Question 1:

function oe = rv2oe\_Hackbardt\_Chris(rPCI,vPCI,mu)

% This function calculates the six orbital elements from the position vector and velocity vector

% Function Call: oe = rv2oe\_Hackbardt\_Chris(rPCI,vPCI,mu)

%

% Inputs:

% rPCI: Cartesian planet-centered inertial (PCI) position (3 by 1)

% vPCI: Cartesian planet-centered inertial (PCI) velocity (3 by 1)

% mu: gravitational parameter of centrally attacting body

% Outputs: orbital elements

% oe(1): semi-major axis

% oe(2): eccentricity

% oe(3): longitude of the ascending node (rad)

% oe(4): inclination (rad)

% oe(5): argument of the periapsis (rad)

% oe(6): true anomaly (rad)

rvec = rPCI;

vvec = vPCI;

Ix = [1;0;0];

Iy = [0;1;0];

Iz = [0;0;1];

hvec = cross(rvec,vvec);

h = norm(hvec);

p = h^2/mu;

r = norm(rvec);

evec = ((cross(vvec,hvec))/mu)-(rvec/r);

e = norm(evec);

a = p/(1-e^2);

nvec = cross(Iz,hvec);

n = norm(nvec);

Omega = atan2(dot(nvec,Iy),dot(nvec,Ix));

if Omega < 0

Omega = Omega + (2\*pi);

end

inc = atan2(dot(hvec,cross(nvec,Iz)),n\*dot(hvec,Iz));

omega = atan2(dot(evec,cross(hvec,nvec)),h\*dot(evec,nvec));

if omega < 0

omega = omega + (2\*pi);

end

nu = atan2(dot(rvec,cross(hvec,evec)),h\*dot(rvec,evec));

if nu < 0

nu = nu + (2\*pi);

end

oe = [a; e; Omega; inc; omega; nu];

end

Question 2:

function [rPCI,vPCI] = oe2rv\_Hackbardt\_Chris(oe,mu)

% This function calculates the postion and velocity vector with respect to the planet using orbital elements.

% Function Call: [rPCI,vPCI] = oe2rv\_Hackbardt\_Chris(oe,mu)

% Input: Orbital Elements: (6 by 1 column vector)

% oe(1): Semi-major axis

% oe(2): Eccentricity

% oe(3): Longitude of the ascending node (rad)

% oe(4): Inclination (rad)

% oe(5): Argument of the periapsis (rad)

% oe(6): True anomaly (rad)

% mu: Planet gravitational parameter (scalar)

% Outputs:

% rPCI: Planet-Centered Inertial (PCI) Cartesian position

% (3 by 1 column vector)

% vPCI: Planet-Centered Inertial (PCI) Cartesian inertial velocity

% (3 by 1 column vector)

a = oe(1);

e = oe(2);

Omega = oe(3);

i = oe(4);

omega = oe(5);

nu = oe(6);

p = a\*(1-e^2);

r = p/(1+(e\*cos(nu)));

rvecP = [r\*cos(nu);r\*sin(nu);0];

vvecP = (sqrt(mu/p))\*[-sin(nu);e+cos(nu);0];

T\_NI = [cos(Omega),-sin(Omega),0;sin(Omega),cos(Omega),0;0,0,1];

T\_QN = [1,0,0;0,cos(i),-sin(i);0,sin(i),cos(i)];

T\_PQ = [cos(omega),-sin(omega),0;sin(omega),cos(omega),0;0,0,1];

T\_PI = T\_NI \* T\_QN \* T\_PQ;

rPCI = T\_PI \* rvecP;

vPCI = T\_PI \* vvecP;

end

Question 3:

Since angles between vectors are undefined when one or more of those vectors have a magnitude of zero.

1. Since the inclination of the orbit, i, is the angle from Iz to , i is undefined when is zero, which happens when the orbit is rectilinear.
2. The longitude of the ascending node, Ω, is the angle between Ix and n, Ω is undefined when n is zero. n is zero when inclination is zero, also known as an equatorial orbit.
3. The argument of the periapsis, ω, is the angle from n to e. This means that ω is undefined when n or e is equal to zero. This happens when the orbit is equatorial or circular.
4. True anomaly, ν, is the angle between e and r. ν is therefore undefined when e is equal to zero, which is a circular orbit.

Question 4:

Finding the rate of change of specific angular momentum:

Taking the derivative and applying the chain rule:

and

Substituting these values in and evaluating:

Finding the rate of change of the eccentricity vector:

Rearranging constants:

Taking the derivative using the chain rule:

Using previous substitutions:

Applying the anticommutative property of the cross product:

Knowing that perpendicular vectors dotted is equal to zero and a vector dotted with itself is equal to the magnitude squared:

Finding the rate of change of the line of nodes:

Taking the derivative and applying the chain rule:

Since is fixed in I, the rate of change is zero, and the rate of change of has already been proven to be zero:

Question 5-6:

clc;clear;

rvec = [0.7;0.6;0.3];

vvec = [-0.8;0.8;0];

mu = 1;

oe = rv2oe\_Hackbardt\_Chris(rvec,vvec,mu);

a = oe(1);

e = oe(2);

Omega = oe(3);

i = oe(4);

omega = oe(5);

nu = oe(6);

tau = (2\*pi)\*sqrt(a^3/mu);

hvec = cross(rvec,vvec);

h = norm(hvec);

p = h^2/mu;

energy = -mu/(2\*a);

calculatedOE = [a; e; Omega; i; omega; nu];

[calculatedRvec,calculatedVvec] = oe2rv\_Hackbardt\_Chris(calculatedOE,mu);

fprintf('Semi-Major Axis (a) [AU]: \t\t\t\t\t\t %16.8f\n',a);

fprintf('Eccentricity (e) [AU]: \t\t\t\t\t\t\t %16.8f\n',e);

fprintf('Longitude of the Ascending Node (Omega) [rad]: %16.8f\n',Omega);

fprintf('Orbital Inclination (i) [rad]: \t\t\t\t\t %16.8f\n',i);

fprintf('Argument of the Periapsis (omega) [rad]: \t\t %16.8f\n',omega);

fprintf('True Anomaly (nu) [rad]: \t\t\t\t\t\t %16.8f\n\n',nu);

fprintf(2,'Orbital Period []: \t\t\t\t\t\t\t\t %16.8f\n\n',tau);

fprintf('Semi-Latus Rectum (p) [AU]: \t\t\t\t\t %16.8f\n\n',p);

fprintf('X Component Angular Momentum Vector [kg m^2/s]: %16.8f\n',hvec(1));

fprintf('Y Component Angular Momentum Vector [kg m^2/s]: %16.8f\n',hvec(2));

fprintf('Z Component Angular Momentum Vector [kg m^2/s]: %16.8f\n\n',hvec(3));

fprintf('Magnitude of Angular Momentum [kg m^2/s]: \t\t %16.8f\n\n',h);

fprintf('Orbital Energy [kg km^2 s^-2]: \t\t\t\t %16.8f\n\n',energy);

fprintf('Using the calculated orbital elements to recalculate the position and velocity vectors:\n');

fprintf('X Component Position [AU]: %16.8f\n',calculatedRvec(1));

fprintf('Y Component Position [AU]: %16.8f\n',calculatedRvec(2));

fprintf('Z Component Position [AU]: %16.8f\n\n',calculatedRvec(3));

fprintf('X Component Velocity [AU/TU]: %16.8f\n',calculatedVvec(1));

fprintf('Y Component Velocity [AU/TU]: %16.8f\n',calculatedVvec(2));

fprintf('Z Component Velocity [AU/TU]: %16.8f\n\n',calculatedVvec(3));

Semi-Major Axis (a) [AU]: 1.27739617

Eccentricity (e) : 0.25118540

Longitude of the Ascending Node (Omega) [rad]: 5.49778714

Orbital Inclination (i) [rad]: 0.31545875

Argument of the Periapsis (omega) [rad]: 1.86539229

True Anomaly (nu) [rad]: 5.91559204

Orbital Period [hours]: 9.07127393

Semi-Latus Rectum (p) [AU]: 1.19680000

X Component Angular Momentum Vector [kg m^2/s]: -0.24000000

Y Component Angular Momentum Vector [kg m^2/s]: -0.24000000

Z Component Angular Momentum Vector [kg m^2/s]: 1.04000000

Magnitude of Angular Momentum [kg m^2/s]: 1.09398355

Orbital Energy [kg km^2 s^-2]: -0.39142125

Using the calculated orbital elements to recalculate the position and velocity vectors:

X Component Position [AU]: 0.70000000

Y Component Position [AU]: 0.60000000

Z Component Position [AU]: 0.30000000

X Component Velocity [AU/TU]: -0.80000000

Y Component Velocity [AU/TU]: 0.80000000

Z Component Velocity [AU/TU]: 0.00000000

Question 7:

clc;clear;

a = 15307.548;

e = 0.7;

Omega = 194;

i = 39;

omega = 85;

nu = 48;

mu = 398600;

Omega = deg2rad(Omega);

i = deg2rad(i);

omega = deg2rad(omega);

nu = deg2rad(nu);

oe = [a; e; Omega; i; omega; nu];

[rvec,vvec] = oe2rv\_Hackbardt\_Chris(oe,mu);

fprintf('X Component Position [km]: %16.8f\n',rvec(1));

fprintf('Y Component Position [km]: %16.8f\n',rvec(2));

fprintf('Z Component Position [km]: %16.8f\n\n',rvec(3));

fprintf('X Component Velocity [km/s]: %16.8f\n',vvec(1));

fprintf('Y Component Velocity [km/s]: %16.8f\n',vvec(2));

fprintf('Z Component Velocity [km/s]: %16.8f\n\n',vvec(3));

X Component Position [km]: 4249.24395473

Y Component Position [km]: -2054.84062287

Z Component Position [km]: 2446.99585787

X Component Velocity [km/s]: 9.07117614

Y Component Velocity [km/s]: 5.81566502

Z Component Velocity [km/s]: -2.79245828

Question 8:

clc;clear;

a = 19133.333;

e = 0.5;

Omega = 30;

i = 45;

omega = 45;

nu = 0;

mu = 398600;

Omega = deg2rad(Omega);

i = deg2rad(i);

omega = deg2rad(omega);

nu = deg2rad(nu);

oe = [a; e; Omega; i; omega; nu];

[rvec,vvec] = oe2rv\_Hackbardt\_Chris(oe,mu);

fprintf('X Component Position [km]: %16.8f\n',rvec(1));

fprintf('Y Component Position [km]: %16.8f\n',rvec(2));

fprintf('Z Component Position [km]: %16.8f\n\n',rvec(3));

fprintf('X Component Velocity [km/s]: %16.8f\n',vvec(1));

fprintf('Y Component Velocity [km/s]: %16.8f\n',vvec(2));

fprintf('Z Component Velocity [km/s]: %16.8f\n\n',vvec(3));

X Component Position [km]: 3466.69624109

Y Component Position [km]: 7524.81548702

Z Component Position [km]: 4783.33325000

X Component Velocity [km/s]: -6.81755776

Y Component Velocity [km/s]: 0.62817226

Z Component Velocity [km/s]: 3.95279202

Question 9:

clc;clear;

a = 20000;

e = 0.45;

Omega = 59;

i = 27;

omega = 94;

nu = 58;

mu = 398600;

Omega = deg2rad(Omega);

i = deg2rad(i);

omega = deg2rad(omega);

nu = deg2rad(nu);

oe = [a; e; Omega; i; omega; nu];

[rvec,vvec] = oe2rv\_Hackbardt\_Chris(oe,mu);

fprintf('X Component Position [km]: %16.8f\n',rvec(1));

fprintf('Y Component Position [km]: %16.8f\n',rvec(2));

fprintf('Z Component Position [km]: %16.8f\n\n',rvec(3));

fprintf('X Component Velocity [km/s]: %16.8f\n',vvec(1));

fprintf('Y Component Velocity [km/s]: %16.8f\n',vvec(2));

fprintf('Z Component Velocity [km/s]: %16.8f\n\n',vvec(3));

X Component Position [km]: -10474.46193267

Y Component Position [km]: -6972.51466993

Z Component Position [km]: 2744.94389648

X Component Velocity [km/s]: 1.12638675

Y Component Velocity [km/s]: -6.03283073

Z Component Velocity [km/s]: -2.07511343

Question 10:

clc;clear;

a = 1.6;

e = 0.4;

Omega = 287;

i = 46;

omega = 28;

nu = 139;

mu = 1;

Omega = deg2rad(Omega);

i = deg2rad(i);

omega = deg2rad(omega);

nu = deg2rad(nu);

oe = [a; e; Omega; i; omega; nu];

[rvec,vvec] = oe2rv\_Hackbardt\_Chris(oe,mu);

fprintf('X Component Position: %16.8f\n',rvec(1));

fprintf('Y Component Position: %16.8f\n',rvec(2));

fprintf('Z Component Position: %16.8f\n\n',rvec(3));

fprintf('X Component Velocity: %16.8f\n',vvec(1));

fprintf('Y Component Velocity: %16.8f\n',vvec(2));

fprintf('Z Component Velocity: %16.8f\n\n',vvec(3));

X Component Position: -0.26075057

Y Component Position: 1.88182968

Z Component Position: 0.31152557

X Component Velocity: -0.46004409

Y Component Velocity: 0.23163947

Z Component Velocity: -0.38544253