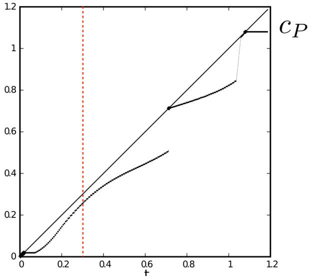

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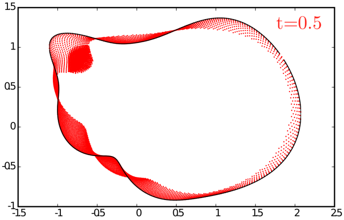

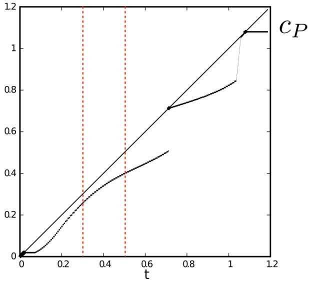

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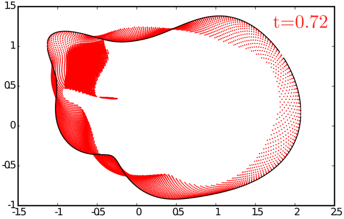

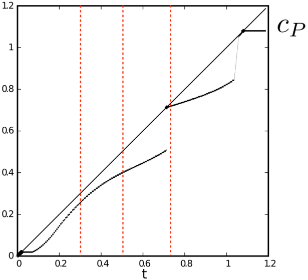

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Homotopy type of the Rips complex

A condition to ensure that the Rips complex has the same homotopy than the Cech complex

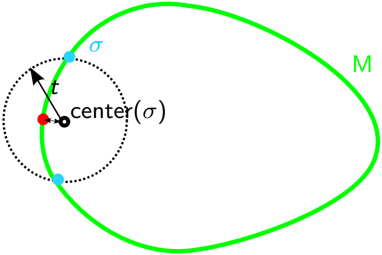
Theorem [Attali Lieutier Salinas 2011]

If $c_P(\theta_D \alpha) < (2 - \theta_D)\alpha$ then $\mathcal{R}(P, \alpha) \simeq \mathcal{C}(P, \alpha)$.
The condition on c_P is optimal (at least in low dimension).

Theorem

If $d_H(P, M) \leq \varepsilon$ then, $\forall t < \mathrm{reach}(M) - \varepsilon$

$$c_P(t) \leq \mathrm{reach}(M) - \sqrt{\mathrm{reach}(M)^2 - (t+\varepsilon)^2} + 2\varepsilon$$

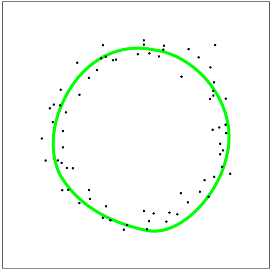


Theorem

If $d_H(P, M) \leq \varepsilon$ and $\frac{\varepsilon}{\text{reach}(M)} < \lambda$ and $\rho = \frac{\alpha}{\varepsilon}$ then $\mathcal{R}(P, \alpha) \simeq M$

Complex	dimension	λ	ρ
Cech complex [NSW04]	$\forall D$	$3 - \sqrt{8} \approx 0.17$	$2 + \sqrt{2} \approx 3.41$
Rips complex	2	0.063	5.00
	3	0.055	5.46
	10	0.041	6.50
	100	0.035	7.22
	$+\infty$	$\frac{2\sqrt{2-\sqrt{2}}-\sqrt{2}}{2+\sqrt{2}} \approx .0340$	7.22

Reconstruction with the Rips complex
Overview



Point cloud $P \subset \mathbb{R}^D$ that approximates a manifold M

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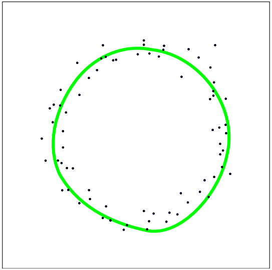

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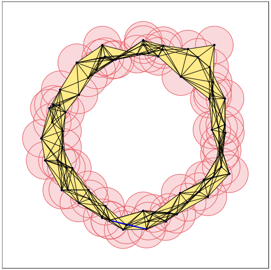

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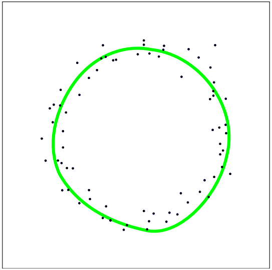

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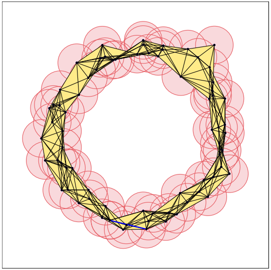

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But in general $\mathcal{R}(P, \alpha) \not\approx M$

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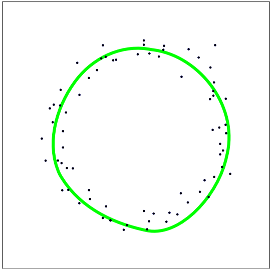

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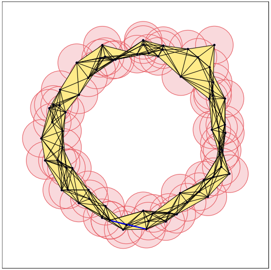

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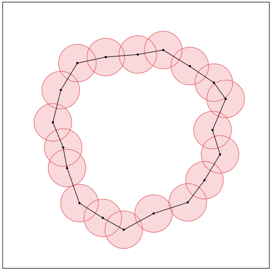

Is it possible to simplify $\mathcal{R}(P, \alpha)$ to a complex homeomorphic to the manifold?

Simplification operations that preserve the homotopy type

We consider two simplification operations :

- the edge contraction of an edge σ
- the collapse of a simplex σ

These two operations preserve the homotopy type when a (local) condition is verified on the link of σ.

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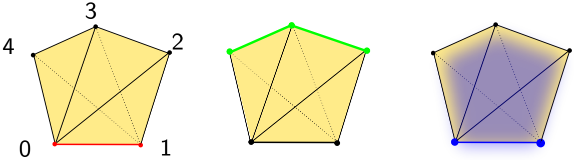
K : a simplicial complex
σ : a simplex of K

Link of σ : $\mathrm{Lk}_K(\sigma) = \{\tau \in K \mid \tau \cap \sigma = \emptyset \text{ and } \tau \cup \sigma \in K\}$
Star of σ : $\mathrm{St}_K(\sigma) = \{\tau \in K \mid \sigma \subset \tau\}$



$\sigma = 01$ $\mathrm{Lk}_K(\sigma) = \{2, 3, 4, 23, 34\}$ $\mathrm{St}_K(\sigma) = \{01, 012, 0123, 01234, ...\}$

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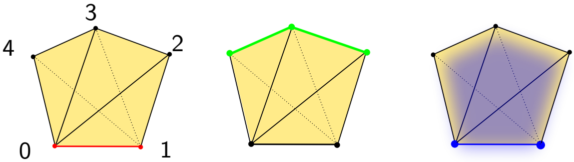
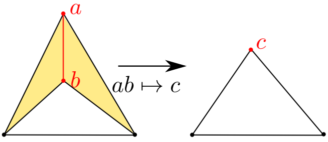


$\sigma = 01$ $\quad$ $\mathrm{Lk}_K(\sigma) = \{2, 3, 4, 23, 34\}$ $\quad$ $\mathrm{St}_K(\sigma) = \{01, 012, 0123, 01234, ...\}$

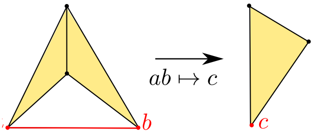
Edge contraction

A condition on the link to ensure that an edge contraction preserves the homotopy type

- Contracting an edge = identify two vertices in the complex
- May change the homotopy type



$ab \mapsto c$ preserves the homotopy type



$ab \mapsto c$ changes the homotopy type

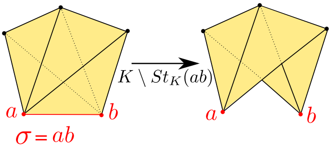
Theorem [Attali Lieutier Salinas 2011]

Let K be a simplicial complex and ab an edge of K.
If $\mathrm{Lk}_K(a) \cap \mathrm{Lk}_K(b) = \mathrm{Lk}_K(ab)$ then the edge contraction $ab \mapsto c$ preserves the homotopy type.

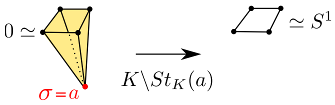
Collapse

Let K be a simplicial complex and σ a simplex of K.

- Removing the star of σ may change the homotopy type



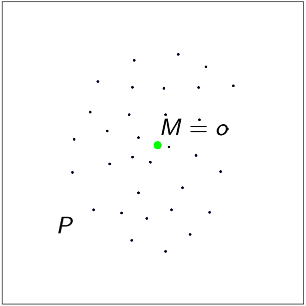
Removing $\mathrm{St}_K(\sigma)$ preserves the homotopy type

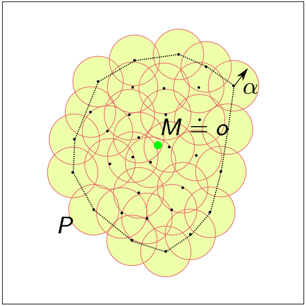


Removing $\mathrm{St}_K(\sigma)$ changes the homotopy type

- If the link of σ is a the closure of a simplex then removing the star of σ preserves the homotopy type
- In this case, we say that removing $\mathrm{St}_K(\sigma)$ from K is a **collapse**

Warm-up : reconstruction of 0-manifold





- Point cloud $P \subset \mathbb{R}^D$ that approximates a point o

- Find conditions on P such that $\mathcal{R}(P, \alpha) \simeq o$
- Find efficiently a sequence of reduction from $\mathcal{R}(P, \alpha)$ to o.

Complexity of deciding if a complex can be reduced to a point

It it really a warm-up?

A complex is said :

- **contractible** if it has the homotopy type of a point
- **collapsible** if it can be reduced to a point by a finite sequence of collapses

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☹ recognizing a contractible complex is undecidable

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- ☺ if $P \subset \mathbb{R}^2$, deciding if $\mathcal{R}(P, \alpha)$ is contractible can be done in polynomial time [Chambers Silva Erickson Ghrist 2010]

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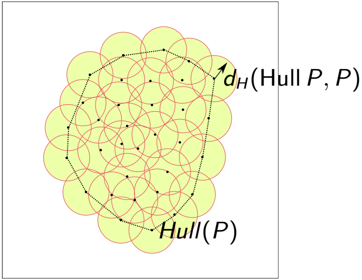
☺ if K is convex and $\dim(K) \leq 3$ then K is collapsible [Chillingworth 67]

Collapsing a Rips complex that approximates a convex

Assume that $d_H(\mathrm{Hull}\, P, P)$ is small regarding to α.



Can you ensure that $\mathcal{R}(P, \alpha)$ is contractible or even collapsible?

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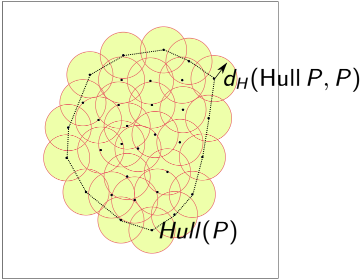

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Theorem [Attali Lieutier Salinas 2013]

If $d_H(\text{Hull}\, P, P) < (2 - \theta_D)\alpha$ then $\mathcal{R}(P, \alpha)$ is collapsible.
The constant $(2 - \theta_D)$ is optimal at least in low dimensions.

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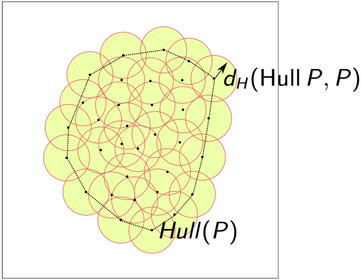

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- ☹ But the proof does not give an efficient algorithm

Reducing a complex to a point

- We propose three efficient strategies to try to reduce a complex L to a point:
  - $\mathrm{SWEEP}(L)$ (vertex and edge collapses)
  - $\mathrm{COMPLETE}(L)$ (edge collapses)
  - $\mathrm{EDGE\_CONTRACTIONS}(L)$
- Each one of these strategies return true if it manages to reduce L to a point with topological-preserving elementary operations and false otherwise

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If $\emptyset \neq \mathsf{Hull}\, P \subset P^{(2-\sqrt{3})\alpha}$ *then* $\mathrm{SWEEP}(\mathcal{R}(P,\alpha))$ *and* $\mathrm{COMPLETE}(\mathcal{R}(P,\alpha))$ *are true*

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- In practice, the most efficient strategy to reduce a complex to a point is $\mathrm{EDGE\_CONTRACTIONS}$

- Build a Rips complex such that $\mathcal{R}(P, \alpha) \simeq M$
- Keep collapsing the largest edge whose link can be reduced to a point

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while $Q \neq \emptyset$ **do**
  Remove the edge ab from Q with highest length
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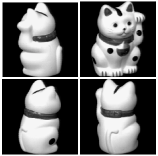
Proposition [Attali Lieutier Salinas 2013]

Let M be a 1-dimensional manifold and $P \subset M$ a finite point cloud.
If $d_H(P, M) < \alpha < \mathrm{reach}(M)/2$ then $\textsc{Simplify}(\mathcal{R}(P, \alpha))$ returns a complex homeomorphic to M.

Experimental results

Data-sets

- Data : a point cloud $P \in \{\texttt{Cat},\texttt{Ramses},\texttt{SO3}\}$ sampling a d-manifold M
  - `Cat`: 72 images of size 128x128
  - `Ramses`: A scan of a statue that consists in 200000 points in $\mathbb{R}^3$
  - `SO3`: 10000 points in $\mathbb{R}^9$ that samples rotational matrices

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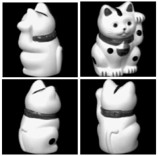

- Input of the simplification algorithm : $\mathcal{R}(P,\alpha)$ such that $\mathcal{R}(P,\alpha) \simeq M$

P	d	D	$\dim(\mathcal{R}(P,\alpha))$
`Cat`	1	16384	19
`Ramses`	2	3	14
`SO3`	3	9	16

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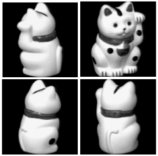

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- Output after simplification K_{out}

P	$\dim(K_{\text{out}})$	$K_{\text{out}} \approx M$	running time
`Cat`	1	YES	2 s
`Ramses`	2	YES	150 min
`SO3`	3	NO	7 min

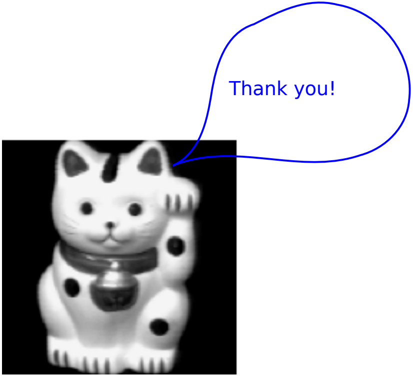
Conclusion

The Rips complex :

- can be computed and stored efficiently
- has the same homotopy type than a sampled manifold under good sampling conditions
- can be simplified (experimentally) to a complex homeomorphic to the manifold

Future work

- Prove that the Rips complex can simplified to a complex homeomorphic to the manifold
- Extend these results for :
  - Manifold with boundary
  - Non-uniform density, presence of outliers


Thank you!