Question 1:

```
function
[posT,velT,timeT,dV1,dV2]=twoImpulseHohmann(r1,r2,i1,i2,Omega1,Omega2,mu)

%This function calculates the two impulses needed to complete a Hohmann
%transfer

%Input: r1, radius of orbit 1
%Input: r2, radius of orbit 2
%Input: i1, inclination of orbit 1
%Input: i2, inclination of orbit 2
%Input: Omega1, longitude of ascending node of orbit 1
%Input: Omega1, longitude of ascending node of orbit 1
%Input: mu, gravitaional parameter

%Output: posT, 500x3 matrix containing postions along the transfer orbit
%Output: timeT, column matrix of times associated with positions in posT
%Output: dV1, impulse at periapsis of transfer
%Output: dV2, impulse at apoapsis of transfer

nu=0:0.001:2*pi;
e=0;
omega=0;
oe1=[r1,e,Omega1,i1,omega,0];
oe2=[r2,e,Omega2,i2,omega,0];
position1=zeros(length(nu),3);
velocity1=zeros(length(nu),3);
position2=zeros(length(nu),3);
velocity2=zeros(length(nu),3);
for i=1:length(nu)
    oe1(6)=nu(i);
    oe2(6)=nu(i);
    [r,v]=oe2rv_Hackbardt_Chris(oe1,mu);
    position1(i,:)=r';
    velocity1(i,:)=v';
    [r,v]=oe2rv_Hackbardt_Chris(oe2,mu);
    position2(i,:)=r';
    velocity2(i,:)=v';
end

v1=sqrt(mu/r1);
v2=sqrt(mu/r2);
hvec1=cross(position1(1,:),velocity1(1,:));
hvec2=cross(position2(1,:),velocity2(1,:));
lvec=cross(hvec1,hvec2);
lvec=lvec/norm(lvec);
position1T=r1*lvec;
aT=(r1+r2)/2;
deltaV1=sqrt(((2*mu)/r1)-(mu/aT))-v1;
u1vec=cross(hvec1,lvec)/norm(hvec1);
V1b4=v1*u1vec;
dV1=deltaV1*u1vec;
V1aft=V1b4+dV1;

M=500;
```

```
tauT=2*pi*sqrt(aT^3/mu);
timeT=linspace(0,tauT/2,M);
posT=zeros(length(timeT),3);
velT=zeros(length(timeT),3);
for i=1:length(timeT)
    [r,v] = propagateKepler_Hackbardt_Chris(position1T,V1aft,0,timeT(i),mu);
    posT(i,:)=r';
    velT(i,:)=v';
end

v2bf=sqrt(((2*mu)/r2)-(mu/aT));
V2b4=-v2bf*u1vec;
u2vec=-cross(hvec2,lvec)/norm(hvec2);
V2aft=v2*u2vec;
dV2=V2aft-V2b4;

end
```

Question 2:

```
function
[posT,timeT,dVP,dVA,totDV]=twoNImpulseOrbitTransfer(r1,r2,i1,i2,Omega1,Omega2
,N,mu)
%This function calculates the impulses needed to complete N Hohmann
%transfers

%Input: r1, radius of orbit 1
%Input: r2, radius of orbit 2
%Input: i1, inclination of orbit 1
%Input: i2, inclination of orbit 2
%Input: Omega1, longitude of ascending node of orbit 1
%Input: Omega1, longitude of ascending node of orbit 1
%Input: N, number of Ho
%Input: mu, gravitaional parameter

%Output: posT, 500x3 matrix containing postions along the transfer orbit
%Output: timeT, column matrix of times associated with positions in posT
%Output: dV1, impulse at periapsis of transfer
%Output: dV2, impulse at apoapsis of transfer
%Output: totDV, the total delta V required to complete the transfer

rn=r1+((1:N)/N)*(r2-r1);
Omegan=Omega1+((1:N)/N)*(Omega2-Omega1);
in=i1+((1:N)/N)*(i2-i1);

rn=[r1,rn];
Omegan=[Omega1,Omegan];
in=[i1,in];

dOmega=Omegan(2)-Omegan(1);

dVP=zeros(N,1);
```

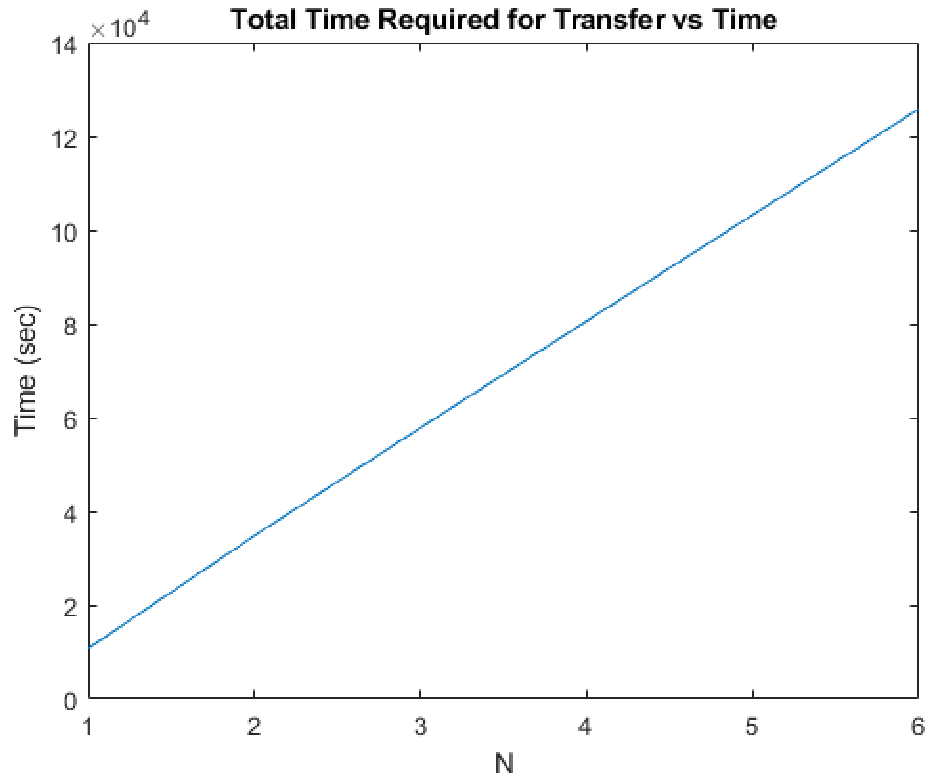
```
dVA=zeros(N,1);
posT=[];
velT=[];
timeT=[0];

N=1:N;
for i=1:length(N)
    %elliptic

[post,velt,timet,dV1,dV2]=twoImpulseHohmann(rn(i),rn(i+1),in(i),in(i+1),Omega
n(i),Omegan(i+1),mu);
    posT=[posT;post];
    velT=[velT;velt];
    timet=timet+timeT(length(timeT));
    timeT=[timeT;timet'];
    dVP(i)=norm(dV1);
    dVA(i)=norm(dV2);
    %circular
    if i<length(N)
    taut=2*pi*sqrt(rn(i+1)^3/mu);
    dOmegaTime=dOmega/sqrt(mu/rn(i+1)^3);
    [timet, post] =
propagateOnCircle(post(end,:),velt(end,:)+dV2,timeT(end),timeT(end)+(taut/2)+
dOmegaTime,mu,500);
    posT=[posT;post];
    timeT=[timeT;timet];
    end
end

timeT(1)=[];

totDV=sum(dVP)+sum(dVA);
fprintf('The total time to complete the transfer is %g seconds or %g
hours\n',timeT(end),timeT(end)/3600)

%Plots 3D view of transfer
figure
hold on
earthSphere
plot3(posT(:,1),posT(:,2),posT(:,3))
hold off

%Plots the ground track;
OmegaE = (2*pi)/((23*3600)+(56*60)+4);
rvValuesECEF = eci2ecef(timeT,posT,OmegaE);
[lonE,lat] = ecef2LonLat(rvValuesECEF);
figure
earth = imread('earth.jpg');
hold on
image('CData',earth,'XData',[-180 180],'YData',[90 -90])
plot(lonE*180/pi,lat*180/pi,'r*');
end
```

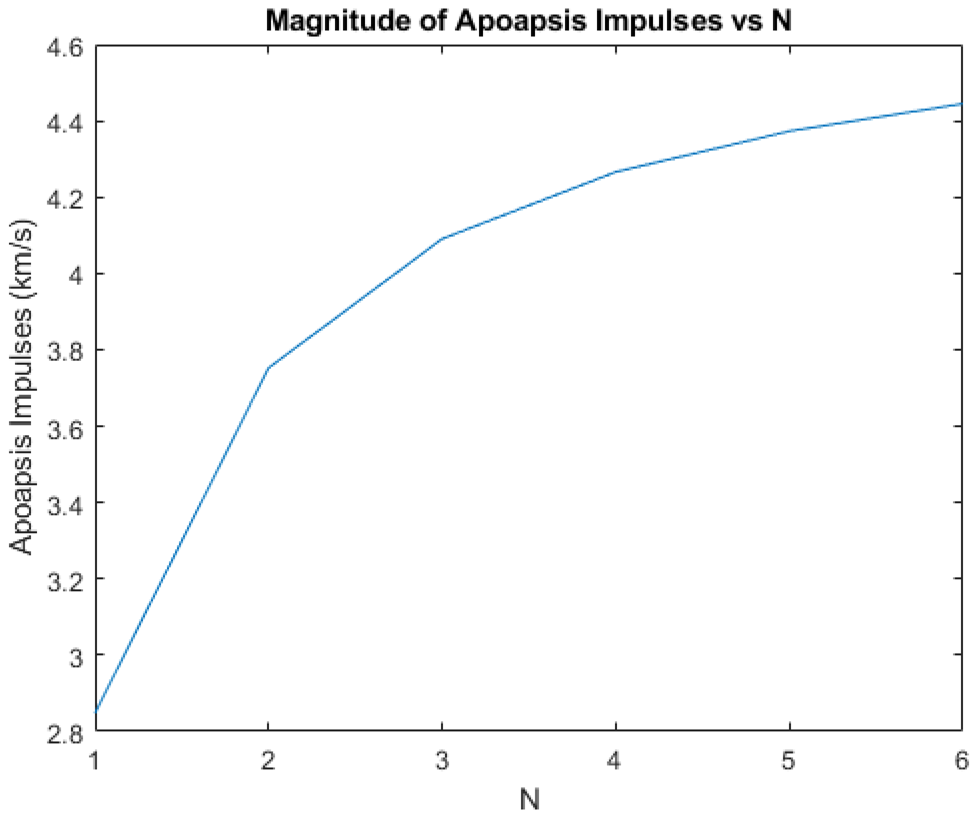
Question 3:

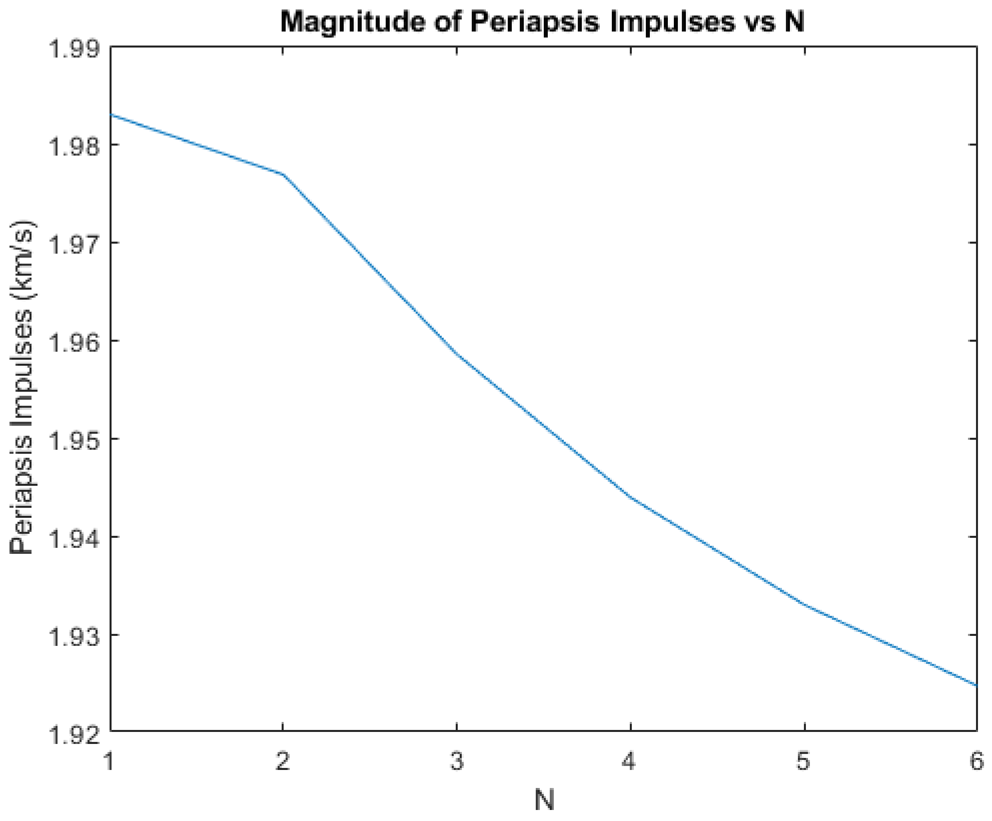
```
clc;clear;
mu=398600;
N=[1,2,3,4,5,6];
r1=500+6378.145;
i1=28.5;
Omega1=60;

r2=20200+6378.145;
i2=57;
Omega2=120;

i1=deg2rad(i1);
i2=deg2rad(i2);
Omega1=deg2rad(Omega1);
Omega2=deg2rad(Omega2);

dVAs=[];
dVPs=[];
totdVs=[];
totTimes=[];
for i=1:length(N)

[posT,timeT,dVP,dVA,totDV]=twoNImpulseOrbitTransfer(r1,r2,i1,i2,Omega1,Omega2
,N(i),mu);
    dVAs(i)=sum(dVA);
    dVPs(i)=sum(dVP);
    totdVs(i)=totDV;
    totTimes(i)=timeT(end);
end

figure
plot(N,dVAs)
title('Magnitude of Apoapsis Impulses vs N')
xlabel('N')
ylabel('Apoapsis Impulses (km/s)')
set(gca, 'XTick', 0:N(end))
figure
plot(N,dVPs)
title('Magnitude of Periapsis Impulses vs N')
xlabel('N')
ylabel('Periapsis Impulses (km/s)')
set(gca, 'XTick', 0:N(end))
figure
plot(N,totdVs)
title('Magnitude of Total Impulse vs N')
xlabel('N')
ylabel('Total Impulse (km/s)')
set(gca, 'XTick', 0:N(end))
figure
plot(N,totTimes)
title('Total Time Required for Transfer vs Time')
xlabel('N')
ylabel('Time (sec)')
set(gca, 'XTick', 0:N(end))
```

```
The total time to complete the transfer is 10766  seconds or 2.99055  hours

The total time to complete the transfer is 34796  seconds or 9.66556  hours

The total time to complete the transfer is 57825.5  seconds or 16.0626 hours

The total time to complete the transfer is 80586  seconds or 22.385  hours

The total time to complete the transfer is 103236  seconds or 28.6766  hours

The total time to complete the transfer is 125829  seconds or 34.9526  hours
```

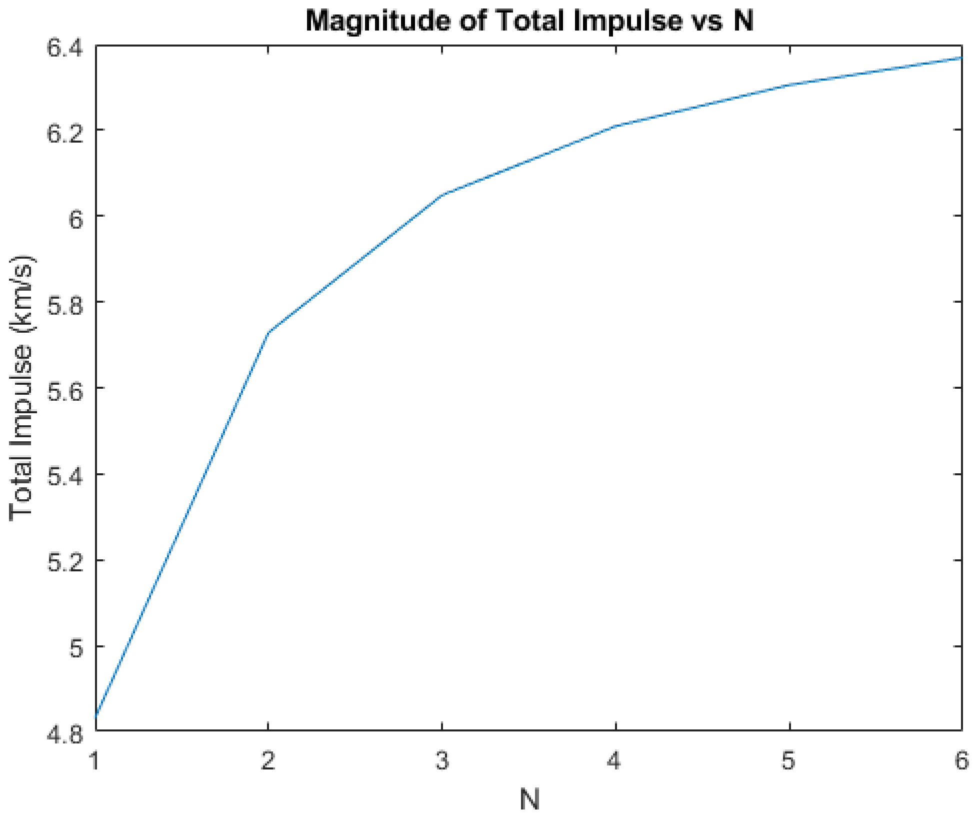
The total impulse required increases as a function of N because it is inefficient to change the plane of an orbit at a lower orbit than the final, highest orbit. So as N increases, more of the plane changes are completed at lower orbits.


Total Time Required for Transfer vs Time
×10^4
Time (sec)
N
0
2
4
6
8
10
12
14
1
2
3
4
5
6


Magnitude of Apoapsis Impulses vs N
Apoapsis Impulses (km/s)
N
2.8
3
3.2
3.4
3.6
3.8
4
4.2
4.4
4.6
1
2
3
4
5
6

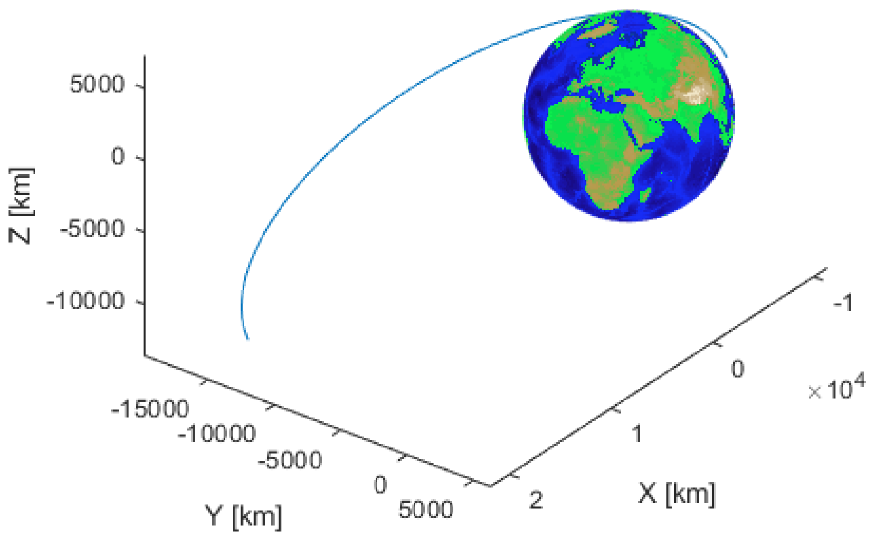


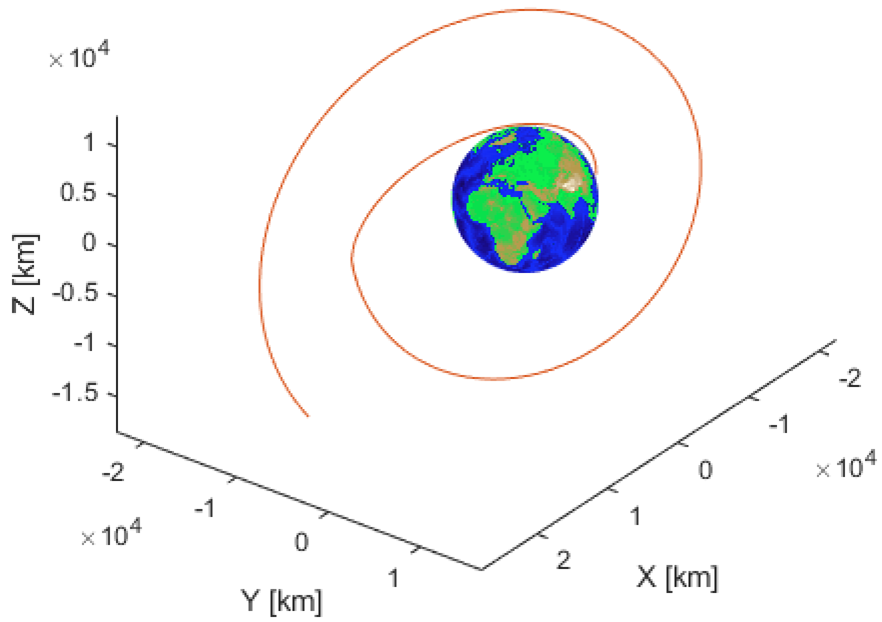
Magnitude of Periapsis Impulses vs N
Periapsis Impulses (km/s)
1.99
1.98
1.97
1.96
1.95
1.94
1.93
1.92
1
2
3
4
5
6
N

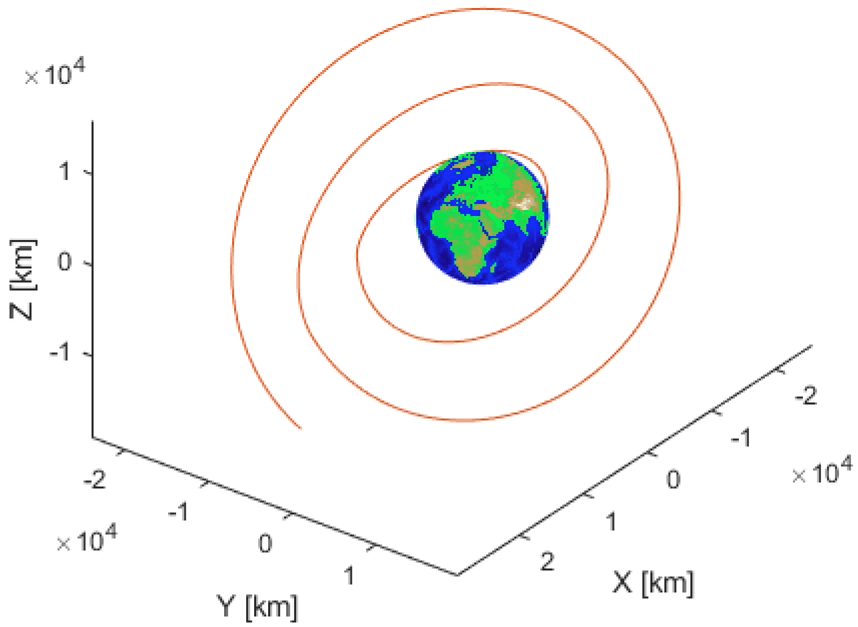


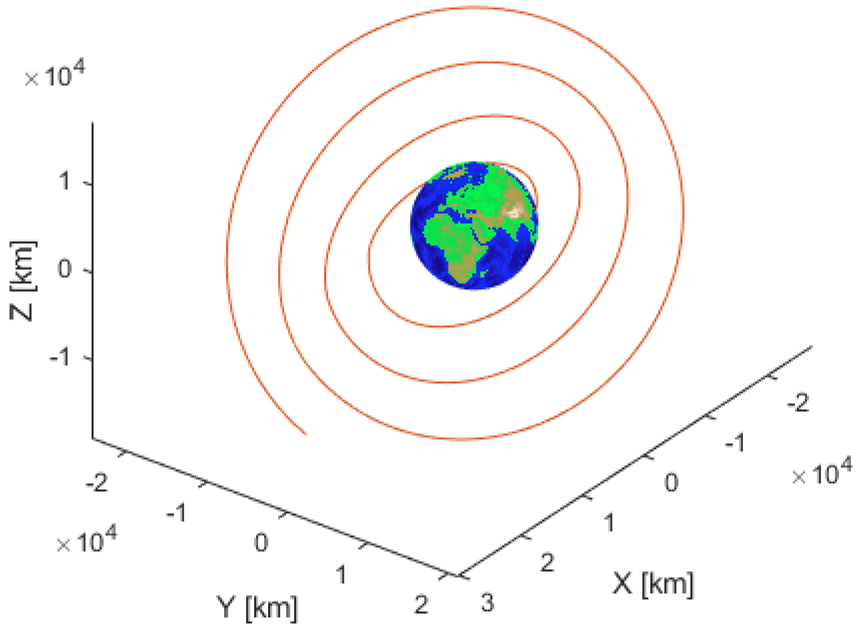
Magnitude of Total Impulse vs N
Total Impulse (km/s)
6.4
6.2
6
5.8
5.6
5.4
5.2
5
4.8
1
2
3
4
5
6
N

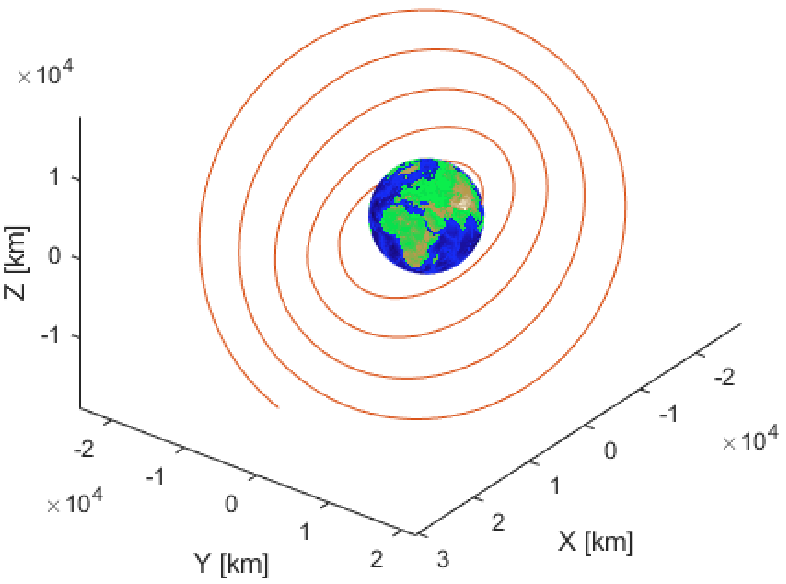
3D Views:


5000
0
-5000
-10000
Z [km]
-15000
-10000
-5000
0
5000
Y [km]
-1
0
×10^4
1
2
X [km]

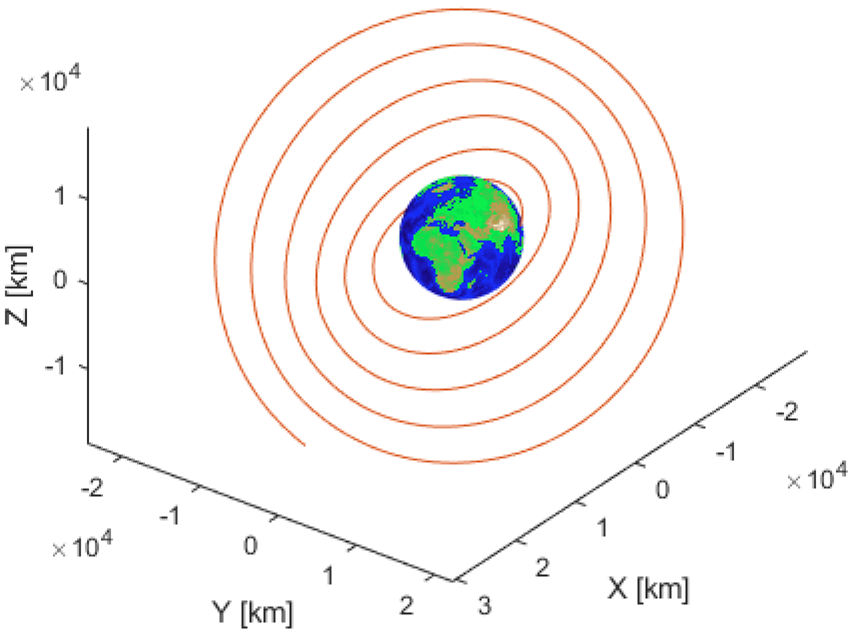

×10^4
1
0.5
0
-0.5
-1
-1.5
Z [km]
-2
-1
×10^4
0
1
Y [km]
-2
-1
×10^4
0
1
2
X [km]


×10^4
1
0
-1
Z [km]
-2
-1
0
1
×10^4
Y [km]
-2
-1
0
1
2
×10^4
X [km]


×10^4
1
0
-1
Z [km]
-2
-1
0
1
2
×10^4
Y [km]
-2
-1
0
1
2
3
×10^4
X [km]

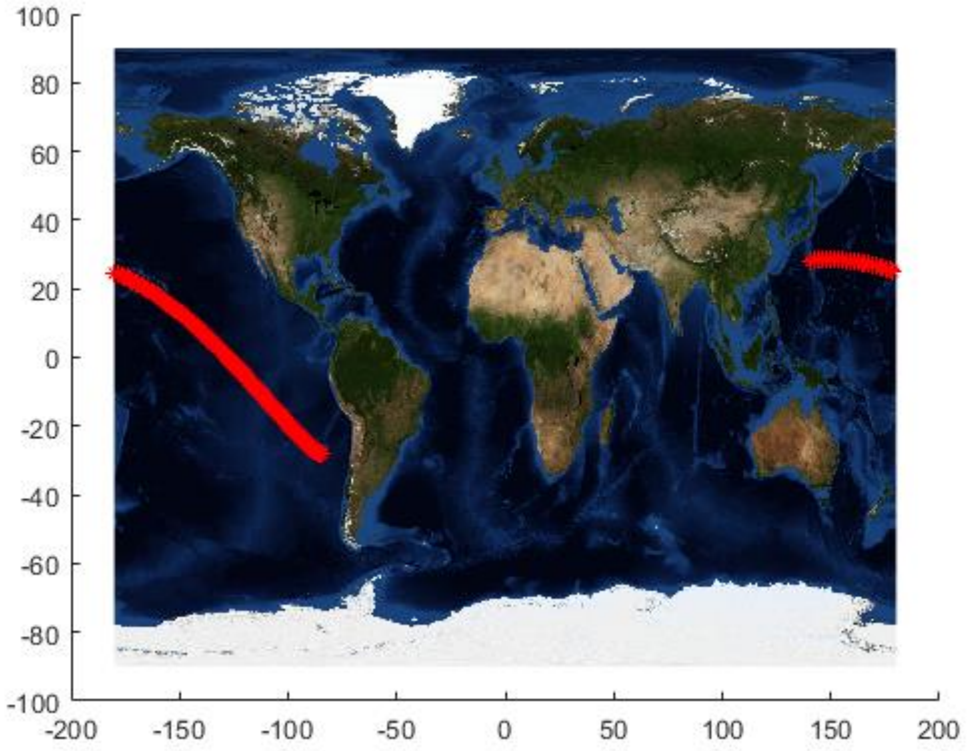


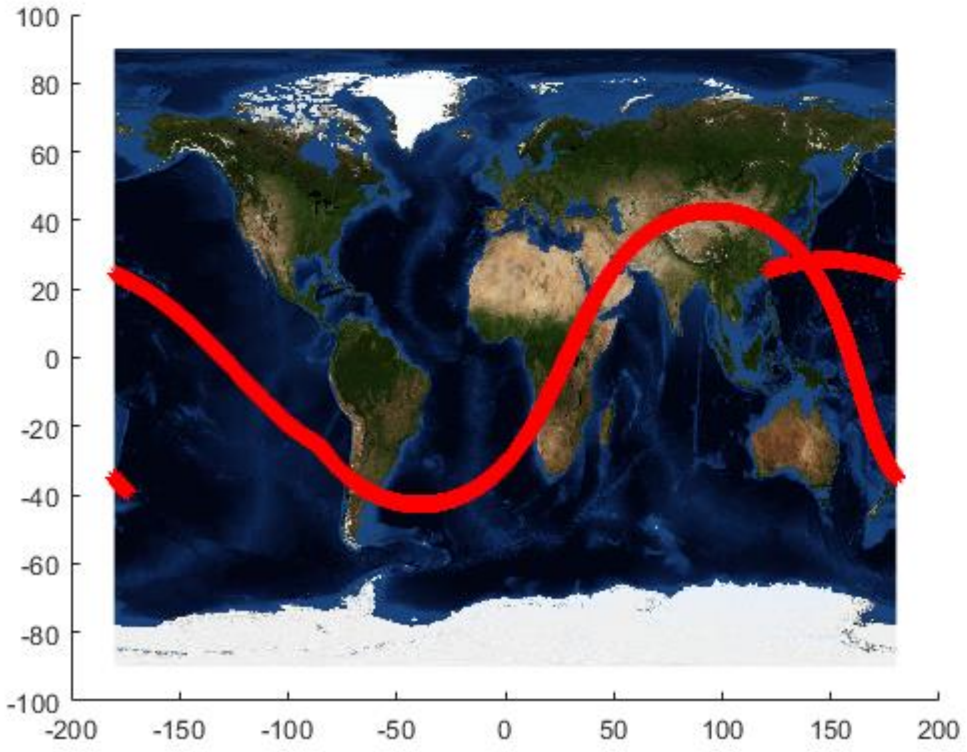
×10^4
1
0
-1
Z [km]
-2
-1
0
1
2
×10^4
Y [km]
3
2
1
0
-1
-2
×10^4
X [km]

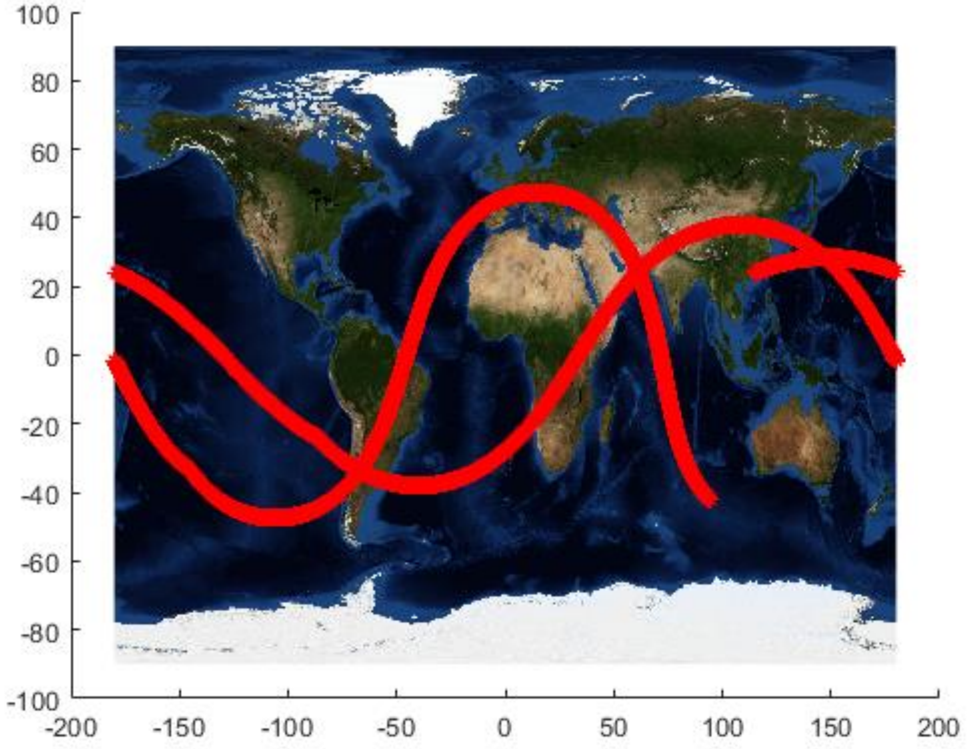


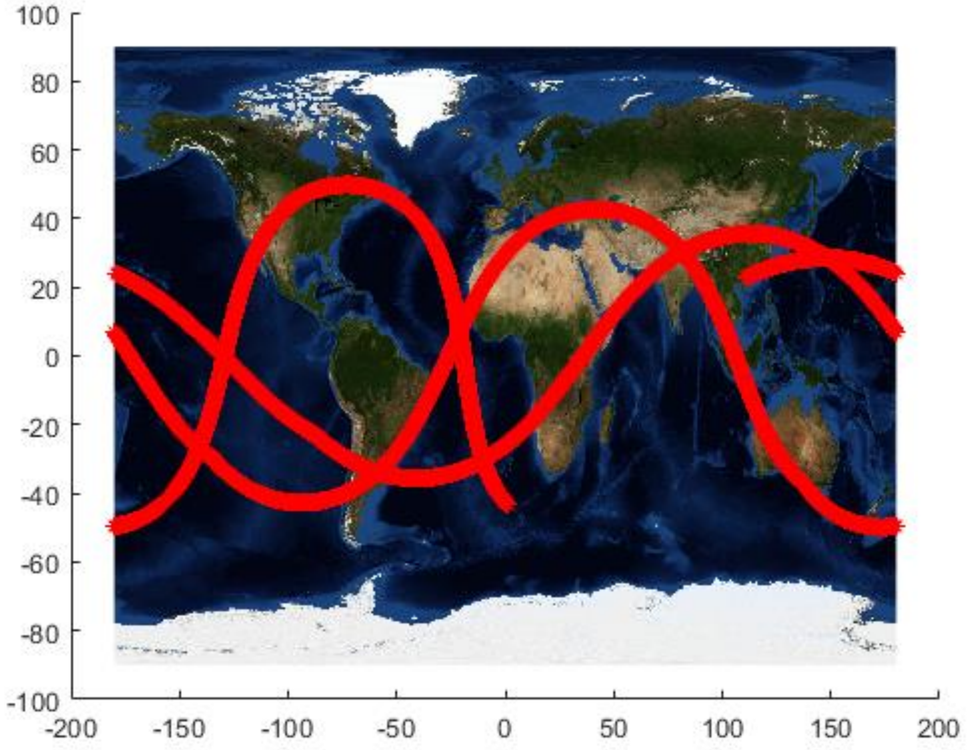
×10^4
1
0
-1
Z [km]
-2
-1
0
1
2
×10^4
Y [km]
3
2
1
0
-1
-2
×10^4
X [km]

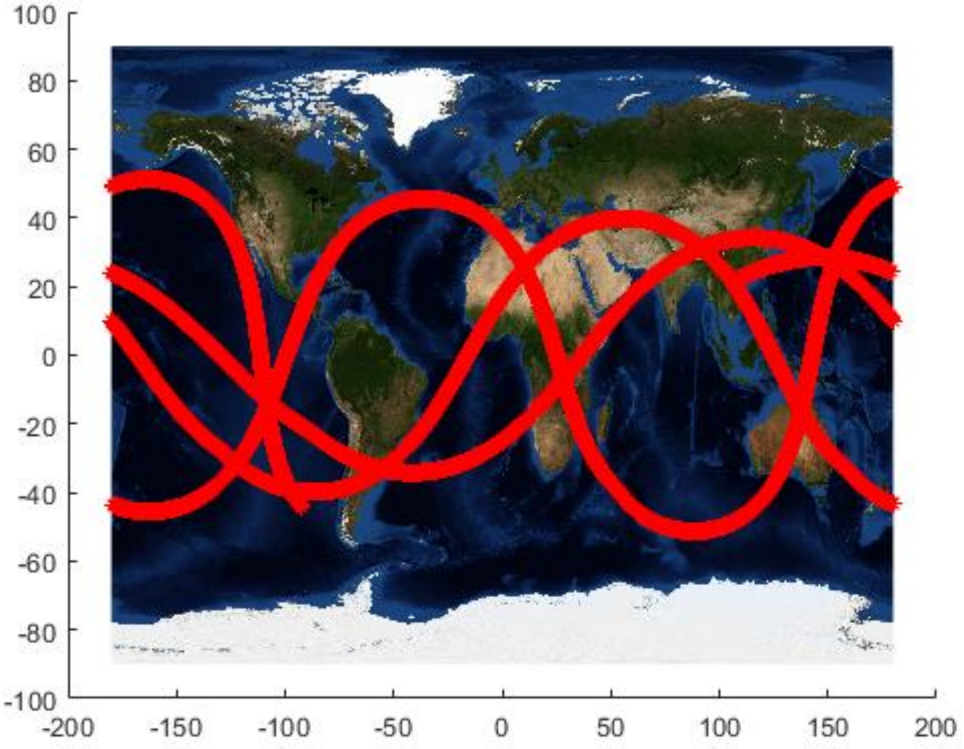
Groundtracks:



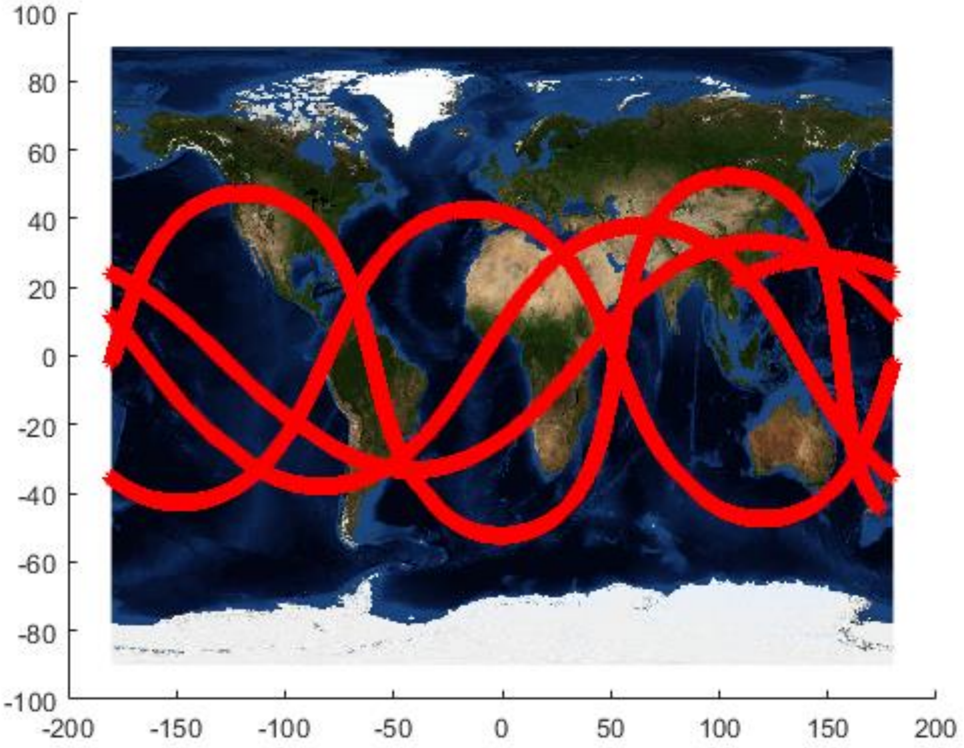



100
80
60
40
20
0
-20
-40
-60
-80
-100
-200
-150
-100
-50
0
50
100
150
200


100
80
60
40
20
0
-20
-40
-60
-80
-100
-200
-150
-100
-50
0
50
100
150
200



100
80
60
40
20
0
-20
-40
-60
-80
-100
-200
-150
-100
-50
0
50
100
150
200



100
80
60
40
20
0
-20
-40
-60
-80
-100
-200
-150
-100
-50
0
50
100
150
200

Additional Code Used:

```
function oe = rv2oe_Hackbardt_Chris(rPCI,vPCI,mu)

% This function calculates the six orbital elements from the position vector
and velocity vector
% Function Call: oe = rv2oe_Hackbardt_Chris(rPCI,vPCI,mu)
%
% Inputs:
%    rPCI:  Cartesian planet-centered inertial (PCI) position (3 by 1)
%    vPCI:  Cartesian planet-centered inertial (PCI) velocity (3 by 1)
%    mu:    gravitational parameter of centrally attacting body
% Outputs:  orbital elements
%    oe(1): semi-major axis
%    oe(2): eccentricity
%    oe(3): longitude of the ascending node (rad)
%    oe(4): inclination (rad)
%    oe(5): argument of the periapsis (rad)
%    oe(6): true anomaly (rad)

rvec = rPCI;
vvec = vPCI;

Ix = [1;0;0];
Iy = [0;1;0];
Iz = [0;0;1];

hvec = cross(rvec,vvec);
h = norm(hvec);
p = h^2/mu;
r = norm(rvec);
evec = ((cross(vvec,hvec))/mu)-(rvec/r);
e = norm(evec);
a = p/(1-e^2);
nvec = cross(Iz,hvec);
n = norm(nvec);
Omega = atan2(dot(nvec,Iy),dot(nvec,Ix));
if Omega < 0
    Omega = Omega + (2*pi);
end
inc = atan2(dot(hvec,cross(nvec,Iz)),n*dot(hvec,Iz));
omega = atan2(dot(evec,cross(hvec,nvec)),h*dot(evec,nvec));
if omega < 0
    omega = omega + (2*pi);
end
nu = atan2(dot(rvec,cross(hvec,evec)),h*dot(rvec,evec));
if nu < 0
    nu = nu + (2*pi);
end

oe = [a; e; Omega; inc; omega; nu];
end
```

```matlab
function [rPCI,vPCI]  = oe2rv_Hackbardt_Chris(oe,mu)

% This function calculates the postion and velocity vector with respect to
the planet using orbital elements.
% Function Call: [rPCI,vPCI]  = oe2rv_Hackbardt_Chris(oe,mu)
% Input:  Orbital Elements: (6 by 1 column vector)
%   oe(1): Semi-major axis
%   oe(2): Eccentricity
%   oe(3): Longitude of the ascending node (rad)
%   oe(4): Inclination (rad)
%   oe(5): Argument of the periapsis (rad)
%   oe(6): True anomaly (rad)
%   mu:    Planet gravitational parameter     (scalar)
% Outputs:
%   rPCI:  Planet-Centered Inertial (PCI) Cartesian position
%          (3 by 1 column vector)
%   vPCI:  Planet-Centered Inertial (PCI) Cartesian inertial velocity
%          (3 by 1 column vector)

a = oe(1);
e = oe(2);
Omega = oe(3);
i = oe(4);
omega = oe(5);
nu = oe(6);

p = a*(1-e^2);
r = p/(1+(e*cos(nu)));

rvecP = [r*cos(nu);r*sin(nu);0];
vvecP = (sqrt(mu/p))*[-sin(nu);e+cos(nu);0];

T_NI = [cos(Omega),-sin(Omega),0;sin(Omega),cos(Omega),0;0,0,1];
T_QN = [1,0,0;0,cos(i),-sin(i);0,sin(i),cos(i)];
T_PQ = [cos(omega),-sin(omega),0;sin(omega),cos(omega),0;0,0,1];
T_PI = T_NI * T_QN * T_PQ;

rPCI = T_PI * rvecP;
vPCI = T_PI * vvecP;
end
```

```matlab
function E = KeplerSolver_Hackbardt_Chris(a,e,mu,t0,t,E0,k)
%This function solves Kepler's equation using fixed point iteration
%
%Input: a, semi-major axis
%Input: e, eccentricity
%Input: mu, gravitational parameter
%Input: t0, the initial time in seconds
%Input: t, the final time in seconds
%Input: E0, initial eccentric anamoly
%Input: k, number of periapsis crossings
%
%Output: E, eccentric anamoly

C=sqrt(mu/a^3)*(t-t0)-(2*pi*k)+(E0-(e*sin(E0)));
f = @(x) e*sin(x)+C;
n=10*ceil(1/(1-e));
guessE=E0-(e*sin(E0));
for i=1:n
    guessE=f(guessE);
end
E=guessE;
end
```

```matlab
function [r,v,E,nu] = propagateKepler_Hackbardt_Chris(r0,v0,t0,t,mu)
%This function finds the position and velocity in an orbit at final time, t
%
%Input: r0, a column vector with the initial position
%Input: v0, a column vector with the initial velocity
%Input: t0, the initial time in seconds
%Input: t, the final time in seconds
%Input: mu, gravitational parameter
%
%Output: r, a column vector with the position
%Output: v, a column vector with the velocity
%Output: E, eccentric anamoly
%Output: nu, true anamoly

oe = rv2oe_Hackbardt_Chris(r0,v0,mu);
a = oe(1);
e = oe(2);
nu0 = oe(6);
E0 = 2*atan2(sqrt(1-e)*sin(nu0/2),sqrt(1+e)*cos(nu0/2));
tau=2*pi*sqrt(a^3/mu);
k=floor((t)/tau);
E = KeplerSolver_Hackbardt_Chris(a,e,mu,t0,t,E0,k);
nu= 2*atan2(sqrt(1+e)*sin(E/2),sqrt(1-e)*cos(E/2));
oe(6)=nu;
[r,v]  = oe2rv_Hackbardt_Chris(oe,mu);
end
```

```
function [times, positions, velocities] =
propagateOnCircle(pos,vel,t0,tf,mu,N)
%This function calculates position and veolocity N times evenly spaced betwen
t0 and tf
%
%Function call: [times, postions, velocities] =
propagateOnCircle(pos,vel,t0,tf,mu,N);
%
%Input: pos, a column vector with the initial ECI postion
%Input: vel, a column vector with the initial ECI inertial velocity
%Input: t0, the initial time. Time from apoapsis to initial postion
%Input: tf, the final time
%Input: mu, gravitational parameter
%Input: N, number of time intervals
%
%Output: times, column vector of times, starting at t0 and ending at tf.
Length of N with even spacing
%Output: postions, matrix of size Nx3 containing ECI positions stored row-
wise at every time in the times vector
%Output: velocities, matrix of size Nx3 containing ECI velocities stored row-
wise at every time in the times vector
%

times=linspace(t0,tf,N);
times=times';

oe = rv2oe_Hackbardt_Chris(pos,vel,mu);
nui=oe(6);
a=oe(1);
thetaDot=sqrt(mu/a^3);

positions=zeros(N,3);
velocities=zeros(N,3);

positions(1,:)=pos';
velocities(1,:)=vel';

for i=2:N
    theta=thetaDot*(times(i)-times(i-1));
    nuOfT=nui+theta;
    oe(6)=nuOfT;
    [rPCI,vPCI]  = oe2rv_Hackbardt_Chris(oe,mu);
    positions(i,:)=rPCI';
    velocities(i,:)=vPCI';
    nui=nuOfT;
end
end
```