Function:

```
function [dv1,dv2,dv3,posT,time] = biEllipticWithPlaneChange(oei,oef,S,f,mu)
%This function calculates the 3 impulses needed to complete a bi-elliptic
orbit transfer

%Input: oei, vector containing the orbital elements of the initial orbit
%Input: oef, vector containing the orbital elements of the final orbit
%Input: S, the ratio ri/rf. The ratio of apoapsis of transfer orbit to radius
of final orbit
%Input: f, fraction of orbit crank performed at first impulse
%Input: mu, gravitaional constant

%Output: dv1, magnitude of impulse one
%Output: dv2, magnitude of impulse two
%Output: dv3, magnitude of impulse three
%Output: posT, matrix containing position of transfer orbit
%Output: time, vector containing times associated with postitions in posT
[ri,vi] = oe2rv_Hackbardt_Chris(oei,mu);
[rf,vf] = oe2rv_Hackbardt_Chris(oef,mu);
hvec1=cross(ri,vi);
hvec2=cross(rf,vf);
lvec=cross(hvec1,hvec2);
lvec=lvec/norm(lvec);
u1vec=cross(hvec1,lvec)/norm(hvec1);
u2vec=cross(hvec2,lvec)/norm(hvec2);

r0=oei(1);
rf=oef(1);
ra=S*rf;
v0=sqrt(mu/r0);
vf=sqrt(mu/rf);

posdV1=r0*lvec;
veldV1b4=v0*u1vec;
veldV3after=vf*u2vec;

%First Transfer Orbit
%Uses Rodrigues' rotation formula to find vel vector at periapsis of first
transfer orbit
at1=(r0+ra)/2;
vpT1=sqrt(mu)*sqrt((2/r0)-(1/at1));
dv1afterDirection=angleBetweenVectors(hvec1,hvec2)*f;
I=eye(3);
K=[0,-lvec(3),lvec(2);lvec(3),0,-lvec(1);-lvec(2),lvec(1),0];
R=I+(sin(dv1afterDirection).*K)+((1-cos(dv1afterDirection)).*K^2);
vel1T1=R*veldV1b4;
vel1T1=(vel1T1/norm(vel1T1))*vpT1;

oet1=rv2oe_Hackbardt_Chris(posdV1,vel1T1,mu);

tauT1=2*pi*sqrt(oet1(1)^3/mu);
timeT1=0:60:tauT1/2;
for i=1:length(timeT1)
    [r,v] = propagateKepler_Hackbardt_Chris(posdV1,vel1T1,0,timeT1(i),mu);
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    posT1(i,:)=r';
    velT1(i,:)=v';
end

%Second Transfer Orbit
%Uses Rodrigues' rotation formula to find vel vector at apoapsis transfer
orbit
at2=(rf+ra)/2;
vAT2=sqrt(mu)*sqrt((2/ra)-(1/at2));
hvecT1=cross(posdV1,vel1T1);
dv2afterDirection=-angleBetweenVectors(hvecT1,hvec2);
posdV2=posT1(end,:);
posDV2dir=posdV2/norm(posdV2);
K=[0,-posDV2dir(3),posDV2dir(2);posDV2dir(3),0,-posDV2dir(1);-
posDV2dir(2),posDV2dir(1),0];
R=I+(sin(dv2afterDirection).*K)+((1-cos(dv2afterDirection)).*K^2);

vellastT1=velT1(end,:)';
vel1T2dir=R*(vellastT1/norm(vellastT1));
vel1T2=vel1T2dir*vAT2;
oet2=rv2oe_Hackbardt_Chris(posdV2,vel1T2,mu);

tauT2=2*pi*sqrt(oet2(1)^3/mu);
timeT2=0:60:tauT2/2;
for i=1:length(timeT2)
    [r,v] = propagateKepler_Hackbardt_Chris(posdV2,vel1T2,0,timeT2(i),mu);
    posT2(i,:)=r';
    velT2(i,:)=v';
end
posT=[posT1;posT2];
timeT2=timeT2+timeT1(end);
vellastT2=velT2(end,:);

%Finds the delta vs
dv1=norm(vel1T1-veldV1b4);
dv2=norm(vel1T2-vellastT1);
dv3=norm(veldV3after-vellastT2');
time=[timeT1';timeT2'];


end
```
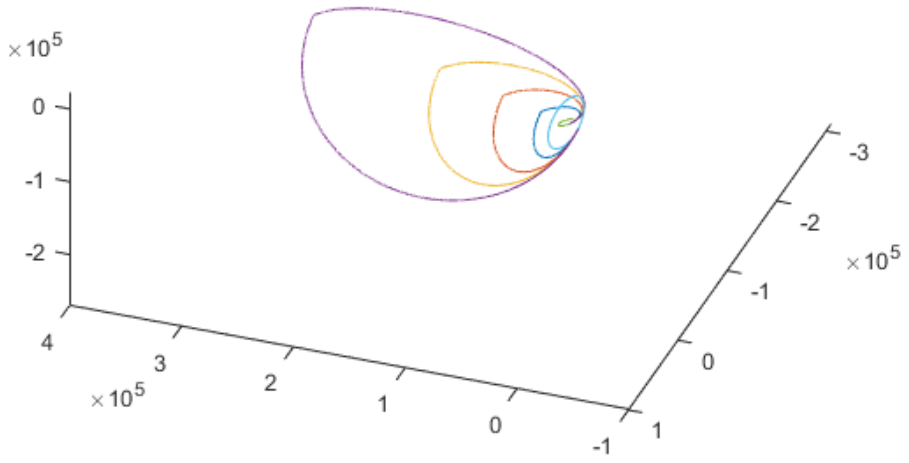

Main:

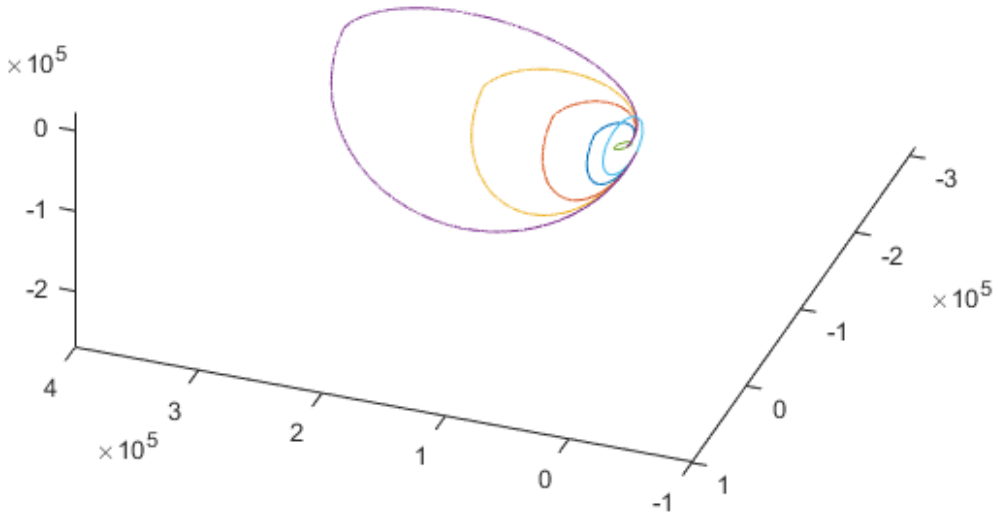
```
clc;clear;close all;
mu=398600;
S=[2,5,10,20];
f=[0,0.5,1];

r1=350+6378.145;
i1=deg2rad(28);
Omega1=deg2rad(60);
oei=[r1,0,Omega1,i1,0,0];

r2=20200+6378.145;
i2=deg2rad(57);
Omega2=deg2rad(120);
oef=[r2,0,Omega2,i2,0,0];

[pos1,v1]  = oe2rv_Hackbardt_Chris(oei,mu);
tau1=2*pi*sqrt(r1^3/mu);
[times1, pos1, vel1] = propagateOnCircle(pos1,v1,0,tau1,mu,500);

[pos2,v2]  = oe2rv_Hackbardt_Chris(oef,mu);
tau2=2*pi*sqrt(r2^3/mu);
[times2, pos2, vel2] = propagateOnCircle(pos2,v2,0,tau2,mu,500);

R=r2/r1;
dVBP=norm(v1)*((sqrt(2)-1)+sqrt(1/R)*(sqrt(2)-1));
dvp=zeros(1,length(f));
dvp(:)=dVBP;

totdV=zeros(length(S),length(f));
for i=1:length(f)
    figure
    hold on
    for j=1:length(S)

[dv1,dv2,dv3,posT,time]=biEllipticWithPlaneChange(oei,oef,S(j),f(i),mu);
        plot3(posT(:,1),posT(:,2),posT(:,3))
        totdV(j,i)=dv1+dv2+dv3;
    end
    plot3(pos1(:,1),pos1(:,2),pos1(:,3))
    plot3(pos2(:,1),pos2(:,2),pos2(:,3))
    view(-160,55)
end
figure
hold on
plot(f,totdV(1,:),f,totdV(2,:),f,totdV(3,:),f,totdV(4,:),f,dvp)
xlabel('f')
ylabel('dV')
legend('S = 2','S = 5','S = 10','S = 20','Bi-
Parabolic','Location','northwest')
```

3D plots:

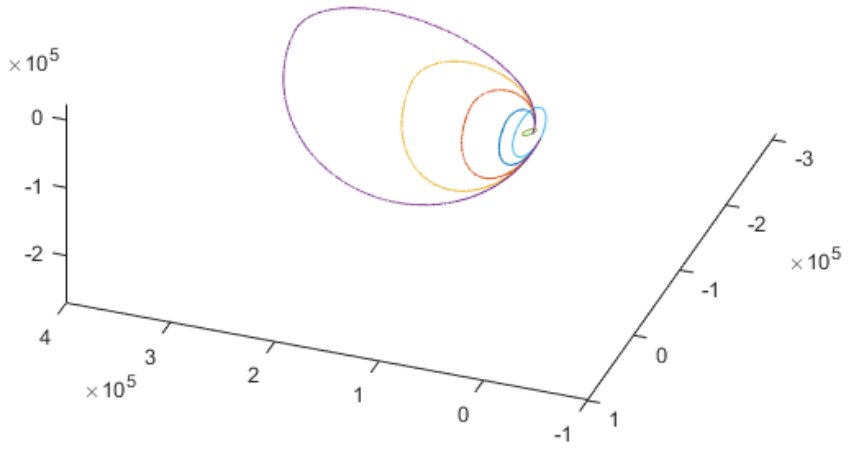
f = 0:



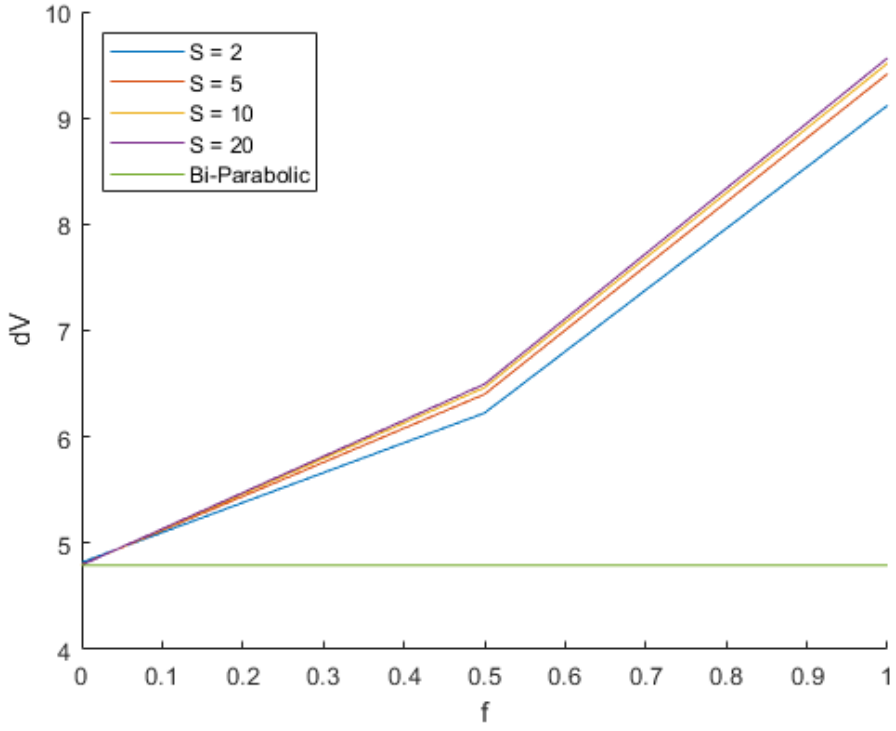
f = 0.5:



f = 1:



f vs ΔV:



The minimum ΔV is used for every value of S when f = 0. This is because it is more efficient to change the orbit plane at apoapsis of the transfer orbit instead of periapsis.