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SELEN: a program for solving the “*Sea Level Equation*”

User Manual for version 2.9

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Figure 1: The SELEN logo is an artwork by Florence Colleoni (2009).

1 Introduction

The open source program SELEN solves numerically the so-called “Sea Level Equation” (SLE) for a spherical, layered, non-rotating Earth with Maxwell viscoelastic rheology.

The SLE is an integral equation that was introduced in the 70s to model the sea level variations in response to the melting of late-Pleistocene ice-sheets, but it can also be employed for predictions of geodetic quantities in response to present-day melting of continental ice-sheets. SELEN can compute vertical and horizontal surface displacements, gravity variations and sea level changes on a global and regional scale.

SELEN (acronym of SEa Level EquatioN solver) is particularly oriented to scientists at their first approach to the glacial isostatic adjustment (GIA) problem and, according to our experience, it can be successfully used in teaching. The current release (2.9) considerably improves the previous version of the code in terms of computational efficiency, portability and versatility. As far as we know, SELEN is the only open source program designed for solving the SLE.

SELEN 2.9 solves the SLE following the classical theory of Farrell and Clark (1976). In the future release of SELEN (SELEN 3.0), which is under development, two important new features will be introduced: the rotational feedback on sea level and the horizontal migration of shorelines in response to sea level change.

This User guide describes the essentials of the theory behind the SLE, and provides instructions for the configuration and execution of SELEN.

2 Acknowledgements

We thank all the SELEN users and a number of colleagues for the numerous feedbacks that have greatly contributed to improve the program. We thank Max Tegmark for making available the code for the icosahedral pixelization of the sphere. For computations involving spherical harmonic functions, SELEN implements some SHTOOLS routines developed by Mark Wieczorek. All the figures have been drawn using the GMT public domain software (Wessel and Smith, 1998). The development of SELEN has been supported by ISCRA (Italian SuperComputing Resource Allocation) with contracts n. HP10CS9J50 (SEASIM) and HP10CUX77T (SEAMED). Partly funded by COST Action ES0701 “Improved Constraints on Models of Glacial Isostatic Adjustment” and by the European Commission’s 7th Framework Programme through grant number 226375 (ice2sea project). CIG (Computational Infrastructure for Geodynamics) is thanked for hosting SELEN (<http://geodynamics.org/cig/software/seLEN>). Eric Heien from CIG has greatly improved the SELEN usability by implementing the autoconf system into the original code.

3 Theoretical background

Here we briefly illustrate the background theory of the SLE, the integral equation that describes the sea level variations and solid Earth deformations associated with GIA. Essentially, the material that follows is a condensed summary of the SLE theory first exposed by Farrell and Clark (1976).

For further details, including the spatiotemporal discretization of the SLE and its numerical implementation, the reader is referred to the review of Spada and Stocchi (2006) and to references therein.

3.1 Sea Level and Sea Level change

Introducing the SLE requires some basic definition involving the general concept of "sea level". *Absolute sea level* is:

$$\text{SL}(\omega, t) = R_{ss}(\omega, t) - R_{se}(\omega, t), \quad (1)$$

where $\omega \equiv (\theta, \lambda)$, θ is colatitude and λ is longitude, t is time, and R_{ss} and R_{se} denote the radii of the (equipotential) sea surface and of the solid surface of the Earth, both relative to the Earth's center of mass (CM).

The quantity involved in the SLE is *sea level change*

$$S(\omega, t) = \text{SL}(\omega, t) - \text{SL}(\omega, t_r), \quad (2)$$

where $t_r \leq t$ denotes a remote time. For studies of past sea level variations, it is convenient to introduce *relative sea level* (RSL):

$$\text{RSL}(\omega, t_{BP}) = \text{SL}(\omega, t_{BP}) - \text{SL}(\omega, t_p), \quad (3)$$

where $t = t_{BP}$ is a given time before present (BP) and $t = t_p$ is present time. Using (2), RSL is easily expressed in terms of sea level change:

$$\text{RSL}(\omega, t_{BP}) = S(\omega, t_{BP}) - S(\omega, t_p). \quad (4)$$

An expression for $S(\omega, t)$ which is suitable in GIA studies is obtained using (1) in (2):

$$S(\omega, t) = N(\omega, t) - U(\omega, t), \quad (5)$$

where $N(\omega, t) = R_{ss}(\omega, t) - R_{ss}(\omega, t_r)$ is the *sea surface variation* relative to the CM (sometimes referred to as *absolute sea level change*) and $U(\omega, t) = R_{se}(\omega, t) - R_{se}(\omega, t_r)$ is the *vertical displacement* of the solid surface of the Earth. Note that $S(\omega, t)$ is defined over the whole Earth surface, including the continental masses.

Eq. (5) represents the SLE in its simplest form; what follows is aimed to illustrate the relationship between S and the variations of the ice thickness through time, in order to obtain a form of the SLE amenable to a numerical approach.

3.2 The Sea Level Equation

According to Farrell and Clark (1976), the sea surface variation N in Eq. (5) can be written as

$$N(\omega, t) = G(\omega, t) + c(t), \quad (6)$$

where c is yet undetermined and the *geoid height variation* is

$$G(\omega, t) = \frac{\Phi}{\gamma}(\omega, t), \quad (7)$$

in which γ is the reference gravity at the surface of the Earth and $\Phi(\omega, t)$ is the *total variation of the gravity potential*.

Hence, using Eq. (6) into (5) gives

$$S(\omega, t) = \frac{\Phi}{\gamma} - U + c, \quad (8)$$

where we have dropped the (ω, t) dependence in right hand side to simplify notation. Mass conservation of the system (Ice sheets + Oceans) is ensured taking

$$\bar{c}(t) = -\frac{m_i}{\rho_w A_o} - \overline{\left(\frac{\Phi}{\gamma} - U\right)}, \quad (9)$$

where ρ_w is the (constant) density of water, m_i is the mass variation of the ice sheets, A_o is the (constant) area of the present-day oceans and the overbar indicates the average over the surface of the oceans

$$\overline{(\dots)} = \frac{1}{A_o} \int_o (\dots) dA, \quad (10)$$

where $dA = a^2 \sin \theta d\theta d\lambda$ is the area element and a is Earth average radius. From Eq. (8), the SLE can be therefore written as

$$S(\omega, t) = \left(\frac{\Phi}{\gamma} - U \right) + S^E - \overline{\left(\frac{\Phi}{\gamma} - U \right)}, \quad (11)$$

where the “*eustatic*” sea level variation:

$$S^E(t) = -\frac{m_i}{\rho_w A_o}, \quad (12)$$

shows the remarkable property

$$S^E(t) = \bar{S}. \quad (13)$$

The SLE has solution $S = S^E$ only in the case of a rigid, non self-gravitating Earth ($U = \Phi = 0$ in Eq. 11).

Functions $U(\omega, t)$ and $\Phi(\omega, t)$ will depend on the spatiotemporal variations of the *surface load*:

$$\mathcal{L}(\omega, t) = \rho_i I + \rho_w S \mathcal{O}, \quad (14)$$

where the two terms on the right hand side are associated with the waxing and waning of the ice sheets (*ice load*), and with the redistribution of meltwater in the ocean basins (*ocean load*), respectively. In Eq. (14), ρ_i is ice density, \mathcal{O} is the *ocean function*

$$\mathcal{O}(\omega) = \begin{cases} 1, & \text{if } \omega \in \text{oceans} \\ 0, & \text{if } \omega \in \text{land}, \end{cases} \quad (15)$$

(hereinafter referred to as OF), and

$$I(\omega, t) = T - T_0, \quad (16)$$

is the *ice thickness variation*, where $T(\omega, t)$ is *ice thickness*, and $T_0(\omega)$ is a reference thickness at a remote time (e.g. the thickness at the Last Glacial Maximum, LGM, 21 kyrs ago). The mass variation in Eq. (12) is obtained from (16) by integration over the ice-covered regions:

$$m_i(t) = \int_i \rho_i I \, dA. \quad (17)$$

Following Eq. (14), vertical displacement stems from two terms

$$U(\omega, t) = \rho_i G_u \otimes_i I + \rho_w G_u \otimes_o S, \quad (18)$$

where G_u is the Green's function for vertical displacement, \otimes_i and \otimes_o are spatiotemporal convolutions over the ice- and ocean-covered regions, respectively (these convolutions and the Green's functions are formally defined in Spada and Stocchi 2006). Similarly, the total variation of the gravity potential is

$$\Phi(\omega, t) = \rho_i G_\phi \otimes_i I + \rho_w G_\phi \otimes_o S, \quad (19)$$

where G_ϕ is the corresponding Green's function. Explicit expressions for G_u and G_ϕ are given in e.g. Spada and Stocchi (2006) in terms of the load-deformation coefficients (LDCs) $h(t)$ and $k(t)$, respectively. The sea level Green's function is defined as the difference:

$$\frac{G_s}{\gamma}(\omega, t) = \frac{G_\phi}{\gamma} - G_u. \quad (20)$$

Substitution of Eqs. (18) and (19) into (11) using (20) gives

$$S(\omega, t) = \frac{\rho_i}{\gamma} G_s \otimes_i I + \frac{\rho_w}{\gamma} G_s \otimes_o S + S^E - \frac{\rho_i}{\gamma} \overline{G_s \otimes_i I} - \frac{\rho_w}{\gamma} \overline{G_s \otimes_o S}, \quad (21)$$

which represents the SLE in the “gravitationally self-consistent” form. Since the unknown $S(\omega, t)$ also appears in the spatiotemporal convolutions at the right hand side, the SLE is an integral equation, which in general cannot be solved in closed form. The SLE is a linear equation as long as shorelines are not allowed to migrate horizontally, i.e. if \mathcal{O} (and consequently A_o) is not dependent on S itself. Sea level variations are sensitive to mantle rheology through G_s , since this is determined by the viscoelastic LDCs h and k (Spada, 2003a, Spada and Stocchi, 2006). Solutions of Eq. (21) in special cases, discussed in detail by Spada and Stocchi (2006), are also available via SELEN .

It is worth to observe that the SLE (21) does not involve absolute quantities and can, consequently, only provide *variations* of geophysical and geodetic quantities relative to a reference state.

3.3 Solving the Sea Level Equation

The form (21) of the SLE is not suitable for a numerical implementation. In fact, the spherical harmonic (SH) decomposition of terms like $G_s \otimes_o S$, which imply an integration over the oceans and not over the whole Earth, would demand evaluating coupling coefficients between sets of SHs, which may severely limit the maximum degree of the harmonic analysis (SH conventions in SELEN are summarized in Section 7.6).

The *pseudo-spectral approach* introduced by Mitrovica and Peltier (1991) and Mitrovica et al. (1994), and implemented in SELEN, overcomes this difficulty, since it allows for a direct spectral analysis (Milne, 1998). At the core of the method is a change of variable in the SLE, where $S(\omega, t)$ is substituted by its projection on the OF:

$$Z(\omega, t) \equiv \mathcal{O}S, \quad (22)$$

where we define Z as the *modified sea level change*. Multiplying both sides of Eq. (21) by \mathcal{O} gives the SLE in the form:

$$Z(\omega, t) = H + Z^E + K(Z), \quad (23)$$

where:

$$Z^E(t) \equiv \mathcal{O}S^E \quad (24)$$

$$H(\omega, t) \equiv \mathcal{O}(A - \bar{A}) \quad (25)$$

$$K(Z; \omega, t) \equiv \mathcal{O}(B - \bar{B}), \quad (26)$$

and variables

$$A(\omega, t) \equiv \frac{\rho_i}{\gamma} G_s \otimes_e I \quad (27)$$

$$B(\omega, t) \equiv \frac{\rho_w}{\gamma} G_s \otimes_e Z, \quad (28)$$

now involve an integration (denoted by \otimes_e) over the whole Earth's surface, which can be tackled using a standard spectral approach. Using this formalism, the SLE (21) reads

$$S(\omega, t) = A - \bar{A} + S^E + B - \bar{B} \quad (29)$$

and from (18) vertical displacement is

$$U(\omega, t) = A_u + B_u, \quad (30)$$

where A_u and B_u have the same form of A and B in (27) and (28), but with G_s replaced by G_u . Similarly, from (19) and (7), the geoid height variation is

$$G(\omega, t) = A_g + B_g, \quad (31)$$

where A_g and B_g have the same form of A and B , with G_s replaced by G_ϕ .

Since term K in (23) depends on Z , the SLE is an integral (implicit) equation, in which the unknown Z appears explicitly at the left-hand side but it is convolved in space and time with G_s on the right hand side. This suggests an iterative approach to

the SLE, similar to that employed to solve the one-dimensional inhomogeneous integral Fredholm equations of the second kind. However, the complexity of the SLE (and its three dimensionality) hinders an analytical study of the convergence conditions of the iteration scheme.

Following Spada and Stocchi (2007), to which the author is referred for more details, the iteration scheme implemented in SELEN reads

$$\begin{cases} Z_{\ell m,i}^{(0)} = Z_{\ell m,i}^E \\ Z_{\ell m,i}^{(k)} = H_{\ell m,i} + Z_{\ell m,i}^E + K_{\ell m,i} (Z_{\ell' m',j}^{(k-1)}) \end{cases}, \quad (32)$$

where $Z_{\ell m,i}^{(k)}$ is the k -th approximation to the degree ℓ ($0 \leq \ell \leq \ell_{max}$) and order m ($0 \leq m \leq \ell$) component of $Z(\omega, t)$, evaluated at the i -th time step (a piecewise constant behavior is assumed for all the variables involved in the SLE, with a typical time step of 1 kilo-year). The iteration continues until the ratio $|Z_{\ell m,i}^{(k)} - Z_{\ell m,i}^{(k-1)}| / |Z_{\ell m,i}^{(k-1)}|$ becomes sufficiently small (according to the numerical tests carried out by Spada and Stocchi (2007), three to five iterations normally suffice for convergence). Once the SLE is solved, all the other geophysical and geodetic quantities (see e.g. Spada et al. 2012a) are immediately obtained from the spectral components of Eqs. (29), (30), and (31).

For the numerical evaluation of surface integrals embedded in terms H and K of the SLE (Eq. 23), SELEN takes advantage of the equal-area, icosahedron-shaped, spherical pixelization by Tegmark (1996), which is particularly convenient for the manipulation of harmonic functions. The number of pixels on the grid is determined by

$$N_p = 40R(R+1) + 12, \quad (33)$$

where the integer R is the *grid resolution parameter* (Tegmark, 1996). The pixels are the centers of slightly distorted hexagonal grid cells. Assuming that N_p is sufficiently large, the area of the surface of the cells is $A_{cell} \sim 4\pi a^2 / N_p$ and their radius is $r_{cell} \sim a\sqrt{4/N_p}$, where a is Earth's radius. As discussed by Tegmark (1996), an accurate numerical integration of harmonic functions up to degree ℓ_{max} demands a sufficiently large number of grid pixels, with

$$N_p \geq \frac{\ell_{max}^2}{3}, \quad (34)$$

where ℓ_{max} is the maximum harmonic degree of the SH expansion of the SLE.

4 How SELEN works

4.1 Software and hardware requirements

Software. Running SELEN requires a standard UNIX environment (including Linux and Mac OS X) and a Fortran 90 compiler. On Windows systems, SELEN can run within the Cygwin environment¹. SELEN also requires the GMT² (Generic Mapping Tools) public domain mapping software (Wessel and Smith, 1998).

The configuration and build process of SELEN 2.9 is handled by standard GNU autoconf scripts³. SELEN 2.9 has been successfully tested using both the freely available g95⁴ and gfortran⁵ compilers, or the commercial Intel Fortran compiler⁶. Additional operating system and compiler configurations can be implemented by modifying the `flags.guess` file and providing suitable options to the `configure` script.

In SELEN 2.9, the most computationally intensive portions of code have been parallelized with OpenMP (2005) directives. The corresponding program units can therefore take advantage of multi-threading on modern CPUs, resulting in a substantial performance improvement⁷.

Porting SELEN on “unsupported” systems should be an easy task. The only tricky aspect is that program unit `tb.F90` (the LDCs calculator) needs quad-precision floating-point (i.e. `REAL*16`), which is not supported on all platforms. The Intel fortran compiler and the g95 compiler fully support `REAL*16` objects. On x86 systems, the GNU gfortran compiler does not support `REAL*16`; however it supports `REAL*10`, essentially a double-precision fraction with a quad-precision exponent, which is adequate for floating-point computations in `tb.F90`. When porting SELEN to other platforms, the support and range of `REAL*16` variables have to be carefully checked.

Hardware. The hardware requirements of SELEN are related to the desired grid resolution parameter R and to the maximum harmonic degree ℓ_{max} . For low or medium resolutions (approximately $R \leq 50$ and $\ell_{max} \leq 128$), SELEN can run without trouble on a notebook. High-resolution runs might require a multi-processor system and/or large amounts of RAM. The TEST SELEN run, described in Section 6, runs in about 34.5 minutes on an Apple MacBook with a 2GHz dual-core Intel Core 2 Duo CPU and in about 10.5 minutes on an Apple Mac Pro with two 2.8Ghz quad-core Intel Xeon CPUs.

The execution time of SELEN scales with the number of grid pixels N_p and the maximum harmonic degree ℓ_{max} as $t_{exe} \sim N_p N_h$ where $N_h = (\ell_{max} + 1)(\ell_{max} + 2)/2$ is the number of SHs for $\ell \leq \ell_{max}$. In view of Eq. (33), as a rough approximation

$$t_{exe} \sim (R\ell_{max})^2, \quad (35)$$

¹Cygwin is freely downloadable from: <http://www.cygwin.com>

²The Generic Mapping Tools can be downloaded from: <http://gmt.soest.hawaii.edu/>

³<http://www.gnu.org/software/autoconf/>

⁴Available from: <http://www.g95.org>

⁵Available from: <http://gcc.gnu.org/gfortran>

⁶More information at: <http://software.intel.com/en-us/fortran-compilers>

⁷The g95 compiler does not support OpenMP, so it is not possible to run SELEN in parallel with this compiler.

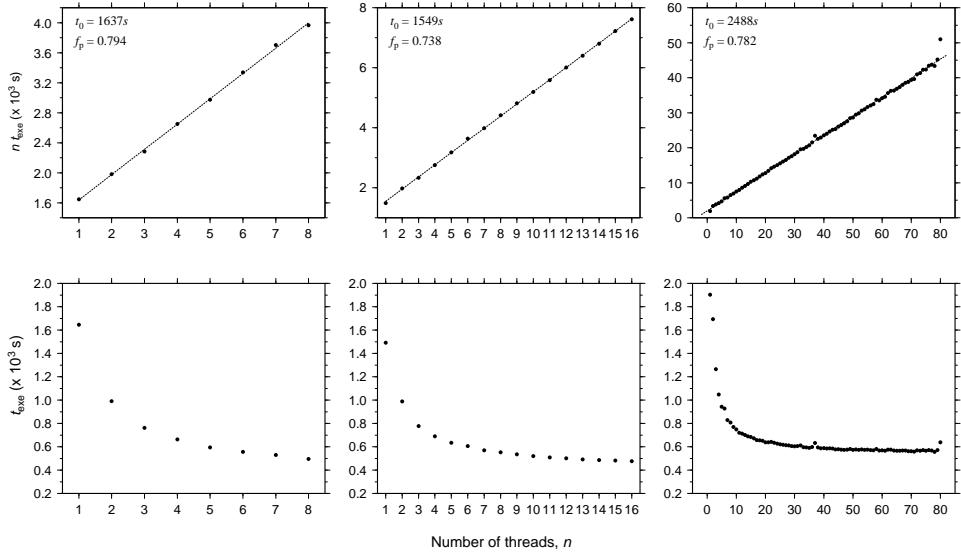


Figure 2: Scaling of execution time for the TEST run with the number of threads. Test systems are: an Apple Mac Pro configured with two 2.8GHz quad-core Intel Xeon CPUs (“Harpertown”–class, model number E5462) and 16GB DDR2 RAM, running Mac OS X 10.5.8 (left frames); an HP ProLiant BL460c G8 configured with two 2.70GHz eight-core Intel Xeon CPUs (“Sandy Bridge”–class, model number E5-2680) and 128GB DDR3 RAM, running Red Hat Enterprise Linux 5.8 (middle frames); an HP ProLiant DL98G7 configured with eight 2.0GHz 10-core Intel Xeon CPUs (“Westmere”–class, model number E7-2850) and 2TB DDR3 RAM, running Scientific Linux release 6.3 (right frames). In top frames, a dashed line represent the fit of Eq. (37) to the measured points; corresponding values of t_0 and f_p are given for each system.

where R is the grid resolution parameter. The memory footprint of SELEN follows a similar scaling law. For high resolution runs (large values of ℓ_{\max} and consequently of R , see Eq. 34), disk I/O can become a considerable fraction of t_{exe} , and these relations may no longer be valid.

On multi-core systems, SELEN can use multi-threading to reduce computation time (multi-threading was not implemented in previous version of SELEN). Multi-threading will automatically be enabled if it is supported by the system and, as a default, all the available cores are used. If the user wishes to leave some resources free for other tasks, the `OMP_NUM_THREADS` environment variable has to be set to the number of usable cores before launching SELEN .

Fig. 2 shows the execution time t_{exe} of the TEST as a function of the number of threads n for three test systems. If the inter-process communication overhead is negligible, the relationship between t_{exe} and n is well approximated by

$$t_{\text{exe}}(n) = t_0 \left(f_s + \frac{f_p}{n} \right), \quad (36)$$

where f_s and f_p are the serial and parallel fractions of the code ($f_s + f_p = 1$), and t_0 is

the serial execution time (Amdahl, 1967). Eq. (36) can be rewritten as

$$nt_{exe}(n) = t_0(f_p + f_sn), \quad (37)$$

showing that when the speedup of the parallel portion of the code is linear, the relation between nt_{exe} and n is also linear. Measured values of nt_{exe} for the TEST run are shown in Fig. 2. By fitting Eq. (37) to the points in Fig. 2 it is possible to evaluate the parallel fraction f_p of the code, which for the TEST run is within 0.7 and 0.8. This value depends on the specific options selected in the config.dat; for instance, enabling the execution of the GMT scripts (which are sequential tasks) results in a lower parallelism level.

4.2 Configuration and execution files

The configuration and the execution of the program involve several files and scripts in the SELEN directory. For a normal execution of SELEN, the user only needs to edit and modify the configuration file config.dat.

The relevant scripts and programs for configuration and execution are:

config.dat is the configuration file of SELEN. It is a plain text file that can be easily configured by the user according to the specific tasks to be accomplished. The structure of the file and the configuration rules are described in detail in Section 5.2 below. A sample config.dat, configured for a TEST run, is provided in Section 7.4,

config.f90 is the SELEN setup program. This program *i*) parses config.dat, *ii*) creates the shell script selen.sh which physically executes the SELEN run, *iii*) creates the “include file” data.inc, which contains declarations useful to all the Fortran units needed for the execution (the full list is given in Table 4), *iv*) creates the GMT scripts needed for visualization of the results. The implementation of new features in SELEN will demand, in general, adding new subprograms to config.f90 or modifying the existing ones. The setup program also performs several checks on the input parameters, and exits with an error message if inconsistencies are found. T4

selen.sh is a shell script, created by config.f90, containing execution commands for the Fortran units of SELEN which are required to fulfill the user’s tasks. This script also creates the depot directory structure where the SELEN outputs are stored, and purges the working directory from temporary files before and after the execution of the program.

4.3 Input and output data

Input. The SELEN input data are organized in several sub-directories:

1. Folder DATA collects various files with information about geophysical and geodetic sites of interest and other miscellanea data,
2. Folder ICE-MODELS contains the files describing the time chronology of each the ice models available by SELEN,

3. Folder VSC stores a collection of viscosity profiles.
4. Folder INPUTS stores user-generated input files, such as pixel table files, pre-computed SH coefficients at the grid pixels and SH coefficients of the ice thickness data.

Output. The SELEN outputs consist of:

1. ASCII files with various formats,
2. PostScript and PDF graphic files,
3. GMT scripts.

After each run, SELEN outputs are stored in a subfolder `depot-name`, where `name` is a 4-character label defined by the user in the `config.dat` file (see Section 5.2). The depot output subfolders are created into the `DEPOTS` folder of the SELEN directory, and their structure is described in Table 5.

Some of the SELEN intermediate outputs, such as SH coefficients at the grid pixels, SH coefficients of the ice thickness data or pixel table files, can be reused as input files for subsequent SELEN runs. After a SELEN run, these files are stored in the `run` folder of the SELEN working directory; in order to make them available for later reuse, they have to be manually moved into the `INPUTS` folder.

5 Setting up and running SELEN

5.1 Installation

The installation of SELEN is straightforward. The user just needs to un–zip the SELEN package in any chosen folder. If not already available on the system, a supported Fortran compiler and the GMT public domain software (Wessel and Smith, 1998) must be also installed.

5.2 Configuration

The configuration file of SELEN is the plain text file config.dat. Here we describe its structure and illustrate the options available to the user. A sample configuration file, employed for a TEST run in Section 6, is listed in Section 7.4.

File config.dat is composed of two sections:

- SECTION 1 contains the general settings of SELEN. It defines the approximation (mode of solution) employed to solve the SLE, the rheological profile of the Earth model and the spatiotemporal features of the surface load (ice model).

SECTION 1 is marked by the header:

```
...
!!!!!!!!!!!!!!  
This is SECTION (1) of "config.dat": SELEN settings  
!!!!!!!!!!!!!!  
...
```

The options for SECTION 1 are described in Section 5.2.1.

- SECTION 2 can be configured to schedule the outputs of SELEN. These are ASCII files and tables, plots and diagrams. The particular set of outputs will of course depend on the kind of analysis planned by the user. After execution, the outputs are collected in a depot folder (see Table 5).

SECTION 2 is marked by the header:

```
...
!!!!!!!!!!!!!!  
This is SECTION (2) of "config.dat": SELEN outputs  
!!!!!!!!!!!!!!  
...
```

The options for SECTION 2 are described in Section 5.2.2.

In both sections, the options are organized in text blocks like this:

```

...
====> TITLE for this block of options
lab-1  Option-1      Description      switches and more data
lab-2  Option-2      Description      ...
...
...
...
lab-n  Option-n      ...
...
...

```

where the three-character string lab-k labels Option-k, which comes with a short Description. The field switches and more data has the form

'y' 'filename'

or

'n' 'parameter1' 'parameter2'

where 'y' and 'n' stand for yes and no, respectively.

The only formatting requirements of file config.dat are: *i*) labels lab-k must be placed in the first three columns of the file; *ii*) ALL the options, switches and data must be enclosed in primes and can be freely placed on the corresponding line. Lines in config.dat which do not begin with a valid label (i.e., a label recognized by the setup program config.f90) are ignored, and therefore can be used for user comments.

5.2.1 Program settings

The options of SECTION 1 of config.dat are organized in eight text blocks:

- 1. Environment settings,**
- 2. Mode of solution,**
- 3. Spatial resolution and reference frame,**
- 4. Choice of the rheological model,**
- 5. Choice of the ice model,**
- 6. Computation of SHs at grid pixels,**
- 7. Ocean Function,**
- 8. Output data archive,**

which are sequentially described below in separate paragraphs.

1. Environment settings

```

...
====> PURGING option -----
110 Purging the wdir before & after execution          'y'
      [see config.f90 for purged filenames extensions ]
...

```

In this first section of `config.dat`, option 110 can be used to remove auxiliary files from the working directory before SELEN is executed (this is particularly useful in the case the previous run of SELEN has crashed, possibly leaving some junk files around).

2. Mode of solution

```
...
=====> SOLUTION of the SLE -----
130 Iterations & mode of solution           '5'   '1'
Modes:  1= Gravitationally self-consistent (GSC)
        2= Elastic GSC / 3="Eustatic" / 4="Woodward" / 5="No Ice"
```

Here, we solve the SLE in “mode 1”, i.e. in its full “Gravitationally self-consistent” form expressed by Eq. (21), and we configure SELEN for five iterations (generally three ensure convergence of the iterative scheme, see Spada and Stocchi 2007).

Other modes of solution available through option 130 above are discussed by Spada and Stocchi (2007); experimenting different solution modes can indeed be very useful to understand the physics behind the SLE and how the program works.

3. Spatial resolution and reference frame

```
...
=====> MAXIMUM HARMONIC DEGREE -----
140 LMAX           '128'

=====> REFERENCE FRAME -----
145 Includes degree 1 Love numbers (CM/CE frames)      'y'   'CM'

=====> TEGMARK RESOLUTION -----
150 R             '33'
151 Prepare a new pixel table (y/n, filename)          'y'   'px-table-r44.dat' ...
```

In this example, the SLE is solved to maximum degree $\ell_{max} = 128$ (option 140).

LDCs of harmonic degree $\ell = 0$ are not computed since they vanish identically because of incompressibility (the volume of the Earth is constant). However, they would not play any role even for a compressible Earth since the SLE includes explicitly the constraint of mass conservation, expressed by Eq. (9).

If the computation of LDCs of harmonic degree $\ell = 1$ is requested (145) (LDCs of harmonic degree one are discussed in e.g. Spada et al. 2011), the program outputs can be expressed in two possible reference frames. The first (CM) is the reference frame with origin in the center of mass of the whole Earth (including the solid and fluid portions), while the second is the reference frame with origin in the center of the undeformed Earth (CE). It is important to note that the solution of the SLE, i.e. sea level change $S(\omega, t)$ (see Eq. 5), is the same in the two frames. The same holds for RSL (4). However, surface displacements, the sea surface variation and the gravity potential variations are indeed sensitive to this choice.

The grid resolution parameter R (option 150) determines the number of pixels of the Tegmark grid by Eq. (33) and the spatial resolution of the oceans/continents distribution (see caption of Fig. 3 for more details). Note that parameters LMAX and R must obey

constraint (34), which ensures an optimal integration on the sphere. This condition is checked by config.f90 and an error message is displayed if it is not satisfied.

When a new Tegmark grid is generated, SELEN identifies oceanic and continental grid pixels by invoking the GMT utility gmtselect, and creates a “pixel table” file (in the example above, px-table-r44.dat) where grid pixels are flagged as “wet” or “dry”. Since this process can be relatively time consuming, especially for large values of grid resolution R, a previously computed pixel table file can be reused in subsequent runs (if R is unchanged) by setting ‘n’ in option 161. In this case, the pixel table file has to be placed in the INPUTS folder.

4. Choice of the rheological model

```
...
=====> RHEOLOGICAL MODEL -----
160    Rheological profile info:                      '2' '2' 'vsc_BJ97.dat'
...

```

The choice of the Earth rheological profile, necessary for the computation of the LDCs, is made through option 160. In SELEN, the computation of the LDCs is accomplished by TABOO (Spada, 2003a, Spada et al., 2004), a program based on the Viscoelastic Normal Modes theory (Peltier, 1974).

Option 160 above allows to provide the number of desired mantle layers NV (here NV=2) and a code (here CODE=2). These parameters identify the values of the density and of the shear modulus of each layer as well as the radii of the interfaces between layers. The reader is referred to the TABOO User guide for information about the options available for NV and a model CODE (see, in particular, Appendix 5.4 of the TABOO program user guide⁸, Spada 2003b). Note that all models available by TABOO (hence by SELEN) assume an elastic lithosphere and a perfectly homogeneous and inviscid core.

In the sample configuration above, the Maxwell viscosities of the mantle layers and the lithospheric thickness are provided by the user-supplied ASCII file vsc_BJ97.dat (this file must be placed in folder VSC before execution).

5. Choice of the ice model

```
...
=====> ICE MODEL -----
170    Ice file name                           'ice3g.dat'
171    Prepare a new SH ice file (y/n, filename)   'y' 'ice3g-l128.dat'
172    Ice history time step (kyrs)                '1.0'
...

```

The ice file name is set by option 170 (this name identifies model ICE-3G of Tushingham and Peltier 1991).

With option 171, we are scheduling the computation of a new set of SH coefficients of the ice thickness distribution of ICE-3G to harmonic degree LMAX (consistently with option 140). These will be written on the ASCII file ice3g-l128.dat in the run directory. Since

⁸http://samizdat.mines.edu/taboo/user_guide.pdf

the computation of the SH coefficients is relatively CPU intensive, the same file can be utilized in subsequent runs if the LMAX parameter is not changed, setting 'n' in option 171 and copying it into the INPUTS folder.

In folder ICE-MODELS, various ice models are available (see Table 6). These include some of the previous ICE-X models developed by Prof. W. R. Peltier and co-workers (ICE-1 and ICE-5G). Folder ICE-MODELS also contains individual regional components of these ice models and some other "test" ice models. Using specific Fortran formats described in the SELEN setup program config.f90, the user can indeed introduce other *ad hoc* ice models according to specific purposes.

6. Computation of SHs at grid pixels

```
...
=====> SPHERICAL HARMONICS (SH) FILE AT PIXELS -----
180    A new SH file  (y/n, filename)           'y'  'sh-r33-1128.bin'
...

```

Using option 180, the user can specify whether to schedule a new computation of SHs at the grid pixels or to employ a pre-computed SH file. Since SH computation is an intensive task, re-using pre-computed SHs can be particularly convenient. The SH set is sensitive to the R and LMAX values (options 140 and 150). We experimented that using a SH binary file generated on a different system (or even with a different compiler) can lead to unexpected results.

In the case a pre-computed file with name sh-r33-1128.bin exists already in the INPUTS folder, the switch 'n' can be set in option 180 to avoid a new, time-consuming computation.

7. Ocean Function

```
...
=====> OCEAN FUNCTION (OF) -----
190    A new OF SH decomposition (y/n, filename)   'y'  'of-1128.dat'
...

```

With option 190, the user can supply the name of the file that will store the coefficients of the SH decomposition of the OF (the ocean function $\mathcal{O}(\omega)$ is defined by Eq. 15). The chosen filename contains information about the values the LMAX parameter (option 140).

If a pre-computed file of SH OF coefficients with name of-1128.dat exists in INPUTS, the switch 'n' can be set to avoid a new computation.

8. Output data archive (depot)

```
...
=====> REPOSITORY LABEL -----
195    The depot name  (four characters)          'T001'

```

The name of the depot where the output data will be stored is given using option 195. It must be four-characters long (in the example above, the name of the depot folder will be depot-T001). The general structure of the SELEN depot is described in Table 5.

By default, all the depot subfolders will be created by SELEN, regardless of the effective configuration of the corresponding tasks.

The depot folder will be created in the DEPOTS directory. If a depot named depot-T001 already exists in DEPOTS, the user is warned by a message on the monitor that existing data will be overwritten.

5.2.2 Output settings

The options of SECTION 2 of config.dat are organized in eight text blocks:

1. *Execution of the GMT scripts,*
2. *Pixelization and grid,*
3. *Ocean function,*
4. *Ice model,*
5. *Spectral properties,*
6. *Relative Sea Level,*
7. *Sea level change at tide gauges,*
8. *Present geodetic variations*

which are sequentially described below in separate paragraphs.

1. Execution of the GMT scripts

```
...
=====> EXECUTION of the GMT SCRIPTS -----
200   Execution of GMT scripts during the SELEN run (y/n)    'y'
...
```

During execution, SELEN creates various GMT scripts that have the purpose of plotting maps, diagrams, etc. based on the program outputs (various examples are given in Section 6.2 for a TEST run). Since for high-resolution computations the execution of the GMT scripts can be quite time consuming, the user can control their execution by option 200 above.

When 'n' is set, SELEN will produce the GMT scripts but these will not be executed. In this case, the outputs of SELEN can be easily transformed in plots at a later stage (the GMT scripts, generated by the setup program config.f90, are in fact copied into the same depot folder where the SELEN output data are stored). Alternatively, they can be visualized with other graphical tools preferred by the user. In particular, for high-resolution runs, some of the default GMT scripts created by SELEN may be not efficient.

2. Pixelization and grid

```

...
=====> PIXELIZATION & WINDOW -----
205  Pixelization maps (y/n)                      'y'
206  Window function evaluation & plot (y/n)      'n'
...

```

This text box can be configured to visualize the geometrical properties of the grid corresponding to the grid resolution parameter R (SECTION 1, option 150). Plotting the grid (option 205) can be useful to familiarize with the numerical solution of the SLE or for teaching purposes.

By the switch with label 206 the user can perform a numerical test of the SH orthogonality on the Tegmark grid, based on the evaluation of a “window function” (Tegmark, 1996). When condition (34) is met, the SH orthogonality is verified to a very high-precision (otherwise SELEN will stop).

Data concerning the pixelization and the window function are stored, after execution, in folders depot-T001/px and depot-T001/wnw, respectively. The grid employed in the TEST run described in Section 6 is visualized in Fig. 3. The caption provides more details on how the pixelization is performed by SELEN.

3. Ocean function

```

...
=====> OCEAN FUNCTION (OF) -----
210  Present-day OF map & reconstruction (y/n)      'y'
215  Plot of OF degree variance (y/n)                 'n'
...

```

In SELEN, the ocean function (OF, defined by Eq. 15) plays an important role, since it defines the domain upon which the ocean load is acting (see Eq. 14). Option 210 can be used for plotting the OF and its SH reconstruction (at degree LMAX), based on the coefficients computed by option 190 above. The comparison between the original and the reconstructed OF can be useful for appreciating the effects of the truncation of the SH series at the chosen level of spatial resolution (an example is given in Figure 4). A plot of the degree variance of the OF can be obtained by switch 215.

All data (ASCII tables and plots, if required) concerning the OF are available in folders depot-T001/of after the execution of SELEN.

4. Ice model

```

...
=====> ICE MODEL -----
220  Maps of original ice sheets (y/n)                'y'
221  Plot of Equivalent Sea Level (ESL) (y/n)        'y'
222  Reconstruction & mapping of the ice sheets (y/n) 'n'
...

```

With these three switches, the users can control some outputs concerning the ice sheet model chosen (see SECTION 1, label 170). These include plots of the spatial distribution of the ice thickness (option 220), and the equivalent sea level (ESL) function (221).

The reconstruction of the ice sheets thickness distribution from the SH coefficients is controlled by option 222.

After execution, all the ice model data (ASCII tables and plots, if required) are stored in the depot-T001/ICE3G (the folder name ICE3G is set by SELEN according to the ice model name in option 170).

5. Spectral properties

```
...
=====> EARTH MODEL SPECTRAL PROPERTIES -----
230   Plot LDCs, relaxation spectrum & residues for normal modes (y/n)  'y'
...
```

This option controls various outputs and plots showing the spectral properties of the Earth model chosen (see SECTION 1, option 160), as a function of harmonic degree ℓ . With term “spectral properties” we denote the isostatic relaxation spectrum, and the elastic, fluid, and viscous components of the LDCs. These quantities and their physical meaning are discussed in detail in e.g. Spada et al. (2011).

Information about the LDCs (tables with ASCII data and plots, if requested) are made available to the user in the depot-T001/Love-Numbers-by-TABOO folder.

6. Relative Sea Level

```
...
=====> RSL PREDICTIONS AT SPECIFIC SITES -----
240   RSL analysis (y/n), database & format  'y' 'my_sealevel_database.txt'  '7'
241   Plot of RSL sites distribution (y/n)          'y'
242   Site-by-site RSL predictions vs data & plots (y/n)  'y' 'y'
243   Scatterplot of RSL data & predictions (y/n)      'n'
244   Misfit between RSL data & predictions (y/n)      'n'
245   Table with all RSL data & predictions (y/n)      'y'

=====> RSL REGIONS -----
250   Global RSL zones           'n'
251   Regional RSL contour lines 'n'  'rsl-region.dat'
...
```

When option 240 is set to y, the lines above allow for a Relative Sea Level (RSL) analysis at specific sites of a user-supplied database with a format recognizable by the setup program config.f90 and by programs rsl.f90 and sh_rsl.f90. A sample is given in file DATA/sealevel.dat (the format label is 0, in this case), which contains information about sites for which radiocarbon-controlled RSL data are available, according to the compilation of Tushingham and Peltier (1992, 1993). Options 240–245 are self-explanatory (examples of the corresponding outputs will be given in Section 6 for a TEST run).

By option 250, global visualizations of the so-called “Clark zones” (Clark et al., 1978) or “RSL zones” can be obtained. These are regions of the globe where the RSL curves show similar features. A regional RSL study can be obtained by switch 251, where RSL contour lines are plotted at a given time. Data about the time of the analysis, the region of interest, and the plot style are supplied in file DATA/rsl-region.dat.

Information about the RSL analyses (tables with ASCII data and plots, if requested) are made available, after the execution, in various sub-folders of depot-T001/rsl.

7. Sea level change at tide gauges

```
...
=====> SEA LEVEL CHANGE AT TIDE-GAUGE STATIONS -----
260  Tide-gauge (TG) analysis & database          'y' 'rlr-trends.txt'
261  Plot of TG stations distribution             'y'
262  TG data scatterplot                         'y'
263  Table of S, N, and U-dot predictions at TG sites   'y'
...
```

These options are designed for a study of the GIA effects at tide gauges (TGs). These are listed in the user-supplied file DATA/rlr-trends.txt, obtained from the Permanent Service for the Mean Sea Level, PSMSL. The Earth model and ice sheets distribution are those chosen by options 160 and 170, respectively.

If the switch 260 is set, the user can obtain a plot of the TGs distribution (option 261), a scatterplot of the TG data (262) and a summary table where rates of change of S , N and U at present time are given for each TG (option 263).

All data about this analysis are stored, after the execution, in the depot-T001/tg folder.

8. Present geodetic variations

```
...
=====> GLOBAL PRESENT-DAY RATES -----
270  Global maps of dot S, U & N                  'y'

=====> 3D VELOCITY -----
275  -Up, North, East, S, and N rates for sites in file   'n' 'NA_KK.txt'

=====> REGIONAL PRESENT-DAY RATES -----
280  Regional maps of dot S, U, & N                 'n'
281  -1 Italy                                     'y'
282  -2 Mediterranean                            'y'
283  -3 Europe                                    'y'
284  -4 Fennoscandia                            'n'
285  -5 Greenland                                'y'
286  -6 North America                           'y'
287  -7 Antarctica                               'y'

=====> STOKES COEFFICIENTS (SC) -----
290  Rate of change of SC & range of degrees for plot   'y' '2' '20'
```

This section of the configuration file is dedicated to predictions of present-day variations of various geodetic quantities.

Option 270 gives access to global maps of *fingeprints* showing the rates of change of S , U and N , for the Earth model and ice sheets distribution chosen by options 160 and 170, respectively (the maps showing the fingerprints are available in folder depot-T001/gmaps). Regional plots are available by switch 280 and the regional options 281–287, with results provided in folder depot-T001/rmaps.

For sites listed in the user-supplied DATA/NA_KK.txt, switch 275 gives access to the Up, North, and East components of the present-day velocity field (this is particularly useful for the interpretation of GPS signals) and to \dot{S} , \dot{U} and \dot{N} as well (the dot indicates the time derivative). All these predictions are collected in folder depot-TEST/geod/sites.

By option 290, the user can obtain predictions for the GIA component of the present-day rates of change of the Stokes coefficients of the Earth's gravity field in a specific range of harmonic degrees (here, the range is $2 \leq \ell \leq 20$). The coefficients, which are normalized according to the GRACE conventions for spherical harmonics and expressed in units of 10^{-11} yr^{-1} , are available in folder depot-T001/stokes⁹.

5.3 Execution

Before launching SELEN, the configuration script has to be invoked with the command:

```
$ configure
```

The configure script will automatically identify the available Fortran compiler on the system and configure SELEN accordingly. If several Fortran compilers are installed, the user can explicitly select a compiler with the command

```
$ configure FC=name
```

where name is the Fortran compiler command (for instance, gfortran or ifort). The configure script will also check the availability of the GMT utilities. If no error is found, SELEN can be launched with

```
$ make run
```

This command will compile the SELEN program units, set up the working folder run in the SELEN directory and launch SELEN. A typical SELEN run consists of three main execution steps. At the end of execution, the SELEN outputs are available in the depot folder (see Table 5).

Step 1: Pre-computation of gridded SHs and of LDCs. At step 1, SELEN builds the spatial grid (see Section 3.3 and Fig. 3), obtains a discrete realization of the OF, performs the SH expansions of the ice thickness variation I and of the OF, and computes the LDCs. The SHs at degree ℓ_{max} are computed at all grid pixels,

Step 2: Numerical solution of the SLE. At step 2, SELEN solves numerically the SLE by the pseudo-spectral method, employing the iterative scheme described by Eqs. (32), and computes the SH coefficients of surface displacements and gravity potential variations,

⁹See rev. 2.3 of the Level-2 Gravity Field Product User Handbook at page 5, available from [ftp://podaac.jpl.nasa.gov/allData/grace/docs/L2-UserHandbook_v2.3.pdf](http://podaac.jpl.nasa.gov/allData/grace/docs/L2-UserHandbook_v2.3.pdf).

Step 3: Computation of geophysical and geodetic quantities. In the last step, SELEN uses the solution of the SLE in the spectral domain to obtain predictions of geophysical and geodetic quantities by SH synthesis, according to the settings in the configuration file config.dat (see Section 5.2).

The execution of SELEN should produce various messages on the monitor, reporting the most important stages of the run. A sample is given in Section 7.5 below. SELEN can be reset by entering

```
$ make clean
```

which clears the SELEN configuration and deletes the SELEN working directory run. This command can be useful if the user wants to restart from scratch, for instance to recover from a previous SELEN crash.

6 A TEST run

To provide an example of the use of SELEN 2.9, we have configured the program for a TEST run.

We advise the user to run this test on his/her own PC and to compare the results with those obtained on our machine (an Apple MacBook configured with a 2GHz Intel Core 2 Duo and running Mac OS X 10.5.8).

6.1 Settings

The configuration file config.dat is reproduced in Section 7.4. A summary of the settings is given in Table 1.

Table 1: Summary of the TEST run settings.

Option(s)	Program settings		See Figure or Table:
130	Mode/iterations	1/3	
140, 150	Resolution (LMAX/R)	128/44	
145	Reference frame	CM	
160	Rheology (NV/CODE)	3/2	
160	Viscosity profile	vsc_VM2a.dat	Table 2
170-172	Ice model		ICE-5G

Option(s)	Output settings	y/n	See Figure or Table:
205	Pixelization maps	y	Fig. 3
206	Window function	n	
210	OF maps	y	Fig. 4
215	OF degree variance	n	
220	Ice sheets maps	y	Fig. 5
221	ESL plot	y	Fig. 6
230	Spectral plots	y	Figs. 7, 8
240-245	Global RSL	y	Figs. 9, 10, 11, 12
250	RSL zones	n	
251	Regional RSL	y	Fig. 13
260-263	TG analysis	y	Fig. 14, Table 3
270	Global fingerprints	y	Figs. 15, 16, 17
275	3D crustal velocity	n	
280-287	Regional fingerprints	n	
290	Stokes coefficients	y	Fig. 18

6.2 Outputs

In this section we collect some of the Figures obtained with SELEN for the TEST run. The whole set of outputs for this run are available in the depot-TEST folder that comes with the SELEN package.



RES= 44, N= 75692

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Figure 3: Pixelization of the sphere for the TEST run ($\ell_{max} = 128$, $R = 44$). Data about the grid, including spherical coordinates of “wet” (oceanic, blue) and “dry” (continental, green) pixels and postscript figures are found in folder depot-TEST/px. In SELEN, wet pixels are separated by dry ones using the GMT program `gmtselect` (Wessel and Smith, 1998). By default, SELEN employs the full resolution coastlines of GMT (`-Df`), and dry (wet) pixels are selected using option `-Ns/k/s/k/s` (`-Nk/s/k/s/k`) of `gmtselect`.

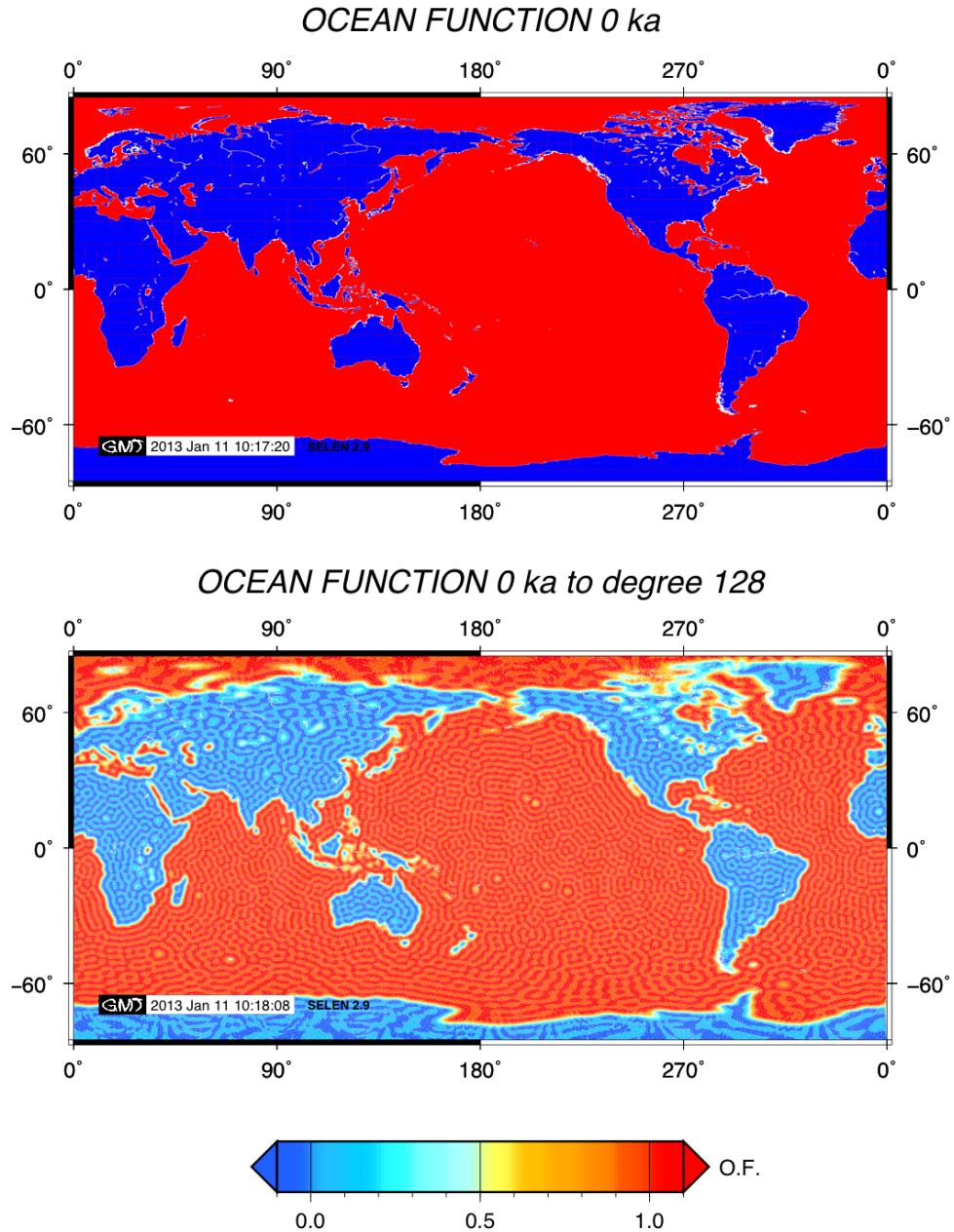


Figure 4: Original OF (top) and its reconstruction (bottom) obtained by synthesis of the SH coefficients to maximum degree $\ell_{max} = 128$ (TEST run). All the OF data are stored in folder depot-TEST/of.

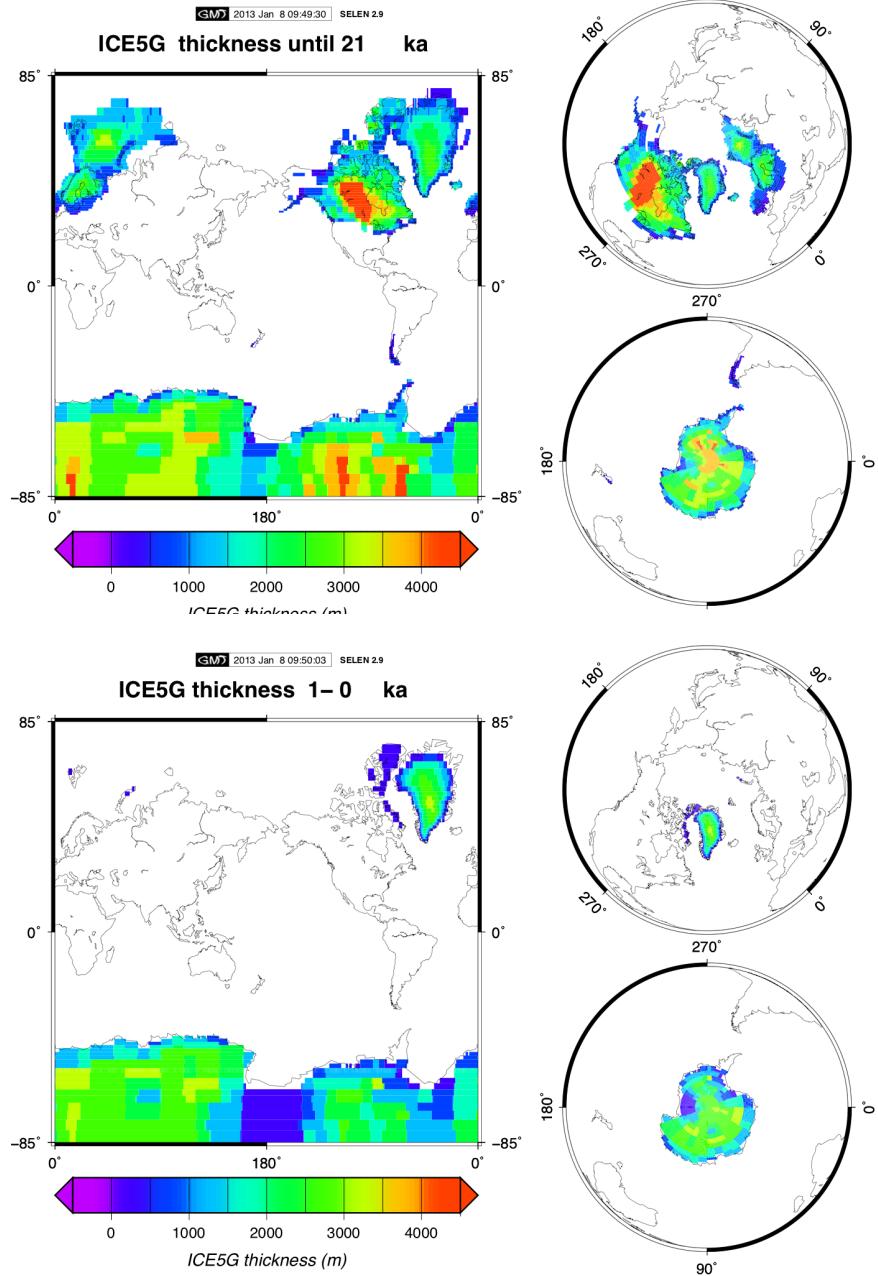


Figure 5: Ice thickness $T(\omega, t)$ at the LGM (21 kyrs ago, top) and during the most recent time increment, between 1 kyr BP and present time (bottom), according to model ICE-5G (Peltier, 2004) (TEST run). The time-history of the Equivalent Sea Level for this model is shown in Fig. 6. Maps for all the other time steps between LGM and present are available in folder depot-TEST/ICE5G/original.

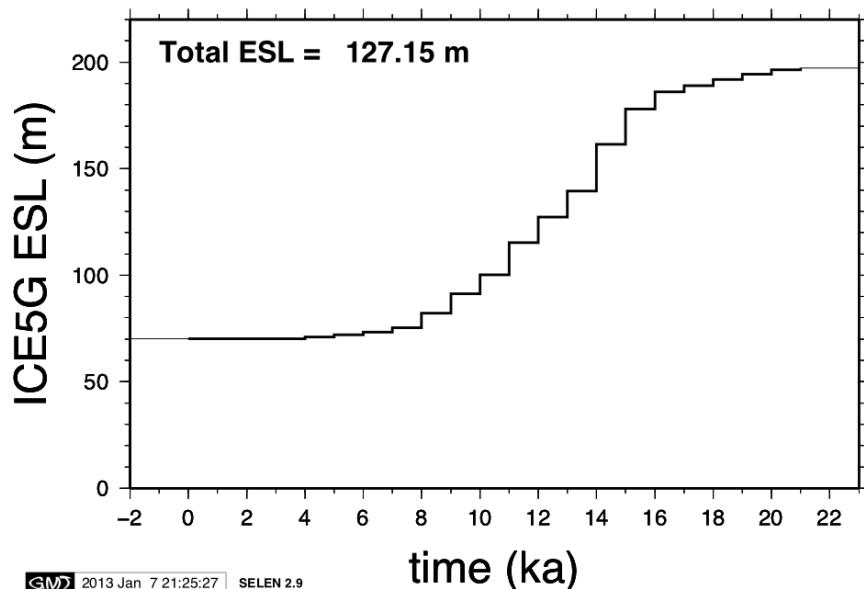


Figure 6: Equivalent Sea Level for model ICE-5G (Peltier, 2004) (TEST run). At a given time t before present, $\text{ESL}(t) = (\rho_i/\rho_w)(V_i(t) - V_i(t_p))/A_o$, where $V_i(t)$ is the ice volume, $V_i(t_p)$ is present day volume, and A_o is the area of the oceans surface. Hence, according to Eq. (12), the plot of ESL mirrors that of $S^E(t)$. The total ESL variation (~ 127 m) represents the difference between ESL at the LGM (21 kyrs ago) and the present day value.

Table 2: Model parameters for model VM2a, employed for the TEST run of SELEN. These correspond to the model with NV=3, CODE=2 and to the rheological profile described in file DATA/vsc_VM2a.dat. This model represents a volume-averaged version of the VM2 viscosity profile associated with the ICE-5G model of Peltier (2004).

Layer	Radius (km)	Density (kg m ⁻³)	Shear modulus ($\times 10^{11}$ Pa)	Viscosity ($\times 10^{21}$ Pa s)	Gravity (m s ⁻²)
Lithosphere	6281–6371	4120	0.73	∞	9.707
Upper mantle	5951–6281	4120	0.95	0.5	9.672
Transition zone	5701–5951	4220	1.10	0.5	9.571
Lower mantle	3480–5701	4508	2.00	2.7	9.505
Core	0–3480	10925	0	0	10.622

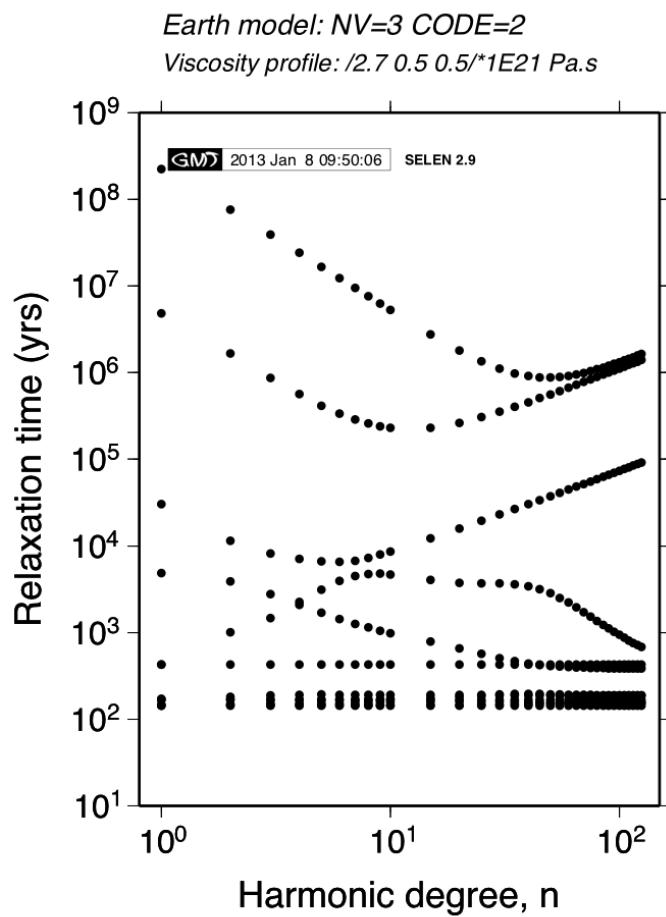


Figure 7: Isostatic relaxation spectrum for the rheological profile VM2a (see TEST run data in Table 2), showing the relaxation times as a function of harmonic degree $\ell = n$ in the range $1 \leq \ell \leq 128$. The spectrum data are stored in folder depot-TEST/Love-Numbers-by-TABOO. The physical meaning of the spectrum is discussed by e.g. Peltier (1974), Spada (2003a), and Spada et al. (2011).

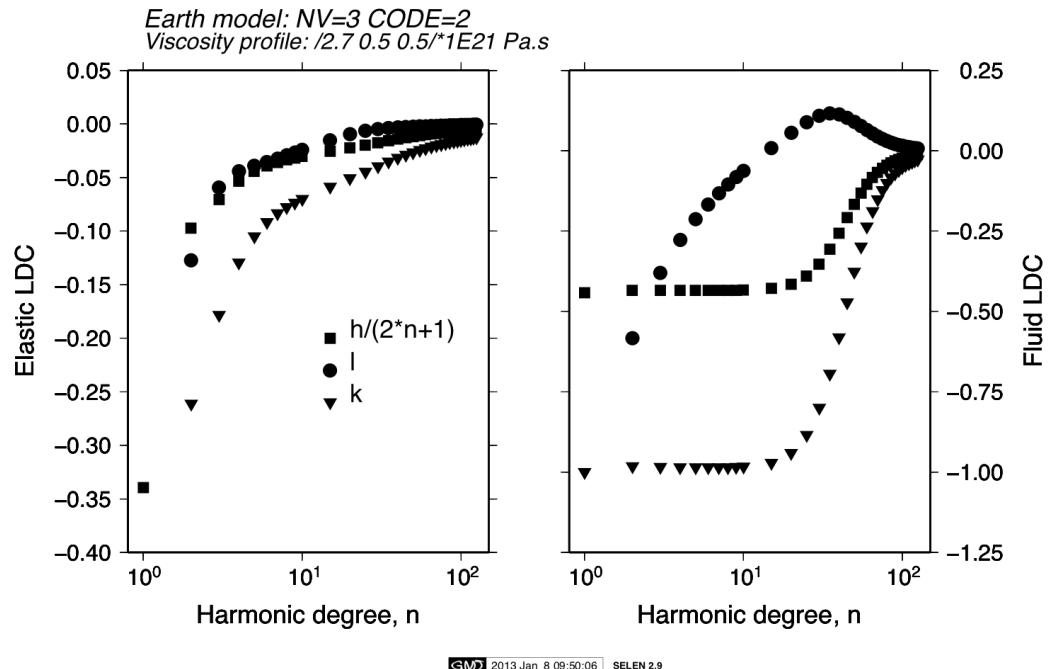


Figure 8: Elastic (left) and fluid (right) values of the LDCs h (associated with vertical displacement), l (horizontal displacement) and k (incremental gravitational potential) for the rheological model VM2a in the TEST run (see Table 2), as a function of harmonic degree ℓ . LDC h is normalized by $(2\ell + 1)$. For the definition of the LDCs see e.g. Spada (2003a), and Spada et al. (2011).

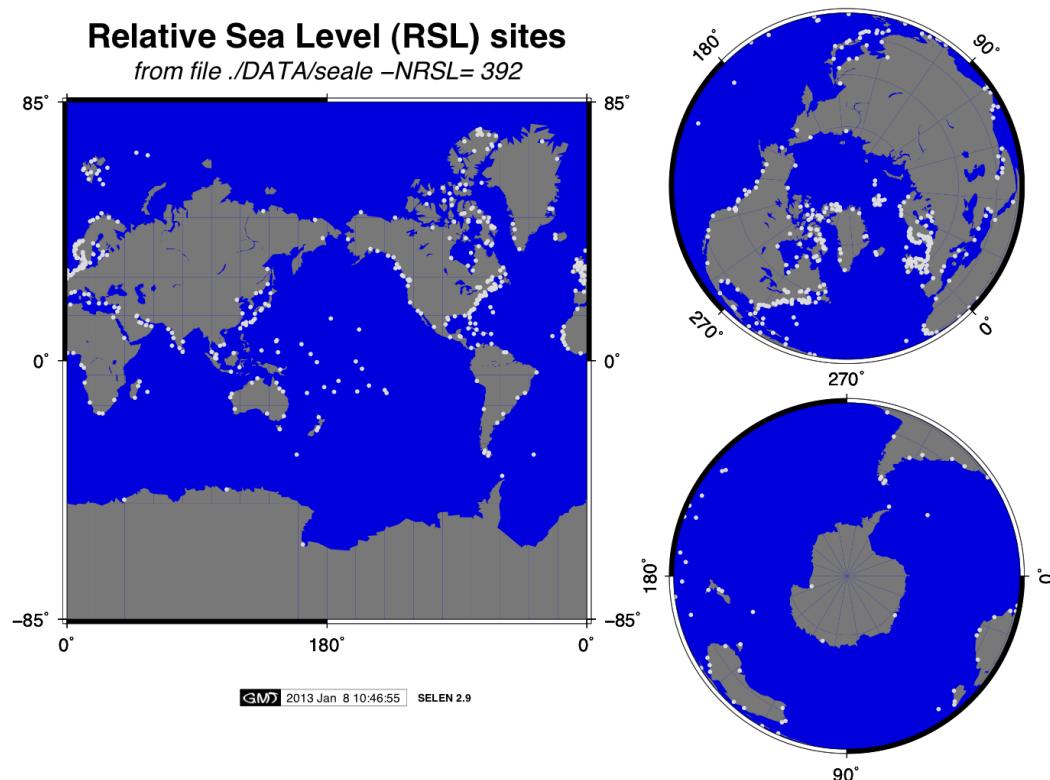
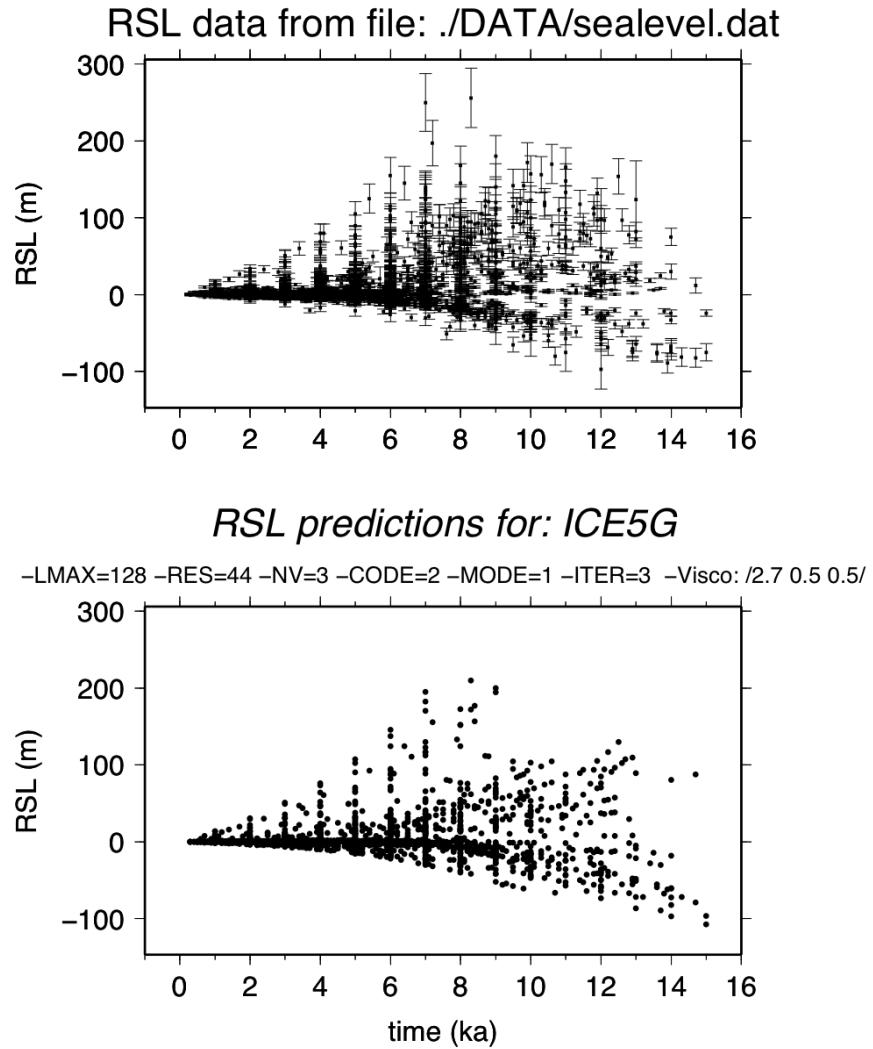


Figure 9: Geographical distribution of the 392 sites in file `sealevel.dat`, from which information about the history of RSL during the last $\sim 15,000$ years is available. This plot and more data about these sites are available from the folder `depot-TEST/rsl/rsl-sites`.



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Figure 10: Scatterplot showing RSL observations (top) from sites of the compilation of Tushingham and Peltier (1992, 1993). RSL predictions at all sites, obtained solving the SLE in our TEST run of SELEN, are shown in the bottom frame. Data are stored in folder depot-TEST/rsl/rsl-scplot.

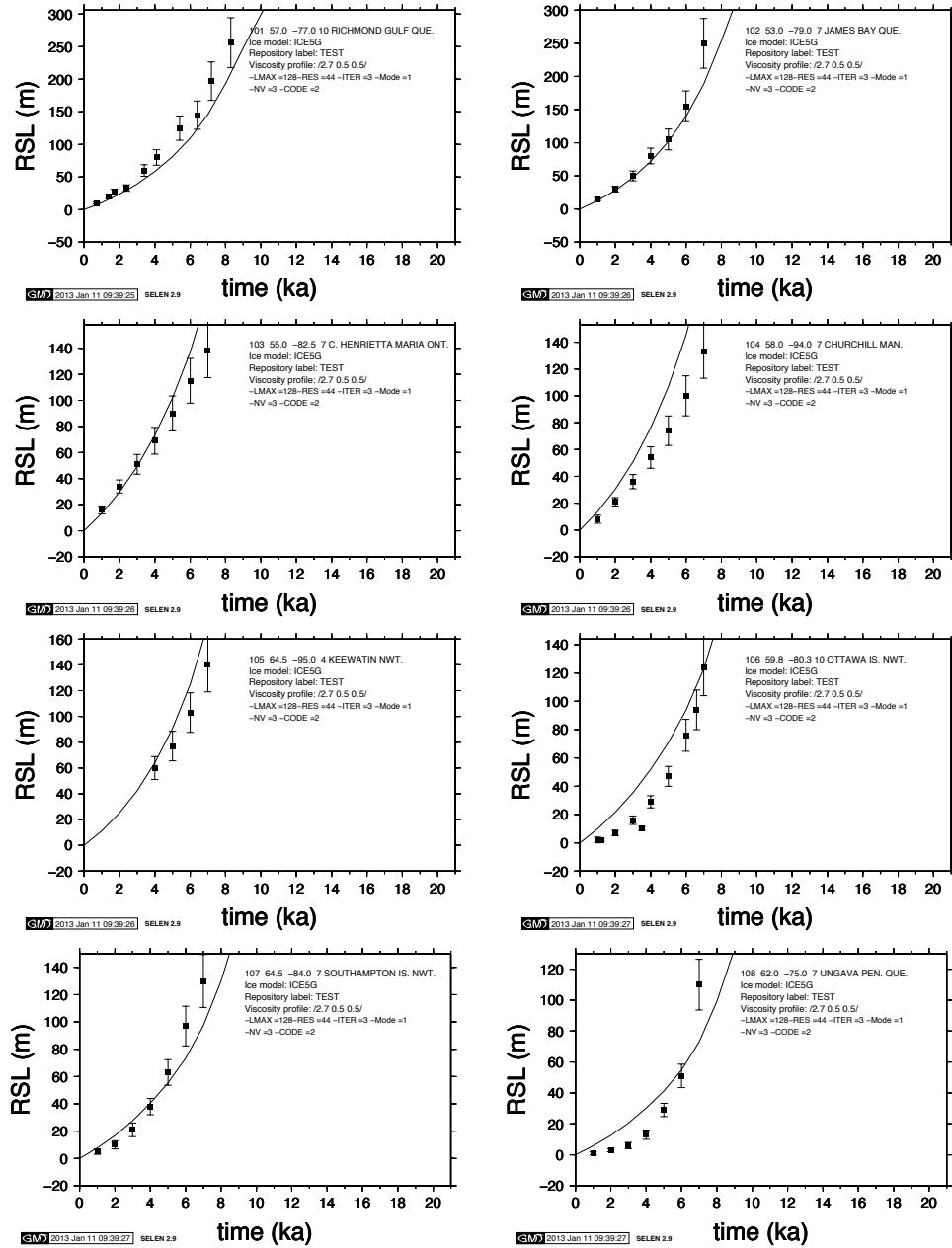


Figure 11: RSL observations (with error bars) pertaining to the eight sites of Hudson bay in file sealevel.dat, compared with SELEN predictions (solid curves). Basic parameters for the TEST run are summarized in each frame. Postscript and PDF figures are located in folders depot-TEST/rsl/rsl-curves/ps and depot-TEST/rsl/rsl-curves/pdf, respectively.

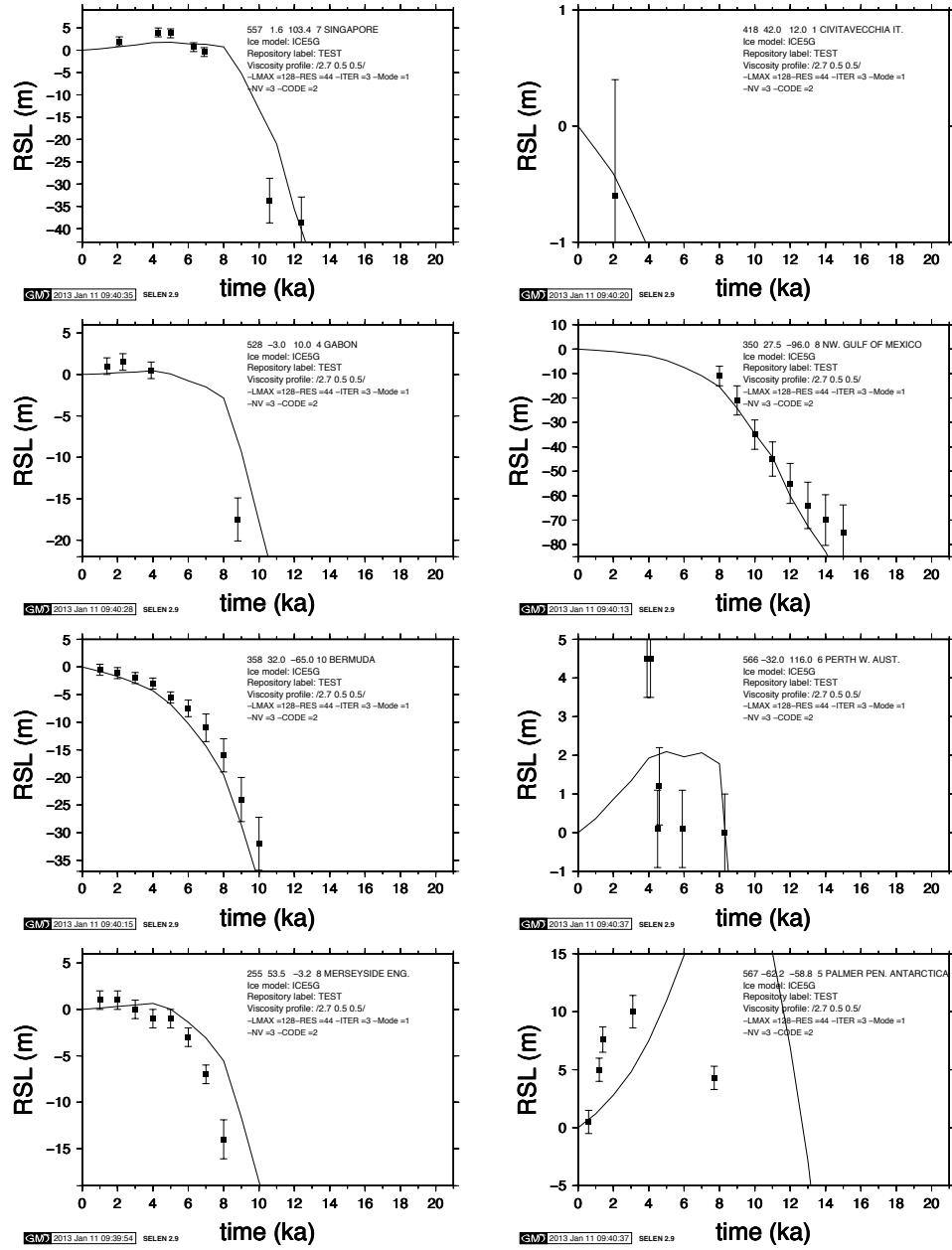


Figure 12: RSL curves at eight miscellanea sites located in the far field of the former ice sheets: Singapore, Civitavecchia (Italy), Gabon, NW Gulf of Mexico, Bermuda, Perth W. Australia, Merseyside (England), and Palmer peninsula (Antarctica). All the RSL predictions for the TEST run are found in `depot-TEST/rsl/rsl-curves`.

Mediterranean RSL

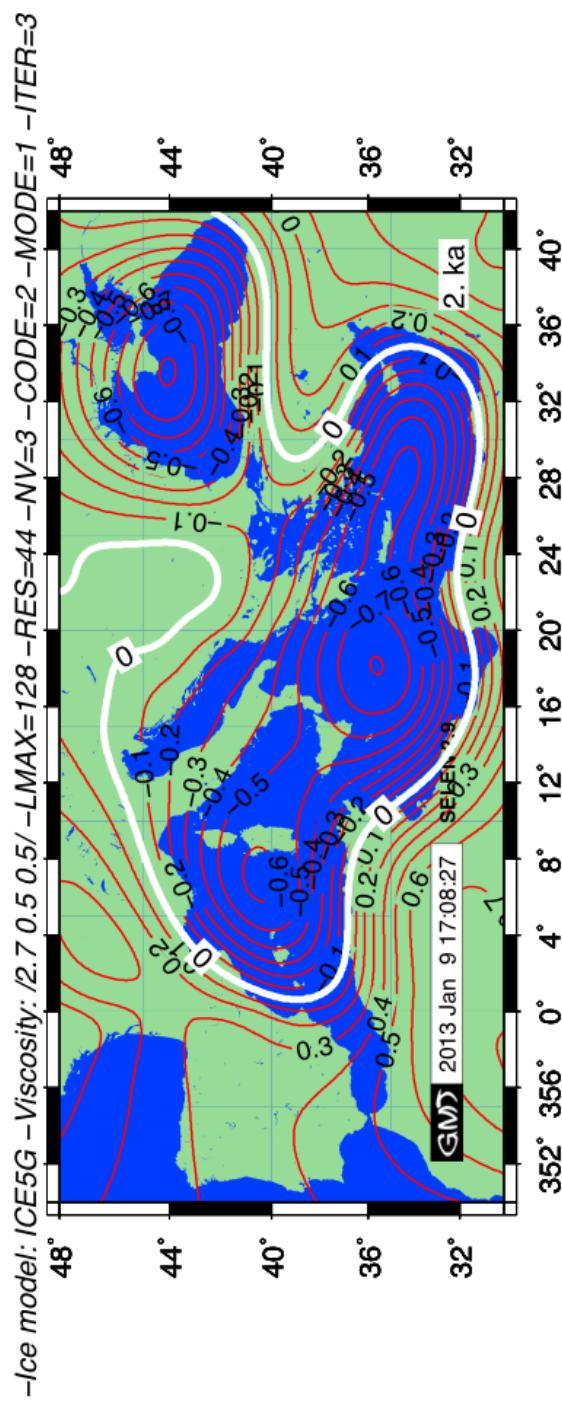
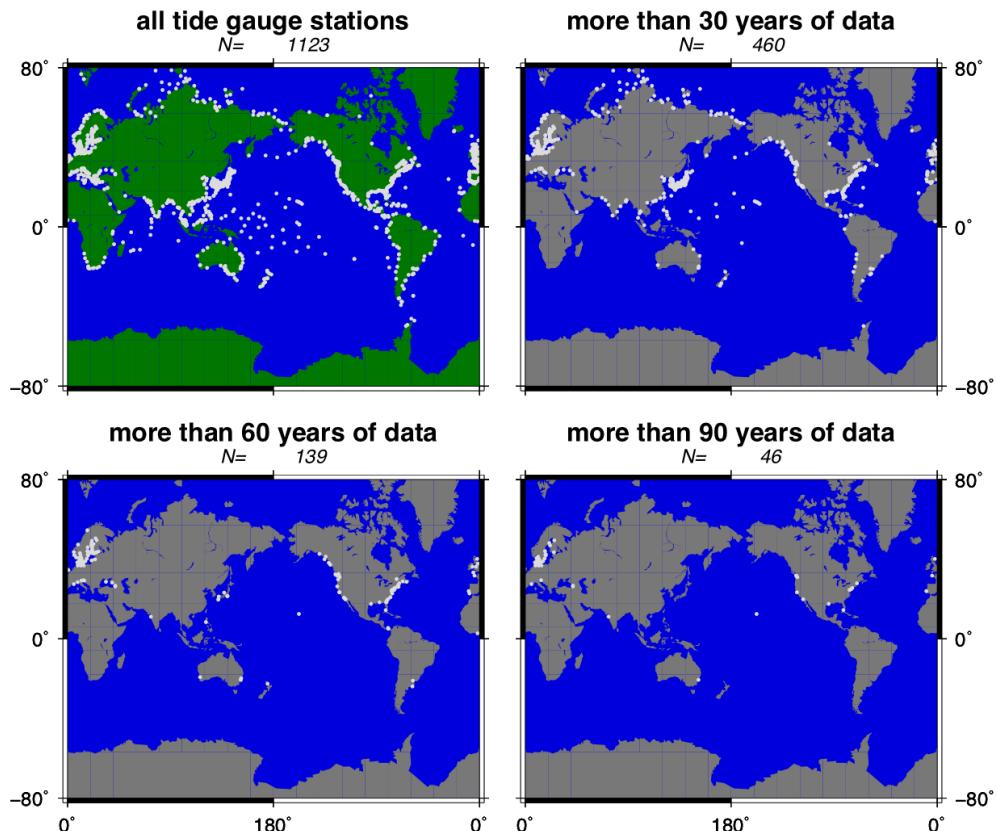


Figure 13: Example of RSL contour plot for the Mediterranean region, obtained at time 2 kyr BP setting the configurations file DATA/rsl-region.dat (RSL is given in units of meters). The computations are based on the SELEN parameters employed in the TEST run (ice model ICE-5G and rheological parameters in Table 2).

Distribution of tide gauge stations as of 1/22/07

GMED 2013 Jan 9 17:08:28 SELEN 2.9

Figure 14: Geographical distribution of the TGs considered in run TEST, according to the number of annual records available from each station in the user-supplied input file DATA/rler-trends.txt. Data and plots for this analysis are stored in folder in depot-TEST/tgauges/tgauges-sites after the execution of SELEN.

Table 3: Present-day trends of GIA-induced sea level change (s-dot), sea surface variation (n-dot), and vertical velocity of the solid Earth (u-dot), expressed in units of millimeters per year, at RLR (Revised Local Reference) PSMSL TGs in the user-supplied file DATA/rllr-trends.txt employed for the TEST run). The table also contains basic information on the TGs and an header with a short summary of the run parameters. This is an excerpt of the ASCII table depot-TEST/tgauges/tgauges-predictions/ptidegauges.

2013.01.09 time=17.09.10											
Ice model: ice5g.dat											
Number of mantle layers: 3											
Model code (see TABOO User guide): 2											
Viscosity model: ./VSC/vsc_VM2a.dat											
Thickness of the lithosphere (km): '90.'											
Viscosity (bottom-to-top, Haskell units) '2.7'											
Viscosity (bottom-to-top, Haskell units) '0.5'											
Viscosity (bottom-to-top, Haskell units) '0.5'											
SLE iterations: 3											
SLE mode of solution: 1											
Maximum harmonic degree: 128											
Tegmark resolution: 44											
PSMSL code	yrs	range yrs	trend mm/yr	error mm/yr	lon deg	lat deg	s-dot mm/yr	n-dot mm/yr	u-dot mm/yr	PSMSL station name	=====
010/001	40	1957 - 2003	2.23	0.46	338.067	64.150	0.147	-0.210	-0.357	REYKJAVIK	
010/011	6	1958 - 1965	8.07	8.35	337.567	63.833	0.274	-0.219	-0.494	GRINDAVIK	
015/011	22	1958 - 2001	1.50	0.34	353.233	62.017	0.645	-0.188	-0.826	TORSHAVN	
025/001	44	1949 - 2003	-2.65	0.42	14.250	78.067	-0.888	0.057	0.974	BARENTSBURG	
025/002	42	1949 - 1993	-1.36	0.46	14.250	78.067	-0.888	0.057	0.974	BARENTSBURG II (SPITSBERGEN)	
025/021	20	1977 - 2004	-3.10	0.64	11.933	78.933	-0.312	0.031	0.354	NY-ALESUND	
030/001	36	1953 - 1989	-1.26	0.62	62.583	76.200	-2.878	0.093	3.070	RUSSKAYA GAVAN	
030/003	36	1953 - 1989	-1.28	0.62	62.583	76.183	-2.867	0.093	3.059	RUSSKAYA GAVAN II	
030/007	19	1960 - 1978	2.43	2.52	60.217	69.600	-0.219	-0.064	0.148	BELYI NOS	
030/014	28	1962 - 1990	-4.89	1.07	58.050	80.617	-2.807	0.070	2.992	KRENKELIA (HEISA OSTROV)	
030/016	23	1950 - 1976	-3.10	1.91	52.700	72.367	-2.132	0.101	2.305	MALYE KARMAKULY	
030/018	48	1952 - 2005	3.14	0.67	33.050	68.967	-3.090	0.290	3.403	MURMANSK	
...											
[omitted records]											
...											
240/001	10	1896 - 1913	1.29	1.43	9.367	41.233	0.267	-0.317	-0.587	LA MADDALENA	
240/011	26	1897 - 1934	1.64	0.39	9.167	39.200	0.250	-0.324	-0.577	CAGLIARI	
250/001	23	1897 - 1921	1.60	0.59	8.017	43.867	0.103	-0.297	-0.402	PORTO MAURIZIO	
250/011	78	1884 - 1992	1.20	0.07	8.900	44.400	0.070	-0.290	-0.362	GENOVA	
250/031	21	1897 - 1920	1.10	0.68	11.817	42.050	0.202	-0.308	-0.512	CIVITAVECCHIA	
250/041	11	1901 - 1921	2.28	1.42	14.267	40.867	0.199	-0.314	-0.514	NAPOLI (ARSENALE)	
250/051	18	1897 - 1921	2.18	0.66	14.267	40.867	0.199	-0.314	-0.514	NAPOLI (MANDRACCIO)	
250/061	9	1951 - 1964	-2.69	3.08	15.650	38.100	0.276	-0.326	-0.605	REGGIO CALABRIA	
260/011	15	1897 - 1919	0.98	0.63	13.333	38.133	0.250	-0.326	-0.579	PALERMO	
260/028	9	1957 - 1968	8.54	3.68	15.300	36.667	0.314	-0.329	-0.647	CAPO PASSERO	
260/031	12	1960 - 1971	-3.02	2.78	15.133	37.500	0.289	-0.328	-0.620	CATANIA	
265/001	11	1991 - 2004	0.63	2.62	14.517	35.900	0.290	-0.329	-0.622	VALLETTA	
270/006	6	1906 - 1911	-3.49	6.73	17.267	40.433	0.179	-0.313	-0.494	TARANTO	
270/011	4	1961 - 1970	-0.58	1.63	18.500	40.133	0.176	-0.314	-0.492	OTRANTO	
270/035	3	1970 - 1972	9.00	2.89	12.283	44.500	0.082	-0.284	-0.367	PORTO CORSINI	
270/041	18	1889 - 1913	2.04	1.20	12.350	45.417	0.039	-0.273	-0.314	VENEZIA (ARSENALE)	
270/051	45	1872 - 1919	2.55	0.42	12.333	45.417	0.039	-0.273	-0.314	VENEZIA (S. STEFANO)	
270/054	82	1909 - 2000	2.39	0.16	12.333	45.433	0.038	-0.273	-0.313	VENEZIA (PUNTA DELLA SALUTE)	
270/061	96	1905 - 2006	1.17	0.12	13.750	45.650	0.037	-0.268	-0.307	TRIESTE	
...											
[omitted records]											
...											
970/211	6	1962 - 2005	2.50	1.19	227.033	69.417	1.393	0.104	-1.295	TUKTOYAKTUK	
A /001	4	1967 - 1977	0.92	6.43	303.083	-63.300	-1.313	-0.050	1.272	BAHIA ESPERANZA	
A /003	43	1960 - 2004	1.57	0.37	295.733	-65.250	-1.326	0.030	1.361	ARGENTINE ISLANDS	
A /005	15	1985 - 2002	5.75	1.85	300.367	-62.483	-1.304	-0.061	1.251	PUERTO SOBERANIA	
A /024	8	1958 - 1978	-1.22	1.82	297.133	-64.900	-1.454	0.020	1.480	ALMIRANTE BROWN	

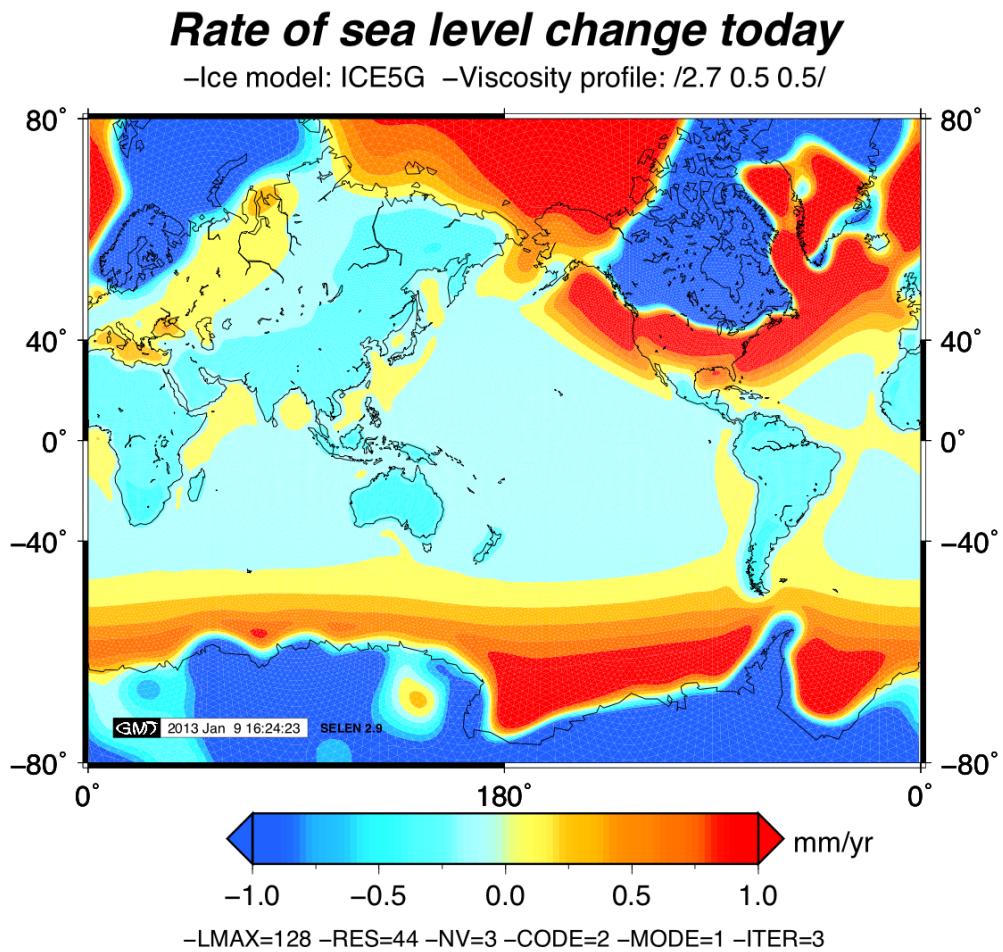


Figure 15: Global map showing the present-day rate of sea level change (*sea level fingerprint*) associated with GIA (\dot{S}) for our TEST run. Data and plots for this analysis are found in folder depot-TEST/gmaps. In this map, the \dot{S} values vary in the range $[-17.01, +3.67]$ mm/yr.

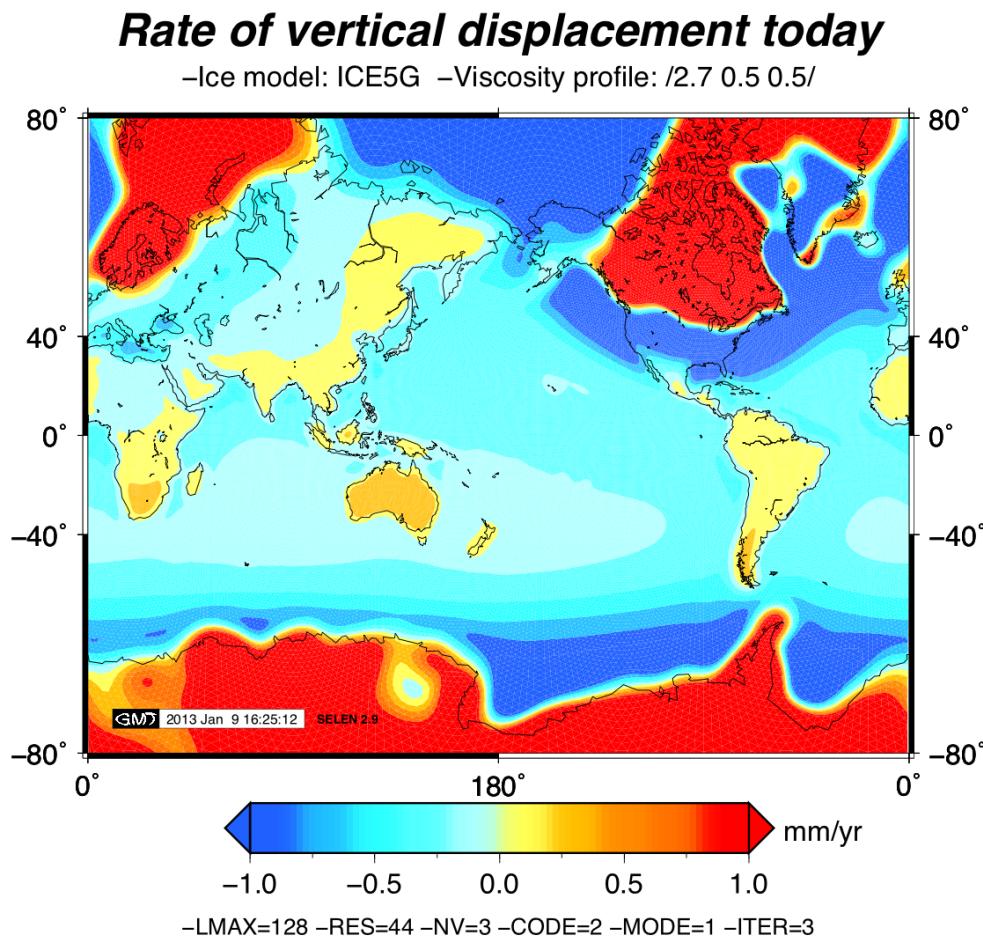


Figure 16: Global map of the GIA-induced vertical velocity (\dot{U}) for the TEST run. Subsiding and uplifting areas are shown by blue and red hues, respectively. The range of the \dot{U} values is $[-3.53, +19.24]$ mm/yr.

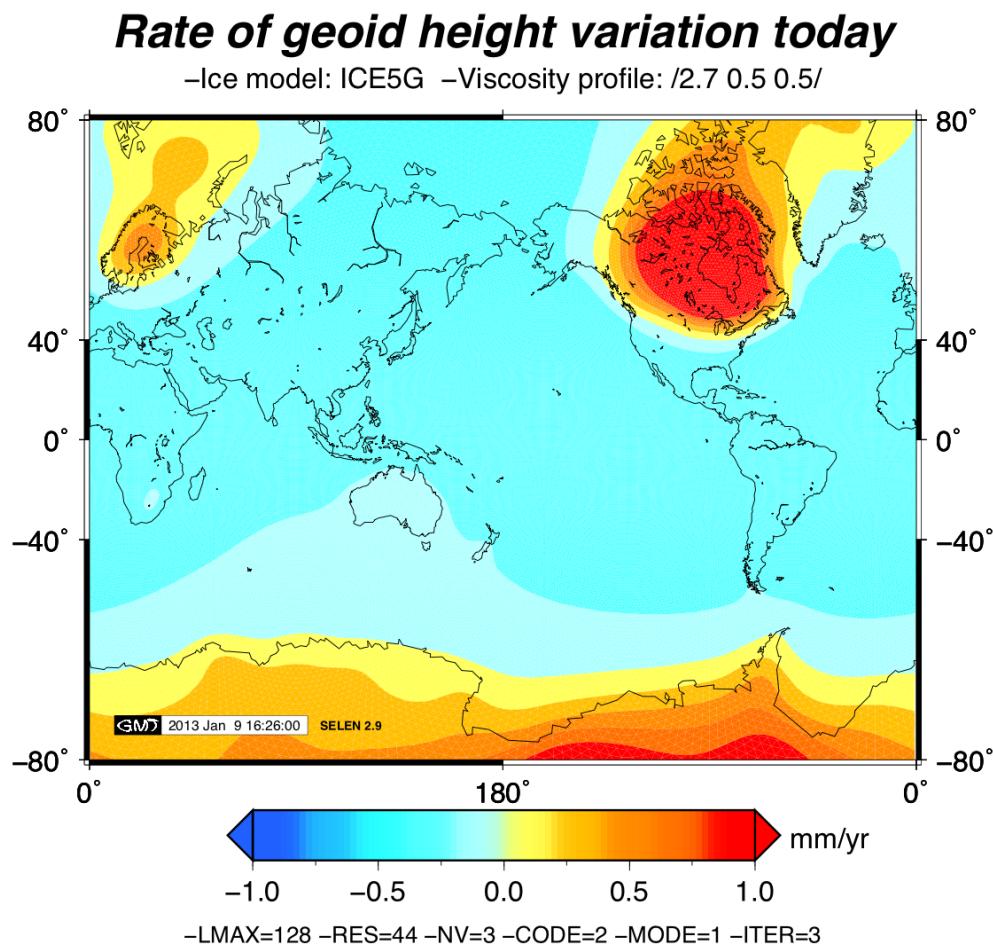


Figure 17: Rate of sea surface variation induced by GIA (*absolute* sea level variation), relative to the Earth's center of mass (\dot{N}), for run TEST. In this map, the range of variation of \dot{N} is $[-0.40/ + 2.35] \text{ mm/yr}$.

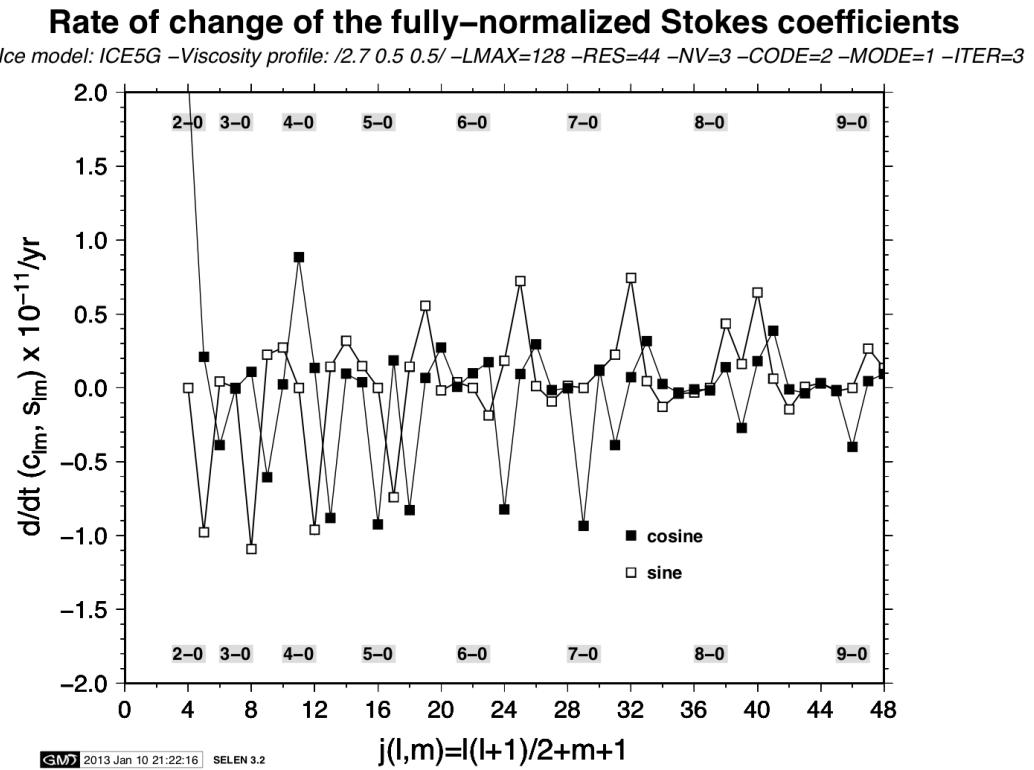


Figure 18: Time-derivatives of the Stokes coefficients of the Earth's gravity field associated with GIA, as a function of the generalized harmonic degree $j = \ell(\ell + 1)/2 + m + 1$ for $2 \leq \ell \leq 9$ and $0 \leq m \leq \ell$, for the TEST run of SELEN. Output data for this analysis are stored in folder depot-TEST/stokes after the execution of SELEN.

7 Appendices

7.1 SELEN Fortran units

Table 4: SELEN Fortran 90 program units. Source files are stored in the `src` folder.

Unit name	Purpose and comments
<code>config.f90</code>	SELEN setup program
<code>esl.f90</code>	Equivalent Sea Level
<code>geo.f90</code>	Time variations of geodetic quantities
<code>gmaps.f90</code>	Synthesis of geodetic quantities on global maps
<code>harmonics.f90</code>	Include file with various SH tools and utilities
<code>ms.f90</code>	GMT multi-segment files from ice data
<code>of_dv.f90</code>	Degree variance of the Ocean Function
<code>px.f90</code>	Pixelization tools (including the Tegmark algorithm)
<code>px_rebuild.f90</code>	Retrieves pixelization data from an existing pixel table file
<code>px_rec.f90</code>	Reorganizes the pixelization data
<code>rec_ice.f90</code>	SH reconstruction of the ice thickness
<code>rec_of.f90</code>	SH reconstruction of the Ocean Function
<code>rmaps.f90</code>	Synthesis of geodetic quantities on regional maps
<code>rsl.f90</code>	Relative Sea Level curves
<code>rsl_zones.f90</code>	Geometry of the Relative Sea Level “Clark’s zones”
<code>rslc.f90</code>	Relative Sealevel Contour lines for regional analyses
<code>sh.f90</code>	SHs at the grid pixels
<code>sh_of.f90</code>	SH coefficients for the Ocean Function
<code>sh_rsl.f90</code>	SHs at the RSL sites
<code>sh_rslc.f90</code>	SHs for regional analysis and RSL contours
<code>sh_tgauges.f90</code>	SHs at the TG sites
<code>shape_factors.f90</code>	“Shape factors” for the ice elements
<code>shice.f90</code>	SH decomposition of the ice model
<code>shtools.f90</code>	A SHTOOLS interface for the SH analysis
<code>sle.f90</code>	The SLE solver
<code>stokes.f90</code>	Variations of the Stokes coefficients of the gravity field
<code>tgauges.f90</code>	Present-day rate of sea level change at the TG sites
<code>tb.F90</code>	The TABOO code
<code>wnw.f90</code>	Numerical test for the SH orthogonality (‘window function’)

7.2 Structure of the SELEN output data

Table 5: Structure of the output data in the depot folder of SELEN.

Folder	Content
ICE5G	Data about the ice model
ICE5G/esl	ESL
ICE5G/original	Ice thickness data
ICE5G/reconstructed	SH reconstruction of ice thickness data
ICE5G/sh	SH coefficients of the ice model
Love-Numbers-by-TABOO	Love numbers data
TABOO	TABOO input files
geod	Predictions at geodetic sites
geod/3dmaps	Regional maps (in progress)
geod/sites	Geodetic predictions at specific sites
gmaps	Global fingerprints (data and plots)
log	Log files of SELEN and TABOO
of	Ocean function data and plots
of/degree_variance	Ocean function degree variance
px	Various pixelization data and plots
rmaps	Regional fingerprints (data and maps)
rsl	Relative Sea Level (RSL) data folder
rsl-contours	RSL contour plot
rsl-curves	RSL curves at specific sites
rsl-misfit	Misfit between RSL data and predictions
rsl-scplot	Scatterplot of RSL data
rsl-sites	Data and plots regarding RSL sites
rsl-table	Summary table of RSL data and predictions
rsl-zones	RSL zones
stokes	Stokes coefficients data
tgauges	Tide gauges (TGs)
tgauges-predictions	Predictions at TGs
tgauges-scplots	TG data scatterplot
tgauges-sites	Maps of TG sites
wnw	“Window test” for the ocean function

7.3 Ice sheets models in SELEN

Table 6: Ice sheets models available in the ICE-MODELS folder of SELEN. For all these models, the details of the chronology and the history of the ice thickness are given in the file headers. A rigorous definition of disk- and cap-shaped ice sheets geometries are given by Spada et al. (2011). The volume of the ice sheets can be assessed running SELEN and with option 221 set to 'y', which will provide the ESL curve (an example is given in Fig. 6 for model ICE-5G.

Ice model file	Description	Notes
alpsc.dat	Alpine ice sheet	See Spada et al. (2009)
alpsf.dat	a few variants	...
alpsh.dat
alpst.dat
disk_off.dat	disk-shaped ice sheet	Instantaneous melting
disk_on.dat	...	Instantaneous freezing
icap_off.dat	cap-shaped ice sheet	Instantaneous melting
icap_on.dat	...	Instantaneous freezing
ice1.dat	The ICE-1 ice model	Peltier and Andrews (1976)
ice1_eup.dat	ICE-1 sub-aggregate	Europe
ice1_gre.dat	...	Greenland
ice1_nam.dat	...	North America and Canada
ice3g.dat	The ICE-3G ice model	Tushingham and Peltier (1991)
ice3g_and.dat	ICE-3G sub-aggregate	Andes
ice3g_ant.dat	...	Antarctica
ice3g_bal.dat	...	Baltic region
ice3g_bar.dat	...	Barents Sea
ice3g_bri.dat	...	British Isles
ice3g_gre.dat	...	Greenland
ice3g_ice.dat	...	Iceland
ice3g_nam.dat	...	North America and Canada
ice3g_sib.dat	...	Siberia
ice5g.dat	The ICE-5G ice model	Peltier (2004)
ice5g_and.dat	ICE-5G sub-aggregate	Andes
ice5g_ant.dat	...	Antarctica
ice5g_fen.dat	...	Fennoscandia
ice5g_gre.dat	...	Greenland
ice5g_icl.dat	...	Iceland
ice5g_lau.dat	...	Laurentide
ice5g_nwz.dat	...	New Zealand

7.4 Configuration file for the TEST run

For the sake of the reader's convenience, file config.dat for the TEST run described in Section 6 is copied below. This file is available from the SELEN distribution package.

```
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
This is file "config.dat" for SELEN 2.9 -
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

~~~~~
The user can configure SELEN by the switches below. Any option is to be written within primes
(e.g., 'option'). A three-digits, left aligned numerical code is provided for each entry (e.g., 001).

In section 1 (settings) the user supplies the spatial resolution, the ice sheets distribution,
and the Earth model viscoelastic structure. This allows one to solve the Sea Level Equation but
no graphical output is obtained.

In section 2 (outputs), a number of optional outputs can be scheduled, including tables and plots
of numerical results. The required GMT scripts are automatically generated according to the
options chosen.

For help, comments, or suggestion, you can contact the authors at the addresses below or consult
the SELEN web page at http://geodynamics.org/cig/software/selen

Contact: Giorgio Spada <giorgio DOT spada AT gmail DOT com>
~~~~~

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
This is SECTION (1) of "config.dat": SELEN settings
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

=====> PURGING option -----
110 Purging the wdir before & after execution           'y'
      [see config.f90 for purged filenames extensions ]

=====> SOLUTION of the SLE -----
130 Iterations & mode of solution                      '3'   '1'
Modes: 1= Gravitationally self-consistent (GSC)
       2= Elastic GSC / 3="Eustatic" / 4="Woodward" / 5="No Ice"

=====> MAXIMUM HARMONIC DEGREE -----
140 LMAX                                         '128'

=====> REFERENCE FRAME -----
145 Includes degree 1 Love numbers (CM/CE frames)     'y'   'CM'

=====> TEGMARK RESOLUTION -----
150 R                                         '44'
151 Prepare a new pixel table (y/n, filename)        'y'   'px-table-r44.dat'

=====> RHEOLOGICAL MODEL -----
160 Rheological profile info:                      '3'   '2'   'vsc_VM2a.dat'

=====> ICE MODEL -----
170 Ice file name                                'ice5g.dat'
171 Prepare a new SH ice file (y/n, filename)      'y'   'ice5g-l128.dat'
172 Ice history time step (kyrs)                  '1.0'

=====> SPHERICAL HARMONICS (SH) FILE AT PIXELS -----
180 A new SH file (y/n, filename)                 'y'   'sh-r44-l128.bin'
```

```

=====> OCEAN FUNCTION (OF) -----
190      A new OF SH decomposition (y/n, filename)          'y'  'of-l128.dat'

=====> REPOSITORY LABEL -----
195      The depot name (four characters)                  'TEST'

!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
This is SECTION (2) of "config.dat: SELEN outputs
!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

=====> EXECUTION of the GMT SCRIPTS -----
200      Execution of GMT scripts during the SELEN run (y/n)      'y'

=====> PIXELIZATION & WINDOW -----
205      Pixelization maps (y/n)                                'y'
206      Window function evaluation & plot (y/n)                'n'

=====> OCEAN FUNCTION (OF) -----
210      Present-day OF map & reconstruction (y/n)            'y'
215      Plot of OF degree variance (y/n)                      'n'

=====> ICE MODEL -----
220      Maps of original ice sheets (y/n)                      'y'
221      Plot of Equivalent Sea Level (ESL) (y/n)              'y'
222      Reconstruction & mapping of the ice sheets (y/n)    'n'

=====> EARTH MODEL SPECTRAL PROPERTIES -----
230      Plot LDCs, relaxation spectrum & residues for normal modes (y/n)  'y'

=====> RSL PREDICTIONS AT SPECIFIC SITES -----
240      RSL analysis, database & format                     'y'  'sealevel.dat'  '0'
241      Plot of RSL sites distribution                      'y'
242      Site-by-site RSL predictions vs data & plots       'y'  'y'
243      Scatterplot of RSL data & predictions             'y'
244      Misfit between RSL data & predictions             'n'
245      Table with all RSL data & predictions             'y'

=====> RSL REGIONS -----
250      Global RSL zones                                 'n'
251      Regional RSL contour lines                      'y'  'rsl-region.dat'

=====> SEA LEVEL CHANGE AT TIDE-GAUGE STATIONS -----
260      Tide-gauge (TG) analysis & database               'y'  'rlr-trends.txt'
261      Plot of TG stations distribution                 'y'
262      TG data scatterplot                            'n'
263      Table of S, N, and U-dot predictions at TG sites 'y'

=====> GLOBAL PRESENT-DAY RATES -----
270      Global maps of dot S, U & N                      'y'

=====> 3D VELOCITY -----
275      -Up, North, East, S, and N rates for sites in file 'n'  'NA_KK.txt'

=====> REGIONAL PRESENT-DAY RATES -----
280      Regional maps of dot S, U, & N                   'n'
281      -1 Italy                                         'y'
282      -2 Mediterranean                               'y'
283      -3 Europe                                       'y'
284      -4 Fennoscandia                             'y'
285      -5 Greenland                                  'y'
286      -6 North America                           'y'

```

```
287      -7 Antarctica          'y'  
=====> STOKES COEFFICIENTS (SC) -----  
290      Rate of change of SC & range of degrees for plot      'y'  '2'  '20'
```

7.5 Output flow

The execution of SELEN should produce the following output flow on the monitor. The three main execution steps, described in Section 5.3, are marked on the right side of the page. The symbol [...] indicates output messages that have been omitted for the sake of clarity.

```
- - - - -  
SELEN, a Sea levEL EquatioN solver, Version 2.9  
Send comments, requests of help and suggestions to:  
<giorgio.spada@gmail.com>  
-  
Copyright(C) 2008-2013  
Giorgio Spada, Daniele Melini,  
Florence Colleoni & Paolo Stocchi  
* * *  
This programs comes with ABSOLUTELY NO WARRANTY  
This is free software and you are welcome to distribute  
it under certain conditions.  
For details, visit <http://www.gnu.org/licenses/>  
or edit file COPYING  
- - - - -  
-----  
>>> 1. Executing SELEN ...  
-----  
---> Output data will be stored into directory DEPOTS/depot-TEST  
---> Purging the working directory:  
---> PX.F90: Hicosahedral pixelization of the sphere  
[...]  
-----+-----+  
| - STEP 1: Grid, SHs and LDCs |  
+-----+  
---> px.gmt: Separating wet from dry pixels  
[...]  
---> PX_REC.F90: Merging the wet & dry pixels tables  
---> pxmap.gmt: Producing pixelization maps  
- wet pixels  
- dry pixels  
- Spherical map of wet and dry pixels  
---> SH.F90: Building the spherical harmonics  
- Maximum degree is: 128  
- JMAX is: 8385  
- Resolution is: 44  
[...]
```

```
---> REC_OF.F90: Reconstructing and mapping the ocean function
[...]

---> Importing ice5g.dat from ICE-MODELS/

---> SHAPE_FACTORS.F90: Computing the shape factors for model: ice5g.dat
[...]

---> SHICE.F90: Computing SH coefficients for the ice model
    - Read      11388   elements from ice5g.dat
[...]

---> MS.F90: Creating multi-segment files for ice sheets maps

---> mapice.gmt: Creating ps images of original ice sheets
[...]

---> TB.F90: Load-deformation coefficients by TABOO (Normal Modes)
    - Calling TABOO
[...]
[...]                                     +-----+
[...]                                     | - STEP 2: Solution of the SLE |
[...]                                     +-----+
[...]

---> SLE.F90: Solving the Sea Level Equation - SLE - for FIXED coastlines
[...]
    - Starting the recursion
    - step  1 of 3
    - step  2 of 3
    - step  3 of 3
[...]                                     +-----+
[...]                                     | - STEP 3: Computation of geophysical |
[...]                                     |           and geodetic variables       |
[...]                                     +-----+
[...]

---> RSL.F90: Predicting RSL at the sites of database: ./DATA/sealevel.dat
    - Computing the harmonics at the RSL sites
    - Computing synthetic RSL curves
[...]

---> TGAUGES.F90: Dot-S, U, and N predictions at tide gauges
    - There are      1123 sites in file ./DATA/rlr-trends.txt
    - Rate of sealevel change at site
        -          1 of      1123 REYKJAVIK
[...]

---> GMAPS.F90: Global maps of dot S, U and N at present time
[...]
---> STOKES.F90: Present-time rate of change of Stokes coefficients
[...]

-----+-----+
>>> 2. Cleaning up the directory...
-----+-----+
[...]

-----+-----+
| - Cleanup |
-----+
```

SELEN, a Sea levEL EquatioN solver, Version 2.9

Send comments, requests of help and suggestions to:
<giorgio.spada@gmail.com>

```
-  
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Florence Colleoni & Paolo Stocchi  
* * *  
This programs comes with ABSOLUTELY NO WARRANTY  
This is free software and you are welcome to distribute  
it under certain conditions.  
For details, visit <http://www.gnu.org/licenses/>  
or edit file COPYING  
-----+-----+  
| - End of execution |  
-----+-----+
```

```
>>> Outputs for this run are available in directory: DEPOTS/depot-TEST
```

7.6 Conventions for Spherical Harmonics

In the SLE theory and in program SELEN, we adopt *complex, 4π -normalized spherical harmonic (SH) functions*:

$$\mathcal{Y}_{\ell m}(\omega) = \sqrt{(2\ell + 1) \frac{(\ell - m)!}{(\ell + m)!}} P_{\ell m}(\cos \theta) e^{im\lambda}, \quad (38)$$

where $\omega \equiv (\theta, \lambda)$, θ is colatitude ($0 \leq \theta \leq \pi$), λ is longitude ($0 \leq \lambda \leq 2\pi$), ℓ is the harmonic degree ($\ell = 0, 1, \dots$), m is the order ($m = 0, 1, \dots, \ell$) and $P_{\ell m}(\cos \theta)$ are the associated Legendre polynomials

$$P_{\ell m}(x) = (-1)^m (1 - x^2)^{\frac{m}{2}} \frac{d^m}{dx^m} P_{\ell}(x), \quad (39)$$

where factor $(-1)^m$ is known as the Condon–Shortley phase, and

$$P_{\ell}(x) = \frac{1}{2^{\ell} \ell!} \frac{d^{\ell}}{dx^{\ell}} (x^2 - 1)^{\ell} \quad (40)$$

are the Legendre polynomials.

SHs with negative order are defined by

$$\mathcal{Y}_{\ell,-m}(\omega) = \mathcal{Y}_{\ell m}^*(\omega), \quad (41)$$

where the asterisk denotes the complex conjugate. By definition (38), the orthogonality condition of the SHs reads

$$\int_0^{2\pi} \int_0^{\pi} \mathcal{Y}_{\ell m}(\omega) \mathcal{Y}_{\ell' m'}^*(\omega) d\omega = 4\pi \delta_{\ell\ell'} \delta_{mm'}, \quad (42)$$

where

$$d\omega \equiv \sin \theta d\theta d\lambda \quad (43)$$

and δ_{ij} is Kronecker delta. In SELEN, the SHs are computed numerically using routines from the SHTOOLS package¹⁰.

7.7 Availability of SELEN

Program SELEN (version 2.9) is hosted by the CIG at the page: <http://geodynamics.org/cig/software/seLEN/>

SELEN is also available from the web page <http://hpc.rm.ingv.it/selen>, or from the authors.

¹⁰<http://www.ipgp.fr/wieczor/SHTOOLS/SHTOOLS.html>

7.8 Related software

TABOO. Program TABOO is a post-glacial rebound calculator written in Fortran 90. It is *not* a SLE solver. SELEN includes the basic components of TABOO in order to compute the LDCs (see program tb.F90). TABOO, along with its documentation, is available from the Samizdat press¹¹,

sTABOO. This is a light version of TABOO, written for absolute beginners. It has been written for the students of the PhD course/workshop on Sea Level Rise and Ice Sheets, held at Center for Ice and Climate, University of Copenhagen (21-25 May, 2012). It comes with installation instructions and can be obtained by email from GS,

ALMA. Program ALMA¹² (Spada, 2008) computes the LDCs for multi-layered Earth models with a generalized Maxwell rheology, adopting a special method of inversion for Laplace transforms (Spada and Boschi, 2006).

7.9 Previous reference documents

Previous versions of SELEN and of the theory behind have been described in various papers and documents. In these works, the readers can find more details and hints, as well as various references to previous relevant papers on the subject:

- The theory behind the SLE is described in a booklet¹³ published from the Samizdat Press¹⁴ in 2005. This material has been subsequently revised and reorganized, and published by Spada and Stocchi (2006),
- the work by Spada and Stocchi (2007) contains a condensed theory of the SLE and illustrates the first version of SELEN, which in this document is referred to as SELEN 1.0,
- The current version of SELEN (2.9) is presented in the manuscript by Spada et al. (2012b) (arXiv:1212.5061), where also some applications are discussed. The present manual is essentially a user-oriented version of this paper.
- The Sea Level Equation page on Wikipedia¹⁵ gives a short definition of the SLE.

7.10 SELEN benchmarks

During the last few years, in the framework of the European COST Action ES0701 “Improved Constraints on Models of Glacial Isostatic Adjustment”¹⁶, various benchmark tests on GIA modeling have been performed.

¹¹<http://samizdat.mines.edu/taboo/>

¹²http://www.fis.uniurb.it/spada/ALMA_minipage.html

¹³<http://samizdat.mines.edu/sle/sle.pdf>

¹⁴<http://www.samizdat.edu>

¹⁵http://en.wikipedia.org/wiki/Sea_level_equation

¹⁶<http://www.cost-es0701.geoenvi.org/>

Until present, the results concerning Post Glacial Rebound modeling have been published in the paper by Spada et al. (2011). In this work, program TABOO (based on the Viscoelastic Normal Modes method of Peltier 1974) has been successfully tested against various programs used in the community, often based on completely different techniques. Since SELEN computes the LDCs using TABOO, and the LDCs enter the Green's functions that appear in the SLE, the Spada et al. (2011) benchmark has validated these fundamental components of the program.

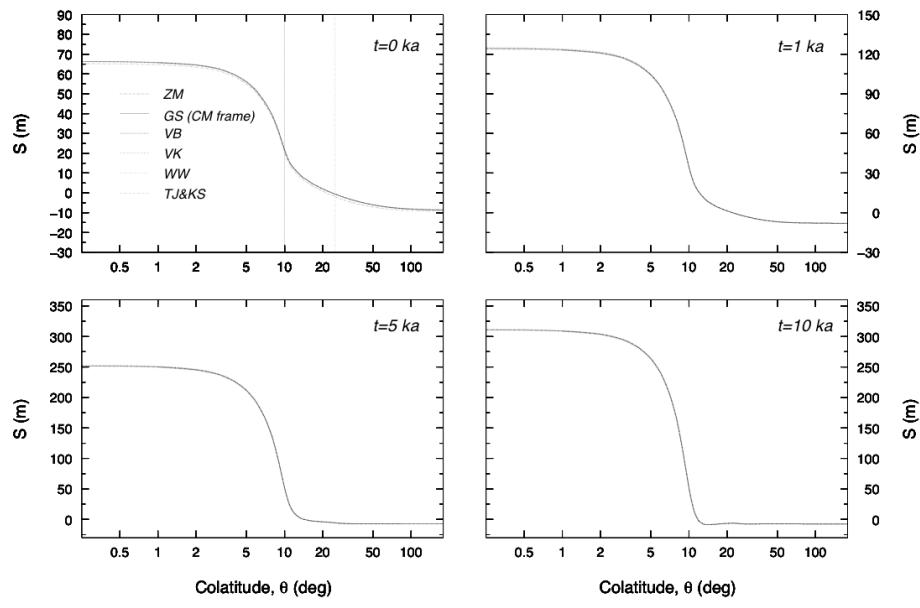


Figure 19: Sea level change $S(\omega, t)$ and a function of colatitude, for various times after the load is imposed ($t = 0$), for the benchmark test described in Spada et al. (2011). Vertical segments in the top left frame mark the margin of the ice load and the (fixed) continent–ocean boundary.

The idea of a SLE benchmark dates back to the mid 1990s, and was first launched by the colleagues Georg Kaufmann and Paul Johnston (the whole story and subsequent developments are reviewed in the Introduction of the Spada et al. (2011) paper, where some useful links are also given). At the current stage (January 2013), a number of tests computations have been performed in the context of the European COST Action ES0701, and a manuscript is in preparation.

In Fig. 19 we show the outcomes of some SLE computations independently performed by Valentina Barletta (VB), Zdenek Martinec (ZM), Tom James and Karen Simons (TJ& KS), Volker Klemann (VK), Wouter van der Wal (WW) and myself (GS, based on a suitably designed version of SELEN). Here, the SLE is solved assuming an ocean–continents distribution with zonal symmetry wrt the axis $\theta = 0^\circ$. The continent, which extends to colatitude $\theta = 25^\circ$, is loaded by a ice sheet with parabolic profile (i.e., a *cap*, see Spada et

al. 2011) having a thickness of 1500 m at the centre and half amplitude $\alpha = 10^\circ$, and characterized by a Heaviside time–history (the load is switched on at time $t = 0$). The ice and water density are $\rho_w = 1000 \text{ kg m}^{-3}$ and $\rho_i = 920 \text{ kg m}^{-3}$, respectively. Shorelines are not allowed to migrate horizontally and the Earth is not rotating. The rheological model is the same employed in the Spada et al. (2011) exercise (i.e., the five-layer, incompressible viscoelastic model M3–L70–V01, see their Table 3). SELEN has been configured for a “gravitationally self–consistent” mode of solutions with three iterations, with a grid resolution parameter $R=44$, and a maximum harmonic degree $LMAX=128$. The SLE is solved in the CM frame.

Fig. 19, which illustrates the history and the spatial pattern of $S(\omega, t)$, indicates that the various numerical solutions of the SLE are, for this axisymmetric test, substantially in agreement. New tests, which are under way, are dealing with the more challenging case of a realistic ocean function and a complex ice sheets distribution.

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