La geometria del triangle

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1 Propietats dels traingles

$\sum angles = 180° = \pi\ rad$ La suma dels catets és més gran que la hipotenusa.
En un angle agut:

$$\sin\theta = \frac{CO}{H}$$

$$\cos\theta = \frac{CA}{H}$$

$$\tan\theta = \frac{CO}{CA}$$

Propietat fonamental de la trigonometria:

$$\cos^2(\alpha) + \sin^2(\alpha) = 1$$

$$\frac{\sin(\alpha)}{\cos(\alpha)} = \tan(\alpha)$$

2 Cicrumferencia trigonometrica

És divideix la cicrumferencia en 4 quadrants.
Els eixos representens $\cos\alpha$ i $\sin\alpha$.
$-1 \leq \cos(\alpha) \leq 1$
$-1 \leq \sin(\alpha) \leq 1$

2.1 Funcions reciporques

$$\sin(x) \to \frac{1}{\sin(x)} = \csc(x)$$

$$\cos(x) \to \frac{1}{\cos(x)} = \sec(x)$$

$$\tan(x) \to \frac{1}{\tan(x)} = \cot(x)$$

2.2 Angle II quadrant

$\alpha + \beta = \pi$, α i β són suplementaris.

$$\sin\beta = \sin(\pi - \beta)$$

$$\cos\beta = -\cos(\pi - \beta)$$

$$\tan\beta = -\tan(\pi - \beta)$$

2.3 Angle III quadrant

$$-\sin\beta = \sin(\pi+\beta)$$
$$-\cos\beta = -\cos(\pi+\beta)$$
$$\tan\beta = \tan(\pi+\beta)$$

2.4 Angle IV quadrant

$$-\sin\beta = \sin(2\pi-\beta)$$
$$\cos\beta = \cos(2\pi-\beta)$$
$$-\tan\beta = \tan(2\pi-\beta)$$

2.5 Angles coplementaris

La suma dels angles és $\frac{\pi}{2}$

$$\sin\beta = \sin(2\pi-\beta)$$
$$\cos\beta = -\cos(2\pi-\beta)$$
$$\tan\beta = \cot(\pi-\beta)$$

3 Equacions trigonomètriques

$$\sin x = \frac{1}{2}$$
$$x = \arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6} \to \frac{\pi}{6} + 2\pi k, k \in \mathbb{Z}$$
$$= \frac{5\pi}{6} \to \frac{5\pi}{6} + 2\pi k, k \in \mathbb{Z}$$

4 Area d'un triangle

$$A_{\triangle} = \frac{1}{2}\cdot b \cdot h$$
$$A_c = \frac{1}{2}\cdot a \cdot b \cdot \sin C$$
$$A_b = \frac{1}{2}\cdot a \cdot c \cdot \sin B$$
$$A_a = \frac{1}{2}\cdot b \cdot c \cdot \sin A$$

4.1 Teorema del sinus

$$\frac{\sin C}{c} = \frac{sinB}{b} = \frac{\sin A}{a}$$

4.2 Teorema del cosinus

$$a^2+ = b^2 + c^2 - 2bc\cdot\cos A$$
$$b^2+ = a^2 + c^2 - 2ac\cdot\cos B$$
$$c^2+ = a^2 + b^2 - 2ab\cdot\cos C$$