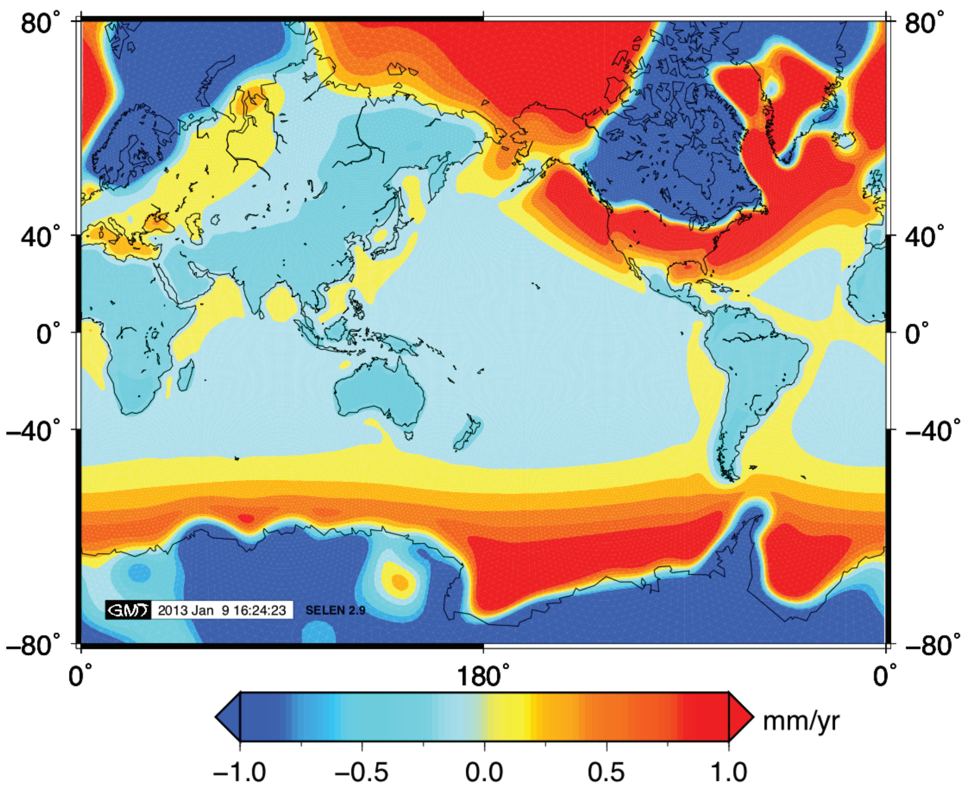
COMPUTATIONAL INFRASTRUCTURE FOR GEODYNAMICS (CIG)

SELEN

User Manual

Version 2.9



SELEN 2.9

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SELEN: a program for solving the *"Sea Level Equation"*

User Manual for version 2.9

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The cover image is a GMT visualization of the rate of sea level change today (*sea level fingerprint*) associated with GIA ($\dot{S}$) calculated by SELEN. Details about parameters and analysis are in Figure 15 in this manual.

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Figure 1: The SELEN logo is an artwork by Florence Colleoni (2009).

1 Introduction

The open source program SELEN solves numerically the so–called "Sea Level Equation" (SLE) for a spherical, layered, non–rotating Earth with Maxwell viscoelastic rheology.

The SLE is an integral equation that was introduced in the 70s to model the sea level variations in response to the melting of late–Pleistocene ice–sheets, but it can also be employed for predictions of geodetic quantities in response to present-day melting of continental ice-sheets. SELEN can compute vertical and horizontal surface displacements, gravity variations and sea level changes on a global and regional scale.

SELEN (acronym of SEa Level EquatioN solver) is particularly oriented to scientists at their first approach to the glacial isostatic adjustment (GIA) problem and, according to our experience, it can be successfully used in teaching. The current release (2.9) considerably improves the previous version of the code in terms of computational efficiency, portability and versatility. As far as we know, SELEN is the only open source program designed for solving the SLE.

SELEN 2.9 solves the SLE following the classical theory of Farrell and Clark (1976). In the future release of SELEN (SELEN 3.0), which is under development, two important new features will be introduced: the rotational feedback on sea level and the horizontal migration of shorelines in response to sea level change.

This User guide describes the essentials of the theory behind the SLE, and provides instructions for the configuration and execution of SELEN.

2 Acknowledgements

We thank all the SELEN users and a number of colleagues for the numerous feedbacks that have greatly contributed to improve the program. We thank Max Tegmark for making available the code for the icosahedral pixelization of the sphere. For computations involving spherical harmonic functions, SELEN implements some SHTOOLS routines developed by Mark Wieczorek. All the figures have been drawn using the GMT public domain software (Wessel and Smith, 1998). The development of SELEN has been supported by ISCRA (Italian SuperComputing Resource Allocation) with contracts n. HP10CS9J50 (SEASIM) and HP10CUX77T (SEAMED). Partly funded by COST Action ES0701 "Improved Constraints on Models of Glacial Isostatic Adjustment" and by the European Commission's 7$^{\mathrm{th}}$ Framework Programme through grant number 226375 (ice2sea project). CIG is thanked for hosting SELEN (`http://geodynamics.org/cig/software/selen`). Eric Heien from CIG has greatly improved the SELEN usability by implementing the autoconf system into the original code.

3 Citation

Computational Infrastructure for Geodynamics (CIG) and the SELEN developers are making the source code to SELEN available to researchers in the hope that it will aid their research and teaching. A number of individuals have contributed a significant amount of time and energy into the development of SELEN. We request that you cite the appropriate papers and make acknowledgements as necessary. The SELEN development team asks that you cite the following papers:

- Spada G, Stocchi P (2006) The Sea Level Equation, Theory and Numerical Examples. Aracne, Roma.
- G. Spada, P. Stocchi, SELEN: A Fortran 90 program for solving the sea-level equation, Computers & Geosciences, Volume 33, Issue 4, May 2007, Pages 538-562, ISSN 0098-3004, http://dx.doi.org/10.1016/j.cageo.2006.08.006 .
- Spada G, Melini D, Galassi G, Colleoni F (2012) Modeling sea level changes and geodetic variations by glacial isostasy: the improved SELEN code (http://arxiv.org/abs/1212.5061)

4 Theoretical background

Here we briefly illustrate the background theory of the SLE, the integral equation that describes the sea level variations and solid Earth deformations associated with GIA. Essentially, the material that follows is a condensed summary of the SLE theory first exposed by Farrell and Clark (1976).

For further details, including the spatiotemporal discretization of the SLE and its numerical implementation, the reader is referred to the review of Spada and Stocchi (2006) and to references therein.

4.1 Sea Level and Sea Level change

Introducing the SLE requires some basic definition involving the general concept of "sea level". *Absolute sea level* is:

$$\mathrm{SL}(\omega, t) = R_{ss}(\omega, t) - R_{se}(\omega, t), \tag{1}$$

where $\omega \equiv (\theta, \lambda)$, θ is colatitude and λ is longitude, t is time, and R_{ss} and R_{se} denote the radii of the (equipotential) sea surface and of the solid surface of the Earth, both relative to the Earth's center of mass (CM).

The quantity involved in the SLE is *sea level change*

$$S(\omega, t) = \mathrm{SL}(\omega, t) - \mathrm{SL}(\omega, t_r), \tag{2}$$

where $t_r \leq t$ denotes a remote time. For studies of past sea level variations, it is convenient to introduce *relative sea level* (RSL):

$$\mathrm{RSL}(\omega, t_{BP}) = \mathrm{SL}(\omega, t_{BP}) - \mathrm{SL}(\omega, t_p), \tag{3}$$

where $t = t_{BP}$ is a given time before present (BP) and $t = t_p$ is present time. Using (2), RSL is easily expressed in terms of sea level change:

$$\mathrm{RSL}(\omega, t_{BP}) = S(\omega, t_{BP}) - S(\omega, t_p). \tag{4}$$

An expression for $S(\omega, t)$ which is suitable in GIA studies is obtained using (1) in (2):

$$S(\omega, t) = N(\omega, t) - U(\omega, t), \tag{5}$$

where $N(\omega, t) = R_{ss}(\omega, t) - R_{ss}(\omega, t_r)$ is the *sea surface variation* relative to the CM (sometimes referred to as *absolute sea level change*) and $U(\omega, t) = R_{se}(\omega, t) - R_{se}(\omega, t_r)$ is the *vertical displacement* of the solid surface of the Earth. Note that $S(\omega, t)$ is defined over the whole Earth surface, including the continental masses.

Eq. (5) represents the SLE in its simplest form; what follows is aimed to illustrate the relationship between S and the variations of the ice thickness through time, in order to obtain a form of the SLE amenable to a numerical approach.

4.2 The Sea Level Equation

According to Farrell and Clark (1976), the sea surface variation N in Eq. (5) can be written as

$$N(\omega,t) = G(\omega,t) + c(t), \tag{6}$$

where c is yet undetermined and the *geoid height variation* is

$$G(\omega,t) = \frac{\Phi}{\gamma}(\omega,t), \tag{7}$$

in which γ is the reference gravity at the surface of the Earth and $\Phi(\omega,t)$ is the *total variation of the gravity potential*.

Hence, using Eq. (6) into (5) gives

$$S(\omega,t) = \frac{\Phi}{\gamma} - U + c, \tag{8}$$

where we have dropped the (ω,t) dependence in right hand side to simplify notation. Mass conservation of the system (Ice sheets + Oceans) is ensured taking

$$c(t) = -\frac{m_i}{\rho_w A_o} - \overline{\left(\frac{\Phi}{\gamma} - U\right)}, \tag{9}$$

where ρ_w is the (constant) density of water, m_i is the mass variation of the ice sheets, A_o is the (constant) area of the present–day oceans and the overbar indicates the average over the surface of the oceans

$$\overline{(\ldots)} = \frac{1}{A_o}\int_o (\ldots)\, dA, \tag{10}$$

where $dA = a^2 \sin\theta d\theta d\lambda$ is the area element and a is Earth average radius. From Eq. (8), the SLE can be therefore written as

$$S(\omega,t) = \left(\frac{\Phi}{\gamma} - U\right) + S^E - \overline{\left(\frac{\Phi}{\gamma} - U\right)}, \tag{11}$$

where the *"eustatic" sea level variation*:

$$S^E(t) = -\frac{m_i}{\rho_w A_o}, \tag{12}$$

shows the remarkable property

$$S^E(t) = \overline{S}. \tag{13}$$

The SLE has solution $S = S^E$ only in the case of a rigid, non self–gravitating Earth ($U = \Phi = 0$ in Eq. 11).

Functions $U(\omega,t)$ and $\Phi(\omega,t)$ will depend on the spatiotemporal variations of the *surface load*:

$$\mathcal{L}(\omega,t) = \rho_i I + \rho_w S \mathcal{O}, \tag{14}$$

where the two terms on the right hand side are associated with the waxing and waning of the ice sheets (*ice load*), and with the redistribution of meltwater in the ocean basins (*ocean load*), respectively. In Eq. (14), ρ_i is ice density, $\mathcal{O}$ is the *ocean function*

$$\mathcal{O}(\omega) = \begin{cases} 1, & \text{if } \omega \in \text{oceans} \\ 0, & \text{if } \omega \in \text{land}, \end{cases} \tag{15}$$

(hereinafter referred to as OF), and

$$I(\omega, t) = T - T_0, \tag{16}$$

is the *ice thickness variation*, where $T(\omega, t)$ is *ice thickness*, and $T_0(\omega)$ is a reference thickness at a remote time (e.g. the thickness at the Last Glacial Maximum, LGM, 21 kyrs ago). The mass variation in Eq. (12) is obtained from (16) by integration over the ice–covered regions:

$$m_i(t) = \int_i \rho_i I \; dA. \tag{17}$$

Following Eq. (14), vertical displacement stems from two terms

$$U(\omega, t) = \rho_i G_u \otimes_i I + \rho_w G_u \otimes_o S, \tag{18}$$

where G_u is the Green's function for vertical displacement, $\otimes_i$ and $\otimes_o$ are spatiotemporal convolutions over the ice– and ocean–covered regions, respectively (these convolutions and the Green's functions are formally defined in Spada and Stocchi 2006). Similarly, the total variation of the gravity potential is

$$\Phi(\omega, t) = \rho_i G_\phi \otimes_i I + \rho_w G_\phi \otimes_o S, \tag{19}$$

where G_ϕ is the corresponding Green's function. Explicit expressions for G_u and G_ϕ are given in e.g. Spada and Stocchi (2006) in terms of the load–deformation coefficients (LDCs) $h(t)$ and $k(t)$, respectively. The sea level Green's function is defined as the difference:

$$\frac{G_s}{\gamma}(\omega, t) = \frac{G_\phi}{\gamma} - G_u. \tag{20}$$

Substitution of Eqs. (18) and (19) into (11) using (20) gives

$$S(\omega, t) = \frac{\rho_i}{\gamma} G_s \otimes_i I + \frac{\rho_w}{\gamma} G_s \otimes_o S + S^E - \frac{\rho_i}{\gamma} \overline{G_s \otimes_i I} - \frac{\rho_w}{\gamma} \overline{G_s \otimes_o S}, \tag{21}$$

which represents the SLE in the *"gravitationally self–consistent"* form. Since the unknown $S(\omega, t)$ also appears in the spatiotemporal convolutions at the right hand side, the SLE is an integral equation, which in general cannot be solved in closed form. The SLE is a linear equation as long as shorelines are not allowed to migrate horizontally, i.e. if $\mathcal{O}$ (and consequently A_o) is not dependent on S itself. Sea level variations are sensitive to mantle rheology through G_s, since this is determined by the viscoelastic LDCs h and k (Spada, 2003a, Spada and Stocchi, 2006). Solutions of Eq. (21) in special cases, discussed in detail by Spada and Stocchi (2006), are also available via SELEN .

It is worth to observe that the SLE (21) does not involve absolute quantities and can, consequently, only provide *variations* of geophysical and geodetic quantities relative to a reference state.

4.3 Solving the Sea Level Equation

The form (21) of the SLE is not suitable for a numerical implementation. In fact, the spherical harmonic (SH) decomposition of terms terms like $G_s \otimes_o S$, which imply an integration over the oceans and not over the whole Earth, would demand evaluating coupling coefficients between sets of SHs, which may severely limit the maximum degree of the harmonic analysis (SH conventions in SELEN are summarized in Section 8.6).

The *pseudo–spectral approach* introduced by Mitrovica and Peltier (1991) and Mitrovica et al. (1994), and implemented in SELEN, overcomes this difficulty, since it allows for a direct spectral analysis (Milne, 1998). At the core of the method is a change of variable in the SLE, where $S(\omega, t)$ is substituted by its projection on the OF:

$$Z(\omega, t) \equiv \mathcal{O}S, \tag{22}$$

where we define Z as the *modified sea level change*. Multiplying both sides of Eq. (21) by $\mathcal{O}$ gives the SLE in the form:

$$Z(\omega, t) = H + Z^E + K(Z), \tag{23}$$

where:

$$\begin{aligned} Z^E(t) &\equiv \mathcal{O}S^E & (24)\\ H(\omega, t) &\equiv \mathcal{O}(A - \overline{A}) & (25)\\ K(Z; \omega, t) &\equiv \mathcal{O}(B - \overline{B}), & (26) \end{aligned}$$

and variables

$$\begin{aligned} A(\omega, t) &\equiv \frac{\rho_i}{\gamma}\, G_s \otimes_e I & (27)\\ B(\omega, t) &\equiv \frac{\rho_w}{\gamma} G_s \otimes_e Z, & (28) \end{aligned}$$

now involve an integration (denoted by $\otimes_e$) over the whole Earth's surface, which can be tackled using a standard spectral approach. Using this formalism, the SLE (21) reads

$$S(\omega, t) = A - \overline{A} + S^E + B - \overline{B} \tag{29}$$

and from (18) vertical displacement is

$$U(\omega, t) = A_u + B_u, \tag{30}$$

where A_u and B_u have the same form of A and B in (27) and (28), but with G_s replaced by G_u. Similarly, from (19) and (7), the geoid height variation is

$$G(\omega, t) = A_g + B_g, \tag{31}$$

where A_g and B_g have the same form of A and B, with G_s replaced by G_ϕ.

Since term K in (23) depends on Z, the SLE is an integral (implicit) equation, in which the unknown Z appears explicitly at the left–hand side but it is convolved in space and time with G_s on the right hand side. This suggests an iterative approach to the SLE, similar to that employed to solve the one–dimensional inhomogeneous integral Fredholm equations of the second kind. However, the complexity of the SLE (and its three dimensionality) hinders an analytical study of the convergence conditions of the iteration scheme.

Following Spada and Stocchi (2007), to which the author is referred for more details, the iteration scheme implemented in SELEN reads

$$\begin{cases} Z^{(0)}_{\ell m,i} = & Z^E_{\ell m,i} \\ Z^{(k)}_{\ell m,i} = & H_{\ell m,i} + Z^E_{\ell m,i} + K_{\ell m,i}\left(Z^{(k-1)}_{\ell' m',j}\right), \end{cases} \tag{32}$$

where $Z^{(k)}_{\ell m,i}$ is the k–th approximation to the degree ℓ ($0 \leq \ell \leq \ell_{max}$) and order m ($0 \leq m \leq \ell$) component of $Z(\omega, t)$, evaluated at the i–th time step (a piecewise constant behavior is assumed for all the variables involved in the SLE, with a typical time step of 1 kilo–year). The iteration continues until the ratio $|Z^{(k)}_{\ell m,i} - Z^{(k-1)}_{\ell m,i}|/|Z^{(k-1)}_{\ell m,i}|$ becomes sufficiently small (according to the numerical tests carried out by Spada and Stocchi (2007), three to five iterations normally suffice for convergence). Once the SLE is solved, all the other geophysical and geodetic quantities (see e.g. Spada et al. 2012a) are immediately obtained from the spectral components of Eqs. (29), (30), and (31).

For the numerical evaluation of surface integrals embedded in terms H and K of the SLE (Eq. 23), SELEN takes advantage of the equal–area, icosahedron–shaped, spherical pixelization by Tegmark (1996), which is particularly convenient for the manipulation of harmonic functions. The number of pixels on the grid is determined by

$$N_p = 40R(R+1) + 12, \tag{33}$$

where the integer R is the *grid resolution parameter* (Tegmark, 1996). The pixels are the centers of slightly distorted hexagonal grid cells. Assuming that N_p is sufficiently large, the area of the surface of the cells is $A_{cell} \sim 4\pi a^2/N_p$ and their radius is $r_{cell} \sim a\sqrt{4/N_p}$, where a is Earth's radius. As discussed by Tegmark (1996), an accurate numerical integration of harmonic functions up to degree ℓ_{max} demands a sufficiently large number of grid pixels, with

$$N_p \geq \frac{\ell^2_{max}}{3}, \tag{34}$$

where ℓ_{max} is the maximum harmonic degree of the SH expansion of the SLE.

5 How SELEN works

5.1 Software and hardware requirements

Software. Running SELEN requires a standard UNIX environment (including Linux and Mac OS X) and a Fortran 90 compiler. On Windows systems, SELEN can run within the Cygwin environment. SELEN also requires the GMT (Generic Mapping Tools) public domain mapping software (Wessel and Smith, 1998).

The configuration and build process of SELEN 2.9 is handled by standard GNU autoconf scripts. SELEN 2.9 has been succesfully tested using both the freely available g95 and gfortran compilers, or the commercial Intel Fortran compiler. Additional operating system and compiler configurations can be implemented by modifying the `flags.guess` file and providing suitable options to the `configure` script.

In SELEN 2.9, the most computationally intensive portions of code have been parallelized with OpenMP (2005) directives. The corresponding program units can therefore take advantage of multi–threading on modern CPUs, resulting in a substantial performance improvement.

Porting SELEN on "unsupported" systems should be an easy task. The only tricky aspect is that program unit `tb.F90` (the LDCs calculator) needs quad–precision floating–point (i.e. `REAL*16`), which is not supported on all platforms. The Intel fortran compiler and the g95 compiler fully support `REAL*16` objects. On x86 systems, the GNU gfortran compiler does not support `REAL*16`; however it supports `REAL*10`, essentially a double-precision fraction with a quad–precision exponent, which is adequate for floating-point computations in `tb.F90`. When porting SELEN to other platforms, the support and range of `REAL*16` variables have to be carefully checked.

Hardware. The hardware requirements of SELEN are related to the desired grid resolution parameter R and to the maximum harmonic degree ℓ_{max}. For low or medium resolutions (approximately $R \leq 50$ and $\ell_{max} \leq 128$), SELEN can run without trouble on a notebook. High-resolution runs might require a multi-processor system and/or large amounts of RAM. The `TEST` SELEN run, described in Section 7, runs in about 34.5 minutes on an Apple MacBook with a 2GHz dual-core Intel Core 2 Duo CPU and in about 10.5 minutes on an Apple Mac Pro with two 2.8Ghz quad-core Intel Xeon CPUs.

The execution time of SELEN scales with the number of grid pixels N_p and the maximum harmonic degree ℓ_{max} as $t_{exe} \sim N_p N_h$ where $N_h = (\ell_{max}+1)(\ell_{max}+2)/2$ is the number

Cygwin is freely downloadable from: http://www.cygwin.com

The Generic Mapping Tools can be downloaded from: http://gmt.soest.hawaii.edu/

http://www.gnu.org/software/autoconf/

Available from: http://www.g95.org

Available from: http://gcc.gnu.org/gfortran

More information at: http://software.intel.com/en-us/fortran-compilers

The g95 compiler does not support OpenMP, so it is not possible to run SELEN in parallel with this compiler.

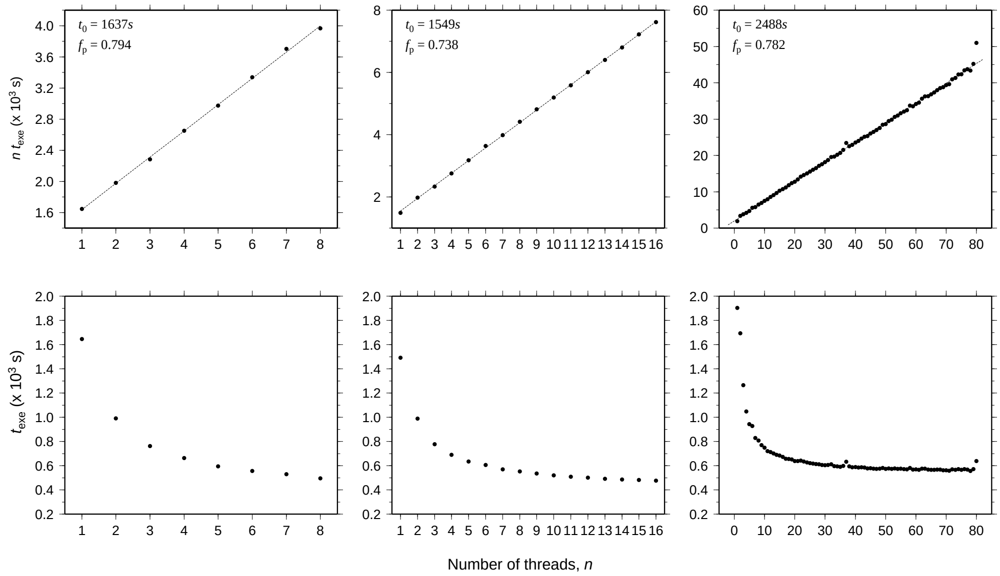

Figure 2: Scaling of execution time for the TEST run with the number of threads. Test systems are: an Apple Mac Pro configured with two 2.8GHz quad-core Intel Xeon CPUs ("Harpertown"–class, model number E5462) and 16GB DDR2 RAM, running Mac OS X 10.5.8 (left frames); an HP ProLiant BL460c G8 configured with two 2.70GHz eight-core Intel Xeon CPUs ("Sandy Bridge"–class, model number E5-2680) and 128GB DDR3 RAM, running Red Hat Enterprise Linux 5.8 (middle frames); an HP ProLiant DL98G7 configured with eight 2.0GHz 10-core Intel Xeon CPUs ("Westmere"–class, model number E7-2850) and 2TB DDR3 RAM, running Scientific Linux release 6.3 (right frames). In top frames, a dashed line represent the fit of Eq. (37) to the measured points; corresponding values of t_0 and f_p are given for each system.

of SHs for $\ell \leq \ell_{max}$. In view of Eq. (33), as a rough approximation

$$t_{exe} \sim (R\ell_{max})^2\,, \tag{35}$$

where R is the grid resolution parameter. The memory footprint of SELEN follows a similar scaling law. For high resolution runs (large values of ℓ_{max} and consequently of R, see Eq. 34), disk I/O can become a considerable fraction of t_{exe}, and these relations may no longer be valid.

On multi–core systems, SELEN can use multi–threading to reduce computation time (multi–threading was not implemented in previous version of SELEN). Multi–threading will automatically be enabled if it is supported by the system and, as a default, all the available cores are used. If the user wishes to leave some resources free for other tasks, the `OMP_NUM_THREADS` environment variable has to be set to the number of usable cores before launching SELEN .

Fig. 2 shows the execution time t_{exe} of the TEST as a function of the number of threads n for three test systems. If the inter-process communication overhead is negligible, the relationship between t_{exe} and n is well approximated by

$$t_{exe}(n) = t_0 \left(f_s + \frac{f_p}{n} \right), \tag{36}$$

where f_s and f_p are the serial and parallel fractions of the code ($f_s + f_p = 1$), and t_0 is the serial execution time (Amdahl, 1967). Eq. (36) can be rewritten as

$$n t_{exe}(n) = t_0 \left(f_p + f_s n \right), \tag{37}$$

showing that when the speedup of the parallel portion of the code is linear, the relation between nt_{exe} and n is also linear. Measured values of nt_{exe} for the TEST run are shown in Fig. 2. By fitting Eq. (37) to the points in Fig. 2 it is possible to evaluate the parallel fraction f_p of the code, which for the TEST run is within 0.7 and 0.8. This value depends on the specific options selected in the `config.dat`; for instance, enabling the execution of the GMT scripts (which are sequential tasks) results in a lower parallelism level.

5.2 Configuration and execution files

The configuration and the execution of the program involve several files and scripts in the SELEN directory. For a normal execution of SELEN, the user only needs to edit and modify the configuration file `config.dat`.

The relevant scripts and programs for configuration and execution are:

`config.dat` is the configuration file of SELEN. It is a plain text file that can be easily configured by the user according to the specific tasks to be accomplished. The structure of the file and the configuration rules are described in detail in Section 6.2 below. A sample `config.dat`, configured for a TEST run, is provided in Section 8.4,

`config.f90` is the SELEN setup program. This program *i)* parses `config.dat`, *ii)* creates the shell script `selen.sh` which physically executes the SELEN run, *iii)* creates the "include file" `data.inc`, which contains declarations useful to all the Fortran units needed for the execution (the full list is given in Table 4), *iv)* creates the GMT scripts needed T4
for visualization of the results. The implementation of new features in SELEN will demand, in general, adding new subprograms to `config.f90` or modifying the existing ones. The setup program also performs several checks on the input parameters, and exits with an error message if inconsistencies are found.

`selen.sh` is a shell script, created by `config.f90`, containing execution commands for the Fortran units of SELEN which are required to fulfill the user's tasks. This script also creates the `depot` directory structure where the SELEN outputs are stored, and purges the working directory from temporary files before and after the execution of the program.

5.3 Input and output data

Input. The SELEN input data are organized in several sub–directories:

1. Folder `DATA` collects various files with information about geophysical and geodetic sites of interest and other miscellanea data,
2. Folder `ICE-MODELS` contains the files describing the time chronology of each the ice models available by SELEN,
3. Folder `VSC` stores a collection of viscosity profiles.
4. Folder `INPUTS` stores user–generated input files, such as pixel table files, pre–computed SH coefficients at the grid pixels and SH coefficients of the ice thickness data.

Output. The SELEN outputs consist of:

1. ASCII files with various formats,
2. PostScript and PDF graphic files,
3. GMT scripts.

After each run, SELEN outputs are stored in a subfolder `depot-`*name*, where *name* is a 4–character label defined by the user in the `config.dat` file (see Section 6.2). The `depot` output subfolders are created into the `DEPOTS` folder of the SELEN directory, and their structure is described in Table 5.

Some of the SELEN intermediate outputs, such as SH coefficients at the grid pixels, SH coefficients of the ice thickness data or pixel table files, can be reused as input files for subsequent SELEN runs. After a SELEN run, these files are stored in the `run` folder of the SELEN working directory; in order to make them available for later reuse, they have to be manually moved into the `INPUTS` folder.

6 Setting up and running SELEN

6.1 Installation

The installation of SELEN is straightforward. The user just needs to un–zip the SELEN package in any chosen folder. If not already available on the system, a supported Fortran compiler and the GMT public domain software (Wessel and Smith, 1998) must be also installed.

6.2 Configuration

The configuration file of SELEN is the plain text file `config.dat`. Here we describe its structure and illustrate the options available to the user. A sample configuration file, employed for a `TEST` run in Section 7, is listed in Section 8.4.

File `config.dat` is composed of two sections:

- `SECTION 1` contains the general settings of SELEN. It defines the approximation (mode of solution) employed to solve the SLE, the rheological profile of the Earth model and the spatiotemporal features of the surface load (ice model).

  `SECTION 1` is marked by the header:

```
...
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
This is SECTION (1) of "config.dat": SELEN settings
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
...
```

  The options for `SECTION 1` are described in Section 6.2.1.

- `SECTION 2` can be configured to schedule the outputs of SELEN. These are ASCII files and tables, plots and diagrams. The particular set of outputs will of course depend on the kind of analysis planned by the user. After execution, the outputs are collected in a `depot` folder (see Table 5).

  `SECTION 2` is marked by the header:

```
...
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
This is SECTION (2) of "config.dat: SELEN outputs
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
...
```

  The options for `SECTION 2` are described in Section 6.2.2.

In both sections, the options are organized in text blocks like this:

```
...
====> TITLE for this block of options
lab-1   Option-1      Description      switches and more data
lab-2   Option-2      Description              ...
 ..      ...              ...                  ...
 ..      ...              ...                  ...
lab-n   Option-n          ...                  ...
...
```

where the three–character string `lab-k` labels `Option-k`, which comes with a short `Description`. The field `switches and more data` has the form

`'y' 'filename'`

or

`'n' 'parameter1' 'parameter2'`

where `'y'` and `'n'` stand for *yes* and *no*, respectively.

The only formatting requirements of file `config.dat` are: *i)* labels `lab-k` must be placed in the first three columns of the file; *ii) ALL* the options, switches and data *must* be enclosed in primes and can be freely placed on the corresponding line. Lines in `config.dat` which do not begin with a valid label (i.e., a label recognized by the setup program `config.f90`) are ignored, and therefore can be used for user comments.

6.2.1 Program settings

The options of `SECTION 1` of `config.dat` are organized in eight text blocks:

1. *Environment settings,*

2. *Mode of solution,*

3. *Spatial resolution and reference frame,*

4. *Choice of the rheological model,*

5. *Choice of the ice model,*

6. *Computation of SHs at grid pixels,*

7. *Ocean Function,*

8. *Output data archive,*

which are sequentially described below in separate paragraphs.

1. Environment settings

```
...
====> PURGING option ----------------------------------------------------------
110  Purging the wdir before & after execution                          'y'
     [see config.f90 for purged filenames extensions ]
...
```

In this first section of `config.dat`, option 110 can be used to remove auxiliary files from the working directory before SELEN is executed (this is particularly useful in the case the previous run of SELEN has crashed, possibly leaving some junk files around).

2. Mode of solution

```
...
====> SOLUTION of the SLE -----------------------------------------------------
130    Iterations & mode of solution                              '5'    '1'
Modes:  1= Gravitationally self-consistent (GSC)
        2= Elastic GSC / 3="Eustatic" / 4="Woodward" / 5="No Ice"
```

Here, we solve the SLE in "mode 1", i.e. in its full "Gravitationally self-consistent" form expressed by Eq. (21), and we configure SELEN for five iterations (generally three ensure convergence of the iterative scheme, see Spada and Stocchi 2007).

Other modes of solution available through option 130 above are discussed by Spada and Stocchi (2007); experimenting different solution modes can indeed be very useful to understand the physics behind the SLE and how the program works.

3. Spatial resolution and reference frame

```
...
====> MAXIMUM HARMONIC DEGREE -------------------------------------------------
140    LMAX                             '128'

====> REFERENCE FRAME ---------------------------------------------------------
145    Includes degree 1 Love numbers (CM/CE frames)              'y'  'CM'

====> TEGMARK RESOLUTION ------------------------------------------------------
150    R                                '33'
151    Prepare a new pixel table (y/n, filename)        'y' 'px-table-r44.dat'    ...
```

In this example, the SLE is solved to maximum degree $\ell_{max} = 128$ (option 140).

LDCs of harmonic degree $\ell = 0$ are not computed since they vanish identically because of incompressibility (the volume of the Earth is constant). However, they would not play any role even for a compressible Earth since the SLE includes explicitly the constraint of mass conservation, expressed by Eq. (9).

If the computation of LDCs of harmonic degree $\ell = 1$ is requested (145) (LDCs of harmonic degree one are discussed in e.g. Spada et al. 2011), the program outputs can be expressed in two possible reference frames. The first (`CM`) is the reference frame with origin in the center of mass of the whole Earth (including the solid and fluid portions), while the second is the reference frame with origin in the center of the undeformed Earth (`CE`). It is important to note that the solution of the SLE, i.e. sea level change $S(\omega, t)$ (see Eq. 5), is the same in the two frames. The same holds for RSL (4). However, surface displacements, the sea surface variation and the gravity potential variations are indeed sensitive to this choice.

The grid resolution parameter `R` (option 150) determines the number of pixels of the Tegmark grid by Eq. (33) and the spatial resolution of the oceans/continents distribution (see caption of Fig. 3 for more details). Note that parameters `LMAX` and `R` must obey constraint

(34), which ensures an optimal integration on the sphere. This condition is checked by `config.f90` and an error message is displayed if it is not satisfied.

When a new Tegmark grid is generated, SELEN identifies oceanic and continental grid pixels by invoking the GMT utility `gmtselect`, and creates a "pixel table" file (in the example above, `px-table-r44.dat`) where grid pixels are flagged as "wet" or "dry". Since this process can be relatively time consuming, especially for large values of grid resolution `R`, a previously computed pixel table file can be reused in subsequent runs (if `R` is unchanged) by setting `'n'` in option `161`. In this case, the pixel table file has to be placed in the `INPUTS` folder.

4. Choice of the rheological model

```
...
====> RHEOLOGICAL MODEL --------------------------------------------------------
160    Rheological profile info:                          '2' '2' 'vsc_BJ97.dat'
...
```

The choice of the Earth rheological profile, necessary for the computation of the LDCs, is made through option `160`. In SELEN, the computation of the LDCs is accomplished by TABOO (Spada, 2003a, Spada et al., 2004), a program based on the Viscoelastic Normal Modes theory (Peltier, 1974).

Option `160` above allows to provide the number of desired mantle layers `NV` (here `NV=2`) and a code (here `CODE=2`). These parameters identify the values of the density and of the shear modulus of each layer as well as the radii of the interfaces between layers. The reader is referred to the TABOO User guide for information about the options available for `NV` and a model `CODE` (see, in particular, Appendix 5.4 of the TABOO program user guide, Spada 2003b). Note that all models available by TABOO (hence by SELEN) assume an elastic lithosphere and a perfectly homogeneous and inviscid core.

In the sample configuration above, the Maxwell viscosities of the mantle layers and the lithospheric thickness are provided by the user–supplied ASCII file `vsc_BJ97.dat` (this file must be placed in folder `VSC` before execution).

5. Choice of the ice model

```
...
====> ICE MODEL ----------------------------------------------------------------
170    Ice file name                                 'ice3g.dat'
171    Prepare a new SH ice file (y/n, filename)       'y'  'ice3g-l128.dat'
172    Ice history time step (kyrs)                       '1.0'
...
```

The ice file name is set by option `170` (this name identifies model ICE–3G of Tushingham and Peltier 1991).

With option `171`, we are scheduling the computation of a new set of SH coefficients of the ice thickness distribution of ICE–3G to harmonic degree `LMAX` (consistently with option `140`). These will be written on the ASCII file `ice3g-l128.dat` in the `run` directory. Since

http://samizdat.mines.edu/taboo/user_guide.pdf the computation of the SH coefficients is relatively CPU intensive, the same file can be utilized in subsequent runs if the LMAX parameter is not changed, setting 'n' in option 171 and copying it into the INPUTS folder.

In folder ICE-MODELS, various ice models are available (see Table 6). These include some of the previous ICE–X models developed by Prof. W. R. Peltier and co–workers (ICE–1 and ICE–5G). Folder ICE-MODELS also contains individual regional components of these ice models and some other "test" ice models. Using specific Fortran formats described in the SELEN setup program config.f90, the user can indeed introduce other *ad hoc* ice models according to specific purposes.

6. Computation of SHs at grid pixels

```
...
====> SPHERICAL HARMONICS (SH) FILE AT PIXELS ---------------------------------
180    A new SH file  (y/n, filename)                  'y'  'sh-r33-l128.bin'
...
```

Using option 180, the user can specify whether to schedule a new computation of SHs at the grid pixels or to employ a pre–computed SH file. Since SH computation is an intensive task, re–using pre–computed SHs can be particularly convenient. The SH set is sensitive to the R and LMAX values (options 140 and 150). We experimented that using a SH binary file generated on a different system (or even with a different compiler) can lead to unexepected results.

In the case a pre–computed file with name sh-r33-l128.bin exists already in the INPUTS folder, the switch 'n' can be set in option 180 to avoid a new, time–consuming computation.

7. Ocean Function

```
...
====> OCEAN FUNCTION (OF) -----------------------------------------------------
190    A new OF SH decomposition (y/n, filename)       'y'  'of-l128.dat'
...
```

With option 190, the user can supply the name of the file that will store the coefficients of the SH decomposition of the OF (the ocean function $\mathcal{O}(\omega)$ is defined by Eq. 15). The chosen filename contains information about the values the LMAX parameter (option 140).

If a pre–computed file of SH OF coefficients with name of-l128.dat exists in INPUTS, the switch 'n' can be set to avoid a new computation.

8. Output data archive (depot)

```
...
====> REPOSITORY LABEL --------------------------------------------------------
195    The depot name  (four characters)               'T001'
```

The name of the depot where the output data will be stored is given using option 195. It must be four–characters long (in the example above, the name of the depot folder will be depot-T001). The general structure of the SELEN depot is described in Table 5. By default, all the depot subfolders will be created by SELEN, regardless of the effective configuration of the corresponding tasks.

The depot folder will be created in the DEPOTS directory. If a depot named depot-T001 already exists in DEPOTS, the user is warned by a message on the monitor that existing data will be overwritten.

6.2.2 Output settings

The options of SECTION 2 of config.dat are organized in eight text blocks:

1. *Execution of the GMT scripts,*

2. *Pixelization and grid,*

3. *Ocean function,*

4. *Ice model,*

5. *Spectral properties,*

6. *Relative Sea Level,*

7. *Sea level change at tide gauges,*

8. *Present geodetic variations*

which are sequentially described below in separate paragraphs.

1. Execution of the GMT scripts

```
...
====> EXECUTION of the GMT SCRIPTS --------------------------------------------
200    Execution of GMT scripts during the SELEN run (y/n)      'y'
...
```

During execution, SELEN creates various GMT scripts that have the purpose of plotting maps, diagrams, etc. based on the program outputs (various examples are given in Section 7.2 for a TEST run). Since for high–resolution computations the execution of the GMT scripts can be quite time consuming, the user can control their execution by option 200 above.

When 'n' is set, SELEN will produce the GMT scripts but these will not be executed. In this case, the outputs of SELEN can be easily transformed in plots at a later stage (the GMT scripts, generated by the setup program config.f90, are in fact copied into the same depot folder where the SELEN output data are stored). Alternatively, they can be visualized with other graphical tools preferred by the user. In particular, for high–resolution runs, some of the default GMT scripts created by SELEN may be not efficient.

2. Pixelization and grid

```
...
====> PIXELIZATION & WINDOW ---------------------------------------------------
205    Pixelization maps (y/n)                              'y'
206    Window function evaluation & plot (y/n)                  'n'
...
```

This text box can be configured to visualize the geometrical properties of the grid corresponding to the grid resolution parameter `R` (`SECTION 1`, option 150). Plotting the grid (option 205) can be useful to familiarize with the numerical solution of the SLE or for teaching purposes.

By the switch with label 206 the user can perform a numerical test of the SH orthogonality on the Tegmark grid, based on the evaluation of a "window function" (Tegmark, 1996). When condition (34) is met, the SH orthogonality is verified to a very high–precision (otherwise SELEN will stop).

Data concerning the pixelization and the window function are stored, after execution, in folders `depot-T001/px` and `depot-T001/wnw`, respectively. The grid employed in the `TEST` run described in Section 7 is visualized in Fig. 3. The caption provides more details on how the pixelization is performed by SELEN.

3. Ocean function

```
...
====> OCEAN FUNCTION (OF) -----------------------------------------------------
210    Present-day OF map & reconstruction (y/n)               'y'
215    Plot of OF degree variance (y/n)                            'n'
...
```

In SELEN, the ocean function (OF, defined by Eq. 15) plays an important role, since it defines the domain upon which the ocean load is acting (see Eq. 14). Option 210 can be used for plotting the OF and its SH reconstruction (at degree `LMAX`), based on the coefficients computed by option 190 above. The comparison between the original and the reconstructed OF can be useful for appreciating the effects of the truncation of the SH series at the chosen level of spatial resolution (an example is given in Figure 4). A plot of the degree variance of the OF can be obtained by switch 215.

All data (ASCII tables and plots, if required) concerning the OF are available in folders `depot-T001/of` after the execution of SELEN.

4. Ice model

```
...
====> ICE MODEL ---------------------------------------------------------------
220    Maps of original ice sheets  (y/n)                       'y'
221    Plot of Equivalent Sea Level (ESL)  (y/n)           'y'
222    Reconstruction & mapping of the ice sheets (y/n)       'n'
...
```

With these three switches, the users can control some outputs concerning the ice sheet model chosen (see `SECTION 1`, label 170). These include plots of the spatial distribution of the ice thickness (option 220), and the equivalent sea level (ESL) function (221). The reconstruction of the ice sheets thickness distribution from the SH coefficients is controlled by option 222.

After execution, all the ice model data (ASCII tables and plots, if required) are stored in the `depot-T001/ICE3G` (the folder name `ICE3G` is set by SELEN according to the ice model name in option `170`).

5. Spectral properties

```
...
====> EARTH MODEL SPECTRAL PROPERTIES -----------------------------------------
230    Plot LDCs, relaxation spectrum & residues for normal modes  (y/n)   'y'
...
```

This option controls various outputs and plots showing the spectral properties of the Earth model chosen (see `SECTION 1`, option `160`), as a function of harmonic degree ℓ. With term "spectral properties" we denote the isostatic relaxation spectrum, and the elastic, fluid, and viscous components of the LDCs. These quantities and their physical meaning are discussed in detail in e.g. Spada et al. (2011).

Information about the LDCs (tables with ASCII data and plots, if requested) are made available to the user in the `depot-T001/Love-Numbers-by-TABOO` folder.

6. Relative Sea Level

```
...
====> RSL PREDICTIONS AT SPECIFIC SITES ---------------------------------------
240    RSL analysis (y/n), database & format   'y'  'my_sealevel_database.txt'   '7'
241    Plot of RSL sites distribution (y/n)                          'y'
242    Site-by-site RSL predictions vs data & plots (y/n)        'y'  'y'
243    Scatterplot of RSL data & predictions (y/n)          'n'
244    Misfit between RSL data & predictions (y/n)         'n'
245    Table with all RSL data & predictions (y/n)      'y'

====> RSL REGIONS ---------------------------------------------------------------
250    Gobal RSL zones                               'n'
251    Regional RSL contour lines                            'n'   'rsl-region.dat'
...
```

When option `240` is set to `y`, the lines above allow for a Relative Sea Level (RSL) analysis at specific sites of a user–supplied database with a format recognizable by the setup program `config.f90` and by programs `rsl.f90` and `sh_rsl.f90`. A sample is given in file `DATA/sealevel.dat` (the format label is `0`, in this case), which contains information about sites for which radiocarbon–controlled RSL data are available, according to the compilation of Tushingham and Peltier (1992, 1993). Options `240-245` are self–explicatory (examples of the corresponding outputs will be given in Section 7 for a `TEST` run).

By option `250`, global visualizations of the so–called "Clark zones" (Clark et al., 1978) or "RSL zones"can be obtained. These are regions of the globe where the RSL curves show similar features. A regional RSL study can be obtained by switch `251`, where RSL contour lines are plotted at a given time. Data about the time of the analysis, the region of interest, and the plot style are supplied in file `DATA/rsl-region.dat`.

Information about the RSL analyses (tables with ASCII data and plots, if requested) are made available, after the execution, in various sub–folders of `depot-T001/rsl`.

7. Sea level change at tide gauges

```
...
====> SEA LEVEL CHANGE AT TIDE-GAUGE STATIONS --------------------------------
260    Tide-gauge (TG) analysis & database                'y' 'rlr-trends.txt'
261    Plot of TG stations distribution                      'y'
262    TG data scatterplot                                    'y'
263    Table of S, N, and U-dot predictions at TG sites           'y'
...
```

These options are designed for a study of the GIA effects at tide gauges (TGs). These are listed in the user–supplied file `DATA/rlr-trends.txt`, obtained from the Permanent Service for the Mean Sea Level, PSMSL. The Earth model and ice sheets distribution are those chosen by options 160 and 170, respectively.

If the switch 260 is set, the user can obtain a plot of the TGs distribution (option 261), a scatterplot of the TG data (262) and a summary table where rates of change of S, N and U at present time are given for each TG (option 263).

All data about this analysis are stored, after the execution, in the `depot-T001/tg` folder.

8. Present geodetic variations

```
...
====> GLOBAL PRESENT-DAY RATES -----------------------------------------------
270    Global maps of dot S, U & N                    'y'

====> 3D VELOCITY -------------------------------------------------------------
275    -Up, North, East, S, and N rates for sites in file       'n' 'NA_KK.txt'

====> REGIONAL PRESENT-DAY RATES ---------------------------------------------
280    Regional maps of dot S, U, & N            'n'
281      -1 Italy                                    'y'
282      -2 Mediterranean                           'y'
283      -3 Europe                                  'y'
284      -4 Fennoscandia                          'n'
285      -5 Greenland                            'y'
286      -6 North America                       'y'
287      -7 Antarctica                         'y'

====> STOKES COEFFICIENTS (SC) -----------------------------------------------
290    Rate of change of SC & range of degrees for plot      'y'  '2'  '20'
```

This section of the configuration file is dedicated to predictions of present–day variations of various geodetic quantities.

Option 270 gives access to global maps of *fingerprints* showing the rates of change of S, U and N, for the Earth model and ice sheets distribution chosen by options 160 and 170, respectively (the maps showing the fingerprints are available in folder `depot-T001/gmaps`). Regional plots are available by switch 280 and the regional options 281-287, with results provided in folder `depot-T001/rmaps`.

For sites listed in the user–supplied `DATA/NA_KK.txt`, switch 275 gives access to the Up, North, and East components of the present–day velocity field (this is particularly useful for the interpretation of GPS signals) and to $\dot{S}$, $\dot{U}$ and $\dot{N}$ as well (the dot indicates the time derivative). All these predictions are collected in folder `depot-TEST/geod/sites`.

By option 290, the user can obtain predictions for the GIA component of the present–day rates of change of the Stokes coefficients of the Earth's gravity field in a specific range of harmonic degrees (here, the range is $2 \leq \ell \leq 20$). The coefficients, which are normalized according to the GRACE conventions for spherical harmonics and expressed in units of 10^{-11} yr^{-1}, are available in folder `depot-T001/stokes`.

6.3 Execution

Before launching SELEN , the configuration script has to be invoked with the command:

```
$ configure
```

The `configure` script will automatically identify the available Fortran compiler on the system and configure SELEN accordingly. If several Fortran compilers are installed, the user can explictly select a compiler with the command

```
$ configure FC=name
```

where `name` is the Fortran compiler command (for instance, `gfortran` or `ifort`). The `configure` script will also check the availability of the GMT utilities. If no error is found, SELEN can be launched with

```
$ make run
```

This command will compile the SELEN program units, set up the working folder `run` in the SELEN directory and launch SELEN . A typical SELEN run consists of three main execution steps. At the end of execution, the SELEN outputs are available in the `depot` folder (see Table 5).

Step 1: Pre–computation of gridded SHs and of LDCs. At step 1, SELEN builds the spatial grid (see Section 4.3 and Fig. 3), obtains a discrete realization of the OF, performs the SH expansions of the ice thickness variation I and of the OF, and computes the LDCs. The SHs at degree ℓ_{max} are computed at all grid pixels,

Step 2: Numerical solution of the SLE. At step 2, SELEN solves numerically the SLE by the pseudo–spectral method, employing the iterative scheme described by Eqs. (32), and computes the SH coefficients of surface displacements and gravity potential variations,

Step 3: Computation of geophysical and geodetic quantities. In the last step, SELEN uses the solution of the SLE in the spectral domain to obtain predictions of geophysical and geodetic quantities by SH synthesis, according to the settings in the configuration file `config.dat` (see Section 6.2).

See rev. 2.3 of the Level–2 Gravity Field Product User Handbook at page 5, available from ftp://podaac.jpl.nasa.gov/allData/grace/docs/L2-UserHandbook_v2.3.pdf).

The execution of SELEN should produce various messages on the monitor, reporting the most important stages of the run. A sample is given in Section 8.5 below. SELEN can be reset by entering

```
$ make clean
```

which clears the SELEN configuration and deletes the SELEN working directory `run`. This command can be useful if the user wants to restart from scratch, for instance to recover from a previous SELEN crash.

7 A TEST run

To provide an example of the use of SELEN 2.9, we have configured the program for a TEST run.

We advise the user to run this test on his/her own PC and to compare the results with those obtained on our machine (an Apple MacBook configured with a 2GHz Intel Core 2 Duo and running Mac OS X 10.5.8).

7.1 Settings

The configuration file `config.dat` is reproduced in Section 8.4. A summary of the settings is given in Table 1.

Table 1: Summary of the TEST run settings.

Option(s)	Program settings		See Figure or Table:
130	Mode/iterations	1/3	
140, 150	Resolution (LMAX/R)	128/44	
145	Reference frame	CM	
160	Rheology (NV/CODE)	3/2	
160	Viscosity profile	vsc_VM2a.dat	Table 2
170-172	Ice model	ICE-5G	
Option(s)	**Output settings**	**y/n**	**See Figure or Table:**
205	Pixelization maps	y	Fig. 3
206	Window function	n	
210	OF maps	y	Fig. 4
215	OF degree variance	n	
220	Ice sheets maps	y	Fig. 5
221	ESL plot	y	Fig. 6
230	Spectral plots	y	Figs. 7, 8
240-245	Global RSL	y	Figs. 9, 10, 11, 12
250	RSL zones	n	
251	Regional RSL	y	Fig. 13
260-263	TG analysis	y	Fig. 14, Table 3
270	Global fingerprints	y	Figs. 15, 16, 17
275	3D crustal velocity	n	
280-287	Regional fingerprints	n	
290	Stokes coefficients	y	Fig. 18

7.2 Outputs

In this section we collect some of the Figures obtained with SELEN for the `TEST` run. The whole set of outputs for this run are available in the `depot-TEST` folder that comes with the SELEN package.

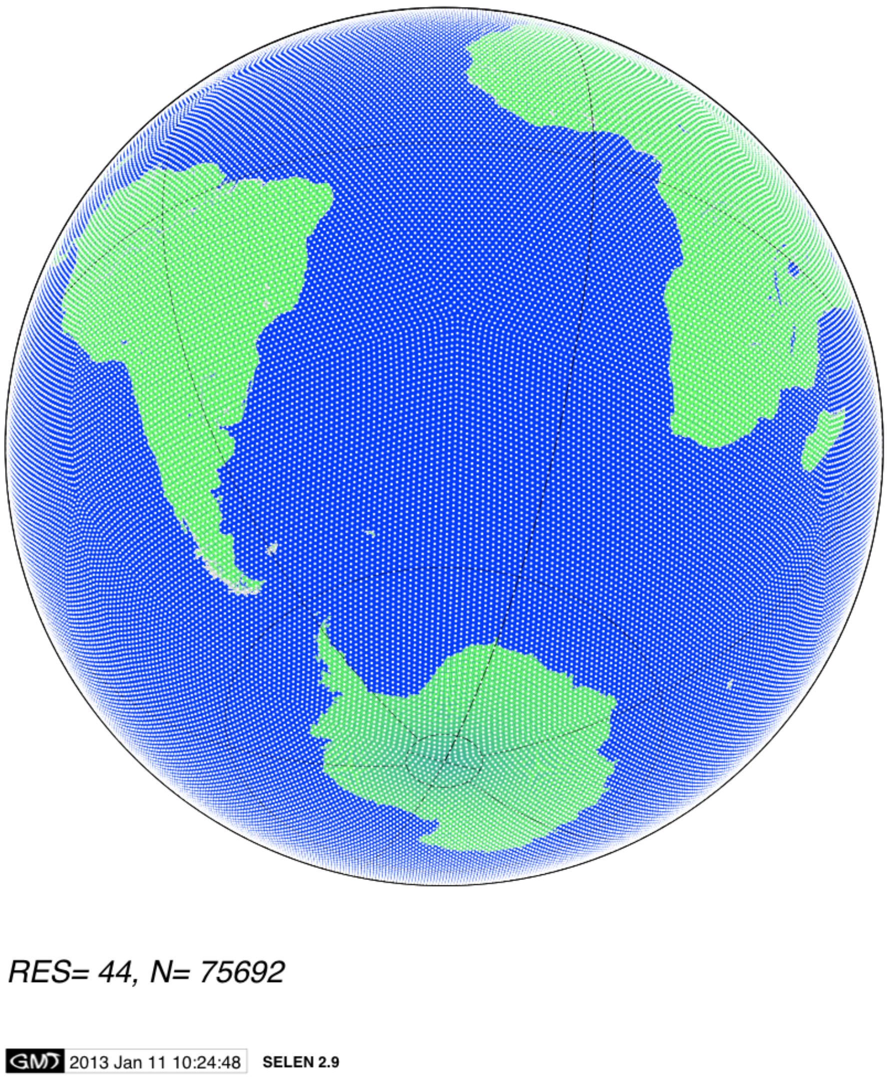

Figure 3: Pixelization of the sphere for the TEST run ($\ell_{max} = 128$, $R = 44$). Data about the grid, including spherical coordinates of "wet" (oceanic, blue) and "dry" (continental, green) pixels and postscript figures are found in folder `depot-TEST/px`. In SELEN, wet pixels are separated by dry ones using the GMT program `gmtselect` (Wessel and Smith, 1998). By default, SELEN employs the full resolution coastlines of GMT (`-Df`), and dry (wet) pixels are selected using option `-Ns/k/s/k/s` (`-Nk/s/k/s/k`) of `gmtselect`.

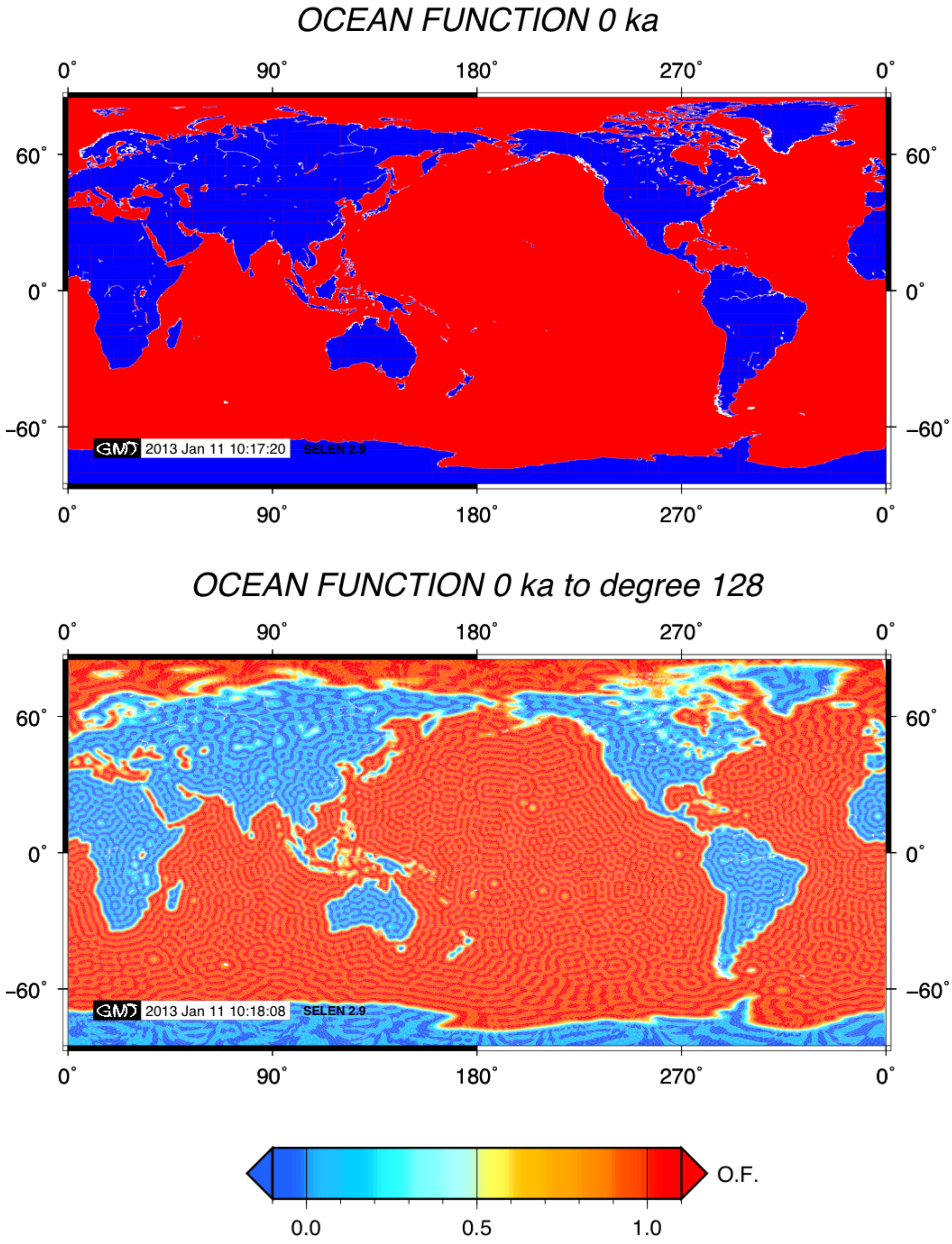

Figure 4: Original OF (top) and its reconstruction (bottom) obtained by synthesis of the SH coefficients to maximum degree $\ell_{max} = 128$ (`TEST` run). All the OF data are stored in folder `depot-TEST/of`.

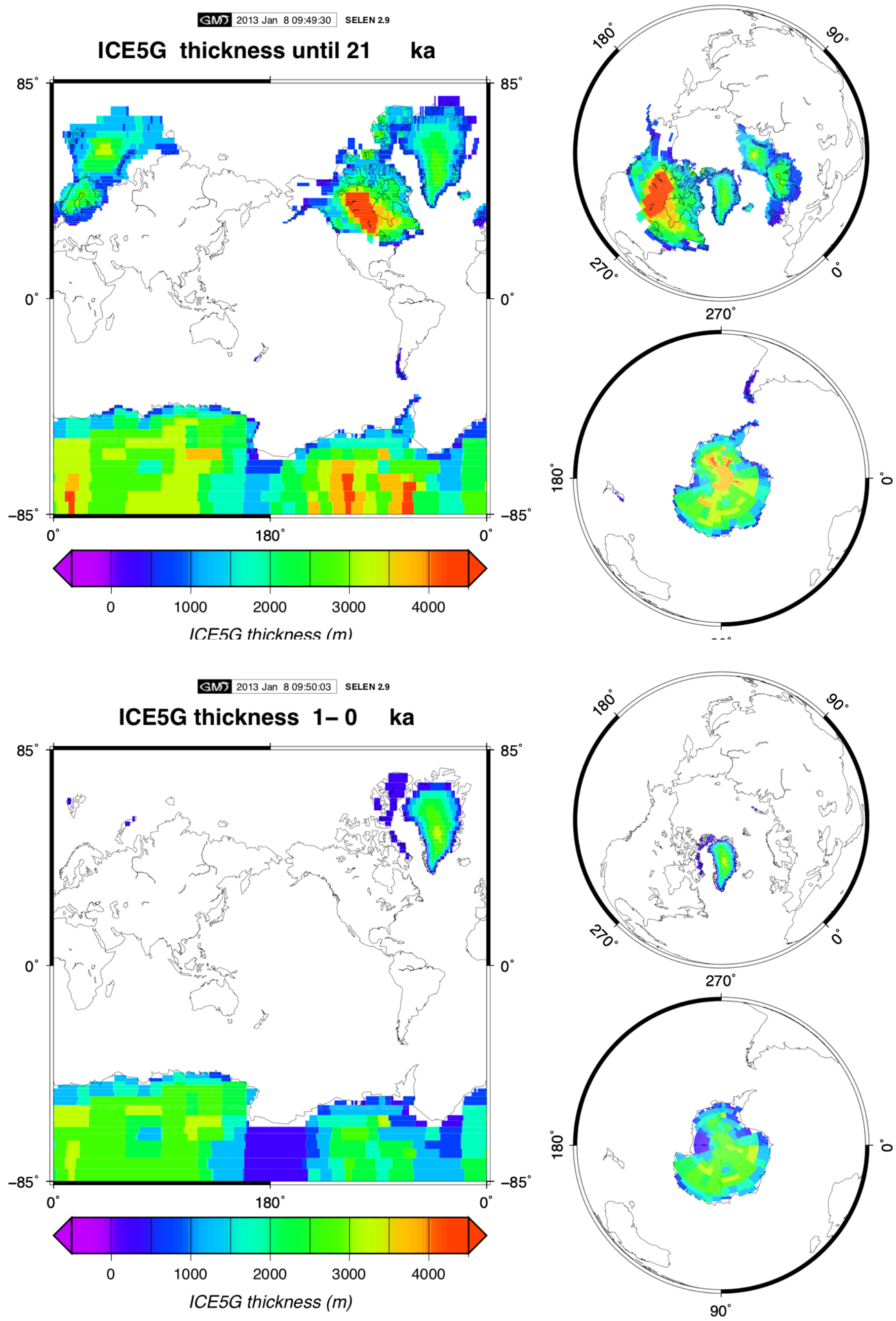

Figure 5: Ice thickness $T(\omega, t)$ at the LGM (21 kyrs ago, top) and during the most recent time increment, between 1 kyr BP and present time (bottom), according to model ICE–5G (Peltier, 2004) (`TEST run`). The time–history of the Equivalent Sea Level for this model is shown in Fig. 6. Maps for all the other time steps between LGM and present are available in folder `depot-TEST/ICE5G/original`.

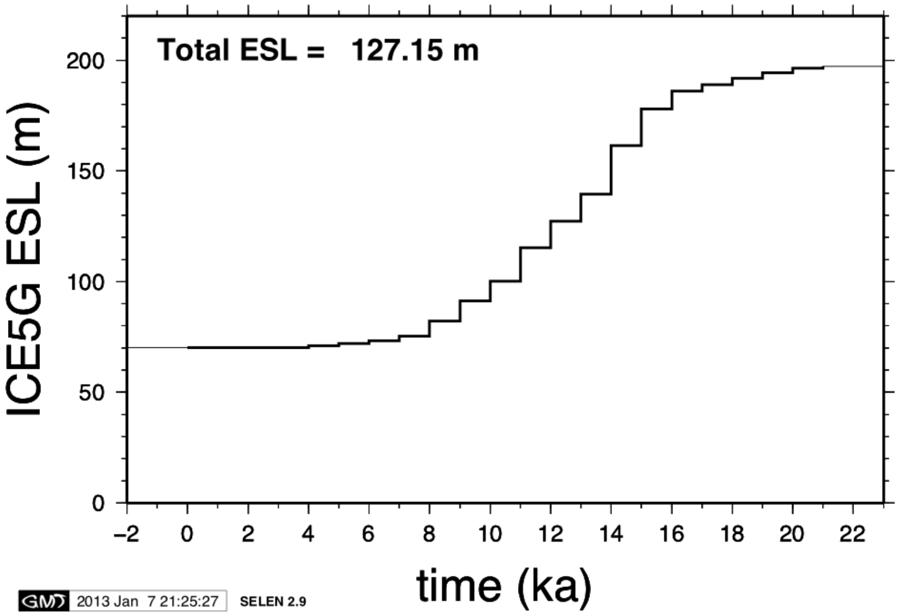

Figure 6: Equivalent Sea Level for model ICE–5G (Peltier, 2004) (TEST run). At a given time t before present, $\mathrm{ESL}(t) = (\rho_i/\rho_w)(V_i(t) - V_i(t_p))/A_o)$, where $V_i(t)$ is the ice volume, $V_i(t_p)$ is present day volume, and A_o is the area of the oceans surface. Hence, according to Eq. (12), the plot of ESL mirrors that of $S^E(t)$. The total ESL variation (~ 127 m) represents the difference between ESL at the LGM (21 kyrs ago) and the present day value.

Table 2: Model parameters for model VM2a, employed for the TEST run of SELEN. These correspond to the model with NV=3, CODE=2 and to the rheological profile described in file DATA/vsc_VM2a.dat. This model represents a volume–averaged version of the VM2 viscosity profile associated with the ICE–5G model of Peltier (2004).

Layer	Radius (km)	Density (kg m^{-3})	Shear modulus ($\times 10^{11}$ Pa)	Viscosity ($\times 10^{21}$Pa s)	Gravity (m s^{-2})
Lithosphere	6281–6371	4120	0.73	∞	9.707
Upper mantle	5951–6281	4120	0.95	0.5	9.672
Transition zone	5701–5951	4220	1.10	0.5	9.571
Lower mantle	3480–5701	4508	2.00	2.7	9.505
Core	0–3480	10925	0	0	10.622

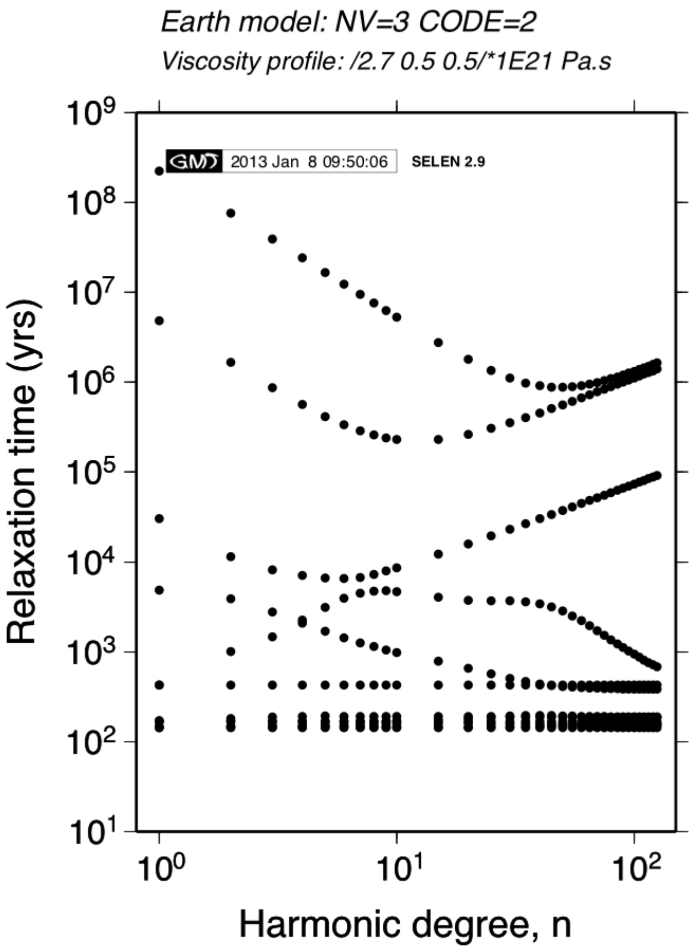

Figure 7: Isostatic relaxation spectrum for the rheological profile VM2a (see `TEST` run data in Table 2), showing the relaxation times as a function of harmonic degree $\ell = n$ in the range $1 \leq \ell \leq 128$. The spectrum data are stored in folder `depot-TEST/Love-Numbers-by-TABOO`. The physical meaning of the spectrum is discussed by e.g. Peltier (1974), Spada (2003a), and Spada et al. (2011).

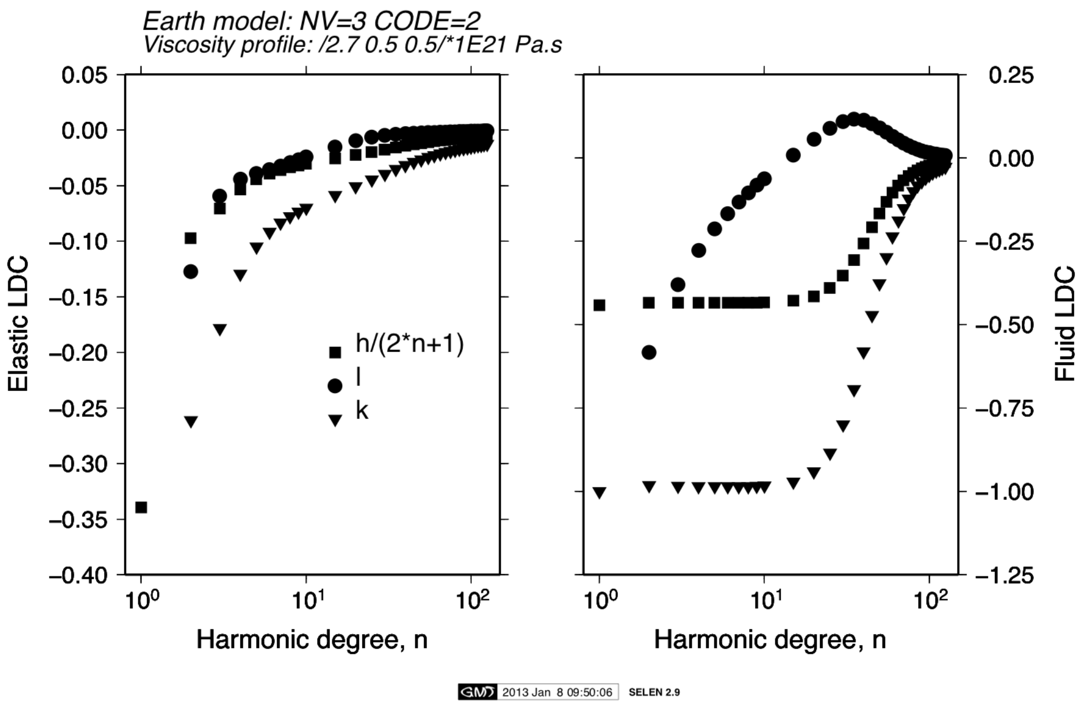

Figure 8: Elastic (left) and fluid (right) values of the LDCs h (associated with vertical displacement), l (horizontal displacement) and k (incremental gravitational potential) for the rheological model VM2a in the `TEST` run (see Table 2), as a function of harmonic degree ℓ. LDC h is normalized by $(2\ell+1)$. For the definition of the LDCs see e.g. Spada (2003a), and Spada et al. (2011).

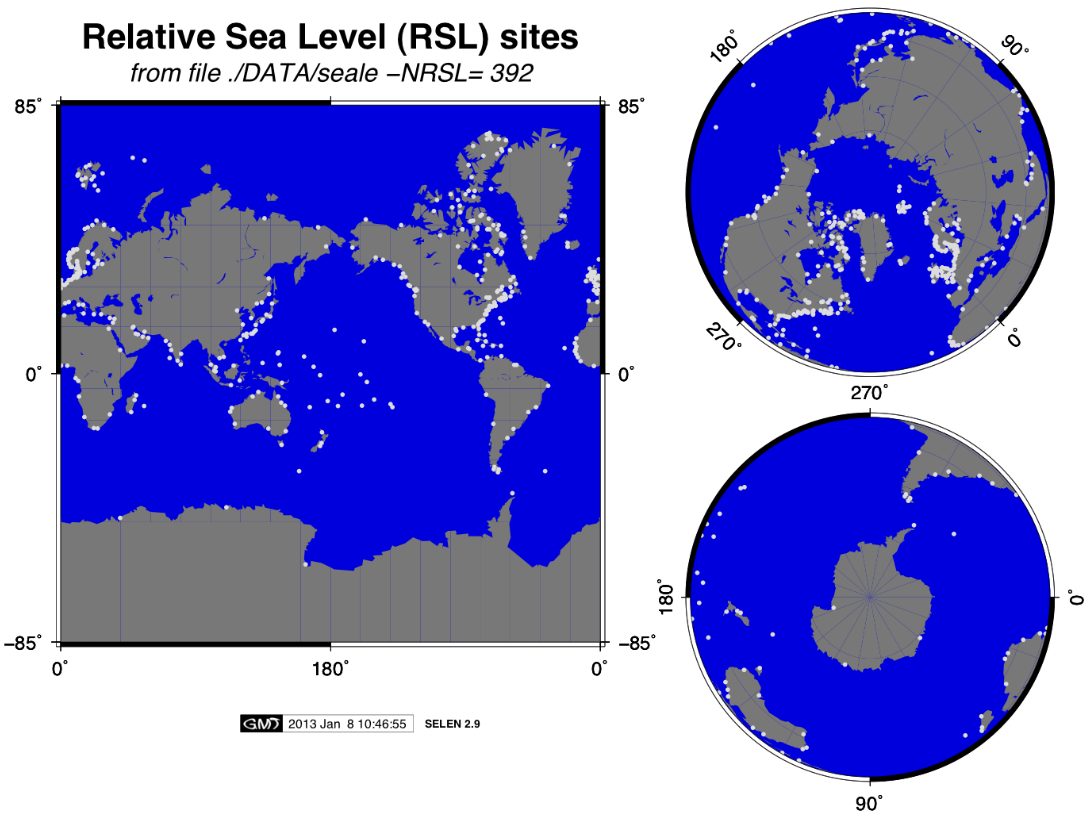

Figure 9: Geographical distribution of the 392 sites in file `sealevel.dat`, from which information about the history of RSL during the last $\sim 15,000$ years is available. This plot and more data about these sites are available from de folder `depot-TEST/rsl/rsl-sites`.

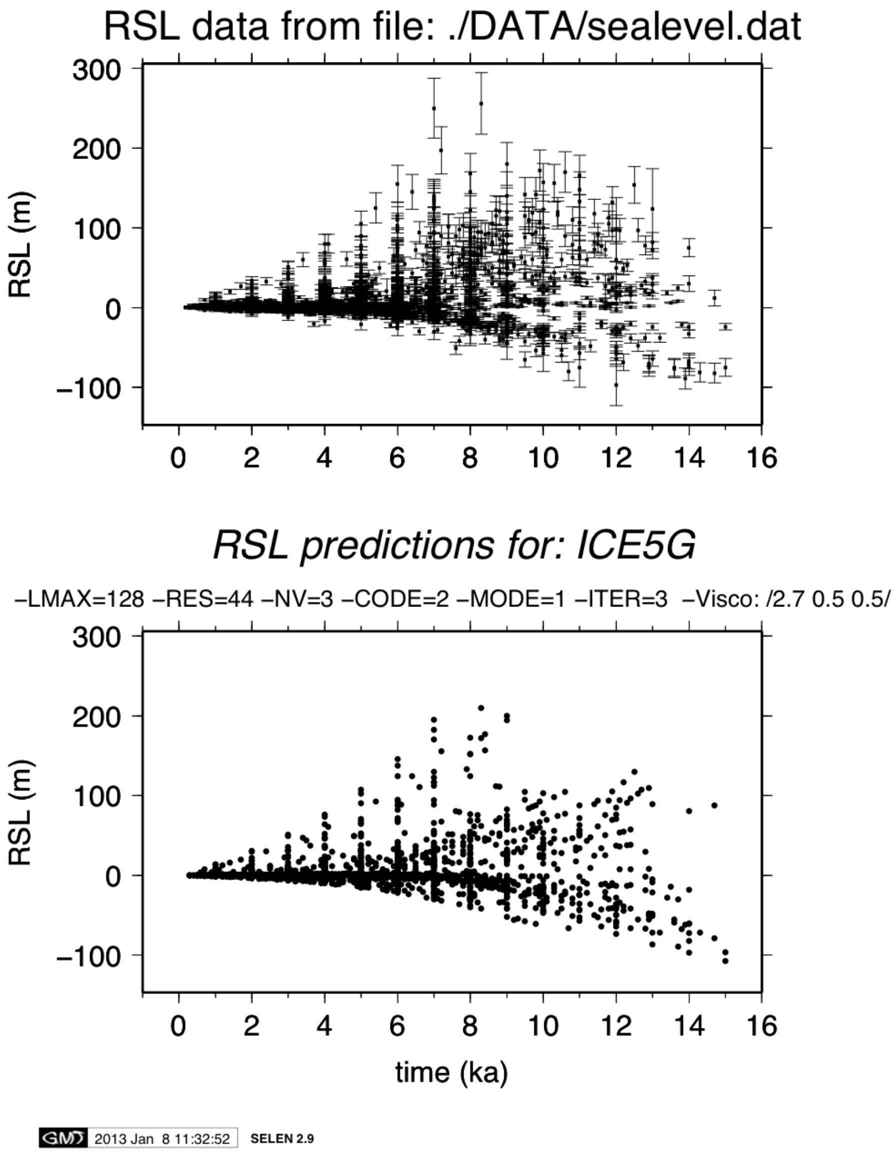

Figure 10: Scatterplot showing RSL observations (top) from sites of the compilation of Tushingham and Peltier (1992, 1993). RSL predictions at all sites, obtained solving the SLE in our TEST run of SELEN, are shown in the bottom frame. Data are stored in folder `depot-TEST/rsl/rsl-scplot`.

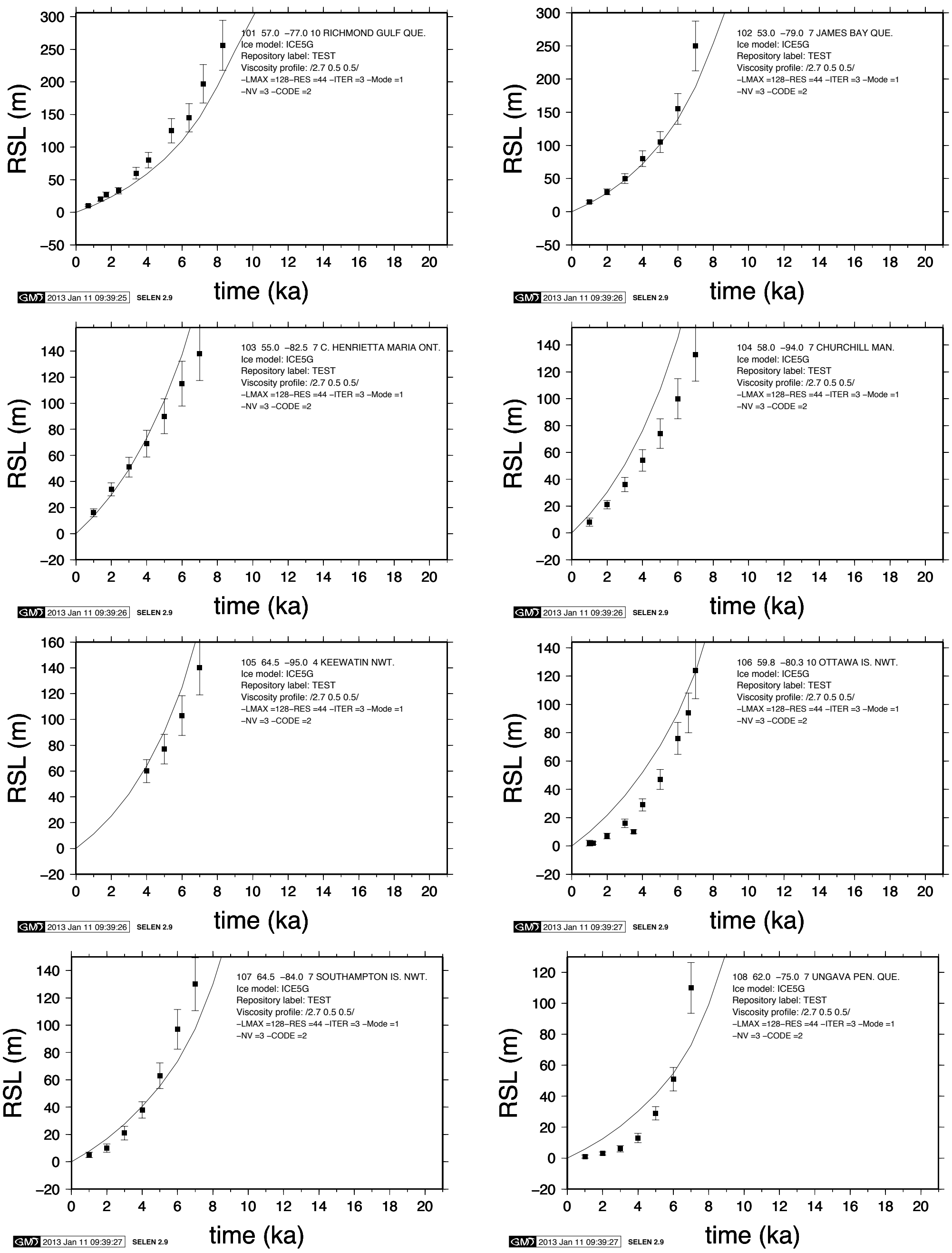

Figure 11: RSL observations (with error bars) pertaining to the eight sites of Hudson bay in file `sealevel.dat`, compared with SELEN predictions (solid curves). Basic parameters for the `TEST` run are summarized in each frame. Postscript and PDF figures are located in folders `depot-TEST/rsl/rsl-curves/ps` and `depot-TEST/rsl/rsl-curves/pdf`, respectively.

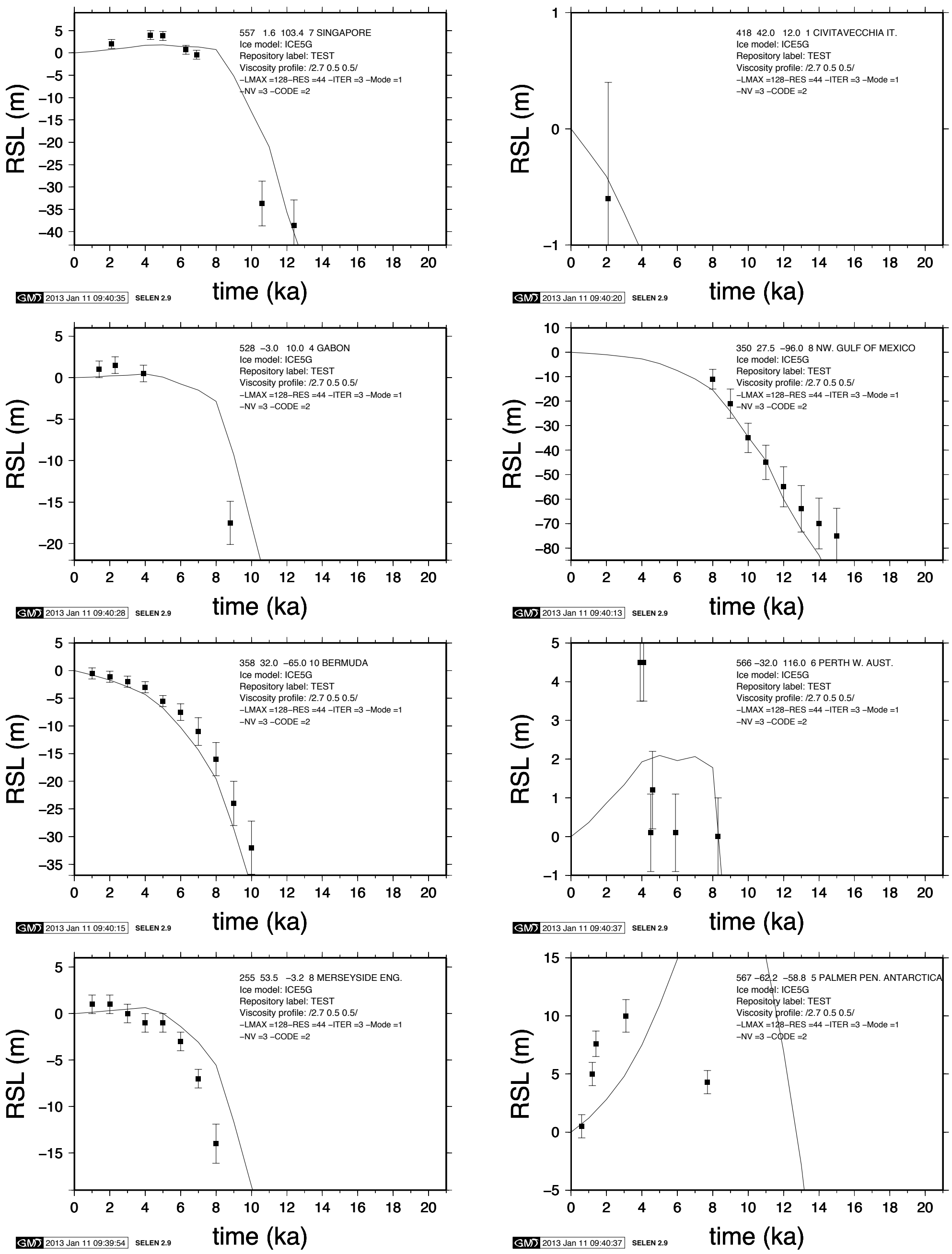

Figure 12: RSL curves at eight miscellanea sites located in the far field of the former ice sheets: Singapore, Civitavecchia (Italy), Gabon, NW Gulf of Mexico, Bermuda, Perth W. Australia, Merseyside (England), and Palmer peninsula (Antarctica). All the RSL predictions for the `TEST` run are found in `depot-TEST/rsl/rsl-curves`.

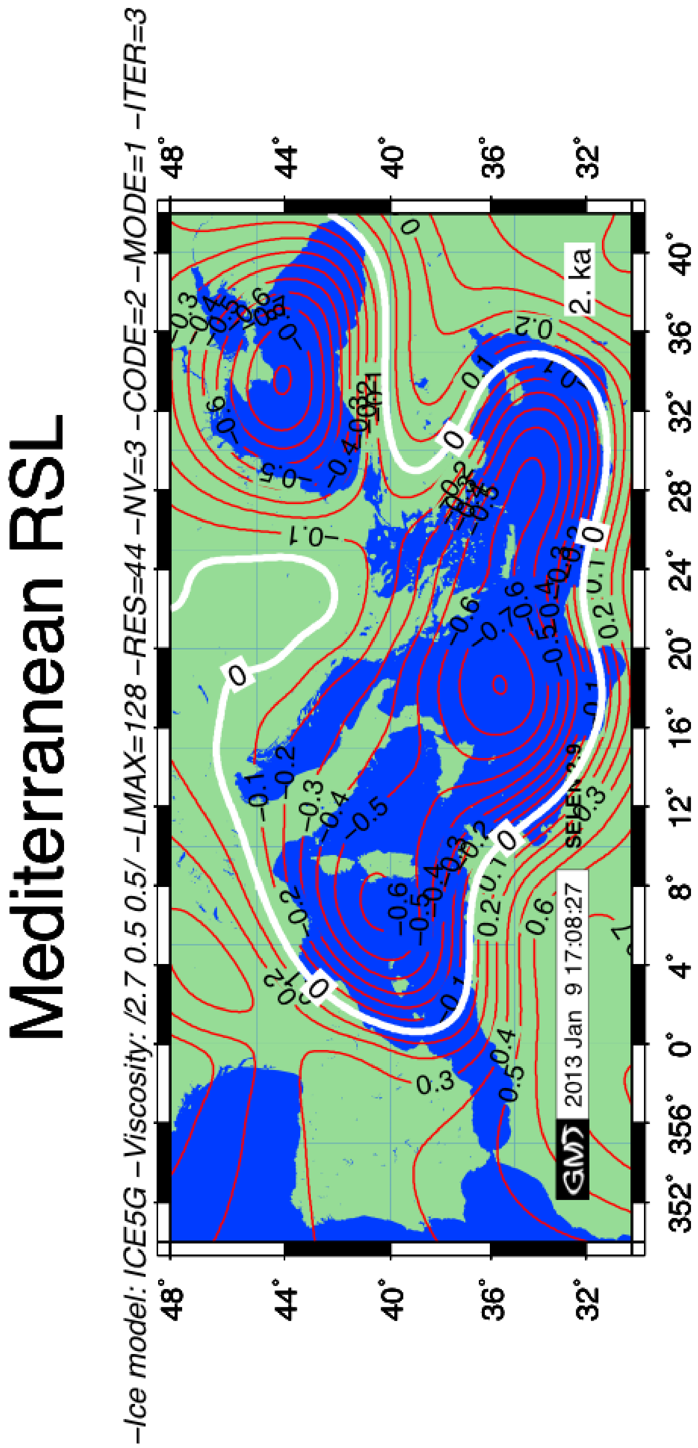

Figure 13: Example of RSL contour plot for the Mediterranean region, obtained at time 2 kyrs BP setting the configurations file `DATA/rsl-region.dat` (RSL is given in units of meters). The computations are based on the SELEN parameters employed in the `TEST` run (ice model ICE–5G and rheological parameters in Table 2).

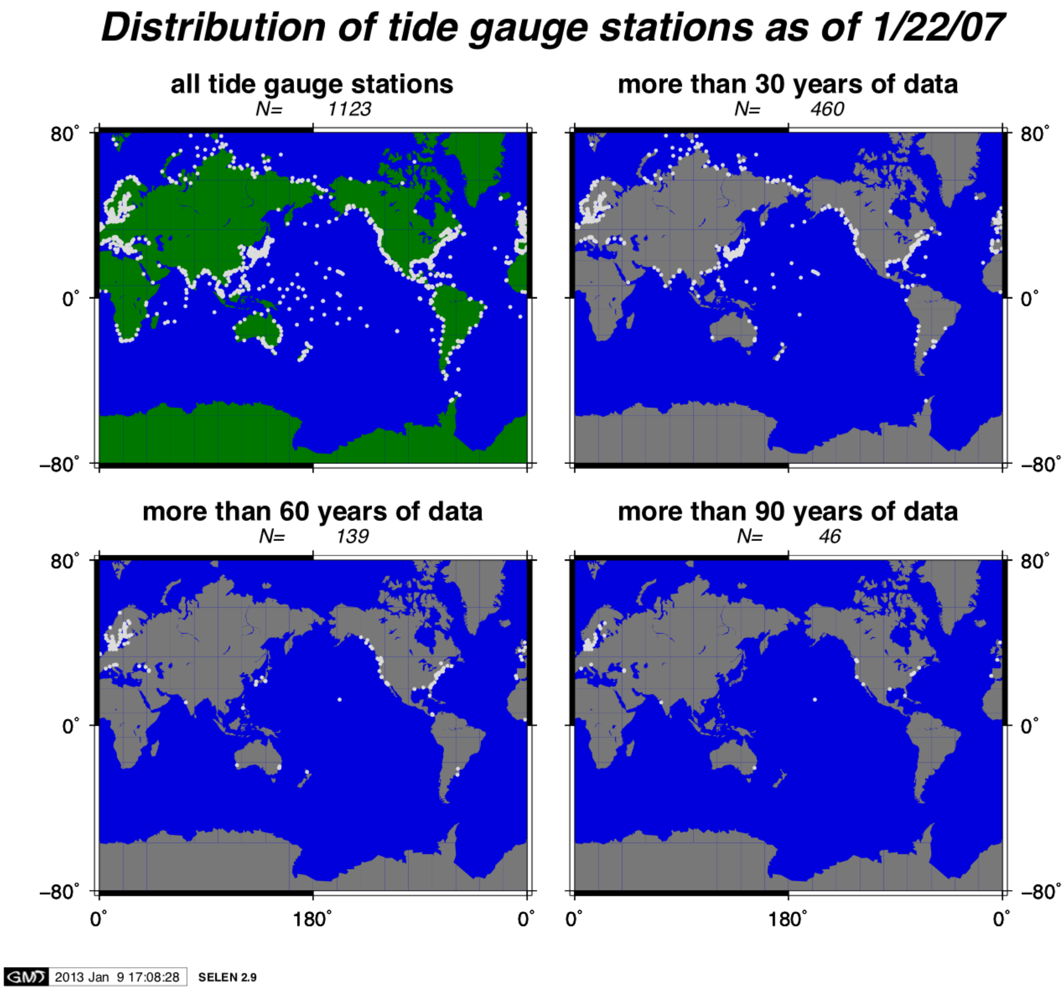

Figure 14: Geographical distribution of the TGs considered in run `TEST`, according to the number of annual records available from each station in the user–supplied input file `DATA/rlr-trends.txt`. Data and plots for this analysis are stored in folder in `depot-TEST/tgauges/tgauges-sites` after the execution of SELEN.

Table 3: Present–day trends of GIA–induced sea level change (`s-dot`), sea surface variation (`n-dot`), and vertical velocity of the solid Earth (`u-dot`), expressed in units of millimeters per year, at RLR (Revised Local Reference) PSMSL TGs in the user–supplied file `DATA/rlr-trends.txt` employed for the `TEST` run). The table also contains basic information on the TGs and an header with a short summary of the run parameters. This is an excerpt of the ASCII table `depot-TEST/tauges/tgauges-predictions/ptidegauges`.

```
2013.01.09 time=17.09.10
Ice model: ice5g.dat
Number of mantle layers:            3
Model code (see TABOO User guide):            2
Viscosity model: ./VSC/vsc_VM2a.dat
Thickness of the lithosphere (km): '90.'
Viscosity (bottom-to-top, Haskell units) '2.7'
Viscosity (bottom-to-top, Haskell units) '0.5'
Viscosity (bottom-to-top, Haskell units) '0.5'
SLE iterations:           3
SLE mode of solution:           1
Maximum harmonic degree:         128
Tegmark resolution:          44
-----------------------------------------------------------------------------------------------------------------------------
 PSMSL    yrs    range       trend  error     lon       lat       s-dot     n-dot     u-dot     PSMSL station name
 code             yrs          mm/yr          deg       deg       mm/yr     mm/yr     mm/yr     ==================
-----------------------------------------------------------------------------------------------------------------------------
010/001   40  1957 - 2003     2.23   0.46   338.067    64.150     0.147    -0.210    -0.357     REYKJAVIK
010/011    6  1958 - 1965     8.07   8.35   337.567    63.833     0.274    -0.219    -0.494     GRINDAVIK
015/011   22  1958 - 2001     1.50   0.34   353.233    62.017     0.645    -0.188    -0.826     TORSHAVN
025/001   44  1949 - 2003    -2.65   0.42    14.250    78.067    -0.888     0.057     0.974     BARENTSBURG
025/002   42  1949 - 1993    -1.36   0.46    14.250    78.067    -0.888     0.057     0.974     BARENTSBURG II (SPITSBERGEN)
025/021   20  1977 - 2004    -3.10   0.64    11.933    78.933    -0.312     0.031     0.354     NY-ALESUND
030/001   36  1953 - 1989    -1.26   0.62    62.583    76.200    -2.878     0.093     3.070     RUSSKAYA GAVAN
030/003   36  1953 - 1989    -1.28   0.62    62.583    76.183    -2.867     0.093     3.059     RUSSKAIA GAVAN II
030/007   19  1960 - 1978     2.43   2.52    60.217    69.600    -0.219    -0.064     0.148     BELYI NOS
030/014   28  1962 - 1990    -4.89   1.07    58.050    80.617    -2.807     0.070     2.992     KRENKELIA (HEISA OSTROV)
030/016   23  1950 - 1976    -3.10   1.91    52.700    72.367    -2.132     0.101     2.305     MALYE KARMAKULY
030/018   48  1952 - 2005     3.14   0.67    33.050    68.967    -3.090     0.290     3.403     MURMANSK
...
[omitted records]
...
240/001   10  1896 - 1913     1.29   1.43     9.367    41.233     0.267    -0.317    -0.587     LA MADDALENA
240/011   26  1897 - 1934     1.64   0.39     9.167    39.200     0.250    -0.324    -0.577     CAGLIARI
250/001   23  1897 - 1921     1.60   0.59     8.017    43.867     0.103    -0.297    -0.402     PORTO MAURIZIO
250/011   78  1884 - 1992     1.20   0.07     8.900    44.400     0.070    -0.290    -0.362     GENOVA
250/031   21  1897 - 1920     1.10   0.68    11.817    42.050     0.202    -0.308    -0.512     CIVITAVECCHIA
250/041   11  1901 - 1921     2.28   1.42    14.267    40.867     0.199    -0.314    -0.514     NAPOLI (ARSENALE)
250/051   18  1897 - 1921     2.18   0.66    14.267    40.867     0.199    -0.314    -0.514     NAPOLI (MANDRACCIO)
250/061    9  1951 - 1964    -2.69   3.08    15.650    38.100     0.276    -0.326    -0.605     REGGIO CALABRIA
260/011   15  1897 - 1919     0.98   0.63    13.333    38.133     0.250    -0.326    -0.579     PALERMO
260/028    9  1957 - 1968     8.54   3.68    15.300    36.667     0.314    -0.329    -0.647     CAPO PASSERO
260/031   12  1960 - 1971    -3.02   2.78    15.133    37.500     0.289    -0.328    -0.620     CATANIA
265/001   11  1991 - 2004     0.63   2.62    14.517    35.900     0.290    -0.329    -0.622     VALLETTA
270/006    6  1906 - 1911    -3.49   6.73    17.267    40.433     0.179    -0.313    -0.494     TARANTO
270/011    4  1961 - 1970    -0.58   1.63    18.500    40.133     0.176    -0.314    -0.492     OTRANTO
270/035    3  1970 - 1972     9.00   2.89    12.283    44.500     0.082    -0.284    -0.367     PORTO CORSINI
270/041   18  1889 - 1913     2.04   1.20    12.350    45.417     0.039    -0.273    -0.314     VENEZIA (ARSENALE)
270/051   45  1872 - 1919     2.55   0.42    12.333    45.417     0.039    -0.273    -0.314     VENEZIA (S.STEFANO)
270/054   82  1909 - 2000     2.39   0.16    12.333    45.433     0.038    -0.273    -0.313     VENEZIA (PUNTA DELLA SALUTE)
270/061   96  1905 - 2006     1.17   0.12    13.750    45.650     0.037    -0.268    -0.307     TRIESTE
...
[omitted records]
...
970/211    6  1962 - 2005     2.50   1.19   227.033    69.417     1.393     0.104    -1.295     TUKTOYAKTUK
A  /001    4  1967 - 1977     0.92   6.43   303.083   -63.300    -1.313    -0.050     1.272     BAHIA ESPERANZA
A  /003   43  1960 - 2004     1.57   0.37   295.733   -65.250    -1.326     0.030     1.361     ARGENTINE ISLANDS
A  /005   15  1985 - 2002     5.75   1.85   300.367   -62.483    -1.304    -0.061     1.251     PUERTO SOBERANIA
A  /024    8  1958 - 1978    -1.22   1.82   297.133   -64.900    -1.454     0.020     1.480     ALMIRANTE BROWN
-----------------------------------------------------------------------------------------------------------------------------
```

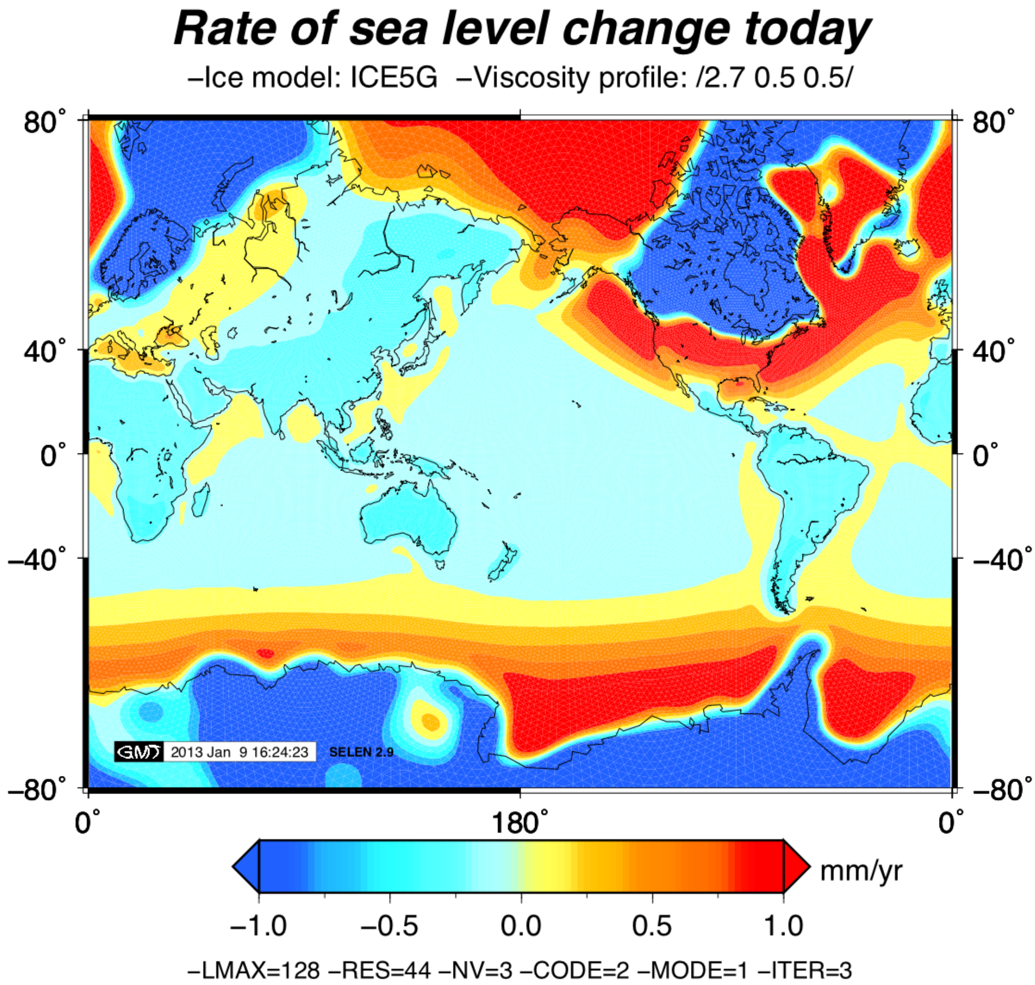

Figure 15: Global map showing the present–day rate of sea level change (*sea level fingerprint*) associated with GIA ($\dot{S}$) for our TEST run. Data and plots for this analysis are found in folder depot-TEST/gmaps. In this map, the $\dot{S}$ values vary in the range $[-17.01/+3.67]$ mm/yr.

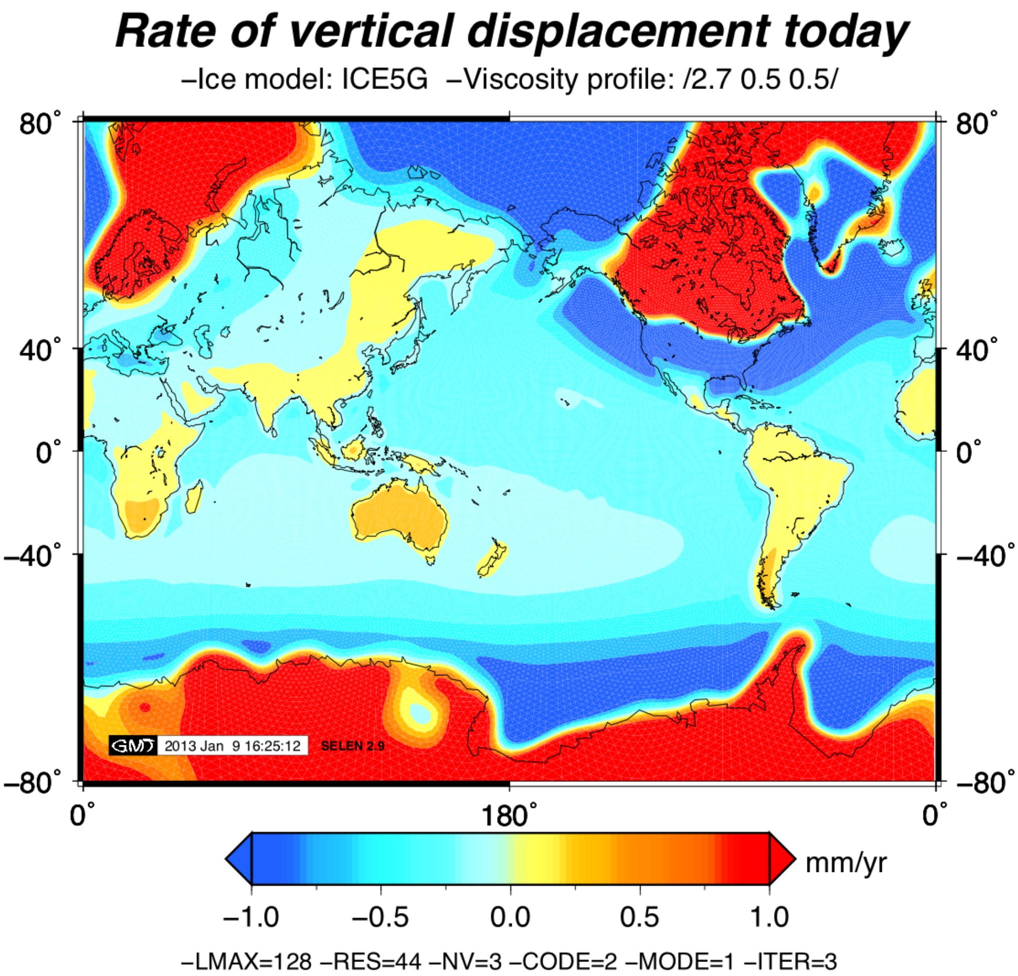

Figure 16: Global map of the GIA–induced vertical velocity ($\dot{U}$) for the TEST run. Subsiding and uplifting areas are shown by blue and red hues, respectively. The range of the $\dot{U}$ values is $[-3.53/+19.24]$ mm/yr.

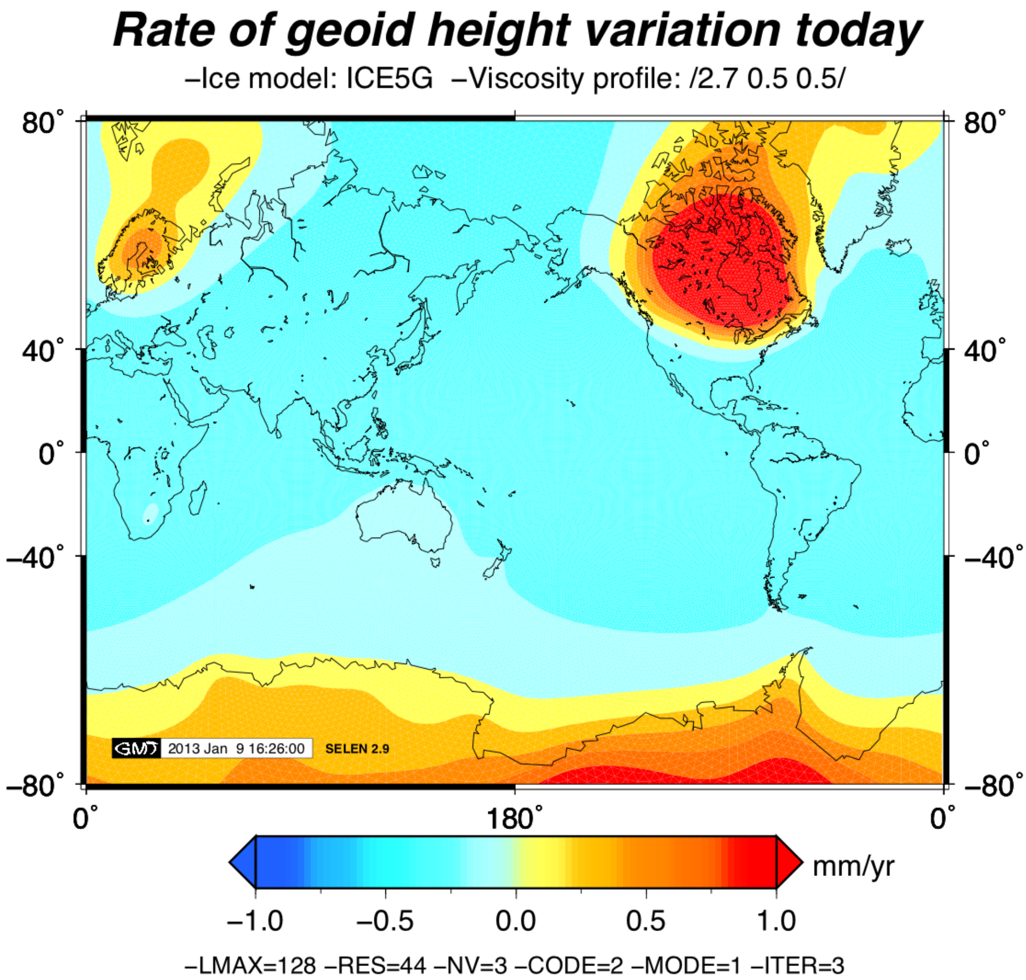

Figure 17: Rate of sea surface variation induced by GIA (*absolute* sea level variation), relative to the Earth's center of mass ($\dot{N}$), for run TEST. In this map, the range of variation of $\dot{N}$ is $[-0.40/+2.35]$ mm/yr.

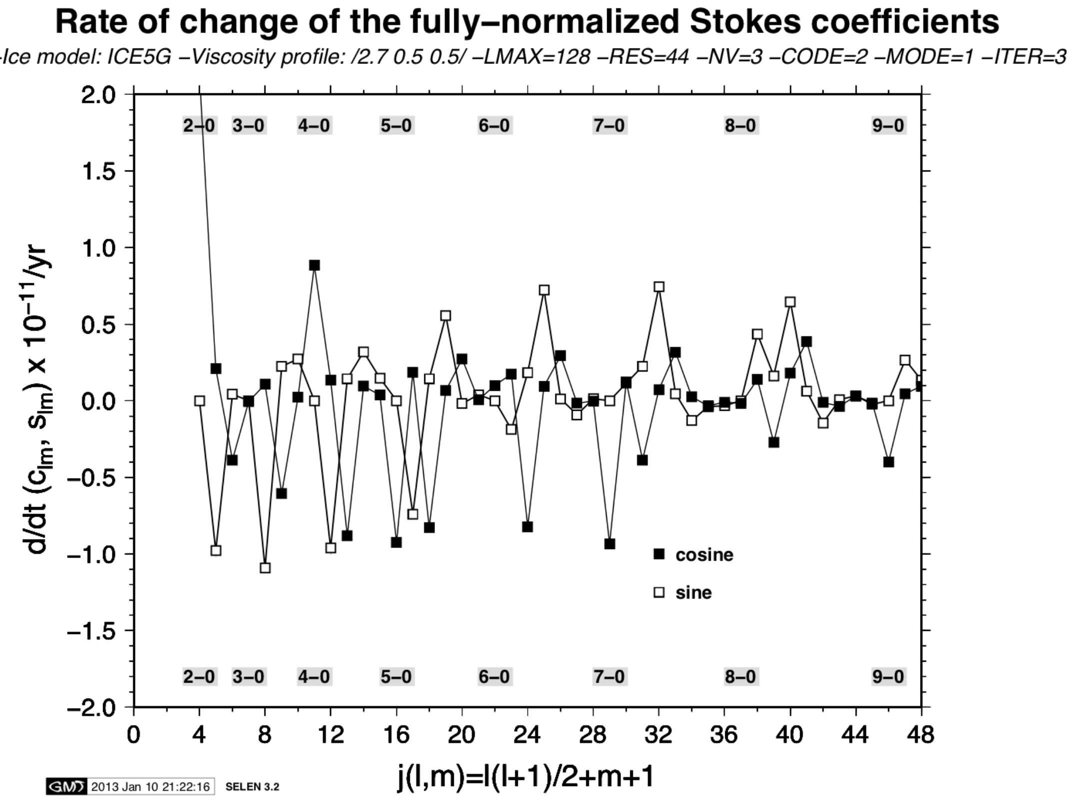

Figure 18: Time–derivatives of the Stokes coefficients of the Earth's gravity field associated with GIA, as a function of the generalized harmonic degree $j = \ell(\ell+1)/2 + m + 1$ for $2 \leq \ell \leq 9$ and $0 \leq m \leq \ell$, for the TEST run of SELEN. Output data for this analysis are stored in folder `depot-TEST/stokes` after the execution of SELEN.

8 Appendices

8.1 SELEN Fortran units

Table 4: SELEN Fortran 90 program units. Source files are stored in the `src` folder.

Unit name	Purpose and comments
config.f90	SELEN setup program
esl.f90	Equivalent Sea Level
geo.f90	Time variations of geodetic quantities
gmaps.f90	Synthesis of geodetic quantities on global maps
harmonics.f90	Include file with various SH tools and utilities
ms.f90	GMT multi–segment files from ice data
of_dv.f90	Degree variance of the Ocean Function
px.f90	Pixelization tools (including the Tegmark algorithm)
px_rebuild.f90	Retrieves pixelization data from an existing pixel table file
px_rec.f90	Reorganizes the pixelization data
rec_ice.f90	SH reconstruction of the ice thickness
rec_of.f90	SH reconstruction of the Ocean Function
rmaps.f90	Synthesis of geodetic quantities on regional maps
rsl.f90	Relative Sea Level curves
rsl_zones.f90	Geometry of the Relative Sea Level "Clark's zones"
rslc.f90	Relative Sealevel Contour lines for regional analyses
sh.f90	SHs at the grid pixels
sh_of.f90	SH coefficients for the Ocean Function
sh_rsl.f90	SHs at the RSL sites
sh_rslc.f90	SHs for regional analysis and RSL contours
sh_tgauges.f90	SHs at the TG sites
shape_factors.f90	"Shape factors" for the ice elements
shice.f90	SH decomposition of the ice model
shtools.f90	A SHTOOLS interface for the SH analysis
sle.f90	The SLE solver
stokes.f90	Variations of the Stokes coefficients of the gravity field
tgauges.f90	Present–day rate of sea level change at the TG sites
tb.F90	The TABOO code
wnw.f90	Numerical test for the SH orthogonality ('window function")

8.2 Structure of the SELEN output data

Table 5: Structure of the output data in the depot folder of SELEN.

Folder	Content
ICE5G	Data about the ice model
ICE5G/esl	ESL
ICE5G/original	Ice thickness data
ICE5G/reconstructed	SH reconstruction of ice thickness data
ICE5G/sh	SH coefficients of the ice model
Love-Numbers-by-TABOO	Love numbers data
TABOO	TABOO input files
geod	Predictions at geodetic sites
geod/3dmaps	Regional maps (in progress)
geod/sites	Geodetic predictions at specific sites
gmaps	Global fingerprints (data and plots)
log	Log files of SELEN and TABOO
of	Ocean function data and plots
of/degree_variance	Ocean function degree variance
px	Various pixelization data and plots
rmaps	Regional fingerprints (data and maps)
rsl	Relative Sea Level (RSL) data folder
rsl-contours	RSL contour plot
rsl-curves	RSL curves at specific sites
rsl-misfit	Misfit between RSL data and predictions
rsl-scplot	Scatterplot of RSL data
rsl-sites	Data and plots regarding RSL sites
rsl-table	Summary table of RSL data and predictions
rsl-zones	RSL zones
stokes	Stokes coefficients data
tgauges	Tide gauges (TGs)
tgauges-predictions	Predictions at TGs
tgauges-scplots	TG data scatterplot
tgauges-sites	Maps of TG sites
wnw	"Window test" for the ocean function

8.3 Ice sheets models in SELEN

Table 6: Ice sheets models available in the ICE-MODELS folder of SELEN. For all these models, the details of the chronology and the history of the ice thickness are given in the file headers. A rigorous definition of disk– and cap–shaped ice sheets geometries are given by Spada et al. (2011). The volume of the ice sheets can be assessed running SELEN and with option 221 set to 'y', which will provide the ESL curve (an example is given in Fig. 6 for model ICE–5G.

Ice model file	Description	Notes
alpsc.dat	Alpine ice sheet	See Spada et al. (2009)
alpsf.dat	a few variants	...
alpsh.dat	...	...
alpst.dat	...	...
disk_off.dat	disk-shaped ice sheet	Instantaneous melting
disk_on.dat	...	Instantaneous freezing
icap_off.dat	cap-shaped ice sheet	Instantaneous melting
icap_on.dat	...	Instantaneous freezing
ice1.dat	The ICE–1 ice model	Peltier and Andrews (1976)
ice1_eup.dat	ICE-1 sub–aggregate	Europe
ice1_gre.dat	...	Greenland
ice1_nam.dat	...	North America and Canada
ice3g.dat	The ICE–3G ice model	Tushingham and Peltier (1991)
ice3g_and.dat	ICE-3G sub–aggregate	Andes
ice3g_ant.dat	...	Antarctica
ice3g_bal.dat	...	Baltic region
ice3g_bar.dat	...	Barents Sea
ice3g_bri.dat	...	British Isles
ice3g_gre.dat	...	Greenland
ice3g_ice.dat	...	Iceland
ice3g_nam.dat	...	North America and Canada
ice3g_sib.dat	...	Siberia
ice5g.dat	The ICE–5G ice model	Peltier (2004)
ice5g_and.dat	ICE-5G sub–aggregate	Andes
ice5g_ant.dat	...	Antarctica
ice5g_fen.dat	...	Fennoscandia
ice5g_gre.dat	...	Greenland
ice5g_icl.dat	...	Iceland
ice5g_lau.dat	...	Laurentide
ice5g_nwz.dat	...	New Zaeland

8.4 Configuration file for the TEST run

For the sake of the reader's convenience, file `config.dat` for the TEST run described in Section 7 is copied below. This file is available from the SELEN distribution package.

```
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
  This is file "config.dat" for SELEN 2.9 -
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
The user can configure SELEN by the switches below. Any option is to be written within primes
(e. g., 'option'). A three-digits, left aligned numerical code is provided for each entry (e.
g., 001).

In section 1 (settings) the user supplies the spatial resolution, the ice sheets distribution,
and the Earth model viscoelastic structure. This allows one to solve the Sea Level Equation but
no graphical output is obtained.

In section 2 (outputs), a number of optional outputs can be scheduled, including tables and plots
of numerical results. The required GMT scripts are automatically generated according to the
options chosen.

For help, comments, or suggestion, you can contact the authors at the addresses below or consult
the SELEN web page at http://geodynamics.org/cig/software/selen

Contact: Giorgio Spada <giorgio DOT spada AT gmail DOT com>
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
 This is SECTION (1) of "config.dat": SELEN settings
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

====> PURGING option ---------------------------------------------------------
110  Purging the wdir before & after execution                     'y'
     [see config.f90 for purged filenames extensions ]

====> SOLUTION of the SLE ----------------------------------------------------
130    Iterations & mode of solution                             '3'    '1'
Modes:  1= Gravitationally self-consistent (GSC)
        2= Elastic GSC / 3="Eustatic" / 4="Woodward" / 5="No Ice"

====> MAXIMUM HARMONIC DEGREE ------------------------------------------------
140    LMAX                            '128'

====> REFERENCE FRAME --------------------------------------------------------
145    Includes degree 1 Love numbers (CM/CE frames)            'y'  'CM'

====> TEGMARK RESOLUTION -----------------------------------------------------
150    R                               '44'
151    Prepare a new pixel table (y/n, filename)        'y' 'px-table-r44.dat'

====> RHEOLOGICAL MODEL ------------------------------------------------------
160    Rheological profile info:                        '3' '2' 'vsc_VM2a.dat'

====> ICE MODEL --------------------------------------------------------------
170    Ice file name                                              'ice5g.dat'
171    Prepare a new SH ice file (y/n, filename)       'y'  'ice5g-l128.dat'
172    Ice history time step (kyrs)                      '1.0'

====> SPHERICAL HARMONICS (SH) FILE AT PIXELS --------------------------------
```

```
180    A new SH file (y/n, filename)                    'y'  'sh-r44-l128.bin'

====> OCEAN FUNCTION (OF) ----------------------------------------------------
190    A new OF SH decomposition (y/n, filename)        'y'  'of-l128.dat'

====> REPOSITORY LABEL ------------------------------------------------------
195    The depot name  (four characters)                'TEST'

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
 This is SECTION (2) of "config.dat: SELEN outputs
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

====> EXECUTION of the GMT SCRIPTS -------------------------------------------
200    Execution of GMT scripts during the SELEN run (y/n)     'y'

====> PIXELIZATION & WINDOW -------------------------------------------------
205    Pixelization maps (y/n)                                  'y'
206    Window function evaluation & plot (y/n)                     'n'

====> OCEAN FUNCTION (OF) ----------------------------------------------------
210    Present-day OF map & reconstruction (y/n)                'y'
215    Plot of OF degree variance (y/n)                            'n'

====> ICE MODEL --------------------------------------------------------------
220    Maps of original ice sheets  (y/n)                       'y'
221    Plot of Equivalent Sea Level (ESL)  (y/n)             'y'
222    Reconstruction & mapping of the ice sheets (y/n)       'n'

====> EARTH MODEL SPECTRAL PROPERTIES ----------------------------------------
230    Plot LDCs, relaxation spectrum & residues for normal modes  (y/n)   'y'

====> RSL PREDICTIONS AT SPECIFIC SITES --------------------------------------
240    RSL analysis, database & format               'y'  'sealevel.dat'   '0'
241    Plot of RSL sites distribution                          'y'
242    Site-by-site RSL predictions vs data & plots          'y'  'y'
243    Scatterplot of RSL data & predictions            'y'
244    Misfit between RSL data & predictions               'n'
245    Table with all RSL data & predictions              'y'

====> RSL REGIONS ------------------------------------------------------------
250    Gobal RSL zones                             'n'
251    Regional RSL contour lines                          'y'   'rsl-region.dat'

====> SEA LEVEL CHANGE AT TIDE-GAUGE STATIONS --------------------------------
260    Tide-gauge (TG) analysis & database                     'y' 'rlr-trends.txt'
261    Plot of TG stations distribution                      'y'
262    TG data scatterplot                                 'n'
263    Table of S, N, and U-dot predictions at TG sites    'y'

====> GLOBAL PRESENT-DAY RATES -----------------------------------------------
270    Global maps of dot S, U & N                 'y'

====> 3D VELOCITY ------------------------------------------------------------
275    -Up, North, East, S, and N rates for sites in file       'n' 'NA_KK.txt'

====> REGIONAL PRESENT-DAY RATES ---------------------------------------------
280    Regional maps of dot S, U, & N            'n'
281      -1 Italy                                   'y'
282      -2 Mediterranean                   'y'
283      -3 Europe                            'y'
284      -4 Fennoscandia                  'y'
285      -5 Greenland                                      'y'
```

```
286      -6 North America            'y'
287      -7 Antarctica                          'y'

====> STOKES COEFFICIENTS (SC) ------------------------------------------------
290     Rate of change of SC & range of degrees for plot      'y'  '2'  '20'
```

8.5 Output flow

The execution of SELEN should produce the following output flow on the monitor. The three main execution steps, described in Section 6.3, are marked on the right side of the page. The symbol [...] indicates output messages that have been omitted for the sake of clarity.

```
- - - - - - - - - - - - - - - - - - - - - - - - - - - - -

     SELEN, a Sea levEL EquatioN solver, Version 2.9

   Send comments, requests of help and suggestions to:
                <giorgio.spada@gmail.com>
                            -
                  Copyright(C) 2008-2013
              Giorgio Spada, Daniele Melini,
            Florence Colleoni & Paolo Stocchi
                          * * *
     This programs comes with  ABSOLUTELY NO WARRANTY
 This is free software and you are welcome to distribute
              it under certain conditions.
    For details, visit  <http://www.gnu.org/licenses/>
                  or edit file COPYING
- - - - - - - - - - - - - - - - - - - - - - - - - - - - -


----------------------------
 >>> 1. Executing SELEN  ...
----------------------------

---> Output data will be stored into directory DEPOTS/depot-TEST

---> Purging the working directory:

---> PX.F90: Hicosahedral pixelization of the sphere      +-------------------------------+
[...]                                                      | - STEP 1: Grid, SHs and LDCs |
                                                           +-------------------------------+
---> px.gmt: Separating wet from dry pixels
[...]

---> PX_REC.F90: Merging the wet & dry pixels tables

---> pxmap.gmt: Producing pixelization maps
     - wet pixels
     - dry pixels
     - Spherical map of wet and dry pixels

---> SH.F90: Building the spherical harmonics
     - Maximum degree is:          128
     - JMAX is:         8385
     - Resolution is:           44
```

```
[...]

---> REC_OF.F90: Reconstructing and mapping the ocean function
[...]

---> Importing ice5g.dat from ICE-MODELS/

---> SHAPE_FACTORS.F90: Computing the shape factors for model: ice5g.dat
[...]

---> SHICE.F90: Computing SH coefficients for the ice model
     - Read        11388   elements from ice5g.dat
[...]

---> MS.F90: Creating multi-segment files for ice sheets maps

---> mapice.gmt: Creating ps images of original ice sheets
[...]

---> TB.F90: Load-deformation coefficients by TABOO (Normal Modes)
     - Calling TABOO
[...]
[...]                                                     +-------------------------------+
[...]                                                     | - STEP 2: Solution of the SLE |
[...]                                                     +-------------------------------+

---> SLE.F90: Solving the Sea Level Equation - SLE - for FIXED coastlines
[...]
     - Starting the recursion
     - step  1 of 3
     - step  2 of 3
     - step  3 of 3                                +---------------------------------------+
[...]                                              | - STEP 3: Computation of geophysical |
[...]                                              |         and geodetic variables       |
[...]                                              +---------------------------------------+

---> RSL.F90: Predicting RSL at the sites of database: ./DATA/sealevel.dat
     - Computing the harmonics at the RSL sites
     - Computing sinthetic RSL curves
[...]

---> TGAUGES.F90: Dot-S, U, and N predictions at tide gauges
     - There are        1123 sites in file ./DATA/rlr-trends.txt
     - Rate of sealevel change at site
     -            1 of        1123 REYKJAVIK
[...]

---> GMAPS.F90: Global maps of dot S, U and N at present time
[...]
---> STOKES.F90: Present-time rate of change of Stokes coefficients
[...]

------------------------------------                                   +-----------+
 >>> 2. Cleaning up the directory...                                   | - Cleanup |
------------------------------------                                   +-----------+
[...]

- - - - - - - - - - - - - - - - - - - - - - - - - - - - -

     SELEN, a Sea levEL EquatioN solver, Version 2.9

   Send comments, requests of help and suggestions to:
```

```
                <giorgio.spada@gmail.com>
                            -
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              Giorgio Spada, Daniele Melini,
            Florence Colleoni & Paolo Stocchi
                          * * *
     This programs comes with  ABSOLUTELY NO WARRANTY
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    For details, visit  <http://www.gnu.org/licenses/>            +--------------------+
                  or edit file COPYING                            | - End of execution |
- - - - - - - - - - - - - - - - - - - - - - - - - - - - -         +--------------------+


 >>> Outputs for this run are available in directory: DEPOTS/depot-TEST
```

8.6 Conventions for Spherical Harmonics

In the SLE theory and in program SELEN, we adopt *complex, 4π–normalized spherical harmonic (SH) functions*:

$$\mathcal{Y}_{\ell m}(\omega) = \sqrt{(2\ell+1)\frac{(\ell-m)!}{(\ell+m)!}} P_{\ell m}(\cos\theta) \mathrm{e}^{im\lambda}, \tag{38}$$

where $\omega \equiv (\theta, \lambda)$, θ is colatitude ($0 \le \theta \le \pi$), λ is longitude ($0 \le \lambda \le 2\pi$), ℓ is the harmonic degree ($\ell = 0, 1, \ldots$), m is the order ($m = 0, 1, \ldots, \ell$) and $P_{\ell m}(\cos\theta)$ are the associated Legendre polynomials

$$P_{\ell m}(x) = (-1)^m (1-x^2)^{\frac{m}{2}} \frac{d^m}{dx^m} P_\ell(x), \tag{39}$$

where factor $(-1)^m$ is known as the Condon–Shortley phase, and

$$P_\ell(x) = \frac{1}{2^\ell \ell!} \frac{d^\ell}{dx^\ell} (x^2-1)^\ell \tag{40}$$

are the Legendre polynomials.

SHs with negative order are defined by

$$\mathcal{Y}_{\ell,-m}(\omega) = \mathcal{Y}^*_{\ell m}(\omega), \tag{41}$$

where the asterisk denotes the complex conjugate. By definition (38), the orthogonality condition of the SHs reads

$$\int_0^{2\pi} \int_0^{\pi} \mathcal{Y}_{\ell m}(\omega) \mathcal{Y}^*_{\ell' m'}(\omega) d\omega = 4\pi \delta_{\ell\ell'} \delta_{mm'}, \tag{42}$$

where

$$d\omega \equiv \sin\theta d\theta d\lambda \tag{43}$$

and δ_{ij} is Kronecker delta. In SELEN, the SHs are computed numerically using routines from the SHTOOLS package.

8.7 Availability of SELEN

Program SELEN (version 2.9) is hosted by the CIG at the page: `http://geodynamics.org/cig/software/sel`

SELEN is also available from the web page *http://hpc.rm.ingv.it/selen*, or from the authors.

http://www.ipgp.fr/wieczor/SHTOOLS/SHTOOLS.html

8.8 Related software

TABOO. Program TABOO is a post–glacial rebound calculator written in Fortran 90. It is *not* a SLE solver. SELEN includes the basic components of TABOO in order to compute the LDCs (see program `tb.F90`). TABOO, along with its documentation, is available from the Samizdat press,

sTABOO. This is a light version of TABOO, written for absolute beginners. It has been written for the students of the PhD course/workshop on Sea Level Rise and Ice Sheets, held at Center for Ice and Climate, University of Copenhagen (21-25 May, 2012). It comes with installation instructions and can be obtained by email from GS,

ALMA. Program ALMA (Spada, 2008) computes the LDCs for multi–layered Earth models with a generalized Maxwell rheology, adopting a special method of inversion for Laplace transforms (Spada and Boschi, 2006).

8.9 Previous reference documents

Previous versions of SELEN and of the theory behind have been described in various papers and documents. In these works, the readers can find more details and hints, as well as various references to previous relevant papers on the subject:

- The theory behind the SLE is described in a booklet published from the Samizdat Press in 2005. This material has been subsequently revised and reorganized, and published by Spada and Stocchi (2006),
- the work by Spada and Stocchi (2007) contains a condensed theory of the SLE and illustrates the first version of SELEN, which in this document is referred to as SELEN 1.0,
- The current version of SELEN (2.9) is presented in the manuscript by Spada et al. (2012b) (arXiv:1212.5061), where also some applications are discussed. The present manual is essentially a user–oriented version of this paper.
- The Sea Level Equation page on Wikipedia gives a short definition of the SLE.

8.10 SELEN benchmarks

During the last few years, in the framework of the European COST Action ES0701 "Improved Constraints on Models of Glacial Isostatic Adjustment", various benchmark tests on GIA modeling have been performed.

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http://samizdat.mines.edu/taboo/
http://www.fis.uniurb.it/spada/ALMA_minipage.html
http://samizdat.mines.edu/sle/sle.pdf
http://www.samizdat.edu
http://en.wikipedia.org/wiki/Sea_level_equation
http://www.cost-es0701.geoenvi.org/

Until present, the results concerning Post Glacial Rebound modeling have been published in the paper by Spada et al. (2011). In this work, program TABOO (based on the Viscoelastic Normal Modes method of Peltier 1974) has been successfully tested against various programs used in the community, often based on completely different techniques. Since SELEN computes the LDCs using TABOO, and the LDCs enter the Green's functions that appear in the SLE, the Spada et al. (2011) benchmark has validated these fundamental components of the program.

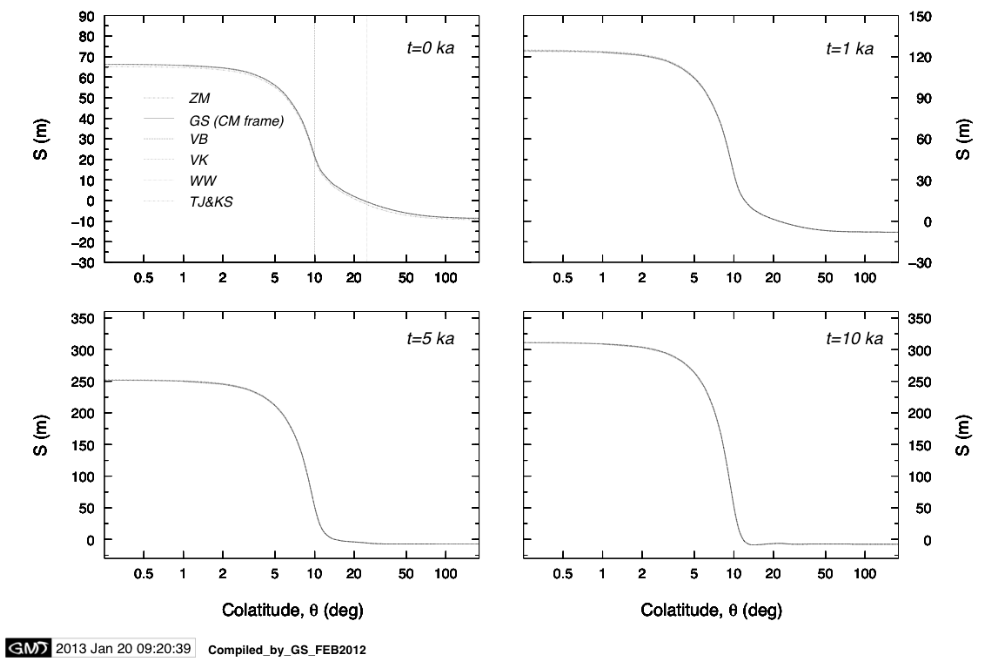

Figure 19: Sea level change $S(\omega, t)$ and a function of colatitude, for various times after the load is imposed ($t = 0$), for the benchmark test described in Spada et al. (2011). Vertical segments in the top left frame mark the margin of the ice load and the (fixed) continent–ocean boundary.

The idea of a SLE benchmark dates back to the mid 1990s, and was first launched by the colleagues Georg Kaufmann and Paul Johnston (the whole story and subsequent developments are reviewed in the Introduction of the Spada et al. (2011) paper, where some useful links are also given). At the current stage (January 2013), a number of tests computations have been performed in the context of the European COST Action ES0701, and a manuscript is in preparation.

In Fig. 19 we show the outcomes of some SLE computations independently performed by Valentina Barletta (VB), Zdenek Martinec (ZM), Tom James and Karen Simons (TJ& KS), Volker Klemann (VK), Wouter van der Wal (WW) and myself (GS, based on a suitably designed version of SELEN). Here, the SLE is solved assuming an ocean–continents distribution with zonal symmetry wrt the axis $\theta = 0^\circ$. The continent, which extends to colatitude

$\theta = 25°$, is loaded by a ice sheet with parabolic profile (i.e., a *cap*, see Spada et al. 2011) having a thickness of 1500 m at the centre and half amplitude $\alpha = 10°$, and characterized by a Heaviside time–history (the load is switched on at time $t = 0$). The ice and water density are $\rho_w = 1000$ kg m^{-3} and $\rho_i = 920$ kg m^{-3}, respectively. Shorelines are not allowed to migrate horizontally and the Earth is not rotating. The rheological model is the same employed in the Spada et al. (2011) exercise (i.e., the five–layer, incompressible viscoelastic model M3–L70–V01, see their Table 3). SELEN has been configured for a "gravitationally self–consistent" mode of solutions with three iterations, with a grid resolution parameter `R=44`, and a maximum harmonic degree `LMAX=128`. The SLE is solved in the CM frame.

Fig. 19, which illustrates the history and the spatial pattern of $S(\omega, t)$, indicates that the various numerical solutions of the SLE are, for this axisymmetric test, substantially in agreement. New tests, which are under way, are dealing with the more challenging case of a realistic ocean function and a complex ice sheets distribution.

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