Physics G6080 – Problem Set 5
Due Monday April 29, 2013

Consider the region shown in the figure. Inside this region, you will first solve an electrostatic problem and then a quantum mechanics eigenvalue problem. Let the region have length 1 in the x and y directions, given in dimensionless units. The solid square in the interior has length 0.25 and height 0.25 and its center is at $(x, y) = (0.625, 0.75)$. The circle at (0.25,0.25) is a point charge.

(If you find it useful, you can do a simple test of your program by removing the cross from the above figure. Then the solutions are known analytically.)

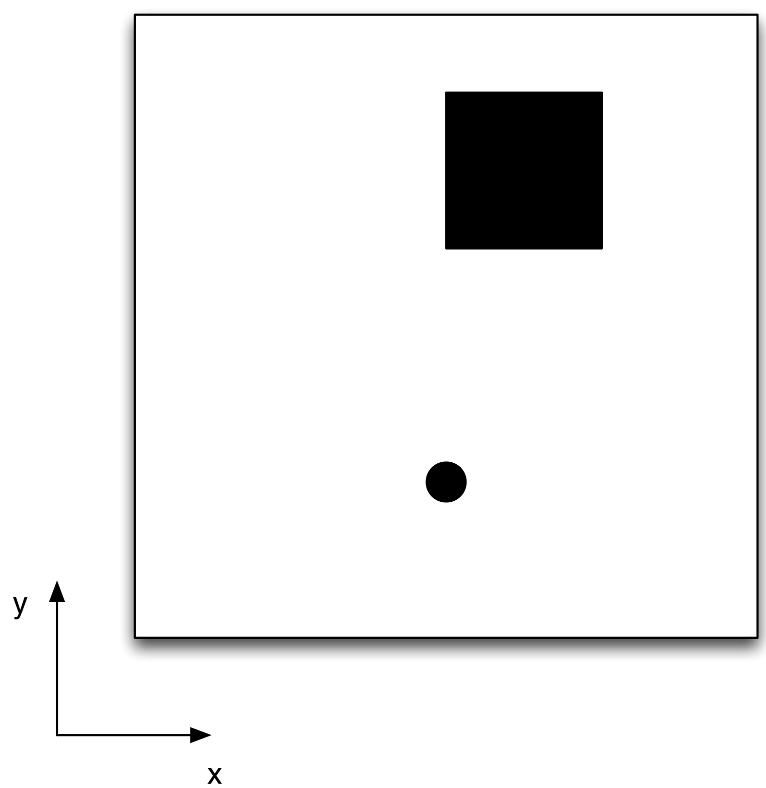

Figure 1: A picture of the regions mentioned in the problems.

1. Using the conjugate gradient algorithm, solve Poisson's equations given that the boundary is held at a potential of zero and the internal rectangle is held at a constant potential of 10 (in dimensionless units). Let the point charge have a value of -2, also in dimensionless units. Plot the potential $\phi(x, y)$ as a surface plot in MATLAB.

2. Now consider the quantum mechanics problem for a particle of mass $m = 1/10$ in the same region. (The mass here is also in dimensionless units, *i.e.* where $\hbar = 1$.) Let the boundary and the internal rectangle be regions where the potential is infinite. (The region outside the boundary also has infinte potential.)

    (a) For the first case, assume the point charge is zero, so you are determining the energy for a free particle in this region. You should use reverse iteration to find the lowest 3 eigenvalues of the energy and the associated eigenvectors for the cases below. Plot $\psi^*(x, y)\phi(x, y)$ for these 3 eigenvectors. (3 significant digit accuracy for the energy are sufficient.)

    (b) We now consider a charged particle and a non-zero charge on the stationary point charge. Choose a value for the stationary point charge at (0.25, 0.25) which produces an attractive potential between it and the particle. Find the value of the attraction that produces a 50% probability for the particle to be in the lower half of the region in the ground state. Plot the probability associated with this ground state eigenvector. (You should shift the point charge location to $(0.25 + a/2, 0.25 + a/2)$, where a is your grid spacing, to avoid calculating the Coulomb potential for zero separation.)