$$G = \frac{E}{2(1+\nu)} \qquad \nu_{21} = \frac{\nu_{12} E_2}{E_1}$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = [T_\sigma] \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \qquad \sigma = \frac{Mc}{I} = \frac{4M}{\pi r^3}$$

$$T_\sigma = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\cos\theta\sin\theta \\ \sin^2\theta & \cos^2\theta & -2\cos\theta\sin\theta \\ -\cos\theta\sin\theta & \cos\theta\sin\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$$

$$\{\sigma\} = \{C\}\{\epsilon\}$$

$$\{\epsilon\} = \{S\}\{\sigma\}$$

$$\{C\}^{-1} = \{S\}$$

$$\{S\}^{-1} = \{C\}$$

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix} = [T_\epsilon] \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$T_\epsilon = \begin{bmatrix} \cos^2\theta & \sin^2\theta & \cos\theta\sin\theta \\ \sin^2\theta & \cos^2\theta & -\cos\theta\sin\theta \\ -2\cos\theta\sin\theta & 2\cos\theta\sin\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$$

$$\bar{S}_{11} = S_{11} m^4 + (2S_{12} + S_{66}) n^2 m^2 + S_{22} n^4$$

$$\bar{S}_{12} = (S_{11} + S_{22} - S_{66}) n^2 m^2 + S_{12}(n^4 + m^4)$$

$$\bar{S}_{16} = (2S_{11} - 2S_{12} - S_{66}) n m^3 - (2S_{22} - 2S_{12} - S_{66}) n^3 m$$

$$\bar{S}_{22} = S_{11} n^4 + (2S_{12} - S_{66}) n^2 m^2 + S_{22} m^4$$

$$\bar{S}_{26} = (2S_{11} - 2S_{12} - S_{66}) n^3 m - (2S_{22} - 2S_{12} - S_{66}) n m^3$$

$$\bar{S}_{66} = 2(2S_{11} + 2S_{22} - 4S_{12} - S_{66}) n^2 m^2 + S_{66}(n^4 + m^4)$$

$$\sigma_i = C_{ij}\epsilon_j$$

$$\epsilon_i = S_{ij}\sigma_j$$

$$[\bar{Q}] = [T_\sigma]^{-1}[Q][T_\epsilon]$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ & \bar{Q}_{22} & \bar{Q}_{26} \\ & & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} \qquad E_x = \frac{1}{\bar{S}_{11}} \quad E_y = \frac{1}{\bar{S}_{22}} \quad G_{xy} = \frac{1}{\bar{S}_{66}}$$

iso

$$S_{11} = S_{22} = S_{33} = \frac{1}{E}$$

$$S_{12} = S_{13} = S_{23} = \frac{-\nu}{E}$$

$$S_{44} = S_{55} = S_{66} = \frac{1}{G}$$

$c = \cos\theta \qquad s = \sin\theta$

$$\bar{Q}_{11} = Q_{11}c^4 + Q_{22}s^4 + 2(Q_{12} + 2Q_{66})s^2c^2 \qquad \nu_{xy} = \frac{-\bar{S}_{12}}{\bar{S}_{11}}$$

$$\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})s^2c^2 + Q_{12}(c^4 + s^4)$$

$$\bar{Q}_{22} = Q_{11}s^4 + Q_{22}c^4 + 2(Q_{12} + 2Q_{66})s^2c^2 \qquad \eta_{xy,x} = \frac{\bar{S}_{16}}{\bar{S}_{66}}$$

$$\bar{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66})c^3s - (Q_{22} - Q_{12} - 2Q_{66})cs^3$$

$$\bar{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66})cs^3 - (Q_{22} - Q_{12} - 2Q_{66})c^3s$$

$$\bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})s^2c^2 + Q_{66}(s^4 + c^4)$$

Lam

$$S_{11} = \frac{1}{E_1} \qquad S_{22} = \frac{1}{E_2}$$

$$S_{21} = S_{12} = \frac{-\nu_{12}}{E_1} = \frac{-\nu_{21}}{E_2} \qquad S_{66} = \frac{1}{G_{12}}$$

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = [\bar{S}] \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \qquad [\bar{S}] = [\bar{Q}]^{-1}$$

$$Q_{11} = \frac{S_{22}}{S_{11}S_{22} - S_{12}^2} = \frac{E_1}{1 - \nu_{12}\nu_{21}}$$

$$Q_{12} = \frac{-S_{12}}{S_{11}S_{22} - S_{12}^2} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} = Q_{21}$$

$$Q_{22} = \frac{S_{11}}{S_{11}S_{22} - S_{12}^2} = \frac{E_2}{1 - \nu_{12}\nu_{21}}$$

$$Q_{66} = \frac{1}{S_{66}} = G_{12}$$

$m = \cos\theta \qquad n = \sin\theta$

$$E_x = \frac{E_1}{m^4 + \left(\frac{E_1}{G_{12}} - 2\nu_{12}\right)n^2m^2 + \frac{E_1}{E_2}n^4} \qquad E_y = \frac{E_2}{n^4 + \left(\frac{E_2}{G_{12}} - 2\nu_{21}\right)n^2m^2 + \frac{E_2}{E_1}m^4}$$

$$\nu_{xy} = \frac{\nu_{12}(n^4 + m^4) - \left(1 + \frac{E_1}{E_2} - \frac{E_1}{G_{12}}\right)n^2m^2}{m^4 + \left(\frac{E_1}{G_{12}} - 2\nu_{12}\right)n^2m^2 + \frac{E_1}{E_2}n^4}$$

$$\nu_{yx} = \frac{\nu_{21}(n^4 + m^4) - \left(1 + \frac{E_2}{E_1} - \frac{E_2}{G_{12}}\right)n^2m^2}{m^4 + \left(\frac{E_2}{G_{12}} - 2\nu_{21}\right)n^2m^2 + \frac{E_2}{E_1}n^4}$$

$$G_{xy} = \frac{G_{12}}{n^4 + m^4 + 2\left(2\frac{G_{12}}{E_1}(1 + 2\nu_{12}) + 2\frac{G_{12}}{E_2} - 1\right)n^2m^2}$$

$$E_1 = E_{f1}V_f + E_m(1 - V_f) \rightarrow E_2 = E_m\left[(1 - \sqrt{V_f}) + \frac{\sqrt{V_f}}{1 - \sqrt{V_f}(1 - E_m/E_{f2})}\right]$$

$$\frac{1}{E_2} = \frac{V_f}{E_{f2}} + \frac{(1 - V_f)}{E_m}$$

$$\frac{E_2}{E_m} = \frac{1 + \xi\eta V_f}{1 - \eta V_f} \qquad \eta = \frac{(E_f/E_m) - 1}{(E_f/E_m) + \xi} \qquad \xi = 2$$

$$\frac{1}{G_{12}} = \frac{V_f}{G_{f12}} + \frac{(1 - V_f)}{G_m}$$

$$\nu_{12} = \nu_{f12}V_f + \nu_m(1 - V_f)$$

Composites | Exam 1 | cum form

1. $E_1 = 200 \quad G_{12} = 5 \quad \nu_{12} = 0.3 \quad \nu_{21} = 0.02 \quad \nu_{23} = 0.2$

$$\nu_{21} = \frac{\nu_{12} E_2}{E_1}$$

$$0.02 = \frac{0.3 \cdot E_2}{200} \Rightarrow E_2 = 13.33 \text{ GPa} = E_3 \rightarrow \text{transversely isotropic}$$

$$G_{23} = \frac{E_2}{2(1+\nu_{23})} = \frac{13.33}{2(1+0.2)} = 5.55 \text{ GPa} = G_{13}$$

$$\boxed{E_2 = E_3 = 13.33 \text{ GPa}} \quad \boxed{G_{23} = G_{13} = 5.55 \text{ GPa}}$$

2.

$$\sigma_1 = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + \tau_{xy}\, 2 \cos\theta \sin\theta$$

$$\sigma_1 = -40 \cos^2 45 + 20 \sin^2 45 + 2(-25) \cos 45 \sin 45$$

$$\sigma_1 = -35 \text{ ksi} \qquad \varepsilon_1 = \frac{\sigma_1}{E_1} = \frac{-35000}{200 \times 10^6} = -1.75 \times 10^{-4}$$

$$\sigma_2 = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\tau_{xy} \cos\theta \sin\theta$$

$$\sigma_2 = -40 \sin^2 45 + 20 \cos^2 45 - 2(-25) \cos 45 \sin 45$$

$$\sigma_2 = 15 \text{ ksi} \qquad \varepsilon_2 = \frac{\sigma_2}{E_2} = \frac{15000}{10 \times 10^6} = 0.0015$$

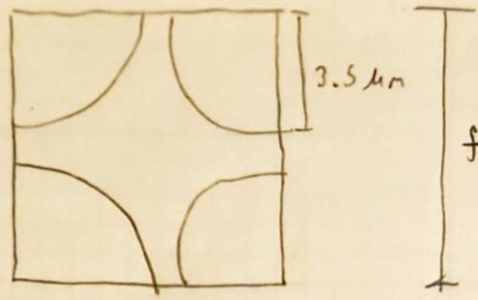
$$\tau_{12} = -\sigma_x \cos\theta \sin\theta + \sigma_y \cos\theta \sin\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$\tau_{12} = 40 \cos 45 \sin 45 + 20 \cos 45 \sin 45 - 25(\cos^2 45 - \sin^2 45)$$

$$\tau_{12} = 30 \text{ ksi} \qquad \gamma_{12} = \frac{\tau_{12}}{G_{12}} = \frac{30000}{5 \times 10^6} = 0.006$$

$$\boxed{\varepsilon_1 = -1.75 \times 10^{-4} \quad \varepsilon_2 = 0.0015 \quad \gamma_{12} = 0.006}$$

3



a

$$A_f = \pi(3.5)^2 \qquad A_M = f_s^2 - A_f$$

$$V_f = \frac{A_f}{f_s^2} = \frac{\pi(3.5^2)}{f_s^2} = 0.7$$

$$\frac{\pi(3.5)^2}{0.7} = f_s^2$$

Fiber Spacing: $\boxed{f_s = 7.41\ \mu m}$

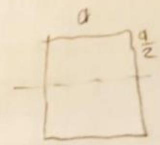
b. $$V_{f_{max}} = \frac{\pi(3.5)^2}{7^2} = \boxed{0.785}$$

c. When the fiber volume fraction is too high, fiber to fiber contact occurs which means the stresses will not distribute as well through the composite

4. Pultrusion is a cheap way to produce long, unidirectional parts. In this method, fibers are pulled through a resin bath and then through a mold to form the desired shape.

Chris [illegible]

5. $y = \frac{FL^3}{48EI}$ $\quad m = a^2 L \rho \quad$ $I = \frac{a^4}{4}$



$$a^2 = \frac{m}{L\rho}$$

$$y = \frac{FL^3}{12Ea^4} \qquad a^4 = \frac{m^2}{L^2\rho^2}$$

$$y = \frac{FL^5\rho^2}{12Em^2}$$

$$\frac{E}{\rho^2} = \frac{FL^5}{12ym^2}$$

$$\frac{E^{1/2}}{\rho} = \sqrt{\frac{FL^5}{12ym^2}}$$

The best material appears to be Unidirectional carbon fiber since wood isn't an option

b. $y = 0.01 \quad E = 200\,GPa \quad L = 1$

$$0.01 = \frac{50}{12(200\times10^9)\cdot a^4}$$

$$\boxed{a = 0.0068\,m}$$

c. $m = a^2 L \rho$

$m = 0.0068^2 \cdot 1.5$

$$\boxed{m = 6.84\times10^{-9}\,Mg = 68.5\,g}$$

6. a. Rule of mixtures

b. $E_1 = E_{f_1} V_f + E_m(1 - V_f)$

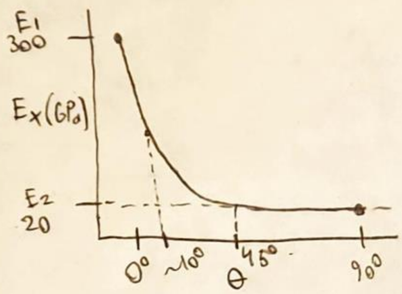
$E_1 = 32\times10^6 (0.65) + 0.32\times10^6(1-0.65) = \boxed{20.9\times10^6 \text{ psi}}$

c. $\xi = 2$

$$\eta = \frac{(2\times10^6/0.32\times10^6)-1}{(2\times10^6/0.32\times10^6)+2} = 0.63636$$

$$E_2 = 0.32\times10^6\left[\frac{1+2(0.6363)\,0.65}{1-0.6363\cdot 0.65}\right] = \boxed{0.997\times10^6 \text{ psi}}$$

7.



a. $E_x|_{\theta=45^\circ} \approx 20\,GPa$

b. Approximately 10°

c. $E_x = \frac{1}{S_{11}}$

8. $[0^\circ/(90^\circ/\mp45^\circ/0^\circ)_2]$

$[(90^\circ)_2/(-45^\circ)_2/\bar{0}^\circ]_s$

$[(90^\circ)_2/(\pm45^\circ)/\bar{0}^\circ]_s$

$[(0^\circ/60^\circ)_2/0^\circ]_s$

$[45^\circ/0/-45^\circ/90^\circ/0^\circ]_s$