Problem 1:

Part a:

9.739e+07	2.8217e+07	-1.8626e-09	2.5466e-11	-4.0359e-12	-1.3642e-12
2.8217e+07	9.739e+07	-1.8626e-09	-4.0359e-12	-1.5461e-11	-1.3642e-12
-1.8626e-09	-1.8626e-09	3.4586e+07	-1.3642e-12	-1.3642e-12	-3.8654e-12
2.5466e-11	-4.0359e-12	-1.3642e-12	34.273	5.0843	2.0935
-4.0359e-12	-1.5461e-11	-1.3642e-12	5.0843	9.1513	2.0935
-1.3642e-12	-1.3642e-12	-3.8654e-12	2.0935	2.0935	6.443

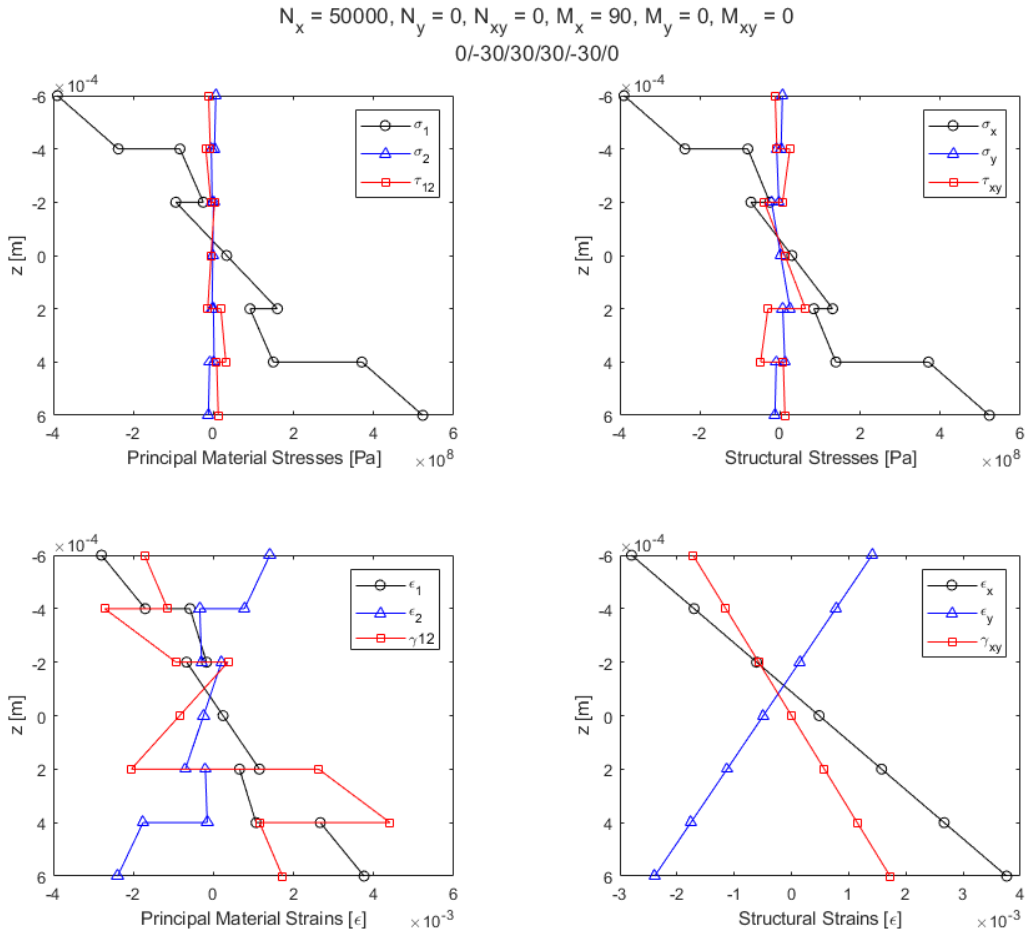
Part b:

1.1209e-08	-3.2476e-09	4.2876e-25	-9.509e-21	3.9398e-21	3.4953e-21
-3.2476e-09	1.1209e-08	4.2876e-25	1.3724e-21	1.7781e-20	-4.5377e-21
4.2876e-25	4.2876e-25	2.8913e-08	4.8029e-23	3.447e-22	1.7218e-20
-9.509e-21	1.3724e-21	4.8029e-23	0.031946	-0.016608	-0.0049835
3.9398e-21	1.7781e-20	3.447e-22	-0.016608	0.12668	-0.035766
3.4953e-21	-4.5377e-21	1.7218e-20	-0.0049835	-0.035766	0.16845

Part c:

Ex = 5.57591e+10, Ey = 5.57591e+10, Gxy = 2.16165e+10, vxy = 0.289736, etax,xy = 3.82512e-17

Problem 2:



Problem 3:

Maximum Stress Theory FoS = 1.0304 Lamina Does Not Fail

Maximum Strain Theory FoS = 1.06227 Lamina Does Not Fail

Tsai-Hill FoS = 0.994632 Lamina Fails

Tsai-Wu FoS = 0.997101 Lamina Fails

Problem 4:

Part a:

Nx = 60400, Ny = 2400, Nxy = 0, Mx = 12.08, My = 0.48, Mxy = 0

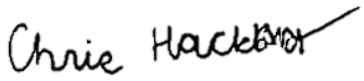
Part b:

ε_{x0} = 0.00100021

Problem 5:

Lamina longitudinal tensile failure strength, SL+ = 1680 MPa

Lamina longitudinal tensile strain to failure is equal to tensile fiber strain to failure, ef = 0.015



Code:

Main:

```
%% Exam 2
clc;clear;close all;

%% Problem 1

laminate = [140e9,10e9,7e9,0.3,0.0002,0,0,0,0,0,0;...
            140e9,10e9,7e9,0.3,0.0002,45,0,0,0,0,0;...
            140e9,10e9,7e9,0.3,0.0002,-45,0,0,0,0,0;...
            140e9,10e9,7e9,0.3,0.0002,90,0,0,0,0,0;...
            140e9,10e9,7e9,0.3,0.0002,90,0,0,0,0,0;...
            140e9,10e9,7e9,0.3,0.0002,-45,0,0,0,0,0;...
            140e9,10e9,7e9,0.3,0.0002,45,0,0,0,0,0;...
            140e9,10e9,7e9,0.3,0.0002,0,0,0,0,0,0];

ABDmatrix = laminateStiffnessMatrix(laminate);
ABDmatrixINV = laminateStiffnessMatrixINV(laminate);
[Ex,Ey,~,~,Gxy,vxy,~,etax_xy,~]=laminateEngineeringConstants(laminate);

fprintf(2,'Problem 1: \n\n')
fprintf('Part a: \n')
disp(array2table(ABDmatrix));
fprintf('\n')
fprintf('Part b: \n')
disp(array2table(ABDmatrixINV));
fprintf('Part c: \n')
fprintf('Ex = %g, Ey = %g, Gxy = %g, vxy = %g, etax,xy =
%g\n\n',Ex,Ey,Gxy,vxy,etax_xy)

%% Problem 2
clear;
laminate = [140e9,10e9,7e9,0.3,0.0002,0,0,0,0,0,0;...
            140e9,10e9,7e9,0.3,0.0002,-30,0,0,0,0,0;...
            140e9,10e9,7e9,0.3,0.0002,30,0,0,0,0,0;...
            140e9,10e9,7e9,0.3,0.0002,30,0,0,0,0,0;...
            140e9,10e9,7e9,0.3,0.0002,-30,0,0,0,0,0;...
            140e9,10e9,7e9,0.3,0.0002,0,0,0,0,0,0];

Nx=50000;
Ny=0;
Nxy=0;
Mx=90;
My=0;
Mxy=0;

forces2StressStrainLaminate(laminate,Nx,Ny,Nxy,Mx,My,Mxy);

fprintf(2,'Problem 2: \n\n')

%% Problem 3
clear;

laminate = [140e9,10e9,7e9,0.3,0.0002,30,1448e6,1172e6,48.3e6,248e6,62.1e6];

Nx=15000;
Ny=7500;
Nxy=0;
```

```
Mx=0;
My=0;
Mxy=0;

FS1=maximumStressTheory(laminate,Nx,Ny,Nxy,Mx,My,Mxy);
FS2 = maximumStrainTheory(laminate,Nx,Ny,Nxy,Mx,My,Mxy);
FS3=tsaiHillFailure(laminate,Nx,Ny,Nxy,Mx,My,Mxy);
FS4=tsaiWuFailure(laminate,Nx,Ny,Nxy,Mx,My,Mxy);

fprintf(2,'\nProblem 3: \n\n')
fprintf('Maximum Stress Theory FoS = %g Lamina Does Not Fail\n',FS1)
fprintf('Maximum Strain Theory FoS = %g Lamina Does Not Fail\n',FS2)
fprintf('Tsai-Hill FoS = %g Lamina Fails\n',FS3)
fprintf('Tsai-Wu FoS = %g Lamina Fails\n',FS4)

%% Problem 4
clear;

laminate = [140e9,10e9,7e9,0.3,0.0002,0,0,0,0,0,0;...
            140e9,10e9,7e9,0.3,0.0002,90,0,0,0,0,0;...
            140e9,10e9,7e9,0.3,0.0002,90,0,0,0,0,0;...
            140e9,10e9,7e9,0.3,0.0002,0,0,0,0,0,0];

sigmax=[140.9e6,140.9e6,10.1e6,10.1e6,10.1e6,10.1e6,140.9e6,140.9e6];
sigmay=[3e6,3e6,3e6,3e6,3e6,3e6,3e6,3e6];
tauxy=[0,0,0,0,0,0,0,0];

[Nx,Ny,Nxy,Mx,My,Mxy] =
structuralStress2ForceResultants(laminate,sigmax,sigmay,tauxy);
[epsx0,~,~,~,~,~] = midPlaneStrainsCurvature(Nx,Ny,Nxy,Mx,My,Mxy,laminate);

fprintf(2,'\nProblem 4: \n\n')
fprintf('Part a: \n')
fprintf('Nx = %g, Ny = %g, Nxy = %g, Mx = %g, My = %g, Mxy =
%g\n',Nx,Ny,Nxy,Mx,My,Mxy)
fprintf('\n')
fprintf('Part b: \n')
fprintf('\x03b5x0 = %g\n',epsx0)

%% Problem 5
clear;

vf=0.7;
Sf=2400e6;
Sm=170e6;
ef=0.015;
em=0.035;

SL=Sf*vf;

fprintf(2,'\nProblem 5: \n\n')
fprintf('Lamina longitudinal tensile failure strength, SL+ = %g MPa\n',SL/10^6)
fprintf('Lamina longitudinal tensile strain to failure is equal to tensile fiber
strain to failure, ef = %g \n',ef)
```

Functions:

```
function [ABDmatrix,distances,Qbars,Sbars] =
laminateStiffnessMatrix(laminate)
    [E1s,E2s,G12s,v12s,thicknesses,thetas] = laminateReader(laminate);
    A=zeros(3);
    B=zeros(3);
    D=zeros(3);
    N=length(thicknesses);
    thickness=sum(thicknesses);
    distances=zeros(1,length(thicknesses)+1);
    distances(1)=-thickness/2;
    distances(end)=thickness/2;
    for i=2:N
        distances(i)=distances(i-1)+thicknesses(i-1);
    end
    Qbars{1}=[];
    Sbars{1}=[];
    for k=1:N
        Qbar =
transReducedStiffnessMatrix(E1s(k),E2s(k),G12s(k),v12s(k),thetas(k));
        Qbars{k}=Qbar;
    end

    for i=1:3
        for j=1:3
            for k=1:N
                A(i,j)=A(i,j)+(Qbars{k}(i,j)*(distances(k+1)-distances(k)));
                B(i,j)=B(i,j)+(Qbars{k}(i,j)*(1/2)*(distances(k+1)^2-
distances(k)^2));
                D(i,j)=D(i,j)+(Qbars{k}(i,j)*(1/3)*(distances(k+1)^3-
distances(k)^3));
            end
        end
    end
    ABDmatrix=[A,B;B,D];
end


function [ABDmatrixINV,distances,Qbars,Sbars] =
laminateStiffnessMatrixINV(laminate)
    [ABDmatrix,distances,Qbars,Sbars] = laminateStiffnessMatrix(laminate);
    ABDmatrixINV=ABDmatrix^-1;
end
```

```
function
[Ex,Ey,Efx,Efy,Gxy,vxy,vyx,etax_xy,etaxy_y]=laminateEngineeringConstants(lami
nate)
    [~,~,~,~,thicknesses,~] = laminateReader(laminate);
    t=sum(thicknesses);
    ABDmatrixINV = laminateStiffnessMatrixINV(laminate);
    Ex=1/(t*ABDmatrixINV(1,1));
    Ey=1/(t*ABDmatrixINV(2,2));
    Efx=12/(t^3*ABDmatrixINV(4,4));
    Efy=12/(t^3*ABDmatrixINV(5,5));
    Gxy=1/(t*ABDmatrixINV(3,3));
    vxy=-ABDmatrixINV(1,2)/ABDmatrixINV(1,1);%nu xy
    vyx=-ABDmatrixINV(1,2)/ABDmatrixINV(2,2);%nu yx
    etax_xy=ABDmatrixINV(1,3)/ABDmatrixINV(1,1);
    etaxy_y=ABDmatrixINV(2,3)/ABDmatrixINV(3,3);
end
```

```
function
[sigmax,sigmay,tauxy,sigma1,sigma2,tau12,epsx,epsy,gammaxy,eps1,eps2,gamma12]
=forces2StressStrainLaminate(laminate,Nx,Ny,Nxy,Mx,My,Mxy)
    [E1s,E2s,G12s,v12s,thicknesses,thetas] = laminateReader(laminate);
    [epsx0,epsy0,gammaxy0,kx,ky,kxy] =
midPlaneStrainsCurvature(Nx,Ny,Nxy,Mx,My,Mxy,laminate);
    [epsx,epsy,gammaxy] =
strainLaminateStructural(epsx0,epsy0,gammaxy0,kx,ky,kxy,thicknesses);
    [eps1,eps2,gamma12] = strainLaminatePrincipal(epsx,epsy,gammaxy,thetas);

[sigmax,sigmay,tauxy]=strain2stressStructural(epsx,epsy,gammaxy,thetas,E1s,E2
s,G12s,v12s);

[sigma1,sigma2,tau12]=stressLaminatePrincipal(sigmax,sigmay,tauxy,thetas);

    distances=laminateDistances(thicknesses);
    figure
    plots = tiledlayout(2,2);
    nexttile
    plot(sigma1,distances,'k-o',sigma2,distances,'b-^',tau12,distances,'r-
square')
    xlabel('Principal Material Stresses [Pa]')
    ylabel('z [m]')
    legend('\sigma_{1}','\sigma_{2}','\tau_{12}')
    set(gca, 'YDir','reverse')
    nexttile
    plot(sigmax,distances,'k-o',sigmay,distances,'b-^',tauxy,distances,'r-
square')
    xlabel('Structural Stresses [Pa]')
    ylabel('z [m]')
    legend('\sigma_{x}','\sigma_{y}','\tau_{xy}')
    set(gca, 'YDir','reverse')
    nexttile
    plot(eps1,distances,'k-o',eps2,distances,'b-^',gamma12,distances,'r-
square')
    xlabel('Principal Material Strains [\epsilon]')
    ylabel('z [m]')
    legend('\epsilon_{1}','\epsilon_{2}','\gamma{12}')
    set(gca, 'YDir','reverse')
    nexttile
    plot(epsx,distances,'k-o',epsy,distances,'b-^',gammaxy,distances,'r-
square')
    xlabel('Structural Strains [\epsilon]')
    ylabel('z [m]')
    legend('\epsilon_{x}','\epsilon_{y}','\gamma_{xy}')
    set(gca, 'YDir','reverse')
    set(gcf,'position',[90,90,1000,800])
       titleLine1=['N_{x} = ',num2str(Nx),', N_{y} = ',num2str(Ny),', N_{xy}
= ',num2str(Nxy),', M_{x} = ',num2str(Mx),', M_{y} = ',num2str(My),', M_{xy}
= ',num2str(Mxy)];
    titleLine2=num2str(thetas(1));
    for i=2:length(thetas)
        titleLine2=[titleLine2,'/',num2str(thetas(i))];
    end
    title(plots,{titleLine1,titleLine2})
end
```

```
function [FS]=maximumStressTheory(laminate,Nx,Ny,Nxy,Mx,My,Mxy)

[~,~,~,~,~,~,longStrengthTen,longStrengthCom,tranStrengthTen,tranStrengthCom,
strengthLT] = laminateReader(laminate);

[~,~,~,sigma1,sigma2,tau12,~,~,~,~,~,~]=forces2StressStrainLaminateNoPlot(lam
inate,Nx,Ny,Nxy,Mx,My,Mxy);
    FS=zeros(length(sigma1),1);
    FSsigma1=zeros(length(sigma1),1);
    FSsigma2=zeros(length(sigma1),1);
    FStau12=zeros(length(sigma1),1);
    for i=2:2:length(sigma1)

        if sigma1(i)>0
            FSsigma1(i)=longStrengthTen(i/2)/sigma1(i);
        else
            FSsigma1(i)=-longStrengthCom(i/2)/sigma1(i);
        end

        if sigma1(i-1)>0
            FSsigma1(i-1)=longStrengthTen(i/2)/sigma1(i-1);
        else
            FSsigma1(i-1)=-longStrengthTen(i/2)/sigma1(i-1);
        end

        if sigma2(i)>0
            FSsigma2(i)=tranStrengthTen(i/2)/sigma2(i);
        else
            FSsigma2(i)=-tranStrengthCom(i/2)/sigma2(i);
        end

        if sigma2(i-1)>0
            FSsigma2(i-1)=tranStrengthTen(i/2)/sigma2(i-1);
        else
            FSsigma2(i-1)=-tranStrengthTen(i/2)/sigma2(i-1);
        end

        FStau12(i)=strengthLT(i/2)/abs(tau12(i));
        FStau12(i-1)=strengthLT(i/2)/abs(tau12(i-1));

        FS(i)=min([FSsigma1(i),FSsigma2(i),FStau12(i)]);
        FS(i-1)=min([FSsigma1(i-1),FSsigma2(i-1),FStau12(i-1)]);
    end
    FS=min(FS);
end
```

```
function [FS]=maximumStrainTheory(laminate,Nx,Ny,Nxy,Mx,My,Mxy)

[E1s,E2s,~,v12s,~,~,longStrengthTen,longStrengthCom,tranStrengthTen,tranStren
gthCom,strengthLT] = laminateReader(laminate);

[~,~,~,sigma1,sigma2,tau12,~,~,~,~,~,~]=forces2StressStrainLaminateNoPlot(lam
inate,Nx,Ny,Nxy,Mx,My,Mxy);
    FS=zeros(length(longStrengthTen),1);
    FSeps1=zeros(length(longStrengthTen),1);
    FSeps2=zeros(length(longStrengthTen),1);
    FSgamma12=zeros(length(longStrengthTen),1);
    for i=2:2:length(sigma1)

        if sigma1(i)>0
            FSeps1(i)=longStrengthTen(i/2)/(sigma1(i)-v12s(i/2)*sigma2(i));
        else
            FSeps1(i)=-longStrengthCom(i/2)/(sigma1(i)-v12s(i/2)*sigma2(i));
        end

        if sigma1(i-1)>0
            FSeps1(i-1)=longStrengthTen(i/2)/(sigma1(i-1)-v12s(i/2)*sigma2(i-
1));
        else
            FSeps1(i-1)=-longStrengthCom(i/2)/(sigma1(i-1)-
v12s(i/2)*sigma2(i-1));
        end

        v21=(v12s(i/2)/E1s(i/2))*E2s(i/2);
        if sigma2(i)>0
            FSeps2(i)=tranStrengthTen(i/2)/(sigma2(i)-v21*sigma1(i));
        else
            FSeps2(i)=-tranStrengthCom(i/2)/(sigma2(i)-v21*sigma1(i));
        end

        if sigma2(i-1)>0
            FSeps2(i-1)=tranStrengthTen(i/2)/(sigma2(i-1)-v21*sigma1(i-1));
        else
            FSeps2(i-1)=-tranStrengthCom(i/2)/(sigma2(i-1)-v21*sigma1(i-1));
        end

        FSgamma12(i)=strengthLT(i/2)/abs(tau12(i));
        FSgamma12(i-1)=strengthLT(i/2)/abs(tau12(i-1));

        FS(i)=min([FSeps1(i),FSeps2(i),FSgamma12(i)]);
        FS(i-1)=min([FSeps1(i-1),FSeps2(i-1),FSgamma12(i-1)]);
    end
    FS=min(FS);
end
```

```
function [FS]=tsaiHillFailure(laminate,Nx,Ny,Nxy,Mx,My,Mxy)

[~,~,~,~,~,~,longStrengthTen,longStrengthCom,tranStrengthTen,tranStrengthCom,
strengthLT] = laminateReader(laminate);

[~,~,~,sigma1,sigma2,tau12,~,~,~,~,~,~]=forces2StressStrainLaminateNoPlot(lam
inate,Nx,Ny,Nxy,Mx,My,Mxy);
    FS=zeros(length(longStrengthTen),1);
    for i=2:2:length(sigma1)
        if sigma1(i)>0
            longStrength=longStrengthTen(i/2);
        else
            longStrength=longStrengthCom(i/2);
        end

        if sigma1(i-1)>0
            longStrength2=longStrengthTen(i/2);
        else
            longStrength2=longStrengthCom(i/2);
        end

        if sigma2(i)>0
            transStrength=tranStrengthTen(i/2);
        else
            transStrength=tranStrengthCom(i/2);
        end

        if sigma2(i-1)>0
            transStrength2=tranStrengthTen(i/2);
        else
            transStrength2=tranStrengthCom(i/2);
        end

        A=((sigma1(i)/longStrength)^2)+((sigma2(i)/transStrength)^2)-
((sigma1(i)*sigma2(i))/(longStrength)^2)+(tau12(i)/strengthLT)^2;
        FS(i)=1/sqrt(A);

        A2=(sigma1(i-1)/longStrength2)^2+(sigma2(i-1)/transStrength2)^2-
((sigma1(i-1)*sigma2(i-1))/(longStrength2)^2)+(tau12(i-1)/strengthLT)^2;
        FS(i-1)=1/sqrt(A2);
    end
    FS=min(FS);
end
```

```
function [FS]=tsaiWuFailure(laminate,Nx,Ny,Nxy,Mx,My,Mxy)

[~,~,~,~,~,~,longStrengthTen,longStrengthCom,tranStrengthTen,tranStrengthCom,
strengthLT] = laminateReader(laminate);

[~,~,~,sigma1,sigma2,tau12,~,~,~,~,~,~]=forces2StressStrainLaminateNoPlot(lam
inate,Nx,Ny,Nxy,Mx,My,Mxy);
    FS=zeros(length(longStrengthTen),1);

    F11=1/(longStrengthTen*longStrengthCom);
    F22=1/(tranStrengthTen*tranStrengthCom);
    F66=1/strengthLT^2;
    F12=-sqrt(F11*F22)/2;
    F1=(1/longStrengthTen)-(1/longStrengthCom);
    F2=(1/tranStrengthTen)-(1/tranStrengthCom);

    for i=1:length(sigma1)

C1=(F11*sigma1(i)^2+F22*sigma2(i)^2+F66*tau12(i)^2+2*F12*sigma1(i)*sigma2(i))
;
        C2=F1*sigma1(i)+F2*sigma2(i);
        C3=-1;
        C=[C1,C2,C3];
        sol=roots(C);
        FS(i)=sol(sol>=0);
    end
    FS=min(FS);
end



function [Nx,Ny,Nxy,Mx,My,Mxy] =
structuralStress2ForceResultants(laminate,sigmax,sigmay,tauxy)
    [~,~,~,~,thicknesses,~] = laminateReader(laminate);

    Nx=0;
    Ny=0;
    Nxy=0;
    Mx=0;
    My=0;
    Mxy=0;

    for i=2:2:length(sigmax)
        Nx=Nx+(((sigmax(i)+sigmax(i-1))/2)*thicknesses(i/2));
        Ny=Ny+(((sigmay(i)+sigmay(i-1))/2)*thicknesses(i/2));
        Nxy=Nxy+(((tauxy(i)+tauxy(i-1))/2)*thicknesses(i/2));
        Mx=Mx+(((sigmax(i)+sigmax(i-1))/2)*thicknesses(i/2)^2);
        My=My+(((sigmay(i)+sigmay(i-1))/2)*thicknesses(i/2)^2);
        Mxy=Mxy+(((tauxy(i)+tauxy(i-1))/2)*thicknesses(i/2)^2);
    end
end
```

```
function [epsx0,epsy0,gammaxy0,kx,ky,kxy] =
midPlaneStrainsCurvature(Nx,Ny,Nxy,Mx,My,Mxy,laminate)
    ABDmatrixINV = laminateStiffnessMatrixINV(laminate);
    epsk=ABDmatrixINV*[Nx;Ny;Nxy;Mx;My;Mxy];
    epsx0=epsk(1);
    epsy0=epsk(2);
    gammaxy0=epsk(3);
    kx=epsk(4);
    ky=epsk(5);
    kxy=epsk(6);
end
```