Problem 1:

a) Using the vis-viva equation, the magnitude of the first impulse is given as:

$$\Delta v_1 = \sqrt{\frac{2\mu}{r_1} - \frac{\mu}{a}} - \sqrt{\frac{\mu}{r_1}}$$

Which is equivalent to the speed of the elliptic transfer orbit minus the speed of the original circular orbit. Since the transfer orbit has periapsis at the original circular orbit and apoapsis at the second circular orbit,

$$a = \frac{r_1 + r_2}{2}$$

Substituting the expression for a:

$$\Delta v_1 = \sqrt{\frac{2\mu}{r_1} - \frac{2\mu}{r_1 + r_2}} - \sqrt{\frac{\mu}{r_1}} = \sqrt{\frac{2\mu(r_1 + r_2 - r_1)}{r_1(r_1 + r_2)}} - \sqrt{\frac{\mu}{r_1}} = \sqrt{\frac{\mu}{r_1}}\left(\sqrt{\frac{2r_2}{r_1 + r_2}} - 1\right)$$

The magnitude of the second impulse is found using the same method:

$$\Delta v_2 = \sqrt{\frac{\mu}{r_2}} - \sqrt{\frac{2\mu}{r_2} - \frac{2\mu}{r_1 + r_2}}$$

$$\Delta v_2 = \sqrt{\frac{\mu}{r_2}} - \sqrt{\frac{2\mu(r_1 + r_2 - r_2)}{r_2(r_1 + r_2)}} = \sqrt{\frac{\mu}{r_2}}\left(1 - \sqrt{\frac{2r_1}{r_1 + r_2}}\right)$$

We also know that,

$$R = \frac{r_2}{r_1}$$

$$v_{c1} = \sqrt{\frac{\mu}{r_1}}$$

We can then nondimensionalize the expressions for the two impulses. Starting with rewriting in terms of R:

$$\Delta v_1 = \sqrt{\frac{\mu}{r_1}}\left(\sqrt{\frac{2R}{1 + R}} - 1\right)$$

$$\Delta v_2 = \sqrt{\frac{\mu}{r_1}}\sqrt{\frac{1}{R}}\left(1 - \sqrt{\frac{2}{1 + R}}\right)$$

Now we can divide each expression by v_{c1}:

$$\frac{\Delta v_1}{v_{c1}} = \sqrt{\frac{2R}{1+R}} - 1$$

$$\frac{\Delta v_2}{v_{c1}} = \sqrt{\frac{1}{R}}\left(1 - \sqrt{\frac{2}{1+R}}\right)$$

b) Finding the three impulses for the bi-elliptic transfer:

$$\Delta v_1 = \sqrt{\frac{2\mu}{r_1} - \frac{\mu}{a_1}} - \sqrt{\frac{\mu}{r_1}}$$

a_1 can be calculated as the semi-major axis of the first transfer orbit:

$$a_1 = \frac{r_1 + r_i}{2}$$

Δv_1 can then be written as:

$$\Delta v_1 = \sqrt{\frac{2\mu}{r_1} - \frac{2\mu}{r_1 + r_i}} - \sqrt{\frac{\mu}{r_1}} = \sqrt{\frac{\mu}{r_1}}\left(\sqrt{\frac{2r_i}{r_1 + r_i}} - 1\right)$$

Finding Δv_2:

$$\Delta v_2 = \sqrt{\frac{2\mu}{r_i} - \frac{\mu}{a_2}} - \sqrt{\frac{2\mu}{r_i} - \frac{\mu}{a_1}}$$

Where,

$$a_2 = \frac{r_2 + r_i}{2}$$

Substituting a_2 and a_1:

$$\Delta v_2 = \sqrt{\frac{2\mu}{r_i} - \frac{2\mu}{r_2 + r_i}} - \sqrt{\frac{2\mu}{r_i} - \frac{2\mu}{r_1 + r_i}} = \sqrt{\frac{\mu}{r_i}}\left(\sqrt{\frac{2r_2}{r_2 + r_i}} - \sqrt{\frac{2r_1}{r_1 + r_i}}\right)$$

Finding Δv_3:

$$\Delta v_3 = \sqrt{\frac{2\mu}{r_2} - \frac{\mu}{a_2}} - \sqrt{\frac{\mu}{r_2}}$$

Δv_3 can then be written as:

$$\Delta v_3 = \sqrt{\frac{2\mu}{r_2} - \frac{2\mu}{r_2 + r_i}} - \sqrt{\frac{\mu}{r_2}} = \sqrt{\frac{\mu}{r_2}}\left(\sqrt{\frac{2r_i}{r_2 + r_i}} - 1\right)$$

The quantities can be defined:

$$R = \frac{r_2}{r_1}$$

$$S = \frac{r_i}{r_2}$$

$$v_{c1} = \sqrt{\frac{\mu}{r_1}}$$

The expressions can then be written as:

$$\Delta v_1 = \sqrt{\frac{\mu}{r_1}}\left(\sqrt{\frac{2RS}{1+RS}} - 1\right)$$

$$\Delta v_2 = \sqrt{\frac{\mu}{r_i}}\sqrt{\frac{1}{RS}}\left(\sqrt{\frac{2}{1+S}} - \sqrt{\frac{2}{1+RS}}\right)$$

$$\Delta v_3 = \sqrt{\frac{\mu}{r_1}}\left(\sqrt{\frac{2S}{R+RS}} - \sqrt{\frac{1}{R}}\right)$$

Normalizing the expressions:

$$\frac{\Delta v_1}{v_{c1}} = \sqrt{\frac{2RS}{1+RS}} - 1$$

$$\frac{\Delta v_2}{v_{c1}} = \sqrt{\frac{1}{RS}}\left(\sqrt{\frac{2}{1+S}} - \sqrt{\frac{2}{1+RS}}\right)$$

$$\frac{\Delta v_3}{v_{c1}} = \sqrt{\frac{2S}{R+RS}} - \sqrt{\frac{1}{R}}$$

c) The bi-parabolic transfer is a case of the bi-elliptic transfer where $r_i \to \infty$ and consequently $S \to \infty$. Starting with rewriting the normalized expressions from the bi-elliptic transfer:

$$\frac{\Delta v_1}{v_{c1}} = \sqrt{\frac{2R}{1/S + R}} - 1$$

$$\frac{\Delta v_2}{v_{c1}} = \sqrt{\frac{1}{RS}}\left(\sqrt{\frac{2/S}{1/S + 1}} - \sqrt{\frac{2/S}{1/S + R}}\right)$$

$$\frac{\Delta v_3}{v_{c1}} = \sqrt{\frac{2}{R/S + R}} - \sqrt{\frac{1}{R}}$$

Taking the limit as $S \to \infty$:

$$\frac{\Delta v_1}{v_{c1}} = \sqrt{2} - 1$$

$$\frac{\Delta v_2}{v_{c1}} = 0$$

$$\frac{\Delta v_3}{v_{c1}} = \sqrt{\frac{1}{R}}\left(\sqrt{2} - 1\right)$$

The expression for the magnitudes can be found by multiplying by v_{c1}:

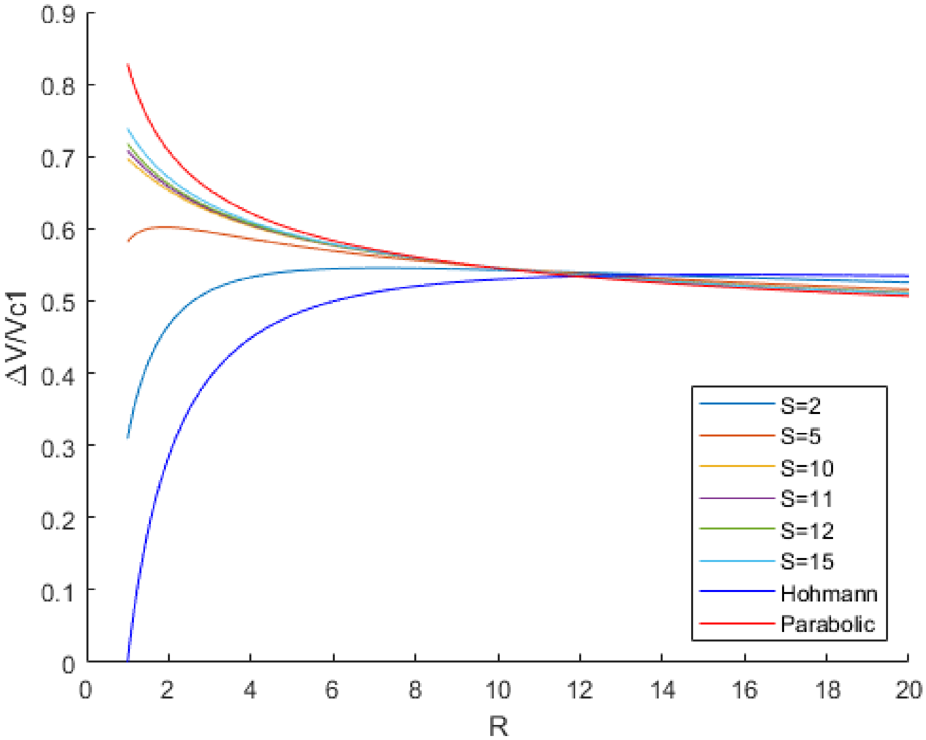
$$\Delta v_1 = v_{c1}\left(\sqrt{2} - 1\right)$$

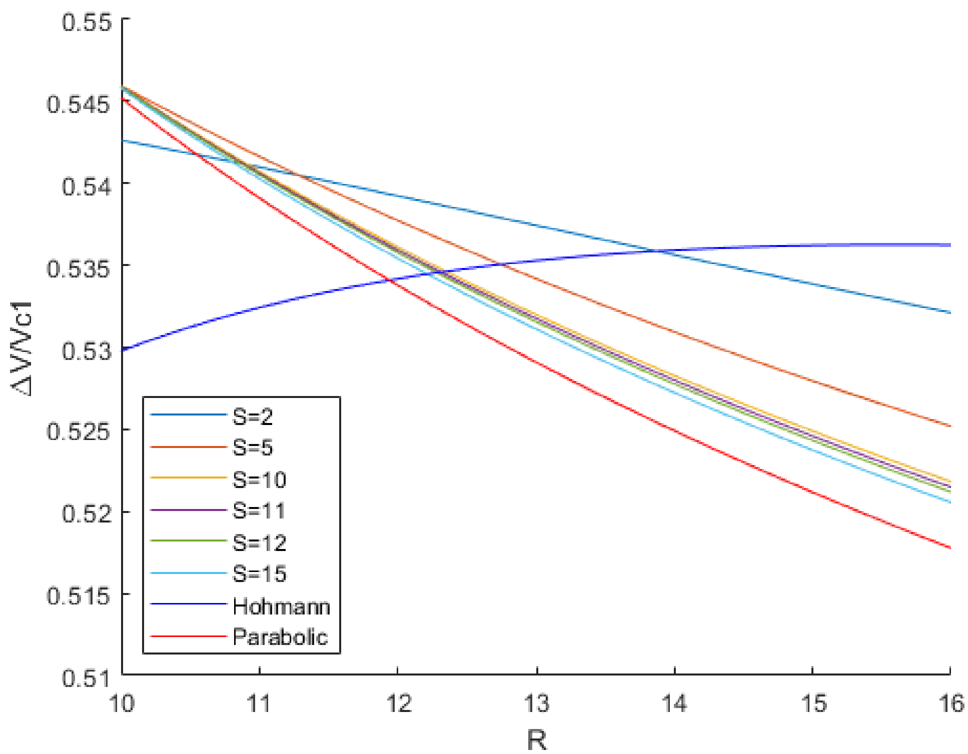
$$\Delta v_2 = 0$$

$$\Delta v_3 = v_{c1}\sqrt{\frac{1}{R}}\left(\sqrt{2} - 1\right)$$

d)

```
clc;clear;close all;
R1=1:0.01:20;
R2=10:0.01:16;
S=[2,5,10,11,12,15];

dvH=@(R) (sqrt((2.*R)./(1+R))-1)+(sqrt(1./R).*(1-sqrt(2./(1+R))));
dvE=@(R,S) (sqrt((2.*R.*S)./(1+R.*S))-
1)+(sqrt(1./(R.*S)).*(sqrt(2./(1+S))-
sqrt(2./(1+R.*S))))+(sqrt(2.*S./(R+R.*S))-sqrt(1./R));
dvP=@(R) (sqrt(2)-1)+(sqrt(1./R).*(sqrt(2)-1));

figure(1)
hold on
p1H=dvH(R1);
p1P=dvP(R1);
p1E=zeros(length(R1),length(S));
for i=1:length(S)
    p1E(:,i)=dvE(R1,S(i))';
    plot(R1,p1E(:,i))
end
legend('','','','','','')
plot(R1,p1H,'b','DisplayName','Hohmann')
plot(R1,p1P,'r','DisplayName','Parabolic')
xlabel('R')
ylabel('\DeltaV/Vc1')
legend

figure(2)
hold on
p1H=dvH(R2);
p1P=dvP(R2);
p1E=zeros(length(R2),length(S));
marker=['r','y','m','c','k','g'];
for i=1:length(S)
    p1E(:,i)=dvE(R2,S(i))';
    plot(R2,p1E(:,i))
end
legend('','','','','','')
plot(R2,p1H,'b','DisplayName','Hohmann')
plot(R2,p1P,'r','DisplayName','Parabolic')
xlabel('R')
ylabel('\DeltaV/Vc1')
axis([-inf inf 0.51 0.55])
```


0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
ΔV/Vc1
0
2
4
6
8
10
12
14
16
18
20
R
S=2
S=5
S=10
S=11
S=12
S=15
Hohmann
Parabolic

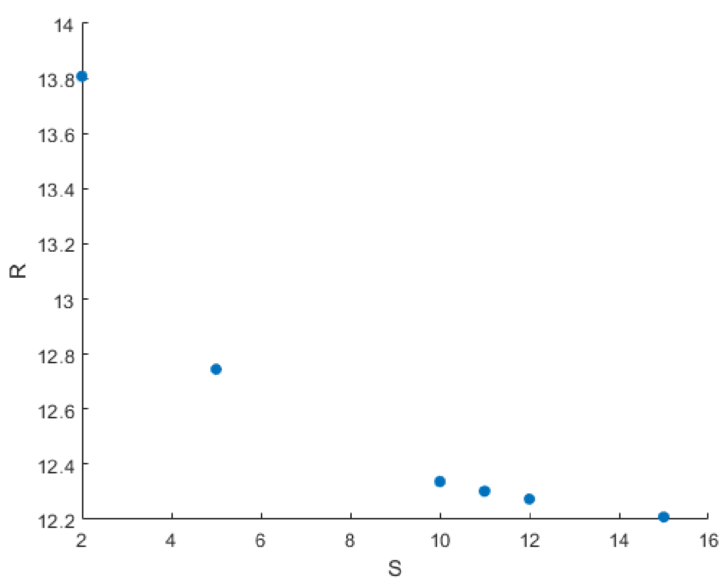

0.55
0.545
0.54
0.535
0.53
0.525
0.52
0.515
0.51
ΔV/Vc1
10
11
12
13
14
15
16
R
S=2
S=5
S=10
S=11
S=12
S=15
Hohmann
Parabolic

Problem 2:

```
clc;clear;
HminusP = @(R) (sqrt((2.*R)./(1+R))-1)+(sqrt(1./R).*(1-sqrt(2./(1+R))))-
((sqrt(2)-1)+(sqrt(1./R).*(sqrt(2)-1)));
transR=fsolve(HminusP,10);
S=[2,5,10,11,12,15];
Rvals=zeros(1,length(S));
for i=1:length(S)
    s=S(i);
    HminusE = @(R) (sqrt((2.*R)./(1+R))-1)+(sqrt(1./R).*(1-sqrt(2./(1+R))))-
((sqrt((2.*R.*s)./(1+R.*s))-1)+(sqrt(1./(R.*s)).*(sqrt(2./(1+s))-
sqrt(2./(1+R.*s))))+(sqrt(2.*s./(R+R.*s))-sqrt(1./R)));
    Rvals(i)=fsolve(HminusE,12);
end
clc;
scatter(S,Rvals,'filled')
xlabel('S')
ylabel('R')
fprintf('The value of R where the total impulse of the Hohmann transfer is
the same as the total impulse of the bi-parabolic transfer is: %g\n',transR)
fprintf('The values of R where the total impulse of the Hohmann transfer is
the same as the total impulse of the bi-elliptic transfer at
S=2,5,10,11,12,15 is \n')
fprintf('%g, ',Rvals)
```

The value of R where the total impulse of the Hohmann transfer is the same as the total impulse of the bi-parabolic transfer is: 11.938

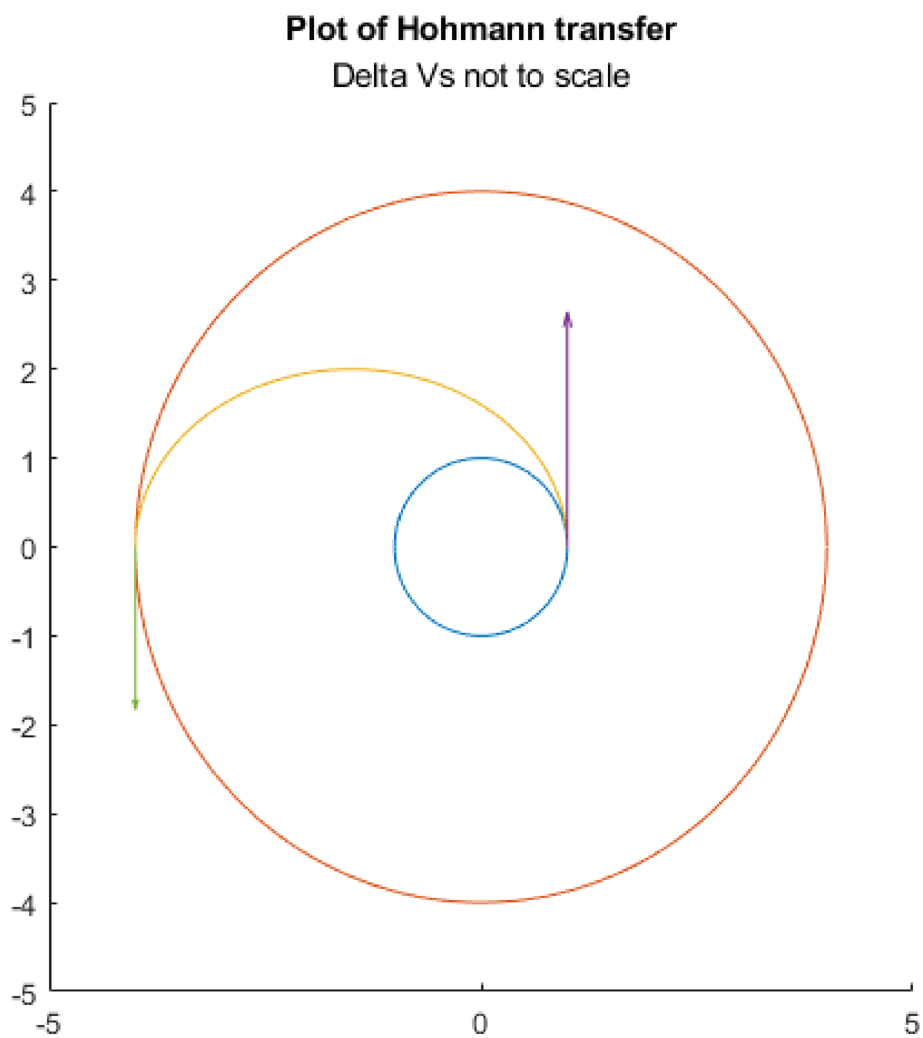
The values of R where the total impulse of the Hohmann transfer is the same as the total impulse of the bi-elliptic transfer at S=2,5,10,11,12,15 is
13.8076, 12.7452, 12.3373, 12.3029, 12.2739, 12.2093



Problem 3:

```
clc;clear;
v1=1;
mu=1;
v2=0.5;
r1=mu/v1^2;
r2=mu/v2^2;
R=r2/r1;
if R<11.938
    fprintf('The Hohmann transfer is the most efficient\n')
end

%Finds properties of the transfer orbit
eT=(r2-r1)/(r2+r1);
aT=(r1+r2)/2;
pT=aT*(1-eT^2);
nuT=0:0.01:pi;
radiusT=pT./(1+eT*cos(nuT));
[xT,yT]=pol2cart(nuT,radiusT);

%Finds delta V
dv1=sqrt(mu/r1)*(sqrt((2*R)/(1+R))-1);
dv2=sqrt(mu/r1)*sqrt(1/R)*(1-sqrt((2)/(1+R)));

%Finds coords of circular orbits
nu=0:0.01:2*pi;
radius1=zeros(1,length(nu));
radius1(:)=r1;
radius2=zeros(1,length(nu));
radius2(:)=r2;
[x1,y1]=pol2cart(nu,radius1);
[x2,y2]=pol2cart(nu,radius2);

hold on
plot(x1,y1,x2,y2,xT,yT)
quiver(xT(1),yT(1),0,dv1)
quiver(xT(length(nuT)),yT(length(nuT)),0,-dv2)
axis([-5 5 -5 5])
set(gcf,'position',[300,300,500,500])
```

The Hohmann transfer is the most efficient

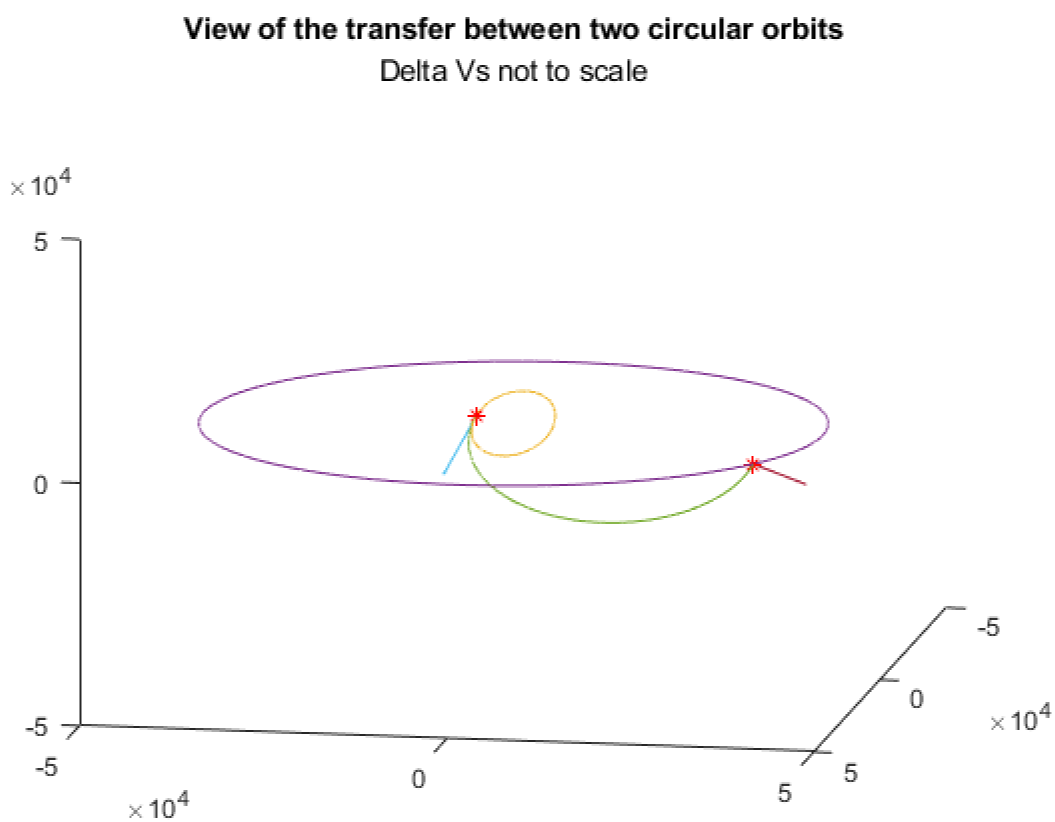

Plot of Hohmann transfer
Delta Vs not to scale
5
4
3
2
1
0
-1
-2
-3
-4
-5
-5
0
5

Problem 4:

```
clc;clear;
%Defines givens
mu=398600;
r1=300+6378.145;
i1=57;
i1=deg2rad(i1);
e1=0;
Omega1=60;
Omega1=deg2rad(Omega1);
omega=0;

%Calculates position of first orbit
nu=0:0.001:2*pi;
position1=zeros(length(nu),3);
velocity1=zeros(length(nu),3);
for i=1:length(nu)
    oe1=[r1,e1,Omega1,i1,omega,nu(i)];
    [r,v]=oe2rv_Hackbardt_Chris(oe1,mu);
    position1(i,:)=r';
    velocity1(i,:)=v';
end

%Calculates pos of second orbit
i2=0;
period=23.934*60*60;
a2=(mu*(period/(2*pi))^2)^(1/3);
r2=a2;
e2=0;
Omega2=0;
position2=zeros(length(nu),3);
velocity2=zeros(length(nu),3);
for i=1:length(nu)
    oe2=[a2,e2,Omega2,i2,omega,nu(i)];
    [r,v]=oe2rv_Hackbardt_Chris(oe2,mu);
    position2(i,:)=r';
    velocity2(i,:)=v';
end

%calculates the first impulse
v1=sqrt(mu/r1);
v2=sqrt(mu/r2);
hvec1=cross(position1(1,:),velocity1(1,:));
hvec2=cross(position2(1,:),velocity2(1,:));
lvec=cross(hvec1,hvec2);
lvec=lvec/norm(lvec);
position1T=r1*lvec;
aT=(r1+r2)/2;
eT=(r2-r1)/(r2+r1);
deltaV1=sqrt(((2*mu)/r1)-(mu/aT))-v1;
u1vec=cross(hvec1,lvec)/norm(hvec1);
V1b4=v1*u1vec;
deltaV1vec=deltaV1*u1vec;
V1aft=V1b4+deltaV1vec;

%calculates pos of transfer orbit
nuT=0:0.001:pi;
oeT=rv2oe_Hackbardt_Chris(position1T,V1aft,mu);
positionT=zeros(length(nuT),3);
velocityT=zeros(length(nuT),3);
for i=1:length(nuT)
```

```
    oeT(6)=nu(i);
    [r,v]=oe2rv_Hackbardt_Chris(oeT,mu);
    positionT(i,:)=r';
    velocityT(i,:)=v';
end

%calculates second impulse
v2bf=sqrt(((2*mu)/r2)-(mu/aT));
V2b4=-v2bf*u1vec;
u2vec=-cross(hvec2,lvec)/norm(hvec2);
V2aft=v2*u2vec;
deltaV2=v2-v2bf;
deltaV2vec=V2aft-V2b4;

%Calculates time and mass for transfer
tauT=2*pi*sqrt(aT^3/mu);
timeForTrans=tauT/2;
g0=9.80665;
Isp=320;
massRatio1=exp(deltaV1/(g0*Isp));
massRatio2=exp(deltaV2/(g0*Isp));

%plots
hold on
scale=10;
plot3(position1T(1),position1T(2),position1T(3),'r*')
plot3(positionT(end,1),positionT(end,2),positionT(end,3),'r*')
plot3(position1(:,1),position1(:,2),position1(:,3),position2(:,1),position2(:,2),posit
ion2(:,3))
plot3(positionT(:,1),positionT(:,2),positionT(:,3))
quiver3(position1T(1),position1T(2),position1T(3),deltaV1vec(1),deltaV1vec(2),deltaV1v
ec(3),5000)
quiver3(positionT(end,1),positionT(end,2),positionT(end,3),deltaV2vec(1),deltaV2vec(2)
,deltaV2vec(3),5000)
axis([-50000 50000 -50000 50000 -50000 50000])
title('View of the transfer between two circular orbits')
subtitle('Delta Vs not to scale')
view(-259.6770,16.8)

%Prints results
DeltaLet=char(916);
subScr1=char(8321);
subScr2=char(8322);
fprintf(['The magnitude of ' DeltaLet 'V' subScr1 ' = %g km/s\n'],deltaV1)
fprintf(['The magnitude of ' DeltaLet 'V' subScr2 ' = %g km/s\n'],deltaV2)
fprintf(['The total ' DeltaLet 'V required is %g km/s\n'],deltaV1+deltaV2)
fprintf('The transfer takes %g seconds to complete\n',timeForTrans)
fprintf('The mass ratio for impulse 1 is %g\n',massRatio1)
fprintf('The mass ratio for impulse 2 is %g\n',massRatio2)
fprintf('Changing the longitude of the ascending node does change where the orbit
transfer begins\n')
fprintf('since that would rotate the orbit plane about the z-axis.\n')
```

The magnitude of ΔV_1 = 2.42572 km/s
The magnitude of ΔV_2 = 1.46682 km/s
The total ΔV required is 3.89254 km/s
The transfer takes 18989.9 seconds to complete
The mass ratio for impulse 1 is 1.00077
The mass ratio for impulse 2 is 1.00047
Changing the longitude of the ascending node does change where the orbit transfer begins since that would rotate the orbit plane about the z-axis.



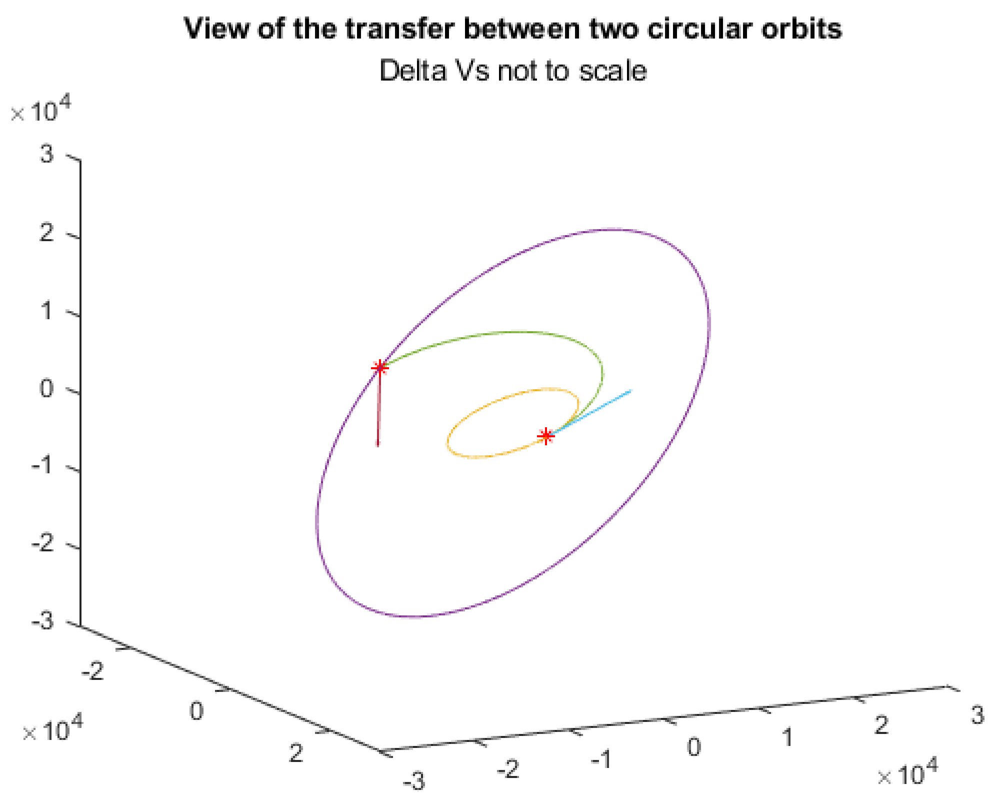
Problem 5:

```
clc;clear;
%Defines givens
mu=398600;
r1=350+6378.145;
i1=28;
i1=deg2rad(i1);
e1=0;
Omega1=0;
Omega1=deg2rad(Omega1);
omega=0;

%Calculates position of first orbit
nu=0:0.001:2*pi;
position1=zeros(length(nu),3);
velocity1=zeros(length(nu),3);
for i=1:length(nu)
    oe1=[r1,e1,Omega1,i1,omega,nu(i)];
    [r,v]=oe2rv_Hackbardt_Chris(oe1,mu);
    position1(i,:)=r';
    velocity1(i,:)=v';
end

%Calculates pos of second orbit
i2=55;
i2=deg2rad(i2);
a2=26558;
r2=a2;
e2=0;
Omega2=0;
position2=zeros(length(nu),3);
velocity2=zeros(length(nu),3);
for i=1:length(nu)
    oe2=[a2,e2,Omega2,i2,omega,nu(i)];
    [r,v]=oe2rv_Hackbardt_Chris(oe2,mu);
    position2(i,:)=r';
    velocity2(i,:)=v';
end

%calculates the first impulse
v1=sqrt(mu/r1);
v2=sqrt(mu/r2);
hvec1=cross(position1(1,:),velocity1(1,:));
hvec2=cross(position2(1,:),velocity2(1,:));
lvec=cross(hvec1,hvec2);
lvec=lvec/norm(lvec);
position1T=r1*lvec;
aT=(r1+r2)/2;
eT=(r2-r1)/(r2+r1);
deltaV1=sqrt(((2*mu)/r1)-(mu/aT))-v1;
u1vec=cross(hvec1,lvec)/norm(hvec1);
V1b4=v1*u1vec;
deltaV1vec=deltaV1*u1vec;
V1aft=V1b4+deltaV1vec;
```

```
%calculates pos of transfer orbit
nuT=0:0.001:pi;
oeT=rv2oe_Hackbardt_Chris(position1T,V1aft,mu);
positionT=zeros(length(nuT),3);
velocityT=zeros(length(nuT),3);
for i=1:length(nuT)
    oeT(6)=nu(i);
    [r,v]=oe2rv_Hackbardt_Chris(oeT,mu);
    positionT(i,:)=r';
    velocityT(i,:)=v';
end

%calculates second impulse
v2bf=sqrt(((2*mu)/r2)-(mu/aT));
V2b4=-v2bf*u1vec;
u2vec=-cross(hvec2,lvec)/norm(hvec2);
V2aft=v2*u2vec;
deltaV2=v2-v2bf;
deltaV2vec=V2aft-V2b4;

%Calculates time and mass for transfer
tauT=2*pi*sqrt(aT^3/mu);
timeForTrans=tauT/2;

%Calculates theta
theta=acos((dot(hvec1,hvec2)/(norm(hvec1)*norm(hvec2))));
theta=rad2deg(theta);

%plots
hold on
scale=10;
plot3(position1T(1),position1T(2),position1T(3),'r*')
plot3(positionT(end,1),positionT(end,2),positionT(end,3),'r*')
plot3(position1(:,1),position1(:,2),position1(:,3),position2(:,1),position2(:
,2),position2(:,3))
plot3(positionT(:,1),positionT(:,2),positionT(:,3))
quiver3(position1T(1),position1T(2),position1T(3),deltaV1vec(1),deltaV1vec(2)
,deltaV1vec(3),5000)
quiver3(positionT(end,1),positionT(end,2),positionT(end,3),deltaV2vec(1),delt
aV2vec(2),deltaV2vec(3),5000)
axis([-30000 30000 -30000 30000 -30000 30000])
title('View of the transfer between two circular orbits')
subtitle('Delta Vs not to scale')
view(62.1,16.85)

%Prints results
DeltaLet=char(916);
subScr1=char(8321);
subScr2=char(8322);
fprintf('The line of intersection is along %g %g
%g\n',lvec(1),lvec(2),lvec(3))
fprintf(['The magnitude of ' DeltaLet 'V' subScr1 ' = %g km/s\n'],deltaV1)
fprintf(['The magnitude of ' DeltaLet 'V' subScr2 ' = %g km/s\n'],deltaV2)
fprintf(['The total ' DeltaLet 'V required is %g km/s\n'],deltaV1+deltaV2)
fprintf('The transfer takes %g hours to complete\n',timeForTrans/3600)
```

```
fprintf('The location of impulse one is: %g km %g km %g
km\n',position1T(1),position1T(2),position1T(3))
fprintf('The location of impulse two is: %g km %g km %g
km\n',positionT(end,1),positionT(end,2),positionT(end,3))
fprintf('The eccentricity of the transfer orbit is %g\n',eT)
fprintf('Theta = %g deg',theta)
```

The line of intersection is along 1 0 0
The magnitude of ΔV_1 = 2.02605 km/s
The magnitude of ΔV_2 = 1.41089 km/s
The total ΔV required is 3.43693 km/s
The transfer takes 2.96776 hours to complete
The location of impulse one is: 6728.15 km 0 km 0 km
The location of impulse two is: -26558 km 13.8973 km 7.38934 km
The eccentricity of the transfer orbit is 0.595739
Theta = 27 deg



Problem 6:

```
clc;clear;close all;
%Defines givens
mu=398600;
r1=300+6378.145;
i1=28.5;
i1=deg2rad(i1);
e1=0;
Omega1=0;
omega=0;

%Calculates position of first orbit
nu=0:0.001:2*pi;
position1=zeros(length(nu),3);
velocity1=zeros(length(nu),3);
for i=1:length(nu)
    oe1=[r1,e1,Omega1,i1,omega,nu(i)];
    [r,v]=oe2rv_Hackbardt_Chris(oe1,mu);
    position1(i,:)=r';
    velocity1(i,:)=v';
end

%Calculates pos of second orbit
i2=0;
period=23.934*60*60;
a2=(mu*(period/(2*pi))^2)^(1/3);
r2=a2;
e2=0;
Omega2=0;
position2=zeros(length(nu),3);
velocity2=zeros(length(nu),3);
for i=1:length(nu)
    oe2=[a2,e2,Omega2,i2,omega,nu(i)];
    [r,v]=oe2rv_Hackbardt_Chris(oe2,mu);
    position2(i,:)=r';
    velocity2(i,:)=v';
end

%calculates the first impulse (not changing i)
v1=sqrt(mu/r1);
v2=sqrt(mu/r2);
hvec1=cross(position1(1,:),velocity1(1,:));
hvec2=cross(position2(1,:),velocity2(1,:));
lvec=cross(hvec1,hvec2);
lvec=lvec/norm(lvec);
position1T=r1*lvec;
aT=(r1+r2)/2;
eT=(r2-r1)/(r2+r1);
deltaV1=sqrt(((2*mu)/r1)-(mu/aT))-v1;
u1vec=cross(hvec1,lvec)/norm(hvec1);
V1b4=v1*u1vec;
deltaV1vec=deltaV1*u1vec;
V1aft=V1b4+deltaV1vec;

%calculates pos of transfer orbit (not chnging i)
nuT=0:0.001:pi;
oeT=rv2oe_Hackbardt_Chris(position1T,V1aft,mu);
positionT=zeros(length(nuT),3);
velocityT=zeros(length(nuT),3);
for i=1:length(nuT)
    oeT(6)=nuT(i);
```

```
    [r,v]=oe2rv_Hackbardt_Chris(oeT,mu);
    positionT(i,:)=r';
    velocityT(i,:)=v';
end

%calculates second impulse (part of first transfer orbit)
v2bf=sqrt(((2*mu)/r2)-(mu/aT));
V2b4=-v2bf*u1vec;
u2vec=-cross(hvec2,lvec)/norm(hvec2);
V2aft=v2*u2vec;
deltaV2vec=V2aft-V2b4;
deltaV2=norm(deltaV2vec);

%Calculates first impulse (changes i)
oeT2=oeT;
oeT2(4)=i2;
oeT2(6)=0;
[r,v]=oe2rv_Hackbardt_Chris(oeT2,mu);
position1T2=r';
velocity1T2=v';
V1T2aft=velocity1T2;
deltaV1T2vec=V1T2aft-V1b4;
deltaV1T2=norm(deltaV1T2vec);

positionT2=zeros(length(nuT),3);
velocityT2=zeros(length(nuT),3);
nuT2=0:0.001:pi;
for j=1:length(nuT2)
    oeT2(6)=nuT2(j);
    [r,v]=oe2rv_Hackbardt_Chris(oeT2,mu);
    positionT2(j,:)=r';
    velocityT2(j,:)=v';
end

%Calculates second impules raises orbit
V2T2b4=velocityT2(end,:);
V2T2aft=v2*u2vec;
deltaV2T2vec=V2T2aft-V2T2b4;
deltaV2T2=norm(deltaV2T2vec);

percentInc=0:0.01:1;
deltaInc=(i2-i1);
deltaVs=zeros(1,length(percentInc));
for i=1:length(percentInc)
    %Calculates first impulse (changes i)
    oeT3=oeT;
    oeT3(4)=i1+(percentInc(i)*deltaInc);
    oeT3(6)=0;
    [r,v]=oe2rv_Hackbardt_Chris(oeT3,mu);
    position1T3=r';
    velocity1T3=v';
    V1T3aft=velocity1T3;
    deltaV1T3vec=V1T3aft-V1b4;
    deltaV1T3=norm(deltaV1T3vec);
    oeT3(5)=pi;
    oeT3(6)=pi;

    %Calculates second impules raises orbit and finishes inclination change
    [r,v]=oe2rv_Hackbardt_Chris(oeT3,mu);
    positionendT3=r';
    velocityendT3=v';
    V2T3b4=velocityendT3;
```

```matlab
    V2T3aft=v2*u2vec;
    deltaV2T3vec=V2T3aft-V2T3b4;
    deltaV2T3=norm(deltaV2T3vec);

    deltaVs(i)=deltaV1T3+deltaV2T3;
end
deltaVs=deltaVs/v1;
minDV=min(deltaVs);
mostEff=deltaVs==minDV;
mostEff=percentInc(mostEff);

figure(1)
plot(percentInc,deltaVs)
xlabel('Percent of inclination change completed at periapsis of transfer orbit')
ylabel('\DeltaV / vc1')

%Prints results
DeltaLet=char(916);
subScr1=char(8321);
subScr2=char(8322);
fprintf(2,'For a transfer with inclination change occuring at apoapsis of transfer
orbit\n')
fprintf(['The magnitude of ' DeltaLet 'V' subScr1 ' = %g km/s\n'],deltaV1)
fprintf(['The magnitude of ' DeltaLet 'V' subScr2 ' = %g km/s\n'],deltaV2)
fprintf(['The total ' DeltaLet 'V required is %g km/s\n'],deltaV1+deltaV2)
fprintf(2,'For a transfer with inclination change occuring at periapsis of transfer
orbit\n')
fprintf(['The magnitude of ' DeltaLet 'V' subScr1 ' = %g km/s\n'],deltaV1T2)
fprintf(['The magnitude of ' DeltaLet 'V' subScr2 ' = %g km/s\n'],deltaV2T2)
fprintf(['The total ' DeltaLet 'V required is %g km/s\n'],deltaV1T2+deltaV2T2)
fprintf('\n')
fprintf(['The smallest total impulse occurs when f = %g with a value of ' DeltaLet
'V/vc1 = %g\n'],mostEff,minDV)
```

For a transfer with inclination change occuring at apoapsis of transfer orbit

The magnitude of ΔV_1 = 2.42572 km/s

The magnitude of ΔV_2 = 1.83023 km/s

The total ΔV required is 4.25595 km/s

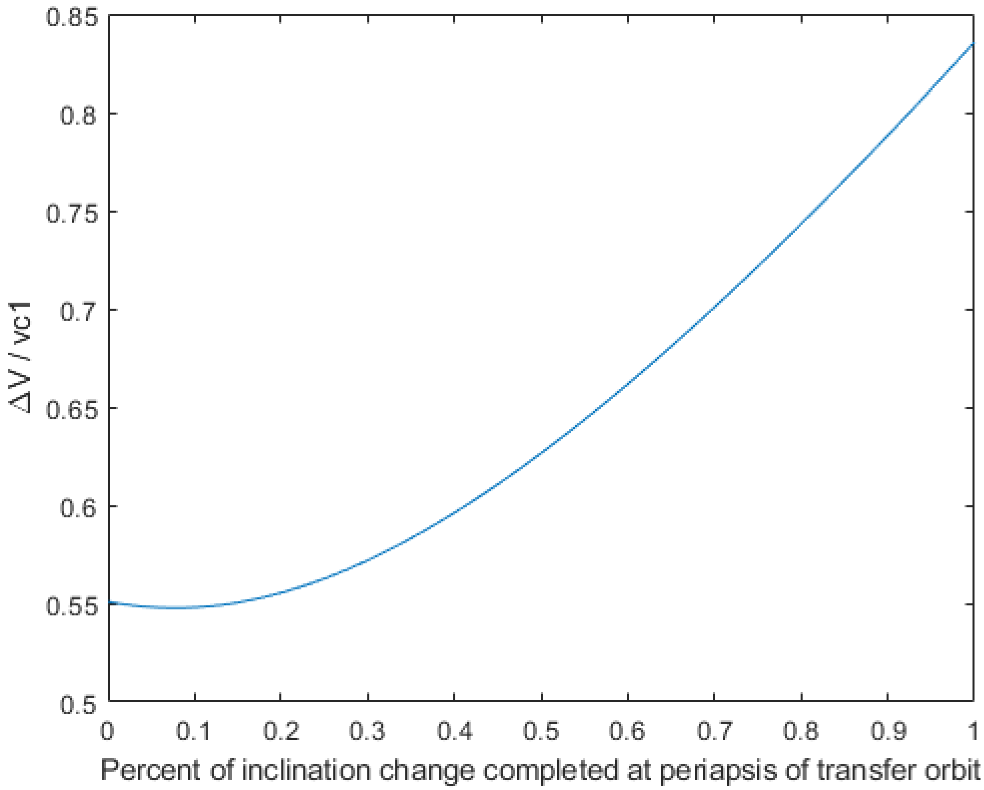
For a transfer with inclination change occuring at periapsis of transfer orbit

The magnitude of ΔV_1 = 4.98922 km/s

The magnitude of ΔV_2 = 1.46683 km/s

The total ΔV required is 6.45605 km/s

The smallest total impulse occurs when f = 0.08 with a value of ΔV/vc1 = 0.547692


0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
ΔV / vc1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Percent of inclination change completed at periapsis of transfer orbit