Probabilistic inverse Theory. Assignment 4 – Probabilistic model of a geological structure

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1 Objective

From the image of a geological structure, build a new image which preserves the same statistics.

2 Considerations

The initial (training) image is a black and white image as it is shown in figure 1. It simulates a pre-historic river system, consisting of sandstone channels (black pixels) surrounded by shale (white pixels).

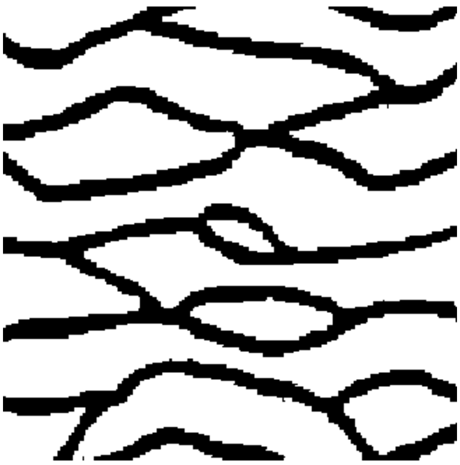

Figure 1: Training image

Initially I create a 2x2 moving window such that it scans all possible configurations of black and white patterns given in the image. There are 16 possible configurations. Figure 2 shows the three configurations which repeat the most in the image. In my case configuration 16 and 1 are the most frequent. Configuration 16 means that all 4 pixels are white, this make sense by a simple inspection of the training image, most of the spots are white. The second more frequent is the configuration 1 which is the case when all the 4 pixels are black.

The marginal probability can be obtained from the histogram which represents the frequency with which each pattern is repeated in the training image. Then, when dividing the frequency by the total number of possible events that a given pattern can occur within the image we obtain the marginal probability. It is presented in the figure 3.

3 The new image

We will construct the new image based on the previous information. From known pixels we will obtain new ones based on the conditional probabilities. This procedure consist in the following steps:

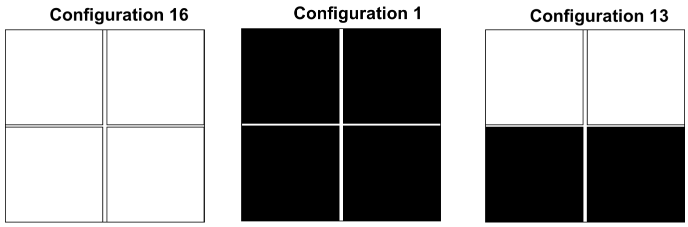

Figure 2: Three most frequent configurations (from left to right) in the training image

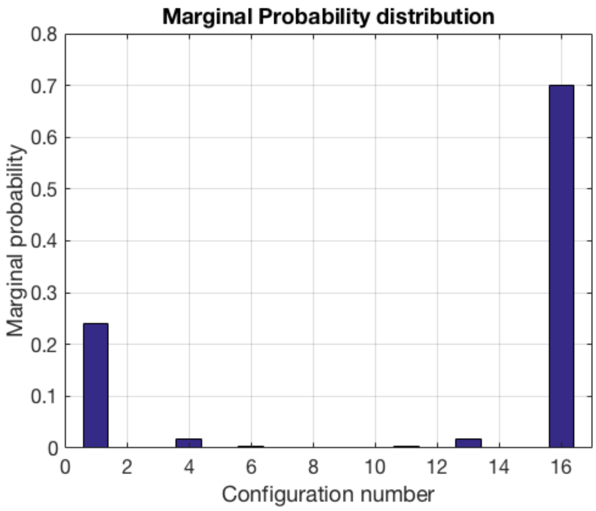

Figure 3: Marginal probability of each pattern obtained by scanning 2x2 squares in the training image

Step 1

We pick a random 2x2 pattern (based on the marginal probability) and place it in the upper left part of what it will be the new image. The first column of this randomly selected pattern is assumed to be known. Then, we compute the conditional probability of having 2 pixels in the second column such that the first column matches with my current pattern. This is like, for a given pattern, we take a subset of the distribution in figure 3 which satisfy my desired condition and then we pick randomly one event within this subset. Figure 4 (left) shows graphically the condition in this first step. We keep selecting pixels until we fill the first two rows of the new image.

Step 2

Now I am placed in the second row of my image but I still keep making decisions based on 2x2 squares. We assume that the first row of my current square is known, then I compute the conditional probability of having 2 pixels in the second row such that the pixels in the first row satisfy my condition.

Step 3

Here I make a similar procedure, but I am assuming that the first row (2 pixels) and the left bottom pixel are all known (observe right hand side of figure 4) . Then I compute the conditional probability and pick randomly from the conditional distribution a pattern which satisfy my desired condition in order to fill the missing pixel.

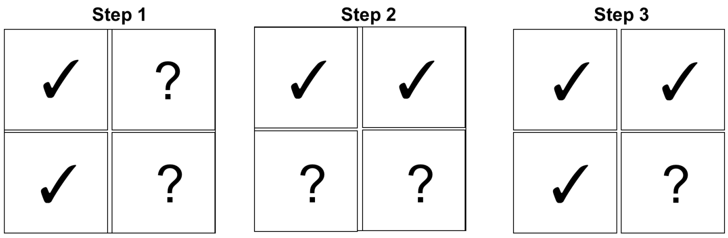

Figure 4: Representation of the three steps performed when creating the new image

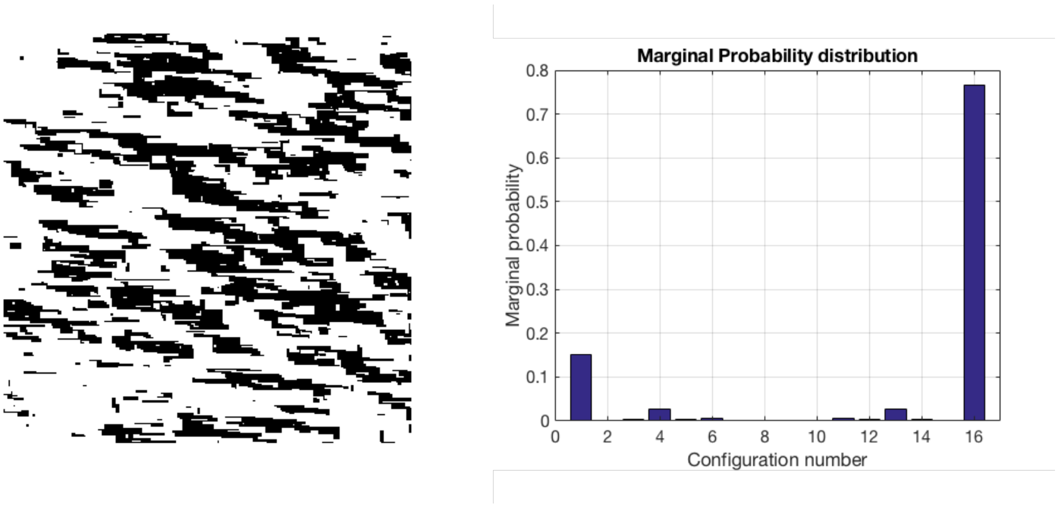

Figure 5: New realization of the image and its marginal probability.

I keep repeating step 2 and 3 until I fill (make) my new image. I can generate as many images as I want but all of them should have similar statistics. Then, if I compute again the marginal probability of my new images, its distribution will be similar to the one of the original (training) image. This is observed in figures 5 and 6, where the new images are presented with its marginal probability. Observe that, although they are in strictly sense different, the distribution is similar to our original image in figure 3.

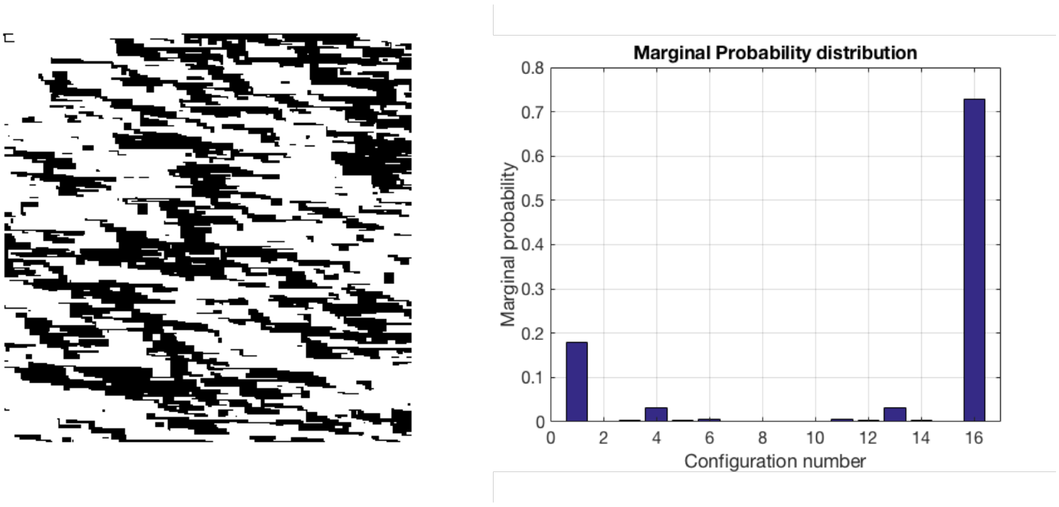

Figure 6: New realization of the image and its marginal probability.