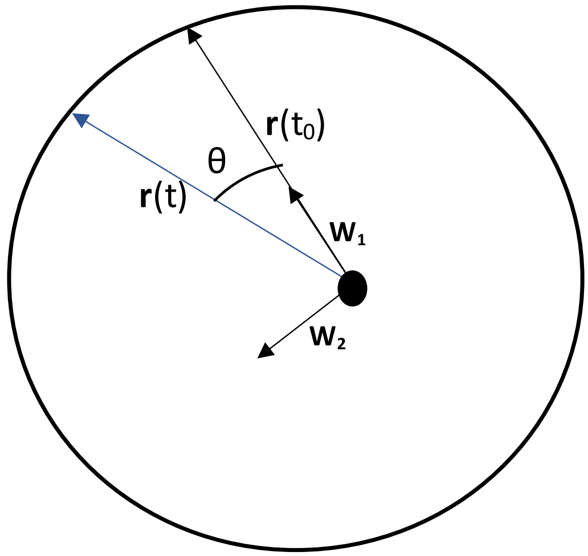
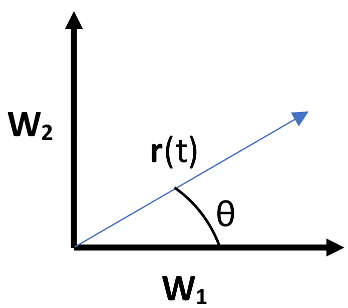
a)





Using the images above, **r**(t) in terms of the basis {**w₁, w₂, w₂**} can be expressed using vector decomposition as:

$$\boldsymbol{r}(t) = r\cos\theta\,\boldsymbol{w}_1 + r\sin\theta\,\boldsymbol{w}_2$$

b) The inertial velocity can be found by taking the derivative in I of **r**(t).

$$^{I}\boldsymbol{v}(t) = \frac{^{I}d}{dt}\big(\boldsymbol{r}(t)\big) = \frac{^{I}d}{dt}(r\cos\theta\,\boldsymbol{w}_1 + r\sin\theta\,\boldsymbol{w}_2)$$

Computing the derivative, using the product and chain rule, results in:

$$^{I}\boldsymbol{v}(t) = \big(r\dot{\theta}\sin\theta\big)\boldsymbol{w}_1 + \big(r\dot{\theta}\cos\theta\big)\boldsymbol{w}_2$$

c) Since the orbit is circular, the semi-major axis and radius are equal, speed of the orbit can be determined using the vis-viva equation:

$$v = \sqrt{\mu}\sqrt{\frac{2}{r}-\frac{1}{a}}$$

Substituting a in for r and simplifying:

$$v = \sqrt{\mu}\sqrt{\frac{2}{a}-\frac{1}{a}}$$

$$v = \sqrt{\mu}\sqrt{\frac{1}{a}} = \sqrt{\frac{\mu}{a}}$$

We can also find speed in a different fashion. We can take the derivative of the position vector and apply the transport theorem. Starting with the position vector:

$$\boldsymbol{r} = a\, \boldsymbol{e}_r$$

Where $\boldsymbol{e}_r$ is part of an orthonormal basis in which $\boldsymbol{e}_r$ is always pointing towards the spacecraft, $\boldsymbol{e}_z$ is along ${}^I\boldsymbol{h}$, and $\boldsymbol{e}_\theta$ is equal to $\boldsymbol{e}_z \times \boldsymbol{e}_r$. This basis rotates with the spacecraft with an angular velocity of $\dot{\theta}(t)$.

Taking the derivative in I of **r:**

$$\frac{{}^I d\boldsymbol{r}}{dt} = \frac{{}^I d}{dt}(a\, \boldsymbol{e}_r)$$

Since $\boldsymbol{e}_r$ isn't expressed in I, the transport theorem must be applied.

$$\frac{{}^I d\boldsymbol{r}}{dt} = {}^I\boldsymbol{v} = \frac{{}^I d}{dt}(a\, \boldsymbol{e}_r) = \frac{{}^I d}{dt}(a\, \boldsymbol{e}_r) + \left(\dot{\theta}(t)\boldsymbol{e}_z \times a\, \boldsymbol{e}_r\right)$$

$${}^I\boldsymbol{v} = a\dot{\theta}(t)\, \boldsymbol{e}_\theta$$

Taking the magnitude of this velocity vector to find speed:

$$v = a\, \dot{\theta}(t)$$

Equating the two velocity equations:

$$a\, \dot{\theta}(t) = \sqrt{\frac{\mu}{a}}$$

Solving for $\dot{\theta}(t)$:

$$\dot{\theta}(t) = \frac{1}{a}\sqrt{\frac{\mu}{a}}$$

$$\dot{\theta}(t) = \frac{1}{\sqrt{a^2}}\sqrt{\frac{\mu}{a}}$$

$$\dot{\theta}(t) = \sqrt{\frac{\mu}{a^3}}$$

a and μ are constants in all orbits which means $\dot{\theta}(t)$ is also constant.

Since $\dot{\theta}$ has the units of radians per second, θ can be found by multiplying by time elapsed:

$$time\ elapsed = (t - t_0)$$

$$\theta(t) = \sqrt{\frac{\mu}{a^3}}(t - t_0)$$

d)

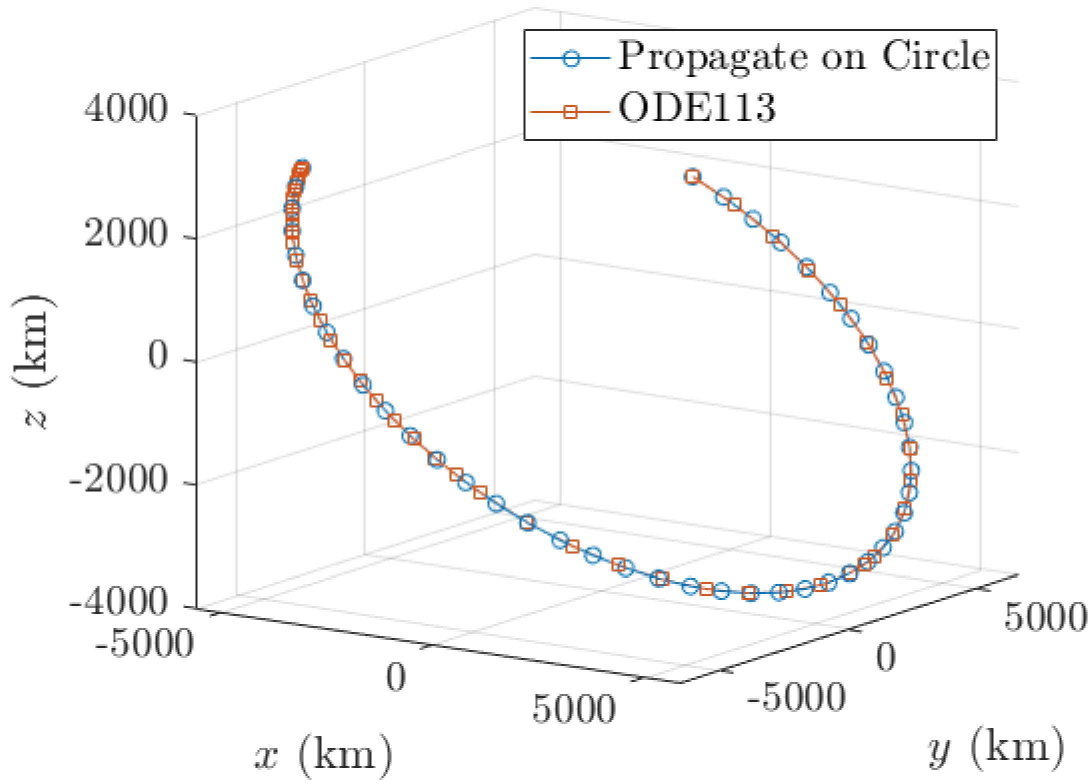
```
function [times, positions, velocities] =
propagateOnCircle(pos,vel,t0,tf,mu,N)
%This function calculates position and veolocity N times evenly spaced betwen
t0 and tf
%
%Function call: [times, postions, velocities] =
propagateOnCircle(pos,vel,t0,tf,mu,N);
%
%Input: pos, a column vector with the initial ECI postion
%Input: vel, a column vector with the initial ECI inertial velocity
%Input: t0, the initial time. Time from apoapsis to initial postion
%Input: tf, the final time
%Input: mu, gravitational parameter
%Input: N, number of time intervals
%
%Output: times, column vector of times, starting at t0 and ending at tf.
Length of N with even spacing
%Output: postions, matrix of size Nx3 containing ECI positions stored row-
wise at every time in the times vector
%Output: velocities, matrix of size Nx3 containing ECI velocities stored row-
wise at every time in the times vector
%

times=linspace(t0,tf,N);
times=times';

oe = rv2oe_Hackbardt_Chris(pos,vel,mu);
nui=oe(6);
a=oe(1);
thetaDot=sqrt(mu/a^3);

positions=zeros(N,3);
velocities=zeros(N,3);

positions(1,:)=pos';
velocities(1,:)=vel';

for i=2:N
    theta=thetaDot*(times(i)-times(i-1));
    nuOfT=nui+theta;
    oe(6)=nuOfT;
    [rPCI,vPCI]  = oe2rv_Hackbardt_Chris(oe,mu);
    positions(i,:)=rPCI';
    velocities(i,:)=vPCI';
    nui=nuOfT;
end
end
```

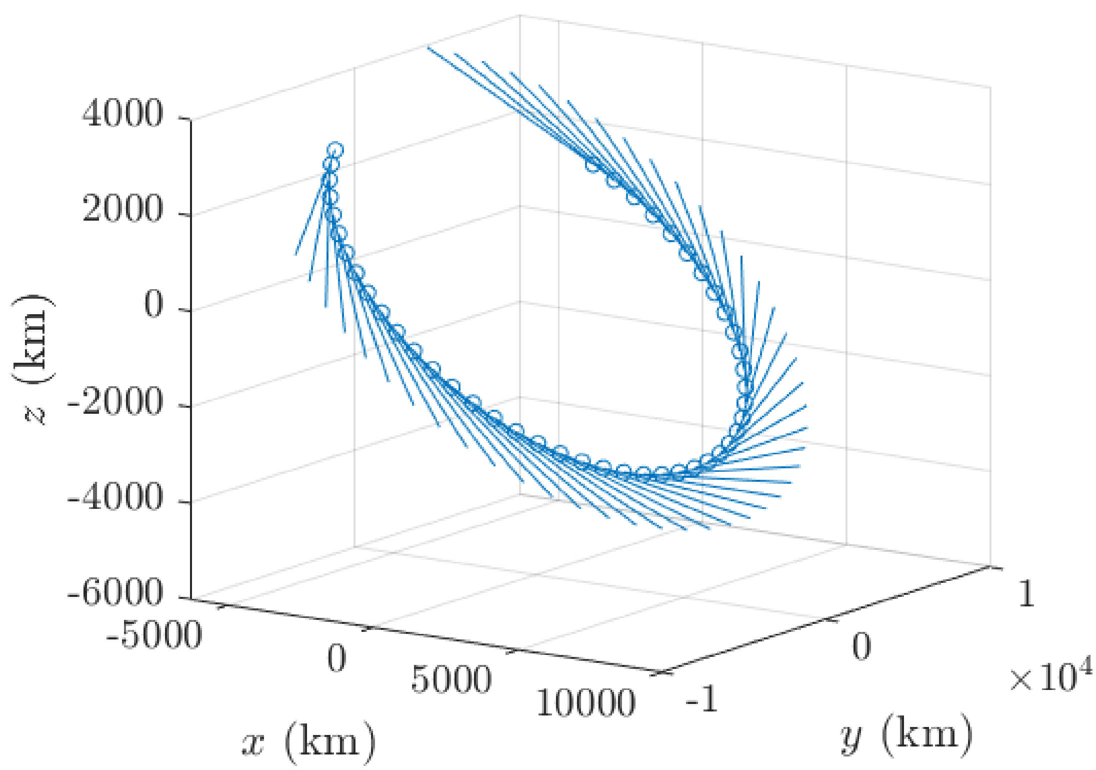
e)

Time	X Position	Y Position	Z Position	X Velocity	Y Velocity	Z Velocity
1992	-5614	-2446.4	2600.5	2.1276	-7.1342	-2.1184
2083.4	-5388.3	-3083.2	2392.6	2.8086	-6.792	-2.4274
2174.7	-5101.7	-3685.2	2157.8	3.4578	-6.3731	-2.709
2266.1	-4757.6	-4245.6	1898.5	4.0679	-5.8823	-2.9601
2357.5	-4359.8	-4758	1617.9	4.6322	-5.3251	-3.1777
2448.8	-3912.8	-5216.8	1319	5.1442	-4.7077	-3.3595
2540.2	-3421.6	-5616.6	1005.2	5.5981	-4.0373	-3.5033
2631.5	-2891.8	-5953.1	680.04	5.9888	-3.3212	-3.6076
2722.9	-2329.3	-6222.4	347.22	6.312	-2.5677	-3.6712
2814.3	-1740.6	-6421.4	10.479	6.5638	-1.7852	-3.6933
2905.6	-1132.2	-6548	-326.38	6.7416	-0.98257	-3.6738
2997	-511.03	-6600.6	-659.55	6.8433	-0.16883	-3.6127
3088.4	115.91	-6578.8	-985.28	6.8678	0.64682	-3.5109
3179.7	741.54	-6482.6	-1299.9	6.8147	1.4552	-3.3695
3271.1	1358.8	-6313.4	-1599.8	6.6847	2.2471	-3.1901
3362.5	1960.7	-6072.8	-1881.7	6.4793	3.0137	-2.9746
3453.8	2540.5	-5763.7	-2142.3	6.2007	3.7462	-2.7256
3545.2	3091.6	-5389.6	-2378.8	5.8522	4.4365	-2.4457
3636.5	3607.8	-4954.6	-2588.4	5.4375	5.0766	-2.1383
3727.9	4083.3	-4463.7	-2768.8	4.9615	5.6595	-1.8068
3819.3	4512.7	-3922.4	-2917.9	4.4295	6.1785	-1.4548
3910.6	4891.2	-3336.8	-3034.1	3.8475	6.6278	-1.0865
4002	5214.5	-2713.6	-3116.1	3.2221	7.0022	-0.70585
4093.4	5478.9	-2059.7	-3162.9	2.5603	7.2976	-0.31726
4184.7	5681.4	-1382.6	-3174	1.8696	7.5107	0.074919
4276.1	5819.9	-689.9	-3149.2	1.1578	7.6389	0.46625

4367.5	5892.6	10.6	-3088.9	0.43297	7.681	0.85232
4458.8	5898.8	710.98	-2993.8	-0.29678	7.6363	1.2288
4550.2	5838.5	1403.3	-2864.8	-1.0232	7.5055	1.5913
4641.5	5712.2	2079.9	-2703.5	-1.738	7.2899	1.936
4732.9	5521.5	2732.9	-2511.7	-2.4333	6.9921	2.2587
4824.3	5268.4	3355.1	-2291.6	-3.101	6.6153	2.556
4915.6	4955.9	3939.4	-2045.6	-3.7338	6.1638	2.8244
5007	4587.4	4479.3	-1776.4	-4.3244	5.6428	3.0609
5098.4	4167.2	4968.6	-1487.3	-4.8662	5.0581	3.2629
5189.7	3699.9	5401.8	-1181.3	-5.3531	4.4163	3.4281
5281.1	3190.9	5774	-862.06	-5.7796	3.7246	3.5546
5372.5	2645.8	6081.1	-533.05	-6.1408	2.991	3.6409
5463.8	2070.9	6319.5	-198.03	-6.4327	2.2235	3.6861
5555.2	1472.6	6486.6	139.24	-6.652	1.4309	3.6898
5646.5	857.68	6580.5	474.93	-6.7962	0.62224	3.6518
5737.9	233.08	6600.1	805.26	-6.8637	-0.19348	3.5725
5829.3	-394.15	6545.2	1126.5	-6.8538	-1.007	3.453
5920.6	-1016.9	6416.5	1435	-6.7665	-1.8092	3.2945
6012	-1628.2	6215.3	1727.4	-6.6027	-2.591	3.0987

f-g)


Propagate on Circle
ODE113
4000
2000
0
-2000
-4000
z (km)
-5000
0
5000
x (km)
5000
0
-5000
y (km)


4000
2000
0
-2000
-4000
-6000
z (km)
-5000
0
5000
10000
x (km)
1
0
-1
$\times 10^4$
y (km)