Register Allocation, ii

Liveness analysis & Graph coloring & their interplay

Liveness analysis

Fixed point computation:

- Define which variables are set and which are used, for each instruction: kill & gen functions
- Specify how nearby instructions transmit live values around the program: in & out functions
- Iterate until nothing changes

kill, gen: i → register set

kill(s) = {all assigned registers in stm s}

gen(s) = {all referenced registers in stm s}

Gen & Kill

		gen	kill
l:	:f	0	()
2:	(x2 <- eax)	(eax)	(x2)
3:	(x2 *= x2)	(x2)	(x2)
4 :	(2x2 < - x2)	(x2)	(2×2)
5:	(2x2 *= 2)	(2x2)	(2×2)
6:	(3x < - eax)	(eax)	(3x)
7:	(3x *= 3)	(3x)	(3x)
8:	(eax <- 2x2)	(2x2)	(eax)
9:	(eax += 3x)	(3x eax)	(eax)
10:	(eax += 4)	(eax)	(eax)
11:	(return)	(eax)	0

in, out: Nat \rightarrow register set in(n) = gen(n-th-inst) \cup (out (n) - kill(n-th-inst)) out(n) = \cup {in(m) | m \in succ(n)}

Intuition behind the definitions: if $x \in \mathbf{in}(n)$ then we know that x is live between instructions n-I and n. Similarly, if $x \in \mathbf{out}(n)$ then we know that x is live between instructions n and n+I.

in, out : Nat \rightarrow register set in(n) = gen(n-th-inst) \cup (out (n) - kill(n-th-inst))

out(n) = $U\{in(m) \mid m \in succ(n)\}$

One possible solution: every variable is in **in**(n) and in **out**(n) for every n.

But that isn't super helpful... since we want to know what values don't need to be in registers at each point.

in, out: Nat \rightarrow register set in(n) = gen(n-th-inst) \cup (out (n) - kill(n-th-inst)) out(n) = \cup {in(m) | m \in succ(n)}

Instead, compute **in** and **out** by iterating the equations until we reach a fixed point. Starting out with the assumption that they map everything to the empty set. Then repeatedly apply the right hand sides until nothing changes.

That result will satisfy the definitions of **in** and **out** but will it be the smallest solution?

		in	out
1:	:f	()	()
2:	$(x2 \leftarrow eax)$	0	0
3:	(x2 *= x2)	()	0
4:	(2x2 < - x2)	()	()
5:	(2x2 *= 2)	()	()
6:	$(3x \leftarrow eax)$	()	()
7:	(3x *= 3)	()	()
8:	(eax < - 2x2)	()	()
9:	(eax += 3x)	()	()
10:	(eax += 4)	()	()
11:	(return)	()	()

		in	out
l:	:f	()	()
2:	(x2 <- eax)	(eax)	()
3:	(x2 *= x2)	(x2)	()
4 :	(2x2 < - x2)	(x2)	()
5:	(2x2 *= 2)	(2x2)	()
6:	(3x <- eax)	(eax)	()
7:	(3x *= 3)	(3x)	()
8:	(eax <- 2x2)	(2x2)	()
9:	(eax += 3x)	(3x eax)	()
10:	(eax += 4)	(eax)	()
11:	(return)	(eax)	()

		in		out
l:	:f	()	(6	eax)
2:	(x2 <- eax)	(eax)	(2	×2)
3:	(x2 *= x2)	(x2)	()	×2)
4:	(2x2 < - x2)	(x2)		2×2)
5:	(2x2 *= 2)	(2×2)	(6	eax)
6:	(3x <- eax)	(eax)		3×)
7:	(3x *= 3)	(3x)		2×2)
8:	(eax <- 2x2)	(2×2)		3x eax)
9:	(eax += 3x)	(3x eax)	(6	eax)
10:	(eax += 4)	(eax)	(6	eax)
11:	(return)	(eax)	()	

		in	out
l:	:f	(eax)	(eax)
2:	$(x2 \leftarrow eax)$	(eax)	(x2)
3:	(x2 *= x2)	(x2)	(x2)
4:	(2x2 < - x2)	(x2)	(2×2)
5:	(2x2 *= 2)	(2x2 eax)	(eax)
6:	(3x <- eax)	(eax)	(3x)
7:	(3x *= 3)	$(2\times2 \ 3\times)$	(2x2)
8:	(eax <- 2x2)	$(2\times2 \ 3\times)$	(3x eax)
9:	(eax += 3x)	(3x eax)	(eax)
10:	(eax += 4)	(eax)	(eax)
11:	(return)	(eax)	()

		in	out
1:	:f	(eax)	(eax)
2:	$(x2 \leftarrow eax)$	(eax)	(x2)
3:	(x2 *= x2)	(x2)	(x2)
4:	(2x2 < - x2)	(x2)	(2x2 eax)
5:	(2x2 *= 2)	(2x2 eax)	(eax)
6:	(3x < - eax)	(eax)	$(2\times2 \ 3\times)$
7:	(3x *= 3)	(2x2 3x)	$(2\times2 \ 3\times)$
8:	(eax <- 2x2)	(2x2 3x)	(3x eax)
9:	(eax += 3x)	(3x eax)	(eax)
10:	(eax += 4)	(eax)	(eax)
11:	(return)	(eax)	()

		in	out
1:	:f	(eax)	(eax)
2:	(x2 <- eax)	(eax)	(x2)
3:	(x2 *= x2)	(x2)	(x2)
4:	(2x2 < - x2)	(eax x2)	(2x2 eax)
5:	(2x2 *= 2)	(2x2 eax)	(eax)
6:	(3x <- eax)	(2x2 eax)	$(2\times2 \ 3\times)$
7:	(3x *= 3)	(2x2 3x)	$(2\times2 \ 3\times)$
8:	(eax <- 2x2)	(2x2 3x)	(3x eax)
9:	(eax += 3x)	(3x eax)	(eax)
10:	(eax += 4)	(eax)	(eax)
11:	(return)	(eax)	()

		in	out
1:	:f	(eax)	(eax)
2:	(x2 < - eax)	(eax)	(x2)
3:	(x2 *= x2)	(x2)	$(eax \times 2)$
4:	(2x2 < - x2)	(eax x2)	(2x2 eax)
5:	(2x2 *= 2)	(2x2 eax)	(2x2 eax)
6:	(3x <- eax)	(2x2 eax)	(2x2 3x)
7:	(3x *= 3)	(2x2 3x)	(2x2 3x)
8:	(eax <- 2x2)	(2x2 3x)	(3x eax)
9:	(eax += 3x)	(3x eax)	(eax)
10:	(eax += 4)	(eax)	(eax)
11:	(return)	(eax)	()

		in	out
1:	:f	(eax)	(eax)
2:	(x2 <- eax)	(eax)	(x2)
3:	(x2 *= x2)	(eax x2)	$(eax \times 2)$
4:	(2x2 < - x2)	(eax x2)	(2x2 eax)
5:	(2x2 *= 2)	(2x2 eax)	(2x2 eax)
6:	(3x <- eax)	(2x2 eax)	(2x2 3x)
7:	(3x *= 3)	(2x2 3x)	(2x2 3x)
8:	(eax <- 2x2)	(2x2 3x)	(3x eax)
9:	(eax += 3x)	(3x eax)	(eax)
10:	(eax += 4)	(eax)	(eax)
11:	(return)	(eax)	()

		in	out
1:	:f	(eax)	(eax)
2:	$(x2 \leftarrow eax)$	(eax)	(eax x2)
3:	(x2 *= x2)	(eax x2)	(eax x2)
4:	(2x2 < - x2)	(eax x2)	(2x2 eax)
5:	(2x2 *= 2)	(2x2 eax)	(2x2 eax)
6:	(3x < - eax)	(2x2 eax)	(2x2 3x)
7:	(3x *= 3)	(2x2 3x)	(2x2 3x)
8:	(eax <- 2x2)	$(2\times2\ 3\times)$	(3x eax)
9:	(eax += 3x)	(3x eax)	(eax)
10:	(eax += 4)	(eax)	(eax)
11:	(return)	(eax)	0

Graph coloring

Reduce register allocation to the graph coloring problem

- Nodes represent variables
- Edges connect nodes that cannot share a register
- Colors represent registers

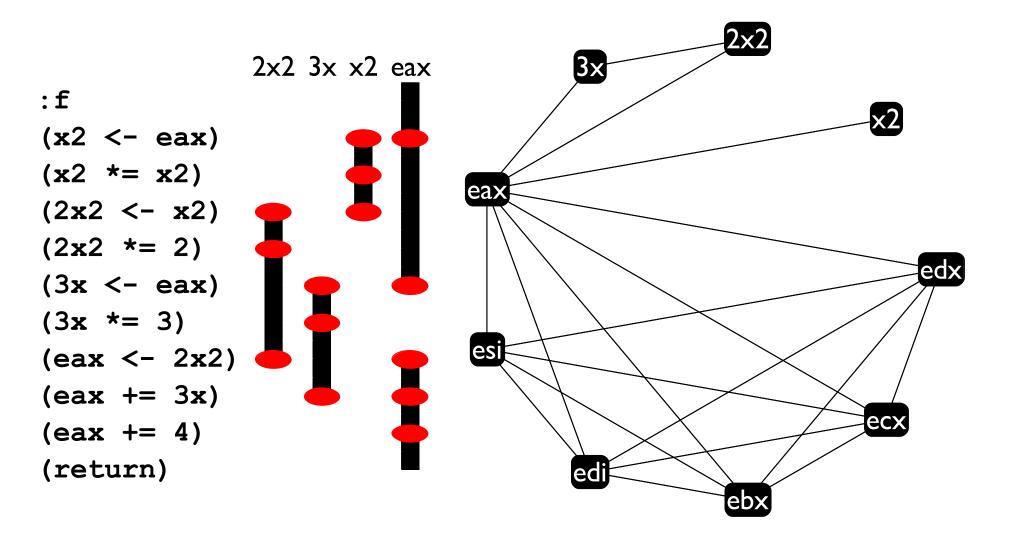
If we can color the graph (where no adjacent nodes share a color) in N colors, then we can compile this program to use N registers

Graph coloring

Build interference graph from the liveness information

- Two variable live at the same time interfere with each other
- Killed variables interferes with all live variables at that point, unless it is a (x <- y) instruction (in which case it is fine if x and y share a register)
- All real registers interfere with each other

Liveness ⇒ Interference graph



Check yourself

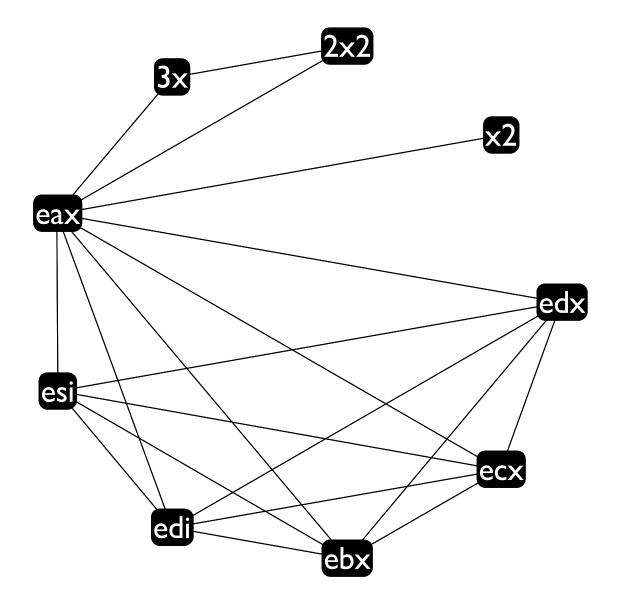
For each pair of x2, 2x2, 3x, and eax:

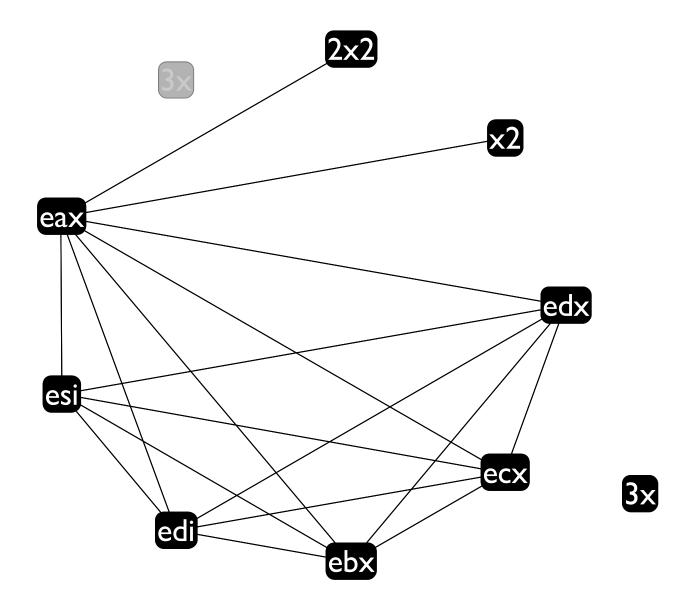
- do they interfere?;
- should they?

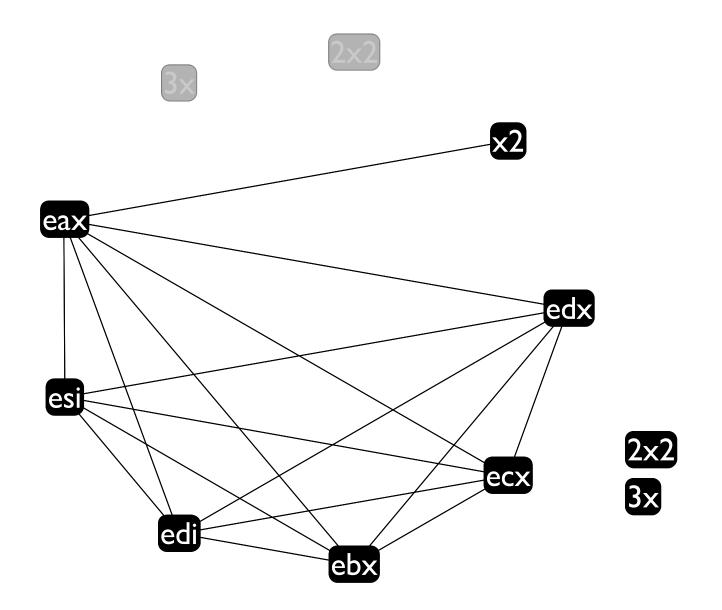
Graph coloring

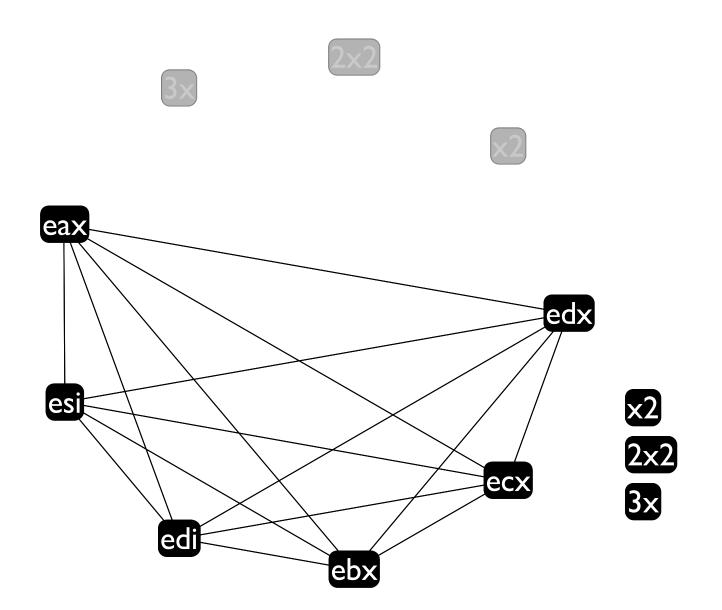
Algorithm:

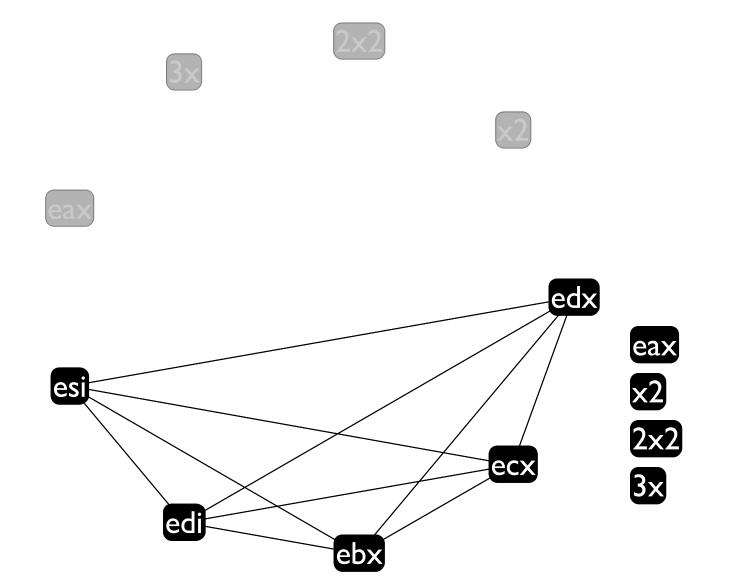
- Repeatedly select a node and remove it from the graph, putting it onto a stack
- When the graph is empty, rebuild it, putting a new color on each node as it comes back into the graph, making sure no adjacent nodes have the same color
- If there are not enough colors, the algorithm fails (spilling comes in here)
- Make sure real registers come out of the graph last so they get the first 6 colors

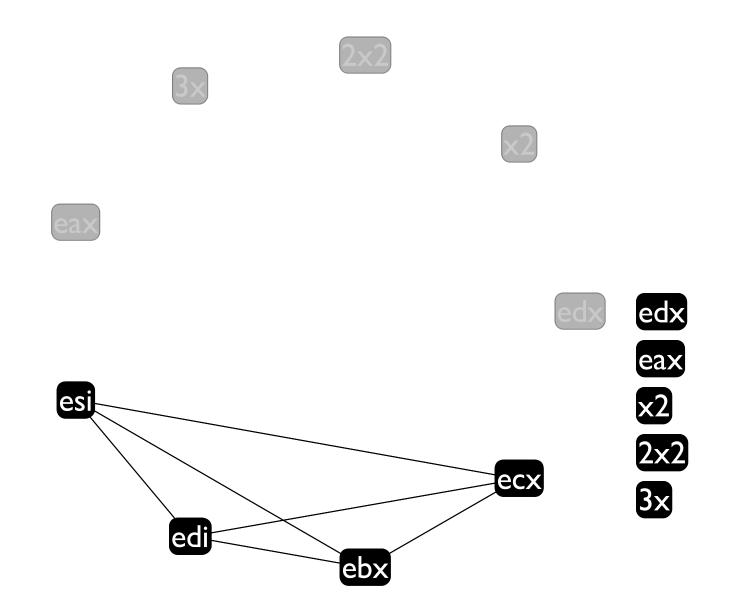


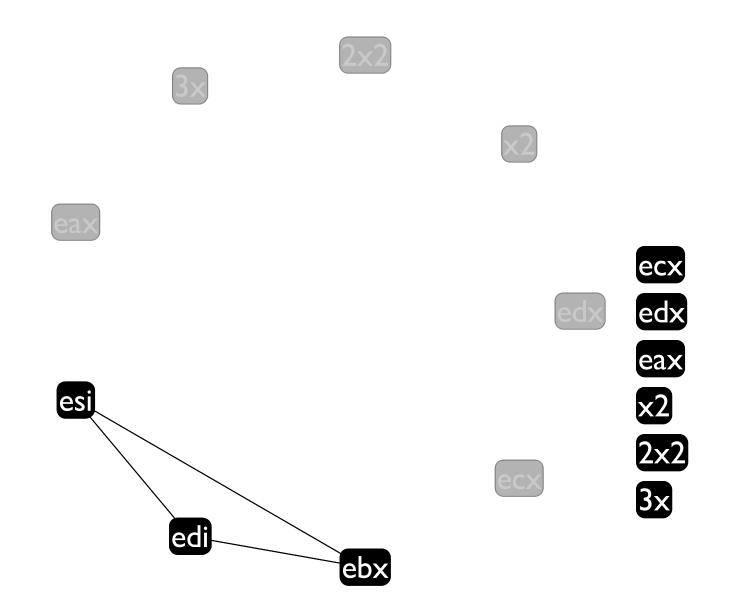


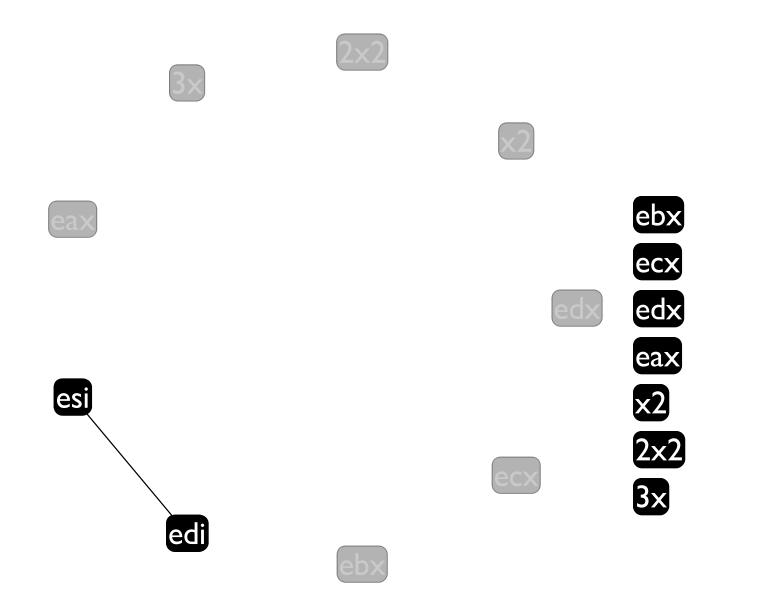


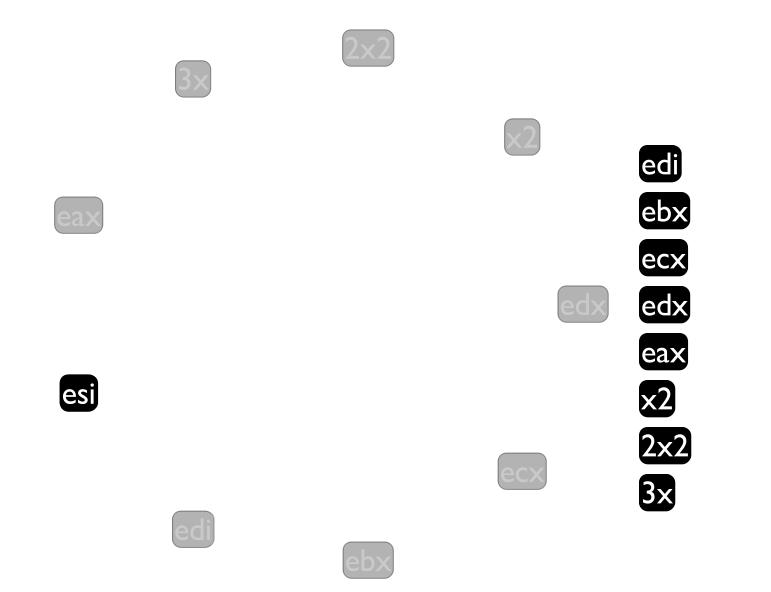


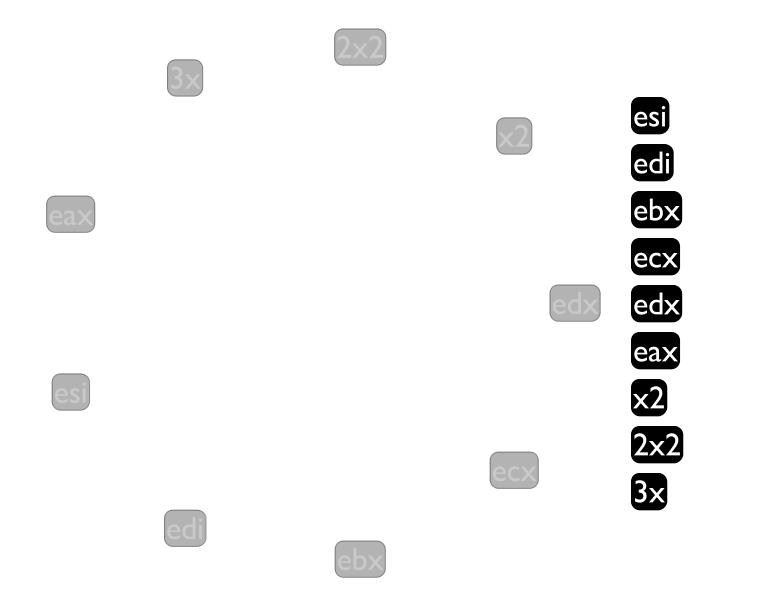


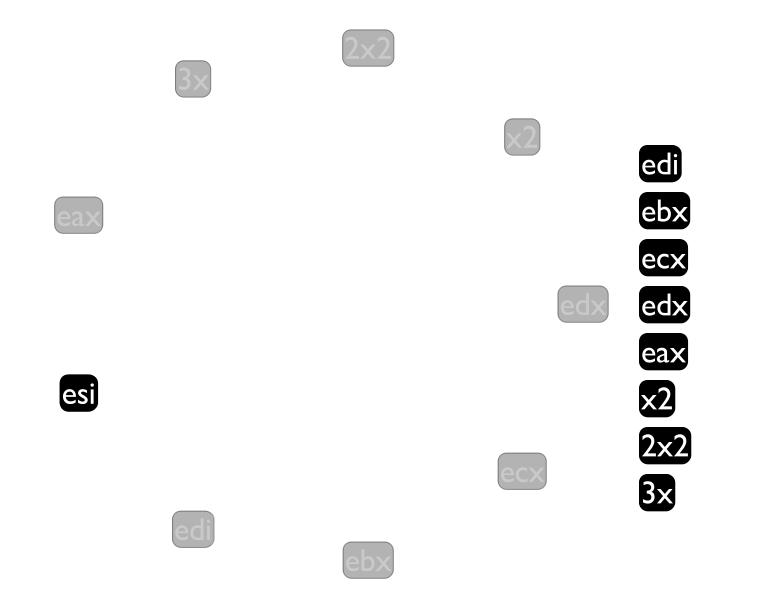


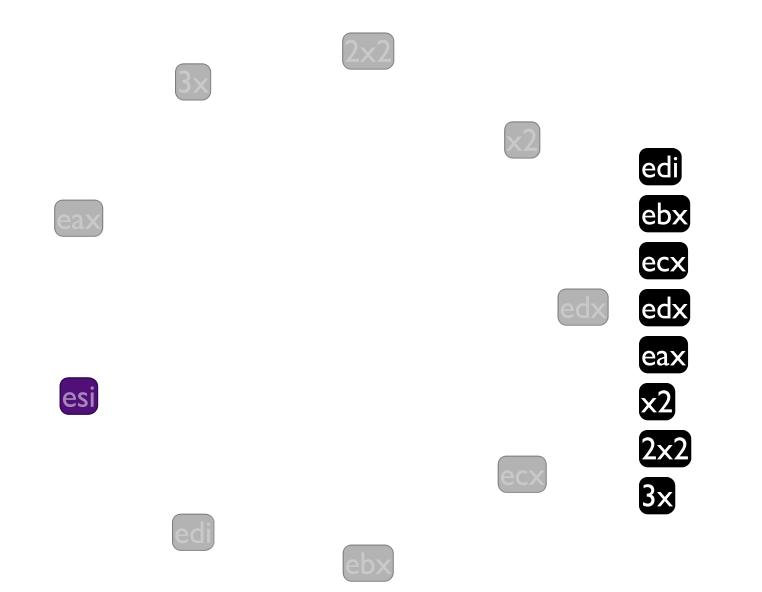


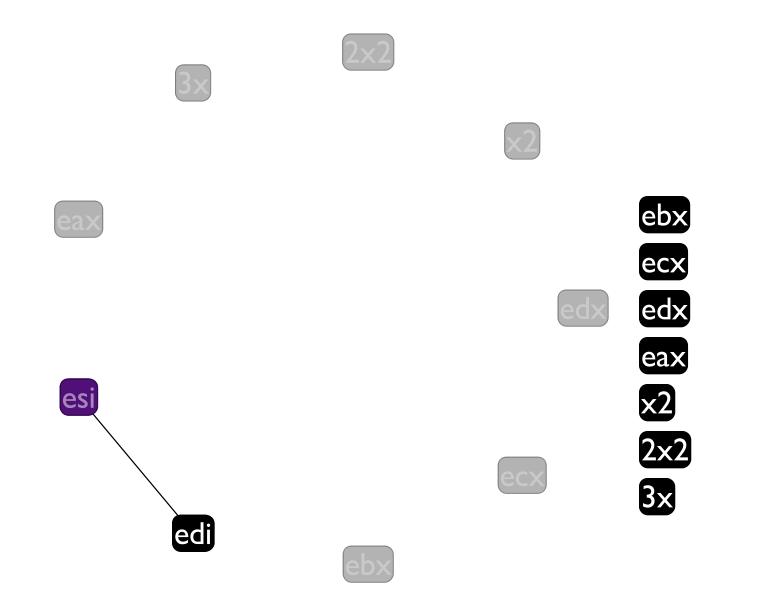


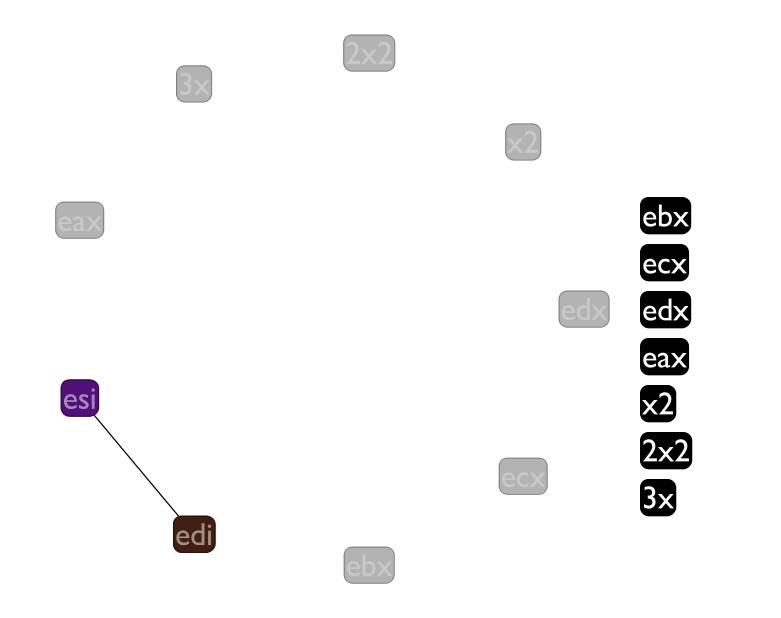


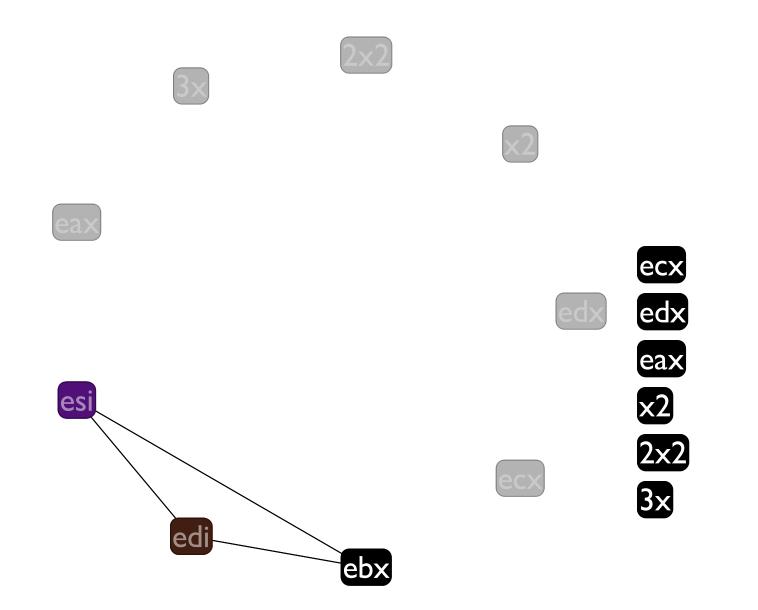


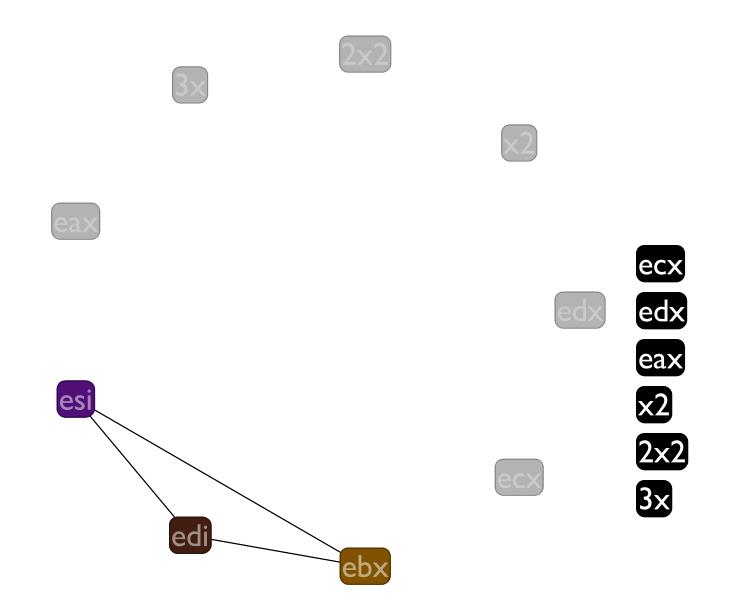


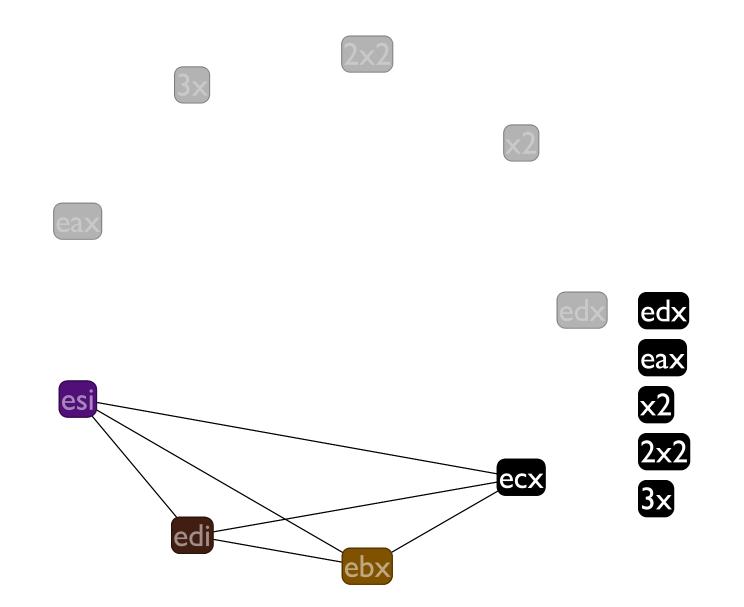


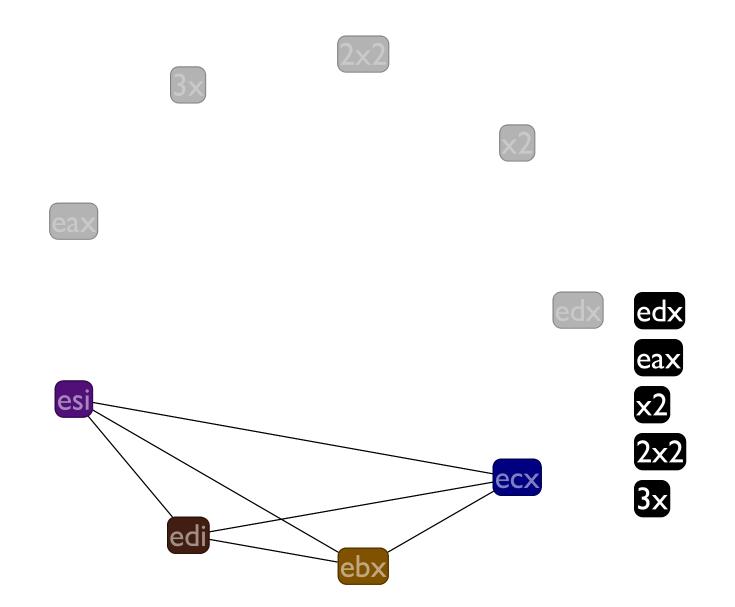


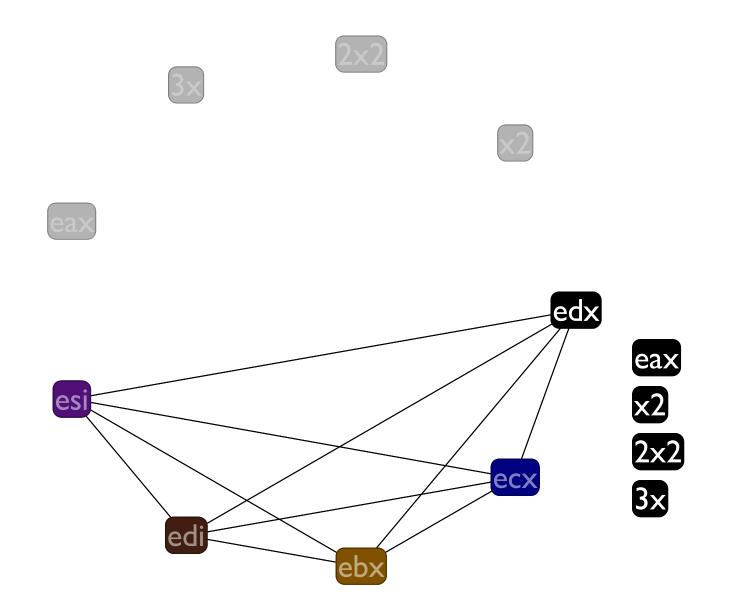


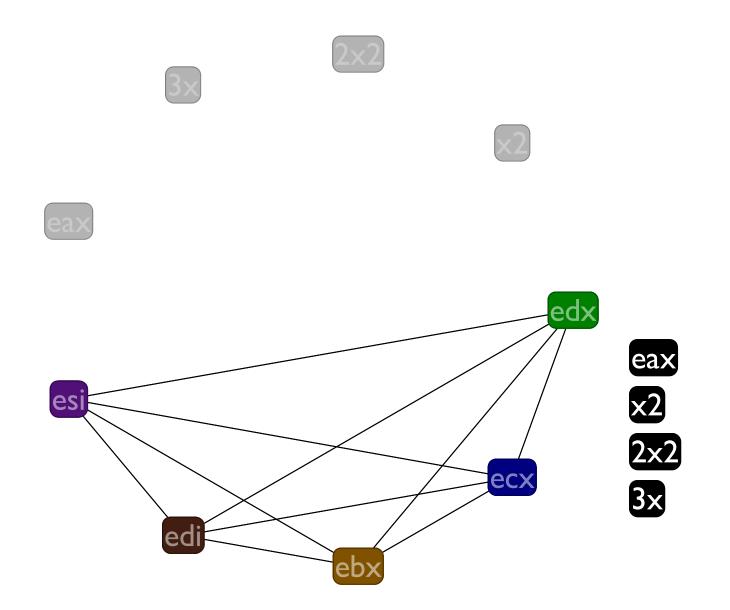


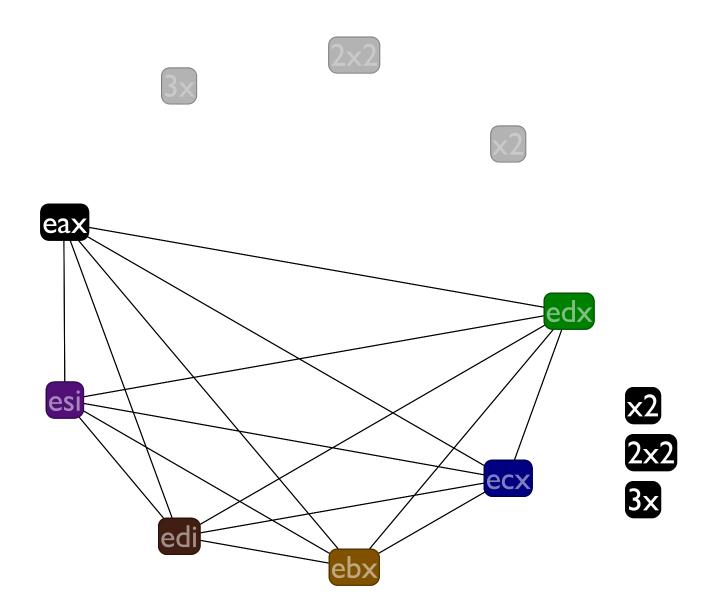


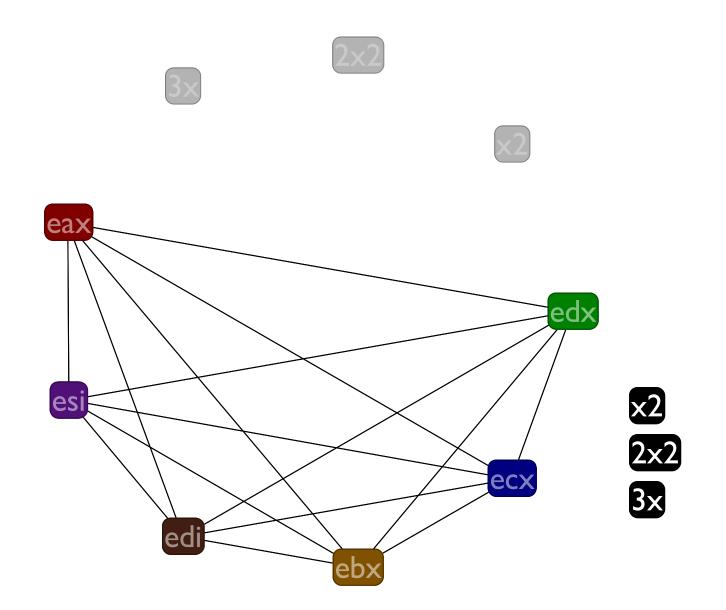


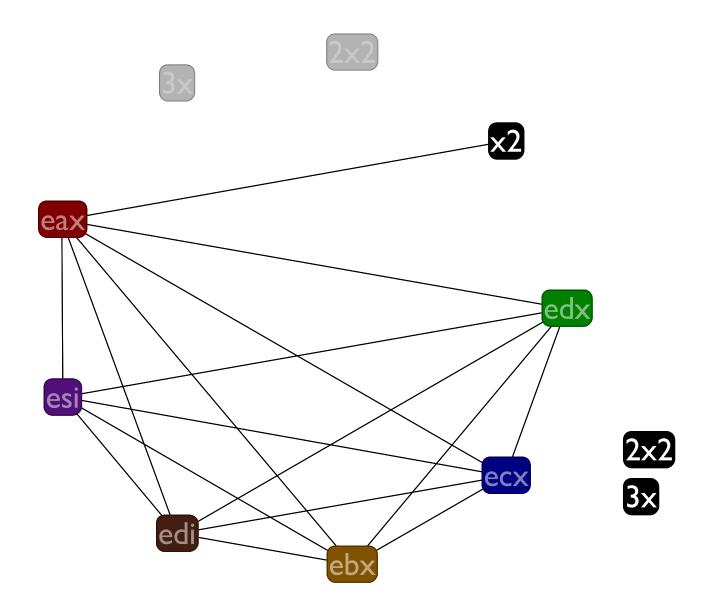


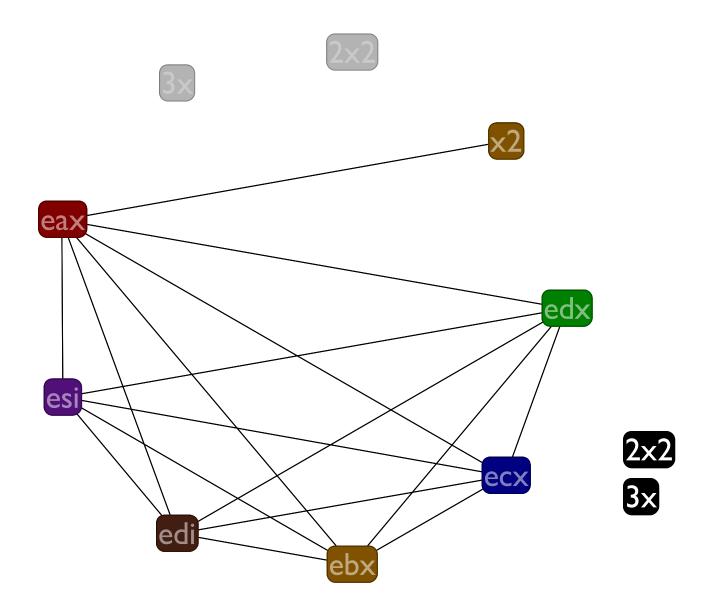


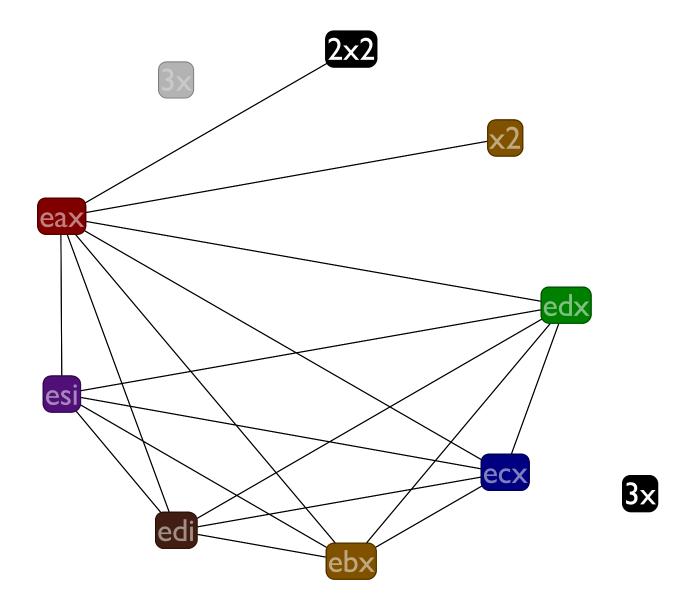


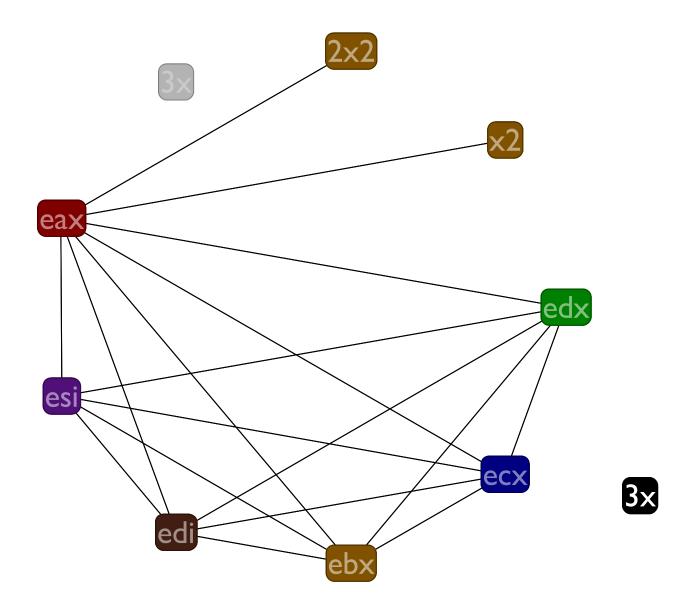


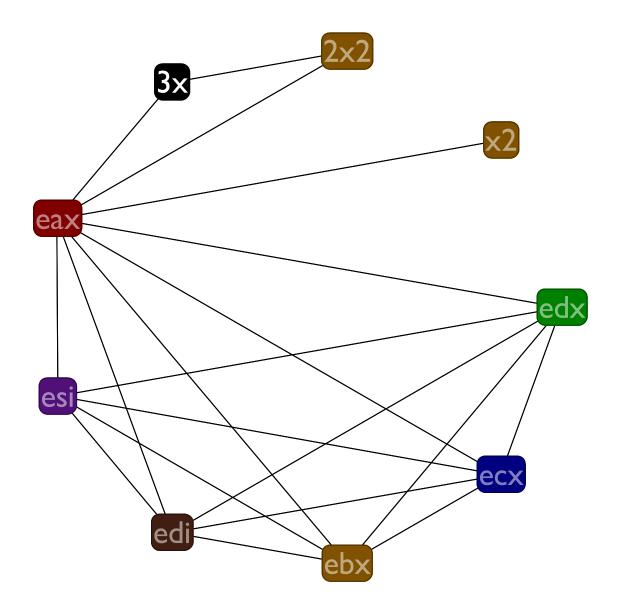


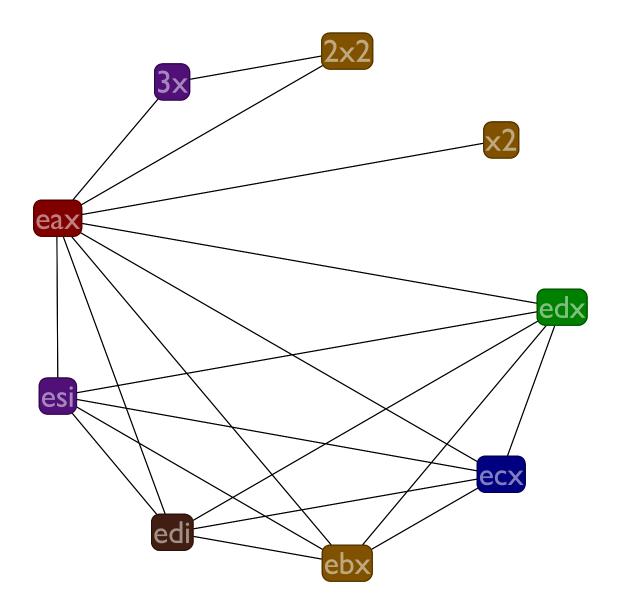












Graph coloring algorithm

Some of the finer details of the graph coloring algorithm

- If possible, prefer to pull out nodes that have 5 or fewer edges at each stage
- When inserting & coloring nodes, always use the "smallest" color possible (according to any ordering on the colors you want). That is, imagine an ordering on the registers (say, alphabetical) and use the color of the first register in that order that works
- Ignore ebp and esp registers when building graph

Caller and callee save registers

Uhoh: the result of coloring the example graph is wrong, since we didn't account for the callee save registers! For example, 3x clobbers esi, breaking the calling convention.

Gen/Kill overview

caller save ecx edx eax ebx
args eax edx ecx
callee save esi edi
result eax

	gen	kill
(call s)	s args	caller save result
(tail-call s)	s args callee save	
(return)	result callee save	

Call may kill caller save regs

	gen	kill
(call s)	s args	caller save
(Cail s)	s args	result
(tail-call s)	s args	
	s args callee save	
(return)	result	
	callee save	

```
(a <- 2)
(eax <- 1)
(call :f)
(a *= a)</pre>
```

Since the caller is responsible for saving the caller save registers, e.g., ecx then : f is free to clobber it. Thus we must be sure not to use ecx for a.

Return gens result

	gen	kill
(call s)	s args	caller save result
(tail-call s)	s args callee save	
(return)	<mark>result</mark> callee save	

```
(eax <- 1)
(a <- edx)
(a *= a)
((mem b 4) <- a)
(return)</pre>
```

Must make sure that a is not allocated into the result register eax. With a reference (aka gen) at the return then the live range spans the assignment to a and thus a won't go into eax

Ensure callee saves are saved

	ge	n	kill
(call s)	S	args	caller save result
(tail-call s)	s a	args save	
(return)	result callee save		

```
(eax <- 1)
(a <- edx)
(a *= a)
((mem b 4) <- a)
(return)</pre>
```

Enforces the calling convention; specifically that the callee save registers are actually saved. In the code on the left, a must not be in esi

Call kills return

	gen	kill
(call s)	s args	caller save result
(tail-call s)	s args callee save	
(return)	result callee save	

(a <- 1) (call :f) (b <- a)

If we put a into eax then the call to : f would clobber it to save the return value, so we must avoid that by treating the call as a kill to the return register, eax (Yep, this is the second reason to do this; other calling conventions may not have both reasons tho)

Call/tail-call gen arguments

	gen		kill
(call s)	Ø	args	caller save result
(tail-call s)	s args callee save		
(return)	result callee save		

(ecx < -1)If we put a into ecx then the (a < - 2)call to : f would get the (call:f) wrong arguments. Avoid this by making call and tail-call refer to (i.e., gen) the argument registers. (Probably your compiler won't generate code like this, but spilling could put you into this kind of situation.)

Constrained arithmetic operators

The (cx <- s cmp s) instruction in LI is limited to only 4 possible destinations.

The (x sop= sx) instruction in LI is limited to only shifting by the value of ecx (or by a constant in the other form).

Constrained arithmetic operators

Add interference edges to disallow the illegal registers when building the interference graph, before starting the coloring.

E.g., if you have this instruction (a <- y < x) then add edges between a and the registers edi and esi, ensuring a ends up in eax, ecx, edx, ebx, or spilled.

Do over

Lets redo the coloring, now with the callee and caller save register information in the graph

Gen & Kill

		gen	kill
1:	:f	0	0
2:	$(x2 \leftarrow eax)$	(eax)	(x2)
3:	$(x2 \neq x2)$	(x2)	(x2)
4:	(2x2 < - x2)	(x2)	(2x2)
5:	(2x2 *= 2)	(2x2)	(2x2)
6:	(3x < - eax)	(eax)	(3x)
7:	(3x *= 3)	(3x)	(3x)
8:	(eax <- 2x2)	(2x2)	(eax)
9:	(eax += 3x)	(3x eax)	(eax)
10:	(eax += 4)	(eax)	(eax)
11:	(return)	(eax edi esi)	()

		in	out
1:	:f	()	()
2:	(x2 <- eax)	Ö	0
3:	$(x2 \neq x2)$	0	()
4:	(2x2 < - x2)	0	()
5:	(2x2 *= 2)	0	()
6:	$(3x \leftarrow eax)$	0	()
7:	(3x *= 3)	()	()
8:	(eax <- 2x2)	()	()
9:	(eax += 3x)	0	()
10:	(eax += 4)	()	()
11:	(return)	()	()

	in	OL
:f	()	()
(x2 <- eax)	(eax)	()
(x2 *= x2)	(x2)	()
(2x2 < - x2)	(x2)	()
(2x2 *= 2)	(2x2)	()
(3x <- eax)	(eax)	()
(3x *= 3)	(3x)	()
(eax <- 2x2)	(2x2)	()
(eax += 3x)	(3x eax)	()
(eax += 4)	(eax)	()
(return)	(eax edi esi)	()
	(x2 <- eax) (x2 *= x2) (2x2 <- x2) (2x2 *= 2) (3x <- eax) (3x *= 3) (eax <- 2x2) (eax += 3x) (eax += 4)	(x2 <- eax) (eax) (x2 *= x2) (x2) (2x2 <- x2) (x2) (2x2 *= 2) (2x2) (3x <- eax) (eax) (3x *= 3) (3x) (eax <- 2x2) (2x2) (eax += 3x) (3x eax) (eax += 4) (eax)

```
in
                                                  out
|: :f
                                                  (eax)
                   ()
                                                  (x2)
2: (x2 < - eax)
                   (eax)
                   (x2)
                                                  (x2)
3: (x2 *= x2)
4: (2x2 < - x2)
                  (x2)
                                                  (2x2)
5: (2x2 *= 2)
                  (2x2)
                                                  (eax)
6: (3x < -eax)
                  (eax)
                                                  (3x)
7: (3x *= 3)
                   (3x)
                                                  (2x2)
8: (eax < - 2x2) (2x2)
                                                  (3x eax)
9: (eax += 3x) (3x eax)
                                                  (eax)
10: (eax += 4)
                  (eax)
                                                  (eax edi esi)
                  (eax edi esi)
||: (return)
                                                  ()
```

```
in
                                                 out
|: :f
                   (eax)
                                                 (eax)
                   (eax)
2: (x2 < - eax)
                                                 (x2)
                  (x2)
                                                 (x2)
3: (x2 *= x2)
4: (2x2 < - x2) (x2)
                                                 (2x2)
5: (2x2 *= 2) (2x2 eax)
                                                 (eax)
6: (3x < -eax) (eax)
                                                 (3x)
7: (3x *= 3) (2x2 3x)
                                                 (2x2)
8: (eax < -2x2) (2x2 3x)
                                                 (3x eax)
9: (eax += 3x) (3x eax)
                                                 (eax)
10: (eax += 4) (eax edi esi)
                                                 (eax edi esi)
| I: (return) (eax edi esi)
                                                 ()
```

	in	out
:f	(eax)	(eax)
$(x2 \leftarrow eax)$	(eax)	(x2)
(x2 *= x2)	(x2)	(x2)
(2x2 < - x2)	(x2)	(2x2 eax)
(2x2 *= 2)	(2x2 eax)	(eax)
(3x <- eax)	(eax)	$(2\times2 \ 3\times)$
(3x *= 3)	(2x2 3x)	$(2\times2 \ 3\times)$
(eax <- 2x2)	(2x2 3x)	(3x eax)
(eax += 3x)	(3x eax)	(eax edi esi)
(eax += 4)	(eax edi esi)	(eax edi esi)
(return)	(eax edi esi)	()
	<pre>:f (x2 <- eax) (x2 *= x2) (2x2 <- x2) (2x2 *= 2) (3x <- eax) (3x *= 3) (eax <- 2x2) (eax += 3x) (eax += 4) (return)</pre>	<pre>:f (eax) (x2 <- eax) (eax) (x2 *= x2) (x2) (2x2 <- x2) (x2) (2x2 *= 2) (2x2 eax) (3x <- eax) (eax) (3x *= 3) (2x2 3x) (eax <- 2x2) (2x2 3x) (eax += 3x) (3x eax) (eax += 4) (eax edi esi)</pre>

```
in
                                                    out
|: :f
                    (eax)
                                                   (eax)
                    (eax)
2: (x2 < - eax)
                                                   (x2)
                   (x2)
                                                    (x2)
3: (x2 *= x2)
4: (2x2 < - x2) (eax x2)
                                                   (2x2 eax)
5: (2x2 *= 2) (2x2 eax)
                                                    (eax)
6: (3x < -eax) (2x2 eax)
                                                    (2x2 3x)
               (2\times2\ 3\times)
                                                    (2x2 3x)
7: (3x *= 3)
8: (eax < -2x2) (2x2 3x)
                                                   (3x eax)
9: (eax += 3x) (3x eax edi esi)
                                                    (eax edi esi)
10: (eax += 4) (eax edi esi)
                                                    (eax edi esi)
                  (eax edi esi)
||: (return)
                                                    ()
```

```
in
                                                    out
|: :f
                    (eax)
                                                    (eax)
2: (x2 < - eax)
                    (eax)
                                                    (x2)
                   (x2)
3: (x2 *= x2)
                                                    (eax x2)
4: (2x2 < - x2) (eax x2)
                                                    (2x2 eax)
5: (2x2 *= 2) (2x2 eax)
                                                    (2x2 eax)
6: (3x < -eax) (2x2 eax)
                                                    (2x2 3x)
               (2\times2 \ 3\times)
                                                    (2x2 3x)
7: (3x *= 3)
8: (eax < -2x2) (2x2 3x)
                                                    (3x eax edi esi)
9: (eax += 3x) (3x eax edi esi)
                                                    (eax edi esi)
10: (eax += 4) (eax edi esi)
                                                    (eax edi esi)
                   (eax edi esi)
||: (return)
                                                    ()
```

```
in
                                                    out
|: :f
                    (eax)
                                                    (eax)
2: (x2 < - eax)
                    (eax)
                                                    (x2)
                   (eax x2)
3: (x2 *= x2)
                                                    (eax x2)
4: (2x2 < - x2) (eax x2)
                                                    (2x2 eax)
5: (2x2 *= 2) (2x2 eax)
                                                    (2x2 eax)
6: (3x < -eax) (2x2 eax)
                                                    (2x2 3x)
               (2\times2\ 3\times)
                                                    (2x2 3x)
7: (3x *= 3)
8: (eax < - 2x2) (2x2 3x edi esi)
                                                    (3x eax edi esi)
9: (eax += 3x) (3x eax edi esi)
                                                    (eax edi esi)
10: (eax += 4) (eax edi esi)
                                                    (eax edi esi)
                   (eax edi esi)
||: (return)
                                                    ()
```

```
in
                                                     out
 |: :f
                    (eax)
                                                     (eax)
2: (x2 < - eax)
                    (eax)
                                                     (eax x2)
                   (eax x2)
                                                      (eax x2)
 3: (x2 *= x2)
4: (2x2 < - x2) (eax x2)
                                                     (2x2 eax)
 5: (2x2 *= 2)
                   (2x2 eax)
                                                     (2x2 eax)
 6: (3x < -eax) (2x2 eax)
                                                     (2x2 3x)
                   (2\times2\ 3\times)
                                                     (2x2 3x edi esi)
7: (3x *= 3)
8: (eax < - 2x2) (2x2 3x edi esi)
                                                     (3x eax edi esi)
9: (eax += 3x) (3x eax edi esi)
                                                     (eax edi esi)
10: (eax += 4)
                   (eax edi esi)
                                                     (eax edi esi)
                    (eax edi esi)
||: (return)
                                                     ()
```

```
in
                                                    out
|: :f
                    (eax)
                                                    (eax)
2: (x2 < - eax)
                    (eax)
                                                    (eax x2)
                   (eax x2)
3: (x2 *= x2)
                                                    (eax x2)
4: (2x2 < - x2) (eax x2)
                                                    (2x2 eax)
5: (2x2 *= 2)
                   (2x2 eax)
                                                    (2x2 eax)
6: (3x < -eax) (2x2 eax)
                                                    (2x2 3x)
                  (2x2 3x edi esi)
                                                    (2x2 3x edi esi)
7: (3x *= 3)
8: (eax < - 2x2) (2x2 3x edi esi)
                                                    (3x eax edi esi)
9: (eax += 3x) (3x eax edi esi)
                                                    (eax edi esi)
10: (eax += 4)
                  (eax edi esi)
                                                    (eax edi esi)
                   (eax edi esi)
||: (return)
                                                    ()
```

```
in
                                                    out
|: :f
                    (eax)
                                                    (eax)
2: (x2 < - eax)
                    (eax)
                                                    (eax x2)
                   (eax x2)
                                                    (eax x2)
3: (x2 *= x2)
4: (2x2 < - x2) (eax x2)
                                                    (2x2 eax)
5: (2x2 *= 2)
                   (2x2 eax)
                                                    (2x2 eax)
6: (3x < -eax) (2x2 eax)
                                                    (2x2 3x edi esi)
                (2x2 3x edi esi)
                                                    (2x2 3x edi esi)
7: (3x *= 3)
8: (eax < - 2x2) (2x2 3x edi esi)
                                                    (3x eax edi esi)
9: (eax += 3x) (3x eax edi esi)
                                                    (eax edi esi)
10: (eax += 4)
                  (eax edi esi)
                                                    (eax edi esi)
                   (eax edi esi)
||: (return)
                                                    ()
```

```
in
                                                     out
 |: :f
                    (eax)
                                                     (eax)
2: (x2 < - eax)
                    (eax)
                                                     (eax x2)
                   (eax x2)
 3: (x2 *= x2)
                                                     (eax x2)
4: (2x2 < - x2) (eax x2)
                                                     (2x2 eax)
 5: (2x2 *= 2)
                   (2x2 eax)
                                                     (2x2 eax)
 6: (3x \leftarrow eax) (2x2 eax edi esi)
                                                     (2x2 3x edi esi)
                (2x2 3x edi esi)
                                                     (2x2 3x edi esi)
7: (3x *= 3)
8: (eax < - 2x2) (2x2 3x edi esi)
                                                     (3x eax edi esi)
9: (eax += 3x) (3x eax edi esi)
                                                     (eax edi esi)
10: (eax += 4)
                   (eax edi esi)
                                                     (eax edi esi)
                    (eax edi esi)
||: (return)
                                                     ()
```

```
in
                                                      out
 |: :f
                     (eax)
                                                      (eax)
2: (x2 < - eax)
                     (eax)
                                                      (eax x2)
                    (eax x2)
                                                      (eax x2)
 3: (x2 *= x2)
                                                      (2x2 eax)
4: (2x2 < - x2) (eax x2)
 5: (2x2 *= 2)
                   (2x2 eax)
                                                      (2x2 eax edi esi)
 6: (3x \leftarrow eax) (2x2 eax edi esi)
                                                      (2x2 3x edi esi)
                (2x2 3x edi esi)
                                                      (2x2 3x edi esi)
7: (3x *= 3)
8: (eax < - 2x2) (2x2 3x edi esi)
                                                      (3x eax edi esi)
9: (eax += 3x) (3x eax edi esi)
                                                      (eax edi esi)
10: (eax += 4)
                   (eax edi esi)
                                                      (eax edi esi)
                    (eax edi esi)
||: (return)
                                                      ()
```

```
in
                                                      out
 |: :f
                     (eax)
                                                      (eax)
2: (x2 < - eax)
                     (eax)
                                                      (eax x2)
                   (eax x2)
                                                      (eax x2)
 3: (x2 *= x2)
                                                      (2x2 eax)
4: (2x2 < - x2) (eax x2)
 5: (2x2 *= 2)
                   (2x2 eax edi esi)
                                                      (2x2 eax edi esi)
 6: (3x \leftarrow eax) (2x2 eax edi esi)
                                                      (2x2 3x edi esi)
                (2x2 3x edi esi)
                                                      (2x2 3x edi esi)
7: (3x *= 3)
8: (eax < - 2x2) (2x2 3x edi esi)
                                                      (3x eax edi esi)
9: (eax += 3x) (3x eax edi esi)
                                                      (eax edi esi)
10: (eax += 4)
                   (eax edi esi)
                                                      (eax edi esi)
                    (eax edi esi)
||: (return)
                                                      ()
```

```
in
                                                      out
 |: :f
                     (eax)
                                                      (eax)
 2: (x2 < - eax)
                     (eax)
                                                      (eax x2)
                    (eax x2)
                                                      (eax x2)
 3: (x2 *= x2)
4: (2x2 < - x2) (eax x2)
                                                      (2x2 eax edi esi)
 5: (2x2 *= 2)
                    (2x2 eax edi esi)
                                                      (2x2 eax edi esi)
 6: (3x \leftarrow eax) (2x2 eax edi esi)
                                                      (2x2 3x edi esi)
                   (2x2 3x edi esi)
                                                      (2x2 3x edi esi)
7: (3x *= 3)
8: (eax < - 2x2) (2x2 3x edi esi)
                                                      (3x eax edi esi)
9: (eax += 3x) (3x eax edi esi)
                                                      (eax edi esi)
10: (eax += 4)
                   (eax edi esi)
                                                      (eax edi esi)
                    (eax edi esi)
||: (return)
                                                      ()
```

```
in
                                                       out
 |: :f
                     (eax)
                                                       (eax)
 2: (x2 < - eax)
                     (eax)
                                                       (eax x2)
                    (eax x2)
                                                       (eax x2)
 3: (x2 *= x2)
4: (2x2 < - x2) (eax edi esi x2)
                                                       (2x2 eax edi esi)
 5: (2x2 *= 2)
                   (2x2 eax edi esi)
                                                       (2x2 eax edi esi)
 6: (3x \leftarrow eax) (2x2 eax edi esi)
                                                       (2x2 3x edi esi)
7: (3x *= 3)
                   (2x2 3x edi esi)
                                                       (2x2 3x edi esi)
8: (eax < - 2x2) (2x2 3x edi esi)
                                                      (3x eax edi esi)
 9: (eax += 3x) (3x eax edi esi)
                                                       (eax edi esi)
10: (eax += 4)
                   (eax edi esi)
                                                       (eax edi esi)
                    (eax edi esi)
||: (return)
                                                       ()
```

```
in
                                                      out
 |: :f
                     (eax)
                                                      (eax)
 2: (x2 < - eax)
                     (eax)
                                                      (eax x2)
                   (eax x2)
                                                      (eax edi esi x2)
 3: (x2 *= x2)
4: (2x2 < - x2) (eax edi esi x2)
                                                      (2x2 eax edi esi)
 5: (2x2 *= 2) (2x2 eax edi esi)
                                                      (2x2 eax edi esi)
 6: (3x \leftarrow eax) (2x2 eax edi esi)
                                                      (2x2 3x edi esi)
                (2x2 3x edi esi)
                                                      (2x2 3x edi esi)
7: (3x *= 3)
8: (eax < - 2x2) (2x2 3x edi esi)
                                                      (3x eax edi esi)
 9: (eax += 3x) (3x eax edi esi)
                                                      (eax edi esi)
10: (eax += 4)
                   (eax edi esi)
                                                      (eax edi esi)
                    (eax edi esi)
||: (return)
                                                      ()
```

```
in
                                                       out
 |: :f
                     (eax)
                                                       (eax)
 2: (x2 < - eax)
                     (eax)
                                                       (eax x2)
                    (eax edi esi x2)
                                                       (eax edi esi x2)
 3: (x2 *= x2)
4: (2x2 < - x2) (eax edi esi x2)
                                                       (2x2 eax edi esi)
 5: (2x2 *= 2)
                   (2x2 eax edi esi)
                                                       (2x2 eax edi esi)
 6: (3x \leftarrow eax) (2x2 eax edi esi)
                                                       (2x2 3x edi esi)
                (2x2 3x edi esi)
                                                       (2x2 3x edi esi)
7: (3x *= 3)
8: (eax < - 2x2) (2x2 3x edi esi)
                                                      (3x eax edi esi)
 9: (eax += 3x) (3x eax edi esi)
                                                       (eax edi esi)
10: (eax += 4)
                   (eax edi esi)
                                                       (eax edi esi)
                    (eax edi esi)
||: (return)
                                                       ()
```

```
in
                                                       out
 |: :f
                     (eax)
                                                       (eax)
 2: (x2 < - eax)
                     (eax)
                                                       (eax edi esi x2)
                    (eax edi esi x2)
                                                       (eax edi esi x2)
 3: (x2 *= x2)
4: (2x2 < - x2) (eax edi esi x2)
                                                       (2x2 eax edi esi)
 5: (2x2 *= 2)
                    (2x2 eax edi esi)
                                                       (2x2 eax edi esi)
 6: (3x \leftarrow eax) (2x2 eax edi esi)
                                                       (2x2 3x edi esi)
7: (3x *= 3)
                (2x2 3x edi esi)
                                                       (2x2 3x edi esi)
8: (eax < - 2x2) (2x2 3x edi esi)
                                                       (3x eax edi esi)
 9: (eax += 3x) (3x eax edi esi)
                                                       (eax edi esi)
10: (eax += 4)
                   (eax edi esi)
                                                       (eax edi esi)
                    (eax edi esi)
||: (return)
                                                       ()
```

```
in
                                                       out
 |: :f
                     (eax)
                                                       (eax)
                     (eax edi esi)
 2: (x2 < - eax)
                                                       (eax edi esi x2)
                    (eax edi esi x2)
                                                       (eax edi esi x2)
 3: (x2 *= x2)
4: (2x2 < - x2) (eax edi esi x2)
                                                       (2x2 eax edi esi)
 5: (2x2 *= 2)
                    (2x2 eax edi esi)
                                                       (2x2 eax edi esi)
 6: (3x \leftarrow eax) (2x2 eax edi esi)
                                                       (2x2 3x edi esi)
7: (3x *= 3)
                    (2x2 3x edi esi)
                                                       (2x2 3x edi esi)
8: (eax < - 2x2) (2x2 3x edi esi)
                                                       (3x eax edi esi)
 9: (eax += 3x) (3x eax edi esi)
                                                       (eax edi esi)
10: (eax += 4)
                    (eax edi esi)
                                                       (eax edi esi)
                     (eax edi esi)
||: (return)
                                                       ()
```

in out |: :f (eax) (eax edi esi) 2: (x2 < - eax)(eax edi esi) (eax edi esi x2) (eax edi esi x2) (eax edi esi x2) 3: (x2 *= x2)4: (2x2 < - x2) (eax edi esi x2) (2x2 eax edi esi) 5: (2x2 *= 2)(2x2 eax edi esi) (2x2 eax edi esi) 6: $(3x \leftarrow eax)$ (2x2 eax edi esi)(2x2 3x edi esi)(2x2 3x edi esi)(2x2 3x edi esi)7: (3x *= 3)8: (eax < - 2x2) (2x2 3x edi esi)(3x eax edi esi) (eax edi esi) 9: (eax += 3x) (3x eax edi esi) 10: (eax += 4)(eax edi esi) (eax edi esi) ||: (return) (eax edi esi) ()

in out |: :f (eax edi esi) (eax edi esi) 2: (x2 < - eax)(eax edi esi) (eax edi esi x2) (eax edi esi x2) (eax edi esi x2) 3: (x2 *= x2)4: (2x2 < - x2) (eax edi esi x2) (2x2 eax edi esi) 5: (2x2 *= 2)(2x2 eax edi esi) (2x2 eax edi esi) 6: $(3x \leftarrow eax)$ (2x2 eax edi esi)(2x2 3x edi esi)(2x2 3x edi esi)(2x2 3x edi esi)7: (3x *= 3)8: (eax < - 2x2) (2x2 3x edi esi)(3x eax edi esi) (eax edi esi) 9: (eax += 3x) (3x eax edi esi) 10: (eax += 4)(eax edi esi) (eax edi esi) ||: (return) (eax edi esi) ()

