Probabilistic inverse Theory. Assignment 3 – Mass of an asteroid

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1 Objective

Build a probabilistic density function of the mass and logarithm of the mass, given some information of the asteroid 2004 BN41, by the Near Earth Object Program of NASA.

2 Considerations

Ideally, a probabilistic density function should be constructed based on empirical information. Then, if we are able to obtain an histogram from some collected data we can built an accurate distribution by increasing the number of samples. In our current problem we know there are 21 samples, however we have not access of each individual data, we only know the estimate. An estimate of a given parameter in this context means its average (remember, we have 21 samples) weighted by the impact probability (how likely is that the asteroid hits the Earth) . Thus, we have no chance to construct such histogram and we need a different approach.

3 Solution

Using only the information from the red rectangles in the exercise we have:

- We have information of the estimate of the mass which is 1.9x10^7 kg.
- The estimation of the mass is accurate to within a factor of 3. This is between $(\frac{1}{3}m_0, 3m_0)$.

The accurate estimation of the mass lies between a non symmetric interval. However, this can be solved by taking the logarithm of the mass. Let's make $y = \ln{(\frac{m}{m_0})}$. We can assume initially a Gaussian distribution in which the maximum probability is reached when $m = m_0$.

$$\rho_y(y) = \exp(\frac{-y^2}{2\sigma^2}). \tag{1}$$

If we consider our variable y by the interval $(\frac{1}{3}m_0, 3m_0))$ in which the mass is accurate we get:

$$y \in [Ln(1/3), Ln(3)] = [-1.09, 1.09] \tag{2}$$

This means that by using the variable y we get a symmetric distribution, we can use also 1.09 as the magnitude of the standard deviation. The value m_0 is a reference value which in our case corresponds to the estimate m_0 =1.9x10^7 kg. Figure 1 shows the probability density of y according to equation 1. The function takes its maximum (the function is not normalized) at $y = 0$, this is, when $m = m_0$ which is the mass estimate.

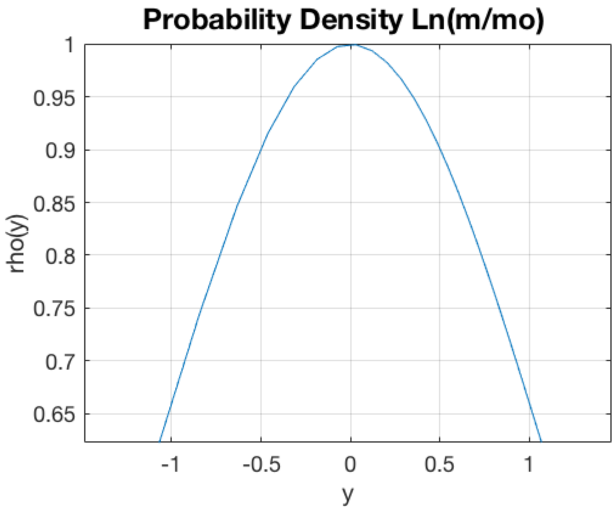

Figure 1: Probability density of the variable $y = \ln\left(\frac{m}{m_0}\right)$

In order to obtain the probability density of the mass we can rename some variables for simplicity. Let's consider the variable y and $z \equiv m$, with pdf $\rho_y(y)$ and $\rho_z(z)$ respectively. Then, the mas distribution will be obtained from:

$$\rho_z(z) = |\frac{dy}{dz}|\rho_y(y) \tag{3}$$

When operating this equation we end up with

$$\rho_z(z) \equiv \rho_m(m) = \frac{1}{m}\rho_y(y) \tag{4}$$

Thus, the pdf of the mass is $\frac{1}{m}$ the pdf of tha variable y. To put everything in terms of the mass we obtain:

$$\Rightarrow \rho_m(m) = \frac{1}{m}\exp(\frac{-(\ln(m/m_0))^2}{2\sigma^2}). \tag{5}$$