Problem 1:

$r_{p1} = r_{p2} = r_p$ and $a_1 < a_2$

$$V = \sqrt{\mu}\sqrt{\frac{2}{r} - \frac{1}{a}}$$

For each orbit at periapsis, the speed is:

$$V_1 = \sqrt{\mu}\sqrt{\frac{2}{r_p} - \frac{1}{a_1}} \qquad \text{and} \qquad V_2 = \sqrt{\mu}\sqrt{\frac{2}{r_p} - \frac{1}{a_2}}$$

Since $a_1 < a_2$,

$$\frac{1}{a_1} > \frac{1}{a_2}$$

Therefore:

$$\frac{2}{r_p} - \frac{1}{a_1} < \frac{2}{r_p} - \frac{1}{a_2}$$

Since μ is a constant for both orbits, we can conclude that $V_2 > V_1$ at periapsis.

For each orbit at apoapsis,

$r_{a1} < r_{a2}$

Therefore, each speed is:

$$V_1 = \sqrt{\mu}\sqrt{\frac{2}{r_{a1}} - \frac{1}{a_1}} \qquad \text{and} \qquad V_2 = \sqrt{\mu}\sqrt{\frac{2}{r_{a2}} - \frac{1}{a_2}}$$

Since $r_{a1} < r_{a2}$ and $a_1 < a_2$,

$$\frac{1}{a_1} > \frac{1}{a_2} \qquad \text{and} \qquad \frac{2}{r_{a1}} > \frac{2}{r_{a2}}$$

Therefore:

$$\frac{2}{r_{a1}} - \frac{1}{a_1} > \frac{2}{r_{a2}} - \frac{1}{a_2}$$

Since μ is a constant for both orbits, we can conclude that $V_1 > V_2$ at apoapsis.

Problem 1 can also be solved numerically.

```
clc;clear;
mu = 1;
rp = 10;
a2 = 20;
a1 = 15;
e1 = 1-(rp/a1);
e2 = 1-(rp/a2);
ra1 = (a1*(1-e1^2))/(1-e1);
ra2 = (a2*(1-e2^2))/(1-e2);
vp1 = sqrt(mu)*sqrt((2/rp)-(1/a1));
vp2 = sqrt(mu)*sqrt((2/rp)-(1/a2));
va1 = sqrt(mu)*sqrt((2/ra1)-(1/a1));
va2 = sqrt(mu)*sqrt((2/ra2)-(1/a2));
fprintf('The speeds at apoapsis are %g and %g for orbits 1 and 2\n',va1,va2);
fprintf('The speeds at periapsis are %g and %g for orbits 1 and 
2\n',vp1,vp2);
```

The speeds at apoapsis are 0.182574 and 0.129099 for orbits 1 and 2

The speeds at periapsis are 0.365148 and 0.387298 for orbits 1 and 2

Problem 2:

Function:

```
function [a, e, p, h, va, vp] = orbitAltitudes(aa,ap,rb,mu)
    %This function calculates properties of a earth orbit with inputs of
    altitude at apoapsis and periapsis
    %Function call: [a, e, p, h, va, vp] = orbitAltitudes(aa,ap,rb,mu)
    %
    %Input: aa, altitude at apoapsis
    %Input: ap, altitude at periapsis
    %Input: rb, radius of body
    %Input: mu, gravitational parameter
    %
    %Output: a, semi-major axis
    %Output: e, eccentricity
    %Output: p, semi-latus rectum
    %Output: h, magnitude of the specific angular momentum
    %Output: va, velocity at apoapsis
    %Output: vp, velocity at periapsis

    ra = aa+rb;
    rp = ap+rb;
    a = (ra+rp)/2;
    e = (ra-rp)/(ra+rp);
    p = a*(1-e^2);
    h = sqrt(mu*p);
    vp = sqrt(mu)*sqrt((2/rp)-(1/a));
    va = sqrt(mu)*sqrt((2/ra)-(1/a));
end
```

Main:

```
clc;clear;
aa = 800;
ap = 500;
rb = 6378.145;
mu = 398600;
[a, e, p, h, va, vp] = orbitAltitudes(aa,ap,rb,mu);
fprintf('a = %g km\n',a);
fprintf('e = %g\n',e);
fprintf('p = %g km\n',p);
fprintf('h = %g km^2/s\n',h);
fprintf('Va = %g km/s\n',va);
fprintf('Vp = %g km/s\n',vp);
```

Output:

a = 7028.15 km

e = 0.0213428

p = 7024.94 km

h = 52916.4 km^2/s

Va = 7.37187 km/s

Vp = 7.69341 km/s

Problem 3:

Orbit B would allow for longer visualization of point Q. The solution to problem 1 proved that an orbit with the same periapsis but a larger apoapsis have a slower speed at apoapsis and a faster speed at periapsis. This is the same as orbit B. The spacecraft would travel slowly over point Q while the spacecraft is nearing apoapsis, and is observing point Q, and would travel faster near periapsis, when point Q cannot be observed.

Problem 4:

Given information:

$$a_1 = a_2 = a \quad \text{and} \quad e_1 = 0$$

Since Orbit 1 is circular:

$$r_{p1} = r_1$$

Substituting known values into the radius of periapsis equation:

$$r_1 = r_{p1} = a(1 - {e_1}^2)$$

$$r_1 = a$$

Using the speed at periapsis of Orbit 2 and the vis-viva equation to find r_{p2}:

$$v_{p2} = \sqrt{\mu}\sqrt{\frac{2}{r_{p2}} - \frac{1}{a}}$$

$$v_{p2} = \sqrt{\mu}\sqrt{\frac{2}{r_{p2}} - \frac{1}{r_1}}$$

Rearranging for r_{p2}:

$$r_{p2} = \frac{2}{\frac{{v_{p2}}^2}{\mu} + \frac{1}{r_1}}$$

We also know that:

$$r_{p2} = r_1(1 - e_2)$$

Equating the two previous equations:

$$\frac{2}{\frac{{v_{p2}}^2}{\mu} + \frac{1}{r_1}} = r_1(1 - e_2)$$

Solving for e_2:

$$e_2 = 1 - \frac{2}{\frac{r_1 {v_{p2}}^2}{\mu} + 1}$$

Problem 5:

Function:

```
function [nu, e, energy, p, h, rp, ra] =
flightPathSpeedRadius(v,r,gamma,mu)
    %This functions calculates true anomaly, eccentricity, and total
    energy given speed, flight path angle, and radius
    %Function call: [nu e energy p h rp ra] =
    flightPathSpeedRadius(v,r,gamma,mu)
    %
    %Input: v, speed
    %Input: r, radius
    %Input: gamma, flight path angle
    %Input: mu, gravitational parameter
    %
    %Output: nu, true anomaly
    %Output: e, eccentricity
    %Output: energy, total mechanical energy
    %Output: p, semi-latus rectum
    %Output: h, magnitude of the specific angular momentum
    %Output: rp, periapsis radius
    %Output: ra, apoapsis radius

    a = ((((v/sqrt(mu))^2)-(2/r))^-1)*-1;
    h = r*v*cos(gamma);
    p = (h^2)/mu;
    e=sqrt(1-(p/a));
    nu = acos((p/(r*e))-(1/e));
    energy = -(mu)/(2*a);
    rp = p/(1+e);
    ra = p/(1-e);
end
```

Main:

```
clc;clear;
v=7.5;
r=9500;
flightPath=18;
gamma=deg2rad(flightPath);
mu=398600;
[nu, e, energy, p, h, rp, ra] = flightPathSpeedRadius(v,r,gamma,mu);
fprintf('true anomaly = %g rad\n',nu);
fprintf('e = %g\n',e);
fprintf('orbital energy = %g kg km^2 s^-2\n',energy);
```

true anomaly = 1.07596 rad

e = 0.447706

orbital energy = -13.8329 kg km^2 s^-2

Problem 6:

```
clc;clear;
rvec = [-12 -20 15];
rvec = (1/20)*rvec;
mu = 1;
r = norm(rvec);
inertialAccel = -(mu/r^3)*rvec;
fprintf('Inertial Acceleration in I is: ');
fprintf('%g, %g, %g',inertialAccel(1),inertialAccel(2),inertialAccel(3));
```

Inertial Acceleration in I is: 0.225088, 0.375146, -0.28136

Problem 7:

From the problem we know that:

$$r_e = r_c = a_c = r \qquad \text{and} \qquad v_c = v_e = v$$

Using the vis-viva equation to solve for the speed of a circular orbit where r = a:

$$v_c = \sqrt{\mu}\sqrt{\frac{2}{r} - \frac{1}{a_c}} = \sqrt{\frac{\mu}{r}}$$

Solving for r in the vis-viva equation for an ellipse results in:

$$r = \frac{2}{\frac{v^2}{\mu} + \frac{1}{a_e}}$$

Substituting v into the previous equation:

$$r = \frac{2}{\frac{1}{r} + \frac{1}{a_e}}$$

We also know from the orbit equation that:

$$r = \frac{a_e(1 - e^2)}{1 + e\cos\nu}$$

Equating the previous two equations:

$$\frac{a_e(1 - e^2)}{1 + e\cos\nu} = \frac{2}{\frac{1}{r} + \frac{1}{a_e}}$$

Substituting r into the right side and solving for ν results in:

$$\nu = \cos^{-1}(-e)$$

Problem 8:

Starting with the equation for angular momentum and taking the magnitude:

$$^{I}\boldsymbol{h} = \boldsymbol{r}\; x\; ^{I}\boldsymbol{v} = rv\boldsymbol{u}_z \sin\phi$$

$$\|^{I}\boldsymbol{h}\| = rv\sin\phi$$

Know that the zenith angle, ϕ, and flight path angle, γ, is equal to:

$$\gamma = \frac{\pi}{2} - \phi$$

$$\phi = \frac{\pi}{2} - \gamma$$

Substituting the equation for zenith angle into the equation for angular momentum:

$$h = rv\sin\frac{\pi}{2} - \gamma = rv\cos\gamma$$

Taking the scaler product of $\boldsymbol{r}$ and $^{I}\boldsymbol{v}$:

$$\boldsymbol{r}\cdot\, ^{I}\boldsymbol{v} = \|\boldsymbol{r}\|\; \|^{I}\boldsymbol{v}\|\cos\phi = rv\cos\phi$$

Since $\cos\phi = \cos\left(\frac{\pi}{2} - \gamma\right) = \sin\gamma$, the scaler product can be rewritten as:

$$\boldsymbol{r}\cdot\, ^{I}\boldsymbol{v} = rv\sin\gamma$$

Combining the results leaves:

$$\tan\gamma = \frac{\boldsymbol{r}\cdot\, ^{I}\boldsymbol{v}}{h}$$

We also know that:

$$\boldsymbol{r}\cdot\, ^{I}\boldsymbol{v} = \frac{1}{2}\frac{d}{dt}(\boldsymbol{r}\cdot\boldsymbol{r}) = \frac{1}{2}\frac{d}{dt}(r^2) = r\dot{r}$$

Substituting this result into the previous equation:

$$\tan\gamma = \frac{r\dot{r}}{h}$$

Knowing that $\dot{r}$ and r equal:

$$r = \frac{p}{1+e\cos\nu} \qquad \text{and} \qquad \dot{r} = \frac{he\sin\nu}{p}$$

We can substitute for r and $\dot{r}$ to result in:

$$\tan\gamma = \frac{e\sin\nu}{1+e\cos\nu}$$

Since the zenith angle is between 0 and π, we can do a substitution to find the range for flight path angle

$$0 \le \phi \le \pi \qquad \text{so} \qquad 0 \le \frac{\pi}{2} - \gamma \le \pi \qquad \text{therefore} \qquad -\frac{\pi}{2} \le \gamma \le \frac{\pi}{2}$$

Problem 9:

Function:

```
function [hVec, eVec, hDote, p, a, nu] = positionVelocity(rVec, vVec, mu)
    %This function takes postion and velocity vectors as inputs and calculates
orbital quantities
    %Function call: [hVec, eVec, hDote, p, a, nu] = positionVelocity(rVec,
vVec, mu)
    %
    %Input: rVec, postion vector
    %Input: vVec, velocity vector
    %Input: mu, gravitational parameter
    %
    %Output: hVec, specific angular momentum vector
    %Output: eVec, eccentricity vector
    %Output: hDote, dot product of specific angular momentum vector and
eccentricity vector
    %Output: p, semi-latus rectum
    %Output: a, semi-major axis
    %Output: nu, true anomaly

    hVec = cross(rVec,vVec);
    r = norm(rVec);
    eVec = (cross(vVec,hVec)/mu)-(rVec/r);
    hDote = dot(hVec,eVec);
    h = norm(hVec);
    p = h^2/mu;
    e = norm(eVec);
    a = abs(p/(1-e^2));
    nu = acos((p/(r*e))-(1/e));
end
```

Main

```
clc;clear;
rVec = [0 2 0];
vVec = [(-1/sqrt(3)) (sqrt(2)/sqrt(3)) 0];
mu = 1;
[hVec, eVec, hDote, p, a, nu] = positionVelocity(rVec, vVec, mu);
fprintf('The specific angular momentum vector is: ');
fprintf('%g, %g, %g\n',hVec(1),hVec(2),hVec(3));
fprintf('The eccentricity vector is: ');
fprintf('%g, %g, %g\n',eVec(1),eVec(2),eVec(3));
fprintf('h in I dotted with the eccentricity vector = %g\n',hDote);
fprintf('p = %g km\n',p);
fprintf('a = %g km\n',a);
fprintf('true anomaly = %g rad\n',nu);
```

The specific angular momentum vector is: 0, -0, 1.1547

The eccentricity vector is: 0.942809, -0.333333, 0

h in I dotted with the eccentricity vector = 0

p = 1.33333 km

a = 3.0024e+15 km

true anomaly = 1.91063 rad

Problem 10:

The orbit equation with semi-latus rectum substituted in is equivalent to:

$$r = \frac{a(1 - e^2)}{1 + e\cos\nu}$$

Since a is equal to r:

$$r = \frac{r(1 - e^2)}{1 + e\cos\nu}$$

Then solving for ν:

$$1 + e\cos\nu = 1 - e^2$$

$$\cos\nu = -e$$

$$\nu = \cos^{-1}(-e)$$

Speed is given by the vis-viva equation:

$$v = \sqrt{\mu}\sqrt{\frac{2}{r} - \frac{1}{a}}$$

Since r is equal to a:

$$v = \sqrt{\mu}\sqrt{\frac{2}{r} - \frac{1}{r}}$$

$$v = \sqrt{\mu}\sqrt{\frac{1}{r}} = \sqrt{\frac{\mu}{r}}$$

Problem 11:

Function:

```
function [h, p, a, rp, ra]=energyEccentricity(energy,e,mu)
    %This function takes orbital energy and eccentricity to calculate
h, p, a, rp, ra
    %Function Call: [h, p, a, rp, ra]=energyEccentricity(energy,e,mu)
    %
    %Input: energy, orbital energy
    %Input: e, eccentricity
    %Input: mu, gravitational parameter
    %
    %Output: h, magnitude of the specific angular momentum
    %Output: p, semi-latus rectum
    %Output: a, semi-major axis
    %Output: rp, periapsis radius
    %Output: ra, apoapsis radius

    a = -mu/(2*energy);
    p = a*(1-e^2);
    h = sqrt(mu*p);
    rp = a*(1-e);
    ra = a*(1+e);
end
```

Main:

```
clc;clear;
givenEnergy = -2*10^8; %ft^2/s
e = 0.2;
mu = 398600;
energy = givenEnergy / 10763910.41671; %ft^2/s^2 to km^2/s^2
[h, p, a, rp, ra]=energyEccentricity(energy,e,mu);
fprintf('h = %g m^2/s\n',h);
fprintf('p = %g km\n',p);
fprintf('a = %g km\n',rp);
fprintf('rp = %g km\n',rp);
fprintf('ra = %g km\n',ra);
```

h = 64066.1 m^2/s

p = 10297.2 km

a = 8580.99 km

rp = 8580.99 km

ra = 12871.5 km

Problem 12:

Function:

```
function [aa, energy, h, p] = periapsisAltEccentricity(ap,e,rb,mu)
    %This function takes altitude at periapsis and eccentricity to
calculate altitude at apoapsis, orbital energy, h, and p
    %Function Call: [aa, energy, h, p] =
periapsisAltEccentricity(ap,e,rb,mu)
    %
    %Input: ap, altitude at periapsis
    %Input: e, eccentricity
    %Input: rb, radius of body
    %Input: mu, gravitational parameter
    %
    %Output: aa, apoapsis altitude
    %Output: energy, orbital energy
    %Output: h, magnitude of the specific angular momentum
    %Output: p, semi-latus rectum

    rp = ap + rb;
    a = rp/(1-e);
    ra = a*(1+e);
    aa = ra - rb;
    p = a*(1-e^2);
    h = sqrt(mu*p);
    energy = -(mu)/(2*a);
end
```

Main:

```
clc;clear;
e = 0.1;
ap = 370;
rb = 6378.145;
mu = 398600;
[aa, energy, h, p] = periapsisAltEccentricity(ap,e,rb,mu);
fprintf('apoapsis altitude = %g km\n',aa);
fprintf('orbital energy = %g kg km^2 s^-2\n',energy);
fprintf('h = %g m^2/s\n',h);
fprintf('p = %g km\n',p);
```

apoapsis altitude = 1869.59 km

orbital energy = -26.5806 kg km^2 s^-2

h = 54394.8 m^2/s

p = 7422.96 km

Problem 13:

Function:

```
function [nu, e, energy, p, h, rp, ra] = flightPathSpeedRadius(v,r,gamma,mu)
    %This functions calculates true anomaly, eccentricity, and total energy
given speed, flight path angle, and radius
    %Function call: [nu e energy p h rp ra] =
flightPathSpeedRadius(v,r,gamma,mu)
    %
    %Input: v, speed
    %Input: r, radius
    %Input: gamma, flight path angle
    %Input: mu, gravitational parameter
    %
    %Output: nu, true anomaly
    %Output: e, eccentricity
    %Output: energy, total mechanical energy
    %Output: p, semi-latus rectum
    %Output: h, magnitude of the specific angular momentum
    %Output: rp, periapsis radius
    %Output: ra, apoapsis radius

    a = ((((v/sqrt(mu))^2)-(2/r))^-1)*-1;
    h = r*v*cos(gamma);
    p = (h^2)/mu;
    e=sqrt(1-(p/a));
    nu = acos((p/(r*e))-(1/e));
    energy = -(mu)/(2*a);
    rp = p/(1+e);
    ra = p/(1-e);
end
```

Main:

```
clc;clear;
v = 0.8;
gamma = 0;
altitude = 4000;
earthRadius = 6378.145;
r = altitude+earthRadius;
mu = 398600;
[nu, e, energy, p, h, rp, ra] = flightPathSpeedRadius(v,r,gamma,mu);
fprintf('orbital energy = %g kg km^2 s^-2\n',energy);
fprintf('h = %g m^2/s\n',h);
fprintf('p = %g km\n',p);
fprintf('rp = %g km\n',rp);
fprintf('ra = %g km\n',ra);
```

orbital energy = -38.0876 kg km^2 s^-2

h = 8302.52 m^2/s

p = 172.935 km

rp = 87.1938 km

ra = 10378.1 km

Problem 14:

Function:

```
function [hVec, eVec, hDote, p, a, nu] = positionVelocity(rVec, vVec, mu)
    %This function takes postion and velocity vectors as inputs and calculates
orbital quantities
    %Function call: [hVec, eVec, hDote, p, a, nu] = positionVelocity(rVec,
vVec, mu)
    %
    %Input: rVec, postion vector
    %Input: vVec, velocity vector
    %Input: mu, gravitational parameter
    %
    %Output: hVec, specific angular momentum vector
    %Output: eVec, eccentricity vector
    %Output: hDote, dot product of specific angular momentum vector and
eccentricity vector
    %Output: p, semi-latus rectum
    %Output: a, semi-major axis
    %Output: nu, true anomaly

    hVec = cross(rVec,vVec);
    r = norm(rVec);
    eVec = (cross(vVec,hVec)/mu)-(rVec/r);
    hDote = dot(hVec,eVec);
    h = norm(hVec);
    p = h^2/mu;
    e = norm(eVec);
    a = abs(p/(1-e^2));
    nu = acos((p/(r*e))-(1/e));
end
```

Main:

```
clc;clear;
rVec = [-0.6 -1 0.75];
vVec = [0.8 -0.45 0.45];
mu = 1;
[hVec, eVec, hDote, p, a, nu] = positionVelocity(rVec, vVec, mu);
fprintf('The specific angular momentum vector is: ');
fprintf('%g, %g, %g\n',hVec(1),hVec(2),hVec(3));
fprintf('The eccentricity vector is: ');
fprintf('%g, %g, %g\n',eVec(1),eVec(2),eVec(3));
fprintf('h in I dotted with the eccentricity vector = %g\n',hDote);
fprintf('p = %g km\n',p);
fprintf('a = %g km\n',a);
fprintf('true anomaly = %g rad\n',nu);
```

The specific angular momentum vector is: -0.1125, 0.87, 1.07

The eccentricity vector is: -0.440269, -0.185407, 0.104461

h in I dotted with the eccentricity vector = 5.55112e-17

p = 1.91446 km

a = 2.51612 km

true anomaly = 0.678355 rad

Problem 15:

Function:

```
function [e, ap, vp] = radiusSpeedTrueAnomaly(r,v,nu,rb,mu)
    %This function takes radius, speed, and true anomaly as inputs and
calculates periapsis altitude, periapsis speed, and eccentricity
    %Function call: [e, ap, vp] = radiusSpeedTrueAnomaly(r,v,nu,mu)
    %
    %Input: r, radius
    %Input: v, speed
    %Input: rb, radius of body
    %Input: nu, true anomaly
    %
    %Output: e, eccentricity
    %Output: ap, periapsis altitude
    %Output: vp, periapsis speed

    a = ((((v/sqrt(mu))^2)-(2/r))^-1)*-1;

    eccen = @(e) a*e^2+r*e*cos(nu)-a+r;
    e = fzero(eccen,1);
    rp = a*(1-e);
    ap = rp-rb;
    vp = sqrt(mu)*sqrt((2/rp)-(1/a));
end
```

Main:

```
clc;clear;
r = 403000;
trueAnomaly = 151;
nu = deg2rad(trueAnomaly);
v = 2.25;
mu = 398600;
rb=6378.145;
[e, ap, vp] = radiusSpeedTrueAnomaly(r,v,nu,rb,mu);
fprintf('e = %g\n',e);
fprintf('altitude at periapsis = %g km\n',ap);
fprintf('speed at periapsis = %g km/s\n',vp);
```

e = 1.08131

altitude at periapsis = 4129.52 km

speed at periapsis = 8.88554 km/s

Problem 16:

(a). True

(b). False

(c). True

(d). False

(e). False