Optimizing Leg Lengths of Quadruped Robots for Stair Climbing

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***Abstract*—Legged robots are becoming increasingly capable of traversing complex terrains, making them suitable for industrial inspection, search and rescue, and guide dog applications. However, small-scale robots often lack the ability to climb stairs reliably due to their shorter leg lengths, while large-scale robots are not portable for daily use. This paper presents the optimization of the link lengths of Vivo, a quadruped robot that balances portability and stair-climbing capability. Using kinematic analysis, we evaluated various leg configurations to achieve optimal stair surface coverage while minimizing the body height of Vivo. The results demonstrate that Vivo achieves over 30% stair coverage for both front and hind legs, meeting metrics for effective climbing. These findings highlight the potential of bio-inspired, dimensionally optimized designs for improving the versatility and accessibility of legged robots.**

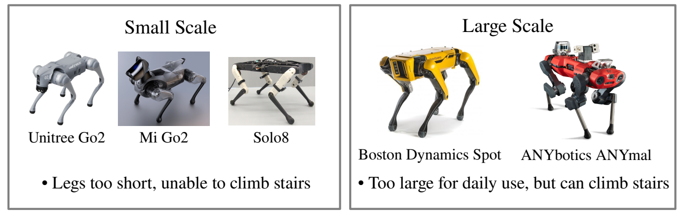

Fig. 1. Comparison of small scale and large scale quadruped robots. Small scale robots are small enough for daily use, but are not capable of reliably climbing standard stairs. Meanwhile, large scale robots excel at climbing stairs, but they are too big for daily use.

I. Introduction

LEGGED robots excel at traversing complex terrains compared to their wheeled counterparts, which makes them applicable for a wide variety of applications, such as industrial inspection and search and rescue [1] [2]. Recent advancements in the locomotion capability of quadruped robots have enabled their commercialization [3] [4]. An applicable use case of quadruped robots is guide dog robots for blind and low vision (BLV) individuals. Plenty of innovative robot dogs already exist on the market. For instance, the Unitree Go2 is great for dynamic applications such as jumping and flipping [4], while the Boston Dynamics spot has proven unprecedented capabilities, especially with stair climbing [3]. However, Spot's size makes it unsuitable as a guide dog, as guide dog owners typically prefer a more portable option and the Unitree Go 2 has relatively shorter legs making it unable to reliably climb stairs of standard height. In response, we are currently developing Vivo, a guide dog robot that is both portable and has long enough legs for smooth stair climbing. In this paper, we introduced the methods for stair climbing analysis in section III B. Analysis results can be found in section IV.

A. Large Scale vs. Small Scale Quadruped Robots

II. Related Works

A. Stair Climbing of Quadruped Robots

Large scale robots like the Boston Dynamics Spot robot [3] and ANYbotics ANYmal [5] robot exhibit extremely stable stair climbing, and with a proper controller, they are able to traverse high obstacles. In particular, the ANYmal robot has been shown to perform parkour, jumping over obstacles beyond its standing height, and even climbing ladders [6] [7].

On the other hand, recent works in the motion planning of small scale quadruped robots have investigated the use of learning based control methods for generalizable control policies to traverse complex terrain [8] [9]. These works utilize small scale robots like the Unitree Go2 robot, and as it can be seen in the demonstration videos of these works, these robots do not have stable stair climbing due to their shorter leg lengths.

III. Stair Climbing Analysis of Quadruped Robots

Past works have studied the optimization of the dynamics of quadruped robot stair climbing [10] [11]. However, to our knowledge, these works do not extensively analyze how the design of the robot leg affects stair climbing ability. In this project, we refer to a stair climbing analysis presented by Barasuol et al, which is a streamlined kinematic analysis algorithm that assesses stair climbing capability of legged robots based on their dimensions [12]. Using this analysis, we evaluate the stair climbing capability of our robot, Vivo.

IV. Experimental Setup

A. Leg Designs

Similar to other robots in the market, our robot features three Degrees of Freedom in each of the front leg and hind leg. However, the front leg and hind leg designs are different for Vivo. The front leg comprises two links with the motors mounted at the shoulder tip. The torque is transmitted to the elbow joint through a transmission link configured in a redundantly actuated, parallel configuration. The hind leg, on the other hand, is formed using a mechanically-coupled three-link leg design. It also features a similar parallel transmission link mechanism where the middle link and lower link are coupled. As De Vincenti et al stated in his paper, bio-inspired leg designs could potentially offer significant performance and efficiency advantages compared to the commonly used two-link leg designs in most quadrupedal robots today [13]. This insight is the primary reason we opted for a bio-inspired design imitating dog anatomy.

It is important to note that, since a link transmission mechanism is used, the mapping of the actuator angle is 1-1

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with the joint angle. The forward kinematics equations for the foot (end-effector) were derived as such:

Two-link front leg:

$$x = l_1 cos(q_1) + l_2 cos(q_1 + q_2 - \pi) \tag{1}$$

$$y = l_1 sin(q_1) + l_2 sin(q_1 + q_2 - \pi) \tag{2}$$

Three-link hind legs:

$$x = l_1 cos(q_1) + l_2 cos(q_1 + q_2 + \pi) + l_3 cos(q_1) \tag{3}$$

$$y = l_1 sin(q_1) + l_2 sin(q_1 + q_2 + \pi) + l_3 sin(q_1) \tag{4}$$

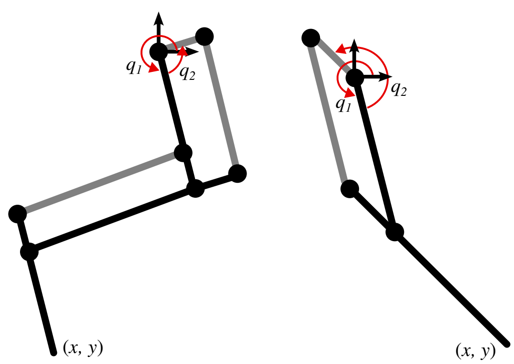

Fig. 2. Analyzed leg designs. Three-link, 2 DoF hind leg (left) and two-link, 2 DoF front leg (right)

B. Stair Climbing Analysis

The stair climbing analysis setup presented by Barasuol et al is illustrated in Fig. 3 [12]. The analysis is conducted in 2D for each leg design, with the adjustable parameters listed in Table I.

TABLE I
PARAMETERS AND DESIGNATIONS

Parameter	Designation
Belly thickness	b
Link thickness	d
Link Lengths	l_i
Foot radius	r_f

Four steps of stairs are analyzed with their dimensions set to reflect ADA standards, having a height of 18 cm and and tread depth of 28 cm, as shown in Fig. 4 [14]. The staircase outline is divided up into discrete points that are each 1 mm apart. These discrete points are used as the target points for the inverse kinematics calculations.

The following assumptions were made during the analysis:

- Robot travels parallel to the stair incline
- The hip roll actuator remains at a constant angular position
- All links have constant thickness
- Links are straight line segments connected via the joint positions

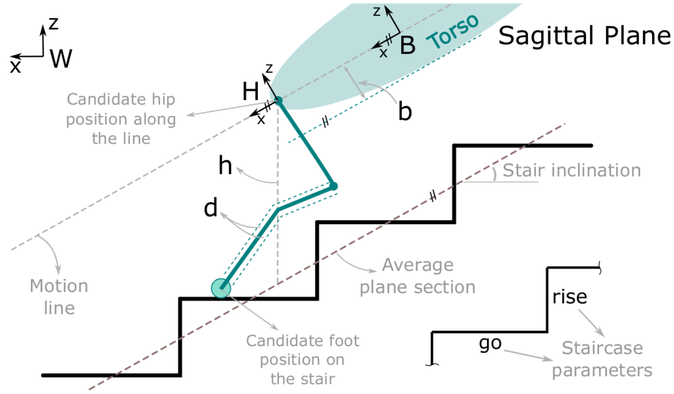

Fig. 3. Diagram of stair climbing analysis presented by Barasuol et al. (Figure from [12])

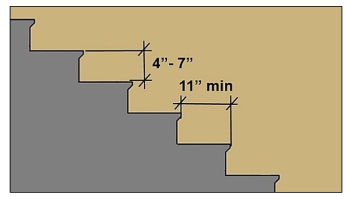

Fig. 4. Stair dimensions established by the ADA. (Figure from [14])

- 30% Stair surface coverage constitutes a stair climbing capable leg design

The Newton-Raphson method was utilized to solve the numerical inverse kinematics algorithm for each discrete point along the staircase [15]. In the inverse kinematics algorithm, a damped Jacobian term was introduced to avoid solving to singular positions due to the parallel redundant nature of the leg designs [16]. Without this damped Jacobian, the numerical inverse kinematics algorithm was more likely to reach singular points. This damped Jacobian term is defined by

$$J_{damped} = J^T (JJ^T + \lambda^2 I)^{-1} \tag{5}$$

where J_{damped} is the damped Jacobian of the leg in the world frame, J is the normal world Jacobian, and λ is the damping factor. This damping factor λ is chosen by the user. In our experiments, we set $\lambda = 0.2$ because it maximized convergence in our tests. Using the numerical inverse kinematics

Algorithm 1 Numerical Inverse Kinematics using Newton-Raphson

Initialize joint angles $\mathbf{q}$ to a reasonable guess.
repeat
  Compute the error $\mathbf{e}(\mathbf{q})$.
  Compute the Jacobian $\mathbf{J}(\mathbf{q})$.
  Computed the damped Jacobian $\mathbf{J_{damped}}(\mathbf{q})$
  Update joint angles:

$$\mathbf{q}_{new} = \mathbf{q}_{old} - \mathbf{J_{damped}}(\mathbf{q}_{old})^{-1} \mathbf{e}(\mathbf{q}_{old}) \tag{6}$$

  Set $\mathbf{q}_{old} = \mathbf{q}_{new}$.
until $\|\mathbf{e}(\mathbf{q})\| \leq \epsilon$ (tolerance)
Return $\mathbf{q}_{new}$ as the solution.

calculations, the percentage of stair surface area coverage was determined by determining the number of successful foothold positions, then dividing it by the total number of attempted foothold positions. A foothold position is deemed to be successful if it meets the following two conditions:

- Joint limit is reached
- Any leg link penetrates the stair surface

To determine whether a leg link penetrates the stair surface after the inverse kinematics calculation, a generic segment intersection algorithm was implemented that determines line intersection based on the orientation of two lines [17].

Using the calculated stair surface area coverage, various link lengths and body heights were tested to optimize

- Stair surface coverage must be greater than 20 % (metric used by Barasuol et al. [12])
- Minimize body height for stable walking
- Link lengths must allow for robot to fit within carry-on suitcase dimensions

The carry-on suitcase dimensions constraint was imposed to allow the robot to be brought on public transportation.

Through a trial and error process, the stair surface coverage of various hand-picked link length combinations were tested for using the stair climbing analysis shown in the previous section. Initially, the leg lengths of the front leg were set to be the same as the Boston Dynamics Spot robot, while the hind legs started with the dimensions of the MIT Cheetah robot [12] [18]. The dimensions of each leg were iteratively decreased to lower the body height, while attempting to maintain greater than 30% stair surface coverage.

V. Results and Discussion

The final results of the iterative trial and error link optimization process are shown in Table II. The final front leg link lengths are most similar to the KAIST Hound robot [19], as listed in Table III. Compared to the MIT Cheetah robot, the hind leg link lengths are longer by 55 and 30 mm for the upper link and middle link, respectively, while the lower link is shortened by 80 mm. The body height for each of the links were set at 0.6 m in these experiments.

With these dimensions, the front leg covers 31% of the staircase, while the hind leg covers 52.49%. It can be noticed that the front leg stair surface coverage is significantly lower than the hind leg, although it is still deemed to be capable of stair climbing due to it being greater than 30% (from the assumptions). We believe this lower coverage value stems from the straight line assumption of the lower link. Since the lower link is straight, it is susceptible to collisions with the stair edge. Modeling the lower link to be curved could be a solution to this, as shown in Fig. 5.

These results indicate that shorter lower link lengths may contribute to increased staircase surface coverage.

VI. Conclusion and Future Work

The algorithms outlined in this paper provide a robust foundation for optimizing the size and design of Vivo, a new quadruped robot tailored for guide-dog applications. Future work will focus on the optimization of Vivo's link lengths.

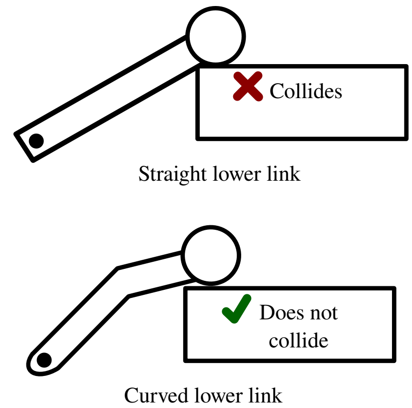

Fig. 5. Collision of straight lower link (top) vs. avoidance of collision of curved lower link (lower)

TABLE II
Link Lengths for Front and Hind Legs

	Front Leg	Hind Leg
Upper Link [m]	0.35	0.30
Middle Link [m]		0.25
Lower Link [m]	0.30	0.10

The constraints on this optimization problem will include minimizing the body height for stability, respecting joint limit specifications, and achieving the target stair surface coverage of 20-40%. A critical next step will also involve accurately calculating the ground reaction forces for each leg during stair climbing. These forces are essential for modeling the robot's interaction with its environment, but their accurate prediction remains a challenge due to the inherent variability and complexity of real-world conditions. By integrating these advancements, we aim to further enhance the design and functionality of Vivo, paving the way for guide-dog robots that are universally applicable, reliable, and capable of providing substantial assistance in diverse environments.

References

[1] T. Yoshiike, M. Kuroda, R. Ujino, H. Kaneko, H. Higuchi, S. Iwasaki, Y. Kanemoto, M. Asatani, and T. Koshiishi, "Development of experimental legged robot for inspection and disaster response in plants," in *2017 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, 2017, pp. 4869–4876.

[2] C. Gehring, P. Fankhauser, L. Isler, R. Diethelm, S. Bachmann, M. Potz, L. Gerstenberg, and M. Hutter, "Anymal in the field: Solving industrial inspection of an offshore hvdc platform with a quadrupedal robot," in *International Symposium on Field and Service Robotics*, 2019. [Online]. Available: https://api.semanticscholar.org/CorpusID:213713448

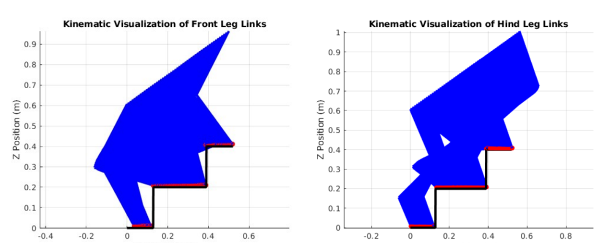

Fig. 6. Kinematic visualization of stair climbing analysis for front leg (left) and hind leg (right). The hip positions travel parallel with the stair incline.

TABLE III
LEG MODEL PARAMETERS FROM BARASUOL ET AL'S WORK [12]

Robot	l_1 **[m]**	l_2 **[m]**	r **[m]**	d **[m]**
AlienGo	0.25	0.25	0.025	0.015
Anymal	0.285	0.35	0.024	0.02*
Hound	0.33	0.35	0.025	0.02*
HyQ	0.35	0.35	0.02	0.02
HyqReal	0.36	0.38	0.03	0.015
MiniCheetah	0.21	0.18	0.015	0.01
Solo	0.16	0.16	0.013	0.01
Spot	0.38	0.38	0.03	0.015
B1	0.33	0.33	0.025	0.015*

[3] "Spot® - the agile mobile robot," Boston Dynamics, 2024. [Online]. Available: https://bostondynamics.com/products/spot/

[4] "Unitree go1," Unitree Robotics, May 2024. [Online]. Available: https://www.unitree.com/go1

[5] M. Hutter, C. Gehring, D. Jud, A. Lauber, C. D. Bellicoso, V. Tsounis, J. Hwangbo, K. Bodie, P. Fankhauser, M. Bloesch, R. Diethelm, S. Bachmann, A. Melzer, and M. Hoepflinger, "Anymal - a highly mobile and dynamic quadrupedal robot," in *2016 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, 2016, pp. 38–44.

[6] D. Hoeller, N. Rudin, D. Sako, and M. Hutter, "Anymal parkour: Learning agile navigation for quadrupedal robots," *Science Robotics*, vol. 9, no. 88, p. eadi7566, 2024. [Online]. Available: https://www.science.org/doi/abs/10.1126/scirobotics.adi7566

[7] D. Vogel, R. Baines, J. Church, J. Lotzer, K. Werner, and M. Hutter, "Robust ladder climbing with a quadrupedal robot," *arXiv preprint arXiv:2409.17731*, 2024.

[8] G. B. Margolis and P. Agrawal, "Walk these ways: Tuning robot control for generalization with multiplicity of behavior," *Conference on Robot Learning*, 2022.

[9] S. Luo, S. Li, R. Yu, Z. Wang, J. Wu, and Q. Zhu, "Pie: Parkour with implicit-explicit learning framework for legged robots," *IEEE Robotics and Automation Letters*, vol. 9, no. 11, pp. 9986–9993, 2024.

[10] L. Ye, Y. Wang, X. Wang, H. Liu, and B. Liang, "Optimized static gait for quadruped robots walking on stairs," in *2021 IEEE 17th International Conference on Automation Science and Engineering (CASE)*, 2021, pp. 921–927.

[11] D. Yoon, B. Kim, I. Jo, and W. Jeong, "A dynamic locomotion strategy for stair walking of a quadruped robot," in *2021 18th International Conference on Ubiquitous Robots (UR)*, 2021, pp. 223–227.

[12] V. Barasuol, S. Emre, and C. Semini, "Stair-climbing charts: On the optimal body height for quadruped robots to walk on stairs," in *Synergetic Cooperation Between Robots and Humans*, E. S. E. Youssef, M. O. Tokhi, M. F. Silva, and L. M. Rincon, Eds. Cham: Springer Nature Switzerland, 2024, pp. 251–262.

[13] F. De Vincenti, D. Kang, and S. Coros, "Control-aware design optimization for bio-inspired quadruped robots," in *2021 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. IEEE, 2021, pp. 1354–1361.

[14] "Guide to the ada accessibility standards," United States Access Board, 2010. [Online]. Available: https://www.access-board.gov/ada/guides/chapter-5-stairways/

[15] K. M. Lynch and F. C. Park, *Modern robotics*. Cambridge, England: Cambridge University Press, May 2017.

[16] A. S. Deo and I. D. Walker, "Overview of damped least-squares methods for inverse kinematics of robot manipulators," *Journal of Intelligent and Robotic Systems*, vol. 14, pp. 43–68, 1995.

[17] T. H. Cormen, C. E. Leiserson, R. L. Rivest, and C. Stein, *Introduction to algorithms*. MIT press, 2022.

[18] P. M. Wensing, A. Wang, S. Seok, D. Otten, J. Lang, and S. Kim, "Proprioceptive actuator design in the mit cheetah: Impact mitigation and high-bandwidth physical interaction for dynamic legged robots," *Ieee transactions on robotics*, vol. 33, no. 3, pp. 509–522, 2017.

[19] Y.-H. Shin, S. Hong, S. Woo, J. Choe, H. Son, G. Kim, J.-H. Kim, K. Lee, J. Hwangbo, and H.-W. Park, "Design of kaist hound, a quadruped robot platform for fast and efficient locomotion with mixed-integer nonlinear optimization of a gear train," in *2022 International Conference on Robotics and Automation (ICRA)*. IEEE, 2022, pp. 6614–6620.